Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.9-P-x-d+e-x-^m-a+b-x+c-x^2-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [400]. This is test number [38].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (400)	% 0. (0)
Mathematica	% 98.5 (394)	% 1.5 (6)
Maple	% 97. (388)	% 3. (12)
Maxima	% 54.25 (217)	% 45.75 (183)
Fricas	% 81. (324)	% 19. (76)
Sympy	% 36.25 (145)	% 63.75 (255)
Giac	% 79.25 (317)	% 20.75 (83)

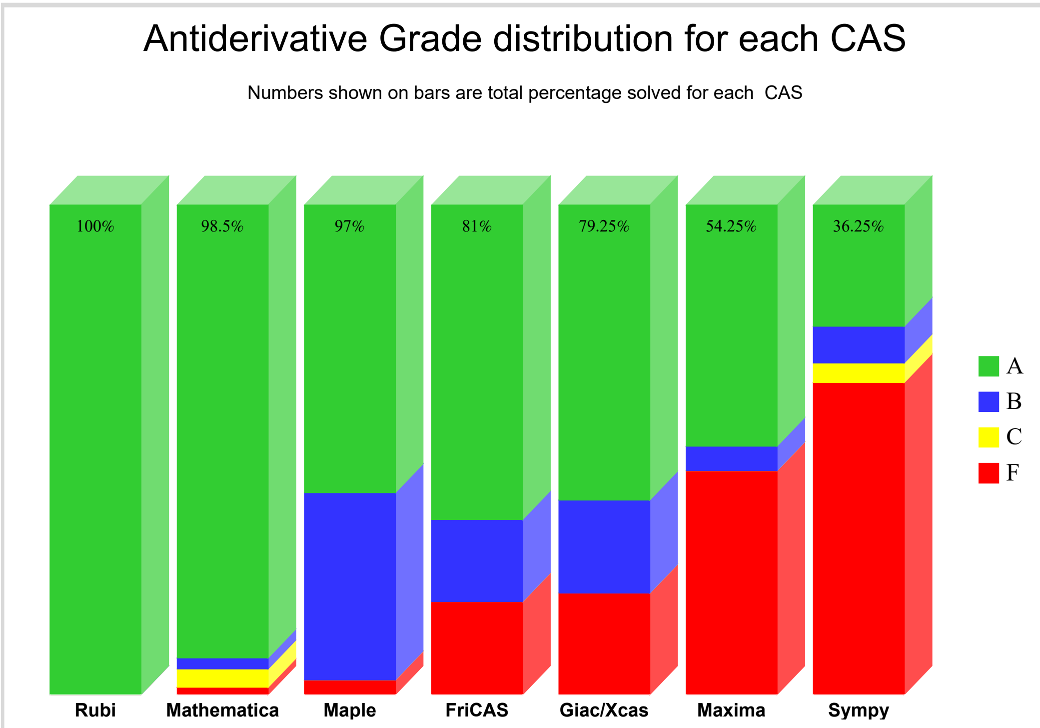
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

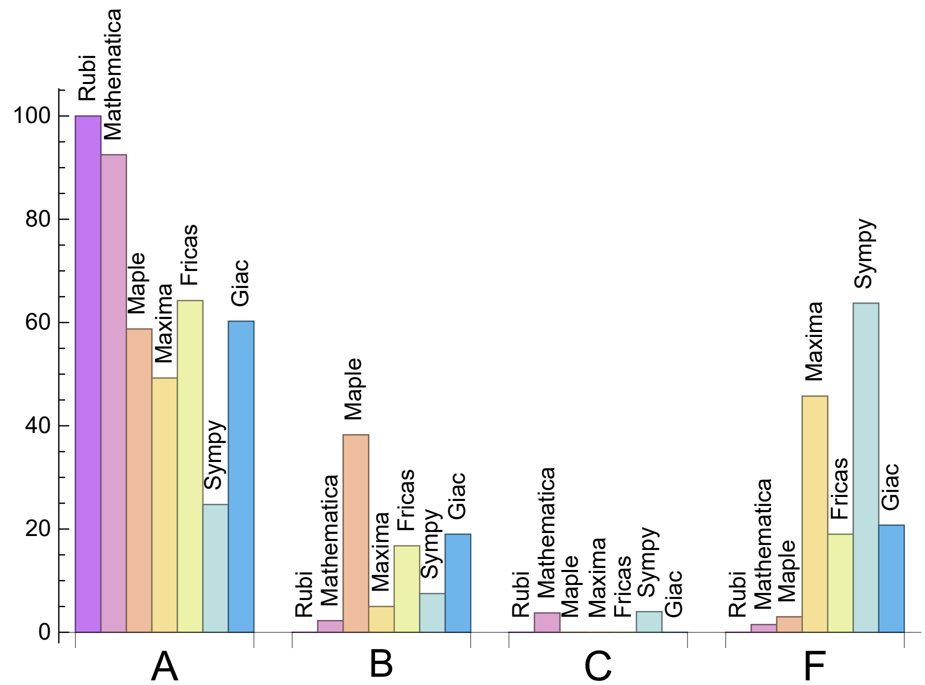
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	92.5	2.25	3.75	1.5
Maple	58.75	38.25	0.	3.
Maxima	49.25	5.	0.	45.75
Fricas	64.25	16.75	0.	19.
Sympy	24.75	7.5	4.	63.75
Giac	60.25	19.	0.	20.75

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.46	238.8	1.	166.5	1.
Mathematica	0.98	687.51	1.43	132.	0.92
Maple	0.1	3404.39	6.48	209.	1.46
Maxima	1.39	233.1	2.23	170.	1.36
Fricas	5.17	1029.29	5.72	462.	3.66
Sympy	11.89	621.85	4.1	226.	1.58
Giac	1.32	651.1	2.95	243.	1.54

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {259, 260, 261, 262, 263, 264, 265, 266, 270, 271, 383, 398, 399, 400}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

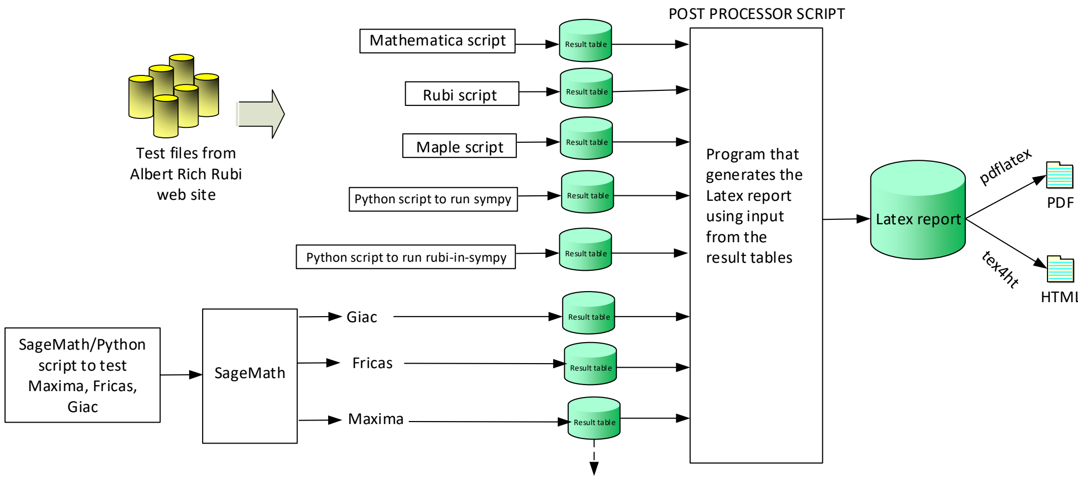
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

B grade: { 39, 40, 41, 42, 202, 203, 204, 239, 278 }

C grade: { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 275, 366 }

F grade: { 136, 137, 138, 272, 273, 274}

2.1.3 Maple

A grade: { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 134, 135, 140, 141, 142, 143, 144, 145, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 181, 182, 183, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 254, 256, 257, 258, 275, 276, 277, 279, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 312, 313, 314, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 374, 375, 376, 380, 381, 382, 386, 387, 388, 389, 392, 393, 394 }

B grade: { 4, 5, 6, 7, 15, 38, 39, 40, 41, 42, 48, 49, 50, 51, 54, 55, 56, 57, 61, 62, 63, 64, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 112, 113, 114, 127, 133, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 178, 179, 180, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 249, 252, 253, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 278, 280, 281, 282, 286, 287, 288, 309, 310, 315, 316, 317, 322, 323, 358, 359, 360, 361, 362, 365, 366, 367, 368, 369, 372, 373, 377, 378, 379, 383, 384, 385, 390, 391, 395, 396, 397 }

C grade: { }

F grade: { 136, 137, 138, 139, 272, 273, 274, 370, 371, 398, 399, 400 }

2.1.4 Maxima

A grade: { 1, 2, 3, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 69, 70, 71, 72, 73, 74, 75, 76, 77, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 275, 276, 277, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394

B grade: { 10, 39, 40, 41, 42, 131, 177, 252, 253, 278, 317, 323, 343, 358, 359, 360, 377, 383, 389, 395 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 136, 137, 138, 139, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 285, 286, 287, 288, 365, 366, 367, 368, 369, 370, 371, 372, 373, 378, 379, 384, 385, 390, 391, 396, 397, 398, 399, 400 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 186, 187, 188, 189, 199, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 312, 313, 314, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade: { 31, 38, 39, 40, 41, 42, 50, 51, 57, 58, 64, 65, 66, 67, 107, 112, 113, 114, 145, 146, 147, 155, 156, 157, 177, 178, 182, 183, 184, 196, 197, 198, 233, 234, 235, 236, 237, 253, 258, 277, 278, 310, 311, 315, 316, 317, 318, 319, 322, 323, 360, 367, 368, 369, 372, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

C grade: { }

F grade: { 48, 49, 54, 55, 56, 61, 62, 63, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 136, 137, 138, 139, 152, 153, 154, 158, 159, 185, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 207, 230, 231, 232, 238, 239, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 365, 366, 370, 371, 398, 399, 400 }

2.1.6 Sympy

A grade: { 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 53, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 110, 111, 115, 118, 119, 120, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 276, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 307, 314, 321 }

B grade: { 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 58, 116, 117, 126, 132, 144, 145, 146, 147, 148, 149, 150, 151, 156, 157, 177, 275, 277, 278 }

C grade: { 1, 2, 3, 304, 305, 306, 308, 309, 311, 312, 313, 315, 316, 318, 319, 320 }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 47, 48, 49, 54, 55, 56, 57, 61, 62, 63, 64, 65, 66, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 112, 113, 114, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 152, 153, 154, 155, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 285, 286, 287, 288, 310, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

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A grade: { 1, 2, 3, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 88, 89, 90, 91, 92, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 186, 187, 188, 189, 199, 208, 209, 210, 211, 214, 215, 216, 217, 220, 221, 222, 223, 226, 227, 228, 229, 235, 236, 240, 241, 242, 243, 246, 247, 248, 249, 252, 253, 254, 255, 279, 280, 281, 282, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 334, 335, 336, 338, 344, 345, 346, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 372, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade: { 39, 40, 41, 42, 61, 63, 64, 65, 84, 85, 86, 87, 94, 95, 97, 98, 99, 107, 112, 114, 121, 123, 129, 147, 154, 158, 159, 178, 183, 184, 185, 196, 197, 198, 203, 212, 218, 224, 232, 233, 234, 237, 239, 245, 251, 257, 275, 276, 277, 278, 285, 286, 287, 288, 327, 328, 329, 330, 331, 332, 333, 337, 339, 340, 341, 342, 343, 348, 349, 356, 357, 363, 364, 367, 368, 369 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 83, 93, 96, 106, 113, 122, 128, 134, 136, 137, 138, 139, 190, 191, 192, 193, 194, 195, 200, 201, 202, 204, 205, 206, 207, 213, 219, 225, 230, 231, 238, 244, 250, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 283, 347, 350, 355, 362, 370, 371, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397, 398, 399, 400 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	226	371	505	464	1239	266
normalized size	1	1.	0.96	1.57	2.14	1.97	5.25	1.13
time (sec)	N/A	0.47	0.518	0.087	1.515	2.398	21.467	1.15

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	190	304	304	375	675	216
normalized size	1	1.	1.02	1.63	1.63	2.02	3.63	1.16
time (sec)	N/A	0.227	0.309	0.059	1.788	2.511	11.5	1.182

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	121	154	188	227	347	115
normalized size	1	1.	0.97	1.23	1.5	1.82	2.78	0.92
time (sec)	N/A	0.069	0.144	0.051	1.53	2.036	6.477	1.141

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	103	384	0	236	0	0
normalized size	1	1.	0.7	2.59	0.	1.59	0.	0.
time (sec)	N/A	0.176	0.226	0.058	0.	1.873	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	109	439	0	409	0	0
normalized size	1	1.	0.64	2.58	0.	2.41	0.	0.
time (sec)	N/A	0.203	0.243	0.06	0.	1.881	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	114	318	0	533	0	0
normalized size	1	1.	0.77	2.13	0.	3.58	0.	0.
time (sec)	N/A	0.189	0.236	0.062	0.	1.824	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	112	453	0	637	0	0
normalized size	1	1.	0.57	2.31	0.	3.25	0.	0.
time (sec)	N/A	0.184	0.296	0.06	0.	1.916	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	109	116	0	678	0	0
normalized size	1	1.	0.61	0.64	0.	3.77	0.	0.
time (sec)	N/A	0.21	0.207	0.048	0.	1.968	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	144	152	0	852	0	0
normalized size	1	1.	0.62	0.65	0.	3.64	0.	0.
time (sec)	N/A	0.264	0.228	0.049	0.	2.55	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	174	374	575	406	1277	224
normalized size	1	1.	0.74	1.58	2.44	1.72	5.41	0.95
time (sec)	N/A	0.657	0.454	0.066	1.505	2.353	20.234	1.233

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	139	301	390	319	898	177
normalized size	1	1.	0.73	1.58	2.04	1.67	4.7	0.93
time (sec)	N/A	0.378	0.183	0.062	1.518	2.388	14.176	1.177

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	103	234	234	236	488	131
normalized size	1	1.	0.72	1.64	1.64	1.65	3.41	0.92
time (sec)	N/A	0.199	0.116	0.056	1.552	1.761	7.577	1.191

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	67	108	126	153	264	70
normalized size	1	1.	0.77	1.24	1.45	1.76	3.03	0.8
time (sec)	N/A	0.051	0.042	0.105	1.526	1.777	3.224	1.213

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	83	149	0	315	0	0
normalized size	1	1.	0.81	1.45	0.	3.06	0.	0.
time (sec)	N/A	0.121	0.161	0.057	0.	1.85	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	95	355	0	451	0	0
normalized size	1	1.	0.58	2.18	0.	2.77	0.	0.
time (sec)	N/A	0.169	0.221	0.061	0.	1.828	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	103	116	0	510	0	0
normalized size	1	1.	0.57	0.64	0.	2.83	0.	0.
time (sec)	N/A	0.205	0.199	0.049	0.	1.781	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	139	152	0	689	0	0
normalized size	1	1.	0.59	0.65	0.	2.94	0.	0.
time (sec)	N/A	0.249	0.224	0.049	0.	2.056	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	173	208	217	273	595	257	327
normalized size	1	0.99	1.19	1.24	1.56	3.4	1.47	1.87
time (sec)	N/A	0.313	0.09	0.044	1.022	1.534	0.092	1.145

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	173	150	148	190	423	173	231
normalized size	1	0.99	0.86	0.85	1.09	2.42	0.99	1.32
time (sec)	N/A	0.216	0.059	0.046	1.064	1.474	0.083	1.173

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	79	108	250	97	135
normalized size	1	1.	1.	0.92	1.26	2.91	1.13	1.57
time (sec)	N/A	0.106	0.029	0.043	0.981	1.469	0.071	1.163

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	51	104	42	54
normalized size	1	1.	1.	0.85	1.11	2.26	0.91	1.17
time (sec)	N/A	0.029	0.012	0.047	0.995	1.489	0.062	1.149

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	143	136	210	215	344	143	230
normalized size	1	0.99	0.94	1.45	1.48	2.37	0.99	1.59
time (sec)	N/A	0.245	0.078	0.049	1.001	1.689	0.766	1.124

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	151	142	234	228	545	184	324
normalized size	1	0.99	0.93	1.53	1.49	3.56	1.2	2.12
time (sec)	N/A	0.205	0.158	0.056	0.999	1.583	1.567	1.148

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	154	176	257	239	585	204	225
normalized size	1	0.99	1.13	1.65	1.53	3.75	1.31	1.44
time (sec)	N/A	0.199	0.103	0.056	0.996	1.686	5.848	1.139

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	301	335	385	486	992	445	571
normalized size	1	0.99	1.1	1.27	1.6	3.26	1.46	1.88
time (sec)	N/A	0.535	0.135	0.046	1.026	1.442	0.119	1.149

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	216	241	268	347	709	311	408
normalized size	1	1.	1.11	1.24	1.6	3.27	1.43	1.88
time (sec)	N/A	0.313	0.092	0.045	1.005	1.405	0.105	1.128

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	144	151	208	428	180	244
normalized size	1	1.	1.12	1.18	1.62	3.34	1.41	1.91
time (sec)	N/A	0.159	0.051	0.047	0.989	1.574	0.091	1.168

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	69	75	100	185	83	103
normalized size	1	1.	1.03	1.12	1.49	2.76	1.24	1.54
time (sec)	N/A	0.04	0.029	0.046	1.019	1.509	0.077	1.159

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	295	285	490	509	786	350	562
normalized size	1	0.99	0.96	1.65	1.71	2.65	1.18	1.89
time (sec)	N/A	0.64	0.168	0.054	0.999	1.755	1.469	1.139

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	289	272	527	529	1185	411	671
normalized size	1	0.99	0.93	1.8	1.81	4.06	1.41	2.3
time (sec)	N/A	0.525	0.291	0.058	1.019	1.74	3.764	1.178

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	292	274	563	543	1305	471	536
normalized size	1	0.99	0.93	1.91	1.84	4.42	1.6	1.82
time (sec)	N/A	0.495	0.136	0.061	1.03	1.731	16.442	1.162

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	400	459	553	691	1413	646	818
normalized size	1	0.99	1.14	1.37	1.71	3.5	1.6	2.02
time (sec)	N/A	0.691	0.205	0.046	1.011	1.431	0.137	1.142

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	288	329	388	495	996	447	583
normalized size	1	1.	1.14	1.34	1.71	3.45	1.55	2.02
time (sec)	N/A	0.424	0.132	0.044	1.003	1.396	0.121	1.167

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	196	223	300	603	265	352
normalized size	1	1.	1.16	1.32	1.78	3.57	1.57	2.08
time (sec)	N/A	0.187	0.075	0.044	1.321	1.457	0.102	1.14

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	100	111	146	261	122	150
normalized size	1	1.	1.15	1.28	1.68	3.	1.4	1.72
time (sec)	N/A	0.058	0.032	0.044	1.227	1.507	0.08	1.144

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	487	498	880	907	1385	658	1031
normalized size	1	0.99	1.02	1.8	1.85	2.83	1.34	2.1
time (sec)	N/A	1.098	0.505	0.054	1.065	1.723	2.321	1.148

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	483	641	928	933	1987	731	1131
normalized size	1	0.99	1.32	1.91	1.92	4.09	1.5	2.33
time (sec)	N/A	0.98	0.358	0.059	0.999	1.861	5.691	1.21

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	463	438	978	946	2198	799	981
normalized size	1	0.99	0.94	2.1	2.03	4.72	1.71	2.11
time (sec)	N/A	0.967	0.235	0.061	1.086	1.828	36.916	1.151

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	62	76	111	153	75	150
normalized size	1	1.	3.65	4.47	6.53	9.	4.41	8.82
time (sec)	N/A	0.032	0.026	0.049	0.991	1.596	0.565	1.172

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	62	76	111	153	75	150
normalized size	1	1.	3.65	4.47	6.53	9.	4.41	8.82
time (sec)	N/A	0.019	0.013	0.047	0.959	1.672	0.546	1.169

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	90	157	216	234	155	292
normalized size	1	1.	5.29	9.24	12.71	13.76	9.12	17.18
time (sec)	N/A	0.046	0.039	0.05	1.004	1.687	0.778	1.162

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	90	157	216	234	155	292
normalized size	1	1.	5.29	9.24	12.71	13.76	9.12	17.18
time (sec)	N/A	0.025	0.021	0.05	0.959	1.606	0.761	1.174

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	237	223	399	0	1239	1000	377
normalized size	1	0.99	0.93	1.66	0.	5.16	4.17	1.57
time (sec)	N/A	0.471	0.224	0.052	0.	1.882	6.769	1.144

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	166	155	256	0	855	638	238
normalized size	1	0.99	0.92	1.52	0.	5.09	3.8	1.42
time (sec)	N/A	0.262	0.193	0.069	0.	1.853	4.463	1.137

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	86	133	0	468	335	123
normalized size	1	1.	0.92	1.43	0.	5.03	3.6	1.32
time (sec)	N/A	0.117	0.11	0.05	0.	1.843	2.202	1.165

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	56	59	0	296	156	65
normalized size	1	1.	1.02	1.07	0.	5.38	2.84	1.18
time (sec)	N/A	0.053	0.038	0.046	0.	1.739	0.674	1.166

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	120	247	0	575	0	169
normalized size	1	1.	0.9	1.86	0.	4.32	0.	1.27
time (sec)	N/A	0.162	0.115	0.053	0.	47.1	0.	1.133

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	188	462	0	0	0	365
normalized size	1	1.	0.88	2.16	0.	0.	0.	1.71
time (sec)	N/A	0.355	0.358	0.058	0.	0.	0.	1.154

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	277	754	0	0	0	660
normalized size	1	1.	0.91	2.47	0.	0.	0.	2.16
time (sec)	N/A	0.651	0.348	0.061	0.	0.	0.	1.163

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	233	484	0	1881	949	390
normalized size	1	1.	1.08	2.24	0.	8.71	4.39	1.81
time (sec)	N/A	0.505	0.214	0.056	0.	2.081	39.307	1.176

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	175	323	0	1276	593	248
normalized size	1	1.	1.2	2.21	0.	8.74	4.06	1.7
time (sec)	N/A	0.245	0.151	0.051	0.	2.101	19.217	1.164

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	102	134	0	717	318	151
normalized size	1	1.	1.05	1.38	0.	7.39	3.28	1.56
time (sec)	N/A	0.082	0.104	0.052	0.	1.769	6.495	1.151

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	68	76	0	412	116	81
normalized size	1	1.	0.99	1.1	0.	5.97	1.68	1.17
time (sec)	N/A	0.043	0.053	0.049	0.	1.773	0.84	1.153

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	195	742	0	0	0	473
normalized size	1	1.	0.86	3.28	0.	0.	0.	2.09
time (sec)	N/A	0.435	0.261	0.06	0.	0.	0.	1.204

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	371	320	1036	0	0	0	821
normalized size	1	0.99	0.86	2.77	0.	0.	0.	2.2
time (sec)	N/A	0.95	0.467	0.067	0.	0.	0.	1.203

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	524	466	1588	0	0	0	1292
normalized size	1	1.	0.89	3.03	0.	0.	0.	2.47
time (sec)	N/A	1.552	0.706	0.074	0.	0.	0.	1.189

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	281	402	0	2325	0	470
normalized size	1	1.	1.34	1.92	0.	11.12	0.	2.25
time (sec)	N/A	0.304	0.324	0.056	0.	2.662	0.	1.165

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	175	211	283	0	1644	389	343
normalized size	1	1.12	1.35	1.81	0.	10.54	2.49	2.2
time (sec)	N/A	0.231	0.154	0.053	0.	2.487	163.039	1.16

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	137	157	0	991	240	205
normalized size	1	1.	1.05	1.21	0.	7.62	1.85	1.58
time (sec)	N/A	0.108	0.113	0.052	0.	2.	34.287	1.148

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	90	96	0	656	156	113
normalized size	1	1.	0.92	0.98	0.	6.69	1.59	1.15
time (sec)	N/A	0.063	0.074	0.049	0.	1.764	1.567	1.153

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	321	1598	0	0	0	965
normalized size	1	1.	0.91	4.53	0.	0.	0.	2.73
time (sec)	N/A	0.734	0.486	0.069	0.	0.	0.	1.181

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	571	566	498	2159	0	0	0	1494
normalized size	1	0.99	0.87	3.78	0.	0.	0.	2.62
time (sec)	N/A	1.925	0.818	0.076	0.	0.	0.	1.214

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	753	753	672	2737	0	0	0	2068
normalized size	1	1.	0.89	3.63	0.	0.	0.	2.75
time (sec)	N/A	3.143	1.226	0.081	0.	0.	0.	1.22

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	437	647	0	3753	0	859
normalized size	1	1.	1.87	2.76	0.	16.04	0.	3.67
time (sec)	N/A	0.291	0.305	0.056	0.	1.713	0.	1.167

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	288	350	464	0	2807	0	641
normalized size	1	1.13	1.38	1.83	0.	11.05	0.	2.52
time (sec)	N/A	0.542	0.322	0.053	0.	1.637	0.	1.188

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	266	333	0	2163	0	443
normalized size	1	1.	1.18	1.48	0.	9.61	0.	1.97
time (sec)	N/A	0.398	0.169	0.053	0.	1.557	0.	1.154

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	171	182	0	1341	298	262
normalized size	1	1.	1.04	1.1	0.	8.13	1.81	1.59
time (sec)	N/A	0.136	0.162	0.049	0.	1.455	127.102	1.159

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	112	113	0	900	196	147
normalized size	1	1.	0.89	0.9	0.	7.14	1.56	1.17
time (sec)	N/A	0.078	0.089	0.05	0.	1.005	2.229	1.158

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	29	30	39	122	29	39
normalized size	1	1.	0.67	0.7	0.91	2.84	0.67	0.91
time (sec)	N/A	0.05	0.018	0.046	1.477	0.975	0.119	1.155

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	27	24	31	111	20	31
normalized size	1	1.	0.9	0.8	1.03	3.7	0.67	1.03
time (sec)	N/A	0.04	0.01	0.061	1.528	0.984	0.111	1.154

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	24	31	89	20	31
normalized size	1	1.	0.79	0.83	1.07	3.07	0.69	1.07
time (sec)	N/A	0.025	0.008	0.046	1.467	0.989	0.114	1.152

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	58	10	16
normalized size	1	1.	1.	0.93	1.14	4.14	0.71	1.14
time (sec)	N/A	0.011	0.007	0.045	1.484	0.98	0.105	1.162

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	26	34	117	24	35
normalized size	1	1.	0.9	0.84	1.1	3.77	0.77	1.13
time (sec)	N/A	0.039	0.01	0.049	1.461	0.994	0.136	1.156

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	46	138	31	47
normalized size	1	1.	1.	0.91	1.39	4.18	0.94	1.42
time (sec)	N/A	0.045	0.016	0.053	1.502	0.986	0.137	1.181

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	38	55	159	41	58
normalized size	1	1.	0.87	0.84	1.22	3.53	0.91	1.29
time (sec)	N/A	0.063	0.016	0.051	1.474	1.037	0.159	1.155

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	22	12	13	16	50	8	16
normalized size	1	1.83	1.	1.08	1.33	4.17	0.67	1.33
time (sec)	N/A	0.007	0.012	0.046	1.462	0.958	0.102	1.177

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	28	74	19	28
normalized size	1	1.	1.	0.78	1.04	2.74	0.7	1.04
time (sec)	N/A	0.014	0.01	0.048	1.494	1.029	0.117	1.178

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	387	362	661	0	1889	1088	641
normalized size	1	0.99	0.93	1.69	0.	4.84	2.79	1.64
time (sec)	N/A	0.83	0.471	0.058	0.	1.646	25.418	1.188

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	279	256	446	0	1319	738	433
normalized size	1	1.	0.91	1.59	0.	4.71	2.64	1.55
time (sec)	N/A	0.498	0.697	0.054	0.	1.608	19.043	1.198

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	153	230	0	770	384	243
normalized size	1	1.	0.87	1.31	0.	4.4	2.19	1.39
time (sec)	N/A	0.268	0.439	0.053	0.	1.426	10.094	1.192

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	98	111	0	448	170	117
normalized size	1	1.	0.92	1.05	0.	4.23	1.6	1.1
time (sec)	N/A	0.064	0.2	0.05	0.	1.352	6.024	1.153

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	224	1265	0	0	0	375
normalized size	1	1.	1.09	6.14	0.	0.	0.	1.82
time (sec)	N/A	0.392	0.477	0.301	0.	0.	0.	1.266

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	303	264	2818	0	0	0	0
normalized size	1	0.98	0.86	9.15	0.	0.	0.	0.
time (sec)	N/A	0.509	0.284	0.234	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	295	318	4432	0	0	0	1246
normalized size	1	1.	1.07	14.97	0.	0.	0.	4.21
time (sec)	N/A	0.546	0.653	0.239	0.	0.	0.	1.423

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	382	5565	0	0	0	2321
normalized size	1	1.	1.22	17.72	0.	0.	0.	7.39
time (sec)	N/A	0.505	0.985	0.244	0.	0.	0.	1.478

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	312	439	7237	0	0	0	2869
normalized size	1	1.	1.4	23.12	0.	0.	0.	9.17
time (sec)	N/A	0.429	1.316	0.25	0.	0.	0.	3.613

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	432	583	8546	0	0	0	5686
normalized size	1	1.	1.35	19.74	0.	0.	0.	13.13
time (sec)	N/A	0.743	1.528	0.25	0.	0.	0.	1.623

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	481	794	0	2634	1916	880
normalized size	1	1.	1.04	1.72	0.	5.7	4.15	1.9
time (sec)	N/A	1.134	0.573	0.061	0.	2.58	67.618	1.229

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	345	346	552	0	1878	1304	610
normalized size	1	1.	1.	1.6	0.	5.43	3.77	1.76
time (sec)	N/A	0.524	1.149	0.057	0.	2.116	50.033	1.189

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	212	209	287	0	1107	768	356
normalized size	1	1.	0.98	1.35	0.	5.2	3.61	1.67
time (sec)	N/A	0.271	0.688	0.052	0.	1.473	25.584	1.171

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	125	146	0	618	348	174
normalized size	1	1.	0.91	1.07	0.	4.51	2.54	1.27
time (sec)	N/A	0.083	0.269	0.049	0.	1.395	14.993	1.193

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	348	2420	0	0	0	744
normalized size	1	1.	1.07	7.42	0.	0.	0.	2.28
time (sec)	N/A	0.766	1.426	0.228	0.	0.	0.	1.229

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	428	392	5121	0	0	0	0
normalized size	1	0.99	0.91	11.85	0.	0.	0.	0.
time (sec)	N/A	0.901	0.582	0.226	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	480	435	7817	0	0	0	1399
normalized size	1	0.98	0.89	16.02	0.	0.	0.	2.87
time (sec)	N/A	0.921	0.706	0.239	0.	0.	0.	1.47

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	475	469	517	9835	0	0	0	2565
normalized size	1	0.99	1.09	20.71	0.	0.	0.	5.4
time (sec)	N/A	0.845	1.401	0.252	0.	0.	0.	1.684

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	511	511	575	12481	0	0	0	0
normalized size	1	1.	1.13	24.42	0.	0.	0.	0.
time (sec)	N/A	1.092	2.522	0.254	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	507	507	639	14169	0	0	0	5951
normalized size	1	1.	1.26	27.95	0.	0.	0.	11.74
time (sec)	N/A	0.86	2.608	0.26	0.	0.	0.	2.139

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	403	696	17026	0	0	0	8265
normalized size	1	1.	1.72	42.14	0.	0.	0.	20.46
time (sec)	N/A	0.552	2.475	0.27	0.	0.	0.	2.112

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	531	863	19093	0	0	0	10714
normalized size	1	1.	1.62	35.89	0.	0.	0.	20.14
time (sec)	N/A	0.889	2.636	0.307	0.	0.	0.	2.142

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	150	181	0	809	510	227
normalized size	1	1.	0.89	1.08	0.	4.82	3.04	1.35
time (sec)	N/A	0.102	0.334	0.05	0.	2.161	30.56	1.176

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	323	252	528	0	1284	796	424
normalized size	1	0.99	0.78	1.62	0.	3.95	2.45	1.3
time (sec)	N/A	0.664	0.394	0.062	0.	2.084	19.693	1.173

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	222	164	339	0	871	518	278
normalized size	1	1.	0.74	1.52	0.	3.91	2.32	1.25
time (sec)	N/A	0.372	0.25	0.061	0.	2.184	13.787	1.173

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	135	96	172	0	482	282	149
normalized size	1	0.99	0.71	1.26	0.	3.54	2.07	1.1
time (sec)	N/A	0.179	0.113	0.055	0.	2.206	7.047	1.211

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	76	0	305	150	78
normalized size	1	1.	0.85	1.03	0.	4.12	2.03	1.05
time (sec)	N/A	0.048	0.041	0.052	0.	2.003	3.071	1.173

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	125	453	0	0	0	186
normalized size	1	1.	0.96	3.48	0.	0.	0.	1.43
time (sec)	N/A	0.174	0.231	0.236	0.	0.	0.	1.217

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	218	923	0	0	0	0
normalized size	1	1.	1.3	5.49	0.	0.	0.	0.
time (sec)	N/A	0.235	0.419	0.249	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	224	254	1574	0	2209	0	1145
normalized size	1	1.	1.13	7.	0.	9.82	0.	5.09
time (sec)	N/A	0.292	0.503	0.246	0.	62.253	0.	1.252

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	228	246	516	0	1635	0	458
normalized size	1	1.	1.07	2.25	0.	7.14	0.	2.
time (sec)	N/A	0.325	0.503	0.06	0.	2.206	0.	1.201

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	177	327	0	1142	0	296
normalized size	1	1.	1.19	2.19	0.	7.66	0.	1.99
time (sec)	N/A	0.184	0.302	0.058	0.	2.052	0.	1.188

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	102	163	0	606	209	157
normalized size	1	1.	1.02	1.63	0.	6.06	2.09	1.57
time (sec)	N/A	0.087	0.151	0.056	0.	1.9	12.768	1.169

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	74	69	0	396	87	85
normalized size	1	1.	1.21	1.13	0.	6.49	1.43	1.39
time (sec)	N/A	0.036	0.063	0.052	0.	1.718	5.61	1.185

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	137	862	0	1431	0	397
normalized size	1	1.	0.99	6.25	0.	10.37	0.	2.88
time (sec)	N/A	0.141	0.203	0.256	0.	12.217	0.	1.185

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	285	1663	0	3160	0	0
normalized size	1	1.	1.19	6.96	0.	13.22	0.	0.
time (sec)	N/A	0.418	0.727	0.234	0.	21.091	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	372	404	2584	0	5783	0	1944
normalized size	1	0.99	1.08	6.91	0.	15.46	0.	5.2
time (sec)	N/A	1.028	1.272	0.258	0.	110.411	0.	1.39

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	47	112	139	194	65
normalized size	1	1.	0.75	0.7	1.67	2.07	2.9	0.97
time (sec)	N/A	0.042	0.034	0.052	0.979	1.596	13.825	1.17

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	71	72	159	215	638	108
normalized size	1	1.	0.73	0.74	1.64	2.22	6.58	1.11
time (sec)	N/A	0.057	0.052	0.048	0.992	1.622	39.559	1.201

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	92	96	207	289	1880	151
normalized size	1	1.	0.72	0.76	1.63	2.28	14.8	1.19
time (sec)	N/A	0.087	0.067	0.052	0.999	1.712	122.907	1.219

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	54	79	105	174	94	73
normalized size	1	1.	0.51	0.75	0.99	1.64	0.89	0.69
time (sec)	N/A	0.114	0.068	0.062	1.448	1.566	2.181	1.216

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	48	65	86	147	75	65
normalized size	1	1.	0.59	0.79	1.05	1.79	0.91	0.79
time (sec)	N/A	0.088	0.043	0.055	1.496	1.529	1.152	1.194

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	44	51	68	136	63	59
normalized size	1	1.	0.71	0.82	1.1	2.19	1.02	0.95
time (sec)	N/A	0.052	0.028	0.049	1.469	1.536	0.57	1.213

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	60	55	78	240	0	134
normalized size	1	1.	0.9	0.82	1.16	3.58	0.	2.
time (sec)	N/A	0.081	0.03	0.052	1.497	1.58	0.	1.328

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	64	65	88	285	0	0
normalized size	1	1.	0.9	0.92	1.24	4.01	0.	0.
time (sec)	N/A	0.07	0.107	0.058	1.493	1.654	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	55	74	103	234	0	243
normalized size	1	1.	0.71	0.96	1.34	3.04	0.	3.16
time (sec)	N/A	0.067	0.068	0.058	1.478	1.577	0.	1.266

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	58	79	105	208	0	73
normalized size	1	1.	0.67	0.91	1.21	2.39	0.	0.84
time (sec)	N/A	0.104	0.057	0.056	1.502	1.606	0.	1.32

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	53	65	86	184	0	66
normalized size	1	1.	0.75	0.92	1.21	2.59	0.	0.93
time (sec)	N/A	0.084	0.053	0.054	1.53	1.555	0.	1.26

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	48	51	68	170	114	59
normalized size	1	1.	0.87	0.93	1.24	3.09	2.07	1.07
time (sec)	N/A	0.044	0.03	0.052	1.473	1.85	12.906	1.209

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	88	78	215	0	111
normalized size	1	1.	0.96	1.66	1.47	4.06	0.	2.09
time (sec)	N/A	0.06	0.024	0.053	1.504	1.812	0.	1.208

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	98	113	266	0	0
normalized size	1	1.	0.95	1.31	1.51	3.55	0.	0.
time (sec)	N/A	0.077	0.042	0.057	1.482	1.7	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	107	167	321	0	265
normalized size	1	1.	0.8	1.1	1.72	3.31	0.	2.73
time (sec)	N/A	0.123	0.088	0.059	1.54	1.656	0.	1.301

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	91	142	232	0	72
normalized size	1	1.	0.86	1.25	1.95	3.18	0.	0.99
time (sec)	N/A	0.085	0.074	0.062	1.504	1.544	0.	1.214

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	58	77	123	212	0	65
normalized size	1	1.	0.97	1.28	2.05	3.53	0.	1.08
time (sec)	N/A	0.075	0.053	0.057	1.464	1.584	0.	1.275

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	27	68	101	180	34
normalized size	1	1.	0.73	0.66	1.66	2.46	4.39	0.83
time (sec)	N/A	0.053	0.018	0.049	0.971	1.549	111.998	1.251

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	58	133	109	273	0	123
normalized size	1	1.	0.79	1.82	1.49	3.74	0.	1.68
time (sec)	N/A	0.085	0.051	0.056	1.495	1.563	0.	1.273

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	91	143	144	356	0	0
normalized size	1	1.	0.96	1.51	1.52	3.75	0.	0.
time (sec)	N/A	0.169	0.063	0.059	1.505	1.654	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	75	140	198	417	0	247
normalized size	1	1.	0.64	1.2	1.69	3.56	0.	2.11
time (sec)	N/A	0.206	0.105	0.063	1.523	1.7	0.	1.325

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	420	417	0	0	0	0	0	0
normalized size	1	0.99	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.604	1.199	0.796	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	403	401	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.453	0.726	0.742	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	474	474	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.519	2.91	0.754	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	165	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.407	0.361	1.615	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	256	343	355	718	320	416
normalized size	1	1.	1.01	1.35	1.4	2.83	1.26	1.64
time (sec)	N/A	0.326	0.097	0.046	1.009	1.222	0.113	1.174

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	163	223	223	444	197	252
normalized size	1	1.	1.01	1.39	1.39	2.76	1.22	1.57
time (sec)	N/A	0.193	0.044	0.046	0.976	1.307	0.092	1.215

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	90	117	244	102	134
normalized size	1	1.	1.	0.94	1.22	2.54	1.06	1.4
time (sec)	N/A	0.099	0.021	0.048	0.964	1.31	0.078	1.247

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	51	104	42	54
normalized size	1	1.	1.	0.85	1.11	2.26	0.91	1.17
time (sec)	N/A	0.03	0.009	0.05	0.983	1.316	0.064	1.169

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	84	140	0	595	413	105
normalized size	1	1.	1.04	1.73	0.	7.35	5.1	1.3
time (sec)	N/A	0.1	0.092	0.184	0.	1.572	1.221	1.316

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	98	146	0	1085	376	146
normalized size	1	1.	0.98	1.46	0.	10.85	3.76	1.46
time (sec)	N/A	0.068	0.084	0.19	0.	1.602	1.287	1.244

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	160	373	0	2592	774	293
normalized size	1	1.	0.99	2.32	0.	16.1	4.81	1.82
time (sec)	N/A	0.114	0.22	0.183	0.	1.735	2.665	1.272

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	204	643	0	4547	1224	549
normalized size	1	1.	0.99	3.12	0.	22.07	5.94	2.67
time (sec)	N/A	0.19	0.4	0.187	0.	1.756	5.196	1.178

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	591	585	1738	0	4321	4962	1041
normalized size	1	1.	0.99	2.94	0.	7.31	8.4	1.76
time (sec)	N/A	1.428	0.631	0.181	0.	9.137	83.989	1.285

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	345	1028	0	2600	2839	575
normalized size	1	1.	0.99	2.95	0.	7.47	8.16	1.65
time (sec)	N/A	0.677	0.364	0.196	0.	3.852	37.39	1.259

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	173	510	0	1361	1265	271
normalized size	1	1.	0.98	2.88	0.	7.69	7.15	1.53
time (sec)	N/A	0.35	0.21	0.18	0.	2.022	12.071	1.239

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	95	196	0	670	488	120
normalized size	1	1.	1.03	2.13	0.	7.28	5.3	1.3
time (sec)	N/A	0.156	0.071	0.175	0.	1.627	2.039	1.133

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	193	622	0	0	0	275
normalized size	1	1.	0.98	3.17	0.	0.	0.	1.4
time (sec)	N/A	0.349	0.243	0.183	0.	0.	0.	1.276

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	281	1125	0	0	0	606
normalized size	1	1.	0.89	3.56	0.	0.	0.	1.92
time (sec)	N/A	0.761	0.657	0.192	0.	0.	0.	1.317

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	504	1945	0	0	0	1353
normalized size	1	1.	0.99	3.82	0.	0.	0.	2.66
time (sec)	N/A	1.251	0.902	0.194	0.	0.	0.	1.261

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	398	1712	0	5646	0	729
normalized size	1	1.	1.38	5.94	0.	19.6	0.	2.53
time (sec)	N/A	0.7	0.95	0.194	0.	4.94	0.	1.308

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	225	500	0	2957	1535	385
normalized size	1	1.	1.26	2.81	0.	16.61	8.62	2.16
time (sec)	N/A	0.266	0.54	0.176	0.	2.704	53.152	1.341

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	114	194	0	1335	459	169
normalized size	1	1.	0.97	1.64	0.	11.31	3.89	1.43
time (sec)	N/A	0.098	0.114	0.187	0.	1.558	2.314	1.165

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	405	3202	0	0	0	1161
normalized size	1	1.	1.	7.87	0.	0.	0.	2.85
time (sec)	N/A	1.087	1.15	0.199	0.	0.	0.	1.34

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	673	650	4716	0	0	0	1940
normalized size	1	1.	0.97	7.01	0.	0.	0.	2.88
time (sec)	N/A	2.559	2.476	0.215	0.	0.	0.	1.425

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	60	53	69	204	60	69
normalized size	1	1.	0.97	0.85	1.11	3.29	0.97	1.11
time (sec)	N/A	0.074	0.033	0.049	1.433	2.34	0.143	1.199

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	46	62	192	54	62
normalized size	1	1.	1.	0.84	1.13	3.49	0.98	1.13
time (sec)	N/A	0.066	0.027	0.048	1.523	2.329	0.133	1.174

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	58	167	51	58
normalized size	1	1.	1.	0.87	1.12	3.21	0.98	1.12
time (sec)	N/A	0.052	0.02	0.047	1.497	2.349	0.138	1.338

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	34	43	115	41	43
normalized size	1	1.	0.95	0.83	1.05	2.8	1.	1.05
time (sec)	N/A	0.033	0.022	0.044	1.518	1.903	0.125	1.285

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	48	63	203	54	65
normalized size	1	1.	1.	0.86	1.12	3.62	0.96	1.16
time (sec)	N/A	0.089	0.025	0.048	1.535	1.627	0.179	1.235

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	55	73	225	65	74
normalized size	1	1.	1.	0.9	1.2	3.69	1.07	1.21
time (sec)	N/A	0.128	0.023	0.049	1.463	1.851	0.177	1.273

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	60	85	250	71	85
normalized size	1	1.	0.97	0.88	1.25	3.68	1.04	1.25
time (sec)	N/A	0.11	0.03	0.048	1.507	1.746	0.187	1.268

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	23	7	11
normalized size	1	1.	1.	1.1	1.4	2.3	0.7	1.1
time (sec)	N/A	0.011	0.005	0.042	1.028	1.614	0.087	1.207

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	36	95	36	36
normalized size	1	1.	1.	0.9	1.16	3.06	1.16	1.16
time (sec)	N/A	0.032	0.008	0.042	1.487	1.644	0.099	1.228

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	28	69	22	28
normalized size	1	1.	1.	0.96	1.22	3.	0.96	1.22
time (sec)	N/A	0.035	0.005	0.05	1.484	1.701	0.101	1.338

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	23	69	14	24
normalized size	1	1.	0.9	0.86	1.1	3.29	0.67	1.14
time (sec)	N/A	0.013	0.009	0.049	1.012	1.671	0.081	1.273

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	47	14	22
normalized size	1	1.	1.	0.83	1.06	2.61	0.78	1.22
time (sec)	N/A	0.018	0.004	0.049	0.991	1.658	0.1	1.274

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	42	12	19
normalized size	1	1.	1.	0.93	1.14	3.	0.86	1.36
time (sec)	N/A	0.017	0.005	0.049	0.991	1.611	0.097	1.246

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	22	28	70	22	28
normalized size	1	1.	1.15	0.81	1.04	2.59	0.81	1.04
time (sec)	N/A	0.028	0.005	0.047	1.526	1.608	0.101	1.243

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	30	49	157	46	61
normalized size	1	1.	1.	0.62	1.02	3.27	0.96	1.27
time (sec)	N/A	0.045	0.039	0.05	1.577	1.769	0.112	1.162

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	26	46	15	26
normalized size	1	1.	1.	0.81	1.24	2.19	0.71	1.24
time (sec)	N/A	0.012	0.007	0.048	1.004	1.648	0.113	1.227

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	41	117	37	41
normalized size	1	1.	1.	0.87	1.05	3.	0.95	1.05
time (sec)	N/A	0.023	0.025	0.048	1.565	1.684	0.123	1.246

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	45	70	31	15
normalized size	1	1.	1.	1.09	4.09	6.36	2.82	1.36
time (sec)	N/A	0.008	0.006	0.047	1.029	1.621	0.13	1.269

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	344	997	0	2342	0	651
normalized size	1	1.	1.29	3.73	0.	8.77	0.	2.44
time (sec)	N/A	0.238	0.851	0.055	0.	2.888	0.	1.25

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	267	613	0	1451	0	401
normalized size	1	1.	1.26	2.89	0.	6.84	0.	1.89
time (sec)	N/A	0.183	0.55	0.057	0.	2.306	0.	1.321

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	144	327	0	846	0	216
normalized size	1	1.	0.92	2.08	0.	5.39	0.	1.38
time (sec)	N/A	0.124	0.2	0.053	0.	2.098	0.	1.252

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	86	136	0	494	0	113
normalized size	1	1.	0.83	1.31	0.	4.75	0.	1.09
time (sec)	N/A	0.08	0.146	0.051	0.	1.98	0.	1.345

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	104	169	0	892	0	149
normalized size	1	1.	1.06	1.72	0.	9.1	0.	1.52
time (sec)	N/A	0.077	0.763	0.052	0.	3.141	0.	1.34

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	107	137	0	520	0	293
normalized size	1	1.	0.94	1.2	0.	4.56	0.	2.57
time (sec)	N/A	0.093	0.89	0.049	0.	9.072	0.	1.243

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	148	316	0	1224	0	659
normalized size	1	1.	0.89	1.89	0.	7.33	0.	3.95
time (sec)	N/A	0.107	1.854	0.051	0.	77.5	0.	1.287

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	199	555	0	0	0	1152
normalized size	1	1.	0.9	2.52	0.	0.	0.	5.24
time (sec)	N/A	0.142	1.782	0.053	0.	0.	0.	1.354

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	930	927	1093	3543	0	6407	0	2298
normalized size	1	1.	1.18	3.81	0.	6.89	0.	2.47
time (sec)	N/A	3.013	2.447	0.067	0.	12.168	0.	1.324

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	584	581	436	2179	0	4019	0	1366
normalized size	1	0.99	0.75	3.73	0.	6.88	0.	2.34
time (sec)	N/A	1.44	0.966	0.059	0.	6.63	0.	1.271

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	258	1117	0	2275	0	668
normalized size	1	1.	0.8	3.47	0.	7.07	0.	2.07
time (sec)	N/A	0.504	0.493	0.053	0.	3.7	0.	1.349

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	173	453	0	1061	0	286
normalized size	1	1.	0.99	2.59	0.	6.06	0.	1.63
time (sec)	N/A	0.167	0.286	0.052	0.	1.734	0.	1.248

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	331	2549	0	0	0	0
normalized size	1	1.	1.03	7.94	0.	0.	0.	0.
time (sec)	N/A	0.778	0.782	0.352	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	453	486	6218	0	0	0	0
normalized size	1	0.99	1.06	13.55	0.	0.	0.	0.
time (sec)	N/A	1.1	1.695	0.273	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	446	645	12139	0	0	0	0
normalized size	1	1.	1.44	27.1	0.	0.	0.	0.
time (sec)	N/A	0.875	3.952	0.285	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	603	601	439	19321	0	0	0	0
normalized size	1	1.	0.73	32.04	0.	0.	0.	0.
time (sec)	N/A	1.449	2.287	0.3	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	499	447	29161	0	0	0	0
normalized size	1	1.	0.9	58.67	0.	0.	0.	0.
time (sec)	N/A	0.855	4.795	0.284	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	824	826	1129	40336	0	0	0	0
normalized size	1	1.	1.37	48.95	0.	0.	0.	0.
time (sec)	N/A	2.335	6.328	0.292	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1169	1166	721	5881	0	11167	0	4019
normalized size	1	1.	0.62	5.03	0.	9.55	0.	3.44
time (sec)	N/A	3.698	3.005	0.078	0.	30.47	0.	1.354

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	753	749	468	3769	0	7302	0	2500
normalized size	1	0.99	0.62	5.01	0.	9.7	0.	3.32
time (sec)	N/A	2.104	1.678	0.065	0.	13.636	0.	1.204

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	285	2026	0	4232	0	1289
normalized size	1	1.	0.68	4.85	0.	10.12	0.	3.08
time (sec)	N/A	0.645	0.806	0.059	0.	4.759	0.	1.223

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	392	862	0	1947	0	563
normalized size	1	1.	1.66	3.65	0.	8.25	0.	2.39
time (sec)	N/A	0.242	0.697	0.053	0.	2.13	0.	1.197

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	660	660	635	6715	0	0	0	0
normalized size	1	1.	0.96	10.17	0.	0.	0.	0.
time (sec)	N/A	1.825	2.407	0.263	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	754	750	756	14734	0	0	0	0
normalized size	1	0.99	1.	19.54	0.	0.	0.	0.
time (sec)	N/A	2.503	4.603	0.264	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	824	819	4162	26596	0	0	0	0
normalized size	1	0.99	5.05	32.28	0.	0.	0.	0.
time (sec)	N/A	2.141	6.293	0.265	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	833	829	7806	40092	0	0	0	9881
normalized size	1	1.	9.37	48.13	0.	0.	0.	11.86
time (sec)	N/A	2.264	6.501	0.299	0.	0.	0.	25.736

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1097	1096	46895	57957	0	0	0	0
normalized size	1	1.	42.75	52.83	0.	0.	0.	0.
time (sec)	N/A	3.123	6.632	0.28	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1226	1223	1111	76693	0	0	0	0
normalized size	1	1.	0.91	62.56	0.	0.	0.	0.
time (sec)	N/A	3.997	6.305	0.288	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	657	660	765	100754	0	0	0	0
normalized size	1	1.	1.16	153.35	0.	0.	0.	0.
time (sec)	N/A	1.216	6.245	0.322	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1062	1062	1222	126612	0	0	0	0
normalized size	1	1.	1.15	119.22	0.	0.	0.	0.
time (sec)	N/A	3.004	6.419	0.396	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	70	115	170	285	0	105
normalized size	1	1.	0.49	0.8	1.19	1.99	0.	0.73
time (sec)	N/A	0.138	0.047	0.057	1.528	1.908	0.	1.349

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	65	98	147	243	0	99
normalized size	1	1.	0.55	0.83	1.25	2.06	0.	0.84
time (sec)	N/A	0.112	0.039	0.055	1.489	1.556	0.	1.16

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	60	81	124	228	0	92
normalized size	1	1.	0.65	0.87	1.33	2.45	0.	0.99
time (sec)	N/A	0.066	0.029	0.051	1.536	1.63	0.	1.162

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	86	95	130	321	0	170
normalized size	1	1.	0.85	0.94	1.29	3.18	0.	1.68
time (sec)	N/A	0.117	0.056	0.054	1.529	1.621	0.	1.261

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	92	123	139	363	0	513
normalized size	1	1.	0.85	1.14	1.29	3.36	0.	4.75
time (sec)	N/A	0.116	0.088	0.055	1.639	1.63	0.	1.672

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	93	125	154	405	0	0
normalized size	1	1.	0.81	1.09	1.34	3.52	0.	0.
time (sec)	N/A	0.117	0.082	0.057	1.497	1.652	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	80	134	209	354	0	119
normalized size	1	1.	0.51	0.85	1.32	2.24	0.	0.75
time (sec)	N/A	0.201	0.05	0.055	1.54	1.407	0.	1.18

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	75	117	186	308	0	112
normalized size	1	1.	0.53	0.83	1.32	2.18	0.	0.79
time (sec)	N/A	0.121	0.043	0.055	1.504	1.63	0.	1.154

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	70	100	163	277	0	105
normalized size	1	1.	0.6	0.86	1.41	2.39	0.	0.91
time (sec)	N/A	0.082	0.036	0.051	1.503	1.568	0.	1.17

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	96	151	169	367	0	184
normalized size	1	1.	0.77	1.22	1.36	2.96	0.	1.48
time (sec)	N/A	0.144	0.058	0.051	1.484	1.608	0.	1.285

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	103	179	178	396	0	770
normalized size	1	1.	0.79	1.37	1.36	3.02	0.	5.88
time (sec)	N/A	0.14	0.09	0.055	1.496	1.714	0.	1.85

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	103	162	193	433	0	0
normalized size	1	1.	0.75	1.17	1.4	3.14	0.	0.
time (sec)	N/A	0.139	0.104	0.063	1.492	1.69	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	90	153	248	444	0	132
normalized size	1	1.	0.48	0.81	1.31	2.35	0.	0.7
time (sec)	N/A	0.156	0.06	0.059	1.491	1.451	0.	1.278

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	85	136	225	390	0	126
normalized size	1	1.	0.52	0.83	1.37	2.38	0.	0.77
time (sec)	N/A	0.133	0.055	0.056	1.48	1.328	0.	1.192

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	80	119	203	344	0	119
normalized size	1	1.	0.58	0.86	1.46	2.47	0.	0.86
time (sec)	N/A	0.092	0.045	0.055	1.565	1.359	0.	1.212

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	106	207	208	440	0	197
normalized size	1	1.	0.72	1.41	1.41	2.99	0.	1.34
time (sec)	N/A	0.16	0.074	0.052	1.513	1.372	0.	1.347

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	113	235	217	462	0	1026
normalized size	1	1.	0.73	1.53	1.41	3.	0.	6.66
time (sec)	N/A	0.163	0.115	0.061	1.569	1.567	0.	2.011

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	113	199	232	487	0	0
normalized size	1	1.	0.7	1.24	1.44	3.02	0.	0.
time (sec)	N/A	0.163	0.108	0.059	1.593	1.491	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	693	692	588	1869	0	3267	0	1110
normalized size	1	1.	0.85	2.7	0.	4.71	0.	1.6
time (sec)	N/A	2.102	1.281	0.063	0.	3.633	0.	1.267

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	418	343	1069	0	1953	0	617
normalized size	1	1.	0.82	2.55	0.	4.65	0.	1.47
time (sec)	N/A	1.011	0.67	0.063	0.	2.177	0.	1.246

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	215	505	0	1060	0	284
normalized size	1	1.	0.96	2.26	0.	4.75	0.	1.27
time (sec)	N/A	0.303	0.256	0.056	0.	2.484	0.	1.222

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	96	185	0	549	0	132
normalized size	1	1.	0.83	1.59	0.	4.73	0.	1.14
time (sec)	N/A	0.106	0.152	0.054	0.	1.429	0.	1.316

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	172	599	0	0	0	0
normalized size	1	1.	0.96	3.35	0.	0.	0.	0.
time (sec)	N/A	0.293	0.286	0.317	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	239	227	1671	0	0	0	0
normalized size	1	0.99	0.94	6.93	0.	0.	0.	0.
time (sec)	N/A	0.371	0.477	0.279	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	367	3615	0	0	0	3114
normalized size	1	1.	1.09	10.76	0.	0.	0.	9.27
time (sec)	N/A	0.656	1.349	0.285	0.	0.	0.	1.363

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	504	502	715	2780	0	6238	0	1423
normalized size	1	1.	1.42	5.52	0.	12.38	0.	2.82
time (sec)	N/A	1.176	1.648	0.064	0.	132.617	0.	1.245

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	288	412	1557	0	3730	0	783
normalized size	1	1.	1.43	5.39	0.	12.91	0.	2.71
time (sec)	N/A	0.392	0.832	0.063	0.	90.979	0.	1.335

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	205	735	0	1939	0	366
normalized size	1	1.	1.1	3.95	0.	10.42	0.	1.97
time (sec)	N/A	0.227	0.781	0.057	0.	53.432	0.	1.222

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	113	249	0	941	0	165
normalized size	1	1.	1.02	2.24	0.	8.48	0.	1.49
time (sec)	N/A	0.066	0.319	0.056	0.	4.565	0.	1.211

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	271	2079	0	3934	0	971
normalized size	1	1.	1.2	9.24	0.	17.48	0.	4.32
time (sec)	N/A	0.266	0.584	0.293	0.	131.668	0.	1.205

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	418	487	4930	0	0	0	0
normalized size	1	0.99	1.16	11.71	0.	0.	0.	0.
time (sec)	N/A	0.797	3.262	0.301	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	713	707	2046	9126	0	0	0	7610
normalized size	1	0.99	2.87	12.8	0.	0.	0.	10.67
time (sec)	N/A	2.67	6.212	0.327	0.	0.	0.	1.784

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	60	96	131	224	0	92
normalized size	1	1.	0.5	0.8	1.09	1.87	0.	0.77
time (sec)	N/A	0.135	0.047	0.056	1.516	1.623	0.	1.183

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	55	79	108	201	0	85
normalized size	1	1.	0.58	0.83	1.14	2.12	0.	0.89
time (sec)	N/A	0.099	0.033	0.056	1.542	1.703	0.	1.188

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	50	62	85	178	0	78
normalized size	1	1.	0.71	0.89	1.21	2.54	0.	1.11
time (sec)	N/A	0.058	0.021	0.057	1.537	1.616	0.	1.169

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	60	90	289	0	157
normalized size	1	1.	1.	0.77	1.15	3.71	0.	2.01
time (sec)	N/A	0.097	0.037	0.051	1.514	1.659	0.	1.247

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	67	100	336	0	0
normalized size	1	1.	0.99	0.81	1.2	4.05	0.	0.
time (sec)	N/A	0.095	0.052	0.056	1.543	1.701	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	69	74	111	252	0	275
normalized size	1	1.	0.78	0.83	1.25	2.83	0.	3.09
time (sec)	N/A	0.088	0.045	0.055	1.595	1.287	0.	1.221

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	69	115	131	269	0	90
normalized size	1	1.	0.67	1.12	1.27	2.61	0.	0.87
time (sec)	N/A	0.124	0.041	0.053	1.547	1.098	0.	1.199

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	61	98	108	244	0	84
normalized size	1	1.	0.74	1.2	1.32	2.98	0.	1.02
time (sec)	N/A	0.108	0.032	0.053	1.554	1.127	0.	1.151

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	81	85	224	0	77
normalized size	1	1.	0.79	1.29	1.35	3.56	0.	1.22
time (sec)	N/A	0.06	0.13	0.049	1.526	0.94	0.	1.196

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	73	102	86	247	0	123
normalized size	1	1.	1.18	1.65	1.39	3.98	0.	1.98
time (sec)	N/A	0.074	0.022	0.061	1.934	1.01	0.	1.215

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	74	109	130	285	0	0
normalized size	1	1.	0.85	1.25	1.49	3.28	0.	0.
time (sec)	N/A	0.092	0.046	0.055	1.501	1.151	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	79	111	196	344	0	301
normalized size	1	1.	0.71	0.99	1.75	3.07	0.	2.69
time (sec)	N/A	0.155	0.058	0.057	1.507	1.132	0.	1.245

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	71	163	273	325	0	90
normalized size	1	1.	0.83	1.9	3.17	3.78	0.	1.05
time (sec)	N/A	0.113	0.063	0.056	1.529	1.143	0.	1.156

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	146	250	304	0	84
normalized size	1	1.	0.97	2.15	3.68	4.47	0.	1.24
time (sec)	N/A	0.095	0.148	0.054	1.462	1.063	0.	1.159

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	103	136	0	38
normalized size	1	1.	0.7	0.64	2.19	2.89	0.	0.81
time (sec)	N/A	0.048	0.06	0.046	1.004	1.002	0.	1.183

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	72	158	126	344	0	136
normalized size	1	1.	0.85	1.86	1.48	4.05	0.	1.6
time (sec)	N/A	0.094	0.048	0.056	1.473	1.108	0.	1.209

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	111	165	169	397	0	0
normalized size	1	1.	1.01	1.5	1.54	3.61	0.	0.
time (sec)	N/A	0.151	0.062	0.057	1.482	1.085	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	89	148	235	460	0	315
normalized size	1	1.	0.66	1.1	1.74	3.41	0.	2.33
time (sec)	N/A	0.213	0.076	0.061	1.494	1.199	0.	1.232

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	219	324	0	941	0	0
normalized size	1	1.	1.05	1.56	0.	4.52	0.	0.
time (sec)	N/A	0.424	0.52	0.06	0.	134.36	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	906	905	15669	19955	0	0	0	0
normalized size	1	1.	17.29	22.03	0.	0.	0.	0.
time (sec)	N/A	2.7	15.29	0.426	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	668	668	9965	12761	0	0	0	0
normalized size	1	1.	14.92	19.1	0.	0.	0.	0.
time (sec)	N/A	1.188	14.439	0.426	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	749	746	13240	8221	0	0	0	0
normalized size	1	1.	17.68	10.98	0.	0.	0.	0.
time (sec)	N/A	1.518	14.111	0.379	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	712	711	8456	21038	0	0	0	0
normalized size	1	1.	11.88	29.55	0.	0.	0.	0.
time (sec)	N/A	1.271	14.451	0.452	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	992	989	12997	48427	0	0	0	0
normalized size	1	1.	13.1	48.82	0.	0.	0.	0.
time (sec)	N/A	1.916	14.864	0.566	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1363	1363	19853	88790	0	0	0	0
normalized size	1	1.	14.57	65.14	0.	0.	0.	0.
time (sec)	N/A	4.2	16.486	0.738	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1904	1904	29140	153623	0	0	0	0
normalized size	1	1.	15.3	80.68	0.	0.	0.	0.
time (sec)	N/A	6.243	19.21	1.017	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	724	724	9972	14084	0	0	0	0
normalized size	1	1.	13.77	19.45	0.	0.	0.	0.
time (sec)	N/A	1.779	14.573	0.438	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	992	8161	0	0	0	0
normalized size	1	1.	1.78	14.65	0.	0.	0.	0.
time (sec)	N/A	0.889	11.719	0.361	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	470	1080	4251	0	0	0	0
normalized size	1	1.	2.29	9.03	0.	0.	0.	0.
time (sec)	N/A	0.482	12.587	0.404	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	506	772	6053	0	0	0	0
normalized size	1	1.	1.52	11.92	0.	0.	0.	0.
time (sec)	N/A	0.649	7.383	0.376	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	684	680	1194	20481	0	0	0	0
normalized size	1	0.99	1.75	29.94	0.	0.	0.	0.
time (sec)	N/A	1.192	12.204	0.463	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	944	942	12295	46695	0	0	0	0
normalized size	1	1.	13.02	49.47	0.	0.	0.	0.
time (sec)	N/A	2.256	15.282	0.628	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	510	508	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.833	2.318	1.428	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	496	494	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.672	1.466	1.338	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	590	588	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.755	3.636	1.356	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	119	36	80	166	221	190
normalized size	1	1.	2.9	0.88	1.95	4.05	5.39	4.63
time (sec)	N/A	0.052	0.095	0.044	1.111	1.395	9.565	1.226

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	39	89	182	280	258
normalized size	1	1.	0.74	0.85	1.93	3.96	6.09	5.61
time (sec)	N/A	0.072	0.124	0.049	1.116	1.375	95.992	1.178

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	51	132	269	483	424
normalized size	1	1.	0.75	0.89	2.32	4.72	8.47	7.44
time (sec)	N/A	0.121	0.3	0.049	1.146	1.37	113.673	1.224

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	167	8419	2402	5253	2281	3217
normalized size	1	1.	8.35	420.95	120.1	262.65	114.05	160.85
time (sec)	N/A	0.42	0.439	0.049	1.056	1.121	0.394	1.208

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	24	58	20	27
normalized size	1	1.	1.	0.73	0.92	2.23	0.77	1.04
time (sec)	N/A	0.025	0.005	0.047	0.999	1.365	0.103	1.192

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	282	783	0	1666	0	446
normalized size	1	1.	0.82	2.26	0.	4.82	0.	1.29
time (sec)	N/A	0.812	0.727	0.058	0.	2.159	0.	1.231

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	199	532	0	1173	0	308
normalized size	1	1.	0.81	2.17	0.	4.79	0.	1.26
time (sec)	N/A	0.438	0.459	0.055	0.	1.622	0.	1.199

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	141	333	0	803	0	201
normalized size	1	1.	0.8	1.88	0.	4.54	0.	1.14
time (sec)	N/A	0.235	0.272	0.086	0.	1.535	0.	1.187

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	134	220	0	1759	0	0
normalized size	1	1.	0.86	1.42	0.	11.35	0.	0.
time (sec)	N/A	0.256	0.385	0.052	0.	19.893	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	127	173	0	1658	0	231
normalized size	1	1.	0.91	1.24	0.	11.93	0.	1.66
time (sec)	N/A	0.235	0.403	0.054	0.	12.899	0.	1.255

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	137	241	0	1837	0	475
normalized size	1	1.	0.86	1.52	0.	11.55	0.	2.99
time (sec)	N/A	0.244	0.371	0.056	0.	18.345	0.	1.296

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	150	375	0	848	0	930
normalized size	1	1.	0.81	2.02	0.	4.56	0.	5.
time (sec)	N/A	0.32	0.313	0.057	0.	21.001	0.	1.186

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	212	591	0	1216	0	1955
normalized size	1	1.	0.79	2.19	0.	4.5	0.	7.24
time (sec)	N/A	0.488	0.556	0.061	0.	42.742	0.	1.213

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	299	859	0	1708	0	2939
normalized size	1	1.	0.81	2.32	0.	4.6	0.	7.92
time (sec)	N/A	0.817	0.78	0.062	0.	115.592	0.	1.261

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	212	208	278	560	230	311
normalized size	1	1.	0.82	0.81	1.08	2.17	0.89	1.21
time (sec)	N/A	0.257	0.041	0.043	0.985	0.831	0.105	1.125

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	136	146	196	389	158	216
normalized size	1	1.	0.87	0.93	1.25	2.48	1.01	1.38
time (sec)	N/A	0.166	0.033	0.044	0.954	0.84	0.095	1.123

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	84	107	217	87	122
normalized size	1	1.	1.	0.9	1.15	2.33	0.94	1.31
time (sec)	N/A	0.108	0.014	0.043	0.991	0.84	0.083	1.143

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	46	84	37	46
normalized size	1	1.	1.	0.83	1.1	2.	0.88	1.1
time (sec)	N/A	0.025	0.001	0.043	0.97	0.869	0.063	1.146

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	179	286	308	520	221	308
normalized size	1	1.	0.79	1.25	1.35	2.28	0.97	1.35
time (sec)	N/A	0.193	0.059	0.05	1.	0.997	0.574	1.161

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	223	313	316	728	226	416
normalized size	1	1.	0.98	1.37	1.39	3.19	0.99	1.82
time (sec)	N/A	0.191	0.089	0.057	0.994	0.952	1.04	1.17

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	204	336	324	819	238	292
normalized size	1	1.	0.88	1.45	1.4	3.55	1.03	1.26
time (sec)	N/A	0.204	0.068	0.051	0.994	0.924	1.822	1.167

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	277	264	355	778	298	400
normalized size	1	1.	0.71	0.68	0.91	1.99	0.76	1.02
time (sec)	N/A	0.386	0.043	0.058	0.963	0.926	0.119	1.145

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	201	186	250	540	206	278
normalized size	1	1.	1.	0.93	1.24	2.69	1.02	1.38
time (sec)	N/A	0.24	0.027	0.041	0.995	0.825	0.105	1.153

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	121	108	142	302	112	157
normalized size	1	1.	1.	0.89	1.17	2.5	0.93	1.3
time (sec)	N/A	0.16	0.018	0.043	0.965	0.871	0.089	1.139

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	45	59	132	56	59
normalized size	1	1.	1.	0.75	0.98	2.2	0.93	0.98
time (sec)	N/A	0.036	0.001	0.043	0.995	0.916	0.069	1.153

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	262	465	494	875	347	510
normalized size	1	1.	0.74	1.32	1.4	2.49	0.99	1.45
time (sec)	N/A	0.316	0.122	0.052	1.016	1.054	0.718	1.141

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	342	500	502	1211	367	620
normalized size	1	1.	0.97	1.42	1.42	3.43	1.04	1.76
time (sec)	N/A	0.328	0.136	0.058	0.996	0.985	1.456	1.166

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	311	531	510	1323	379	478
normalized size	1	1.	0.88	1.5	1.44	3.74	1.07	1.35
time (sec)	N/A	0.343	0.104	0.056	0.986	1.	2.829	1.151

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	344	558	527	1434	391	466
normalized size	1	1.	0.96	1.55	1.46	3.98	1.09	1.29
time (sec)	N/A	0.358	0.121	0.056	1.013	1.022	5.233	1.142

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	178	291	278	626	450	286
normalized size	1	1.	0.81	1.32	1.26	2.83	2.04	1.29
time (sec)	N/A	0.19	0.129	0.051	1.496	1.004	1.247	1.147

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	130	191	190	436	303	196
normalized size	1	1.	0.83	1.22	1.22	2.79	1.94	1.26
time (sec)	N/A	0.162	0.084	0.052	1.54	1.007	0.998	1.136

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	86	102	113	275	163	119
normalized size	1	1.	0.87	1.03	1.14	2.78	1.65	1.2
time (sec)	N/A	0.108	0.05	0.046	1.565	1.312	0.686	1.122

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	44	58	157	61	58
normalized size	1	1.	0.89	0.79	1.04	2.8	1.09	1.04
time (sec)	N/A	0.049	0.018	0.048	1.543	1.233	0.123	1.172

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	146	298	216	414	4106	213
normalized size	1	1.	0.87	1.77	1.29	2.46	24.44	1.27
time (sec)	N/A	0.194	0.104	0.058	1.472	1.526	11.239	1.125

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	233	538	397	995	8391	479
normalized size	1	1.	1.	2.31	1.7	4.27	36.01	2.06
time (sec)	N/A	0.251	0.15	0.063	1.474	1.827	20.706	1.171

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	278	819	672	1674	0	548
normalized size	1	1.	0.88	2.58	2.12	5.28	0.	1.73
time (sec)	N/A	0.29	0.432	0.062	1.508	2.305	0.	1.166

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	209	283	286	1046	444	278
normalized size	1	1.	1.11	1.5	1.51	5.53	2.35	1.47
time (sec)	N/A	0.255	0.155	0.055	1.639	1.289	2.556	1.134

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	150	189	198	747	298	196
normalized size	1	1.	1.07	1.35	1.41	5.34	2.13	1.4
time (sec)	N/A	0.208	0.109	0.056	1.544	1.217	1.743	1.166

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	96	106	122	462	163	127
normalized size	1	1.	0.99	1.09	1.26	4.76	1.68	1.31
time (sec)	N/A	0.193	0.066	0.055	1.496	1.207	1.113	1.144

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	51	70	244	63	70
normalized size	1	1.	0.94	0.81	1.11	3.87	1.	1.11
time (sec)	N/A	0.077	0.037	0.048	1.5	1.24	0.171	1.162

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	186	691	390	1191	8322	383
normalized size	1	1.	0.83	3.08	1.74	5.32	37.15	1.71
time (sec)	N/A	0.34	0.16	0.1	1.604	1.68	18.377	1.175

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	270	986	740	2264	13362	771
normalized size	1	1.	0.86	3.15	2.36	7.23	42.69	2.46
time (sec)	N/A	0.498	0.255	0.072	1.576	1.919	27.34	1.225

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	363	1314	1149	3853	0	803
normalized size	1	1.	0.88	3.19	2.79	9.35	0.	1.95
time (sec)	N/A	0.715	0.37	0.073	1.66	2.904	0.	1.257

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	209	267	300	1339	469	271
normalized size	1	1.	1.22	1.56	1.75	7.83	2.74	1.58
time (sec)	N/A	0.336	0.201	0.059	1.55	1.258	5.288	1.149

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	146	179	209	933	304	194
normalized size	1	1.	1.09	1.34	1.56	6.96	2.27	1.45
time (sec)	N/A	0.239	0.176	0.053	1.543	1.254	3.385	1.14

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	107	102	136	558	163	131
normalized size	1	1.	1.04	0.99	1.32	5.42	1.58	1.27
time (sec)	N/A	0.145	0.08	0.053	1.485	1.263	1.652	1.165

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	76	243	61	62
normalized size	1	1.	0.83	0.73	1.19	3.8	0.95	0.97
time (sec)	N/A	0.05	0.038	0.047	1.5	1.29	0.185	1.148

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	282	1437	771	2678	0	621
normalized size	1	1.	0.86	4.37	2.34	8.14	0.	1.89
time (sec)	N/A	0.496	0.297	0.071	1.578	2.324	0.	1.158

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	389	1850	1237	4680	0	1029
normalized size	1	1.	0.88	4.18	2.79	10.56	0.	2.32
time (sec)	N/A	0.893	0.481	0.077	1.705	3.376	0.	1.264

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	70	115	170	296	0	105
normalized size	1	1.	0.49	0.8	1.19	2.07	0.	0.73
time (sec)	N/A	0.155	0.155	0.054	1.522	1.309	0.	1.166

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	65	98	147	258	0	99
normalized size	1	1.	0.52	0.79	1.19	2.08	0.	0.8
time (sec)	N/A	0.092	0.101	0.055	1.577	1.355	0.	1.174

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	91	127	173	394	0	174
normalized size	1	1.	0.61	0.85	1.16	2.64	0.	1.17
time (sec)	N/A	0.24	0.148	0.056	1.591	1.386	0.	1.164

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	98	152	178	431	0	717
normalized size	1	1.	0.66	1.02	1.19	2.89	0.	4.81
time (sec)	N/A	0.237	0.163	0.059	1.536	1.415	0.	1.312

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	98	158	193	479	0	348
normalized size	1	1.	0.65	1.05	1.28	3.17	0.	2.3
time (sec)	N/A	0.228	0.156	0.063	1.503	1.411	0.	1.275

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	98	165	216	543	0	410
normalized size	1	1.	0.62	1.04	1.37	3.44	0.	2.59
time (sec)	N/A	0.226	0.159	0.064	1.562	1.42	0.	1.398

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	98	167	244	618	0	441
normalized size	1	1.	0.59	1.01	1.48	3.75	0.	2.67
time (sec)	N/A	0.234	0.179	0.066	1.598	1.458	0.	1.338

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	98	188	300	694	0	522
normalized size	1	1.	0.59	1.14	1.82	4.21	0.	3.16
time (sec)	N/A	0.229	0.212	0.074	1.616	1.407	0.	1.299

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	91	195	338	555	0	547
normalized size	1	1.	0.54	1.15	2.	3.28	0.	3.24
time (sec)	N/A	0.218	0.173	0.089	1.605	1.36	0.	1.308

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	96	216	406	649	0	616
normalized size	1	1.	0.49	1.11	2.09	3.35	0.	3.18
time (sec)	N/A	0.268	0.207	0.078	1.547	1.44	0.	1.341

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	80	134	209	375	0	119
normalized size	1	1.	0.48	0.81	1.26	2.26	0.	0.72
time (sec)	N/A	0.194	0.191	0.059	1.584	1.323	0.	1.224

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	75	117	186	316	0	112
normalized size	1	1.	0.51	0.8	1.27	2.15	0.	0.76
time (sec)	N/A	0.121	0.13	0.055	1.478	1.286	0.	1.388

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	101	183	212	462	0	188
normalized size	1	1.	0.59	1.06	1.23	2.69	0.	1.09
time (sec)	N/A	0.268	0.188	0.056	1.615	1.449	0.	1.22

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	108	208	217	504	0	954
normalized size	1	1.	0.63	1.21	1.26	2.93	0.	5.55
time (sec)	N/A	0.282	0.225	0.06	1.583	1.468	0.	1.363

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	108	214	232	524	0	362
normalized size	1	1.	0.62	1.23	1.33	3.01	0.	2.08
time (sec)	N/A	0.273	0.212	0.061	1.589	1.398	0.	1.254

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	108	221	255	581	0	424
normalized size	1	1.	0.6	1.22	1.41	3.21	0.	2.34
time (sec)	N/A	0.267	0.22	0.062	1.557	1.484	0.	1.246

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	108	204	284	657	0	679
normalized size	1	1.	0.57	1.09	1.51	3.49	0.	3.61
time (sec)	N/A	0.265	0.239	0.062	1.565	1.491	0.	1.336

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	108	225	339	747	0	548
normalized size	1	1.	0.55	1.15	1.74	3.83	0.	2.81
time (sec)	N/A	0.263	0.247	0.066	1.653	1.521	0.	1.302

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	108	246	401	830	0	610
normalized size	1	1.	0.55	1.26	2.06	4.26	0.	3.13
time (sec)	N/A	0.268	0.259	0.073	1.541	1.446	0.	1.266

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	108	267	470	915	0	660
normalized size	1	1.	0.55	1.37	2.41	4.69	0.	3.38
time (sec)	N/A	0.262	0.272	0.078	1.611	1.535	0.	1.261

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	60	95	130	223	0	92
normalized size	1	1.	0.5	0.79	1.08	1.86	0.	0.77
time (sec)	N/A	0.136	0.116	0.05	1.475	1.299	0.	1.208

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	55	79	108	205	0	85
normalized size	1	1.	0.54	0.78	1.07	2.03	0.	0.84
time (sec)	N/A	0.08	0.075	0.053	1.476	1.306	0.	1.154

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	81	92	134	344	0	161
normalized size	1	1.	0.64	0.73	1.06	2.73	0.	1.28
time (sec)	N/A	0.21	0.103	0.056	1.522	1.402	0.	1.164

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	88	96	139	389	0	0
normalized size	1	1.	0.7	0.76	1.1	3.09	0.	0.
time (sec)	N/A	0.202	0.114	0.056	1.521	1.444	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	88	102	154	443	0	335
normalized size	1	1.	0.69	0.8	1.2	3.46	0.	2.62
time (sec)	N/A	0.208	0.13	0.065	1.509	1.428	0.	1.203

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	88	109	177	502	0	385
normalized size	1	1.	0.65	0.81	1.31	3.72	0.	2.85
time (sec)	N/A	0.205	0.151	0.059	1.494	1.345	0.	1.203

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	81	116	201	405	0	0
normalized size	1	1.	0.58	0.83	1.45	2.91	0.	0.
time (sec)	N/A	0.191	0.137	0.07	1.561	1.34	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	74	132	154	300	0	97
normalized size	1	1.	0.6	1.06	1.24	2.42	0.	0.78
time (sec)	N/A	0.152	0.492	0.056	1.648	1.363	0.	1.15

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	60	115	131	270	0	90
normalized size	1	1.	0.58	1.12	1.27	2.62	0.	0.87
time (sec)	N/A	0.102	0.181	0.064	1.456	1.322	0.	1.146

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	55	98	108	244	0	84
normalized size	1	1.	0.67	1.2	1.32	2.98	0.	1.02
time (sec)	N/A	0.055	0.129	0.051	1.542	1.351	0.	1.159

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	86	148	134	413	0	159
normalized size	1	1.	0.85	1.47	1.33	4.09	0.	1.57
time (sec)	N/A	0.151	0.349	0.061	1.527	1.383	0.	1.169

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	104	152	157	458	0	0
normalized size	1	1.	0.96	1.41	1.45	4.24	0.	0.
time (sec)	N/A	0.153	0.318	0.057	1.526	1.377	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	84	144	201	379	0	297
normalized size	1	1.	0.75	1.29	1.79	3.38	0.	2.65
time (sec)	N/A	0.146	0.297	0.058	1.513	1.341	0.	1.157

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	95	151	293	458	0	366
normalized size	1	1.	0.69	1.1	2.14	3.34	0.	2.67
time (sec)	N/A	0.204	0.164	0.063	1.601	1.334	0.	1.185

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	65	180	296	359	0	96
normalized size	1	1.	0.62	1.71	2.82	3.42	0.	0.91
time (sec)	N/A	0.131	0.711	0.059	1.552	1.333	0.	1.162

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	60	163	273	331	0	89
normalized size	1	1.	0.7	1.9	3.17	3.85	0.	1.03
time (sec)	N/A	0.082	0.26	0.055	1.502	1.343	0.	1.177

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	146	250	300	0	84
normalized size	1	1.	0.81	2.15	3.68	4.41	0.	1.24
time (sec)	N/A	0.052	0.226	0.053	1.465	1.348	0.	1.148

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	190	149	362	0	124
normalized size	1	1.	0.94	2.24	1.75	4.26	0.	1.46
time (sec)	N/A	0.128	0.467	0.056	1.657	1.319	0.	1.173

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	92	194	171	425	0	0
normalized size	1	1.	0.84	1.76	1.55	3.86	0.	0.
time (sec)	N/A	0.153	0.407	0.058	1.558	1.317	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	97	200	240	502	0	308
normalized size	1	1.	0.72	1.48	1.78	3.72	0.	2.28
time (sec)	N/A	0.221	0.307	0.062	1.613	1.404	0.	1.217

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	89	207	332	582	0	377
normalized size	1	1.	0.56	1.29	2.08	3.64	0.	2.36
time (sec)	N/A	0.283	0.232	0.062	1.592	1.399	0.	1.249

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	316	1406	0	0	0	628
normalized size	1	1.	0.89	3.97	0.	0.	0.	1.77
time (sec)	N/A	0.377	1.235	0.056	0.	0.	0.	1.166

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	319	1453	0	0	0	659
normalized size	1	1.	0.9	4.12	0.	0.	0.	1.87
time (sec)	N/A	0.386	1.178	0.059	0.	0.	0.	1.197

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	588	588	537	5924	0	14657	0	14796
normalized size	1	1.	0.91	10.07	0.	24.93	0.	25.16
time (sec)	N/A	0.364	0.41	0.082	0.	1.848	0.	1.409

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	432	391	3222	0	7602	0	8401
normalized size	1	1.	0.91	7.46	0.	17.6	0.	19.45
time (sec)	N/A	0.243	0.255	0.062	0.	1.587	0.	1.319

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	261	1504	0	3494	0	4182
normalized size	1	1.	0.89	5.15	0.	11.97	0.	14.32
time (sec)	N/A	0.189	0.182	0.052	0.	1.448	0.	1.211

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	221	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.481	0.673	3.62	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	441	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.9	1.757	2.375	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	528	488	1244	0	7274	0	887
normalized size	1	1.	0.92	2.36	0.	13.78	0.	1.68
time (sec)	N/A	1.312	1.319	0.191	0.	1.842	0.	1.154

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	765	765	754	1960	0	5392	0	1326
normalized size	1	1.	0.99	2.56	0.	7.05	0.	1.73
time (sec)	N/A	5.825	0.778	0.192	0.	3.615	0.	1.292

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	85	166	239	417	0	124
normalized size	1	1.	0.41	0.8	1.15	2.	0.	0.6
time (sec)	N/A	0.352	0.315	0.075	1.494	1.133	0.	1.286

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	75	132	193	324	0	111
normalized size	1	1.	0.45	0.8	1.16	1.95	0.	0.67
time (sec)	N/A	0.203	0.195	0.06	1.498	1.105	0.	1.178

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	65	98	147	255	0	97
normalized size	1	1.	0.52	0.79	1.19	2.06	0.	0.78
time (sec)	N/A	0.116	0.107	0.055	1.591	1.03	0.	1.247

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	189	403	675	1148	0	0
normalized size	1	1.	1.01	2.16	3.61	6.14	0.	0.
time (sec)	N/A	0.355	0.912	0.142	1.77	1.233	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	354	1084	0	1499	0	0
normalized size	1	1.	1.78	5.45	0.	7.53	0.	0.
time (sec)	N/A	0.25	1.3	0.124	0.	1.174	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	334	2342	0	1700	0	0
normalized size	1	1.	1.57	11.	0.	7.98	0.	0.
time (sec)	N/A	0.237	1.493	0.127	0.	1.256	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	95	185	278	518	0	138
normalized size	1	1.	0.41	0.8	1.2	2.24	0.	0.6
time (sec)	N/A	0.364	0.399	0.074	1.533	1.487	0.	1.201

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	85	151	232	397	0	124
normalized size	1	1.	0.45	0.8	1.23	2.1	0.	0.66
time (sec)	N/A	0.227	0.252	0.061	1.586	1.286	0.	1.21

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	75	117	186	323	0	111
normalized size	1	1.	0.51	0.8	1.27	2.2	0.	0.76
time (sec)	N/A	0.13	0.149	0.056	1.479	1.264	0.	1.304

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	202	730	722	1442	0	0
normalized size	1	1.	0.96	3.48	3.44	6.87	0.	0.
time (sec)	N/A	0.304	0.948	0.116	1.878	1.491	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	354	1828	0	1515	0	0
normalized size	1	1.	1.59	8.23	0.	6.82	0.	0.
time (sec)	N/A	0.322	1.873	0.118	0.	1.55	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	376	3828	0	1914	0	0
normalized size	1	1.	1.61	16.36	0.	8.18	0.	0.
time (sec)	N/A	0.319	2.343	0.125	0.	1.722	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	75	147	200	332	0	111
normalized size	1	1.	0.41	0.79	1.08	1.79	0.	0.6
time (sec)	N/A	0.312	0.261	0.067	1.531	1.41	0.	1.164

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	65	113	154	267	0	97
normalized size	1	1.	0.45	0.79	1.08	1.87	0.	0.68
time (sec)	N/A	0.203	0.153	0.058	1.452	1.363	0.	1.192

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	55	79	108	207	0	84
normalized size	1	1.	0.54	0.78	1.07	2.05	0.	0.83
time (sec)	N/A	0.145	0.086	0.052	1.531	1.249	0.	1.196

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	157	204	628	1126	0	0
normalized size	1	1.	0.96	1.24	3.83	6.87	0.	0.
time (sec)	N/A	0.23	0.485	0.106	1.653	1.548	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	313	510	0	1393	0	0
normalized size	1	1.	1.76	2.87	0.	7.83	0.	0.
time (sec)	N/A	0.196	1.024	0.117	0.	1.366	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	371	1194	0	1897	0	0
normalized size	1	1.	1.63	5.26	0.	8.36	0.	0.
time (sec)	N/A	0.271	1.282	0.131	0.	1.561	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	75	166	200	386	0	109
normalized size	1	1.	0.45	1.	1.2	2.33	0.	0.66
time (sec)	N/A	0.241	0.461	0.069	1.485	1.442	0.	1.183

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	65	132	154	305	0	96
normalized size	1	1.	0.52	1.06	1.24	2.46	0.	0.77
time (sec)	N/A	0.161	0.274	0.054	1.579	1.306	0.	1.161

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	55	98	108	254	0	84
normalized size	1	1.	0.67	1.2	1.32	3.1	0.	1.02
time (sec)	N/A	0.087	0.155	0.05	1.504	1.387	0.	1.184

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	174	489	1049	1269	0	0
normalized size	1	1.	1.05	2.95	6.32	7.64	0.	0.
time (sec)	N/A	0.216	1.177	0.104	1.801	1.406	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	351	1214	0	1759	0	0
normalized size	1	1.	1.63	5.65	0.	8.18	0.	0.
time (sec)	N/A	0.316	1.198	0.111	0.	1.482	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	381	2600	0	2311	0	0
normalized size	1	1.	1.52	10.4	0.	9.24	0.	0.
time (sec)	N/A	0.324	1.574	0.12	0.	2.547	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	242	0	0	0	0	0
normalized size	1	1.	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	0.424	0.681	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	236	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	0.331	0.631	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	252	252	302	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.481	0.531	0.618	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [168] had the largest ratio of [0.3571]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.	34	0.147
2	A	6	5	1.	32	0.156
3	A	5	5	1.	27	0.185
4	A	5	5	1.	34	0.147
5	A	5	5	1.	34	0.147
6	A	5	5	1.	34	0.147
7	A	8	6	1.	34	0.176
8	A	4	4	1.	34	0.118
9	A	5	4	1.	34	0.118
10	A	7	4	1.	34	0.118
11	A	6	4	1.	34	0.118
12	A	5	4	1.	32	0.125
13	A	4	4	1.	27	0.148
14	A	4	4	1.	34	0.118
15	A	7	5	1.	34	0.147
16	A	4	4	1.	34	0.118
17	A	5	4	1.	34	0.118
18	A	2	1	0.99	25	0.04
19	A	2	1	0.99	25	0.04
20	A	2	1	1.	23	0.043
21	A	2	1	1.	18	0.056
22	A	2	1	0.99	25	0.04
23	A	2	1	0.99	25	0.04
24	A	2	1	0.99	25	0.04
25	A	3	2	0.99	27	0.074
26	A	3	2	1.	27	0.074
27	A	3	2	1.	25	0.08
28	A	3	2	1.	20	0.1
29	A	2	1	0.99	27	0.037
30	A	2	1	0.99	27	0.037
31	A	2	1	0.99	27	0.037

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	3	2	0.99	27	0.074
33	A	3	2	1.	27	0.074
34	A	3	2	1.	25	0.08
35	A	3	2	1.	20	0.1
36	A	2	1	0.99	27	0.037
37	A	2	1	0.99	27	0.037
38	A	2	1	0.99	27	0.037
39	A	1	1	1.	32	0.031
40	A	1	1	1.	31	0.032
41	A	1	1	1.	34	0.029
42	A	1	1	1.	33	0.03
43	A	5	4	0.99	27	0.148
44	A	5	4	0.99	27	0.148
45	A	5	4	1.	25	0.16
46	A	5	4	1.	20	0.2
47	A	5	4	1.	27	0.148
48	A	5	4	1.	27	0.148
49	A	5	4	1.	27	0.148
50	A	6	5	1.	27	0.185
51	A	5	5	1.	27	0.185
52	A	4	4	1.	25	0.16
53	A	3	3	1.	20	0.15
54	A	6	5	1.	27	0.185
55	A	6	5	0.99	27	0.185
56	A	6	5	1.	27	0.185
57	A	5	5	1.	27	0.185
58	A	3	3	1.12	27	0.111
59	A	3	3	1.	25	0.12
60	A	4	4	1.	20	0.2
61	A	7	6	1.	27	0.222
62	A	7	5	0.99	27	0.185
63	A	7	5	1.	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	4	4	1.	27	0.148
65	A	4	4	1.13	27	0.148
66	A	4	4	1.	27	0.148
67	A	4	4	1.	25	0.16
68	A	5	4	1.	20	0.2
69	A	6	5	1.	17	0.294
70	A	5	5	1.	17	0.294
71	A	4	4	1.	15	0.267
72	A	3	3	1.	14	0.214
73	A	6	5	1.	17	0.294
74	A	6	5	1.	17	0.294
75	A	6	5	1.	17	0.294
76	A	3	3	1.83	16	0.188
77	A	3	3	1.	18	0.167
78	A	7	6	0.99	29	0.207
79	A	6	6	1.	29	0.207
80	A	5	5	1.	27	0.185
81	A	5	5	1.	22	0.227
82	A	7	6	1.	29	0.207
83	A	7	6	0.98	29	0.207
84	A	7	6	1.	29	0.207
85	A	7	6	1.	29	0.207
86	A	5	5	1.	29	0.172
87	A	6	6	1.	29	0.207
88	A	8	6	1.	29	0.207
89	A	7	6	1.	29	0.207
90	A	6	5	1.	27	0.185
91	A	6	5	1.	22	0.227
92	A	8	6	1.	29	0.207
93	A	8	6	0.99	29	0.207
94	A	8	7	0.98	29	0.241
95	A	8	6	0.99	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	8	7	1.	29	0.241
97	A	8	6	1.	29	0.207
98	A	6	5	1.	29	0.172
99	A	7	6	1.	29	0.207
100	A	7	5	1.	22	0.227
101	A	6	5	0.99	29	0.172
102	A	5	5	1.	29	0.172
103	A	4	4	0.99	27	0.148
104	A	4	4	1.	22	0.182
105	A	6	5	1.	29	0.172
106	A	6	5	1.	29	0.172
107	A	4	4	1.	29	0.138
108	A	5	5	1.	29	0.172
109	A	4	4	1.	29	0.138
110	A	4	4	1.	27	0.148
111	A	4	4	1.	22	0.182
112	A	4	4	1.	29	0.138
113	A	4	4	1.	29	0.138
114	A	5	5	0.99	29	0.172
115	A	3	3	1.	22	0.136
116	A	4	4	1.	22	0.182
117	A	5	4	1.	22	0.182
118	A	5	4	1.	29	0.138
119	A	4	4	1.	29	0.138
120	A	3	3	1.	27	0.111
121	A	5	5	1.	29	0.172
122	A	5	5	1.	29	0.172
123	A	4	4	1.	29	0.138
124	A	5	4	1.	29	0.138
125	A	4	4	1.	29	0.138
126	A	3	3	1.	27	0.111
127	A	4	4	1.	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	4	4	1.	29	0.138
129	A	5	5	1.	29	0.172
130	A	4	3	1.	29	0.103
131	A	4	3	1.	29	0.103
132	A	2	2	1.	27	0.074
133	A	5	5	1.	29	0.172
134	A	5	4	1.	29	0.138
135	A	6	5	1.	29	0.172
136	A	6	4	0.99	27	0.148
137	A	6	4	1.	29	0.138
138	A	5	5	1.	31	0.161
139	A	6	6	1.	70	0.086
140	A	2	1	1.	20	0.05
141	A	2	1	1.	20	0.05
142	A	2	1	1.	20	0.05
143	A	2	1	1.	18	0.056
144	A	6	5	1.	20	0.25
145	A	4	4	1.	20	0.2
146	A	5	5	1.	20	0.25
147	A	6	5	1.	20	0.25
148	A	6	5	1.	30	0.167
149	A	6	5	1.	30	0.167
150	A	6	5	1.	28	0.179
151	A	6	5	1.	23	0.217
152	A	6	5	1.	30	0.167
153	A	6	5	1.	30	0.167
154	A	6	5	1.	30	0.167
155	A	6	6	1.	30	0.2
156	A	5	5	1.	28	0.179
157	A	4	4	1.	23	0.174
158	A	7	6	1.	30	0.2
159	A	7	6	1.	30	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
160	A	7	6	1.	20	0.3
161	A	7	6	1.	20	0.3
162	A	5	5	1.	18	0.278
163	A	4	4	1.	17	0.235
164	A	7	6	1.	20	0.3
165	A	7	6	1.	20	0.3
166	A	7	6	1.	20	0.3
167	A	1	1	1.	16	0.062
168	A	6	5	1.	14	0.357
169	A	6	5	1.	16	0.312
170	A	3	2	1.	18	0.111
171	A	5	3	1.	16	0.188
172	A	5	3	1.	19	0.158
173	A	6	5	1.	23	0.217
174	A	5	3	1.	19	0.158
175	A	2	2	1.	23	0.087
176	A	4	4	1.	17	0.235
177	A	1	1	1.	19	0.053
178	A	7	5	1.	22	0.227
179	A	6	5	1.	22	0.227
180	A	5	5	1.	22	0.227
181	A	4	4	1.	22	0.182
182	A	4	4	1.	22	0.182
183	A	3	3	1.	22	0.136
184	A	4	4	1.	22	0.182
185	A	5	4	1.	22	0.182
186	A	7	6	1.	32	0.188
187	A	6	6	0.99	32	0.188
188	A	5	5	1.	30	0.167
189	A	5	5	1.	25	0.2
190	A	7	6	1.	32	0.188
191	A	7	6	0.99	32	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	7	6	1.	32	0.188
193	A	7	6	1.	32	0.188
194	A	5	5	1.	32	0.156
195	A	6	6	1.	32	0.188
196	A	8	6	1.	32	0.188
197	A	7	6	0.99	32	0.188
198	A	6	5	1.	30	0.167
199	A	6	5	1.	25	0.2
200	A	8	6	1.	32	0.188
201	A	8	6	0.99	32	0.188
202	A	8	7	0.99	32	0.219
203	A	8	6	1.	32	0.188
204	A	8	7	1.	32	0.219
205	A	8	6	1.	32	0.188
206	A	6	5	1.	32	0.156
207	A	7	6	1.	32	0.188
208	A	7	6	1.	32	0.188
209	A	6	6	1.	32	0.188
210	A	5	5	1.	30	0.167
211	A	7	7	1.	32	0.219
212	A	7	7	1.	32	0.219
213	A	7	7	1.	32	0.219
214	A	9	6	1.	32	0.188
215	A	7	6	1.	32	0.188
216	A	6	5	1.	30	0.167
217	A	8	7	1.	32	0.219
218	A	8	7	1.	32	0.219
219	A	8	8	1.	32	0.25
220	A	9	6	1.	32	0.188
221	A	8	6	1.	32	0.188
222	A	7	5	1.	30	0.167
223	A	9	7	1.	32	0.219

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	9	7	1.	32	0.219
225	A	9	8	1.	32	0.25
226	A	6	5	1.	32	0.156
227	A	5	5	1.	32	0.156
228	A	4	4	1.	30	0.133
229	A	4	4	1.	25	0.16
230	A	6	5	1.	32	0.156
231	A	6	5	0.99	32	0.156
232	A	4	4	1.	32	0.125
233	A	5	5	1.	32	0.156
234	A	4	4	1.	32	0.125
235	A	4	4	1.	30	0.133
236	A	4	4	1.	25	0.16
237	A	4	4	1.	32	0.125
238	A	4	4	0.99	32	0.125
239	A	5	5	0.99	32	0.156
240	A	6	5	1.	32	0.156
241	A	5	5	1.	32	0.156
242	A	4	4	1.	30	0.133
243	A	6	6	1.	32	0.188
244	A	6	6	1.	32	0.188
245	A	4	4	1.	32	0.125
246	A	6	5	1.	32	0.156
247	A	5	5	1.	32	0.156
248	A	4	4	1.	30	0.133
249	A	4	4	1.	32	0.125
250	A	4	4	1.	32	0.125
251	A	5	5	1.	32	0.156
252	A	5	4	1.	32	0.125
253	A	5	4	1.	32	0.125
254	A	2	2	1.	30	0.067
255	A	5	5	1.	32	0.156

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	5	4	1.	32	0.125
257	A	6	5	1.	32	0.156
258	A	3	3	1.	47	0.064
259	A	8	7	1.	34	0.206
260	A	7	6	1.	34	0.176
261	A	7	6	1.	34	0.176
262	A	7	6	1.	34	0.176
263	A	7	6	1.	34	0.176
264	A	8	7	1.	34	0.206
265	A	9	7	1.	34	0.206
266	A	8	6	1.	34	0.176
267	A	7	6	1.	34	0.176
268	A	6	5	1.	34	0.147
269	A	6	5	1.	34	0.147
270	A	7	6	0.99	34	0.176
271	A	8	6	1.	34	0.176
272	A	6	4	1.	30	0.133
273	A	6	4	1.	32	0.125
274	A	5	5	1.	34	0.147
275	A	3	3	1.	42	0.071
276	A	2	2	1.	46	0.043
277	A	2	2	1.	69	0.029
278	A	2	2	1.	75	0.027
279	A	6	4	1.	16	0.25
280	A	6	5	1.	33	0.152
281	A	5	4	1.	31	0.129
282	A	5	4	1.	30	0.133
283	A	7	5	1.	33	0.152
284	A	7	6	1.	33	0.182
285	A	7	5	1.	33	0.152
286	A	5	4	1.	33	0.121
287	A	6	5	1.	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	7	5	1.	33	0.152
289	A	2	1	1.	36	0.028
290	A	2	1	1.	36	0.028
291	A	2	1	1.	34	0.029
292	A	2	1	1.	29	0.034
293	A	2	1	1.	36	0.028
294	A	2	1	1.	36	0.028
295	A	2	1	1.	36	0.028
296	A	2	1	1.	38	0.026
297	A	2	1	1.	38	0.026
298	A	2	1	1.	36	0.028
299	A	2	1	1.	31	0.032
300	A	2	1	1.	38	0.026
301	A	2	1	1.	38	0.026
302	A	2	1	1.	38	0.026
303	A	2	1	1.	38	0.026
304	A	6	5	1.	38	0.132
305	A	6	5	1.	38	0.132
306	A	6	5	1.	36	0.139
307	A	6	5	1.	31	0.161
308	A	6	5	1.	38	0.132
309	A	6	5	1.	38	0.132
310	A	6	5	1.	38	0.132
311	A	7	6	1.	38	0.158
312	A	7	6	1.	38	0.158
313	A	7	6	1.	36	0.167
314	A	7	6	1.	31	0.194
315	A	7	6	1.	38	0.158
316	A	7	6	1.	38	0.158
317	A	7	6	1.	38	0.158
318	A	8	6	1.	38	0.158
319	A	8	6	1.	38	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
320	A	6	6	1.	36	0.167
321	A	5	4	1.	31	0.129
322	A	8	6	1.	38	0.158
323	A	8	6	1.	38	0.158
324	A	7	5	1.	38	0.132
325	A	7	5	1.	33	0.152
326	A	9	7	1.	40	0.175
327	A	9	8	1.	40	0.2
328	A	9	8	1.	40	0.2
329	A	9	7	1.	40	0.175
330	A	9	7	1.	40	0.175
331	A	9	7	1.	40	0.175
332	A	7	5	1.	40	0.125
333	A	8	6	1.	40	0.15
334	A	8	5	1.	38	0.132
335	A	8	5	1.	33	0.152
336	A	10	7	1.	40	0.175
337	A	10	8	1.	40	0.2
338	A	10	8	1.	40	0.2
339	A	10	7	1.	40	0.175
340	A	10	8	1.	40	0.2
341	A	10	7	1.	40	0.175
342	A	10	8	1.	40	0.2
343	A	10	7	1.	40	0.175
344	A	6	4	1.	38	0.105
345	A	6	4	1.	33	0.121
346	A	8	6	1.	40	0.15
347	A	8	7	1.	40	0.175
348	A	8	7	1.	40	0.175
349	A	8	6	1.	40	0.15
350	A	6	4	1.	40	0.1
351	A	7	5	1.	40	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	6	5	1.	38	0.132
353	A	5	5	1.	33	0.152
354	A	7	7	1.	40	0.175
355	A	7	7	1.	40	0.175
356	A	5	5	1.	40	0.125
357	A	6	5	1.	40	0.125
358	A	6	5	1.	40	0.125
359	A	5	4	1.	38	0.105
360	A	5	4	1.	33	0.121
361	A	5	4	1.	40	0.1
362	A	5	4	1.	40	0.1
363	A	6	5	1.	40	0.125
364	A	7	5	1.	40	0.125
365	A	5	4	1.	35	0.114
366	A	5	4	1.	36	0.111
367	A	2	1	1.	38	0.026
368	A	2	1	1.	38	0.026
369	A	2	1	1.	36	0.028
370	A	4	2	1.	38	0.053
371	A	5	3	1.	38	0.079
372	A	6	5	1.	38	0.132
373	A	6	5	1.	53	0.094
374	A	11	5	1.	35	0.143
375	A	9	5	1.	35	0.143
376	A	7	5	1.	33	0.152
377	A	9	7	1.	35	0.2
378	A	9	7	1.	35	0.2
379	A	7	5	1.	35	0.143
380	A	12	5	1.	35	0.143
381	A	10	5	1.	35	0.143
382	A	8	5	1.	33	0.152
383	A	10	7	1.	35	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
384	A	10	8	1.	35	0.229
385	A	10	7	1.	35	0.2
386	A	10	4	1.	35	0.114
387	A	8	4	1.	35	0.114
388	A	6	4	1.	33	0.121
389	A	8	6	1.	35	0.171
390	A	6	4	1.	35	0.114
391	A	7	4	1.	35	0.114
392	A	9	5	1.	35	0.143
393	A	7	5	1.	35	0.143
394	A	5	5	1.	33	0.152
395	A	6	4	1.	35	0.114
396	A	7	4	1.	35	0.114
397	A	8	4	1.	35	0.114
398	A	7	5	1.	26	0.192
399	A	7	6	1.	24	0.25
400	A	11	8	1.	29	0.276

Chapter 3

Listing of integrals

3.1 $\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=236

$$\frac{d^2x\sqrt{d^2-e^2x^2}\left(10Ae^2+4Bde+3Cd^2\right)}{16e^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7Bd)+4Cd^2\right)}{15e^3}-\frac{x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+3\right.}{8e^2}$$

```
[Out] (d^2*(3*C*d^2 + 4*B*d*e + 10*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) - (d*(4
*C*d^2 + e*(7*B*d + 10*A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((3*C*d^2 +
2*e*(2*B*d + A*e))*x*(d^2 - e^2*x^2)^(3/2))/(8*e^2) - ((2*C*d + B*e)*x^2*(d
^2 - e^2*x^2)^(3/2))/(5*e) - (C*x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^4*(3*C*d^
2 + 4*B*d*e + 10*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rubi [A] time = 0.469868, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1815, 641, 195, 217, 203}

$$\frac{d^2x\sqrt{d^2-e^2x^2}\left(10Ae^2+4Bde+3Cd^2\right)}{16e^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7Bd)+4Cd^2\right)}{15e^3}-\frac{x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+3\right.}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^2*(3*C*d^2 + 4*B*d*e + 10*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) - (d*(4
*C*d^2 + e*(7*B*d + 10*A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((3*C*d^2 +
```

```
2*e*(2*B*d + A*e))*x*(d^2 - e^2*x^2)^(3/2))/(8*e^2) - ((2*C*d + B*e)*x^2*(d
^2 - e^2*x^2)^(3/2))/(5*e) - (C*x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^4*(3*C*d^
2 + 4*B*d*e + 10*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \sqrt{d^2-e^2x^2}\left(-6Ad^2e^2-6de^2(Bd+2Ae)x-3e^2\right.}{6e}$$

$$= -\frac{(2Cd+Be)x^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \sqrt{d^2-e^2x^2}\,(30A}{}$$

$$= -\frac{\left(3Cd^2+2e(2Bd+Ae)\right)x\left(d^2-e^2x^2\right)^{3/2}}{8e^2} - \frac{(2Cd+Be)x^2\left(d^2-e^2x^2\right)^{3/2}}{5e} -$$

$$= -\frac{d\left(4Cd^2+e(7Bd+10Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} - \frac{\left(3Cd^2+2e(2Bd+Ae)\right)x\left(d^2\right.}{8e^2}$$

$$= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d\left(4Cd^2+e(7Bd+10Ae)\right)\left(d^2-\right.}{15e^3}$$

$$= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d\left(4Cd^2+e(7Bd+10Ae)\right)\left(d^2-\right.}{15e^3}$$

$$= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d\left(4Cd^2+e(7Bd+10Ae)\right)\left(d^2-\right.}{15e^3}$$

Mathematica [A] time = 0.51806, size = 226, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(2e\left(5Ae\left(9d^2ex-16d^3+16de^2x^2+6e^3x^3\right)+B\left(32d^2e^2x^2-30d^3ex-56d^4+60de^3x^3+24e^4x^4\right)\right)\right.\right.}{240e^3\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(C*(-64*d^5 - 45*d^4*e*x - 32
*d^3*e^2*x^2 + 50*d^2*e^3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5) + 2*e*(5*A*e*(-1
6*d^3 + 9*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3) + B*(-56*d^4 - 30*d^3*e*x + 3
2*d^2*e^2*x^2 + 60*d*e^3*x^3 + 24*e^4*x^4))) + 15*(3*C*d^5 + 2*d^3*e*(2*B*d
 + 5*A*e))*ArcSin[(e*x)/d]))/(240*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.087, size = 371, normalized size = 1.6

$$-\frac{Cx^3}{6}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} - \frac{3\,Cd^2x}{8\,e^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} + \frac{3\,Cd^4x}{16\,e^2}\sqrt{-x^2e^2+d^2} + \frac{3\,Cd^6}{16\,e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{x^2B}{5}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/6*C*x^3*(-e^2*x^2+d^2)^(3/2)-3/8/e^2*C*d^2*x*(-e^2*x^2+d^2)^(3/2)+3/16/e
^2*C*d^4*x*(-e^2*x^2+d^2)^(1/2)+3/16/e^2*C*d^6/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2))-1/5*x^2*(-e^2*x^2+d^2)^(3/2)*B-2/5*x^2*(-e^2*x^2
+d^2)^(3/2)/e*d*C-7/15*d^2/e^2*(-e^2*x^2+d^2)^(3/2)*B-4/15*d^3/e^3*(-e^2*x^
2+d^2)^(3/2)*C-1/4*x*(-e^2*x^2+d^2)^(3/2)*A-1/2*x*(-e^2*x^2+d^2)^(3/2)/e*B*
d+5/8*d^2*x*(-e^2*x^2+d^2)^(1/2)*A+1/4*d^3/e*x*(-e^2*x^2+d^2)^(1/2)*B+5/8*d
^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*A+1/4*d^5/e/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*B-2/3*(-e^2*x^2+d^2)^(3/2
)/e*A*d
```

Maxima [A] time = 1.51519, size = 505, normalized size = 2.14

$$-\frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^3+\frac{Ad^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\sqrt{e^2}}+\frac{Cd^6\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{16\sqrt{e^2}e^2}+\frac{1}{2}\sqrt{-e^2x^2+d^2}Ad^2x+\frac{\sqrt{-e^2x^2+d^2}Cd^4x}{16\,e^2}-\frac{\left(-e^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*(-e^2*x^2 + d^2)^(3/2)*C*x^3 + 1/2*A*d^4*arcsin(e^2*x/sqrt(d^2*e^2))/s
qrt(e^2) + 1/16*C*d^6*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) + 1/2*sqr
t(-e^2*x^2 + d^2)*A*d^2*x + 1/16*sqrt(-e^2*x^2 + d^2)*C*d^4*x/e^2 - 1/8*(-e
^2*x^2 + d^2)^(3/2)*C*d^2*x/e^2 + 1/8*(C*d^2 + 2*B*d*e + A*e^2)*d^4*arcsin(
e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) - 1/3*(-e^2*x^2 + d^2)^(3/2)*B*d^2/e^2
 - 2/3*(-e^2*x^2 + d^2)^(3/2)*A*d/e + 1/8*sqrt(-e^2*x^2 + d^2)*(C*d^2 + 2*B
*d*e + A*e^2)*d^2*x/e^2 - 1/5*(-e^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^2)*x^2/
e^2 - 1/4*(-e^2*x^2 + d^2)^(3/2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/15*(-e
^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^2)*d^2/e^4
```

Fricas [A] time = 2.39827, size = 464, normalized size = 1.97

$$-\frac{30\left(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(40\,Ce^5x^5-64\,Cd^5-112\,Bd^4e-160\,Ad^3e^2+48\left(2\,Cde^4+Be\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*(30*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arctan(-(d - sqrt(-e^2*x^2
+ d^2))/(e*x)) - (40*C*e^5*x^5 - 64*C*d^5 - 112*B*d^4*e - 160*A*d^3*e^2 + 4
8*(2*C*d*e^4 + B*e^5)*x^4 + 10*(5*C*d^2*e^3 + 12*B*d*e^4 + 6*A*e^5)*x^3 - 3
2*(C*d^3*e^2 - 2*B*d^2*e^3 - 5*A*d*e^4)*x^2 - 15*(3*C*d^4*e + 4*B*d^3*e^2 -
 6*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [C] time = 21.467, size = 1239, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d
**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
 + 2*A*d*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)
**(3/2)/(3*e**2), True)) + A*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3)
+ I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**
2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)
/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*
x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1
 - e**2*x**2/d**2)), True)) + B*d**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2,
 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) + 2*B*d*e*Piecewise((-I*
d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) -
3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*
x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d*
*3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**
2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + B*e**2*Piecewise((
-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)
/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4,
True)) + C*d**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2
*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*
e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (
d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*
x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)
```

```
), True)) + 2*C*d*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d
**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5,
Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + C*e**2*Piecewise((-I*d**6*acosh(e*x
/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/
(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d*
*2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2
) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d*
*2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.15015, size = 266, normalized size = 1.13

$$\frac{1}{16}\left(3\,Cd^6 + 4\,Bd^5e + 10\,Ad^4e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}\left(d\right) + \frac{1}{240}\,\sqrt{-x^2e^2 + d^2}\left(\left(2\left(\left(4\left(5\,Cxe^2 + 6\left(2\,Cde^9 + Be^{10}\right)e^{(-8)}\right)x + 5\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/2
40*sqrt(-x^2*e^2 + d^2)*((2*((4*(5*C*x*e^2 + 6*(2*C*d*e^9 + B*e^10)*e^(-8))
*x + 5*(5*C*d^2*e^8 + 12*B*d*e^9 + 6*A*e^10)*e^(-8))*x - 16*(C*d^3*e^7 - 2*
B*d^2*e^8 - 5*A*d*e^9)*e^(-8))*x - 15*(3*C*d^4*e^6 + 4*B*d^3*e^7 - 6*A*d^2*
e^8)*e^(-8))*x - 16*(4*C*d^5*e^5 + 7*B*d^4*e^6 + 10*A*d^3*e^7)*e^(-8))
```

3.2 $\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=186

$$\frac{dx\sqrt{d^2-e^2x^2}\left(e(4Ae+Bd)+Cd^2\right)}{8e^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{15e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(4Ae+Bd)+Cd^2\right)}{8e^3}$$

```
[Out] (d*(C*d^2 + e*(B*d + 4*A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d^2 + 5
*e*(B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((C*d + B*e)*x*(d^2 - e^2
*x^2)^(3/2))/(4*e^2) - (C*x^2*(d^2 - e^2*x^2)^(3/2))/(5*e) + (d^3*(C*d^2 +
e*(B*d + 4*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rubi [A] time = 0.227334, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1815, 641, 195, 217, 203}

$$\frac{dx\sqrt{d^2-e^2x^2}\left(e(4Ae+Bd)+Cd^2\right)}{8e^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{15e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(4Ae+Bd)+Cd^2\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d*(C*d^2 + e*(B*d + 4*A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d^2 + 5
*e*(B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((C*d + B*e)*x*(d^2 - e^2
*x^2)^(3/2))/(4*e^2) - (C*x^2*(d^2 - e^2*x^2)^(3/2))/(5*e) + (d^3*(C*d^2 +
e*(B*d + 4*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{\int \sqrt{d^2-e^2x^2}\left(-5Ade^2-e\left(2Cd^2+5e(Bd+Ae)\right)x-5\right.}{5e^2}$$

$$= -\frac{(Cd+Be)x\left(d^2-e^2x^2\right)^{3/2}}{4e^2} - \frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} + \frac{\int \left(5de^2\left(Cd^2+e(Bd+4A\right.\right.}{}$$

$$= -\frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} - \frac{(Cd+Be)x\left(d^2-e^2x^2\right)^{3/2}}{4e^2} - \frac{Cx^2\left(d^2\right.}{}$$

$$= \frac{d\left(Cd^2+e(Bd+4Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} -$$

$$= \frac{d\left(Cd^2+e(Bd+4Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} -$$

$$= \frac{d\left(Cd^2+e(Bd+4Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} -$$

Mathematica [A] time = 0.308719, size = 190, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(5e\left(4Ae\left(-2d^2+3dex+2e^2x^2\right)+B\left(-3d^2ex-8d^3+8de^2x^2+6e^3x^3\right)\right)+C\left(-8d^2e^2x^2-15d^3ex\right.\right.\right.}{120e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(C*(-16*d^4 - 15*d^3*e*x - 8*
d^2*e^2*x^2 + 30*d*e^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(-2*d^2 + 3*d*e*x + 2
*e^2*x^2) + B*(-8*d^3 - 3*d^2*e*x + 8*d*e^2*x^2 + 6*e^3*x^3))) + 15*(C*d^4
+ d^2*e*(B*d + 4*A*e))*ArcSin[(e*x)/d]))/(120*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.059, size = 304, normalized size = 1.6

$$-\frac{Cx^2}{5e}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{2Cd^2}{15e^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{Bx}{4e}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{Cdx}{4e^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{d^2xB}{8e}\sqrt{-x^2e^2+d^2}+\frac{Cd^3x}{8e^2}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*C*x^2*(-e^2*x^2+d^2)^(3/2)/e-2/15/e^3*C*d^2*(-e^2*x^2+d^2)^(3/2)-1/4*x
*(-e^2*x^2+d^2)^(3/2)/e*B-1/4*x*(-e^2*x^2+d^2)^(3/2)/e^2*C*d+1/8*d^2/e*x*(-
e^2*x^2+d^2)^(1/2)*B+1/8*d^3/e^2*x*(-e^2*x^2+d^2)^(1/2)*C+1/8*d^4/e/(e^2)^(
1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*B+1/8*d^5/e^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*C-1/3*(-e^2*x^2+d^2)^(3/2)/e*A-1/
3*(-e^2*x^2+d^2)^(3/2)/e^2*B*d+1/2*A*d*x*(-e^2*x^2+d^2)^(1/2)+1/2*A*d^3/(e^
2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.78755, size = 304, normalized size = 1.63

$$\frac{Ad^3\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\sqrt{e^2}}+\frac{1}{2}\sqrt{-e^2x^2+d^2}Adx-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^2}{5e}+\frac{(Cd+Be)d^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2}e^2}+\frac{\sqrt{-e^2x^2+d^2}(Cd+Be)d}{8e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*d^3*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) + 1/2*sqrt(-e^2*x^2 + d^2)*
A*d*x - 1/5*(-e^2*x^2 + d^2)^(3/2)*C*x^2/e + 1/8*(C*d + B*e)*d^4*arcsin(e^2
*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) + 1/8*sqrt(-e^2*x^2 + d^2)*(C*d + B*e)*d^
2*x/e^2 - 2/15*(-e^2*x^2 + d^2)^(3/2)*C*d^2/e^3 - 1/3*(-e^2*x^2 + d^2)^(3/2
)*B*d/e^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)*A/e - 1/4*(-e^2*x^2 + d^2)^(3/2)*(C*
d + B*e)*x/e^2
```

Fricas [A] time = 2.51104, size = 375, normalized size = 2.02

$$-\frac{30\left(Cd^5+Bd^4e+4\,Ad^3e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,Ce^4x^4-16\,Cd^4-40\,Bd^3e-40\,Ad^2e^2+30\left(Cde^3+Be^4\right)x^3-8\right.}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (24*C*e^4*x^4 - 16*C*d^4 - 40*B*d^3*e - 40*A*d^2*e^2 + 30*(C*d*
e^3 + B*e^4)*x^3 - 8*(C*d^2*e^2 - 5*B*d*e^3 - 5*A*e^4)*x^2 - 15*(C*d^3*e +
B*d^2*e^2 - 4*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [C] time = 11.4995, size = 675, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d*
*2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2
) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) +
A*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)
/(3*e**2), True)) + B*d*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2
 - e**2*x**2)**(3/2)/(3*e**2), True)) + B*e*Piecewise((-I*d**4*acosh(e*x/d)
/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqr
```

```
t(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(
e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt
(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(
4*d*sqrt(1 - e**2*x**2/d**2)), True)) + C*d*Piecewise((-I*d**4*acosh(e*x/d)
/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqr
t(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(
e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt
(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(
4*d*sqrt(1 - e**2*x**2/d**2)), True)) + C*e*Piecewise((-2*d**4*sqrt(d**2 -
e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sq
rt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

Giac [A] time = 1.18175, size = 216, normalized size = 1.16

$$\frac{1}{8}\left(Cd^5 + Bd^4e + 4\,Ad^3e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}\left(d\right) + \frac{1}{120}\,\sqrt{-x^2e^2 + d^2}\left(\left(2\left(3\left(4\,Cxe + 5\left(Cde^6 + Be^7\right)e^{(-6)}\right)x - 4\left(Cd^2e^5\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/120*sqr
t(-x^2*e^2 + d^2)*((2*(3*(4*C*x*e + 5*(C*d*e^6 + B*e^7)*e^(-6))*x - 4*(C*d^
2*e^5 - 5*B*d*e^6 - 5*A*e^7)*e^(-6))*x - 15*(C*d^3*e^4 + B*d^2*e^5 - 4*A*d*
e^6)*e^(-6))*x - 8*(2*C*d^4*e^3 + 5*B*d^3*e^4 + 5*A*d^2*e^5)*e^(-6))
```

3.3 $\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=125

$$\frac{1}{8}x\sqrt{d^2-e^2x^2}\left(4A+\frac{Cd^2}{e^2}\right)+\frac{d^2\left(4Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

```
[Out] ((4*A + (C*d^2)/e^2)*x*Sqrt[d^2 - e^2*x^2])/8 - (B*(d^2 - e^2*x^2)^(3/2))/(
3*e^2) - (C*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2*(C*d^2 + 4*A*e^2)*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rubi [A] time = 0.068973, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1815, 641, 195, 217, 203}

$$\frac{1}{8}x\sqrt{d^2-e^2x^2}\left(4A+\frac{Cd^2}{e^2}\right)+\frac{d^2\left(4Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] ((4*A + (C*d^2)/e^2)*x*Sqrt[d^2 - e^2*x^2])/8 - (B*(d^2 - e^2*x^2)^(3/2))/(
3*e^2) - (C*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2*(C*d^2 + 4*A*e^2)*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\int\left(-Cd^2-4Ae^2-4Be^2x\right)\sqrt{d^2-e^2x^2}\,dx}{4e^2}$$

$$= -\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\left(-Cd^2-4Ae^2\right)\int\sqrt{d^2-e^2x^2}\,dx}{4e^2}$$

$$= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\left(d^2\left(-Cd^2-4\right.\right.}{}$$

$$= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\left(d^2\left(-Cd^2-4\right.\right.}{}$$

$$= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{d^2\left(Cd^2+4Ae\right.}{}$$

Mathematica [A] time = 0.143962, size = 121, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\left(12Ae^2x-8Bd^2+8Be^2x^2-3Cd^2x+6Ce^2x^3\right)+3\left(4Ade^2+Cd^3\right)\sin^{-1}\left(\frac{ex}{d}\right)\right)}{24e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(e*Sqrt[1 - (e^2*x^2)/d^2]*(-8*B*d^2 - 3*C*d^2*x + 12*
A*e^2*x + 8*B*e^2*x^2 + 6*C*e^2*x^3) + 3*(C*d^3 + 4*A*d*e^2)*ArcSin[(e*x)/d
]))/(24*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.051, size = 154, normalized size = 1.2

$$-\frac{Cx}{4e^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{Cd^2x}{8e^2}\sqrt{-x^2e^2+d^2}+\frac{Cd^4}{8e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{B}{3e^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{Ax}{2}\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*C*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/8*C*d^2/e^2*x*(-e^2*x^2+d^2)^(1/2)+1/8*
C*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/3*B*(-e^
2*x^2+d^2)^(3/2)/e^2+1/2*A*x*(-e^2*x^2+d^2)^(1/2)+1/2*A*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.52992, size = 188, normalized size = 1.5

$$\frac{Ad^2\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\sqrt{e^2}}+\frac{Cd^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2}e^2}+\frac{1}{2}\sqrt{-e^2x^2+d^2}Ax+\frac{\sqrt{-e^2x^2+d^2}Cd^2x}{8e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx}{4e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) + 1/8*C*d^4*arcsin(e^2*x/sq
rt(d^2*e^2))/(sqrt(e^2)*e^2) + 1/2*sqrt(-e^2*x^2 + d^2)*A*x + 1/8*sqrt(-e^2
*x^2 + d^2)*C*d^2*x/e^2 - 1/4*(-e^2*x^2 + d^2)^(3/2)*C*x/e^2 - 1/3*(-e^2*x^
2 + d^2)^(3/2)*B/e^2
```

Fricas [A] time = 2.0362, size = 227, normalized size = 1.82

$$-\frac{6\left(Cd^4+4\,Ad^2e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(6\,Ce^3x^3+8\,Be^3x^2-8\,Bd^2e-3\left(Cd^2e-4\,Ae^3\right)x\right)\sqrt{-e^2x^2+d^2}}{24\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*(C*d^4 + 4*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) -
(6*C*e^3*x^3 + 8*B*e^3*x^2 - 8*B*d^2*e - 3*(C*d^2*e - 4*A*e^3)*x)*sqrt(-e^2
*x^2 + d^2))/e^3
```

Sympy [C] time = 6.47671, size = 347, normalized size = 2.78

$$A\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)+B\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+C\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+ \cdots\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3 \cdots}{8e^2\sqrt{1- \cdots}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2
)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2)
> 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + B*
Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*
e**2), True)) + C*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e*
*2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) +
I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1),
 (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*
d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**
2)), True))
```

Giac [A] time = 1.14138, size = 115, normalized size = 0.92

$$\frac{1}{8}\left(Cd^4+4\,Ad^2e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d)-\frac{1}{24}\left(8\,Bd^2e^{(-2)}-\left(2\,(3\,Cx+4\,B)x-3\left(Cd^2e^2-4\,Ae^4\right)e^{(-4)}\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(C*d^4 + 4*A*d^2*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24*(8*B*d^2*e^(-2
) - (2*(3*C*x + 4*B)*x - 3*(C*d^2*e^2 - 4*A*e^4)*e^(-4))*x)*sqrt(-x^2*e^2 +
 d^2)
```

3.4 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{d^2-e^2x^2}\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{\left(d^2-e^2x^2\right)^{3/2}(Cd-Be)}{2e^3(d+ex)}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

```
[Out] ((C*d^2 - e*(B*d - 2*A*e))*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (C*(d^2 - e^2*x^2
)^(3/2))/(3*e^3) + ((C*d - B*e)*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)) +
(d*(C*d^2 - e*(B*d - 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rubi [A] time = 0.1763, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1639, 795, 665, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{\left(d^2-e^2x^2\right)^{3/2}(Cd-Be)}{2e^3(d+ex)}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] ((C*d^2 - e*(B*d - 2*A*e))*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (C*(d^2 - e^2*x^2
)^(3/2))/(3*e^3) + ((C*d - B*e)*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)) +
(d*(C*d^2 - e*(B*d - 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{\int \frac{\left(-3Ae^4+3e^3(Cd-Be)x\right)\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}$$

$$= -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{(Cd-Be)\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)}+\frac{\left(Cd^2-e(Bd-2Ae)\right)\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{2e^2}$$

$$= \frac{\left(Cd^2-e(Bd-2Ae)\right)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{(Cd-Be)\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)}+\frac{(d}{}$$

$$= \frac{\left(Cd^2-e(Bd-2Ae)\right)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{(Cd-Be)\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)}+\frac{(d}{}$$

$$= \frac{\left(Cd^2-e(Bd-2Ae)\right)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{(Cd-Be)\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)}+\frac{d(}{}$$

Mathematica [A] time = 0.226047, size = 103, normalized size = 0.7

$$\frac{\sqrt{d^2-e^2x^2}\left(3e(2Ae-2Bd+Bex)+C\left(4d^2-3dex+2e^2x^2\right)\right)+3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae-Bd)+Cd^2\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*e*(-2*B*d + 2*A*e + B*e*x) + C*(4*d^2 - 3*d*e*x + 2
*e^2*x^2)) + 3*d*(C*d^2 + e*(-(B*d) + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x
^2]])/(6*e^3)
```

Maple [B] time = 0.058, size = 384, normalized size = 2.6

$$-\frac{C}{3e^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{Bx}{2e}\sqrt{-x^2e^2+d^2}+\frac{Bd^2}{2e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{Cdx}{2e^2}\sqrt{-x^2e^2+d^2}-\frac{Cd^3}{2e^2}\arctan\left(x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/3*C*(-e^2*x^2+d^2)^(3/2)/e^3+1/2/e*B*x*(-e^2*x^2+d^2)^(1/2)+1/2/e*B*d^2/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2/e^2*C*d*x*(-e^2*
x^2+d^2)^(1/2)-1/2/e^2*C*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))+1/e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*A-1/e^2*(-(d/e+x)^2*e^2+2
*d*e*(d/e+x))^(1/2)*B*d+1/e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*C*d^2+d/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))*A-1/
e*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
)*B+1/e^2*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x
))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87252, size = 236, normalized size = 1.59

$$-\frac{6\left(Cd^3-Bd^2e+2\,Ade^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(2\,Ce^2x^2+4\,Cd^2-6\,Bde+6\,Ae^2-3\left(Cde-Be^2\right)x\right)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 - B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e
*x)) - (2*C*e^2*x^2 + 4*C*d^2 - 6*B*d*e + 6*A*e^2 - 3*(C*d*e - B*e^2)*x)*sq
rt(-e^2*x^2 + d^2))/e^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(A+Bx+Cx^2\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.5 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{de^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\left(5Cd^2-2e(2Bd-Ae)\right)}{2de^3}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(5Cd^2-2e(2Bd-Ae)\right)}{2e^3}-$$

```
[Out] -((5*C*d^2 - 2*e*(2*B*d - A*e))*Sqrt[d^2 - e^2*x^2])/(2*d*e^3) - ((C*d^2 -
B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^2) - (C*(d^2 - e^2*x
^2)^(3/2))/(2*e^3*(d + e*x)) - ((5*C*d^2 - 2*e*(2*B*d - A*e))*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rubi [A] time = 0.202595, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1639, 793, 665, 217, 203}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{de^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\left(5Cd^2-2e(2Bd-Ae)\right)}{2de^3}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(5Cd^2-2e(2Bd-Ae)\right)}{2e^3}-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] -((5*C*d^2 - 2*e*(2*B*d - A*e))*Sqrt[d^2 - e^2*x^2])/(2*d*e^3) - ((C*d^2 -
B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^2) - (C*(d^2 - e^2*x
^2)^(3/2))/(2*e^3*(d + e*x)) - ((5*C*d^2 - 2*e*(2*B*d - A*e))*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = -\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} - \frac{\int \frac{\left(e^2(Cd^2-2Ae^2)+e^3(3Cd-2Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{2e^4}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} - \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} - \frac{\left(-3e^5(Cd^2-2Ae^2)-2(\cdots\right.}{\cdots}$$

$$= -\frac{\left(5Cd^2-2e(2Bd-Ae)\right)\sqrt{d^2-e^2x^2}}{2de^3} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} - \frac{C(d^2\cdots}{2e^3(\cdots}$$

$$= -\frac{\left(5Cd^2-2e(2Bd-Ae)\right)\sqrt{d^2-e^2x^2}}{2de^3} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} - \frac{C(d^2\cdots}{2e^3(\cdots}$$

$$= -\frac{\left(5Cd^2-2e(2Bd-Ae)\right)\sqrt{d^2-e^2x^2}}{2de^3} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} - \frac{C(d^2\cdots}{2e^3(\cdots}$$

Mathematica [A] time = 0.242847, size = 109, normalized size = 0.64

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(2e(-2Ae+3Bd+Bex)+C(-8d^2-3dex+e^2x^2)\right)}{d+ex} - \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(2e(Ae-2Bd)+5Cd^2\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(2*e*(3*B*d - 2*A*e + B*e*x) + C*(-8*d^2 - 3*d*e*x +
e^2*x^2)))/(d + e*x) - (5*C*d^2 + 2*e*(-2*B*d + A*e))*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(2*e^3)
```

Maple [B] time = 0.06, size = 439, normalized size = 2.6

$$\frac{Cx}{2e^2}\sqrt{-x^2e^2+d^2} + \frac{Cd^2}{2e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}} + 2\frac{B}{e^2}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)} - 3\frac{dC}{e^3}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x)
```

```
[Out] 1/2*C*x*(-e^2*x^2+d^2)^(1/2)/e^2+1/2*C/e^2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2))+2/e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*B-3/e
^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*d*C+2/e*d/(e^2)^(1/2)*arctan((e^2)^
(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))*B-3/e^2*d^2/(e^2)^(1/2)*arcta
n((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))*C-1/e^3/d/(d/e+x)^2*(
-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*A+1/e^4/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e
*(d/e+x))^(3/2)*B-1/e^5*d/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*C-
1/e/d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*A-1/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88055, size = 409, normalized size = 2.41

$$-\frac{8\,Cd^3-6\,Bd^2e+4\,Ade^2+2\left(4\,Cd^2e-3\,Bde^2+2\,Ae^3\right)x-2\left(5\,Cd^3-4\,Bd^2e+2\,Ade^2+\left(5\,Cd^2e-4\,Bde^2+2\,Ae^3\right)x\right)\text{ar}}{2\left(e^4x+de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(8*C*d^3 - 6*B*d^2*e + 4*A*d*e^2 + 2*(4*C*d^2*e - 3*B*d*e^2 + 2*A*e^3)
*x - 2*(5*C*d^3 - 4*B*d^2*e + 2*A*d*e^2 + (5*C*d^2*e - 4*B*d*e^2 + 2*A*e^3)
*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (C*e^2*x^2 - 8*C*d^2 + 6*B*
d*e - 4*A*e^2 - (3*C*d*e - 2*B*e^2)*x)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d*e^3
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}\left(A+Bx+Cx^2\right)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.6 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{3de^3(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}(3Cd-Be)}{e^3(d+ex)}+\frac{(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2}$$

```
[Out] (2*(3*C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A
*e^2)*(d^2 - e^2*x^2)^(3/2))/(3*d*e^3*(d + e*x)^3) - (C*(d^2 - e^2*x^2)^(3/
2))/(e^3*(d + e*x)^2) + ((3*C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
^3
```

Rubi [A] time = 0.188542, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1639, 793, 663, 217, 203}

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{3de^3(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}(3Cd-Be)}{e^3(d+ex)}+\frac{(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```


```
[Out] (2*(3*C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A
*e^2)*(d^2 - e^2*x^2)^(3/2))/(3*d*e^3*(d + e*x)^3) - (C*(d^2 - e^2*x^2)^(3/
2))/(e^3*(d + e*x)^2) + ((3*C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
^3
```


Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}dx = -\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2} - \frac{\int \frac{(e^2(2Cd^2-Ae^2)+e^3(3Cd-Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^3}dx}{e^4}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} - \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2} - \frac{(3Cd-Be)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}dx}{e^2}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} - \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2} + \frac{(3\cdots}{\cdots}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} - \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2} + \frac{(3\cdots}{\cdots}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} - \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2} + \frac{(3\cdots}{\cdots}$$

Mathematica [A] time = 0.23575, size = 114, normalized size = 0.77

$$\frac{\frac{\sqrt{d^2-e^2x^2}(e(Ae(ex-d)-Bd(5d+7ex))+Cd(14d^2+19dex+3e^2x^2))}{d(d+ex)^2} + 3(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(C*d*(14*d^2 + 19*d*e*x + 3*e^2*x^2) + e*(A*e*(-d + e
*x) - B*d*(5*d + 7*e*x))))/(d*(d + e*x)^2) + 3*(3*C*d - B*e)*ArcTan[(e*x)/S
qrt[d^2 - e^2*x^2]])/(3*e^3)
```

Maple [B] time = 0.062, size = 318, normalized size = 2.1

$$3\frac{C}{e^3}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)} + 3\frac{Cd}{e^2\sqrt{e^2}}\arctan\left(\sqrt{e^2}x\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}}\right) - \frac{B}{e^4d}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x)
```

```
[Out] 3*C/e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+3*C/e^2*d/(e^2)^(1/2)*arctan((
e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1/e^4/d/(d/e+x)^2*(-(d/e
+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*B+2/e^5/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(3/2)*C-1/e^2/d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*B-1/e/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))*B-1/3*(A*e^2-B*
d*e+C*d^2)/e^6/d/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82373, size = 533, normalized size = 3.58

$$\frac{14\,Cd^4 - 5\,Bd^3e - Ad^2e^2 + \left(14\,Cd^2e^2 - 5\,Bde^3 - Ae^4\right)x^2 + 2\left(14\,Cd^3e - 5\,Bd^2e^2 - Ade^3\right)x - 6\left(3\,Cd^4 - Bd^3e + \left(3\,Cd^2e^2\right.\right.}{3\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="fricas
")
```

```
[Out] 1/3*(14*C*d^4 - 5*B*d^3*e - A*d^2*e^2 + (14*C*d^2*e^2 - 5*B*d*e^3 - A*e^4)*
x^2 + 2*(14*C*d^3*e - 5*B*d^2*e^2 - A*d*e^3)*x - 6*(3*C*d^4 - B*d^3*e + (3*
C*d^2*e^2 - B*d*e^3)*x^2 + 2*(3*C*d^3*e - B*d^2*e^2)*x)*arctan(-(d - sqrt(-
e^2*x^2 + d^2))/(e*x)) + (3*C*d*e^2*x^2 + 14*C*d^3 - 5*B*d^2*e - A*d*e^2 +
(19*C*d^2*e - 7*B*d*e^2 + A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^2 + 2*d^
2*e^4*x + d^3*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}\left(A+Bx+Cx^2\right)}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.7 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=196

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{15d^2e^3(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^4}+\frac{\left(d^2-e^2x^2\right)^{3/2}(2Cd-Be)}{3de^3(d+ex)^3}-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-$$

```
[Out] (-2*C*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2)*(d^2
- e^2*x^2)^(3/2))/(5*d*e^3*(d + e*x)^4) + ((2*C*d - B*e)*(d^2 - e^2*x^2)^(3
/2))/(3*d*e^3*(d + e*x)^3) - ((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2)
)/(15*d^2*e^3*(d + e*x)^3) - (C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rubi [A] time = 0.183946, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1637, 659, 651, 663, 217, 203}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{15d^2e^3(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^4}+\frac{\left(d^2-e^2x^2\right)^{3/2}(2Cd-Be)}{3de^3(d+ex)^3}-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*C*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2)*(d^2
- e^2*x^2)^(3/2))/(5*d*e^3*(d + e*x)^4) + ((2*C*d - B*e)*(d^2 - e^2*x^2)^(3
/2))/(3*d*e^3*(d + e*x)^3) - ((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2)
)/(15*d^2*e^3*(d + e*x)^3) - (C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
```

```
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \left(\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4} + \frac{(-2Cd+Be)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3} + \frac{C\sqrt{d^2-e^2x^2}}{e^2(d+ex)^2}\right) dx$$

$$= \frac{C\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2} - \frac{(2Cd-Be)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2} + \frac{(Cd^2-Bde+Ae^2)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^2}$$

$$= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-Be)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} -$$

$$= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-Be)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} -$$

$$= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-Be)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} -$$

Mathematica [A] time = 0.295722, size = 112, normalized size = 0.57

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(e(d-ex)(Ae(4d+ex)+Bd(d+4ex))+3Cd^2\left(8d^2+19dex+13e^2x^2\right)\right)}{d^2(d+ex)^3}+15C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(3*C*d^2*(8*d^2 + 19*d*e*x + 13*e^2*x^2) + e*(d - e*
x)*(A*e*(4*d + e*x) + B*d*(d + 4*e*x))))/(d^2*(d + e*x)^3) + 15*C*ArcTan[(e
*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^3)
```

Maple [B] time = 0.06, size = 453, normalized size = 2.3

$$-\frac{A}{5e^5d}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}\left(\frac{d}{e}+x\right)^{-4}+\frac{B}{5e^6}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}\left(\frac{d}{e}+x\right)^{-4}-\frac{Cd}{5e^7}\left(-\left(\frac{d}{e}+x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/5/e^5/d/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*A+1/5/e^6/(d/e+x)
^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*B-1/5/e^7*d/(d/e+x)^4*(-(d/e+x)^2*e
^2+2*d*e*(d/e+x))^(3/2)*C-1/15/e^4/d^2/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(3/2)*A+1/15/e^5/d/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*B-1/
15/e^6/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*C-C/e^5/d/(d/e+x)^2*(
-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-C/e^3/d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^
(1/2)-C/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))
^(1/2))-1/3*(B*e-2*C*d)/e^6/d/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91587, size = 637, normalized size = 3.25

$$-\frac{24\,Cd^5 + Bd^4e + 4\,Ad^3e^2 + \left(24\,Cd^2e^3 + Bde^4 + 4\,Ae^5\right)x^3 + 3\left(24\,Cd^3e^2 + Bd^2e^3 + 4\,Ade^4\right)x^2 + 3\left(24\,Cd^4e + Bd^3e^2 + 4\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas
")
```

```
[Out] -1/15*(24*C*d^5 + B*d^4*e + 4*A*d^3*e^2 + (24*C*d^2*e^3 + B*d*e^4 + 4*A*e^5
)*x^3 + 3*(24*C*d^3*e^2 + B*d^2*e^3 + 4*A*d*e^4)*x^2 + 3*(24*C*d^4*e + B*d^
3*e^2 + 4*A*d^2*e^3)*x - 30*(C*d^2*e^3*x^3 + 3*C*d^3*e^2*x^2 + 3*C*d^4*e*x
+ C*d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*C*d^4 + B*d^3*e +
4*A*d^2*e^2 + (39*C*d^2*e^2 - 4*B*d*e^3 - A*e^4)*x^2 + 3*(19*C*d^3*e + B*d^
2*e^2 - A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^2*e^6*x^3 + 3*d^3*e^5*x^2 + 3*
d^4*e^4*x + d^5*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}\,(A+Bx+Cx^2)}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.8 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(e(2Ae+5Bd)+23Cd^2\right)}{105d^3e^3(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(e(2Ae+5Bd)+23Cd^2\right)}{35d^2e^3(d+ex)^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{7de^3(d+ex)^5}+$$

```
[Out] -((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(7*d*e^3*(d + e*x)^5) + (C
*(d^2 - e^2*x^2)^(3/2))/(e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d + 2*A*e))
*(d^2 - e^2*x^2)^(3/2))/(35*d^2*e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d +
2*A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^3)
```

Rubi [A] time = 0.210378, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 659, 651}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(e(2Ae+5Bd)+23Cd^2\right)}{105d^3e^3(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(e(2Ae+5Bd)+23Cd^2\right)}{35d^2e^3(d+ex)^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{7de^3(d+ex)^5}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(7*d*e^3*(d + e*x)^5) + (C
*(d^2 - e^2*x^2)^(3/2))/(e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d + 2*A*e))
*(d^2 - e^2*x^2)^(3/2))/(35*d^2*e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d +
2*A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^3)
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} + \frac{\int \frac{\left(e^2\left(4Cd^2+Ae^2\right)+e^3(3Cd+Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx}{e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} + \frac{\left(23Cd^2+e(5Bd+2Ae)\right)}{7de^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} - \frac{\left(23Cd^2+e(5Bd+2Ae)\right)(}{35d^2e^3(d+ex}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} - \frac{\left(23Cd^2+e(5Bd+2Ae)\right)(}{35d^2e^3(d+ex}$$

Mathematica [A] time = 0.206715, size = 109, normalized size = 0.61

$$-\frac{(d-ex)\sqrt{d^2-e^2x^2}\left(e\left(Ae\left(23d^2+10dex+2e^2x^2\right)+5Bd\left(d^2+5dex+e^2x^2\right)\right)+Cd^2\left(2d^2+10dex+23e^2x^2\right)\right)}{105d^3e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 10*d*e*x + 23*e^2*x^2) + e*
(5*B*d*(d^2 + 5*d*e*x + e^2*x^2) + A*e*(23*d^2 + 10*d*e*x + 2*e^2*x^2))))/(
105*d^3*e^3*(d + e*x)^4)
```

Maple [A] time = 0.048, size = 116, normalized size = 0.6

$$-\frac{\left(2\,Ae^4x^2+5\,Bde^3x^2+23\,Cd^2e^2x^2+10\,Ade^3x+25\,Bd^2e^2x+10\,Cd^3ex+23\,Ad^2e^2+5\,Bd^3e+2\,Cd^4\right)(-ex+d)}{105\,(ex+d)^4\,d^3e^3}\sqrt{-x^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x)
```

```
[Out] -1/105*(-e*x+d)*(2*A*e^4*x^2+5*B*d*e^3*x^2+23*C*d^2*e^2*x^2+10*A*d*e^3*x+25
*B*d^2*e^2*x+10*C*d^3*e*x+23*A*d^2*e^2+5*B*d^3*e+2*C*d^4)*(-e^2*x^2+d^2)^(1
/2)/(e*x+d)^4/d^3/e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96755, size = 678, normalized size = 3.77

$$-\frac{2\,Cd^6+5\,Bd^5e+23\,Ad^4e^2+\left(2\,Cd^2e^4+5\,Bde^5+23\,Ae^6\right)x^4+4\left(2\,Cd^3e^3+5\,Bd^2e^4+23\,Ade^5\right)x^3+6\left(2\,Cd^4e^2+5\,Bd^3e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="fricas
")
```

```
[Out] -1/105*(2*C*d^6 + 5*B*d^5*e + 23*A*d^4*e^2 + (2*C*d^2*e^4 + 5*B*d*e^5 + 23*
A*e^6)*x^4 + 4*(2*C*d^3*e^3 + 5*B*d^2*e^4 + 23*A*d*e^5)*x^3 + 6*(2*C*d^4*e^
2 + 5*B*d^3*e^3 + 23*A*d^2*e^4)*x^2 + 4*(2*C*d^5*e + 5*B*d^4*e^2 + 23*A*d^3
*e^3)*x + (2*C*d^5 + 5*B*d^4*e + 23*A*d^3*e^2 - (23*C*d^2*e^3 + 5*B*d*e^4 +
 2*A*e^5)*x^3 + (13*C*d^3*e^2 - 20*B*d^2*e^3 - 8*A*d*e^4)*x^2 + (8*C*d^4*e
+ 20*B*d^3*e^2 - 13*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^4 + 4*d^
4*e^6*x^3 + 6*d^5*e^5*x^2 + 4*d^6*e^4*x + d^7*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}\left(A+Bx+Cx^2\right)}{(d+ex)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**5,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**5, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.9 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=234

$$-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+11Cd^2)}{315d^4e^3(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+11Cd^2)}{105d^3e^3(d+ex)^4}-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+11Cd^2)}{42d^2e^3(d+ex)^5}$$

```
[Out] -((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(9*d*e^3*(d + e*x)^6) + (C
*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*d + A*e
))*(d^2 - e^2*x^2)^(3/2))/(42*d^2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*
d + A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^4) - ((11*C*d^2 + 2
*e*(2*B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(315*d^4*e^3*(d + e*x)^3)
```

Rubi [A] time = 0.264216, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 659, 651}

$$-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+11Cd^2)}{315d^4e^3(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+11Cd^2)}{105d^3e^3(d+ex)^4}-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+11Cd^2)}{42d^2e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(9*d*e^3*(d + e*x)^6) + (C
*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*d + A*e
))*(d^2 - e^2*x^2)^(3/2))/(42*d^2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*
d + A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^4) - ((11*C*d^2 + 2
*e*(2*B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(315*d^4*e^3*(d + e*x)^3)
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
```

```
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} + \frac{\int \frac{\left(e^2\left(5Cd^2+2Ae^2\right)+e^3(3Cd+2Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx}{2e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} + \frac{\left(11Cd^2+2e(2Bd+Ae)\right)\int}{6de^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} - \frac{\left(11Cd^2+2e(2Bd+Ae)\right)\left(d\right.}{42d^2e^3(d+ex)^5}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} - \frac{\left(11Cd^2+2e(2Bd+Ae)\right)\left(d\right.}{42d^2e^3(d+ex)^5}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} - \frac{\left(11Cd^2+2e(2Bd+Ae)\right)\left(d\right.}{42d^2e^3(d+ex)^5}$$

Mathematica [A] time = 0.227784, size = 144, normalized size = 0.62

$$-\frac{(d-ex)\sqrt{d^2-e^2x^2}\left(e\left(Ae\left(33d^2ex+58d^3+12de^2x^2+2e^3x^3\right)+Bd\left(66d^2ex+11d^3+24de^2x^2+4e^3x^3\right)\right)+Cd^2\left(24d^2ex\right.\right.}{315d^4e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d^3 + 24*d^2*e*x + 66*d*e^2*x^2 +
 11*e^3*x^3) + e*(A*e*(58*d^3 + 33*d^2*e*x + 12*d*e^2*x^2 + 2*e^3*x^3) + B*
d*(11*d^3 + 66*d^2*e*x + 24*d*e^2*x^2 + 4*e^3*x^3))))/(315*d^4*e^3*(d + e*x
)^5)
```

Maple [A] time = 0.049, size = 152, normalized size = 0.7

$$-\frac{\left(2\,Ae^5x^3+4\,Bde^4x^3+11\,Cd^2e^3x^3+12\,Ade^4x^2+24\,Bd^2e^3x^2+66\,Cd^3e^2x^2+33\,Ad^2e^3x+66\,Bd^3e^2x+24\,Cd^4ex+58\,A\right.}{315\,(ex+d)^5\,d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x)
```

```
[Out] -1/315*(-e*x+d)*(2*A*e^5*x^3+4*B*d*e^4*x^3+11*C*d^2*e^3*x^3+12*A*d*e^4*x^2+
24*B*d^2*e^3*x^2+66*C*d^3*e^2*x^2+33*A*d^2*e^3*x+66*B*d^3*e^2*x+24*C*d^4*e*
x+58*A*d^3*e^2+11*B*d^4*e+4*C*d^5)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5/d^4/e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.5496, size = 852, normalized size = 3.64

$$-\frac{4\,Cd^7 + 11\,Bd^6e + 58\,Ad^5e^2 + \left(4\,Cd^2e^5 + 11\,Bde^6 + 58\,Ae^7\right)x^5 + 5\left(4\,Cd^3e^4 + 11\,Bd^2e^5 + 58\,Ade^6\right)x^4 + 10\left(4\,Cd^4e^3 + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas
")
```

```
[Out] -1/315*(4*C*d^7 + 11*B*d^6*e + 58*A*d^5*e^2 + (4*C*d^2*e^5 + 11*B*d*e^6 + 5
8*A*e^7)*x^5 + 5*(4*C*d^3*e^4 + 11*B*d^2*e^5 + 58*A*d*e^6)*x^4 + 10*(4*C*d^
4*e^3 + 11*B*d^3*e^4 + 58*A*d^2*e^5)*x^3 + 10*(4*C*d^5*e^2 + 11*B*d^4*e^3 +
 58*A*d^3*e^4)*x^2 + 5*(4*C*d^6*e + 11*B*d^5*e^2 + 58*A*d^4*e^3)*x + (4*C*d
^6 + 11*B*d^5*e + 58*A*d^4*e^2 - (11*C*d^2*e^4 + 4*B*d*e^5 + 2*A*e^6)*x^4 -
 5*(11*C*d^3*e^3 + 4*B*d^2*e^4 + 2*A*d*e^5)*x^3 + 21*(2*C*d^4*e^2 - 2*B*d^3
*e^3 - A*d^2*e^4)*x^2 + 5*(4*C*d^5*e + 11*B*d^4*e^2 - 5*A*d^3*e^3)*x)*sqrt(
-e^2*x^2 + d^2))/(d^4*e^8*x^5 + 5*d^5*e^7*x^4 + 10*d^6*e^6*x^3 + 10*d^7*e^5
*x^2 + 5*d^8*e^4*x + d^9*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.10 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=236

$$-\frac{x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+19Cd^2\right)}{15e}-\frac{dx\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{8e^2}-\frac{d^2\sqrt{d^2-e^2x^2}\left(55Ae^2+45Bde+\right.}{15e^3}$$

```
[Out] -(d^2*(38*C*d^2 + 45*B*d*e + 55*A*e^2)*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*(
13*C*d^2 + 15*B*d*e + 12*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((19*C*d^2
 + 5*e*(3*B*d + A*e))*x^2*Sqrt[d^2 - e^2*x^2])/(15*e) - ((3*C*d + B*e)*x^3*
Sqrt[d^2 - e^2*x^2])/4 - (C*e*x^4*Sqrt[d^2 - e^2*x^2])/5 + (d^3*(13*C*d^2 +
 15*B*d*e + 20*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rubi [A] time = 0.656543, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1815, 641, 217, 203}

$$-\frac{x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+19Cd^2\right)}{15e}-\frac{dx\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{8e^2}-\frac{d^2\sqrt{d^2-e^2x^2}\left(55Ae^2+45Bde+\right.}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```


```
[Out] -(d^2*(38*C*d^2 + 45*B*d*e + 55*A*e^2)*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*(
13*C*d^2 + 15*B*d*e + 12*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((19*C*d^2
 + 5*e*(3*B*d + A*e))*x^2*Sqrt[d^2 - e^2*x^2])/(15*e) - ((3*C*d + B*e)*x^3*
Sqrt[d^2 - e^2*x^2])/4 - (C*e*x^4*Sqrt[d^2 - e^2*x^2])/5 + (d^3*(13*C*d^2 +
 15*B*d*e + 20*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```


Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}-\frac{\int \frac{-5Ad^3e^2-5d^2e^2(Bd+3Ae)x-5de^2\left(Cd^2+3e(Bd+Ae)\right)x^2-e^3\left(19Cd^2+5e(3Bd+Ae)\right)x^3}{\sqrt{d^2-e^2x^2}}}{5e^2}\\
&= -\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}+\frac{\int \frac{20Ad^3e^4+20d^2e^4(Bd+3Ae)x+5de^4\left(13Cd^2+\right.}{\sqrt{d^2}}}{2}\\
&= -\frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2-e^2x^2}}{15e}-\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2}-\frac{1}{5}Cex^4\sqrt{d^2-e}\\
&= -\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2-e^2x^2}}{15e}-\\
&= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}\\
&= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}\\
&= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}
\end{aligned}$$

Mathematica [A] time = 0.453851, size = 174, normalized size = 0.74

$$\frac{15d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(5e(4Ae+3Bd)+13Cd^2\right)-\sqrt{d^2-e^2x^2}\left(5e\left(4Ae\left(22d^2+9dex+2e^2x^2\right)+3B\left(15d^2ex+24d^3+8de\right.\right.\right.}{120e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(C*(304*d^4 + 195*d^3*e*x + 152*d^2*e^2*x^2 + 90*d*e
^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(22*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*B*(24*
d^3 + 15*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3)))) + 15*d^3*(13*C*d^2 + 5*e*(3*
B*d + 4*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(120*e^3)
```

Maple [A] time = 0.066, size = 374, normalized size = 1.6

$$-\frac{Cex^4}{5}\sqrt{-x^2e^2+d^2}-\frac{19\,Cd^2x^2}{15\,e}\sqrt{-x^2e^2+d^2}-\frac{38\,Cd^4}{15\,e^3}\sqrt{-x^2e^2+d^2}-\frac{x^3eB}{4}\sqrt{-x^2e^2+d^2}-\frac{3\,dx^3C}{4}\sqrt{-x^2e^2+d^2}-\frac{15\,d^2x}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*C*e*x^4*(-e^2*x^2+d^2)^(1/2)-19/15/e*C*d^2*x^2*(-e^2*x^2+d^2)^(1/2)-38
/15/e^3*C*d^4*(-e^2*x^2+d^2)^(1/2)-1/4*x^3*e*(-e^2*x^2+d^2)^(1/2)*B-3/4*x^3
*(-e^2*x^2+d^2)^(1/2)*d*C-15/8*d^2/e*x*(-e^2*x^2+d^2)^(1/2)*B-13/8*d^3/e^2*
x*(-e^2*x^2+d^2)^(1/2)*C+15/8*d^4/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*
x^2+d^2)^(1/2))*B+13/8*d^5/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d
^2)^(1/2))*C-1/3*x^2*e*(-e^2*x^2+d^2)^(1/2)*A-x^2*(-e^2*x^2+d^2)^(1/2)*d*B-
11/3*d^2/e*(-e^2*x^2+d^2)^(1/2)*A-3*d^3/e^2*(-e^2*x^2+d^2)^(1/2)*B-3/2*A*d*
x*(-e^2*x^2+d^2)^(1/2)+5/2*A*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))
```

Maxima [B] time = 1.5055, size = 575, normalized size = 2.44

$$-\frac{1}{5}\sqrt{-e^2x^2+d^2}Cex^4-\frac{4\sqrt{-e^2x^2+d^2}Cd^2x^2}{15\,e}+\frac{Ad^3\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}}-\frac{8\sqrt{-e^2x^2+d^2}Cd^4}{15\,e^3}-\frac{\sqrt{-e^2x^2+d^2}Bd^3}{e^2}-\frac{3\sqrt{-e^2x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*C*e*x^4 - 4/15*sqrt(-e^2*x^2 + d^2)*C*d^2*x^2/e +
 A*d^3*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) - 8/15*sqrt(-e^2*x^2 + d^2)*C*
d^4/e^3 - sqrt(-e^2*x^2 + d^2)*B*d^3/e^2 - 3*sqrt(-e^2*x^2 + d^2)*A*d^2/e -
 1/4*(3*C*d*e^2 + B*e^3)*sqrt(-e^2*x^2 + d^2)*x^3/e^2 - 1/3*(3*C*d^2*e + 3*
B*d*e^2 + A*e^3)*sqrt(-e^2*x^2 + d^2)*x^2/e^2 + 3/8*(3*C*d*e^2 + B*e^3)*d^4
*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^4) + 1/2*(C*d^3 + 3*B*d^2*e + 3*A
*d*e^2)*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) - 3/8*(3*C*d*e^2 +
B*e^3)*sqrt(-e^2*x^2 + d^2)*d^2*x/e^4 - 1/2*(C*d^3 + 3*B*d^2*e + 3*A*d*e^2)
*sqrt(-e^2*x^2 + d^2)*x/e^2 - 2/3*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*sqrt(-e^2
*x^2 + d^2)*d^2/e^4
```

Fricas [A] time = 2.35253, size = 406, normalized size = 1.72

$$-\frac{30\left(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(24\,Ce^4x^4+304\,Cd^4+360\,Bd^3e+440\,Ad^2e^2+30\left(3\,Cde^3\right.\right.}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/120*(30*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (24*C*e^4*x^4 + 304*C*d^4 + 360*B*d^3*e + 440*A*d^2*e^2
+ 30*(3*C*d*e^3 + B*e^4)*x^3 + 8*(19*C*d^2*e^2 + 15*B*d*e^3 + 5*A*e^4)*x^2
+ 15*(13*C*d^3*e + 15*B*d^2*e^2 + 12*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] time = 20.234, size = 1277, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**3*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2
 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2)
, (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(
-d**2), (d**2 < 0) & (e**2 < 0))) + 3*A*d**2*e*Piecewise((x**2/(2*sqrt(d**2
)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + 3*A*d*e**2*Piecew
```

```
ise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**
2), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2
*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) +
A*e**3*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2
- e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + B*d**3*Pie
cewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, T
rue)) + 3*B*d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1
 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/
d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e*
*2*x**2/d**2)), True)) + 3*B*d*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**
2)/(3*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqr
t(d**2)), True)) + B*e**3*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*
d**3*x/(8*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2
*x**2/d**2)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d
**2) > 1), (3*d**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x*
*2/d**2)) + d*x**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e
**2*x**2/d**2)), True)) + C*d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) -
 I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2)/Abs(d**2) > 1), (
d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2
*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*C*d**2*e*Piecewise((-2*d**2*sqrt(d
**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0))
, (x**4/(4*sqrt(d**2)), True)) + 3*C*d*e**2*Piecewise((-3*I*d**4*acosh(e*x/
d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e
**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2)/Abs(d**2) > 1), (3*d**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**
4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x*
*5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + C*e**3*Piecewise((-8*d**4*sqrt(
d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**4)
- x**4*sqrt(d**2 - e**2*x**2)/(5*e**2), Ne(e, 0)), (x**6/(6*sqrt(d**2)), Tr
ue))
```

Giac [A] time = 1.23252, size = 224, normalized size = 0.95

$$\frac{1}{8}\left(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}\left(d\right)-\frac{1}{120}\sqrt{-x^2e^2+d^2}\left(\left(2\left(3\left(4\,Cxe+5\left(3\,Cde^6+Be^7\right)e^{(-6)}\right)x+4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/
120*sqrt(-x^2*e^2 + d^2)*((2*(3*(4*C*x*e + 5*(3*C*d*e^6 + B*e^7)*e^(-6))*x
+ 4*(19*C*d^2*e^5 + 15*B*d*e^6 + 5*A*e^7)*e^(-6))*x + 15*(13*C*d^3*e^4 + 15
```

```
*B*d^2*e^5 + 12*A*d*e^6)*e^(-6))*x + 8*(38*C*d^4*e^3 + 45*B*d^3*e^4 + 55*A*
d^2*e^5)*e^(-6))
```

3.11 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal. Leaf size=191

$$-\frac{x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{8e^2}-\frac{d\sqrt{d^2-e^2x^2}\left(e(6Ae+5Bd)+4Cd^2\right)}{3e^3}+\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(12Ae^2+8Bde+7\right.}{8e^3}$$

```
[Out] -(d*(4*C*d^2 + e*(5*B*d + 6*A*e))*Sqrt[d^2 - e^2*x^2])/(3*e^3) - ((7*C*d^2
+ 4*e*(2*B*d + A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d + B*e)*x^2*Sq
rt[d^2 - e^2*x^2])/(3*e) - (C*x^3*Sqrt[d^2 - e^2*x^2])/4 + (d^2*(7*C*d^2 +
8*B*d*e + 12*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rubi [A] time = 0.378195, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1815, 641, 217, 203}

$$-\frac{x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{8e^2}-\frac{d\sqrt{d^2-e^2x^2}\left(e(6Ae+5Bd)+4Cd^2\right)}{3e^3}+\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(12Ae^2+8Bde+7\right.}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(d*(4*C*d^2 + e*(5*B*d + 6*A*e))*Sqrt[d^2 - e^2*x^2])/(3*e^3) - ((7*C*d^2
+ 4*e*(2*B*d + A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d + B*e)*x^2*Sq
rt[d^2 - e^2*x^2])/(3*e) - (C*x^3*Sqrt[d^2 - e^2*x^2])/4 + (d^2*(7*C*d^2 +
8*B*d*e + 12*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2} - \frac{\int \frac{-4Ad^2e^2-4de^2(Bd+2Ae)x-e^2\left(7Cd^2+4e(2Bd+Ae)\right)x^2-4e^3(2Cd+Be)x^3}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= -\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}Cx^3\sqrt{d^2-e^2x^2} + \frac{\int \frac{12Ad^2e^4+4de^3\left(4Cd^2+e(5Bd+6Ae)\right)x+3e^4\left(7Cd\right.}{\sqrt{d^2-e^2x^2}}}{12e^4}$$

$$= -\frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd}{}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd}{}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd}{}$$

Mathematica [A] time = 0.183379, size = 139, normalized size = 0.73

$$\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(4e(3Ae+2Bd)+7Cd^2\right)-\sqrt{d^2-e^2x^2}\left(4e\left(3Ae(4d+ex)+2B\left(5d^2+3dex+e^2x^2\right)\right)+C\left(21d^2ex+3\right.\right.}{24e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(C*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3)
+ 4*e*(3*A*e*(4*d + e*x) + 2*B*(5*d^2 + 3*d*e*x + e^2*x^2)))) + 3*d^2*(7*C*
d^2 + 4*e*(2*B*d + 3*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e^3)
```

Maple [A] time = 0.062, size = 301, normalized size = 1.6

$$-\frac{Cx^3}{4}\sqrt{-x^2e^2+d^2}-\frac{7\,Cd^2x}{8\,e^2}\sqrt{-x^2e^2+d^2}+\frac{7\,Cd^4}{8\,e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{x^2B}{3}\sqrt{-x^2e^2+d^2}-\frac{2\,Cdx^2}{3\,e}\sqrt{-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*C*x^3*(-e^2*x^2+d^2)^(1/2)-7/8*C*d^2/e^2*x*(-e^2*x^2+d^2)^(1/2)+7/8*C*
d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/3*x^2*(-e^
2*x^2+d^2)^(1/2)*B-2/3*x^2/e*(-e^2*x^2+d^2)^(1/2)*d*C-5/3*d^2/e^2*(-e^2*x^2
+d^2)^(1/2)*B-4/3*d^3/e^3*(-e^2*x^2+d^2)^(1/2)*C-1/2*A*x*(-e^2*x^2+d^2)^(1/
2)-x/e*(-e^2*x^2+d^2)^(1/2)*B*d+3/2*A*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2))+d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))*B-2/e*(-e^2*x^2+d^2)^(1/2)*A*d
```

Maxima [A] time = 1.51843, size = 390, normalized size = 2.04

$$-\frac{1}{4}\sqrt{-e^2x^2+d^2}Cx^3+\frac{Ad^2\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}}+\frac{3\,Cd^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2}e^2}-\frac{3\sqrt{-e^2x^2+d^2}Cd^2x}{8\,e^2}-\frac{\sqrt{-e^2x^2+d^2}Bd^2}{e^2}-\frac{2\sqrt{-e^2x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*C*x^3 + A*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^
2) + 3/8*C*d^4*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) - 3/8*sqrt(-e^2*
x^2 + d^2)*C*d^2*x/e^2 - sqrt(-e^2*x^2 + d^2)*B*d^2/e^2 - 2*sqrt(-e^2*x^2 +
 d^2)*A*d/e - 1/3*sqrt(-e^2*x^2 + d^2)*(2*C*d*e + B*e^2)*x^2/e^2 + 1/2*(C*d
^2 + 2*B*d*e + A*e^2)*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) - 1/2
*sqrt(-e^2*x^2 + d^2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/3*sqrt(-e^2*x^2 +
 d^2)*(2*C*d*e + B*e^2)*d^2/e^4
```

Fricas [A] time = 2.38753, size = 319, normalized size = 1.67

$$-\frac{6\left(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,Ce^3x^3+32\,Cd^3+40\,Bd^2e+48\,Ade^2+8\left(2\,Cde^2+Be^3\right)x^2+\right.}{24\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/24*(6*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) + (6*C*e^3*x^3 + 32*C*d^3 + 40*B*d^2*e + 48*A*d*e^2 + 8*(2*C*d
*e^2 + B*e^3)*x^2 + 3*(7*C*d^2*e + 8*B*d*e^2 + 4*A*e^3)*x)*sqrt(-e^2*x^2 +
d^2))/e^3
```

Sympy [A] time = 14.1762, size = 898, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2
 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2)
, (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(
-d**2), (d**2 < 0) & (e**2 < 0))) + 2*A*d*e*Piecewise((x**2/(2*sqrt(d**2)),
 Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + A*e**2*Piecewise((-I
*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs
(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1
 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + B*d**2*
Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2
, True)) + 2*B*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1
 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/
d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e*
*2*x**2/d**2)), True)) + B*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(
3*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d*
*2)), True)) + C*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt
(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e
```

```
*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 -
 e**2*x**2/d**2)), True)) + 2*C*d*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**
2)/(3*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqr
t(d**2)), True)) + C*e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*
d**3*x/(8*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2
*x**2/d**2)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d
**2) > 1), (3*d**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x*
*2/d**2)) + d*x**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e
**2*x**2/d**2)), True))
```

Giac [A] time = 1.17706, size = 177, normalized size = 0.93

$$\frac{1}{8}\left(7\,Cd^4 + 8\,Bd^3e + 12\,Ad^2e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}\left(d\right) - \frac{1}{24}\sqrt{-x^2e^2 + d^2}\left(\left(2\left(3\,Cx + 4\left(2\,Cde^4 + Be^5\right)e^{(-5)}\right)x + 3\left(7\,Cd^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24
*sqrt(-x^2*e^2 + d^2)*((2*(3*C*x + 4*(2*C*d*e^4 + B*e^5)*e^(-5))*x + 3*(7*C
*d^2*e^3 + 8*B*d*e^4 + 4*A*e^5)*e^(-5))*x + 8*(4*C*d^3*e^2 + 5*B*d^2*e^3 +
6*A*d*e^4)*e^(-5))
```

3.12 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{d^2-e^2x^2}\left(3e(Ae+Bd)+2Cd^2\right)}{3e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)}{2e^3}-\frac{x\sqrt{d^2-e^2x^2}(Be+Cd)}{2e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

```
[Out] -((2*C*d^2 + 3*e*(B*d + A*e))*Sqrt[d^2 - e^2*x^2])/(3*e^3) - ((C*d + B*e)*x
*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (C*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) + (d*(C*d
^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rubi [A] time = 0.198975, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1815, 641, 217, 203}

$$-\frac{\sqrt{d^2-e^2x^2}\left(3e(Ae+Bd)+2Cd^2\right)}{3e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)}{2e^3}-\frac{x\sqrt{d^2-e^2x^2}(Be+Cd)}{2e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((2*C*d^2 + 3*e*(B*d + A*e))*Sqrt[d^2 - e^2*x^2])/(3*e^3) - ((C*d + B*e)*x
*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (C*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) + (d*(C*d
^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{\int \frac{-3Ade^2-e\left(2Cd^2+3e(Bd+Ae)\right)x-3e^2(Cd+Be)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{3e^2}$$

$$= -\frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} + \frac{\int \frac{3de^2\left(Cd^2+e(Bd+2Ae)\right)+2e^3\left(2Cd^2+3e(Bd+Ae)\right)x}{\sqrt{d^2-e^2x^2}}\,d}{6e^4}$$

$$= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} + \frac{(d\,(}{}$$

$$= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} + \frac{(d\,(}{}$$

$$= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} + \frac{d\,(}{}$$

Mathematica [A] time = 0.115612, size = 103, normalized size = 0.72

$$\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)-\sqrt{d^2-e^2x^2}\left(3e(2Ae+2Bd+Bex)+C\left(4d^2+3dex+2e^2x^2\right)\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(3*e*(2*B*d + 2*A*e + B*e*x) + C*(4*d^2 + 3*d*e*x +
2*e^2*x^2))) + 3*d*(C*d^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(6*e^3)
```

Maple [A] time = 0.056, size = 234, normalized size = 1.6

$$-\frac{Cx^2}{3e}\sqrt{-x^2e^2+d^2}-\frac{2\,Cd^2}{3e^3}\sqrt{-x^2e^2+d^2}-\frac{Bx}{2e}\sqrt{-x^2e^2+d^2}-\frac{Cdx}{2e^2}\sqrt{-x^2e^2+d^2}+\frac{Bd^2}{2e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*C*x^2*(-e^2*x^2+d^2)^(1/2)/e-2/3/e^3*C*d^2*(-e^2*x^2+d^2)^(1/2)-1/2/e*
B*x*(-e^2*x^2+d^2)^(1/2)-1/2/e^2*C*d*x*(-e^2*x^2+d^2)^(1/2)+1/2/e*B*d^2/(e^
2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/2/e^2*C*d^3/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/e*(-e^2*x^2+d^2)^(1/2)*A-1/
e^2*(-e^2*x^2+d^2)^(1/2)*B*d+A*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))
```

Maxima [A] time = 1.55194, size = 234, normalized size = 1.64

$$-\frac{\sqrt{-e^2x^2+d^2}Cx^2}{3e}+\frac{Ad\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}}+\frac{(Cd+Be)d^2\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\sqrt{e^2}e^2}-\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{3e^3}-\frac{\sqrt{-e^2x^2+d^2}Bd}{e^2}-\frac{\sqrt{-e^2x^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*C*x^2/e + A*d*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^
2) + 1/2*(C*d + B*e)*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) - 2/3*
sqrt(-e^2*x^2 + d^2)*C*d^2/e^3 - sqrt(-e^2*x^2 + d^2)*B*d/e^2 - sqrt(-e^2*x
^2 + d^2)*A/e - 1/2*sqrt(-e^2*x^2 + d^2)*(C*d + B*e)*x/e^2
```

Fricas [A] time = 1.76141, size = 236, normalized size = 1.65

$$-\frac{6\left(Cd^3+Bd^2e+2\,Ade^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,Ce^2x^2+4\,Cd^2+6\,Bde+6\,Ae^2+3\left(Cde+Be^2\right)x\right)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e
*x)) + (2*C*e^2*x^2 + 4*C*d^2 + 6*B*d*e + 6*A*e^2 + 3*(C*d*e + B*e^2)*x)*sq
rt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] time = 7.5767, size = 488, normalized size = 3.41

$$Ad\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)+Ae\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+Bd\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+Be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (
d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d*
*2), (d**2 < 0) & (e**2 < 0))) + A*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**
2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + B*d*Piecewise((x**2/(2*sqrt
(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + B*e*Piecewis
e((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2)
, Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*s
qrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + C*
d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2
)/(2*e**2), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x
/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), T
rue)) + C*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(
d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))
```

Giac [A] time = 1.19084, size = 131, normalized size = 0.92

$$\frac{1}{2}\left(Cd^3+Bd^2e+2\,Ade^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d)-\frac{1}{6}\sqrt{-x^2e^2+d^2}\left(\left(2\,Cxe^{(-1)}+3\left(Cde^3+Be^4\right)e^{(-5)}\right)x+2\left(2\,Cd^2e^2+3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/6*sqrt(-x
^2*e^2 + d^2)*((2*C*x*e^(-1) + 3*(C*d*e^3 + B*e^4)*e^(-5))*x + 2*(2*C*d^2*e
^2 + 3*B*d*e^3 + 3*A*e^4)*e^(-5))
```

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=87

$$\frac{(2Ae^2+Cd^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

```
[Out] -((B*Sqrt[d^2 - e^2*x^2])/e^2) - (C*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + ((C*d^
2 + 2*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rubi [A] time = 0.0510074, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1815, 641, 217, 203}

$$\frac{(2Ae^2+Cd^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((B*Sqrt[d^2 - e^2*x^2])/e^2) - (C*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + ((C*d^
2 + 2*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\int \frac{-Cd^2-2Ae^2-2Be^2x}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\left(-Cd^2-2Ae^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\left(-Cd^2-2Ae^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}\\
&= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{\left(Cd^2+2Ae^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}
\end{aligned}
$$

Mathematica [A] time = 0.0417679, size = 67, normalized size = 0.77

$$
\frac{\left(2Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-e(2B+Cx)\sqrt{d^2-e^2x^2}}{2e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(e*(2*B + C*x)*Sqrt[d^2 - e^2*x^2]) + (C*d^2 + 2*A*e^2)*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/(2*e^3)
```

Maple [A] time = 0.105, size = 108, normalized size = 1.2

$$
-\frac{Cx}{2\,e^2}\sqrt{-x^2e^2+d^2}+\frac{Cd^2}{2\,e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{B}{e^2}\sqrt{-x^2e^2+d^2}+A\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*C*x*(-e^2*x^2+d^2)^(1/2)/e^2+1/2*C/e^2*d^2/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-e^2*x^2+d^2)^(1/2))-B*(-e^2*x^2+d^2)^(1/2)/e^2+A/(e^2)^(1/2)*arctan
((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.52561, size = 126, normalized size = 1.45

$$\frac{A \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{\sqrt{e^2}} + \frac{C d^2 \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{2\sqrt{e^2} e^2} - \frac{\sqrt{-e^2 x^2 + d^2} C x}{2 e^2} - \frac{\sqrt{-e^2 x^2 + d^2} B}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] A*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) + 1/2*C*d^2*arcsin(e^2*x/sqrt(d^2*e
^2))/(sqrt(e^2)*e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*C*x/e^2 - sqrt(-e^2*x^2 + d
^2)*B/e^2
```

Fricas [A] time = 1.77747, size = 153, normalized size = 1.76

$$-\frac{2\left(C d^2 + 2 A e^2\right) \arctan\left(-\frac{d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) + \sqrt{-e^2 x^2 + d^2}(C e x + 2 B e)}{2 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*d^2 + 2*A*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(
-e^2*x^2 + d^2)*(C*e*x + 2*B*e))/e^3
```

Sympy [A] time = 3.22441, size = 264, normalized size = 3.03

$$A\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)+B\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+C\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2}\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 > 0)
 & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d*
*2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2
), (d**2 < 0) & (e**2 < 0))) + B*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0
)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + C*Piecewise((-I*d**2*acosh(e*x/
d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2)/Abs(
d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**
2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.21307, size = 70, normalized size = 0.8

$$\frac{1}{2}\left(Cd^2+2\,Ae^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d)-\frac{1}{2}\sqrt{-x^2e^2+d^2}\left(Cxe^{(-2)}+2\,Be^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(C*d^2 + 2*A*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2
)*(C*x*e^(-2) + 2*B*e^(-2))
```

3.14 $\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

```
[Out] -((C*Sqrt[d^2 - e^2*x^2])/e^3) - ((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^
2])/(d*e^3*(d + e*x)) - ((C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rubi [A] time = 0.12093, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 217, 203}

$$-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*Sqrt[d^2 - e^2*x^2])/e^3) - ((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^
2])/(d*e^3*(d + e*x)) - ((C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{\int \frac{-Ae^4+e^3(Cd-Be)x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\ &= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\ &= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\ &= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.161356, size = 83, normalized size = 0.81

$$\frac{(Be-Cd)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{\sqrt{d^2-e^2x^2}(e(Ae-Bd)+Cd(2d+ex))}{d(d+ex)}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-((Sqrt[d^2 - e^2*x^2]*(e*(-(B*d) + A*e) + C*d*(2*d + e*x)))/(d*(d + e*x))
) + (-(C*d) + B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Maple [A] time = 0.057, size = 149, normalized size = 1.5

$$-\frac{C}{e^3}\sqrt{-x^2e^2+d^2}+\frac{B}{e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{Cd}{e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{Ae^2-Bde+Cd^2}{e^4d}\sqrt{-(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -C*(-e^2*x^2+d^2)^(1/2)/e^3+1/e*B/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))-1/e^2*C*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/
2))-(A*e^2-B*d*e+C*d^2)/e^4/d/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84959, size = 315, normalized size = 3.06

$$-\frac{2\,Cd^3-Bd^2e+Ade^2+\left(2\,Cd^2e-Bde^2+Ae^3\right)x-2\left(Cd^3-Bd^2e+\left(Cd^2e-Bde^2\right)x\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(Cdex+\right.}{de^4x+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*C*d^3 - B*d^2*e + A*d*e^2 + (2*C*d^2*e - B*d*e^2 + A*e^3)*x - 2*(C*d^3
- B*d^2*e + (C*d^2*e - B*d*e^2)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + (C*d*e*x + 2*C*d^2 - B*d*e + A*e^2)*sqrt(-e^2*x^2 + d^2))/(d*e^4*x + d^
2*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx + Cx^2}{\sqrt{-\left(-d + ex\right)\left(d + ex\right)}\left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.15 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3d^2e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3de^3(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}(2Cd-Be)}{de^3(d+ex)}+\frac{C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(3*d*e^3*(d + e*x)^2) + ((2*
C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2
)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e^3*(d + e*x)) + (C*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/e^3
```

Rubi [A] time = 0.169427, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1637, 217, 203, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3d^2e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3de^3(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}(2Cd-Be)}{de^3(d+ex)}+\frac{C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```


```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(3*d*e^3*(d + e*x)^2) + ((2*
C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2
)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e^3*(d + e*x)) + (C*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/e^3
```


Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx &= \int \left(\frac{C}{e^2\sqrt{d^2-e^2x^2}}+\frac{Cd^2-Bde+Ae^2}{e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{-2Cd+Be}{e^2(d+ex)\sqrt{d^2-e^2x^2}}\right)dx\\
&= \frac{C\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}-\frac{(2Cd-Be)\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^2}+\frac{\left(Cd^2-Bde+Ae^2\right)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2}+\frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}+\frac{C\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\
&= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2}+\frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{3d^2e^3(d+ex)}
\end{aligned}
$$

Mathematica [A] time = 0.221091, size = 95, normalized size = 0.58

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\left(Cd^2(4d+5ex)-e(Ae(2d+ex)+Bd(d+2ex))\right)}{d^2(d+ex)^2}+3C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d + 5*e*x) - e*(A*e*(2*d + e*x) + B*d*(d +
2*e*x))))/(d^2*(d + e*x)^2) + 3*C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^3
)
```

Maple [B] time = 0.061, size = 355, normalized size = 2.2

$$\frac{C}{e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{Be-2\,Cd}{e^4d}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1}-\frac{A}{3\,de^3}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] C/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/e^4*(B*e-2*C
*d)/d/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-1/3/e^3/d/(d/e+x)^2*(-(d
/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*A+1/3/e^4/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*
(d/e+x))^(1/2)*B-1/3/e^5*d/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*C
-1/3/e^2/d^2/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*A+1/3/e^3/d/(d/e+
x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*B-1/3/e^4/(d/e+x)*(-(d/e+x)^2*e^2+2
*d*e*(d/e+x))^(1/2)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82777, size = 451, normalized size = 2.77

$$\frac{4\,Cd^4 - Bd^3e - 2\,Ad^2e^2 + \left(4\,Cd^2e^2 - Bde^3 - 2\,Ae^4\right)x^2 + 2\left(4\,Cd^3e - Bd^2e^2 - 2\,Ade^3\right)x - 6\left(Cd^2e^2x^2 + 2\,Cd^3ex + Cd^4\right)\mathrm{arc}}{3\left(d^2e^5x^2 + 2\,d^3e^4x + d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3*(4*C*d^4 - B*d^3*e - 2*A*d^2*e^2 + (4*C*d^2*e^2 - B*d*e^3 - 2*A*e^4)*x^
2 + 2*(4*C*d^3*e - B*d^2*e^2 - 2*A*d*e^3)*x - 6*(C*d^2*e^2*x^2 + 2*C*d^3*e*
x + C*d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*C*d^3 - B*d^2*e -
 2*A*d*e^2 + (5*C*d^2*e - 2*B*d*e^2 - A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^2*
e^5*x^2 + 2*d^3*e^4*x + d^4*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx + Cx^2}{\sqrt{-\left(-d + ex\right)\left(d + ex\right)}\left(d + ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.16 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=180

$$-\frac{\sqrt{d^2-e^2x^2}\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^3e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^2e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-}}{e^3(d+}$$

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(5*d*e^3*(d + e*x)^3) + (C*S
qrt[d^2 - e^2*x^2])/(e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*Sqrt
[d^2 - e^2*x^2])/(15*d^2*e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*
Sqrt[d^2 - e^2*x^2])/(15*d^3*e^3*(d + e*x))
```

Rubi [A] time = 0.205141, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^3e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^2e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-}}{e^3(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```


```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(5*d*e^3*(d + e*x)^3) + (C*S
qrt[d^2 - e^2*x^2])/(e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*Sqrt
[d^2 - e^2*x^2])/(15*d^2*e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*
Sqrt[d^2 - e^2*x^2])/(15*d^3*e^3*(d + e*x))
```


Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{\int \frac{e^2\left(2Cd^2+Ae^2\right)+e^3(Cd+Be)x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{\left(7Cd^2+e(3Bd+2Ae)\right)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}}{5de^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}-\frac{\left(7Cd^2+e(3Bd+2Ae)\right)\sqrt{d^2-e^2x^2}}{15d^2e^3(d+ex)^2}+\frac{(7}{}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}-\frac{\left(7Cd^2+e(3Bd+2Ae)\right)\sqrt{d^2-e^2x^2}}{15d^2e^3(d+ex)^2}-\frac{(7}{}$$

Mathematica [A] time = 0.19948, size = 103, normalized size = 0.57

$$-\frac{\sqrt{d^2-e^2x^2}\left(e\left(Ae\left(7d^2+6dex+2e^2x^2\right)+3Bd\left(d^2+3dex+e^2x^2\right)\right)+Cd^2\left(2d^2+6dex+7e^2x^2\right)\right)}{15d^3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 6*d*e*x + 7*e^2*x^2) + e*(3*B*d*(d^2
+ 3*d*e*x + e^2*x^2) + A*e*(7*d^2 + 6*d*e*x + 2*e^2*x^2))))/(15*d^3*e^3*(d
+ e*x)^3)
```

Maple [A] time = 0.049, size = 116, normalized size = 0.6

$$-\frac{(-ex+d)\left(2\,Ae^4x^2+3\,Bde^3x^2+7\,Cd^2e^2x^2+6\,Ade^3x+9\,Bd^2e^2x+6\,Cd^3ex+7\,Ad^2e^2+3\,Bd^3e+2\,Cd^4\right)}{15\,e^3d^3\,(ex+d)^2}\frac{1}{\sqrt{-x^2e^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(2*A*e^4*x^2+3*B*d*e^3*x^2+7*C*d^2*e^2*x^2+6*A*d*e^3*x+9*B*d
^2*e^2*x+6*C*d^3*e*x+7*A*d^2*e^2+3*B*d^3*e+2*C*d^4)/(e*x+d)^2/d^3/e^3/(-e^2
*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78098, size = 510, normalized size = 2.83

$$-\frac{2\,Cd^5+3\,Bd^4e+7\,Ad^3e^2+\left(2\,Cd^2e^3+3\,Bde^4+7\,Ae^5\right)x^3+3\left(2\,Cd^3e^2+3\,Bd^2e^3+7\,Ade^4\right)x^2+3\left(2\,Cd^4e+3\,Bd^3e^2+\cdots\right.}{15\left(d^3e^6x^3+3\,d^4e^{5}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/15*(2*C*d^5 + 3*B*d^4*e + 7*A*d^3*e^2 + (2*C*d^2*e^3 + 3*B*d*e^4 + 7*A*e
^5)*x^3 + 3*(2*C*d^3*e^2 + 3*B*d^2*e^3 + 7*A*d*e^4)*x^2 + 3*(2*C*d^4*e + 3*
B*d^3*e^2 + 7*A*d^2*e^3)*x + (2*C*d^4 + 3*B*d^3*e + 7*A*d^2*e^2 + (7*C*d^2*
e^2 + 3*B*d*e^3 + 2*A*e^4)*x^2 + 3*(2*C*d^3*e + 3*B*d^2*e^2 + 2*A*d*e^3)*x)
*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^6*e^3
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx + Cx^2}{\sqrt{-(-d + ex)(d + ex)}\,(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.17 $\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=234

$$-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{105d^4e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{105d^3e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{70d^2e^3(d+ex)^3}-\sqrt{}$$

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(7*d*e^3*(d + e*x)^4) + (C*S
qrt[d^2 - e^2*x^2])/(2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e^2)*S
qrt[d^2 - e^2*x^2])/(70*d^2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e
^2)*Sqrt[d^2 - e^2*x^2])/(105*d^3*e^3*(d + e*x)^2) - ((13*C*d^2 + 8*B*d*e +
 6*A*e^2)*Sqrt[d^2 - e^2*x^2])/(105*d^4*e^3*(d + e*x))
```

Rubi [A] time = 0.248522, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{105d^4e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{105d^3e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{70d^2e^3(d+ex)^3}-\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(7*d*e^3*(d + e*x)^4) + (C*S
qrt[d^2 - e^2*x^2])/(2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e^2)*S
qrt[d^2 - e^2*x^2])/(70*d^2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e
^2)*Sqrt[d^2 - e^2*x^2])/(105*d^3*e^3*(d + e*x)^2) - ((13*C*d^2 + 8*B*d*e +
 6*A*e^2)*Sqrt[d^2 - e^2*x^2])/(105*d^4*e^3*(d + e*x))
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} + \frac{\int \frac{e^2\left(3Cd^2+2Ae^2\right)+e^3(Cd+2Be)x}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx}{2e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} + \frac{\left(13Cd^2+8Bde+6Ae^2\right)\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}}{14de^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{\left(13Cd^2+8Bde+6Ae^2\right)\sqrt{d^2-e^2x^2}}{70d^2e^3(d+ex)^3} +$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{\left(13Cd^2+8Bde+6Ae^2\right)\sqrt{d^2-e^2x^2}}{70d^2e^3(d+ex)^3} -$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{\left(13Cd^2+8Bde+6Ae^2\right)\sqrt{d^2-e^2x^2}}{70d^2e^3(d+ex)^3} -$$

Mathematica [A] time = 0.224481, size = 139, normalized size = 0.59

$$-\frac{\sqrt{d^2-e^2x^2}\left(e\left(3Ae\left(13d^2ex+12d^3+8de^2x^2+2e^3x^3\right)+Bd\left(52d^2ex+13d^3+32de^2x^2+8e^3x^3\right)\right)+Cd^2\left(32d^2ex+8d^3\right.\right.}{105d^4e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(C*d^2*(8*d^3 + 32*d^2*e*x + 52*d*e^2*x^2 + 13*e^3*x^
3) + e*(3*A*e*(12*d^3 + 13*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3) + B*d*(13*d^3
 + 52*d^2*e*x + 32*d*e^2*x^2 + 8*e^3*x^3))))/(105*d^4*e^3*(d + e*x)^4)
```

Maple [A] time = 0.049, size = 152, normalized size = 0.7

$$-\frac{(-ex+d)\left(6\,Ae^5x^3+8\,Bde^4x^3+13\,Cd^2e^3x^3+24\,Ade^4x^2+32\,Bd^2e^3x^2+52\,Cd^3e^2x^2+39\,Ad^2e^3x+52\,Bd^3e^2x+32\,C\right.}{105\,e^3d^4\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/105*(-e*x+d)*(6*A*e^5*x^3+8*B*d*e^4*x^3+13*C*d^2*e^3*x^3+24*A*d*e^4*x^2+
32*B*d^2*e^3*x^2+52*C*d^3*e^2*x^2+39*A*d^2*e^3*x+52*B*d^3*e^2*x+32*C*d^4*e*
x+36*A*d^3*e^2+13*B*d^4*e+8*C*d^5)/(e*x+d)^3/d^4/e^3/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05572, size = 689, normalized size = 2.94

$$-\frac{8\,Cd^6+13\,Bd^5e+36\,Ad^4e^2+\left(8\,Cd^2e^4+13\,Bde^5+36\,Ae^6\right)x^4+4\left(8\,Cd^3e^3+13\,Bd^2e^4+36\,Ade^5\right)x^3+6\left(8\,Cd^4e^2+13\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/105*(8*C*d^6 + 13*B*d^5*e + 36*A*d^4*e^2 + (8*C*d^2*e^4 + 13*B*d*e^5 + 3
6*A*e^6)*x^4 + 4*(8*C*d^3*e^3 + 13*B*d^2*e^4 + 36*A*d*e^5)*x^3 + 6*(8*C*d^4
*e^2 + 13*B*d^3*e^3 + 36*A*d^2*e^4)*x^2 + 4*(8*C*d^5*e + 13*B*d^4*e^2 + 36*
A*d^3*e^3)*x + (8*C*d^5 + 13*B*d^4*e + 36*A*d^3*e^2 + (13*C*d^2*e^3 + 8*B*d
*e^4 + 6*A*e^5)*x^3 + 4*(13*C*d^3*e^2 + 8*B*d^2*e^3 + 6*A*d*e^4)*x^2 + (32*
C*d^4*e + 52*B*d^3*e^2 + 39*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^4*e^7*x^
4 + 4*d^5*e^6*x^3 + 6*d^6*e^5*x^2 + 4*d^7*e^4*x + d^8*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.18 $\int (d+ex)^3 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=175

$$\frac{(d+ex)^6 \left(aCe^2+c\left(6Cd^2-e(3Bd-Ae)\right)\right)}{6e^5}-\frac{(d+ex)^5 \left(ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)\right)}{5e^5}+\frac{(d+ex)^4 \left(ae^2+\right.}{}$$

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^4)/(4*e^5) - ((a*e^2*(2*
C*d - B*e) + c*d*(4*C*d^2 - e*(3*B*d - 2*A*e)))*(d + e*x)^5)/(5*e^5) + ((a*
C*e^2 + c*(6*C*d^2 - e*(3*B*d - A*e)))*(d + e*x)^6)/(6*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^7)/(7*e^5) + (c*C*(d + e*x)^8)/(8*e^5)
```

Rubi [A] time = 0.313178, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1628}

$$\frac{(d+ex)^6 \left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{6e^5}-\frac{(d+ex)^5 \left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{5e^5}+\frac{(d+ex)^4 \left(ae^2+cd^2\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^4)/(4*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^5)/(5*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^6)/(6*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^7)/(7*e^5) + (c*C*(d + e*x)^8)/(8*e^5)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx = \int \left(\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^3}{e^4}+\frac{\left(-4cCd^3+cde(3Bd-2Ae)\right.}{e^4}\right.$$

$$= \frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^4}{4e^5}-\frac{\left(4cCd^3-cde(3Bd-2Ae)+ae^2(\right.}{5e^5}$$

Mathematica [A] time = 0.0895023, size = 208, normalized size = 1.19

$$\frac{1}{6}ex^6\left(aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{5}x^5\left(ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)+\frac{1}{4}x^4\left(aAe^3+3aBde^2+3aCd^2e+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^3*x + (a*d^2*(B*d + 3*A*e)*x^2)/2 + (d*(a*d*(C*d + 3*B*e) + A*(c*d^2
+ 3*a*e^2))*x^3)/3 + ((B*c*d^3 + 3*A*c*d^2*e + 3*a*C*d^2*e + 3*a*B*d*e^2 +
a*A*e^3)*x^4)/4 + ((c*C*d^3 + 3*c*d*e*(B*d + A*e) + a*e^2*(3*C*d + B*e))*x^
5)/5 + (e*(3*c*C*d^2 + a*C*e^2 + c*e*(3*B*d + A*e))*x^6)/6 + (c*e^2*(3*C*d
+ B*e)*x^7)/7 + (c*C*e^3*x^8)/8
```

Maple [A] time = 0.044, size = 217, normalized size = 1.2

$$\frac{e^3cCx^8}{8}+\frac{\left(e^3cB+3\,de^2cC\right)x^7}{7}+\frac{\left(\left(ae^3+3\,d^2ec\right)C+3\,de^2cB+e^3cA\right)x^6}{6}+\frac{\left(\left(3\,ade^2+cd^3\right)C+\left(ae^3+3\,d^2ec\right)B+3\,Acde^2\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] 1/8*e^3*c*C*x^8+1/7*(B*c*e^3+3*C*c*d*e^2)*x^7+1/6*((a*e^3+3*c*d^2*e)*C+3*d*
e^2*c*B+e^3*c*A)*x^6+1/5*((3*a*d*e^2+c*d^3)*C+(a*e^3+3*c*d^2*e)*B+3*A*c*d*e
^2)*x^5+1/4*(3*d^2*e*a*C+(3*a*d*e^2+c*d^3)*B+(a*e^3+3*c*d^2*e)*A)*x^4+1/3*(
d^3*a*C+3*B*a*d^2*e+(3*a*d*e^2+c*d^3)*A)*x^3+1/2*(3*A*a*d^2*e+B*a*d^3)*x^2+
d^3*a*A*x
```

Maxima [A] time = 1.02217, size = 273, normalized size = 1.56

$$\frac{1}{8}\,Cce^3x^8+\frac{1}{7}\left(3\,Ccde^2+Bce^3\right)x^7+\frac{1}{6}\left(3\,Ccd^2e+3\,Bcde^2+(Ca+Ac)e^3\right)x^6+Aad^3x+\frac{1}{5}\left(Ccd^3+3\,Bcd^2e+Bae^3+3\,(Ca\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c*e^3*x^8 + 1/7*(3*C*c*d*e^2 + B*c*e^3)*x^7 + 1/6*(3*C*c*d^2*e + 3*B*
c*d*e^2 + (C*a + A*c)*e^3)*x^6 + A*a*d^3*x + 1/5*(C*c*d^3 + 3*B*c*d^2*e + B
*a*e^3 + 3*(C*a + A*c)*d*e^2)*x^5 + 1/4*(B*c*d^3 + 3*B*a*d*e^2 + A*a*e^3 +
3*(C*a + A*c)*d^2*e)*x^4 + 1/3*(3*B*a*d^2*e + 3*A*a*d*e^2 + (C*a + A*c)*d^3
)*x^3 + 1/2*(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

Fricas [A] time = 1.53428, size = 595, normalized size = 3.4

$$\frac{1}{8}x^8e^3cC + \frac{3}{7}x^7e^2dcC + \frac{1}{7}x^7e^3cB + \frac{1}{2}x^6ed^2cC + \frac{1}{6}x^6e^3aC + \frac{1}{2}x^6e^2dcB + \frac{1}{6}x^6e^3cA + \frac{1}{5}x^5d^3cC + \frac{3}{5}x^5e^2daC + \frac{3}{5}x^5ed^2cB$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^3*c*C + 3/7*x^7*e^2*d*c*C + 1/7*x^7*e^3*c*B + 1/2*x^6*e*d^2*c*C +
 1/6*x^6*e^3*a*C + 1/2*x^6*e^2*d*c*B + 1/6*x^6*e^3*c*A + 1/5*x^5*d^3*c*C +
3/5*x^5*e^2*d*a*C + 3/5*x^5*e*d^2*c*B + 1/5*x^5*e^3*a*B + 3/5*x^5*e^2*d*c*A
 + 3/4*x^4*e*d^2*a*C + 1/4*x^4*d^3*c*B + 3/4*x^4*e^2*d*a*B + 3/4*x^4*e*d^2*
c*A + 1/4*x^4*e^3*a*A + 1/3*x^3*d^3*a*C + x^3*e*d^2*a*B + 1/3*x^3*d^3*c*A +
 x^3*e^2*d*a*A + 1/2*x^2*d^3*a*B + 3/2*x^2*e*d^2*a*A + x*d^3*a*A
```

Sympy [A] time = 0.092206, size = 257, normalized size = 1.47

$$Aad^3x + \frac{Cce^3x^8}{8} + x^7\left(\frac{Bce^3}{7} + \frac{3Ccde^2}{7}\right) + x^6\left(\frac{Ace^3}{6} + \frac{Bcde^2}{2} + \frac{Cae^3}{6} + \frac{Ccd^2e}{2}\right) + x^5\left(\frac{3Acde^2}{5} + \frac{Bae^3}{5} + \frac{3Bcd^2e}{5} + \frac{3Ca}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**3*x + C*c*e**3*x**8/8 + x**7*(B*c*e**3/7 + 3*C*c*d*e**2/7) + x**6*(A
*c*e**3/6 + B*c*d*e**2/2 + C*a*e**3/6 + C*c*d**2*e/2) + x**5*(3*A*c*d*e**2/
5 + B*a*e**3/5 + 3*B*c*d**2*e/5 + 3*C*a*d*e**2/5 + C*c*d**3/5) + x**4*(A*a*
e**3/4 + 3*A*c*d**2*e/4 + 3*B*a*d*e**2/4 + B*c*d**3/4 + 3*C*a*d**2*e/4) + x
**3*(A*a*d*e**2 + A*c*d**3/3 + B*a*d**2*e + C*a*d**3/3) + x**2*(3*A*a*d**2*
e/2 + B*a*d**3/2)
```

Giac [A] time = 1.14464, size = 327, normalized size = 1.87

$$\frac{1}{8}\,Ccx^8e^3 + \frac{3}{7}\,Ccdx^7e^2 + \frac{1}{2}\,Ccd^2x^6e + \frac{1}{5}\,Ccd^3x^5 + \frac{1}{7}\,Bcx^7e^3 + \frac{1}{2}\,Bcdx^6e^2 + \frac{3}{5}\,Bcd^2x^5e + \frac{1}{4}\,Bcd^3x^4 + \frac{1}{6}\,Cax^6e^3 + \frac{1}{6}\,Acx^6e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c*x^8*e^3 + 3/7*C*c*d*x^7*e^2 + 1/2*C*c*d^2*x^6*e + 1/5*C*c*d^3*x^5 +
 1/7*B*c*x^7*e^3 + 1/2*B*c*d*x^6*e^2 + 3/5*B*c*d^2*x^5*e + 1/4*B*c*d^3*x^4
+ 1/6*C*a*x^6*e^3 + 1/6*A*c*x^6*e^3 + 3/5*C*a*d*x^5*e^2 + 3/5*A*c*d*x^5*e^2
 + 3/4*C*a*d^2*x^4*e + 3/4*A*c*d^2*x^4*e + 1/3*C*a*d^3*x^3 + 1/3*A*c*d^3*x^
3 + 1/5*B*a*x^5*e^3 + 3/4*B*a*d*x^4*e^2 + B*a*d^2*x^3*e + 1/2*B*a*d^3*x^2 +
 1/4*A*a*x^4*e^3 + A*a*d*x^3*e^2 + 3/2*A*a*d^2*x^2*e + A*a*d^3*x
```

3.19 $\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=175

$$\frac{(d+ex)^5 \left(aCe^2+c\left(6Cd^2-e(3Bd-Ae)\right)\right)}{5e^5}-\frac{(d+ex)^4 \left(ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)\right)}{4e^5}+\frac{(d+ex)^3 \left(ae^2+\cdots\right.}{\cdots}$$

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^3)/(3*e^5) - ((a*e^2*(2*
C*d - B*e) + c*d*(4*C*d^2 - e*(3*B*d - 2*A*e)))*(d + e*x)^4)/(4*e^5) + ((a*
C*e^2 + c*(6*C*d^2 - e*(3*B*d - A*e)))*(d + e*x)^5)/(5*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^6)/(6*e^5) + (c*C*(d + e*x)^7)/(7*e^5)
```

Rubi [A] time = 0.216273, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1628}

$$\frac{(d+ex)^5 \left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{5e^5}-\frac{(d+ex)^4 \left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{4e^5}+\frac{(d+ex)^3 \left(ae^2+cd^2\cdots\right.}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^3)/(3*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^4)/(4*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^5)/(5*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^6)/(6*e^5) + (c*C*(d + e*x)^7)/(7*e^5)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx = \int \left(\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^2}{e^4}+\frac{\left(-4cCd^3+cde(3Bd-2Ae)\cdots\right.}{e^4\cdots}\right.$$

$$=\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^3}{3e^5}-\frac{\left(4cCd^3-cde(3Bd-2Ae)+ae^2(\cdots\right.}{4e^5}$$

Mathematica [A] time = 0.0589761, size = 150, normalized size = 0.86

$$\frac{1}{5}x^5\left(aCe^2+Ace^2+2Bcde+cCd^2\right)+\frac{1}{4}x^4\left(aBe^2+2aCde+2Acde+Bcd^2\right)+\frac{1}{3}x^3\left(aAe^2+2aBde+aCd^2+Acd^2\right)+\frac{1}{2}ad$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^2*x + (a*d*(B*d + 2*A*e)*x^2)/2 + ((A*c*d^2 + a*C*d^2 + 2*a*B*d*e + a
*A*e^2)*x^3)/3 + ((B*c*d^2 + 2*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4)/4 + ((c*
C*d^2 + 2*B*c*d*e + A*c*e^2 + a*C*e^2)*x^5)/5 + (c*e*(2*C*d + B*e)*x^6)/6 +
 (c*C*e^2*x^7)/7
```

Maple [A] time = 0.046, size = 148, normalized size = 0.9

$$\frac{ce^2Cx^7}{7}+\frac{\left(ce^2B+2\,decC\right)x^6}{6}+\frac{\left(\left(ae^2+cd^2\right)C+2\,Bcde+Ace^2\right)x^5}{5}+\frac{\left(2\,adeC+\left(ae^2+cd^2\right)B+2\,Acde\right)x^4}{4}+\frac{\left(ad^2C+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] 1/7*c*e^2*C*x^7+1/6*(B*c*e^2+2*C*c*d*e)*x^6+1/5*((a*e^2+c*d^2)*C+2*B*c*d*e+
A*c*e^2)*x^5+1/4*(2*a*d*e*C+(a*e^2+c*d^2)*B+2*A*c*d*e)*x^4+1/3*(a*d^2*C+2*a
*B*d*e+A*(a*e^2+c*d^2))*x^3+1/2*(2*A*a*d*e+B*a*d^2)*x^2+a*d^2*A*x
```

Maxima [A] time = 1.06387, size = 190, normalized size = 1.09

$$\frac{1}{7}\,Cce^2x^7+\frac{1}{6}\left(2\,Ccde+Bce^2\right)x^6+\frac{1}{5}\left(Ccd^2+2\,Bcde+(Ca+Ac)e^2\right)x^5+Aad^2x+\frac{1}{4}\left(Bcd^2+Bae^2+2\,(Ca+Ac)de\right)x^4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*e^2*x^7 + 1/6*(2*C*c*d*e + B*c*e^2)*x^6 + 1/5*(C*c*d^2 + 2*B*c*d*e
+ (C*a + A*c)*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2 + B*a*e^2 + 2*(C*a + A*c)
*d*e)*x^4 + 1/3*(2*B*a*d*e + A*a*e^2 + (C*a + A*c)*d^2)*x^3 + 1/2*(B*a*d^2
```

```
+ 2*A*a*d*e)*x^2
```

Fricas [A] time = 1.47413, size = 423, normalized size = 2.42

$$\frac{1}{7}x^7e^2cC + \frac{1}{3}x^6edcC + \frac{1}{6}x^6e^2cB + \frac{1}{5}x^5d^2cC + \frac{1}{5}x^5e^2aC + \frac{2}{5}x^5edcB + \frac{1}{5}x^5e^2cA + \frac{1}{2}x^4edaC + \frac{1}{4}x^4d^2cB + \frac{1}{4}x^4e^2aB + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^2*c*C + 1/3*x^6*e*d*c*C + 1/6*x^6*e^2*c*B + 1/5*x^5*d^2*c*C + 1/5
*x^5*e^2*a*C + 2/5*x^5*e*d*c*B + 1/5*x^5*e^2*c*A + 1/2*x^4*e*d*a*C + 1/4*x^
4*d^2*c*B + 1/4*x^4*e^2*a*B + 1/2*x^4*e*d*c*A + 1/3*x^3*d^2*a*C + 2/3*x^3*e
*d*a*B + 1/3*x^3*d^2*c*A + 1/3*x^3*e^2*a*A + 1/2*x^2*d^2*a*B + x^2*e*d*a*A
+ x*d^2*a*A
```

Sympy [A] time = 0.082669, size = 173, normalized size = 0.99

$$Aad^2x + \frac{Cce^2x^7}{7} + x^6\left(\frac{Bce^2}{6} + \frac{Ccde}{3}\right) + x^5\left(\frac{Ace^2}{5} + \frac{2Bcde}{5} + \frac{Cae^2}{5} + \frac{Ccd^2}{5}\right) + x^4\left(\frac{Acde}{2} + \frac{Bae^2}{4} + \frac{Bcd^2}{4} + \frac{Cade}{2}\right) + x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**2*x + C*c*e**2*x**7/7 + x**6*(B*c*e**2/6 + C*c*d*e/3) + x**5*(A*c*e*
*2/5 + 2*B*c*d*e/5 + C*a*e**2/5 + C*c*d**2/5) + x**4*(A*c*d*e/2 + B*a*e**2/
4 + B*c*d**2/4 + C*a*d*e/2) + x**3*(A*a*e**2/3 + A*c*d**2/3 + 2*B*a*d*e/3 +
 C*a*d**2/3) + x**2*(A*a*d*e + B*a*d**2/2)
```

Giac [A] time = 1.1732, size = 231, normalized size = 1.32

$$\frac{1}{7}Ccx^7e^2 + \frac{1}{3}Ccdx^6e + \frac{1}{5}Ccd^2x^5 + \frac{1}{6}Bcx^6e^2 + \frac{2}{5}Bcdx^5e + \frac{1}{4}Bcd^2x^4 + \frac{1}{5}Cax^5e^2 + \frac{1}{5}Acx^5e^2 + \frac{1}{2}Cadx^4e + \frac{1}{2}Acdx^4e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c*x^7*e^2 + 1/3*C*c*d*x^6*e + 1/5*C*c*d^2*x^5 + 1/6*B*c*x^6*e^2 + 2/5
*B*c*d*x^5*e + 1/4*B*c*d^2*x^4 + 1/5*C*a*x^5*e^2 + 1/5*A*c*x^5*e^2 + 1/2*C*
a*d*x^4*e + 1/2*A*c*d*x^4*e + 1/3*C*a*d^2*x^3 + 1/3*A*c*d^2*x^3 + 1/4*B*a*x
^4*e^2 + 2/3*B*a*d*x^3*e + 1/2*B*a*d^2*x^2 + 1/3*A*a*x^3*e^2 + A*a*d*x^2*e
+ A*a*d^2*x
```

3.20 $\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=86

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

Rubi [A] time = 0.106489, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx=\int \left(aAd+a(Bd+Ae)x+(Acd+aCd+aBe)x^2+(Bcd+Ace+aCe)x^3+c(\right.$$
$$=aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aCd+aBe)x^3+\frac{1}{4}(Bcd+Ace+aCe)x^4$$

Mathematica [A] time = 0.0288402, size = 86, normalized size = 1.

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

Maple [A] time = 0.043, size = 79, normalized size = 0.9

$$\frac{cCex^6}{6}+\frac{(ceB+cdC)\,x^5}{5}+\frac{(Ace+Bcd+aCe)\,x^4}{4}+\frac{(Acd+aBe+Cad)\,x^3}{3}+\frac{(aAe+adB)\,x^2}{2}+aAdx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] 1/6*c*C*e*x^6+1/5*(B*c*e+C*c*d)*x^5+1/4*(A*c*e+B*c*d+C*a*e)*x^4+1/3*(A*c*d+
B*a*e+C*a*d)*x^3+1/2*(A*a*e+B*a*d)*x^2+a*A*d*x
```

Maxima [A] time = 0.981304, size = 108, normalized size = 1.26

$$\frac{1}{6}\,Ccex^6+\frac{1}{5}\,(Ccd+Bce)x^5+\frac{1}{4}\,(Bcd+(Ca+Ac)e)x^4+Aadx+\frac{1}{3}\,(Bae+(Ca+Ac)d)x^3+\frac{1}{2}\,(Bad+Aae)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*C*c*e*x^6 + 1/5*(C*c*d + B*c*e)*x^5 + 1/4*(B*c*d + (C*a + A*c)*e)*x^4 +
 A*a*d*x + 1/3*(B*a*e + (C*a + A*c)*d)*x^3 + 1/2*(B*a*d + A*a*e)*x^2
```

Fricas [A] time = 1.46919, size = 250, normalized size = 2.91

$$\frac{1}{6}x^6ecC+\frac{1}{5}x^5dcC+\frac{1}{5}x^5ecB+\frac{1}{4}x^4eaC+\frac{1}{4}x^4dcB+\frac{1}{4}x^4ecA+\frac{1}{3}x^3daC+\frac{1}{3}x^3eaB+\frac{1}{3}x^3dcA+\frac{1}{2}x^2daB+\frac{1}{2}x^2eaA+xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*c*C + 1/5*x^5*d*c*C + 1/5*x^5*e*c*B + 1/4*x^4*e*a*C + 1/4*x^4*d*c
*B + 1/4*x^4*e*c*A + 1/3*x^3*d*a*C + 1/3*x^3*e*a*B + 1/3*x^3*d*c*A + 1/2*x^
2*d*a*B + 1/2*x^2*e*a*A + x*d*a*A
```

Sympy [A] time = 0.070829, size = 97, normalized size = 1.13

$$Aadx + \frac{Ccex^6}{6} + x^5\left(\frac{Bce}{5} + \frac{Ccd}{5}\right) + x^4\left(\frac{Ace}{4} + \frac{Bcd}{4} + \frac{Cae}{4}\right) + x^3\left(\frac{Acd}{3} + \frac{Bae}{3} + \frac{Cad}{3}\right) + x^2\left(\frac{Aae}{2} + \frac{Bad}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d*x + C*c*e*x**6/6 + x**5*(B*c*e/5 + C*c*d/5) + x**4*(A*c*e/4 + B*c*d/4
 + C*a*e/4) + x**3*(A*c*d/3 + B*a*e/3 + C*a*d/3) + x**2*(A*a*e/2 + B*a*d/2)
```

Giac [A] time = 1.16253, size = 135, normalized size = 1.57

$$\frac{1}{6}Ccx^6e + \frac{1}{5}Ccdx^5 + \frac{1}{5}Bcx^5e + \frac{1}{4}Bcdx^4 + \frac{1}{4}Cax^4e + \frac{1}{4}Acx^4e + \frac{1}{3}Cadx^3 + \frac{1}{3}Acdx^3 + \frac{1}{3}Bax^3e + \frac{1}{2}Badx^2 + \frac{1}{2}Aax^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/6*C*c*x^6*e + 1/5*C*c*d*x^5 + 1/5*B*c*x^5*e + 1/4*B*c*d*x^4 + 1/4*C*a*x^4
*e + 1/4*A*c*x^4*e + 1/3*C*a*d*x^3 + 1/3*A*c*d*x^3 + 1/3*B*a*x^3*e + 1/2*B*
a*d*x^2 + 1/2*A*a*x^2*e + A*a*d*x
```

3.21 $\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

Rubi [A] time = 0.028997, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1657}

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx &= \int \left(aA+aBx+(Ac+aC)x^2+Bcx^3+cCx^4\right)\,dx\\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5\end{aligned}$$

Mathematica [A] time = 0.0124125, size = 46, normalized size = 1.

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

Maple [A] time = 0.047, size = 39, normalized size = 0.9

$$aAx+\frac{aBx^2}{2}+\frac{(Ac+aC)\,x^3}{3}+\frac{Bcx^4}{4}+\frac{cCx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

Maxima [A] time = 0.994897, size = 51, normalized size = 1.11

$$\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Bcx^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ac)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Fricas [A] time = 1.48862, size = 104, normalized size = 2.26

$$\frac{1}{5}x^5cC+\frac{1}{4}x^4cB+\frac{1}{3}x^3aC+\frac{1}{3}x^3cA+\frac{1}{2}x^2aB+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c*C + 1/4*x^4*c*B + 1/3*x^3*a*C + 1/3*x^3*c*A + 1/2*x^2*a*B + x*a*A
```

Sympy [A] time = 0.061914, size = 42, normalized size = 0.91

$$Aax + \frac{Bax^2}{2} + \frac{Bcx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

Giac [A] time = 1.1493, size = 54, normalized size = 1.17

$$\frac{1}{5}\,Ccx^5 + \frac{1}{4}\,Bcx^4 + \frac{1}{3}\,Cax^3 + \frac{1}{3}\,Acx^3 + \frac{1}{2}\,Bax^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + A*a*x
```

3.22 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal. Leaf size=145

$$\frac{x^2\left(aCe^2+c\left(Cd^2-e(Bd-Ae)\right)\right)}{2e^3}-\frac{x\left(ae^2(Cd-Be)+cd\left(Cd^2-e(Bd-Ae)\right)\right)}{e^4}+\frac{\left(ae^2+cd^2\right)\log(d+ex)\left(Ae^2-Bde+C\right.}{e^5}$$

```
[Out] -((((a*e^2*(C*d - B*e) + c*d*(C*d^2 - e*(B*d - A*e)))*x)/e^4) + ((a*C*e^2 +
c*(C*d^2 - e*(B*d - A*e)))*x^2)/(2*e^3) - (c*(C*d - B*e)*x^3)/(3*e^2) + (c*
C*x^4)/(4*e) + ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^5
```

Rubi [A] time = 0.245168, antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1628}

$$\frac{x^2\left(aCe^2-ce(Bd-Ae)+cCd^2\right)}{2e^3}-\frac{x\left(ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3\right)}{e^4}+\frac{\left(ae^2+cd^2\right)\log(d+ex)\left(Ae^2-Bde+Cd^2\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -((((c*C*d^3 - c*d*e*(B*d - A*e) + a*e^2*(C*d - B*e))*x)/e^4) + ((c*C*d^2 +
a*C*e^2 - c*e*(B*d - A*e))*x^2)/(2*e^3) - (c*(C*d - B*e)*x^3)/(3*e^2) + (c*
C*x^4)/(4*e) + ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^5
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)\left(A+Bx+Cx^2\right)}{d+ex}\,dx=\int\left(\frac{-ae^2(Cd-Be)-c\left(Cd^3-de(Bd-Ae)\right)}{e^4}+\frac{\left(cCd^2+aCe^2-ce(Bd-Ae)\right)x}{e^3}+\frac{c(-}{}\right.$$

$$=-\frac{\left(cCd^3-cde(Bd-Ae)+ae^2(Cd-Be)\right)x}{e^4}+\frac{\left(cCd^2+aCe^2-ce(Bd-Ae)\right)x^2}{2e^3}-\frac{c(C}{}$$

Mathematica [A] time = 0.0782833, size = 136, normalized size = 0.94

$$\frac{ex\left(6ae^2(2Be-2Cd+Cex)+2ce\left(3Ae(ex-2d)+B\left(6d^2-3dex+2e^2x^2\right)\right)+cC\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)\right)+12\left(\right.}{12e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(6*a*e^2*(-2*C*d + 2*B*e + C*e*x) + c*C*(-12*d^3 + 6*d^2*e*x - 4*d*e^2
*x^2 + 3*e^3*x^3) + 2*c*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e*x + 2*e^2*
x^2))) + 12*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/(12*e^
5)
```

Maple [A] time = 0.049, size = 210, normalized size = 1.5

$$\frac{Ccx^4}{4e}+\frac{Bcx^3}{3e}-\frac{Cx^3cd}{3e^2}+\frac{Ax^2c}{2e}-\frac{Bcx^2d}{2e^2}+\frac{Cx^2a}{2e}+\frac{Cx^2cd^2}{2e^3}-\frac{Acdx}{e^2}+\frac{aBx}{e}+\frac{Bcd^2x}{e^3}-\frac{aCdx}{e^2}-\frac{Ccd^3x}{e^4}+\frac{\ln(ex+d)Aa}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x)
```

```
[Out] 1/4*c*C*x^4/e+1/3/e*B*x^3*c-1/3/e^2*C*x^3*c*d+1/2/e*A*x^2*c-1/2/e^2*B*x^2*c
*d+1/2/e*C*x^2*a+1/2/e^3*C*x^2*c*d^2-1/e^2*A*c*d*x+1/e*B*a*x+1/e^3*B*c*d^2*
x-1/e^2*C*a*d*x-1/e^4*C*c*d^3*x+1/e*ln(e*x+d)*A*a+1/e^3*ln(e*x+d)*A*c*d^2-1
/e^2*ln(e*x+d)*B*a*d-1/e^4*ln(e*x+d)*B*c*d^3+1/e^3*ln(e*x+d)*C*a*d^2+1/e^5*
ln(e*x+d)*C*c*d^4
```

Maxima [A] time = 1.00087, size = 215, normalized size = 1.48

$$\frac{3Cce^3x^4-4\left(Ccde^2-Bce^3\right)x^3+6\left(Ccd^2e-Bcde^2+(Ca+Ac)e^3\right)x^2-12\left(Ccd^3-Bcd^2e-Bae^3+(Ca+Ac)de^2\right)x}{12e^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/12*(3*C*c*e^3*x^4 - 4*(C*c*d*e^2 - B*c*e^3)*x^3 + 6*(C*c*d^2*e - B*c*d*e^
2 + (C*a + A*c)*e^3)*x^2 - 12*(C*c*d^3 - B*c*d^2*e - B*a*e^3 + (C*a + A*c)*
```

```
d*e^2)*x)/e^4 + (C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^
2*e^2)*log(e*x + d)/e^5
```

Fricas [A] time = 1.68902, size = 344, normalized size = 2.37

$$\frac{3\,Cce^4x^4 - 4\left(Ccde^3 - Bce^4\right)x^3 + 6\left(Ccd^2e^2 - Bcde^3 + (Ca + Ac)e^4\right)x^2 - 12\left(Ccd^3e - Bcd^2e^2 - Bae^4 + (Ca + Ac)de^3\right)x +}{12\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c*e^4*x^4 - 4*(C*c*d*e^3 - B*c*e^4)*x^3 + 6*(C*c*d^2*e^2 - B*c*d*
e^3 + (C*a + A*c)*e^4)*x^2 - 12*(C*c*d^3*e - B*c*d^2*e^2 - B*a*e^4 + (C*a +
 A*c)*d*e^3)*x + 12*(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c
)*d^2*e^2)*log(e*x + d))/e^5
```

Sympy [A] time = 0.766002, size = 143, normalized size = 0.99

$$\frac{Ccx^4}{4e} - \frac{x^3\left(-Bce + Ccd\right)}{3e^2} + \frac{x^2\left(Ace^2 - Bcde + Cae^2 + Ccd^2\right)}{2e^3} - \frac{x\left(Acde^2 - Bae^3 - Bcd^2e + Cade^2 + Ccd^3\right)}{e^4} + \frac{\left(ae^2 + cd^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c*x**4/(4*e) - x**3*(-B*c*e + C*c*d)/(3*e**2) + x**2*(A*c*e**2 - B*c*d*e
+ C*a*e**2 + C*c*d**2)/(2*e**3) - x*(A*c*d*e**2 - B*a*e**3 - B*c*d**2*e + C
*a*d*e**2 + C*c*d**3)/e**4 + (a*e**2 + c*d**2)*(A*e**2 - B*d*e + C*d**2)*lo
g(d + e*x)/e**5
```

Giac [A] time = 1.12377, size = 230, normalized size = 1.59

$$\left(Ccd^4 - Bcd^3e + Cad^2e^2 + Acd^2e^2 - Bade^3 + Aae^4\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{12}\left(3\,Ccx^4e^3 - 4\,Ccdx^3e^2 + 6\,Ccd^2x^2e - 12\,Ccd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c*d^4 - B*c*d^3*e + C*a*d^2*e^2 + A*c*d^2*e^2 - B*a*d*e^3 + A*a*e^4)*e^(
-5)*log(abs(x*e + d)) + 1/12*(3*C*c*x^4*e^3 - 4*C*c*d*x^3*e^2 + 6*C*c*d^2*x
^2*e - 12*C*c*d^3*x + 4*B*c*x^3*e^3 - 6*B*c*d*x^2*e^2 + 12*B*c*d^2*x*e + 6*
C*a*x^2*e^3 + 6*A*c*x^2*e^3 - 12*C*a*d*x*e^2 - 12*A*c*d*x*e^2 + 12*B*a*x*e^
3)*e^(-4)
```

3.23 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2} dx$

Optimal. Leaf size=153

$$\frac{x\left(aCe^2+c\left(3Cd^2-e(2Bd-Ae)\right)\right)}{e^4}-\frac{\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{e^5(d+ex)}-\frac{\log(d+ex)\left(ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd\right.\right.}{e^5}$$

```
[Out] ((a*C*e^2 + c*(3*C*d^2 - e*(2*B*d - A*e)))*x)/e^4 - (c*(2*C*d - B*e)*x^2)/(
2*e^3) + (c*C*x^3)/(3*e^2) - ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2))/(e^5
*(d + e*x)) - ((a*e^2*(2*C*d - B*e) + c*d*(4*C*d^2 - e*(3*B*d - 2*A*e)))*Lo
g[d + e*x])/e^5
```

Rubi [A] time = 0.204897, antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1628}

$$\frac{x\left(aCe^2-ce(2Bd-Ae)+3cCd^2\right)}{e^4}-\frac{\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{e^5(d+ex)}-\frac{\log(d+ex)\left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((3*c*C*d^2 + a*C*e^2 - c*e*(2*B*d - A*e))*x)/e^4 - (c*(2*C*d - B*e)*x^2)/(
2*e^3) + (c*C*x^3)/(3*e^2) - ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2))/(e^5
*(d + e*x)) - ((4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*Lo
g[d + e*x])/e^5
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)\left(A+Bx+Cx^2\right)}{(d+ex)^2}\,dx = \int \left(\frac{3cCd^2+aCe^2-ce(2Bd-Ae)}{e^4} + \frac{c(-2Cd+Be)x}{e^3} + \frac{cCx^2}{e^2} + \frac{\left(cd^2+ae^2\right)\left(Cd^2-Bd\right.}{e^4(d+ex)^2}\right.$$

$$= \frac{\left(3cCd^2+aCe^2-ce(2Bd-Ae)\right)x}{e^4} - \frac{c(2Cd-Be)x^2}{2e^3} + \frac{cCx^3}{3e^2} - \frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde\right.}{e^5(d+ex)}$$

Mathematica [A] time = 0.158209, size = 142, normalized size = 0.93

$$\frac{6ex\left(aCe^2+ce(Ae-2Bd)+3cCd^2\right) - \frac{6\left(ae^2+cd^2\right)\left(e(Ae-Bd)+Cd^2\right)}{d+ex} + 6\log(d+ex)\left(ae^2(Be-2Cd)+cde(3Bd-2Ae)-4cCd^3\right)}{6e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (6*e*(3*c*C*d^2 + a*C*e^2 + c*e*(-2*B*d + A*e))*x + 3*c*e^2*(-2*C*d + B*e)*
x^2 + 2*c*C*e^3*x^3 - (6*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e
*x) + 6*(-4*c*C*d^3 + c*d*e*(3*B*d - 2*A*e) + a*e^2*(-2*C*d + B*e))*Log[d +
 e*x])/(6*e^5)
```

Maple [A] time = 0.056, size = 234, normalized size = 1.5

$$\frac{Ccx^3}{3\,e^2} + \frac{Bcx^2}{2\,e^2} - \frac{Cx^2cd}{e^3} + \frac{Acx}{e^2} - 2\,\frac{Bcdx}{e^3} + \frac{aCx}{e^2} + 3\,\frac{Ccd^2x}{e^4} - 2\,\frac{\ln(ex+d)\,Acd}{e^3} + \frac{\ln(ex+d)\,Ba}{e^2} + 3\,\frac{\ln(ex+d)\,Bcd^2}{e^4} - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x)
```

```
[Out] 1/3*c*C*x^3/e^2+1/2/e^2*B*x^2*c-1/e^3*C*x^2*c*d+1/e^2*A*c*x-2/e^3*B*c*d*x+1
/e^2*a*C*x+3/e^4*C*c*d^2*x-2/e^3*ln(e*x+d)*A*c*d+1/e^2*ln(e*x+d)*B*a+3/e^4*
ln(e*x+d)*B*c*d^2-2/e^3*ln(e*x+d)*C*a*d-4/e^5*ln(e*x+d)*C*c*d^3-1/e/(e*x+d)
*A*a-1/e^3/(e*x+d)*A*c*d^2+1/e^2/(e*x+d)*B*d*a+1/e^4/(e*x+d)*B*c*d^3-1/e^3/
(e*x+d)*C*a*d^2-1/e^5/(e*x+d)*C*c*d^4
```

Maxima [A] time = 0.998793, size = 228, normalized size = 1.49

$$-\frac{Ccd^4 - Bcd^3e - Bade^3 + Aae^4 + (Ca + Ac)d^2e^2}{e^6x + de^5} + \frac{2\,Cce^2x^3 - 3\left(2\,Ccde - Bce^2\right)x^2 + 6\left(3\,Ccd^2 - 2\,Bcde + (Ca + Ac)e^2\right.}{6\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^2*e^2)/(e^6*x +
 d*e^5) + 1/6*(2*C*c*e^2*x^3 - 3*(2*C*c*d*e - B*c*e^2)*x^2 + 6*(3*C*c*d^2 -
 2*B*c*d*e + (C*a + A*c)*e^2)*x)/e^4 - (4*C*c*d^3 - 3*B*c*d^2*e - B*a*e^3 +
 2*(C*a + A*c)*d*e^2)*log(e*x + d)/e^5
```

Fricas [A] time = 1.58326, size = 545, normalized size = 3.56

$$\frac{2\,Cce^4x^4 - 6\,Ccd^4 + 6\,Bcd^3e + 6\,Bade^3 - 6\,Aae^4 - 6\,(Ca + Ac)d^2e^2 - \left(4\,Ccde^3 - 3\,Bce^4\right)x^3 + 3\left(4\,Ccd^2e^2 - 3\,Bcde^3 + 2\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*c*e^4*x^4 - 6*C*c*d^4 + 6*B*c*d^3*e + 6*B*a*d*e^3 - 6*A*a*e^4 - 6*
(C*a + A*c)*d^2*e^2 - (4*C*c*d*e^3 - 3*B*c*e^4)*x^3 + 3*(4*C*c*d^2*e^2 - 3*
B*c*d*e^3 + 2*(C*a + A*c)*e^4)*x^2 + 6*(3*C*c*d^3*e - 2*B*c*d^2*e^2 + (C*a
+ A*c)*d*e^3)*x - 6*(4*C*c*d^4 - 3*B*c*d^3*e - B*a*d*e^3 + 2*(C*a + A*c)*d^
2*e^2 + (4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x)*lo
g(e*x + d))/(e^6*x + d*e^5)
```

Sympy [A] time = 1.56747, size = 184, normalized size = 1.2

$$\frac{Ccx^3}{3e^2} - \frac{Aae^4 + Acd^2e^2 - Bade^3 - Bcd^3e + Cad^2e^2 + Ccd^4}{de^5 + e^6x} - \frac{x^2\,(-Bce + 2Ccd)}{2e^3} + \frac{x\left(Ace^2 - 2Bcde + Cae^2 + 3Ccd^2\right)}{e^4} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c*x**3/(3*e**2) - (A*a*e**4 + A*c*d**2*e**2 - B*a*d*e**3 - B*c*d**3*e + C
*a*d**2*e**2 + C*c*d**4)/(d*e**5 + e**6*x) - x**2*(-B*c*e + 2*C*c*d)/(2*e**
3) + x*(A*c*e**2 - 2*B*c*d*e + C*a*e**2 + 3*C*c*d**2)/e**4 - (2*A*c*d*e**2
- B*a*e**3 - 3*B*c*d**2*e + 2*C*a*d*e**2 + 4*C*c*d**3)*log(d + e*x)/e**5
```

Giac [A] time = 1.14848, size = 324, normalized size = 2.12

$$\frac{1}{6}\left(2\,Cc - \frac{3\left(4\,Ccde - Bce^2\right)e^{(-1)}}{xe+d} + \frac{6\left(6\,Ccd^2e^2 - 3\,Bcde^3 + Cae^4 + Ace^4\right)e^{(-2)}}{(xe+d)^2}\right)(xe+d)^3e^{(-5)} + \left(4\,Ccd^3 - 3\,Bcd^2e + 2\,Cad\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*C*c - 3*(4*C*c*d*e - B*c*e^2)*e^(-1)/(x*e + d) + 6*(6*C*c*d^2*e^2 -
3*B*c*d*e^3 + C*a*e^4 + A*c*e^4)*e^(-2)/(x*e + d)^2)*(x*e + d)^3*e^(-5) + (
4*C*c*d^3 - 3*B*c*d^2*e + 2*C*a*d*e^2 + 2*A*c*d*e^2 - B*a*e^3)*e^(-5)*log(a
bs(x*e + d)*e^(-1)/(x*e + d)^2) - (C*c*d^4*e^3/(x*e + d) - B*c*d^3*e^4/(x*e
 + d) + C*a*d^2*e^5/(x*e + d) + A*c*d^2*e^5/(x*e + d) - B*a*d*e^6/(x*e + d)
 + A*a*e^7/(x*e + d))*e^(-8)
```

3.24 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

Optimal. Leaf size=156

$$\frac{ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)}{e^5(d+ex)}-\frac{\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{2e^5(d+ex)^2}+\frac{\log(d+ex)\left(aCe^2+c\left(6Cd^2-e(3Bd-\right.\right.}{e^5}$$

```
[Out] -((c*(3*C*d - B*e)*x)/e^4) + (c*C*x^2)/(2*e^3) - ((c*d^2 + a*e^2)*(C*d^2 -
B*d*e + A*e^2))/(2*e^5*(d + e*x)^2) + (a*e^2*(2*C*d - B*e) + c*d*(4*C*d^2 -
 e*(3*B*d - 2*A*e)))/(e^5*(d + e*x)) + ((a*C*e^2 + c*(6*C*d^2 - e*(3*B*d -
A*e)))*Log[d + e*x])/e^5
```

Rubi [A] time = 0.198574, antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1628}

$$\frac{ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3}{e^5(d+ex)}-\frac{\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{2e^5(d+ex)^2}+\frac{\log(d+ex)\left(aCe^2-ce(3Bd-Ae)+6cCd\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(3*C*d - B*e)*x)/e^4) + (c*C*x^2)/(2*e^3) - ((c*d^2 + a*e^2)*(C*d^2 -
B*d*e + A*e^2))/(2*e^5*(d + e*x)^2) + (4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) +
a*e^2*(2*C*d - B*e))/(e^5*(d + e*x)) + ((6*c*C*d^2 + a*C*e^2 - c*e*(3*B*d -
 A*e))*Log[d + e*x])/e^5
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \int \left(\frac{c(-3Cd+Be)}{e^4} + \frac{cCx}{e^3} + \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{e^4(d+ex)^3} + \frac{-4cCd^3+cde(3Bd-2A}{e^4(d+e}\right.$$
$$= -\frac{c(3Cd-Be)x}{e^4} + \frac{cCx^2}{2e^3} - \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{2e^5(d+ex)^2} + \frac{4cCd^3-cde(3Bd-2Ae)}{e^5(d+ex)}$$

Mathematica [A] time = 0.102892, size = 176, normalized size = 1.13

$$\frac{-aBe^3+2aCde^2+2Acde^2-3Bcd^2e+4cCd^3}{e^5(d+ex)} + \frac{-aAe^4+aBde^3-aCd^2e^2-Acd^2e^2+Bcd^3e-cCd^4}{2e^5(d+ex)^2} + \frac{\log(d+ex)\left(aCe^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (c*(-3*C*d + B*e)*x)/e^4 + (c*C*x^2)/(2*e^3) + (-(c*C*d^4) + B*c*d^3*e - A*
c*d^2*e^2 - a*C*d^2*e^2 + a*B*d*e^3 - a*A*e^4)/(2*e^5*(d + e*x)^2) + (4*c*C
*d^3 - 3*B*c*d^2*e + 2*A*c*d*e^2 + 2*a*C*d*e^2 - a*B*e^3)/(e^5*(d + e*x)) +
 ((6*c*C*d^2 - 3*B*c*d*e + A*c*e^2 + a*C*e^2)*Log[d + e*x])/e^5
```

Maple [A] time = 0.056, size = 257, normalized size = 1.7

$$\frac{Ccx^2}{2e^3} + \frac{Bcx}{e^3} - 3\frac{Ccdx}{e^4} - \frac{aA}{2e(ex+d)^2} - \frac{Acd^2}{2e^3(ex+d)^2} + \frac{Bda}{2e^2(ex+d)^2} + \frac{Bcd^3}{2e^4(ex+d)^2} - \frac{Cd^2a}{2e^3(ex+d)^2} - \frac{Ccd^4}{2e^5(ex+d)^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x)
```

```
[Out] 1/2*c*C*x^2/e^3+c/e^3*B*x-3*c/e^4*C*d*x-1/2/e/(e*x+d)^2*A*a-1/2/e^3/(e*x+d)
^2*A*d^2*c+1/2/e^2/(e*x+d)^2*B*d*a+1/2/e^4/(e*x+d)^2*B*c*d^3-1/2/e^3/(e*x+d
)^2*C*d^2*a-1/2/e^5/(e*x+d)^2*C*c*d^4+1/e^3*ln(e*x+d)*A*c-3/e^4*ln(e*x+d)*B
*c*d+1/e^3*ln(e*x+d)*a*C+6/e^5*ln(e*x+d)*C*c*d^2+2/e^3/(e*x+d)*A*c*d-1/e^2/
(e*x+d)*B*a-3/e^4/(e*x+d)*B*c*d^2+2/e^3/(e*x+d)*C*a*d+4/e^5/(e*x+d)*C*c*d^3
```

Maxima [A] time = 0.995817, size = 239, normalized size = 1.53

$$\frac{7\,Ccd^4 - 5\,Bcd^3e - Bade^3 - Aae^4 + 3\,(Ca + Ac)d^2e^2 + 2\left(4\,Ccd^3e - 3\,Bcd^2e^2 - Bae^4 + 2\,(Ca + Ac)de^3\right)x}{2\left(e^7x^2 + 2\,de^6x + d^2e^5\right)} + \frac{Ccex^2 - 2\,(3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a + A*c)*d^2*e^2
+ 2*(4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x)/(e^7*x
^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(C*c*e*x^2 - 2*(3*C*c*d - B*c*e)*x)/e^4 + (
6*C*c*d^2 - 3*B*c*d*e + (C*a + A*c)*e^2)*log(e*x + d)/e^5
```

Fricas [A] time = 1.68601, size = 585, normalized size = 3.75

$$\frac{Cce^4x^4 + 7\,Ccd^4 - 5\,Bcd^3e - Bade^3 - Aae^4 + 3\,(Ca + Ac)d^2e^2 - 2\left(2\,Ccde^3 - Bce^4\right)x^3 - \left(11\,Ccd^2e^2 - 4\,Bcde^3\right)x^2 + 2\,(Cc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*c*e^4*x^4 + 7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a +
 A*c)*d^2*e^2 - 2*(2*C*c*d*e^3 - B*c*e^4)*x^3 - (11*C*c*d^2*e^2 - 4*B*c*d*e
^3)*x^2 + 2*(C*c*d^3*e - 2*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x +
 2*(6*C*c*d^4 - 3*B*c*d^3*e + (C*a + A*c)*d^2*e^2 + (6*C*c*d^2*e^2 - 3*B*c*
d*e^3 + (C*a + A*c)*e^4)*x^2 + 2*(6*C*c*d^3*e - 3*B*c*d^2*e^2 + (C*a + A*c)
*d*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] time = 5.84764, size = 204, normalized size = 1.31

$$\frac{Ccx^2}{2e^3} + \frac{-Aae^4 + 3Acd^2e^2 - Bade^3 - 5Bcd^3e + 3Cad^2e^2 + 7Ccd^4 + x\left(4Acde^3 - 2Bae^4 - 6Bcd^2e^2 + 4Cade^3 + 8Ccd^3e\right)}{2d^2e^5 + 4de^6x + 2e^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c*x**2/(2*e**3) + (-A*a*e**4 + 3*A*c*d**2*e**2 - B*a*d*e**3 - 5*B*c*d**3*
e + 3*C*a*d**2*e**2 + 7*C*c*d**4 + x*(4*A*c*d*e**3 - 2*B*a*e**4 - 6*B*c*d**
2*e**2 + 4*C*a*d*e**3 + 8*C*c*d**3*e))/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x
**2) - x*(-B*c*e + 3*C*c*d)/e**4 + (A*c*e**2 - 3*B*c*d*e + C*a*e**2 + 6*C*c
*d**2)*log(d + e*x)/e**5
```

Giac [A] time = 1.1388, size = 225, normalized size = 1.44

$$\left(6\,Ccd^2 - 3\,Bcde + Cae^2 + Ace^2\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{2}\left(Ccx^2e^3 - 6\,Ccdxe^2 + 2\,Bcxe^3\right)e^{(-6)} + \frac{\left(7\,Ccd^4 - 5\,Bcd^3e + 3\,Cad^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (6*C*c*d^2 - 3*B*c*d*e + C*a*e^2 + A*c*e^2)*e^(-5)*log(abs(x*e + d)) + 1/2*
(C*c*x^2*e^3 - 6*C*c*d*x*e^2 + 2*B*c*x*e^3)*e^(-6) + 1/2*(7*C*c*d^4 - 5*B*c
*d^3*e + 3*C*a*d^2*e^2 + 3*A*c*d^2*e^2 - B*a*d*e^3 - A*a*e^4 + 2*(4*C*c*d^3
*e - 3*B*c*d^2*e^2 + 2*C*a*d*e^3 + 2*A*c*d*e^3 - B*a*e^4)*x)*e^(-5)/(x*e +
d)^2
```

3.25 $\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=304

$$\frac{1}{4}a^2ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^2Ad^3x+\frac{1}{8}cex^8\left(2aCe^2+c\left(e(Ae+3Bd)+3Cd^2\right)\right)+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+cd\left(3e(A\right.\right.$$

```
[Out] a^2*A*d^3*x + (a*d*(a*d*(C*d + 3*B*e) + A*(2*c*d^2 + 3*a*e^2))*x^3)/3 + (a^
2*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e
^2*(3*C*d + B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*e*(a*C*e^2 + 2*c*(3*
C*d^2 + e*(3*B*d + A*e)))*x^6)/6 + (c*(2*a*e^2*(3*C*d + B*e) + c*d*(C*d^2 +
 3*e*(B*d + A*e)))*x^7)/7 + (c*e*(2*a*C*e^2 + c*(3*C*d^2 + e*(3*B*d + A*e))
)*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*x^10)/10 + (d^2*(B*d
+ 3*A*e)*(a + c*x^2)^3)/(6*c)
```

Rubi [A] time = 0.534531, antiderivative size = 301, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1582, 1810}

$$\frac{1}{4}a^2ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^2Ad^3x+\frac{1}{8}cex^8\left(2aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+3cde(Ae+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a*d*(a*d*(C*d + 3*B*e) + A*(2*c*d^2 + 3*a*e^2))*x^3)/3 + (a^
2*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e
^2*(3*C*d + B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*e*(6*c*C*d^2 + a*C*e
^2 + 2*c*e*(3*B*d + A*e))*x^6)/6 + (c*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 2*a*
e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(3*B*d + A*e)
)*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*x^10)/10 + (d^2*(B*d
+ 3*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
```

```
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx = \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a+cx^2\right)^2 \left(-\left(Bd^3+3Ad^2e\right)x+(d+ex)^3 \left(A\right.\right.$$

$$= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a^2Ad^3+ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right.\right.\right.$$

$$= a^2Ad^3x + \frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3 + \frac{1}{4}a^2e\left(3Cd^2+e(3Bd\right.$$

Mathematica [A] time = 0.134529, size = 335, normalized size = 1.1

$$\frac{1}{2}a^2d^2x^2(3Ae+Bd)+a^2Ad^3x+\frac{1}{8}cex^8\left(2aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a^2*d^2*(B*d + 3*A*e)*x^2)/2 + (a*d*(a*d*(C*d + 3*B*e) + A*(
2*c*d^2 + 3*a*e^2))*x^3)/3 + (a*(2*B*c*d^3 + 6*A*c*d^2*e + 3*a*C*d^2*e + 3*
a*B*d*e^2 + a*A*e^3)*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e^2*(3*C*d +
 B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + ((a*C*e*(6*c*d^2 + a*e^2) + A*c*e*
(3*c*d^2 + 2*a*e^2) + B*c*d*(c*d^2 + 6*a*e^2))*x^6)/6 + (c*(c*C*d^3 + 3*c*d
*e*(B*d + A*e) + 2*a*e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^
2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*
x^10)/10
```

Maple [A] time = 0.046, size = 385, normalized size = 1.3

$$\frac{c^2Ce^3x^{10}}{10}+\frac{\left(e^3c^2B+3\,de^2c^2C\right)x^9}{9}+\frac{\left(\left(2\,ace^3+3\,c^2d^2e\right)C+3\,de^2c^2B+e^3c^2A\right)x^8}{8}+\frac{\left(\left(6\,ade^2c+c^2d^3\right)C+\left(2\,ace^3+3\,c^2d\right.\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/10*c^2*C*e^3*x^10+1/9*(B*c^2*e^3+3*C*c^2*d*e^2)*x^9+1/8*((2*a*c*e^3+3*c^2
*d^2*e)*C+3*d*e^2*c^2*B+e^3*c^2*A)*x^8+1/7*((6*a*c*d*e^2+c^2*d^3)*C+(2*a*c*
e^3+3*c^2*d^2*e)*B+3*d*e^2*c^2*A)*x^7+1/6*((a^2*e^3+6*a*c*d^2*e)*C+(6*a*c*d
*e^2+c^2*d^3)*B+(2*a*c*e^3+3*c^2*d^2*e)*A)*x^6+1/5*((3*a^2*d*e^2+2*a*c*d^3)
*C+(a^2*e^3+6*a*c*d^2*e)*B+(6*a*c*d*e^2+c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^2*C+
(3*a^2*d*e^2+2*a*c*d^3)*B+(a^2*e^3+6*a*c*d^2*e)*A)*x^4+1/3*(d^3*a^2*C+3*d^2
*e*a^2*B+(3*a^2*d*e^2+2*a*c*d^3)*A)*x^3+1/2*(3*A*a^2*d^2*e+B*a^2*d^3)*x^2+a
^2*A*d^3*x
```

Maxima [A] time = 1.02582, size = 486, normalized size = 1.6

$$\frac{1}{10}Cc^2e^3x^{10} + \frac{1}{9}\left(3Cc^2de^2 + Bc^2e^3\right)x^9 + \frac{1}{8}\left(3Cc^2d^2e + 3Bc^2de^2 + \left(2Cac + Ac^2\right)e^3\right)x^8 + \frac{1}{7}\left(Cc^2d^3 + 3Bc^2d^2e + 2Bace^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*e^3*x^10 + 1/9*(3*C*c^2*d*e^2 + B*c^2*e^3)*x^9 + 1/8*(3*C*c^2*d^
2*e + 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^8 + 1/7*(C*c^2*d^3 + 3*B*c^2
*d^2*e + 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x^7 + A*a^2*d^3*x + 1/6*(
B*c^2*d^3 + 6*B*a*c*d*e^2 + 3*(2*C*a*c + A*c^2)*d^2*e + (C*a^2 + 2*A*a*c)*e
^3)*x^6 + 1/5*(6*B*a*c*d^2*e + B*a^2*e^3 + (2*C*a*c + A*c^2)*d^3 + 3*(C*a^2
 + 2*A*a*c)*d*e^2)*x^5 + 1/4*(2*B*a*c*d^3 + 3*B*a^2*d*e^2 + A*a^2*e^3 + 3*(
C*a^2 + 2*A*a*c)*d^2*e)*x^4 + 1/3*(3*B*a^2*d^2*e + 3*A*a^2*d*e^2 + (C*a^2 +
 2*A*a*c)*d^3)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

Fricas [A] time = 1.44218, size = 992, normalized size = 3.26

$$\frac{1}{10}x^{10}e^3c^2C + \frac{1}{3}x^9e^2dc^2C + \frac{1}{9}x^9e^3c^2B + \frac{3}{8}x^8ed^2c^2C + \frac{1}{4}x^8e^3caC + \frac{3}{8}x^8e^2dc^2B + \frac{1}{8}x^8e^3c^2A + \frac{1}{7}x^7d^3c^2C + \frac{6}{7}x^7e^2dcaC +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^3*c^2*C + 1/3*x^9*e^2*d*c^2*C + 1/9*x^9*e^3*c^2*B + 3/8*x^8*e*d
^2*c^2*C + 1/4*x^8*e^3*c*a*C + 3/8*x^8*e^2*d*c^2*B + 1/8*x^8*e^3*c^2*A + 1/
7*x^7*d^3*c^2*C + 6/7*x^7*e^2*d*c*a*C + 3/7*x^7*e*d^2*c^2*B + 2/7*x^7*e^3*c
*a*B + 3/7*x^7*e^2*d*c^2*A + x^6*e*d^2*c*a*C + 1/6*x^6*e^3*a^2*C + 1/6*x^6*
d^3*c^2*B + x^6*e^2*d*c*a*B + 1/2*x^6*e*d^2*c^2*A + 1/3*x^6*e^3*c*a*A + 2/5
*x^5*d^3*c*a*C + 3/5*x^5*e^2*d*a^2*C + 6/5*x^5*e*d^2*c*a*B + 1/5*x^5*e^3*a^
2*B + 1/5*x^5*d^3*c^2*A + 6/5*x^5*e^2*d*c*a*A + 3/4*x^4*e*d^2*a^2*C + 1/2*x
^4*d^3*c*a*B + 3/4*x^4*e^2*d*a^2*B + 3/2*x^4*e*d^2*c*a*A + 1/4*x^4*e^3*a^2*
A + 1/3*x^3*d^3*a^2*C + x^3*e*d^2*a^2*B + 2/3*x^3*d^3*c*a*A + x^3*e^2*d*a^2
*A + 1/2*x^2*d^3*a^2*B + 3/2*x^2*e*d^2*a^2*A + x*d^3*a^2*A
```

Sympy [A] time = 0.118716, size = 445, normalized size = 1.46

$$Aa^2d^3x + \frac{Cc^2e^3x^{10}}{10} + x^9\left(\frac{Bc^2e^3}{9} + \frac{Cc^2de^2}{3}\right) + x^8\left(\frac{Ac^2e^3}{8} + \frac{3Bc^2de^2}{8} + \frac{Cace^3}{4} + \frac{3Cc^2d^2e}{8}\right) + x^7\left(\frac{3Ac^2de^2}{7} + \frac{2Bace^3}{7} + \frac{3B}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**3*x + C*c**2*e**3*x**10/10 + x**9*(B*c**2*e**3/9 + C*c**2*d*e**2/
3) + x**8*(A*c**2*e**3/8 + 3*B*c**2*d*e**2/8 + C*a*c*e**3/4 + 3*C*c**2*d**2
*e/8) + x**7*(3*A*c**2*d*e**2/7 + 2*B*a*c*e**3/7 + 3*B*c**2*d**2*e/7 + 6*C*
a*c*d*e**2/7 + C*c**2*d**3/7) + x**6*(A*a*c*e**3/3 + A*c**2*d**2*e/2 + B*a*
c*d*e**2 + B*c**2*d**3/6 + C*a**2*e**3/6 + C*a*c*d**2*e) + x**5*(6*A*a*c*d*
e**2/5 + A*c**2*d**3/5 + B*a**2*e**3/5 + 6*B*a*c*d**2*e/5 + 3*C*a**2*d*e**2
/5 + 2*C*a*c*d**3/5) + x**4*(A*a**2*e**3/4 + 3*A*a*c*d**2*e/2 + 3*B*a**2*d*
e**2/4 + B*a*c*d**3/2 + 3*C*a**2*d**2*e/4) + x**3*(A*a**2*d*e**2 + 2*A*a*c*
d**3/3 + B*a**2*d**2*e + C*a**2*d**3/3) + x**2*(3*A*a**2*d**2*e/2 + B*a**2*
d**3/2)
```

Giac [A] time = 1.14872, size = 571, normalized size = 1.88

$$\frac{1}{10}Cc^2x^{10}e^3 + \frac{1}{3}Cc^2dx^9e^2 + \frac{3}{8}Cc^2d^2x^8e + \frac{1}{7}Cc^2d^3x^7 + \frac{1}{9}Bc^2x^9e^3 + \frac{3}{8}Bc^2dx^8e^2 + \frac{3}{7}Bc^2d^2x^7e + \frac{1}{6}Bc^2d^3x^6 + \frac{1}{4}Cacx^8e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*x^10*e^3 + 1/3*C*c^2*d*x^9*e^2 + 3/8*C*c^2*d^2*x^8*e + 1/7*C*c^2
*d^3*x^7 + 1/9*B*c^2*x^9*e^3 + 3/8*B*c^2*d*x^8*e^2 + 3/7*B*c^2*d^2*x^7*e +
1/6*B*c^2*d^3*x^6 + 1/4*C*a*c*x^8*e^3 + 1/8*A*c^2*x^8*e^3 + 6/7*C*a*c*d*x^7
*e^2 + 3/7*A*c^2*d*x^7*e^2 + C*a*c*d^2*x^6*e + 1/2*A*c^2*d^2*x^6*e + 2/5*C*
a*c*d^3*x^5 + 1/5*A*c^2*d^3*x^5 + 2/7*B*a*c*x^7*e^3 + B*a*c*d*x^6*e^2 + 6/5
*B*a*c*d^2*x^5*e + 1/2*B*a*c*d^3*x^4 + 1/6*C*a^2*x^6*e^3 + 1/3*A*a*c*x^6*e^
3 + 3/5*C*a^2*d*x^5*e^2 + 6/5*A*a*c*d*x^5*e^2 + 3/4*C*a^2*d^2*x^4*e + 3/2*A
*a*c*d^2*x^4*e + 1/3*C*a^2*d^3*x^3 + 2/3*A*a*c*d^3*x^3 + 1/5*B*a^2*x^5*e^3
+ 3/4*B*a^2*d*x^4*e^2 + B*a^2*d^2*x^3*e + 1/2*B*a^2*d^3*x^2 + 1/4*A*a^2*x^4
*e^3 + A*a^2*d*x^3*e^2 + 3/2*A*a^2*d^2*x^2*e + A*a^2*d^3*x
```

3.26 $\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$

Optimal. Leaf size=217

$$a^2Ad^2x+\frac{1}{4}a^2ex^4(Be+2Cd)+\frac{1}{7}cx^7\left(2aCe^2+c\left(e(Ae+2Bd)+Cd^2\right)\right)+\frac{1}{5}x^5\left(Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be+Cd\right.\right.$$

```
[Out] a^2*A*d^2*x + (a*(a*d*(C*d + 2*B*e) + A*(2*c*d^2 + a*e^2))*x^3)/3 + (a^2*e*
(2*C*d + B*e)*x^4)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d +
2*B*e)))*x^5)/5 + (a*c*e*(2*C*d + B*e)*x^6)/3 + (c*(2*a*C*e^2 + c*(C*d^2 +
e*(2*B*d + A*e)))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*e^2*x^9)/9
+ (d*(B*d + 2*A*e)*(a + c*x^2)^3)/(6*c)
```

Rubi [A] time = 0.313161, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1582, 1810}

$$a^2Ad^2x+\frac{1}{4}a^2ex^4(Be+2Cd)+\frac{1}{7}cx^7\left(2aCe^2+ce(Ae+2Bd)+cCd^2\right)+\frac{1}{5}x^5\left(Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be+Cd)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a*(a*d*(C*d + 2*B*e) + A*(2*c*d^2 + a*e^2))*x^3)/3 + (a^2*e*
(2*C*d + B*e)*x^4)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d +
2*B*e)))*x^5)/5 + (a*c*e*(2*C*d + B*e)*x^6)/3 + (c*(c*C*d^2 + 2*a*C*e^2 + c
*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*e^2*x^9)/9
+ (d*(B*d + 2*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx = \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a+cx^2\right)^2 \left(-\left(Bd^2+2Ade\right)x+(d+ex)^2 \left(A\right.\right.$$

$$= \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a^2Ad^2+a\left(ad(Cd+2Be)+A\left(2cd^2+ae^2\right)\right)\right.$$

$$= a^2Ad^2x+\frac{1}{3}a\left(ad(Cd+2Be)+A\left(2cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^2e(2Cd+Be)x^4+\frac{1}{5}$$

Mathematica [A] time = 0.091812, size = 241, normalized size = 1.11

$$\frac{1}{2}a^2dx^2(2Ae+Bd)+a^2Ad^2x+\frac{1}{7}cx^7\left(2aCe^2+ce(Ae+2Bd)+cCd^2\right)+\frac{1}{6}cx^6\left(2aBe^2+4aCde+2Acde+Bcd^2\right)+\frac{1}{5}x^5\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a^2*d*(B*d + 2*A*e)*x^2)/2 + (a*(a*d*(C*d + 2*B*e) + A*(2*c*
d^2 + a*e^2))*x^3)/3 + (a*(2*B*c*d^2 + 4*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4
)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d + 2*B*e)))*x^5)/5 +
 (c*(B*c*d^2 + 2*A*c*d*e + 4*a*C*d*e + 2*a*B*e^2)*x^6)/6 + (c*(c*C*d^2 + 2*
a*C*e^2 + c*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*
e^2*x^9)/9
```

Maple [A] time = 0.045, size = 268, normalized size = 1.2

$$\frac{c^2Ce^2x^9}{9}+\frac{\left(c^2e^2B+2\,c^2deC\right)x^8}{8}+\frac{\left(\left(2\,ace^2+c^2d^2\right)C+2\,c^2deB+c^2e^2A\right)x^7}{7}+\frac{\left(4\,acdeC+\left(2\,ace^2+c^2d^2\right)B+2\,c^2deA\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/9*c^2*C*e^2*x^9+1/8*(B*c^2*e^2+2*C*c^2*d*e)*x^8+1/7*((2*a*c*e^2+c^2*d^2)*
C+2*c^2*d*e*B+c^2*e^2*A)*x^7+1/6*(4*a*c*d*e*C+(2*a*c*e^2+c^2*d^2)*B+2*c^2*d
*e*A)*x^6+1/5*((a^2*e^2+2*a*c*d^2)*C+4*B*a*c*d*e+(2*a*c*e^2+c^2*d^2)*A)*x^5
+1/4*(2*d*e*a^2*C+(a^2*e^2+2*a*c*d^2)*B+4*a*c*d*e*A)*x^4+1/3*(a^2*d^2*C+2*d
*e*a^2*B+(a^2*e^2+2*a*c*d^2)*A)*x^3+1/2*(2*A*a^2*d*e+B*a^2*d^2)*x^2+a^2*A*d
^2*x
```

Maxima [A] time = 1.00544, size = 347, normalized size = 1.6

$$\frac{1}{9}Cc^2e^2x^9 + \frac{1}{8}\left(2\,Cc^2de + Bc^2e^2\right)x^8 + \frac{1}{7}\left(Cc^2d^2 + 2\,Bc^2de + \left(2\,Cac + Ac^2\right)e^2\right)x^7 + \frac{1}{6}\left(Bc^2d^2 + 2\,Bace^2 + 2\left(2\,Cac + Ac^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*e^2*x^9 + 1/8*(2*C*c^2*d*e + B*c^2*e^2)*x^8 + 1/7*(C*c^2*d^2 + 2*
B*c^2*d*e + (2*C*a*c + A*c^2)*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2*B*a*c*e^2 + 2*(
2*C*a*c + A*c^2)*d*e)*x^6 + A*a^2*d^2*x + 1/5*(4*B*a*c*d*e + (2*C*a*c + A*c
^2)*d^2 + (C*a^2 + 2*A*a*c)*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + B*a^2*e^2 + 2*(C*
a^2 + 2*A*a*c)*d*e)*x^4 + 1/3*(2*B*a^2*d*e + A*a^2*e^2 + (C*a^2 + 2*A*a*c)*
d^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d*e)*x^2
```

Fricas [A] time = 1.40483, size = 709, normalized size = 3.27

$$\frac{1}{9}x^9e^2c^2C + \frac{1}{4}x^8edc^2C + \frac{1}{8}x^8e^2c^2B + \frac{1}{7}x^7d^2c^2C + \frac{2}{7}x^7e^2caC + \frac{2}{7}x^7edc^2B + \frac{1}{7}x^7e^2c^2A + \frac{2}{3}x^6edcaC + \frac{1}{6}x^6d^2c^2B + \frac{1}{3}x^6e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c^2*C + 1/4*x^8*e*d*c^2*C + 1/8*x^8*e^2*c^2*B + 1/7*x^7*d^2*c^2
*C + 2/7*x^7*e^2*c*a*C + 2/7*x^7*e*d*c^2*B + 1/7*x^7*e^2*c^2*A + 2/3*x^6*e*
d*c*a*C + 1/6*x^6*d^2*c^2*B + 1/3*x^6*e^2*c*a*B + 1/3*x^6*e*d*c^2*A + 2/5*x
^5*d^2*c*a*C + 1/5*x^5*e^2*a^2*C + 4/5*x^5*e*d*c*a*B + 1/5*x^5*d^2*c^2*A +
2/5*x^5*e^2*c*a*A + 1/2*x^4*e*d*a^2*C + 1/2*x^4*d^2*c*a*B + 1/4*x^4*e^2*a^2
*B + x^4*e*d*c*a*A + 1/3*x^3*d^2*a^2*C + 2/3*x^3*e*d*a^2*B + 2/3*x^3*d^2*c*
a*A + 1/3*x^3*e^2*a^2*A + 1/2*x^2*d^2*a^2*B + x^2*e*d*a^2*A + x*d^2*a^2*A
```

Sympy [A] time = 0.104901, size = 311, normalized size = 1.43

$$Aa^2d^2x + \frac{Cc^2e^2x^9}{9} + x^8\left(\frac{Bc^2e^2}{8} + \frac{Cc^2de}{4}\right) + x^7\left(\frac{Ac^2e^2}{7} + \frac{2Bc^2de}{7} + \frac{2Cace^2}{7} + \frac{Cc^2d^2}{7}\right) + x^6\left(\frac{Ac^2de}{3} + \frac{Bace^2}{3} + \frac{Bc^2d^2}{6} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**2*x + C*c**2*e**2*x**9/9 + x**8*(B*c**2*e**2/8 + C*c**2*d*e/4) +
x**7*(A*c**2*e**2/7 + 2*B*c**2*d*e/7 + 2*C*a*c*e**2/7 + C*c**2*d**2/7) + x*
*6*(A*c**2*d*e/3 + B*a*c*e**2/3 + B*c**2*d**2/6 + 2*C*a*c*d*e/3) + x**5*(2*
A*a*c*e**2/5 + A*c**2*d**2/5 + 4*B*a*c*d*e/5 + C*a**2*e**2/5 + 2*C*a*c*d**2
/5) + x**4*(A*a*c*d*e + B*a**2*e**2/4 + B*a*c*d**2/2 + C*a**2*d*e/2) + x**3
*(A*a**2*e**2/3 + 2*A*a*c*d**2/3 + 2*B*a**2*d*e/3 + C*a**2*d**2/3) + x**2*(
A*a**2*d*e + B*a**2*d**2/2)
```

Giac [A] time = 1.12811, size = 408, normalized size = 1.88

$$\frac{1}{9}Cc^2x^9e^2 + \frac{1}{4}Cc^2dx^8e + \frac{1}{7}Cc^2d^2x^7 + \frac{1}{8}Bc^2x^8e^2 + \frac{2}{7}Bc^2dx^7e + \frac{1}{6}Bc^2d^2x^6 + \frac{2}{7}Cacx^7e^2 + \frac{1}{7}Ac^2x^7e^2 + \frac{2}{3}Cacdx^6e + \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*x^9*e^2 + 1/4*C*c^2*d*x^8*e + 1/7*C*c^2*d^2*x^7 + 1/8*B*c^2*x^8*e
^2 + 2/7*B*c^2*d*x^7*e + 1/6*B*c^2*d^2*x^6 + 2/7*C*a*c*x^7*e^2 + 1/7*A*c^2*
x^7*e^2 + 2/3*C*a*c*d*x^6*e + 1/3*A*c^2*d*x^6*e + 2/5*C*a*c*d^2*x^5 + 1/5*A
*c^2*d^2*x^5 + 1/3*B*a*c*x^6*e^2 + 4/5*B*a*c*d*x^5*e + 1/2*B*a*c*d^2*x^4 +
1/5*C*a^2*x^5*e^2 + 2/5*A*a*c*x^5*e^2 + 1/2*C*a^2*d*x^4*e + A*a*c*d*x^4*e +
 1/3*C*a^2*d^2*x^3 + 2/3*A*a*c*d^2*x^3 + 1/4*B*a^2*x^4*e^2 + 2/3*B*a^2*d*x^
3*e + 1/2*B*a^2*d^2*x^2 + 1/3*A*a^2*x^3*e^2 + A*a^2*d*x^2*e + A*a^2*d^2*x
```

3.27 $\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=128

$$a^2Adx+\frac{1}{4}a^2Cex^4+\frac{1}{5}cx^5(2a(Be+Cd)+Acd)+\frac{1}{3}ax^3(aBe+aCd+2Acd)+\frac{\left(a+cx^2\right)^3(Ae+Bd)}{6c}+\frac{1}{3}acCex^6+\frac{1}{7}c^2x^7($$

```
[Out] a^2*A*d*x + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^2*C*e*x^4)/4 + (c*(A*c
*d + 2*a*(C*d + B*e))*x^5)/5 + (a*c*C*e*x^6)/3 + (c^2*(C*d + B*e)*x^7)/7 +
(c^2*C*e*x^8)/8 + ((B*d + A*e)*(a + c*x^2)^3)/(6*c)
```

Rubi [A] time = 0.15868, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1582, 1810}

$$a^2Adx+\frac{1}{4}a^2Cex^4+\frac{1}{5}cx^5(2a(Be+Cd)+Acd)+\frac{1}{3}ax^3(aBe+aCd+2Acd)+\frac{\left(a+cx^2\right)^3(Ae+Bd)}{6c}+\frac{1}{3}acCex^6+\frac{1}{7}c^2x^7($$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^2*C*e*x^4)/4 + (c*(A*c
*d + 2*a*(C*d + B*e))*x^5)/5 + (a*c*C*e*x^6)/3 + (c^2*(C*d + B*e)*x^7)/7 +
(c^2*C*e*x^8)/8 + ((B*d + A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int(d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)dx=\frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c}+\int\left(a+cx^2\right)^2\left(-(Bd+Ae)x+(d+ex)\left(A+Bx+C\right.\right.$$

$$=\frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c}+\int\left(a^2Ad+a(2Acd+aCd+aBe)x^2+a^2Cex^3+c(\right.$$

$$=a^2Adx+\frac{1}{3}a(2Acd+aCd+aBe)x^3+\frac{1}{4}a^2Cex^4+\frac{1}{5}c(Acd+2a(Cd+Be))x^5$$

Mathematica [A] time = 0.050692, size = 144, normalized size = 1.12

$$\frac{1}{2}a^2x^2(Ae+Bd)+a^2Adx+\frac{1}{6}cx^6(2aCe+Ace+Bcd)+\frac{1}{5}cx^5(2aBe+2aCd+Acd)+\frac{1}{4}ax^4(aCe+2Ace+2Bcd)+\frac{1}{3}ax$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a^2*(B*d + A*e)*x^2)/2 + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 +
 (a*(2*B*c*d + 2*A*c*e + a*C*e)*x^4)/4 + (c*(A*c*d + 2*a*C*d + 2*a*B*e)*x^5
)/5 + (c*(B*c*d + A*c*e + 2*a*C*e)*x^6)/6 + (c^2*(C*d + B*e)*x^7)/7 + (c^2*
C*e*x^8)/8
```

Maple [A] time = 0.047, size = 151, normalized size = 1.2

$$\frac{c^2Cex^8}{8}+\frac{\left(c^2eB+c^2dC\right)x^7}{7}+\frac{\left(c^2eA+c^2dB+2\,aceC\right)x^6}{6}+\frac{\left(c^2dA+2\,aceB+2\,acdC\right)x^5}{5}+\frac{\left(2\,aceA+2\,acdB+a^2eC\right)x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/8*c^2*C*e*x^8+1/7*(B*c^2*e+C*c^2*d)*x^7+1/6*(A*c^2*e+B*c^2*d+2*C*a*c*e)*x
^6+1/5*(A*c^2*d+2*B*a*c*e+2*C*a*c*d)*x^5+1/4*(2*A*a*c*e+2*B*a*c*d+C*a^2*e)*
x^4+1/3*(2*A*a*c*d+B*a^2*e+C*a^2*d)*x^3+1/2*(A*a^2*e+B*a^2*d)*x^2+a^2*A*d*x
```

Maxima [A] time = 0.98917, size = 208, normalized size = 1.62

$$\frac{1}{8}Cc^2ex^8 + \frac{1}{7}\left(Cc^2d + Bc^2e\right)x^7 + \frac{1}{6}\left(Bc^2d + \left(2\,Cac + Ac^2\right)e\right)x^6 + \frac{1}{5}\left(2\,Bace + \left(2\,Cac + Ac^2\right)d\right)x^5 + Aa^2dx + \frac{1}{4}\left(2\,Bacd + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*e*x^8 + 1/7*(C*c^2*d + B*c^2*e)*x^7 + 1/6*(B*c^2*d + (2*C*a*c + A
*c^2)*e)*x^6 + 1/5*(2*B*a*c*e + (2*C*a*c + A*c^2)*d)*x^5 + A*a^2*d*x + 1/4*
(2*B*a*c*d + (C*a^2 + 2*A*a*c)*e)*x^4 + 1/3*(B*a^2*e + (C*a^2 + 2*A*a*c)*d)
*x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

Fricas [A] time = 1.57426, size = 428, normalized size = 3.34

$$\frac{1}{8}x^8ec^2C + \frac{1}{7}x^7dc^2C + \frac{1}{7}x^7ec^2B + \frac{1}{3}x^6ecaC + \frac{1}{6}x^6dc^2B + \frac{1}{6}x^6ec^2A + \frac{2}{5}x^5dcaC + \frac{2}{5}x^5ecaB + \frac{1}{5}x^5dc^2A + \frac{1}{4}x^4ea^2C + \frac{1}{2}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*c^2*C + 1/7*x^7*d*c^2*C + 1/7*x^7*e*c^2*B + 1/3*x^6*e*c*a*C + 1/6
*x^6*d*c^2*B + 1/6*x^6*e*c^2*A + 2/5*x^5*d*c*a*C + 2/5*x^5*e*c*a*B + 1/5*x^
5*d*c^2*A + 1/4*x^4*e*a^2*C + 1/2*x^4*d*c*a*B + 1/2*x^4*e*c*a*A + 1/3*x^3*d
*a^2*C + 1/3*x^3*e*a^2*B + 2/3*x^3*d*c*a*A + 1/2*x^2*d*a^2*B + 1/2*x^2*e*a^
2*A + x*d*a^2*A
```

Sympy [A] time = 0.090758, size = 180, normalized size = 1.41

$$Aa^2dx + \frac{Cc^2ex^8}{8} + x^7\left(\frac{Bc^2e}{7} + \frac{Cc^2d}{7}\right) + x^6\left(\frac{Ac^2e}{6} + \frac{Bc^2d}{6} + \frac{Cace}{3}\right) + x^5\left(\frac{Ac^2d}{5} + \frac{2Bace}{5} + \frac{2Cacd}{5}\right) + x^4\left(\frac{Aace}{2} + \frac{Bacd}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d*x + C*c**2*e*x**8/8 + x**7*(B*c**2*e/7 + C*c**2*d/7) + x**6*(A*c**
2*e/6 + B*c**2*d/6 + C*a*c*e/3) + x**5*(A*c**2*d/5 + 2*B*a*c*e/5 + 2*C*a*c*
```

```
d/5) + x**4*(A*a*c*e/2 + B*a*c*d/2 + C*a**2*e/4) + x**3*(2*A*a*c*d/3 + B*a*
*2*e/3 + C*a**2*d/3) + x**2*(A*a**2*e/2 + B*a**2*d/2)
```

Giac [A] time = 1.16791, size = 244, normalized size = 1.91

$$\frac{1}{8}Cc^2x^8e + \frac{1}{7}Cc^2dx^7 + \frac{1}{7}Bc^2x^7e + \frac{1}{6}Bc^2dx^6 + \frac{1}{3}Cacx^6e + \frac{1}{6}Ac^2x^6e + \frac{2}{5}Cacdx^5 + \frac{1}{5}Ac^2dx^5 + \frac{2}{5}Bacx^5e + \frac{1}{2}Bacdx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*x^8*e + 1/7*C*c^2*d*x^7 + 1/7*B*c^2*x^7*e + 1/6*B*c^2*d*x^6 + 1/3
*C*a*c*x^6*e + 1/6*A*c^2*x^6*e + 2/5*C*a*c*d*x^5 + 1/5*A*c^2*d*x^5 + 2/5*B*
a*c*x^5*e + 1/2*B*a*c*d*x^4 + 1/4*C*a^2*x^4*e + 1/2*A*a*c*x^4*e + 1/3*C*a^2
*d*x^3 + 2/3*A*a*c*d*x^3 + 1/3*B*a^2*x^3*e + 1/2*B*a^2*d*x^2 + 1/2*A*a^2*x^
2*e + A*a^2*d*x
```

3.28 $\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=67

$$a^2Ax+\frac{1}{5}cx^5(2aC+Ac)+\frac{1}{3}ax^3(aC+2Ac)+\frac{B\left(a+cx^2\right)^3}{6c}+\frac{1}{7}c^2Cx^7$$

```
[Out] a^2*A*x + (a*(2*A*c + a*C)*x^3)/3 + (c*(A*c + 2*a*C)*x^5)/5 + (c^2*C*x^7)/7
 + (B*(a + c*x^2)^3)/(6*c)
```

Rubi [A] time = 0.040093, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1582, 373}

$$a^2Ax+\frac{1}{5}cx^5(2aC+Ac)+\frac{1}{3}ax^3(aC+2Ac)+\frac{B\left(a+cx^2\right)^3}{6c}+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*c + a*C)*x^3)/3 + (c*(A*c + 2*a*C)*x^5)/5 + (c^2*C*x^7)/7
 + (B*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx = \frac{B\left(a+cx^2\right)^3}{6c} + \int \left(a+cx^2\right)^2 \left(A+Cx^2\right) dx$$
$$= \frac{B\left(a+cx^2\right)^3}{6c} + \int \left(a^2A + a(2Ac+aC)x^2 + c(Ac+2aC)x^4 + c^2Cx^6\right) dx$$
$$= a^2Ax + \frac{1}{3}a(2Ac+aC)x^3 + \frac{1}{5}c(Ac+2aC)x^5 + \frac{1}{7}c^2Cx^7 + \frac{B\left(a+cx^2\right)^3}{6c}$$

Mathematica [A] time = 0.0289469, size = 69, normalized size = 1.03

$$\frac{1}{210}x\left(35a^2(6A+x(3B+2Cx)) + 7acx^2(20A+3x(5B+4Cx)) + c^2x^4(42A+5x(7B+6Cx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] (x*(35*a^2*(6*A + x*(3*B + 2*C*x)) + 7*a*c*x^2*(20*A + 3*x*(5*B + 4*C*x)) +
 c^2*x^4*(42*A + 5*x*(7*B + 6*C*x))))/210
```

Maple [A] time = 0.046, size = 75, normalized size = 1.1

$$\frac{c^2Cx^7}{7} + \frac{Bc^2x^6}{6} + \frac{\left(Ac^2 + 2\,acC\right)x^5}{5} + \frac{aBcx^4}{2} + \frac{\left(2\,aAc + a^2C\right)x^3}{3} + \frac{a^2Bx^2}{2} + a^2Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/7*c^2*C*x^7+1/6*B*c^2*x^6+1/5*(A*c^2+2*C*a*c)*x^5+1/2*a*B*c*x^4+1/3*(2*A*
a*c+C*a^2)*x^3+1/2*a^2*B*x^2+a^2*A*x
```

Maxima [A] time = 1.01937, size = 100, normalized size = 1.49

$$\frac{1}{7}\,Cc^2x^7 + \frac{1}{6}\,Bc^2x^6 + \frac{1}{2}\,Bacx^4 + \frac{1}{5}\left(2\,Cac + Ac^2\right)x^5 + \frac{1}{2}\,Ba^2x^2 + Aa^2x + \frac{1}{3}\left(Ca^2 + 2\,Aac\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*a*c*x^4 + 1/5*(2*C*a*c + A*c^2)*x^5 +
 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*c)*x^3
```

Fricas [A] time = 1.50887, size = 185, normalized size = 2.76

$$\frac{1}{7}x^7c^2C + \frac{1}{6}x^6c^2B + \frac{2}{5}x^5caC + \frac{1}{5}x^5c^2A + \frac{1}{2}x^4caB + \frac{1}{3}x^3a^2C + \frac{2}{3}x^3caA + \frac{1}{2}x^2a^2B + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^2*C + 1/6*x^6*c^2*B + 2/5*x^5*c*a*C + 1/5*x^5*c^2*A + 1/2*x^4*c*a
*B + 1/3*x^3*a^2*C + 2/3*x^3*c*a*A + 1/2*x^2*a^2*B + x*a^2*A
```

Sympy [A] time = 0.077304, size = 83, normalized size = 1.24

$$Aa^2x + \frac{Ba^2x^2}{2} + \frac{Bacx^4}{2} + \frac{Bc^2x^6}{6} + \frac{Cc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Cac}{5}\right) + x^3\left(\frac{2Aac}{3} + \frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x
**5*(A*c**2/5 + 2*C*a*c/5) + x**3*(2*A*a*c/3 + C*a**2/3)
```

Giac [A] time = 1.15864, size = 103, normalized size = 1.54

$$\frac{1}{7}Cc^2x^7 + \frac{1}{6}Bc^2x^6 + \frac{2}{5}Cacx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{2}Bacx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aacx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*a*c*x
^4 + 1/3*C*a^2*x^3 + 2/3*A*a*c*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

3.29 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal. Leaf size=297

$$\frac{x^2\left(a^2Ce^4+2ace^2\left(Cd^2-e(Bd-Ae)\right)+c^2d^2\left(Cd^2-e(Bd-Ae)\right)\right)}{2e^5}-\frac{x\left(a^2e^4(Cd-Be)+2acde^2\left(Cd^2-e(Bd-Ae)\right)+c^2d\right.}{e^6}$$

```
[Out] -((((a^2*e^4*(C*d - B*e) + c^2*d^3*(C*d^2 - e*(B*d - A*e)) + 2*a*c*d*e^2*(C*
d^2 - e*(B*d - A*e)))*x)/e^6) + ((a^2*C*e^4 + c^2*d^2*(C*d^2 - e*(B*d - A*e
)) + 2*a*c*e^2*(C*d^2 - e*(B*d - A*e)))*x^2)/(2*e^5) - (c*(2*a*e^2*(C*d - B
*e) + c*d*(C*d^2 - e*(B*d - A*e)))*x^3)/(3*e^4) + (c*(2*a*C*e^2 + c*(C*d^2
- e*(B*d - A*e)))*x^4)/(4*e^3) - (c^2*(C*d - B*e)*x^5)/(5*e^2) + (c^2*C*x^6
)/(6*e) + ((c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.6399, antiderivative size = 295, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{x^2\left(a^2Ce^4+2ace^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(Cd^4-d^2e(Bd-Ae)\right)\right)}{2e^5}-\frac{x\left(a^2e^4(Cd-Be)+2acde^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(\right.\right.}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -((((a^2*e^4*(C*d - B*e) + 2*a*c*d*e^2*(C*d^2 - e*(B*d - A*e)) + c^2*(C*d^5
- d^3*e*(B*d - A*e)))*x)/e^6) + ((a^2*C*e^4 + 2*a*c*e^2*(C*d^2 - e*(B*d - A
*e)) + c^2*(C*d^4 - d^2*e*(B*d - A*e)))*x^2)/(2*e^5) - (c*(c*C*d^3 - c*d*e*
(B*d - A*e) + 2*a*e^2*(C*d - B*e))*x^3)/(3*e^4) + (c*(c*C*d^2 + 2*a*C*e^2 -
 c*e*(B*d - A*e))*x^4)/(4*e^3) - (c^2*(C*d - B*e)*x^5)/(5*e^2) + (c^2*C*x^6
)/(6*e) + ((c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)}{d+ex}\,dx = \int\left(\frac{-a^2e^4(Cd-Be)-2acde^2\left(Cd^2-e(Bd-Ae)\right)-c^2\left(Cd^5-d^3e(Bd-Ae)\right)}{e^6}+\frac{\left(a^2C\right.}{}\right.$$

$$= -\frac{\left(a^2e^4(Cd-Be)+2acde^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(Cd^5-d^3e(Bd-Ae)\right)\right)x}{e^6}+\frac{\left(a^2C\right.}{}$$

Mathematica [A] time = 0.16785, size = 285, normalized size = 0.96

$$\frac{ex\left(30a^2e^4(2Be-2Cd+Cex)+10ace^2\left(2e\left(3Ae(ex-2d)+B\left(6d^2-3dex+2e^2x^2\right)\right)+C\left(6d^2ex-12d^3-4de^2x^2+3e^3x\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(30*a^2*e^4*(-2*C*d + 2*B*e + C*e*x) + 10*a*c*e^2*(C*(-12*d^3 + 6*d^2*
e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e
*x + 2*e^2*x^2))) + c^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*
e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2*e*x - 4*d*
e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x
^3 + 12*e^4*x^4)))) + 60*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e))*Log[d
 + e*x])/(60*e^7)
```

Maple [A] time = 0.054, size = 490, normalized size = 1.7

$$-2\,\frac{\ln(ex+d)\,Bacd^3}{e^4}+\frac{\ln(ex+d)\,Ac^2d^4}{e^5}-\frac{Cc^2d^5x}{e^6}-\frac{\ln(ex+d)\,Ba^2d}{e^2}-\frac{\ln(ex+d)\,Bc^2d^5}{e^6}+\frac{\ln(ex+d)\,Ca^2d^2}{e^3}+\frac{\ln(ex}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x)
```

```
[Out] -2/e^4*ln(e*x+d)*B*a*c*d^3+1/e^5*ln(e*x+d)*A*c^2*d^4-1/e^6*C*c^2*d^5*x-1/e^
2*ln(e*x+d)*B*a^2*d-1/e^6*ln(e*x+d)*B*c^2*d^5+1/e^3*ln(e*x+d)*C*a^2*d^2+1/e
^7*ln(e*x+d)*C*c^2*d^6-1/e^2*C*a^2*d*x+1/2/e^5*C*x^2*c^2*d^4-1/e^4*A*c^2*d^
3*x+1/e*A*x^2*a*c+1/3/e^3*B*x^3*c^2*d^2+1/e^5*B*c^2*d^4*x-1/2/e^4*B*x^2*c^2
*d^3+1/2/e^3*A*x^2*c^2*d^2-1/3/e^2*A*x^3*c^2*d+1/4/e^3*C*x^4*c^2*d^2-1/4/e^
2*B*x^4*c^2*d+2/3/e*B*x^3*a*c-1/3/e^4*C*x^3*c^2*d^3+2/e^5*ln(e*x+d)*C*a*c*d
```

```
^4-2/e^4*C*a*c*d^3*x+1/2/e*C*x^4*a*c-1/5/e^2*C*x^5*c^2*d+1/e*a^2*B*x+1/4/e*
A*x^4*c^2+1/5/e*B*x^5*c^2+1/e*ln(e*x+d)*A*a^2+1/2/e*C*x^2*a^2+2/e^3*B*a*c*d
^2*x+2/e^3*ln(e*x+d)*A*a*c*d^2-2/3/e^2*C*x^3*a*c*d-1/e^2*B*x^2*a*c*d+1/e^3*
C*x^2*a*c*d^2-2/e^2*A*a*c*d*x+1/6*c^2*C*x^6/e
```

Maxima [A] time = 0.998783, size = 509, normalized size = 1.71

$$10\,Cc^2e^5x^6 - 12\left(Cc^2de^4 - Bc^2e^5\right)x^5 + 15\left(Cc^2d^2e^3 - Bc^2de^4 + \left(2\,Cac + Ac^2\right)e^5\right)x^4 - 20\left(Cc^2d^3e^2 - Bc^2d^2e^3 - 2\,Bace^5 + \left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/60*(10*C*c^2*e^5*x^6 - 12*(C*c^2*d*e^4 - B*c^2*e^5)*x^5 + 15*(C*c^2*d^2*e
^3 - B*c^2*d*e^4 + (2*C*a*c + A*c^2)*e^5)*x^4 - 20*(C*c^2*d^3*e^2 - B*c^2*d
^2*e^3 - 2*B*a*c*e^5 + (2*C*a*c + A*c^2)*d*e^4)*x^3 + 30*(C*c^2*d^4*e - B*c
^2*d^3*e^2 - 2*B*a*c*d*e^4 + (2*C*a*c + A*c^2)*d^2*e^3 + (C*a^2 + 2*A*a*c)*
e^5)*x^2 - 60*(C*c^2*d^5 - B*c^2*d^4*e - 2*B*a*c*d^2*e^3 - B*a^2*e^5 + (2*C
*a*c + A*c^2)*d^3*e^2 + (C*a^2 + 2*A*a*c)*d*e^4)*x)/e^6 + (C*c^2*d^6 - B*c^
2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c + A*c^2)*d^4
*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d)/e^7
```

Fricas [A] time = 1.75467, size = 786, normalized size = 2.65

$$10\,Cc^2e^6x^6 - 12\left(Cc^2de^5 - Bc^2e^6\right)x^5 + 15\left(Cc^2d^2e^4 - Bc^2de^5 + \left(2\,Cac + Ac^2\right)e^6\right)x^4 - 20\left(Cc^2d^3e^3 - Bc^2d^2e^4 - 2\,Bace^6 + \left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^2*e^6*x^6 - 12*(C*c^2*d*e^5 - B*c^2*e^6)*x^5 + 15*(C*c^2*d^2*e
^4 - B*c^2*d*e^5 + (2*C*a*c + A*c^2)*e^6)*x^4 - 20*(C*c^2*d^3*e^3 - B*c^2*d
^2*e^4 - 2*B*a*c*e^6 + (2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(C*c^2*d^4*e^2 - B
*c^2*d^3*e^3 - 2*B*a*c*d*e^5 + (2*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A*a*c
)*e^6)*x^2 - 60*(C*c^2*d^5*e - B*c^2*d^4*e^2 - 2*B*a*c*d^2*e^4 - B*a^2*e^6
+ (2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*x + 60*(C*c^2*d^6 -
B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c + A*c^2)
```

```
*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d))/e^7
```

Sympy [A] time = 1.46908, size = 350, normalized size = 1.18

$$\frac{Cc^2x^6}{6e} - \frac{x^5\left(-Bc^2e + Cc^2d\right)}{5e^2} + \frac{x^4\left(Ac^2e^2 - Bc^2de + 2Cace^2 + Cc^2d^2\right)}{4e^3} - \frac{x^3\left(Ac^2de^2 - 2Bace^3 - Bc^2d^2e + 2Cacde^2 + Cc^2\right.}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c**2*x**6/(6*e) - x**5*(-B*c**2*e + C*c**2*d)/(5*e**2) + x**4*(A*c**2*e**
2 - B*c**2*d*e + 2*C*a*c*e**2 + C*c**2*d**2)/(4*e**3) - x**3*(A*c**2*d*e**2
 - 2*B*a*c*e**3 - B*c**2*d**2*e + 2*C*a*c*d*e**2 + C*c**2*d**3)/(3*e**4) +
x**2*(2*A*a*c*e**4 + A*c**2*d**2*e**2 - 2*B*a*c*d*e**3 - B*c**2*d**3*e + C*
a**2*e**4 + 2*C*a*c*d**2*e**2 + C*c**2*d**4)/(2*e**5) - x*(2*A*a*c*d*e**4 +
 A*c**2*d**3*e**2 - B*a**2*e**5 - 2*B*a*c*d**2*e**3 - B*c**2*d**4*e + C*a**
2*d*e**4 + 2*C*a*c*d**3*e**2 + C*c**2*d**5)/e**6 + (a*e**2 + c*d**2)**2*(A*
e**2 - B*d*e + C*d**2)*log(d + e*x)/e**7
```

Giac [A] time = 1.13939, size = 562, normalized size = 1.89

$$\left(Cc^2d^6 - Bc^2d^5e + 2\,Cacd^4e^2 + Ac^2d^4e^2 - 2\,Bacd^3e^3 + Ca^2d^2e^4 + 2\,Aacd^2e^4 - Ba^2de^5 + Aa^2e^6\right)e^{(-7)}\log\left(|xe + d|\right) + \frac{1}{60}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c^2*d^6 - B*c^2*d^5*e + 2*C*a*c*d^4*e^2 + A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^
3 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - B*a^2*d*e^5 + A*a^2*e^6)*e^(-7)*log(a
bs(x*e + d)) + 1/60*(10*C*c^2*x^6*e^5 - 12*C*c^2*d*x^5*e^4 + 15*C*c^2*d^2*x
^4*e^3 - 20*C*c^2*d^3*x^3*e^2 + 30*C*c^2*d^4*x^2*e - 60*C*c^2*d^5*x + 12*B*
c^2*x^5*e^5 - 15*B*c^2*d*x^4*e^4 + 20*B*c^2*d^2*x^3*e^3 - 30*B*c^2*d^3*x^2*
e^2 + 60*B*c^2*d^4*x*e + 30*C*a*c*x^4*e^5 + 15*A*c^2*x^4*e^5 - 40*C*a*c*d*x
^3*e^4 - 20*A*c^2*d*x^3*e^4 + 60*C*a*c*d^2*x^2*e^3 + 30*A*c^2*d^2*x^2*e^3 -
 120*C*a*c*d^3*x*e^2 - 60*A*c^2*d^3*x*e^2 + 40*B*a*c*x^3*e^5 - 60*B*a*c*d*x
^2*e^4 + 120*B*a*c*d^2*x*e^3 + 30*C*a^2*x^2*e^5 + 60*A*a*c*x^2*e^5 - 60*C*a
^2*d*x*e^4 - 120*A*a*c*d*x*e^4 + 60*B*a^2*x*e^5)*e^(-6)
```

3.30 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2} dx$

Optimal. Leaf size=292

$$\frac{x\left(a^2Ce^4+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2d^2\left(5Cd^2-e(4Bd-3Ae)\right)\right)}{e^6}+\frac{cx^3\left(2aCe^2+c\left(3Cd^2-e(2Bd-Ae)\right)\right)}{3e^4}-\frac{cx^2\left(2\right.}{}$$

```
[Out] ((a^2*C*e^4 + c^2*d^2*(5*C*d^2 - e*(4*B*d - 3*A*e)) + 2*a*c*e^2*(3*C*d^2 -
e*(2*B*d - A*e)))*x)/e^6 - (c*(2*a*e^2*(2*C*d - B*e) + c*d*(4*C*d^2 - e*(3*
B*d - 2*A*e)))*x^2)/(2*e^5) + (c*(2*a*C*e^2 + c*(3*C*d^2 - e*(2*B*d - A*e))
)*x^3)/(3*e^4) - (c^2*(2*C*d - B*e)*x^4)/(4*e^3) + (c^2*C*x^5)/(5*e^2) - ((
c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2))/(e^7*(d + e*x)) - ((c*d^2 + a*e^2
)*(a*e^2*(2*C*d - B*e) + c*d*(6*C*d^2 - e*(5*B*d - 4*A*e)))*Log[d + e*x])/e
^7
```

Rubi [A] time = 0.525428, antiderivative size = 289, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{x\left(a^2Ce^4+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)\right)}{e^6}+\frac{cx^3\left(2aCe^2-ce(2Bd-Ae)+3cCd^2\right)}{3e^4}-\frac{cx^2\left(2a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^2*C*e^4 + c^2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 2*a*c*e^2*(3*C*d^2 -
e*(2*B*d - A*e)))*x)/e^6 - (c*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 2*a*e^2*
(2*C*d - B*e))*x^2)/(2*e^5) + (c*(3*c*C*d^2 + 2*a*C*e^2 - c*e*(2*B*d - A*e)
)*x^3)/(3*e^4) - (c^2*(2*C*d - B*e)*x^4)/(4*e^3) + (c^2*C*x^5)/(5*e^2) - ((
c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2))/(e^7*(d + e*x)) - ((c*d^2 + a*e^2
)*(6*c*C*d^3 - c*d*e*(5*B*d - 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e
^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^2}\,dx = \int \left(\frac{a^2Ce^4 + c^2\left(5Cd^4 - d^2e(4Bd-3Ae)\right) + 2ace^2\left(3Cd^2 - e(2Bd-Ae)\right)}{e^6} + \frac{c\left(-4cCd\right.}{} \right.$$

$$= \frac{\left(a^2Ce^4 + c^2\left(5Cd^4 - d^2e(4Bd-3Ae)\right) + 2ace^2\left(3Cd^2 - e(2Bd-Ae)\right)\right)x}{e^6} - \frac{c\left(4cCd^3\right.}{}$$

Mathematica [A] time = 0.291287, size = 272, normalized size = 0.93

$$\frac{60ex\left(a^2Ce^4 + 2ace^2\left(e(Ae-2Bd) + 3Cd^2\right) + c^2\left(d^2e(3Ae-4Bd) + 5Cd^4\right)\right) + 20ce^3x^3\left(2aCe^2 + ce(Ae-2Bd) + 3cCd^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (60*e*(a^2*C*e^4 + 2*a*c*e^2*(3*C*d^2 + e*(-2*B*d + A*e)) + c^2*(5*C*d^4 +
d^2*e*(-4*B*d + 3*A*e)))*x - 30*c*e^2*(4*c*C*d^3 + c*d*e*(-3*B*d + 2*A*e) -
 2*a*e^2*(-2*C*d + B*e))*x^2 + 20*c*e^3*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(-2*B*
d + A*e))*x^3 + 15*c^2*e^4*(-2*C*d + B*e)*x^4 + 12*c^2*C*e^5*x^5 - (60*(c*d
^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) - 60*(c*d^2 + a*e^2)*(6
*c*C*d^3 + c*d*e*(-5*B*d + 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/(60*
e^7)
```

Maple [A] time = 0.058, size = 527, normalized size = 1.8

$$-4\frac{Bc^2d^3x}{e^5} + 5\frac{Cc^2d^4x}{e^6} + \frac{\ln(ex+d)Ba^2}{e^2} + \frac{a^2Cx}{e^2} + \frac{Ax^3c^2}{3e^2} + 5\frac{\ln(ex+d)Bc^2d^4}{e^6} + 2\frac{aAcx}{e^2} + \frac{2Cx^3ac}{3e^2} + \frac{Cx^3c^2d^2}{e^4} + \frac{}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x)
```

```
[Out] -4/e^5*B*c^2*d^3*x+5/e^6*C*c^2*d^4*x+1/e^2*ln(e*x+d)*B*a^2+1/e^2*a^2*C*x+1/
3/e^2*A*x^3*c^2+5/e^6*ln(e*x+d)*B*c^2*d^4+2/e^2*A*a*c*x+2/3/e^2*C*x^3*a*c+1
/e^4*C*x^3*c^2*d^2+1/e^2/(e*x+d)*B*d*a^2+1/e^6/(e*x+d)*B*c^2*d^5-1/e^3/(e*x
+d)*C*a^2*d^2-1/e^7/(e*x+d)*C*c^2*d^6+1/e^2*B*x^2*a*c+1/4/e^2*B*x^4*c^2+3/e
```

```
^4*A*c^2*d^2*x-4/e^5*ln(e*x+d)*A*c^2*d^3-1/2/e^3*C*x^4*c^2*d-2/3/e^3*B*x^3*
c^2*d-2/e^3*ln(e*x+d)*C*a^2*d+3/2/e^4*B*x^2*c^2*d^2-2/e^5*C*x^2*c^2*d^3-6/e
^7*ln(e*x+d)*C*c^2*d^5-1/e^5/(e*x+d)*A*c^2*d^4-1/e^3*A*x^2*c^2*d-1/e/(e*x+d
)*A*a^2-8/e^5*ln(e*x+d)*C*a*c*d^3-2/e^3/(e*x+d)*A*a*c*d^2+2/e^4/(e*x+d)*B*a
*c*d^3-2/e^5/(e*x+d)*C*a*c*d^4+6/e^4*C*a*c*d^2*x-4/e^3*ln(e*x+d)*A*a*c*d+6/
e^4*ln(e*x+d)*B*a*c*d^2-2/e^3*C*x^2*a*c*d-4/e^3*B*a*c*d*x+1/5*c^2*C*x^5/e^2
```

Maxima [A] time = 1.01945, size = 529, normalized size = 1.81

$$-\frac{Cc^2d^6 - Bc^2d^5e - 2\,Bacd^3e^3 - Ba^2de^5 + Aa^2e^6 + \left(2\,Cac + Ac^2\right)d^4e^2 + \left(Ca^2 + 2\,Aac\right)d^2e^4}{e^8x + de^7} + \frac{12\,Cc^2e^4x^5 - 15\left(2\,Cc^2de^3 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c^2*d^6 - B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*
C*a*c + A*c^2)*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)/(e^8*x + d*e^7) + 1/60*
(12*C*c^2*e^4*x^5 - 15*(2*C*c^2*d*e^3 - B*c^2*e^4)*x^4 + 20*(3*C*c^2*d^2*e^
2 - 2*B*c^2*d*e^3 + (2*C*a*c + A*c^2)*e^4)*x^3 - 30*(4*C*c^2*d^3*e - 3*B*c^
2*d^2*e^2 - 2*B*a*c*e^4 + 2*(2*C*a*c + A*c^2)*d*e^3)*x^2 + 60*(5*C*c^2*d^4
- 4*B*c^2*d^3*e - 4*B*a*c*d*e^3 + 3*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2*
A*a*c)*e^4)*x)/e^6 - (6*C*c^2*d^5 - 5*B*c^2*d^4*e - 6*B*a*c*d^2*e^3 - B*a^2
*e^5 + 4*(2*C*a*c + A*c^2)*d^3*e^2 + 2*(C*a^2 + 2*A*a*c)*d*e^4)*log(e*x + d
)/e^7
```

Fricas [A] time = 1.73969, size = 1185, normalized size = 4.06

$$\frac{12\,Cc^2e^6x^6 - 60\,Cc^2d^6 + 60\,Bc^2d^5e + 120\,Bacd^3e^3 + 60\,Ba^2de^5 - 60\,Aa^2e^6 - 60\left(2\,Cac + Ac^2\right)d^4e^2 - 60\left(Ca^2 + 2\,Aac\right)d^2e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c^2*e^6*x^6 - 60*C*c^2*d^6 + 60*B*c^2*d^5*e + 120*B*a*c*d^3*e^3
+ 60*B*a^2*d*e^5 - 60*A*a^2*e^6 - 60*(2*C*a*c + A*c^2)*d^4*e^2 - 60*(C*a^2
+ 2*A*a*c)*d^2*e^4 - 3*(6*C*c^2*d*e^5 - 5*B*c^2*e^6)*x^5 + 5*(6*C*c^2*d^2*e
^4 - 5*B*c^2*d*e^5 + 4*(2*C*a*c + A*c^2)*e^6)*x^4 - 10*(6*C*c^2*d^3*e^3 - 5
```

```
*B*c^2*d^2*e^4 - 6*B*a*c*e^6 + 4*(2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(6*C*c^2
*d^4*e^2 - 5*B*c^2*d^3*e^3 - 6*B*a*c*d*e^5 + 4*(2*C*a*c + A*c^2)*d^2*e^4 +
2*(C*a^2 + 2*A*a*c)*e^6)*x^2 + 60*(5*C*c^2*d^5*e - 4*B*c^2*d^4*e^2 - 4*B*a*
c*d^2*e^4 + 3*(2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*x - 60*(
6*C*c^2*d^6 - 5*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 - B*a^2*d*e^5 + 4*(2*C*a*c +
A*c^2)*d^4*e^2 + 2*(C*a^2 + 2*A*a*c)*d^2*e^4 + (6*C*c^2*d^5*e - 5*B*c^2*d^4
*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A*c^2)*d^3*e^3 + 2*(C*a^2
 + 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^8*x + d*e^7)
```

Sympy [A] time = 3.76405, size = 411, normalized size = 1.41

$$\frac{Cc^2x^5}{5e^2} - \frac{Aa^2e^6 + 2Aacd^2e^4 + Ac^2d^4e^2 - Ba^2de^5 - 2Bacd^3e^3 - Bc^2d^5e + Ca^2d^2e^4 + 2Cacd^4e^2 + Cc^2d^6}{de^7 + e^8x} - \frac{x^4\left(-Bc^2e + 2C\right.}{4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**2*x**5/(5*e**2) - (A*a**2*e**6 + 2*A*a*c*d**2*e**4 + A*c**2*d**4*e**2
- B*a**2*d*e**5 - 2*B*a*c*d**3*e**3 - B*c**2*d**5*e + C*a**2*d**2*e**4 + 2*
C*a*c*d**4*e**2 + C*c**2*d**6)/(d*e**7 + e**8*x) - x**4*(-B*c**2*e + 2*C*c*
*2*d)/(4*e**3) + x**3*(A*c**2*e**2 - 2*B*c**2*d*e + 2*C*a*c*e**2 + 3*C*c**2
*d**2)/(3*e**4) - x**2*(2*A*c**2*d*e**2 - 2*B*a*c*e**3 - 3*B*c**2*d**2*e +
4*C*a*c*d*e**2 + 4*C*c**2*d**3)/(2*e**5) + x*(2*A*a*c*e**4 + 3*A*c**2*d**2*
e**2 - 4*B*a*c*d*e**3 - 4*B*c**2*d**3*e + C*a**2*e**4 + 6*C*a*c*d**2*e**2 +
 5*C*c**2*d**4)/e**6 - (a*e**2 + c*d**2)*(4*A*c*d*e**2 - B*a*e**3 - 5*B*c*d
**2*e + 2*C*a*d*e**2 + 6*C*c*d**3)*log(d + e*x)/e**7
```

Giac [A] time = 1.1782, size = 671, normalized size = 2.3

$$\frac{1}{60}\left(12\,Cc^2 - \frac{15\left(6\,Cc^2de - Bc^2e^2\right)e^{(-1)}}{xe + d} + \frac{20\left(15\,Cc^2d^2e^2 - 5\,Bc^2de^3 + 2\,Cace^4 + Ac^2e^4\right)e^{(-2)}}{(xe + d)^2} - \frac{60\left(10\,Cc^2d^3e^3 - 5\,Bc^2d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/60*(12*C*c^2 - 15*(6*C*c^2*d*e - B*c^2*e^2)*e^(-1)/(x*e + d) + 20*(15*C*c
^2*d^2*e^2 - 5*B*c^2*d*e^3 + 2*C*a*c*e^4 + A*c^2*e^4)*e^(-2)/(x*e + d)^2 -
```

```
60*(10*C*c^2*d^3*e^3 - 5*B*c^2*d^2*e^4 + 4*C*a*c*d*e^5 + 2*A*c^2*d*e^5 - B*
a*c*e^6)*e^(-3)/(x*e + d)^3 + 60*(15*C*c^2*d^4*e^4 - 10*B*c^2*d^3*e^5 + 12*
C*a*c*d^2*e^6 + 6*A*c^2*d^2*e^6 - 6*B*a*c*d*e^7 + C*a^2*e^8 + 2*A*a*c*e^8)*
e^(-4)/(x*e + d)^4)*(x*e + d)^5*e^(-7) + (6*C*c^2*d^5 - 5*B*c^2*d^4*e + 8*C
*a*c*d^3*e^2 + 4*A*c^2*d^3*e^2 - 6*B*a*c*d^2*e^3 + 2*C*a^2*d*e^4 + 4*A*a*c*
d*e^4 - B*a^2*e^5)*e^(-7)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - (C*c^2*d^6
*e^5/(x*e + d) - B*c^2*d^5*e^6/(x*e + d) + 2*C*a*c*d^4*e^7/(x*e + d) + A*c^
2*d^4*e^7/(x*e + d) - 2*B*a*c*d^3*e^8/(x*e + d) + C*a^2*d^2*e^9/(x*e + d) +
 2*A*a*c*d^2*e^9/(x*e + d) - B*a^2*d*e^10/(x*e + d) + A*a^2*e^11/(x*e + d))
*e^(-12)
```

3.31 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3} dx$

Optimal. Leaf size=295

$$\frac{\log(d+ex)\left(a^2Ce^4+2ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2d^2\left(15Cd^2-2e(5Bd-3Ae)\right)\right)}{e^7}+\frac{cx^2\left(2aCe^2+c\left(6Cd^2-e(3Bd-\right.\right.}{2e^5}$$

```
[Out] -((c*(2*a*e^2*(3*C*d - B*e) + c*d*(10*C*d^2 - 3*e*(2*B*d - A*e)))*x)/e^6) +
 (c*(2*a*C*e^2 + c*(6*C*d^2 - e*(3*B*d - A*e)))*x^2)/(2*e^5) - (c^2*(3*C*d
- B*e)*x^3)/(3*e^4) + (c^2*C*x^4)/(4*e^3) - ((c*d^2 + a*e^2)^2*(C*d^2 - B*d
*e + A*e^2))/(2*e^7*(d + e*x)^2) + ((c*d^2 + a*e^2)*(a*e^2*(2*C*d - B*e) +
c*d*(6*C*d^2 - e*(5*B*d - 4*A*e))))/(e^7*(d + e*x)) + ((a^2*C*e^4 + c^2*d^2
*(15*C*d^2 - 2*e*(5*B*d - 3*A*e)) + 2*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e)))*
Log[d + e*x])/e^7
```

Rubi [A] time = 0.494568, antiderivative size = 292, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{\log(d+ex)\left(a^2Ce^4+2ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2\left(15Cd^4-2d^2e(5Bd-3Ae)\right)\right)}{e^7}+\frac{cx^2\left(2aCe^2-ce(3Bd-Ae)+6cC\right.}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 2*a*e^2*(3*C*d - B*e))*x)/e^6) +
 (c*(6*c*C*d^2 + 2*a*C*e^2 - c*e*(3*B*d - A*e))*x^2)/(2*e^5) - (c^2*(3*C*d
- B*e)*x^3)/(3*e^4) + (c^2*C*x^4)/(4*e^3) - ((c*d^2 + a*e^2)^2*(C*d^2 - B*d
*e + A*e^2))/(2*e^7*(d + e*x)^2) + ((c*d^2 + a*e^2)*(6*c*C*d^3 - c*d*e*(5*B
*d - 4*A*e) + a*e^2*(2*C*d - B*e)))/(e^7*(d + e*x)) + ((a^2*C*e^4 + c^2*(15
*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 2*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e)))*
Log[d + e*x])/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)}{(d+ex)^3}\,dx = \int\left(\frac{c\left(-10cCd^3+3cde(2Bd-Ae)-2ae^2(3Cd-Be)\right)}{e^6}+\frac{c\left(6cCd^2+2aCe^2-ce(3Bd-\right.}{e^5}\right.$$

$$= -\frac{c\left(10cCd^3-3cde(2Bd-Ae)+2ae^2(3Cd-Be)\right)x}{e^6}+\frac{c\left(6cCd^2+2aCe^2-ce(3Bd-A\right.}{2e^5}$$

Mathematica [A] time = 0.135966, size = 274, normalized size = 0.93

$$\frac{12\log(d+ex)\left(a^2Ce^4+2ace^2\left(e(Ae-3Bd)+6Cd^2\right)+c^2\left(2d^2e(3Ae-5Bd)+15Cd^4\right)\right)+6ce^2x^2\left(2aCe^2+ce(Ae-3Bd)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-12*c*e*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 2*a*e^2*(-3*C*d + B*e))*x +
 6*c*e^2*(6*c*C*d^2 + 2*a*C*e^2 + c*e*(-3*B*d + A*e))*x^2 + 4*c^2*e^3*(-3*C
*d + B*e)*x^3 + 3*c^2*C*e^4*x^4 - (6*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) +
 A*e)))/(d + e*x)^2 + (12*(c*d^2 + a*e^2)*(6*c*C*d^3 + c*d*e*(-5*B*d + 4*A*
e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 12*(a^2*C*e^4 + 2*a*c*e^2*(6*C*d^2 +
 e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B*d + 3*A*e)))*Log[d + e*x
])/(12*e^7)
```

Maple [A] time = 0.061, size = 563, normalized size = 1.9

$$-\frac{3\,Bc^2x^2d}{2\,e^4}+15\,\frac{\ln(ex+d)\,Cc^2d^4}{e^7}+4\,\frac{Ad^3c^2}{e^5\,(ex+d)}-5\,\frac{Bc^2d^4}{e^6\,(ex+d)}+2\,\frac{a^2Cd}{e^3\,(ex+d)}+6\,\frac{Cc^2d^5}{e^7\,(ex+d)}-10\,\frac{\ln(ex+d)\,Bc^2d^3}{e^6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x)
```

```
[Out] -3/2*c^2/e^4*B*x^2*d+15/e^7*ln(e*x+d)*C*c^2*d^4+4/e^5/(e*x+d)*A*c^2*d^3-5/e
^6/(e*x+d)*B*c^2*d^4+2/e^3/(e*x+d)*C*a^2*d+6/e^7/(e*x+d)*C*c^2*d^5-10/e^6*l
n(e*x+d)*B*c^2*d^3+c/e^3*C*x^2*a+3*c^2/e^5*C*x^2*d^2+2/e^3*ln(e*x+d)*A*a*c+
6/e^5*ln(e*x+d)*A*c^2*d^2-1/e^2/(e*x+d)*B*a^2-3*c^2/e^4*A*d*x+2*c/e^3*B*a*x
```

```
+6*c^2/e^5*B*d^2*x-10*c^2/e^6*C*d^3*x-c^2/e^4*C*x^3*d-1/2/e/(e*x+d)^2*A*a^2
+1/3*c^2/e^3*B*x^3+1/2*c^2/e^3*A*x^2+1/e^3*ln(e*x+d)*a^2*C-1/2/e^5/(e*x+d)^
2*A*c^2*d^4+1/e^4/(e*x+d)^2*B*a*c*d^3-1/e^5/(e*x+d)^2*C*a*c*d^4-6/e^4*ln(e*
x+d)*B*a*c*d+12/e^5*ln(e*x+d)*C*a*c*d^2+4/e^3/(e*x+d)*A*a*c*d-6/e^4/(e*x+d)
*B*a*c*d^2+8/e^5/(e*x+d)*C*a*c*d^3-6*c/e^4*C*a*d*x-1/e^3/(e*x+d)^2*A*d^2*a*
c+1/2/e^2/(e*x+d)^2*B*d*a^2+1/2/e^6/(e*x+d)^2*B*c^2*d^5-1/2/e^3/(e*x+d)^2*C
*d^2*a^2-1/2/e^7/(e*x+d)^2*C*c^2*d^6+1/4*c^2*C*x^4/e^3
```

Maxima [A] time = 1.0299, size = 543, normalized size = 1.84

$$\frac{11\,Cc^2d^6 - 9\,Bc^2d^5e - 10\,Bacd^3e^3 - Ba^2de^5 - Aa^2e^6 + 7\left(2\,Cac + Ac^2\right)d^4e^2 + 3\left(Ca^2 + 2\,Aac\right)d^2e^4 + 2\left(6\,Cc^2d^5e - 5\,Bc^2\right.}{2\left(e^9x^2 + 2\,de^8x + d^2e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*C*c^2*d^6 - 9*B*c^2*d^5*e - 10*B*a*c*d^3*e^3 - B*a^2*d*e^5 - A*a^2*
e^6 + 7*(2*C*a*c + A*c^2)*d^4*e^2 + 3*(C*a^2 + 2*A*a*c)*d^2*e^4 + 2*(6*C*c^
2*d^5*e - 5*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A*c^
2)*d^3*e^3 + 2*(C*a^2 + 2*A*a*c)*d*e^5)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
+ 1/12*(3*C*c^2*e^3*x^4 - 4*(3*C*c^2*d*e^2 - B*c^2*e^3)*x^3 + 6*(6*C*c^2*d^
2*e - 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^2 - 12*(10*C*c^2*d^3 - 6*B*c
^2*d^2*e - 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x)/e^6 + (15*C*c^2*d^4
- 10*B*c^2*d^3*e - 6*B*a*c*d*e^3 + 6*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2
*A*a*c)*e^4)*log(e*x + d)/e^7
```

Fricas [B] time = 1.7306, size = 1305, normalized size = 4.42

$$\frac{3\,Cc^2e^6x^6 + 66\,Cc^2d^6 - 54\,Bc^2d^5e - 60\,Bacd^3e^3 - 6\,Ba^2de^5 - 6\,Aa^2e^6 + 42\left(2\,Cac + Ac^2\right)d^4e^2 + 18\left(Ca^2 + 2\,Aac\right)d^2e^4 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c^2*e^6*x^6 + 66*C*c^2*d^6 - 54*B*c^2*d^5*e - 60*B*a*c*d^3*e^3 -
6*B*a^2*d*e^5 - 6*A*a^2*e^6 + 42*(2*C*a*c + A*c^2)*d^4*e^2 + 18*(C*a^2 + 2*
A*a*c)*d^2*e^4 - 2*(3*C*c^2*d*e^5 - 2*B*c^2*e^6)*x^5 + (15*C*c^2*d^2*e^4 -
```

```
10*B*c^2*d*e^5 + 6*(2*C*a*c + A*c^2)*e^6)*x^4 - 4*(15*C*c^2*d^3*e^3 - 10*B*
c^2*d^2*e^4 - 6*B*a*c*e^6 + 6*(2*C*a*c + A*c^2)*d*e^5)*x^3 - 6*(34*C*c^2*d^
4*e^2 - 21*B*c^2*d^3*e^3 - 8*B*a*c*d*e^5 + 11*(2*C*a*c + A*c^2)*d^2*e^4)*x^
2 - 12*(4*C*c^2*d^5*e - B*c^2*d^4*e^2 + 4*B*a*c*d^2*e^4 + B*a^2*e^6 - (2*C*
a*c + A*c^2)*d^3*e^3 - 2*(C*a^2 + 2*A*a*c)*d*e^5)*x + 12*(15*C*c^2*d^6 - 10
*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 + 6*(2*C*a*c + A*c^2)*d^4*e^2 + (C*a^2 + 2*A
*a*c)*d^2*e^4 + (15*C*c^2*d^4*e^2 - 10*B*c^2*d^3*e^3 - 6*B*a*c*d*e^5 + 6*(2
*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A*a*c)*e^6)*x^2 + 2*(15*C*c^2*d^5*e -
10*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 + 6*(2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 +
 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] time = 16.4422, size = 471, normalized size = 1.6

$$\frac{Cc^2x^4}{4e^3} + \frac{-Aa^2e^6 + 6Aacd^2e^4 + 7Ac^2d^4e^2 - Ba^2de^5 - 10Bacd^3e^3 - 9Bc^2d^5e + 3Ca^2d^2e^4 + 14Cacd^4e^2 + 11Cc^2d^6 + x\left(8Aa\right.}{2d^2e^7 + 4de^8x + 2e^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**2*x**4/(4*e**3) + (-A*a**2*e**6 + 6*A*a*c*d**2*e**4 + 7*A*c**2*d**4*e*
*2 - B*a**2*d*e**5 - 10*B*a*c*d**3*e**3 - 9*B*c**2*d**5*e + 3*C*a**2*d**2*e
**4 + 14*C*a*c*d**4*e**2 + 11*C*c**2*d**6 + x*(8*A*a*c*d*e**5 + 8*A*c**2*d*
*3*e**3 - 2*B*a**2*e**6 - 12*B*a*c*d**2*e**4 - 10*B*c**2*d**4*e**2 + 4*C*a*
*2*d*e**5 + 16*C*a*c*d**3*e**3 + 12*C*c**2*d**5*e))/(2*d**2*e**7 + 4*d*e**8
*x + 2*e**9*x**2) - x**3*(-B*c**2*e + 3*C*c**2*d)/(3*e**4) + x**2*(A*c**2*e
**2 - 3*B*c**2*d*e + 2*C*a*c*e**2 + 6*C*c**2*d**2)/(2*e**5) - x*(3*A*c**2*d
*e**2 - 2*B*a*c*e**3 - 6*B*c**2*d**2*e + 6*C*a*c*d*e**2 + 10*C*c**2*d**3)/e
**6 + (2*A*a*c*e**4 + 6*A*c**2*d**2*e**2 - 6*B*a*c*d*e**3 - 10*B*c**2*d**3*
e + C*a**2*e**4 + 12*C*a*c*d**2*e**2 + 15*C*c**2*d**4)*log(d + e*x)/e**7
```

Giac [A] time = 1.16243, size = 536, normalized size = 1.82

$$\left(15\,Cc^2d^4 - 10\,Bc^2d^3e + 12\,Cacd^2e^2 + 6\,Ac^2d^2e^2 - 6\,Bacde^3 + Ca^2e^4 + 2\,Aace^4\right)e^{(-7)}\log\left(|xe + d|\right) + \frac{1}{12}\left(3\,Cc^2x^4e^9 - 12\,C\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (15*C*c^2*d^4 - 10*B*c^2*d^3*e + 12*C*a*c*d^2*e^2 + 6*A*c^2*d^2*e^2 - 6*B*a
*c*d*e^3 + C*a^2*e^4 + 2*A*a*c*e^4)*e^(-7)*log(abs(x*e + d)) + 1/12*(3*C*c^
2*x^4*e^9 - 12*C*c^2*d*x^3*e^8 + 36*C*c^2*d^2*x^2*e^7 - 120*C*c^2*d^3*x*e^6
 + 4*B*c^2*x^3*e^9 - 18*B*c^2*d*x^2*e^8 + 72*B*c^2*d^2*x*e^7 + 12*C*a*c*x^2
*e^9 + 6*A*c^2*x^2*e^9 - 72*C*a*c*d*x*e^8 - 36*A*c^2*d*x*e^8 + 24*B*a*c*x*e
^9)*e^(-12) + 1/2*(11*C*c^2*d^6 - 9*B*c^2*d^5*e + 14*C*a*c*d^4*e^2 + 7*A*c^
2*d^4*e^2 - 10*B*a*c*d^3*e^3 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - B*a^2*d*
e^5 - A*a^2*e^6 + 2*(6*C*c^2*d^5*e - 5*B*c^2*d^4*e^2 + 8*C*a*c*d^3*e^3 + 4*
A*c^2*d^3*e^3 - 6*B*a*c*d^2*e^4 + 2*C*a^2*d*e^5 + 4*A*a*c*d*e^5 - B*a^2*e^6
)*x)*e^(-7)/(x*e + d)^2
```

3.32 $\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=404

$$\frac{1}{6}a^2ex^6\left(aCe^2+3c\left(e(Ae+3Bd)+3Cd^2\right)\right)+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right)+\frac{1}{4}a^3ex^4\left(e(Ae+3Bd)+3Cd^2\right)+$$

```
[Out] a^3*A*d^3*x + (a^2*d*(a*d*(C*d + 3*B*e) + 3*A*(c*d^2 + a*e^2))*x^3)/3 + (a^
3*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*
(a*e^2*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a^2*e*(a*C*e^2 + 3
*c*(3*C*d^2 + e*(3*B*d + A*e)))*x^6)/6 + (c*(A*c*d*(c*d^2 + 9*a*e^2) + 3*a*
(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/7 + (3*a*c*e*(a*C*e^2 + c
*(3*C*d^2 + e*(3*B*d + A*e)))*x^8)/8 + (c^2*(3*a*e^2*(3*C*d + B*e) + c*d*(C
*d^2 + 3*e*(B*d + A*e)))*x^9)/9 + (c^2*e*(3*a*C*e^2 + c*(3*C*d^2 + e*(3*B*d
 + A*e)))*x^10)/10 + (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
+ (d^2*(B*d + 3*A*e)*(a + c*x^2)^4)/(8*c)
```

Rubi [A] time = 0.691394, antiderivative size = 400, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1582, 1810}

$$\frac{1}{6}a^2ex^6\left(aCe^2+3ce(Ae+3Bd)+9cCd^2\right)+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right)+\frac{1}{4}a^3ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^2*d*(a*d*(C*d + 3*B*e) + 3*A*(c*d^2 + a*e^2))*x^3)/3 + (a^
3*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*
(a*e^2*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a^2*e*(9*c*C*d^2 +
 a*C*e^2 + 3*c*e*(3*B*d + A*e))*x^6)/6 + (c*(A*c*d*(c*d^2 + 9*a*e^2) + 3*a*
(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/7 + (3*a*c*e*(3*c*C*d^2 +
 a*C*e^2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e)
+ 3*a*e^2*(3*C*d + B*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*
d + A*e))*x^10)/10 + (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
+ (d^2*(B*d + 3*A*e)*(a + c*x^2)^4)/(8*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)dx=\frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c}+\int \left(a+cx^2\right)^3\left(-\left(Bd^3+3Ad^2e\right)x+(d+ex)^3\left(\right.\right.$$

$$=\frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c}+\int \left(a^3Ad^3+a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae\right.\right.\right.$$

$$=a^3Ad^3x+\frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^3e\left(3Cd^2+e(3Ba\right.$$

Mathematica [A] time = 0.204747, size = 459, normalized size = 1.14

$$\frac{1}{4}a^2x^4\left(aAe^3+3aBde^2+3aCd^2e+9Acd^2e+3Bcd^3\right)+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right)+\frac{1}{2}a^3d^2x^2(3Ae+Bd)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^3*d^2*(B*d + 3*A*e)*x^2)/2 + (a^2*d*(a*d*(C*d + 3*B*e) + 3
*A*(c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^3 + 9*A*c*d^2*e + 3*a*C*d^2*e +
3*a*B*d*e^2 + a*A*e^3)*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2*(3
*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*(3*A*c*e*(3*c*d^2 + a*e^2
) + a*C*e*(9*c*d^2 + a*e^2) + 3*B*c*d*(c*d^2 + 3*a*e^2))*x^6)/6 + (c*(A*c*d
*(c*d^2 + 9*a*e^2) + 3*a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/
7 + (c*(B*c*d*(c*d^2 + 9*a*e^2) + 3*e*(A*c*(c*d^2 + a*e^2) + a*C*(3*c*d^2 +
 a*e^2)))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 3*a*e^2*(3*C*d + B
*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*d + A*e))*x^10)/10 +
 (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
```

Maple [A] time = 0.046, size = 553, normalized size = 1.4

$$\frac{c^3Ce^3x^{12}}{12}+\frac{\left(e^3c^3B+3\,de^2c^3C\right)x^{11}}{11}+\frac{\left(\left(3\,e^3ac^2+3\,d^2ec^3\right)C+3\,de^2c^3B+e^3c^3A\right)x^{10}}{10}+\frac{\left(\left(9\,c^2ade^2+c^3d^3\right)C+\left(3\,e^3ac^2+\right.\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/12*c^3*C*e^3*x^12+1/11*(B*c^3*e^3+3*C*c^3*d*e^2)*x^11+1/10*((3*a*c^2*e^3+
3*c^3*d^2*e)*C+3*d*e^2*c^3*B+e^3*c^3*A)*x^10+1/9*((9*a*c^2*d*e^2+c^3*d^3)*C
+(3*a*c^2*e^3+3*c^3*d^2*e)*B+3*d*e^2*c^3*A)*x^9+1/8*((3*a^2*c*e^3+9*a*c^2*d
^2*e)*C+(9*a*c^2*d*e^2+c^3*d^3)*B+(3*a*c^2*e^3+3*c^3*d^2*e)*A)*x^8+1/7*((9*
a^2*c*d*e^2+3*a*c^2*d^3)*C+(3*a^2*c*e^3+9*a*c^2*d^2*e)*B+(9*a*c^2*d*e^2+c^3
*d^3)*A)*x^7+1/6*((a^3*e^3+9*a^2*c*d^2*e)*C+(9*a^2*c*d*e^2+3*a*c^2*d^3)*B+(
3*a^2*c*e^3+9*a*c^2*d^2*e)*A)*x^6+1/5*((3*a^3*d*e^2+3*a^2*c*d^3)*C+(a^3*e^3
+9*a^2*c*d^2*e)*B+(9*a^2*c*d*e^2+3*a*c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^3*C+(3*
a^3*d*e^2+3*a^2*c*d^3)*B+(a^3*e^3+9*a^2*c*d^2*e)*A)*x^4+1/3*(a^3*d^3*C+3*d^
2*e*a^3*B+(3*a^3*d*e^2+3*a^2*c*d^3)*A)*x^3+1/2*(3*A*a^3*d^2*e+B*a^3*d^3)*x^
2+a^3*A*d^3*x
```

Maxima [A] time = 1.0111, size = 691, normalized size = 1.71

$$\frac{1}{12}\,Cc^3e^3x^{12}+\frac{1}{11}\left(3\,Cc^3de^2+Bc^3e^3\right)x^{11}+\frac{1}{10}\left(3\,Cc^3d^2e+3\,Bc^3de^2+\left(3\,Cac^2+Ac^3\right)e^3\right)x^{10}+\frac{1}{9}\left(Cc^3d^3+3\,Bc^3d^2e+3\,Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*C*c^3*e^3*x^12 + 1/11*(3*C*c^3*d*e^2 + B*c^3*e^3)*x^11 + 1/10*(3*C*c^3
*d^2*e + 3*B*c^3*d*e^2 + (3*C*a*c^2 + A*c^3)*e^3)*x^10 + 1/9*(C*c^3*d^3 + 3
*B*c^3*d^2*e + 3*B*a*c^2*e^3 + 3*(3*C*a*c^2 + A*c^3)*d*e^2)*x^9 + 1/8*(B*c^
3*d^3 + 9*B*a*c^2*d*e^2 + 3*(3*C*a*c^2 + A*c^3)*d^2*e + 3*(C*a^2*c + A*a*c^
2)*e^3)*x^8 + A*a^3*d^3*x + 1/7*(9*B*a*c^2*d^2*e + 3*B*a^2*c*e^3 + (3*C*a*c
^2 + A*c^3)*d^3 + 9*(C*a^2*c + A*a*c^2)*d*e^2)*x^7 + 1/6*(3*B*a*c^2*d^3 + 9
*B*a^2*c*d*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2*e + (C*a^3 + 3*A*a^2*c)*e^3)*x^6
 + 1/5*(9*B*a^2*c*d^2*e + B*a^3*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3 + 3*(C*a^3
+ 3*A*a^2*c)*d*e^2)*x^5 + 1/4*(3*B*a^2*c*d^3 + 3*B*a^3*d*e^2 + A*a^3*e^3 +
3*(C*a^3 + 3*A*a^2*c)*d^2*e)*x^4 + 1/3*(3*B*a^3*d^2*e + 3*A*a^3*d*e^2 + (C*
```

```
a^3 + 3*A*a^2*c)*d^3)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

Fricas [A] time = 1.43126, size = 1413, normalized size = 3.5

$$\frac{1}{12}x^{12}e^3c^3C + \frac{3}{11}x^{11}e^2dc^3C + \frac{1}{11}x^{11}e^3c^3B + \frac{3}{10}x^{10}ed^2c^3C + \frac{3}{10}x^{10}e^3c^2aC + \frac{3}{10}x^{10}e^2dc^3B + \frac{1}{10}x^{10}e^3c^3A + \frac{1}{9}x^9d^3c^3C +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^3*c^3*C + 3/11*x^11*e^2*d*c^3*C + 1/11*x^11*e^3*c^3*B + 3/10*x^
10*e*d^2*c^3*C + 3/10*x^10*e^3*c^2*a*C + 3/10*x^10*e^2*d*c^3*B + 1/10*x^10*
e^3*c^3*A + 1/9*x^9*d^3*c^3*C + x^9*e^2*d*c^2*a*C + 1/3*x^9*e*d^2*c^3*B + 1
/3*x^9*e^3*c^2*a*B + 1/3*x^9*e^2*d*c^3*A + 9/8*x^8*e*d^2*c^2*a*C + 3/8*x^8*
e^3*c*a^2*C + 1/8*x^8*d^3*c^3*B + 9/8*x^8*e^2*d*c^2*a*B + 3/8*x^8*e*d^2*c^3
*A + 3/8*x^8*e^3*c^2*a*A + 3/7*x^7*d^3*c^2*a*C + 9/7*x^7*e^2*d*c*a^2*C + 9/
7*x^7*e*d^2*c^2*a*B + 3/7*x^7*e^3*c*a^2*B + 1/7*x^7*d^3*c^3*A + 9/7*x^7*e^2
*d*c^2*a*A + 3/2*x^6*e*d^2*c*a^2*C + 1/6*x^6*e^3*a^3*C + 1/2*x^6*d^3*c^2*a*
B + 3/2*x^6*e^2*d*c*a^2*B + 3/2*x^6*e*d^2*c^2*a*A + 1/2*x^6*e^3*c*a^2*A + 3
/5*x^5*d^3*c*a^2*C + 3/5*x^5*e^2*d*a^3*C + 9/5*x^5*e*d^2*c*a^2*B + 1/5*x^5*
e^3*a^3*B + 3/5*x^5*d^3*c^2*a*A + 9/5*x^5*e^2*d*c*a^2*A + 3/4*x^4*e*d^2*a^3
*C + 3/4*x^4*d^3*c*a^2*B + 3/4*x^4*e^2*d*a^3*B + 9/4*x^4*e*d^2*c*a^2*A + 1/
4*x^4*e^3*a^3*A + 1/3*x^3*d^3*a^3*C + x^3*e*d^2*a^3*B + x^3*d^3*c*a^2*A + x
^3*e^2*d*a^3*A + 1/2*x^2*d^3*a^3*B + 3/2*x^2*e*d^2*a^3*A + x*d^3*a^3*A
```

Sympy [A] time = 0.136789, size = 646, normalized size = 1.6

$$Aa^3d^3x + \frac{Cc^3e^3x^{12}}{12} + x^{11}\left(\frac{Bc^3e^3}{11} + \frac{3Cc^3de^2}{11}\right) + x^{10}\left(\frac{Ac^3e^3}{10} + \frac{3Bc^3de^2}{10} + \frac{3Cac^2e^3}{10} + \frac{3Cc^3d^2e}{10}\right) + x^9\left(\frac{Ac^3de^2}{3} + \frac{Bac^2e^3}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**3*x + C*c**3*e**3*x**12/12 + x**11*(B*c**3*e**3/11 + 3*C*c**3*d*e
**2/11) + x**10*(A*c**3*e**3/10 + 3*B*c**3*d*e**2/10 + 3*C*a*c**2*e**3/10 +
 3*C*c**3*d**2*e/10) + x**9*(A*c**3*d*e**2/3 + B*a*c**2*e**3/3 + B*c**3*d**
2*e/3 + C*a*c**2*d*e**2 + C*c**3*d**3/9) + x**8*(3*A*a*c**2*e**3/8 + 3*A*c*
*3*d**2*e/8 + 9*B*a*c**2*d*e**2/8 + B*c**3*d**3/8 + 3*C*a**2*c*e**3/8 + 9*C
```

```
*a*c**2*d**2*e/8) + x**7*(9*A*a*c**2*d*e**2/7 + A*c**3*d**3/7 + 3*B*a**2*c*
e**3/7 + 9*B*a*c**2*d**2*e/7 + 9*C*a**2*c*d*e**2/7 + 3*C*a*c**2*d**3/7) + x
**6*(A*a**2*c*e**3/2 + 3*A*a*c**2*d**2*e/2 + 3*B*a**2*c*d*e**2/2 + B*a*c**2
*d**3/2 + C*a**3*e**3/6 + 3*C*a**2*c*d**2*e/2) + x**5*(9*A*a**2*c*d*e**2/5
+ 3*A*a*c**2*d**3/5 + B*a**3*e**3/5 + 9*B*a**2*c*d**2*e/5 + 3*C*a**3*d*e**2
/5 + 3*C*a**2*c*d**3/5) + x**4*(A*a**3*e**3/4 + 9*A*a**2*c*d**2*e/4 + 3*B*a
**3*d*e**2/4 + 3*B*a**2*c*d**3/4 + 3*C*a**3*d**2*e/4) + x**3*(A*a**3*d*e**2
 + A*a**2*c*d**3 + B*a**3*d**2*e + C*a**3*d**3/3) + x**2*(3*A*a**3*d**2*e/2
 + B*a**3*d**3/2)
```

Giac [A] time = 1.14244, size = 818, normalized size = 2.02

$$\frac{1}{12}Cc^3x^{12}e^3 + \frac{3}{11}Cc^3dx^{11}e^2 + \frac{3}{10}Cc^3d^2x^{10}e + \frac{1}{9}Cc^3d^3x^9 + \frac{1}{11}Bc^3x^{11}e^3 + \frac{3}{10}Bc^3dx^{10}e^2 + \frac{1}{3}Bc^3d^2x^9e + \frac{1}{8}Bc^3d^3x^8 + \frac{3}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*C*c^3*x^12*e^3 + 3/11*C*c^3*d*x^11*e^2 + 3/10*C*c^3*d^2*x^10*e + 1/9*C
*c^3*d^3*x^9 + 1/11*B*c^3*x^11*e^3 + 3/10*B*c^3*d*x^10*e^2 + 1/3*B*c^3*d^2*
x^9*e + 1/8*B*c^3*d^3*x^8 + 3/10*C*a*c^2*x^10*e^3 + 1/10*A*c^3*x^10*e^3 + C
*a*c^2*d*x^9*e^2 + 1/3*A*c^3*d*x^9*e^2 + 9/8*C*a*c^2*d^2*x^8*e + 3/8*A*c^3*
d^2*x^8*e + 3/7*C*a*c^2*d^3*x^7 + 1/7*A*c^3*d^3*x^7 + 1/3*B*a*c^2*x^9*e^3 +
 9/8*B*a*c^2*d*x^8*e^2 + 9/7*B*a*c^2*d^2*x^7*e + 1/2*B*a*c^2*d^3*x^6 + 3/8*
C*a^2*c*x^8*e^3 + 3/8*A*a*c^2*x^8*e^3 + 9/7*C*a^2*c*d*x^7*e^2 + 9/7*A*a*c^2
*d*x^7*e^2 + 3/2*C*a^2*c*d^2*x^6*e + 3/2*A*a*c^2*d^2*x^6*e + 3/5*C*a^2*c*d^
3*x^5 + 3/5*A*a*c^2*d^3*x^5 + 3/7*B*a^2*c*x^7*e^3 + 3/2*B*a^2*c*d*x^6*e^2 +
 9/5*B*a^2*c*d^2*x^5*e + 3/4*B*a^2*c*d^3*x^4 + 1/6*C*a^3*x^6*e^3 + 1/2*A*a^
2*c*x^6*e^3 + 3/5*C*a^3*d*x^5*e^2 + 9/5*A*a^2*c*d*x^5*e^2 + 3/4*C*a^3*d^2*x
^4*e + 9/4*A*a^2*c*d^2*x^4*e + 1/3*C*a^3*d^3*x^3 + A*a^2*c*d^3*x^3 + 1/5*B*
a^3*x^5*e^3 + 3/4*B*a^3*d*x^4*e^2 + B*a^3*d^2*x^3*e + 1/2*B*a^3*d^3*x^2 + 1
/4*A*a^3*x^4*e^3 + A*a^3*d*x^3*e^2 + 3/2*A*a^3*d^2*x^2*e + A*a^3*d^3*x
```

3.33 $\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=289

$$\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+a^3Ad^2x+\frac{1}{2}a^2cex^6(Be+2Cd)+\frac{1}{4}a^3ex^4(Be+2Cd)+\frac{1}{9}c^2x^9\left(3aCe^2+c\left(e(Ae\right.\right.$$

```
[Out] a^3*A*d^2*x + (a^2*(a*d*(C*d + 2*B*e) + A*(3*c*d^2 + a*e^2))*x^3)/3 + (a^3*
e*(2*C*d + B*e)*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*
d + 2*B*e)))*x^5)/5 + (a^2*c*e*(2*C*d + B*e)*x^6)/2 + (c*(A*c*(c*d^2 + 3*a*
e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (3*a*c^2*e*(2*C*d + B*e)
*x^8)/8 + (c^2*(3*a*C*e^2 + c*(C*d^2 + e*(2*B*d + A*e)))*x^9)/9 + (c^3*e*(2
*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11 + (d*(B*d + 2*A*e)*(a + c*x^2)^4
)/(8*c)
```

Rubi [A] time = 0.424141, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1582, 1810}

$$\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+a^3Ad^2x+\frac{1}{2}a^2cex^6(Be+2Cd)+\frac{1}{4}a^3ex^4(Be+2Cd)+\frac{1}{9}c^2x^9\left(3aCe^2+ce(Ae\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^2*(a*d*(C*d + 2*B*e) + A*(3*c*d^2 + a*e^2))*x^3)/3 + (a^3*
e*(2*C*d + B*e)*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*
d + 2*B*e)))*x^5)/5 + (a^2*c*e*(2*C*d + B*e)*x^6)/2 + (c*(A*c*(c*d^2 + 3*a*
e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (3*a*c^2*e*(2*C*d + B*e)
*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*(2*B*d + A*e))*x^9)/9 + (c^3*e*(2
*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11 + (d*(B*d + 2*A*e)*(a + c*x^2)^4
)/(8*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
```

```
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx = \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3 \left(-\left(Bd^2+2Ade\right)x+(d+ex)^2\left(A+\right.\right.$$

$$= \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a^3Ad^2+a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)\right.$$

$$= a^3Ad^2x + \frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3 + \frac{1}{4}a^3e(2Cd+Be)x^4 + \frac{1}{5}$$

Mathematica [A] time = 0.131723, size = 329, normalized size = 1.14

$$\frac{1}{4}a^2x^4\left(aBe^2+2aCde+6Acde+3Bcd^2\right)+\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+\frac{1}{2}a^3dx^2(2Ae+Bd)+a^3Ad^2x+\frac{1}{9}c^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^3*d*(B*d + 2*A*e)*x^2)/2 + (a^2*(a*d*(C*d + 2*B*e) + A*(3*
c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^2 + 6*A*c*d*e + 2*a*C*d*e + a*B*e^2)
*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*d + 2*B*e)))*x^
5)/5 + (a*c*(2*(A*c + a*C)*d*e + B*(c*d^2 + a*e^2))*x^6)/2 + (c*(A*c*(c*d^2
 + 3*a*e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (c^2*(B*c*d^2 + 2
*A*c*d*e + 6*a*C*d*e + 3*a*B*e^2)*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*
(2*B*d + A*e))*x^9)/9 + (c^3*e*(2*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11
```

Maple [A] time = 0.044, size = 388, normalized size = 1.3

$$\frac{c^3Ce^2x^{11}}{11}+\frac{\left(e^2c^3B+2\,dec^3C\right)x^{10}}{10}+\frac{\left(\left(3\,ac^2e^2+c^3d^2\right)C+2\,dec^3B+e^2c^3A\right)x^9}{9}+\frac{\left(6\,ac^2deC+\left(3\,ac^2e^2+c^3d^2\right)B+2\,dec^3\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/11*c^3*C*e^2*x^11+1/10*(B*c^3*e^2+2*C*c^3*d*e)*x^10+1/9*((3*a*c^2*e^2+c^3
*d^2)*C+2*d*e*c^3*B+e^2*c^3*A)*x^9+1/8*(6*a*c^2*d*e*C+(3*a*c^2*e^2+c^3*d^2)
*B+2*d*e*c^3*A)*x^8+1/7*((3*a^2*c*e^2+3*a*c^2*d^2)*C+6*a*c^2*d*e*B+(3*a*c^2
*e^2+c^3*d^2)*A)*x^7+1/6*(6*d*e*a^2*c*C+(3*a^2*c*e^2+3*a*c^2*d^2)*B+6*a*c^2
*d*e*A)*x^6+1/5*((a^3*e^2+3*a^2*c*d^2)*C+6*d*e*a^2*c*B+(3*a^2*c*e^2+3*a*c^2
*d^2)*A)*x^5+1/4*(2*d*e*a^3*C+(a^3*e^2+3*a^2*c*d^2)*B+6*d*e*a^2*c*A)*x^4+1/
3*(a^3*d^2*C+2*d*e*a^3*B+(a^3*e^2+3*a^2*c*d^2)*A)*x^3+1/2*(2*A*a^3*d*e+B*a^
3*d^2)*x^2+a^3*A*d^2*x
```

Maxima [A] time = 1.00264, size = 495, normalized size = 1.71

$$\frac{1}{11}Cc^3e^2x^{11}+\frac{1}{10}\left(2Cc^3de+Bc^3e^2\right)x^{10}+\frac{1}{9}\left(Cc^3d^2+2Bc^3de+\left(3Cac^2+Ac^3\right)e^2\right)x^9+\frac{1}{8}\left(Bc^3d^2+3Bac^2e^2+2\left(3Cac^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^3*e^2*x^11 + 1/10*(2*C*c^3*d*e + B*c^3*e^2)*x^10 + 1/9*(C*c^3*d^2
+ 2*B*c^3*d*e + (3*C*a*c^2 + A*c^3)*e^2)*x^9 + 1/8*(B*c^3*d^2 + 3*B*a*c^2*e
^2 + 2*(3*C*a*c^2 + A*c^3)*d*e)*x^8 + 1/7*(6*B*a*c^2*d*e + (3*C*a*c^2 + A*c
^3)*d^2 + 3*(C*a^2*c + A*a*c^2)*e^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c^2*d^2 +
 B*a^2*c*e^2 + 2*(C*a^2*c + A*a*c^2)*d*e)*x^6 + 1/5*(6*B*a^2*c*d*e + 3*(C*a
^2*c + A*a*c^2)*d^2 + (C*a^3 + 3*A*a^2*c)*e^2)*x^5 + 1/4*(3*B*a^2*c*d^2 + B
*a^3*e^2 + 2*(C*a^3 + 3*A*a^2*c)*d*e)*x^4 + 1/3*(2*B*a^3*d*e + A*a^3*e^2 +
(C*a^3 + 3*A*a^2*c)*d^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*x^2
```

Fricas [A] time = 1.39606, size = 996, normalized size = 3.45

$$\frac{1}{11}x^{11}e^2c^3C+\frac{1}{5}x^{10}edc^3C+\frac{1}{10}x^{10}e^2c^3B+\frac{1}{9}x^9d^2c^3C+\frac{1}{3}x^9e^2c^2aC+\frac{2}{9}x^9edc^3B+\frac{1}{9}x^9e^2c^3A+\frac{3}{4}x^8edc^2aC+\frac{1}{8}x^8d^2c^3B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^2*c^3*C + 1/5*x^10*e*d*c^3*C + 1/10*x^10*e^2*c^3*B + 1/9*x^9*d^
2*c^3*C + 1/3*x^9*e^2*c^2*a*C + 2/9*x^9*e*d*c^3*B + 1/9*x^9*e^2*c^3*A + 3/4
*x^8*e*d*c^2*a*C + 1/8*x^8*d^2*c^3*B + 3/8*x^8*e^2*c^2*a*B + 1/4*x^8*e*d*c^
3*A + 3/7*x^7*d^2*c^2*a*C + 3/7*x^7*e^2*c*a^2*C + 6/7*x^7*e*d*c^2*a*B + 1/7
*x^7*d^2*c^3*A + 3/7*x^7*e^2*c^2*a*A + x^6*e*d*c*a^2*C + 1/2*x^6*d^2*c^2*a*
B + 1/2*x^6*e^2*c*a^2*B + x^6*e*d*c^2*a*A + 3/5*x^5*d^2*c*a^2*C + 1/5*x^5*e
^2*a^3*C + 6/5*x^5*e*d*c*a^2*B + 3/5*x^5*d^2*c^2*a*A + 3/5*x^5*e^2*c*a^2*A
+ 1/2*x^4*e*d*a^3*C + 3/4*x^4*d^2*c*a^2*B + 1/4*x^4*e^2*a^3*B + 3/2*x^4*e*d
*c*a^2*A + 1/3*x^3*d^2*a^3*C + 2/3*x^3*e*d*a^3*B + x^3*d^2*c*a^2*A + 1/3*x^
3*e^2*a^3*A + 1/2*x^2*d^2*a^3*B + x^2*e*d*a^3*A + x*d^2*a^3*A
```

Sympy [A] time = 0.121491, size = 447, normalized size = 1.55

$$Aa^3d^2x + \frac{Cc^3e^2x^{11}}{11} + x^{10}\left(\frac{Bc^3e^2}{10} + \frac{Cc^3de}{5}\right) + x^9\left(\frac{Ac^3e^2}{9} + \frac{2Bc^3de}{9} + \frac{Cac^2e^2}{3} + \frac{Cc^3d^2}{9}\right) + x^8\left(\frac{Ac^3de}{4} + \frac{3Bac^2e^2}{8} + \frac{Bc^3d^2}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**2*x + C*c**3*e**2*x**11/11 + x**10*(B*c**3*e**2/10 + C*c**3*d*e/5
) + x**9*(A*c**3*e**2/9 + 2*B*c**3*d*e/9 + C*a*c**2*e**2/3 + C*c**3*d**2/9)
 + x**8*(A*c**3*d*e/4 + 3*B*a*c**2*e**2/8 + B*c**3*d**2/8 + 3*C*a*c**2*d*e/
4) + x**7*(3*A*a*c**2*e**2/7 + A*c**3*d**2/7 + 6*B*a*c**2*d*e/7 + 3*C*a**2*
c*e**2/7 + 3*C*a*c**2*d**2/7) + x**6*(A*a*c**2*d*e + B*a**2*c*e**2/2 + B*a*
c**2*d**2/2 + C*a**2*c*d*e) + x**5*(3*A*a**2*c*e**2/5 + 3*A*a*c**2*d**2/5 +
 6*B*a**2*c*d*e/5 + C*a**3*e**2/5 + 3*C*a**2*c*d**2/5) + x**4*(3*A*a**2*c*d
*e/2 + B*a**3*e**2/4 + 3*B*a**2*c*d**2/4 + C*a**3*d*e/2) + x**3*(A*a**3*e**
2/3 + A*a**2*c*d**2 + 2*B*a**3*d*e/3 + C*a**3*d**2/3) + x**2*(A*a**3*d*e +
B*a**3*d**2/2)
```

Giac [A] time = 1.16747, size = 583, normalized size = 2.02

$$\frac{1}{11}Cc^3x^{11}e^2 + \frac{1}{5}Cc^3dx^{10}e + \frac{1}{9}Cc^3d^2x^9 + \frac{1}{10}Bc^3x^{10}e^2 + \frac{2}{9}Bc^3dx^9e + \frac{1}{8}Bc^3d^2x^8 + \frac{1}{3}Cac^2x^9e^2 + \frac{1}{9}Ac^3x^9e^2 + \frac{3}{4}Cac^2dx^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^3*x^11*e^2 + 1/5*C*c^3*d*x^10*e + 1/9*C*c^3*d^2*x^9 + 1/10*B*c^3*x
^10*e^2 + 2/9*B*c^3*d*x^9*e + 1/8*B*c^3*d^2*x^8 + 1/3*C*a*c^2*x^9*e^2 + 1/9
*A*c^3*x^9*e^2 + 3/4*C*a*c^2*d*x^8*e + 1/4*A*c^3*d*x^8*e + 3/7*C*a*c^2*d^2*
x^7 + 1/7*A*c^3*d^2*x^7 + 3/8*B*a*c^2*x^8*e^2 + 6/7*B*a*c^2*d*x^7*e + 1/2*B
*a*c^2*d^2*x^6 + 3/7*C*a^2*c*x^7*e^2 + 3/7*A*a*c^2*x^7*e^2 + C*a^2*c*d*x^6*
e + A*a*c^2*d*x^6*e + 3/5*C*a^2*c*d^2*x^5 + 3/5*A*a*c^2*d^2*x^5 + 1/2*B*a^2
*c*x^6*e^2 + 6/5*B*a^2*c*d*x^5*e + 3/4*B*a^2*c*d^2*x^4 + 1/5*C*a^3*x^5*e^2
+ 3/5*A*a^2*c*x^5*e^2 + 1/2*C*a^3*d*x^4*e + 3/2*A*a^2*c*d*x^4*e + 1/3*C*a^3
*d^2*x^3 + A*a^2*c*d^2*x^3 + 1/4*B*a^3*x^4*e^2 + 2/3*B*a^3*d*x^3*e + 1/2*B*
a^3*d^2*x^2 + 1/3*A*a^3*x^3*e^2 + A*a^3*d*x^2*e + A*a^3*d^2*x
```

3.34 $\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=169

$$\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+a^3Adx+\frac{1}{2}a^2cCex^6+\frac{1}{4}a^3Cex^4+\frac{1}{7}c^2x^7(3a(Be+Cd)+Acd)+\frac{3}{5}acx^5(aBe+aCd+Acd)+$$

```
[Out] a^3*A*d*x + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^3*C*e*x^4)/4 + (3*a*
c*(A*c*d + a*C*d + a*B*e)*x^5)/5 + (a^2*c*C*e*x^6)/2 + (c^2*(A*c*d + 3*a*(C
*d + B*e))*x^7)/7 + (3*a*c^2*C*e*x^8)/8 + (c^3*(C*d + B*e)*x^9)/9 + (c^3*C*
e*x^10)/10 + ((B*d + A*e)*(a + c*x^2)^4)/(8*c)
```

Rubi [A] time = 0.187031, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1582, 1810}

$$\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+a^3Adx+\frac{1}{2}a^2cCex^6+\frac{1}{4}a^3Cex^4+\frac{1}{7}c^2x^7(3a(Be+Cd)+Acd)+\frac{3}{5}acx^5(aBe+aCd+Acd)+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```



```
[Out] a^3*A*d*x + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^3*C*e*x^4)/4 + (3*a*
c*(A*c*d + a*C*d + a*B*e)*x^5)/5 + (a^2*c*C*e*x^6)/2 + (c^2*(A*c*d + 3*a*(C
*d + B*e))*x^7)/7 + (3*a*c^2*C*e*x^8)/8 + (c^3*(C*d + B*e)*x^9)/9 + (c^3*C*
e*x^10)/10 + ((B*d + A*e)*(a + c*x^2)^4)/(8*c)
```


Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)dx = \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3\left(-(Bd+Ae)x+(d+ex)\left(A+Bx+C\right.\right.$$

$$= \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a^3Ad + a^2(3Acd+aCd+aBe)x^2 + a^3Cex^3 + 3a\right.$$

$$= a^3Adx + \frac{1}{3}a^2(3Acd+aCd+aBe)x^3 + \frac{1}{4}a^3Cex^4 + \frac{3}{5}ac(Acd+aCd+aBe)x^5$$

Mathematica [A] time = 0.0751338, size = 196, normalized size = 1.16

$$\frac{1}{4}a^2x^4(aCe+3Ace+3Bcd) + \frac{1}{3}a^2x^3(aBe+aCd+3Acd) + \frac{1}{2}a^3x^2(Ae+Bd) + a^3Adx + \frac{1}{8}c^2x^8(3aCe+Ace+Bcd) + \frac{1}{7}c$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d*x + (a^3*(B*d + A*e)*x^2)/2 + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3
 + (a^2*(3*B*c*d + 3*A*c*e + a*C*e)*x^4)/4 + (3*a*c*(A*c*d + a*C*d + a*B*e)
*x^5)/5 + (a*c*(B*c*d + A*c*e + a*C*e)*x^6)/2 + (c^2*(A*c*d + 3*a*C*d + 3*a
*B*e)*x^7)/7 + (c^2*(B*c*d + A*c*e + 3*a*C*e)*x^8)/8 + (c^3*(C*d + B*e)*x^9
)/9 + (c^3*C*e*x^10)/10
```

Maple [A] time = 0.044, size = 223, normalized size = 1.3

$$\frac{c^3Cex^{10}}{10} + \frac{\left(c^3eB + c^3dC\right)x^9}{9} + \frac{\left(c^3eA + c^3dB + 3\,eac^2C\right)x^8}{8} + \frac{\left(c^3dA + 3\,eac^2B + 3\,ac^2dC\right)x^7}{7} + \frac{\left(3\,eac^2A + 3\,ac^2dB + 3\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/10*c^3*C*e*x^10+1/9*(B*c^3*e+C*c^3*d)*x^9+1/8*(A*c^3*e+B*c^3*d+3*C*a*c^2*
e)*x^8+1/7*(A*c^3*d+3*B*a*c^2*e+3*C*a*c^2*d)*x^7+1/6*(3*A*a*c^2*e+3*B*a*c^2
```

```
*d+3*C*a^2*c*e)*x^6+1/5*(3*A*a*c^2*d+3*B*a^2*c*e+3*C*a^2*c*d)*x^5+1/4*(3*A*
a^2*c*e+3*B*a^2*c*d+C*a^3*e)*x^4+1/3*(3*A*a^2*c*d+B*a^3*e+C*a^3*d)*x^3+1/2*
(A*a^3*e+B*a^3*d)*x^2+a^3*A*d*x
```

Maxima [A] time = 1.321, size = 300, normalized size = 1.78

$$\frac{1}{10}\,Cc^3ex^{10} + \frac{1}{9}\left(Cc^3d + Bc^3e\right)x^9 + \frac{1}{8}\left(Bc^3d + \left(3\,Cac^2 + Ac^3\right)e\right)x^8 + \frac{1}{7}\left(3\,Bac^2e + \left(3\,Cac^2 + Ac^3\right)d\right)x^7 + \frac{1}{2}\left(Bac^2d + \left(Ca^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^3*e*x^10 + 1/9*(C*c^3*d + B*c^3*e)*x^9 + 1/8*(B*c^3*d + (3*C*a*c^2
 + A*c^3)*e)*x^8 + 1/7*(3*B*a*c^2*e + (3*C*a*c^2 + A*c^3)*d)*x^7 + 1/2*(B*a
*c^2*d + (C*a^2*c + A*a*c^2)*e)*x^6 + A*a^3*d*x + 3/5*(B*a^2*c*e + (C*a^2*c
 + A*a*c^2)*d)*x^5 + 1/4*(3*B*a^2*c*d + (C*a^3 + 3*A*a^2*c)*e)*x^4 + 1/3*(B
*a^3*e + (C*a^3 + 3*A*a^2*c)*d)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

Fricas [A] time = 1.45735, size = 603, normalized size = 3.57

$$\frac{1}{10}x^{10}ec^3C + \frac{1}{9}x^9dc^3C + \frac{1}{9}x^9ec^3B + \frac{3}{8}x^8ec^2aC + \frac{1}{8}x^8dc^3B + \frac{1}{8}x^8ec^3A + \frac{3}{7}x^7dc^2aC + \frac{3}{7}x^7ec^2aB + \frac{1}{7}x^7dc^3A + \frac{1}{2}x^6eca^2C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c^3*C + 1/9*x^9*d*c^3*C + 1/9*x^9*e*c^3*B + 3/8*x^8*e*c^2*a*C +
 1/8*x^8*d*c^3*B + 1/8*x^8*e*c^3*A + 3/7*x^7*d*c^2*a*C + 3/7*x^7*e*c^2*a*B
+ 1/7*x^7*d*c^3*A + 1/2*x^6*e*c*a^2*C + 1/2*x^6*d*c^2*a*B + 1/2*x^6*e*c^2*a
*A + 3/5*x^5*d*c*a^2*C + 3/5*x^5*e*c*a^2*B + 3/5*x^5*d*c^2*a*A + 1/4*x^4*e*
a^3*C + 3/4*x^4*d*c*a^2*B + 3/4*x^4*e*c*a^2*A + 1/3*x^3*d*a^3*C + 1/3*x^3*e
*a^3*B + x^3*d*c*a^2*A + 1/2*x^2*d*a^3*B + 1/2*x^2*e*a^3*A + x*d*a^3*A
```

Sympy [A] time = 0.101808, size = 265, normalized size = 1.57

$$Aa^3dx + \frac{Cc^3ex^{10}}{10} + x^9\left(\frac{Bc^3e}{9} + \frac{Cc^3d}{9}\right) + x^8\left(\frac{Ac^3e}{8} + \frac{Bc^3d}{8} + \frac{3Cac^2e}{8}\right) + x^7\left(\frac{Ac^3d}{7} + \frac{3Bac^2e}{7} + \frac{3Cac^2d}{7}\right) + x^6\left(\frac{Aac^2e}{2} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d*x + C*c**3*e*x**10/10 + x**9*(B*c**3*e/9 + C*c**3*d/9) + x**8*(A*c
**3*e/8 + B*c**3*d/8 + 3*C*a*c**2*e/8) + x**7*(A*c**3*d/7 + 3*B*a*c**2*e/7
+ 3*C*a*c**2*d/7) + x**6*(A*a*c**2*e/2 + B*a*c**2*d/2 + C*a**2*c*e/2) + x**
5*(3*A*a*c**2*d/5 + 3*B*a**2*c*e/5 + 3*C*a**2*c*d/5) + x**4*(3*A*a**2*c*e/4
 + 3*B*a**2*c*d/4 + C*a**3*e/4) + x**3*(A*a**2*c*d + B*a**3*e/3 + C*a**3*d/
3) + x**2*(A*a**3*e/2 + B*a**3*d/2)
```

Giac [A] time = 1.14016, size = 352, normalized size = 2.08

$$\frac{1}{10}Cc^3x^{10}e + \frac{1}{9}Cc^3dx^9 + \frac{1}{9}Bc^3x^9e + \frac{1}{8}Bc^3dx^8 + \frac{3}{8}Cac^2x^8e + \frac{1}{8}Ac^3x^8e + \frac{3}{7}Cac^2dx^7 + \frac{1}{7}Ac^3dx^7 + \frac{3}{7}Bac^2x^7e + \frac{1}{2}Bac$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^3*x^10*e + 1/9*C*c^3*d*x^9 + 1/9*B*c^3*x^9*e + 1/8*B*c^3*d*x^8 + 3
/8*C*a*c^2*x^8*e + 1/8*A*c^3*x^8*e + 3/7*C*a*c^2*d*x^7 + 1/7*A*c^3*d*x^7 +
3/7*B*a*c^2*x^7*e + 1/2*B*a*c^2*d*x^6 + 1/2*C*a^2*c*x^6*e + 1/2*A*a*c^2*x^6
*e + 3/5*C*a^2*c*d*x^5 + 3/5*A*a*c^2*d*x^5 + 3/5*B*a^2*c*x^5*e + 3/4*B*a^2*
c*d*x^4 + 1/4*C*a^3*x^4*e + 3/4*A*a^2*c*x^4*e + 1/3*C*a^3*d*x^3 + A*a^2*c*d
*x^3 + 1/3*B*a^3*x^3*e + 1/2*B*a^3*d*x^2 + 1/2*A*a^3*x^2*e + A*a^3*d*x
```

3.35 $\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=87

$$\frac{1}{3}a^2x^3(aC+3Ac)+a^3Ax+\frac{1}{7}c^2x^7(3aC+Ac)+\frac{3}{5}acx^5(aC+Ac)+\frac{B\left(a+cx^2\right)^4}{8c}+\frac{1}{9}c^3Cx^9$$

```
[Out] a^3*A*x + (a^2*(3*A*c + a*C)*x^3)/3 + (3*a*c*(A*c + a*C)*x^5)/5 + (c^2*(A*c
 + 3*a*C)*x^7)/7 + (c^3*C*x^9)/9 + (B*(a + c*x^2)^4)/(8*c)
```

Rubi [A] time = 0.0578149, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1582, 373}

$$\frac{1}{3}a^2x^3(aC+3Ac)+a^3Ax+\frac{1}{7}c^2x^7(3aC+Ac)+\frac{3}{5}acx^5(aC+Ac)+\frac{B\left(a+cx^2\right)^4}{8c}+\frac{1}{9}c^3Cx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*c + a*C)*x^3)/3 + (3*a*c*(A*c + a*C)*x^5)/5 + (c^2*(A*c
 + 3*a*C)*x^7)/7 + (c^3*C*x^9)/9 + (B*(a + c*x^2)^4)/(8*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx = \frac{B\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3 \left(A+Cx^2\right) dx$$
$$= \frac{B\left(a+cx^2\right)^4}{8c} + \int \left(a^3A + a^2(3Ac+aC)x^2 + 3ac(Ac+aC)x^4 + c^2(Ac+3aC)x^6 + c$$
$$= a^3Ax + \frac{1}{3}a^2(3Ac+aC)x^3 + \frac{3}{5}ac(Ac+aC)x^5 + \frac{1}{7}c^2(Ac+3aC)x^7 + \frac{1}{9}c^3Cx^9 + \frac{B\left(a\right.}{}$$

Mathematica [A] time = 0.0318585, size = 100, normalized size = 1.15

$$\frac{1}{20}a^2cx^3(20A+3x(5B+4Cx)) + \frac{1}{6}a^3x(6A+x(3B+2Cx)) + \frac{1}{70}ac^2x^5(42A+5x(7B+6Cx)) + \frac{1}{504}c^3x^7(72A+7x(9B$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] (a^3*x*(6*A + x*(3*B + 2*C*x)))/6 + (a^2*c*x^3*(20*A + 3*x*(5*B + 4*C*x)))/
20 + (a*c^2*x^5*(42*A + 5*x*(7*B + 6*C*x)))/70 + (c^3*x^7*(72*A + 7*x*(9*B
+ 8*C*x)))/504
```

Maple [A] time = 0.044, size = 111, normalized size = 1.3

$$\frac{c^3Cx^9}{9} + \frac{c^3Bx^8}{8} + \frac{\left(Ac^3+3\,ac^2C\right)x^7}{7} + \frac{ac^2Bx^6}{2} + \frac{\left(3\,aAc^2+3\,a^2cC\right)x^5}{5} + \frac{3\,Ba^2cx^4}{4} + \frac{\left(3\,a^2Ac+a^3C\right)x^3}{3} + \frac{a^3Bx^2}{2} + a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/9*c^3*C*x^9+1/8*c^3*B*x^8+1/7*(A*c^3+3*C*a*c^2)*x^7+1/2*a*c^2*B*x^6+1/5*(
3*A*a*c^2+3*C*a^2*c)*x^5+3/4*B*a^2*c*x^4+1/3*(3*A*a^2*c+C*a^3)*x^3+1/2*a^3*
B*x^2+a^3*A*x
```

Maxima [A] time = 1.22697, size = 146, normalized size = 1.68

$$\frac{1}{9}\,Cc^3x^9+\frac{1}{8}\,Bc^3x^8+\frac{1}{2}\,Bac^2x^6+\frac{3}{4}\,Ba^2cx^4+\frac{1}{7}\left(3\,Cac^2+Ac^3\right)x^7+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\left(Ca^2c+Aac^2\right)x^5+Aa^3x+\frac{1}{3}\left(Ca^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 1/2*B*a*c^2*x^6 + 3/4*B*a^2*c*x^4 + 1/7*(3*
C*a*c^2 + A*c^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*c + A*a*c^2)*x^5 + A*a^3*
x + 1/3*(C*a^3 + 3*A*a^2*c)*x^3
```

Fricas [A] time = 1.50704, size = 261, normalized size = 3.

$$\frac{1}{9}x^9c^3C+\frac{1}{8}x^8c^3B+\frac{3}{7}x^7c^2aC+\frac{1}{7}x^7c^3A+\frac{1}{2}x^6c^2aB+\frac{3}{5}x^5ca^2C+\frac{3}{5}x^5c^2aA+\frac{3}{4}x^4ca^2B+\frac{1}{3}x^3a^3C+x^3ca^2A+\frac{1}{2}x^2a^3B+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^3*C + 1/8*x^8*c^3*B + 3/7*x^7*c^2*a*C + 1/7*x^7*c^3*A + 1/2*x^6*c
^2*a*B + 3/5*x^5*c*a^2*C + 3/5*x^5*c^2*a*A + 3/4*x^4*c*a^2*B + 1/3*x^3*a^3*
C + x^3*c*a^2*A + 1/2*x^2*a^3*B + x*a^3*A
```

Sympy [A] time = 0.079793, size = 122, normalized size = 1.4

$$Aa^3x+\frac{Ba^3x^2}{2}+\frac{3Ba^2cx^4}{4}+\frac{Bac^2x^6}{2}+\frac{Bc^3x^8}{8}+\frac{Cc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Cac^2}{7}\right)+x^5\left(\frac{3Aac^2}{5}+\frac{3Ca^2c}{5}\right)+x^3\left(Aa^2c+\frac{Ca^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/2 + B*c**3*x**
8/8 + C*c**3*x**9/9 + x**7*(A*c**3/7 + 3*C*a*c**2/7) + x**5*(3*A*a*c**2/5 +
 3*C*a**2*c/5) + x**3*(A*a**2*c + C*a**3/3)
```

Giac [A] time = 1.14382, size = 150, normalized size = 1.72

$$\frac{1}{9}Cc^3x^9 + \frac{1}{8}Bc^3x^8 + \frac{3}{7}Cac^2x^7 + \frac{1}{7}Ac^3x^7 + \frac{1}{2}Bac^2x^6 + \frac{3}{5}Ca^2cx^5 + \frac{3}{5}Aac^2x^5 + \frac{3}{4}Ba^2cx^4 + \frac{1}{3}Ca^3x^3 + Aa^2cx^3 + \frac{1}{2}Ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 3/7*C*a*c^2*x^7 + 1/7*A*c^3*x^7 + 1/2*B*a*c
^2*x^6 + 3/5*C*a^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*B*a^2*c*x^4 + 1/3*C*a^3*x^
3 + A*a^2*c*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

3.36 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal. Leaf size=490

$$\frac{c(d+ex)^4\left(3a^2Ce^4+3ace^2\left(15Cd^2-e(5Bd-Ae)\right)+5c^2d^2\left(14Cd^2-e(7Bd-3Ae)\right)\right)}{4e^9}-\frac{c(d+ex)^3\left(3a^2e^4(4Cd-Be)+6ac\right.}{}$$

```
[Out] -((((c*d^2 + a*e^2)^2*(a*e^2*(2*C*d - B*e) + c*d*(8*C*d^2 - e*(7*B*d - 6*A*e
)))*x)/e^8) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*d^2*(28*C*d^2 - 3*e*(7*B*d
- 5*A*e)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*(d + e*x)^2)/(2*e^9) -
(c*(3*a^2*e^4*(4*C*d - B*e) + c^2*d^3*(56*C*d^2 - 5*e*(7*B*d - 4*A*e)) + 6*
a*c*d*e^2*(10*C*d^2 - e*(5*B*d - 2*A*e)))*(d + e*x)^3)/(3*e^9) + (c*(3*a^2*
C*e^4 + 5*c^2*d^2*(14*C*d^2 - e*(7*B*d - 3*A*e)) + 3*a*c*e^2*(15*C*d^2 - e*
(5*B*d - A*e)))*(d + e*x)^4)/(4*e^9) - (c^2*(3*a*e^2*(6*C*d - B*e) + c*d*(5
6*C*d^2 - 3*e*(7*B*d - 2*A*e)))*(d + e*x)^5)/(5*e^9) + (c^2*(3*a*C*e^2 + c*
(28*C*d^2 - e*(7*B*d - A*e)))*(d + e*x)^6)/(6*e^9) - (c^3*(8*C*d - B*e)*(d
+ e*x)^7)/(7*e^9) + (c^3*C*(d + e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2)^3*(C*d^2
 - B*d*e + A*e^2)*Log[d + e*x])/e^9
```

Rubi [A] time = 1.09758, antiderivative size = 487, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{c(d+ex)^4\left(3a^2Ce^4+3ace^2\left(15Cd^2-e(5Bd-Ae)\right)+5c^2\left(14Cd^4-d^2e(7Bd-3Ae)\right)\right)}{4e^9}-\frac{c(d+ex)^3\left(3a^2e^4(4Cd-Be)+6ac\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -((((c*d^2 + a*e^2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*
e))*x)/e^8) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d
- 5*A*e)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*(d + e*x)^2)/(2*e^9) -
(c*(3*a^2*e^4*(4*C*d - B*e) + c^2*(56*C*d^5 - 5*d^3*e*(7*B*d - 4*A*e)) + 6*
a*c*d*e^2*(10*C*d^2 - e*(5*B*d - 2*A*e)))*(d + e*x)^3)/(3*e^9) + (c*(3*a^2*
C*e^4 + 5*c^2*(14*C*d^4 - d^2*e*(7*B*d - 3*A*e)) + 3*a*c*e^2*(15*C*d^2 - e*
(5*B*d - A*e)))*(d + e*x)^4)/(4*e^9) - (c^2*(56*c*C*d^3 - 3*c*d*e*(7*B*d -
2*A*e) + 3*a*e^2*(6*C*d - B*e))*(d + e*x)^5)/(5*e^9) + (c^2*(28*c*C*d^2 + 3
*a*C*e^2 - c*e*(7*B*d - A*e))*(d + e*x)^6)/(6*e^9) - (c^3*(8*C*d - B*e)*(d
+ e*x)^7)/(7*e^9) + (c^3*C*(d + e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2)^3*(C*d^2
```

```
- B*d*e + A*e^2)*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)}{d+ex}\,dx=\int\left(\frac{\left(cd^2+ae^2\right)^2\left(-8cCd^3+cde(7Bd-6Ae)-ae^2(2Cd-Be)\right)}{e^8}+\frac{\left(cd^2+ae^2\right)^3\left(Cd^2\right.}{e^8(d+}\right.$$

$$=-\frac{\left(cd^2+ae^2\right)^2\left(8cCd^3-cde(7Bd-6Ae)+ae^2(2Cd-Be)\right)x}{e^8}+\frac{\left(cd^2+ae^2\right)\left(a^2Ce^4+\right.}{}$$

Mathematica [A] time = 0.504503, size = 498, normalized size = 1.02

$$\frac{x\left(210a^2ce^4\left(2e\left(3Ae(ex-2d)+B\left(6d^2-3dex+2e^2x^2\right)\right)+C\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)\right)+420a^3e^6(2Be-2Cd+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (x*(420*a^3*e^6*(-2*C*d + 2*B*e + C*e*x) + 210*a^2*c*e^4*(C*(-12*d^3 + 6*d^
2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d
*e*x + 2*e^2*x^2))) + 42*a*c^2*e^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^
2 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2
*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 -
 15*d*e^3*x^3 + 12*e^4*x^4))) + c^3*(C*(-840*d^7 + 420*d^6*e*x - 280*d^5*e^
2*x^2 + 210*d^4*e^3*x^3 - 168*d^3*e^4*x^4 + 140*d^2*e^5*x^5 - 120*d*e^6*x^6
 + 105*e^7*x^7) + 2*e*(7*A*e*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^
2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + B*(420*d^6 - 210*d^5*e*x + 140*d^4
*e^2*x^2 - 105*d^3*e^3*x^3 + 84*d^2*e^4*x^4 - 70*d*e^5*x^5 + 60*e^6*x^6))))
)/(840*e^8) + ((c*d^2 + a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/e
^9
```

Maple [A] time = 0.054, size = 880, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x)
```

```
[Out] -3/5/e^2*C*x^5*a*c^2*d-3/4/e^2*B*x^4*a*c^2*d+3/e^3*ln(e*x+d)*A*a^2*c*d^2-3/
e^2*A*a^2*c*d*x-3/e^4*A*a*c^2*d^3*x+3/2/e^5*C*x^2*a*c^2*d^4+3/4/e^3*C*x^4*a
*c^2*d^2-3/2/e^2*B*x^2*a^2*c*d+3/2/e^3*A*x^2*a*c^2*d^2+3/e^3*B*a^2*c*d^2*x+
3/e^5*B*a*c^2*d^4*x-1/7/e^2*C*x^7*c^3*d+1/2/e*C*x^6*a*c^2-1/e^4*C*x^3*a*c^2
*d^3-3/2/e^4*B*x^2*a*c^2*d^3+1/e^3*B*x^3*a*c^2*d^2+3/e^5*ln(e*x+d)*C*a^2*c*
d^4-1/e^2*A*x^3*a*c^2*d+3/2/e^3*C*x^2*a^2*c*d^2+1/7/e*B*x^7*c^3+1/8/e*C*c^3
*x^8+1/2/e*C*x^2*a^3+1/4/e^3*A*x^4*c^3*d^2+3/5/e*B*x^5*a*c^2-1/5/e^4*C*x^5*
c^3*d^3+3/4/e*A*x^4*a*c^2-3/e^6*ln(e*x+d)*B*a*c^2*d^5-3/e^4*ln(e*x+d)*B*a^2
*c*d^3-1/e^2*C*x^3*a^2*c*d+3/e^7*ln(e*x+d)*C*a*c^2*d^6-3/e^4*C*a^2*c*d^3*x-
1/4/e^4*B*x^4*c^3*d^3+1/5/e^3*B*x^5*c^3*d^2+3/4/e*C*x^4*a^2*c-1/3/e^6*C*x^3
*c^3*d^5+3/2/e*A*x^2*a^2*c+1/3/e^5*B*x^3*c^3*d^4-1/2/e^6*B*x^2*c^3*d^5-1/e^
2*C*a^3*d*x+1/2/e^7*C*x^2*c^3*d^6-1/e^6*A*c^3*d^5*x-1/3/e^4*A*x^3*c^3*d^3+1
/e*B*x^3*a^2*c-1/5/e^2*A*x^5*c^3*d+1/4/e^5*C*x^4*c^3*d^4+1/2/e^5*A*x^2*c^3*
d^4-1/6/e^2*B*x^6*c^3*d+1/6/e^3*C*x^6*c^3*d^2-1/e^8*C*c^3*d^7*x+1/e^7*ln(e*
x+d)*A*c^3*d^6-1/e^2*ln(e*x+d)*B*a^3*d-1/e^8*ln(e*x+d)*B*c^3*d^7+1/e^3*ln(e
*x+d)*C*a^3*d^2+1/e^9*ln(e*x+d)*C*c^3*d^8+1/e^7*B*c^3*d^6*x+1/6/e*A*x^6*c^3
+1/e*ln(e*x+d)*A*a^3+1/e*a^3*B*x-3/e^6*C*a*c^2*d^5*x+3/e^5*ln(e*x+d)*A*a*c^
2*d^4
```

Maxima [A] time = 1.0654, size = 907, normalized size = 1.85

$$\frac{105\,Cc^3e^7x^8 - 120\left(Cc^3de^6 - Bc^3e^7\right)x^7 + 140\left(Cc^3d^2e^5 - Bc^3de^6 + \left(3\,Cac^2 + Ac^3\right)e^7\right)x^6 - 168\left(Cc^3d^3e^4 - Bc^3d^2e^5 - 3\,Bac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/840*(105*C*c^3*e^7*x^8 - 120*(C*c^3*d*e^6 - B*c^3*e^7)*x^7 + 140*(C*c^3*d
^2*e^5 - B*c^3*d*e^6 + (3*C*a*c^2 + A*c^3)*e^7)*x^6 - 168*(C*c^3*d^3*e^4 -
B*c^3*d^2*e^5 - 3*B*a*c^2*e^7 + (3*C*a*c^2 + A*c^3)*d*e^6)*x^5 + 210*(C*c^3
*d^4*e^3 - B*c^3*d^3*e^4 - 3*B*a*c^2*d*e^6 + (3*C*a*c^2 + A*c^3)*d^2*e^5 +
3*(C*a^2*c + A*a*c^2)*e^7)*x^4 - 280*(C*c^3*d^5*e^2 - B*c^3*d^4*e^3 - 3*B*a
*c^2*d^2*e^5 - 3*B*a^2*c*e^7 + (3*C*a*c^2 + A*c^3)*d^3*e^4 + 3*(C*a^2*c + A
```

```
*a*c^2)*d*e^6)*x^3 + 420*(C*c^3*d^6*e - B*c^3*d^5*e^2 - 3*B*a*c^2*d^3*e^4 -
 3*B*a^2*c*d*e^6 + (3*C*a*c^2 + A*c^3)*d^4*e^3 + 3*(C*a^2*c + A*a*c^2)*d^2*
e^5 + (C*a^3 + 3*A*a^2*c)*e^7)*x^2 - 840*(C*c^3*d^7 - B*c^3*d^6*e - 3*B*a*c
^2*d^4*e^3 - 3*B*a^2*c*d^2*e^5 - B*a^3*e^7 + (3*C*a*c^2 + A*c^3)*d^5*e^2 +
3*(C*a^2*c + A*a*c^2)*d^3*e^4 + (C*a^3 + 3*A*a^2*c)*d*e^6)*x)/e^8 + (C*c^3*
d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + A
*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C
*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d)/e^9
```

Fricas [A] time = 1.72348, size = 1385, normalized size = 2.83

$$\frac{105\,Cc^3e^8x^8 - 120\left(Cc^3de^7 - Bc^3e^8\right)x^7 + 140\left(Cc^3d^2e^6 - Bc^3de^7 + \left(3\,Cac^2 + Ac^3\right)e^8\right)x^6 - 168\left(Cc^3d^3e^5 - Bc^3d^2e^6 - 3\,Ba\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/840*(105*C*c^3*e^8*x^8 - 120*(C*c^3*d*e^7 - B*c^3*e^8)*x^7 + 140*(C*c^3*d
^2*e^6 - B*c^3*d*e^7 + (3*C*a*c^2 + A*c^3)*e^8)*x^6 - 168*(C*c^3*d^3*e^5 -
B*c^3*d^2*e^6 - 3*B*a*c^2*e^8 + (3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 210*(C*c^3
*d^4*e^4 - B*c^3*d^3*e^5 - 3*B*a*c^2*d*e^7 + (3*C*a*c^2 + A*c^3)*d^2*e^6 +
3*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 280*(C*c^3*d^5*e^3 - B*c^3*d^4*e^4 - 3*B*a
*c^2*d^2*e^6 - 3*B*a^2*c*e^8 + (3*C*a*c^2 + A*c^3)*d^3*e^5 + 3*(C*a^2*c + A
*a*c^2)*d*e^7)*x^3 + 420*(C*c^3*d^6*e^2 - B*c^3*d^5*e^3 - 3*B*a*c^2*d^3*e^5
 - 3*B*a^2*c*d*e^7 + (3*C*a*c^2 + A*c^3)*d^4*e^4 + 3*(C*a^2*c + A*a*c^2)*d^
2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 - 840*(C*c^3*d^7*e - B*c^3*d^6*e^2 - 3
*B*a*c^2*d^4*e^4 - 3*B*a^2*c*d^2*e^6 - B*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^5*
e^3 + 3*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)*x + 840*(C
*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^
7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4
 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d))/e^9
```

Sympy [A] time = 2.32097, size = 658, normalized size = 1.34

$$\frac{Cc^3x^8}{8e} - \frac{x^7\left(-Bc^3e + Cc^3d\right)}{7e^2} + \frac{x^6\left(Ac^3e^2 - Bc^3de + 3Cac^2e^2 + Cc^3d^2\right)}{6e^3} - \frac{x^5\left(Ac^3de^2 - 3Bac^2e^3 - Bc^3d^2e + 3Cac^2de^2 + \right.}{5e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c**3*x**8/(8*e) - x**7*(-B*c**3*e + C*c**3*d)/(7*e**2) + x**6*(A*c**3*e**
2 - B*c**3*d*e + 3*C*a*c**2*e**2 + C*c**3*d**2)/(6*e**3) - x**5*(A*c**3*d*e
**2 - 3*B*a*c**2*e**3 - B*c**3*d**2*e + 3*C*a*c**2*d*e**2 + C*c**3*d**3)/(5
*e**4) + x**4*(3*A*a*c**2*e**4 + A*c**3*d**2*e**2 - 3*B*a*c**2*d*e**3 - B*c
**3*d**3*e + 3*C*a**2*c*e**4 + 3*C*a*c**2*d**2*e**2 + C*c**3*d**4)/(4*e**5)
 - x**3*(3*A*a*c**2*d*e**4 + A*c**3*d**3*e**2 - 3*B*a**2*c*e**5 - 3*B*a*c**
2*d**2*e**3 - B*c**3*d**4*e + 3*C*a**2*c*d*e**4 + 3*C*a*c**2*d**3*e**2 + C*
c**3*d**5)/(3*e**6) + x**2*(3*A*a**2*c*e**6 + 3*A*a*c**2*d**2*e**4 + A*c**3
*d**4*e**2 - 3*B*a**2*c*d*e**5 - 3*B*a*c**2*d**3*e**3 - B*c**3*d**5*e + C*a
**3*e**6 + 3*C*a**2*c*d**2*e**4 + 3*C*a*c**2*d**4*e**2 + C*c**3*d**6)/(2*e*
*7) - x*(3*A*a**2*c*d*e**6 + 3*A*a*c**2*d**3*e**4 + A*c**3*d**5*e**2 - B*a*
*3*e**7 - 3*B*a**2*c*d**2*e**5 - 3*B*a*c**2*d**4*e**3 - B*c**3*d**6*e + C*a
**3*d*e**6 + 3*C*a**2*c*d**3*e**4 + 3*C*a*c**2*d**5*e**2 + C*c**3*d**7)/e**
8 + (a*e**2 + c*d**2)**3*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**9
```

<u>Giac</u> [A] time = 1.14769, size = 1031, normalized size = 2.1

$$\left(Cc^3d^8 - Bc^3d^7e + 3\,Cac^2d^6e^2 + Ac^3d^6e^2 - 3\,Bac^2d^5e^3 + 3\,Ca^2cd^4e^4 + 3\,Aac^2d^4e^4 - 3\,Ba^2cd^3e^5 + Ca^3d^2e^6 + 3\,Aa^2cd^2e^6 - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c^3*d^8 - B*c^3*d^7*e + 3*C*a*c^2*d^6*e^2 + A*c^3*d^6*e^2 - 3*B*a*c^2*d^
5*e^3 + 3*C*a^2*c*d^4*e^4 + 3*A*a*c^2*d^4*e^4 - 3*B*a^2*c*d^3*e^5 + C*a^3*d
^2*e^6 + 3*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 + A*a^3*e^8)*e^(-9)*log(abs(x*e +
d)) + 1/840*(105*C*c^3*x^8*e^7 - 120*C*c^3*d*x^7*e^6 + 140*C*c^3*d^2*x^6*e^
5 - 168*C*c^3*d^3*x^5*e^4 + 210*C*c^3*d^4*x^4*e^3 - 280*C*c^3*d^5*x^3*e^2 +
 420*C*c^3*d^6*x^2*e - 840*C*c^3*d^7*x + 120*B*c^3*x^7*e^7 - 140*B*c^3*d*x^
6*e^6 + 168*B*c^3*d^2*x^5*e^5 - 210*B*c^3*d^3*x^4*e^4 + 280*B*c^3*d^4*x^3*e
^3 - 420*B*c^3*d^5*x^2*e^2 + 840*B*c^3*d^6*x*e + 420*C*a*c^2*x^6*e^7 + 140*
A*c^3*x^6*e^7 - 504*C*a*c^2*d*x^5*e^6 - 168*A*c^3*d*x^5*e^6 + 630*C*a*c^2*d
^2*x^4*e^5 + 210*A*c^3*d^2*x^4*e^5 - 840*C*a*c^2*d^3*x^3*e^4 - 280*A*c^3*d^
3*x^3*e^4 + 1260*C*a*c^2*d^4*x^2*e^3 + 420*A*c^3*d^4*x^2*e^3 - 2520*C*a*c^2
*d^5*x*e^2 - 840*A*c^3*d^5*x*e^2 + 504*B*a*c^2*x^5*e^7 - 630*B*a*c^2*d*x^4*
e^6 + 840*B*a*c^2*d^2*x^3*e^5 - 1260*B*a*c^2*d^3*x^2*e^4 + 2520*B*a*c^2*d^4
*x*e^3 + 630*C*a^2*c*x^4*e^7 + 630*A*a*c^2*x^4*e^7 - 840*C*a^2*c*d*x^3*e^6
- 840*A*a*c^2*d*x^3*e^6 + 1260*C*a^2*c*d^2*x^2*e^5 + 1260*A*a*c^2*d^2*x^2*e
^5 - 2520*C*a^2*c*d^3*x*e^4 - 2520*A*a*c^2*d^3*x*e^4 + 840*B*a^2*c*x^3*e^7
```

```
- 1260*B*a^2*c*d*x^2*e^6 + 2520*B*a^2*c*d^2*x*e^5 + 420*C*a^3*x^2*e^7 + 126
0*A*a^2*c*x^2*e^7 - 840*C*a^3*d*x*e^6 - 2520*A*a^2*c*d*x*e^6 + 840*B*a^3*x*
e^7)*e^(-8)
```

3.37 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

Optimal. Leaf size=486

$$\frac{cx^3\left(3a^2Ce^4+3ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2d^2\left(5Cd^2-e(4Bd-3Ae)\right)\right)}{3e^6}-\frac{cx^2\left(3a^2e^4(2Cd-Be)+3acde^2\left(4Cd^2-e(3B\right.\right.}{2e}$$

```
[Out] ((a^3*C*e^6 + c^3*d^4*(7*C*d^2 - e*(6*B*d - 5*A*e)) + 3*a*c^2*d^2*e^2*(5*C*
d^2 - e*(4*B*d - 3*A*e)) + 3*a^2*c*e^4*(3*C*d^2 - e*(2*B*d - A*e)))*x)/e^8
- (c*(3*a^2*e^4*(2*C*d - B*e) + c^2*d^3*(6*C*d^2 - e*(5*B*d - 4*A*e)) + 3*a
*c*d*e^2*(4*C*d^2 - e*(3*B*d - 2*A*e)))*x^2)/(2*e^7) + (c*(3*a^2*C*e^4 + c^
2*d^2*(5*C*d^2 - e*(4*B*d - 3*A*e)) + 3*a*c*e^2*(3*C*d^2 - e*(2*B*d - A*e))
)*x^3)/(3*e^6) - (c^2*(3*a*e^2*(2*C*d - B*e) + c*d*(4*C*d^2 - e*(3*B*d - 2*
A*e)))*x^4)/(4*e^5) + (c^2*(3*a*C*e^2 + c*(3*C*d^2 - e*(2*B*d - A*e)))*x^5)
/(5*e^4) - (c^3*(2*C*d - B*e)*x^6)/(6*e^3) + (c^3*C*x^7)/(7*e^2) - ((c*d^2
+ a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(e^9*(d + e*x)) - ((c*d^2 + a*e^2)^2*(a
*e^2*(2*C*d - B*e) + c*d*(8*C*d^2 - e*(7*B*d - 6*A*e)))*Log[d + e*x])/e^9
```

Rubi [A] time = 0.980382, antiderivative size = 483, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{cx^3\left(3a^2Ce^4+3ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)\right)}{3e^6}-\frac{cx^2\left(3a^2e^4(2Cd-Be)+3acde^2\left(4Cd^2-e(3B\right.\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^3*C*e^6 + c^3*(7*C*d^6 - d^4*e*(6*B*d - 5*A*e)) + 3*a*c^2*d^2*e^2*(5*C*
d^2 - e*(4*B*d - 3*A*e)) + 3*a^2*c*e^4*(3*C*d^2 - e*(2*B*d - A*e)))*x)/e^8
- (c*(3*a^2*e^4*(2*C*d - B*e) + c^2*(6*C*d^5 - d^3*e*(5*B*d - 4*A*e)) + 3*a
*c*d*e^2*(4*C*d^2 - e*(3*B*d - 2*A*e)))*x^2)/(2*e^7) + (c*(3*a^2*C*e^4 + c^
2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 3*a*c*e^2*(3*C*d^2 - e*(2*B*d - A*e))
)*x^3)/(3*e^6) - (c^2*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 3*a*e^2*(2*C*d -
 B*e))*x^4)/(4*e^5) + (c^2*(3*c*C*d^2 + 3*a*C*e^2 - c*e*(2*B*d - A*e))*x^5)
/(5*e^4) - (c^3*(2*C*d - B*e)*x^6)/(6*e^3) + (c^3*C*x^7)/(7*e^2) - ((c*d^2
+ a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(e^9*(d + e*x)) - ((c*d^2 + a*e^2)^2*(8
*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)}{(d+ex)^2}\,dx = \int\left(\frac{a^3Ce^6+c^3\left(7Cd^6-d^4e(6Bd-5Ae)\right)+3ac^2d^2e^2\left(5Cd^2-e(4Bd-3Ae)\right)+3a^2c}{e^8}\right.$$

$$= \frac{\left(a^3Ce^6+c^3\left(7Cd^6-d^4e(6Bd-5Ae)\right)+3ac^2d^2e^2\left(5Cd^2-e(4Bd-3Ae)\right)+3a^2ce^4\right(}{e^8}$$

Mathematica [A] time = 0.357787, size = 641, normalized size = 1.32

$$\frac{210a^2ce^4\left(3e\left(2Ae\left(-d^2+dex+e^2x^2\right)+B\left(-4d^2ex+2d^3-3de^2x^2+e^3x^3\right)\right)+2C\left(6d^2e^2x^2+9d^3ex-3d^4-2de^3x^3+e^4x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (420*a^3*e^6*(e*(B*d - A*e) + C*(-d^2 + d*e*x + e^2*x^2)) + 210*a^2*c*e^4*(
2*C*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 3*e*(2*A
*e*(-d^2 + d*e*x + e^2*x^2) + B*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3)
)) + 21*a*c^2*e^2*(-6*C*(10*d^6 - 50*d^5*e*x - 30*d^4*e^2*x^2 + 10*d^3*e^3*
x^3 - 5*d^2*e^4*x^4 + 3*d*e^5*x^5 - 2*e^6*x^6) + 5*e*(4*A*e*(-3*d^4 + 9*d^3
*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + B*(12*d^5 - 48*d^4*e*x - 30
*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5))) + c^3*(-4*C*(105
*d^8 - 735*d^7*e*x - 420*d^6*e^2*x^2 + 140*d^5*e^3*x^3 - 70*d^4*e^4*x^4 + 4
2*d^3*e^5*x^5 - 28*d^2*e^6*x^6 + 20*d*e^7*x^7 - 15*e^8*x^8) + 7*e*(6*A*e*(-
10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*d
*e^5*x^5 + 2*e^6*x^6) + B*(60*d^7 - 360*d^6*e*x - 210*d^5*e^2*x^2 + 70*d^4*
e^3*x^3 - 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 - 14*d*e^6*x^6 + 10*e^7*x^7))) -
420*(c*d^2 + a*e^2)^2*(8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d -
B*e))*(d + e*x)*Log[d + e*x])/(420*e^9*(d + e*x))
```

Maple [A] time = 0.059, size = 928, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x)
```

```
[Out] 7/e^8*ln(e*x+d)*B*c^3*d^6-2/e^3*ln(e*x+d)*C*a^3*d+3/e^2*a^2*A*c*x+5/e^6*A*c
^3*d^4*x-6/e^7*B*c^3*d^5*x+5/2/e^6*B*x^2*c^3*d^4-2/e^5*A*x^2*c^3*d^3+3/2/e^
2*B*x^2*a^2*c+15/e^6*ln(e*x+d)*B*a*c^2*d^4-12/e^5*ln(e*x+d)*C*a^2*c*d^3-18/
e^7*ln(e*x+d)*C*a*c^2*d^5-3/e^3/(e*x+d)*A*a^2*c*d^2-3/e^5/(e*x+d)*A*a*c^2*d
^4+3/e^4/(e*x+d)*B*a^2*c*d^3+3/e^6/(e*x+d)*B*a*c^2*d^5-4/3/e^5*B*x^3*c^3*d^
3+5/3/e^6*C*x^3*c^3*d^4-1/3/e^3*C*x^6*c^3*d-2/5/e^3*B*x^5*c^3*d+3/5/e^2*C*x
^5*a*c^2+3/5/e^4*C*x^5*c^3*d^2-1/2/e^3*A*x^4*c^3*d+1/e^4*A*x^3*c^3*d^2+3/4/
e^4*B*x^4*c^3*d^2-1/e^5*C*x^4*c^3*d^3+1/e^2*C*x^3*a^2*c+1/e^2*A*x^3*a*c^2-3
/e^7*C*x^2*c^3*d^5-8/e^9*ln(e*x+d)*C*c^3*d^7-1/e^7/(e*x+d)*A*c^3*d^6+1/e^2/
(e*x+d)*B*d*a^3+1/e^8/(e*x+d)*B*c^3*d^7-1/e^3/(e*x+d)*C*a^3*d^2-1/e^9/(e*x+
d)*C*c^3*d^8-6/e^7*ln(e*x+d)*A*c^3*d^5+7/e^8*C*c^3*d^6*x+3/4/e^2*B*x^4*a*c^
2+1/e^2*a^3*C*x+1/5/e^2*A*x^5*c^3+1/6/e^2*B*x^6*c^3-1/e/(e*x+d)*A*a^3+1/e^2
*ln(e*x+d)*B*a^3-3/e^5/(e*x+d)*C*a^2*c*d^4-3/e^7/(e*x+d)*C*a*c^2*d^6-6/e^3*
ln(e*x+d)*A*a^2*c*d-12/e^5*ln(e*x+d)*A*a*c^2*d^3+9/e^4*ln(e*x+d)*B*a^2*c*d^
2-2/e^3*B*x^3*a*c^2*d+9/e^4*A*a*c^2*d^2*x-6/e^3*B*a^2*c*d*x-3/2/e^3*C*x^4*a
*c^2*d-3/e^3*A*x^2*a*c^2*d+3/e^4*C*x^3*a*c^2*d^2+9/2/e^4*B*x^2*a*c^2*d^2-3/
e^3*C*x^2*a^2*c*d-6/e^5*C*x^2*a*c^2*d^3-12/e^5*B*a*c^2*d^3*x+9/e^4*C*a^2*c*
d^2*x+15/e^6*C*a*c^2*d^4*x+1/7*c^3*C*x^7/e^2
```

Maxima [A] time = 0.999464, size = 933, normalized size = 1.92

$$-\frac{Cc^3d^8 - Bc^3d^7e - 3\,Bac^2d^5e^3 - 3\,Ba^2cd^3e^5 - Ba^3de^7 + Aa^3e^8 + \left(3\,Cac^2 + Ac^3\right)d^6e^2 + 3\,\left(Ca^2c + Aac^2\right)d^4e^4 + \left(Ca^3 + 3\,A\right.}{e^{10}x + de^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d
*e^7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*
e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)/(e^10*x + d*e^9) + 1/420*(60*C*c^3*e^6*x
^7 - 70*(2*C*c^3*d*e^5 - B*c^3*e^6)*x^6 + 84*(3*C*c^3*d^2*e^4 - 2*B*c^3*d*e
^5 + (3*C*a*c^2 + A*c^3)*e^6)*x^5 - 105*(4*C*c^3*d^3*e^3 - 3*B*c^3*d^2*e^4
- 3*B*a*c^2*e^6 + 2*(3*C*a*c^2 + A*c^3)*d*e^5)*x^4 + 140*(5*C*c^3*d^4*e^2 -
```

```
 4*B*c^3*d^3*e^3 - 6*B*a*c^2*d*e^5 + 3*(3*C*a*c^2 + A*c^3)*d^2*e^4 + 3*(C*a
^2*c + A*a*c^2)*e^6)*x^3 - 210*(6*C*c^3*d^5*e - 5*B*c^3*d^4*e^2 - 9*B*a*c^2
*d^2*e^4 - 3*B*a^2*c*e^6 + 4*(3*C*a*c^2 + A*c^3)*d^3*e^3 + 6*(C*a^2*c + A*a
*c^2)*d*e^5)*x^2 + 420*(7*C*c^3*d^6 - 6*B*c^3*d^5*e - 12*B*a*c^2*d^3*e^3 -
6*B*a^2*c*d*e^5 + 5*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2
*e^4 + (C*a^3 + 3*A*a^2*c)*e^6)*x)/e^8 - (8*C*c^3*d^7 - 7*B*c^3*d^6*e - 15*
B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 - B*a^3*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^5
*e^2 + 12*(C*a^2*c + A*a*c^2)*d^3*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d*e^6)*log(e*
x + d)/e^9
```

Fricas [A] time = 1.86145, size = 1987, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/420*(60*C*c^3*e^8*x^8 - 420*C*c^3*d^8 + 420*B*c^3*d^7*e + 1260*B*a*c^2*d^
5*e^3 + 1260*B*a^2*c*d^3*e^5 + 420*B*a^3*d*e^7 - 420*A*a^3*e^8 - 420*(3*C*a
*c^2 + A*c^3)*d^6*e^2 - 1260*(C*a^2*c + A*a*c^2)*d^4*e^4 - 420*(C*a^3 + 3*A
*a^2*c)*d^2*e^6 - 10*(8*C*c^3*d*e^7 - 7*B*c^3*e^8)*x^7 + 14*(8*C*c^3*d^2*e^
6 - 7*B*c^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 21*(8*C*c^3*d^3*e^5 -
7*B*c^3*d^2*e^6 - 15*B*a*c^2*e^8 + 6*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 35*(8
*C*c^3*d^4*e^4 - 7*B*c^3*d^3*e^5 - 15*B*a*c^2*d*e^7 + 6*(3*C*a*c^2 + A*c^3)
*d^2*e^6 + 12*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 70*(8*C*c^3*d^5*e^3 - 7*B*c^3*
d^4*e^4 - 15*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 6*(3*C*a*c^2 + A*c^3)*d^3*e^
5 + 12*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 + 210*(8*C*c^3*d^6*e^2 - 7*B*c^3*d^5*
e^3 - 15*B*a*c^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^4*e^4
+ 12*(C*a^2*c + A*a*c^2)*d^2*e^6 + 2*(C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 420*(7*
C*c^3*d^7*e - 6*B*c^3*d^6*e^2 - 12*B*a*c^2*d^4*e^4 - 6*B*a^2*c*d^2*e^6 + 5*
(3*C*a*c^2 + A*c^3)*d^5*e^3 + 9*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*
a^2*c)*d*e^7)*x - 420*(8*C*c^3*d^8 - 7*B*c^3*d^7*e - 15*B*a*c^2*d^5*e^3 - 9
*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 12*(C*a^2*
c + A*a*c^2)*d^4*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + (8*C*c^3*d^7*e - 7*B
*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*
a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^
2*c)*d*e^7)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

Sympy [A] time = 5.69052, size = 731, normalized size = 1.5

$$\frac{Cc^3x^7}{7e^2} - \frac{Aa^3e^8 + 3Aa^2cd^2e^6 + 3Aac^2d^4e^4 + Ac^3d^6e^2 - Ba^3de^7 - 3Ba^2cd^3e^5 - 3Bac^2d^5e^3 - Bc^3d^7e + Ca^3d^2e^6 + 3Ca^2cd^4e}{de^9 + e^{10}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**3*x**7/(7*e**2) - (A*a**3*e**8 + 3*A*a**2*c*d**2*e**6 + 3*A*a*c**2*d**
4*e**4 + A*c**3*d**6*e**2 - B*a**3*d*e**7 - 3*B*a**2*c*d**3*e**5 - 3*B*a*c*
*2*d**5*e**3 - B*c**3*d**7*e + C*a**3*d**2*e**6 + 3*C*a**2*c*d**4*e**4 + 3*
C*a*c**2*d**6*e**2 + C*c**3*d**8)/(d*e**9 + e**10*x) - x**6*(-B*c**3*e + 2*
C*c**3*d)/(6*e**3) + x**5*(A*c**3*e**2 - 2*B*c**3*d*e + 3*C*a*c**2*e**2 + 3
*C*c**3*d**2)/(5*e**4) - x**4*(2*A*c**3*d*e**2 - 3*B*a*c**2*e**3 - 3*B*c**3
*d**2*e + 6*C*a*c**2*d*e**2 + 4*C*c**3*d**3)/(4*e**5) + x**3*(3*A*a*c**2*e*
*4 + 3*A*c**3*d**2*e**2 - 6*B*a*c**2*d*e**3 - 4*B*c**3*d**3*e + 3*C*a**2*c*
e**4 + 9*C*a*c**2*d**2*e**2 + 5*C*c**3*d**4)/(3*e**6) - x**2*(6*A*a*c**2*d*
e**4 + 4*A*c**3*d**3*e**2 - 3*B*a**2*c*e**5 - 9*B*a*c**2*d**2*e**3 - 5*B*c*
*3*d**4*e + 6*C*a**2*c*d*e**4 + 12*C*a*c**2*d**3*e**2 + 6*C*c**3*d**5)/(2*e
**7) + x*(3*A*a**2*c*e**6 + 9*A*a*c**2*d**2*e**4 + 5*A*c**3*d**4*e**2 - 6*B
*a**2*c*d*e**5 - 12*B*a*c**2*d**3*e**3 - 6*B*c**3*d**5*e + C*a**3*e**6 + 9*
C*a**2*c*d**2*e**4 + 15*C*a*c**2*d**4*e**2 + 7*C*c**3*d**6)/e**8 - (a*e**2
+ c*d**2)**2*(6*A*c*d*e**2 - B*a*e**3 - 7*B*c*d**2*e + 2*C*a*d*e**2 + 8*C*c
*d**3)*log(d + e*x)/e**9
```

Giac [A] time = 1.21041, size = 1131, normalized size = 2.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/420*(60*C*c^3 - 70*(8*C*c^3*d*e - B*c^3*e^2)*e^(-1)/(x*e + d) + 84*(28*C*
c^3*d^2*e^2 - 7*B*c^3*d*e^3 + 3*C*a*c^2*e^4 + A*c^3*e^4)*e^(-2)/(x*e + d)^2
 - 105*(56*C*c^3*d^3*e^3 - 21*B*c^3*d^2*e^4 + 18*C*a*c^2*d*e^5 + 6*A*c^3*d*
e^5 - 3*B*a*c^2*e^6)*e^(-3)/(x*e + d)^3 + 140*(70*C*c^3*d^4*e^4 - 35*B*c^3*
d^3*e^5 + 45*C*a*c^2*d^2*e^6 + 15*A*c^3*d^2*e^6 - 15*B*a*c^2*d*e^7 + 3*C*a^
2*c*e^8 + 3*A*a*c^2*e^8)*e^(-4)/(x*e + d)^4 - 210*(56*C*c^3*d^5*e^5 - 35*B*
c^3*d^4*e^6 + 60*C*a*c^2*d^3*e^7 + 20*A*c^3*d^3*e^7 - 30*B*a*c^2*d^2*e^8 +
```

```
12*C*a^2*c*d*e^9 + 12*A*a*c^2*d*e^9 - 3*B*a^2*c*e^10)*e^(-5)/(x*e + d)^5 +
420*(28*C*c^3*d^6*e^6 - 21*B*c^3*d^5*e^7 + 45*C*a*c^2*d^4*e^8 + 15*A*c^3*d^
4*e^8 - 30*B*a*c^2*d^3*e^9 + 18*C*a^2*c*d^2*e^10 + 18*A*a*c^2*d^2*e^10 - 9*
B*a^2*c*d*e^11 + C*a^3*e^12 + 3*A*a^2*c*e^12)*e^(-6)/(x*e + d)^6)*(x*e + d)
^7*e^(-9) + (8*C*c^3*d^7 - 7*B*c^3*d^6*e + 18*C*a*c^2*d^5*e^2 + 6*A*c^3*d^5
*e^2 - 15*B*a*c^2*d^4*e^3 + 12*C*a^2*c*d^3*e^4 + 12*A*a*c^2*d^3*e^4 - 9*B*a
^2*c*d^2*e^5 + 2*C*a^3*d*e^6 + 6*A*a^2*c*d*e^6 - B*a^3*e^7)*e^(-9)*log(abs(
x*e + d)*e^(-1)/(x*e + d)^2) - (C*c^3*d^8*e^7/(x*e + d) - B*c^3*d^7*e^8/(x*
e + d) + 3*C*a*c^2*d^6*e^9/(x*e + d) + A*c^3*d^6*e^9/(x*e + d) - 3*B*a*c^2*
d^5*e^10/(x*e + d) + 3*C*a^2*c*d^4*e^11/(x*e + d) + 3*A*a*c^2*d^4*e^11/(x*e
 + d) - 3*B*a^2*c*d^3*e^12/(x*e + d) + C*a^3*d^2*e^13/(x*e + d) + 3*A*a^2*c
*d^2*e^13/(x*e + d) - B*a^3*d*e^14/(x*e + d) + A*a^3*e^15/(x*e + d))*e^(-16
)
```

3.38 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

Optimal. Leaf size=466

$$\frac{cx^2\left(3a^2Ce^4+3ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2d^2\left(15Cd^2-2e(5Bd-3Ae)\right)\right)}{2e^7}-\frac{cx\left(3a^2e^4(3Cd-Be)+3acde^2\left(10Cd^2-3\right.\right.}{}$$

```
[Out] -((c*(3*a^2*e^4*(3*C*d - B*e) + c^2*d^3*(21*C*d^2 - 5*e*(3*B*d - 2*A*e)) +
3*a*c*d*e^2*(10*C*d^2 - 3*e*(2*B*d - A*e)))*x)/e^8) + (c*(3*a^2*C*e^4 + c^2
*d^2*(15*C*d^2 - 2*e*(5*B*d - 3*A*e)) + 3*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e
)))*x^2)/(2*e^7) - (c^2*(3*a*e^2*(3*C*d - B*e) + c*d*(10*C*d^2 - 3*e*(2*B*d
 - A*e)))*x^3)/(3*e^6) + (c^2*(3*a*C*e^2 + c*(6*C*d^2 - e*(3*B*d - A*e)))*x
^4)/(4*e^5) - (c^3*(3*C*d - B*e)*x^5)/(5*e^4) + (c^3*C*x^6)/(6*e^3) - ((c*d
^2 + a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(2*e^9*(d + e*x)^2) + ((c*d^2 + a*e^
2)^2*(a*e^2*(2*C*d - B*e) + c*d*(8*C*d^2 - e*(7*B*d - 6*A*e))))/(e^9*(d + e
*x)) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*d^2*(28*C*d^2 - 3*e*(7*B*d - 5*A*e
)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*Log[d + e*x])/e^9
```

Rubi [A] time = 0.966645, antiderivative size = 463, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{cx^2\left(3a^2Ce^4+3ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2\left(15Cd^4-2d^2e(5Bd-3Ae)\right)\right)}{2e^7}-\frac{cx\left(3a^2e^4(3Cd-Be)+3acde^2\left(10Cd^2-3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(3*a^2*e^4*(3*C*d - B*e) + c^2*(21*C*d^5 - 5*d^3*e*(3*B*d - 2*A*e)) +
3*a*c*d*e^2*(10*C*d^2 - 3*e*(2*B*d - A*e)))*x)/e^8) + (c*(3*a^2*C*e^4 + c^2
*(15*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 3*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e
)))*x^2)/(2*e^7) - (c^2*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 3*a*e^2*(3*C*
d - B*e))*x^3)/(3*e^6) + (c^2*(6*c*C*d^2 + 3*a*C*e^2 - c*e*(3*B*d - A*e))*x
^4)/(4*e^5) - (c^3*(3*C*d - B*e)*x^5)/(5*e^4) + (c^3*C*x^6)/(6*e^3) - ((c*d
^2 + a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(2*e^9*(d + e*x)^2) + ((c*d^2 + a*e^
2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e)))/(e^9*(d + e
*x)) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d - 5*A*e
)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)}{(d+ex)^3}\,dx = \int\left(\frac{c\left(-3a^2e^4(3Cd-Be)-c^2\left(21Cd^5-5d^3e(3Bd-2Ae)\right)-3acde^2\left(10Cd^2-3e(2B\right.\right.}{e^8}\right.$$

$$= -\frac{c\left(3a^2e^4(3Cd-Be)+c^2\left(21Cd^5-5d^3e(3Bd-2Ae)\right)+3acde^2\left(10Cd^2-3e(2Bd-\right.\right.}{e^8}$$

Mathematica [A] time = 0.235061, size = 438, normalized size = 0.94

$$\frac{30ce^2x^2\left(3a^2Ce^4+3ace^2\left(e(Ae-3Bd)+6Cd^2\right)+c^2\left(2d^2e(3Ae-5Bd)+15Cd^4\right)\right)-60cex\left(-3a^2e^4(Be-3Cd)+3acde^2\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-60*c*e*(-3*a^2*e^4*(-3*C*d + B*e) + 3*a*c*d*e^2*(10*C*d^2 + 3*e*(-2*B*d +
 A*e)) + c^2*(21*C*d^5 + 5*d^3*e*(-3*B*d + 2*A*e)))*x + 30*c*e^2*(3*a^2*C*e
^4 + 3*a*c*e^2*(6*C*d^2 + e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B
*d + 3*A*e)))*x^2 - 20*c^2*e^3*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 3*a*e
^2*(-3*C*d + B*e))*x^3 + 15*c^2*e^4*(6*c*C*d^2 + 3*a*C*e^2 + c*e*(-3*B*d +
A*e))*x^4 + 12*c^3*e^5*(-3*C*d + B*e)*x^5 + 10*c^3*C*e^6*x^6 - (30*(c*d^2 +
 a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 + (60*(c*d^2 + a*e^2)^2*(
8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 60*(
c*d^2 + a*e^2)*(a^2*C*e^4 + a*c*e^2*(17*C*d^2 + 3*e*(-3*B*d + A*e)) + c^2*(
28*C*d^4 + 3*d^2*e*(-7*B*d + 5*A*e)))*Log[d + e*x])/(60*e^9)
```

Maple [B] time = 0.061, size = 978, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x)
```

```
[Out] -1/2/e^9/(e*x+d)^2*C*c^3*d^8-10*c^3/e^6*A*d^3*x+15*c^3/e^7*B*d^4*x-5*c^3/e^
6*B*x^2*d^3+3*c^3/e^5*A*x^2*d^2-10/3*c^3/e^6*C*x^3*d^3+2*c^3/e^5*B*x^3*d^2+
c^2/e^3*B*x^3*a-c^3/e^4*A*x^3*d-7/e^8/(e*x+d)*B*c^3*d^6+2/e^3/(e*x+d)*C*a^3
*d-3/5*c^3/e^4*C*x^5*d-3/4*c^3/e^4*B*x^4*d-1/2/e^7/(e*x+d)^2*A*c^3*d^6+1/2/
e^2/(e*x+d)^2*B*d*a^3+1/2/e^8/(e*x+d)^2*B*c^3*d^7-1/2/e^3/(e*x+d)^2*C*d^2*a
^3-21*c^3/e^8*C*d^5*x+15/2*c^3/e^7*C*x^2*d^4-3/2/e^5/(e*x+d)^2*C*a^2*c*d^4-
3/2/e^7/(e*x+d)^2*C*a*c^2*d^6+18*c^2/e^5*B*a*d^2*x-3*c^2/e^4*C*x^3*a*d-9/2*
c^2/e^4*B*x^2*a*d+9*c^2/e^5*C*x^2*a*d^2+8/e^9/(e*x+d)*C*c^3*d^7+3/e^3*ln(e*
x+d)*A*a^2*c+15/e^7*ln(e*x+d)*A*c^3*d^4-21/e^8*ln(e*x+d)*B*c^3*d^5+28/e^9*l
n(e*x+d)*C*c^3*d^6+6/e^7/(e*x+d)*A*c^3*d^5+3/2*c^2/e^3*A*x^2*a+3/2*c/e^3*C*
x^2*a^2+3*c/e^3*a^2*B*x+1/5*c^3/e^3*B*x^5-1/2/e/(e*x+d)^2*A*a^3+1/e^3*ln(e*
x+d)*a^3*C-1/e^2/(e*x+d)*B*a^3-3/2/e^3/(e*x+d)^2*A*d^2*a^2*c-3/2/e^5/(e*x+d
)^2*A*a*c^2*d^4+3/2/e^4/(e*x+d)^2*B*a^2*c*d^3-9*c/e^4*C*a^2*d*x-9*c^2/e^4*A
*a*d*x+3/2*c^3/e^5*C*x^4*d^2+3/4*c^2/e^3*C*x^4*a+1/4*c^3/e^3*A*x^4-30*c^2/e
^6*C*a*d^3*x+18/e^7/(e*x+d)*C*a*c^2*d^5+12/e^5/(e*x+d)*C*a^2*c*d^3+45/e^7*l
n(e*x+d)*C*a*c^2*d^4-30/e^6*ln(e*x+d)*B*a*c^2*d^3+18/e^5*ln(e*x+d)*A*a*c^2*
d^2-9/e^4*ln(e*x+d)*B*a^2*c*d+3/2/e^6/(e*x+d)^2*B*a*c^2*d^5-9/e^4/(e*x+d)*B
*a^2*c*d^2+1/6*c^3*C*x^6/e^3+12/e^5/(e*x+d)*A*a*c^2*d^3+18/e^5*ln(e*x+d)*C*
a^2*c*d^2-15/e^6/(e*x+d)*B*a*c^2*d^4+6/e^3/(e*x+d)*A*a^2*c*d
```

Maxima [A] time = 1.08593, size = 946, normalized size = 2.03

$$\frac{15\,Cc^3d^8 - 13\,Bc^3d^7e - 27\,Bac^2d^5e^3 - 15\,Ba^2cd^3e^5 - Ba^3de^7 - Aa^3e^8 + 11\left(3\,Cac^2 + Ac^3\right)d^6e^2 + 21\left(Ca^2c + Aac^2\right)d^4e^4 + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e - 27*B*a*c^2*d^5*e^3 - 15*B*a^2*c*d^3*e^
5 - B*a^3*d*e^7 - A*a^3*e^8 + 11*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 21*(C*a^2*c
+ A*a*c^2)*d^4*e^4 + 3*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + 2*(8*C*c^3*d^7*e - 7*B
*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*
a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^
2*c)*d*e^7)*x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/60*(10*C*c^3*e^5*x^6 -
 12*(3*C*c^3*d*e^4 - B*c^3*e^5)*x^5 + 15*(6*C*c^3*d^2*e^3 - 3*B*c^3*d*e^4 +
 (3*C*a*c^2 + A*c^3)*e^5)*x^4 - 20*(10*C*c^3*d^3*e^2 - 6*B*c^3*d^2*e^3 - 3*
B*a*c^2*e^5 + 3*(3*C*a*c^2 + A*c^3)*d*e^4)*x^3 + 30*(15*C*c^3*d^4*e - 10*B*
c^3*d^3*e^2 - 9*B*a*c^2*d*e^4 + 6*(3*C*a*c^2 + A*c^3)*d^2*e^3 + 3*(C*a^2*c
```

```
+ A*a*c^2)*e^5)*x^2 - 60*(21*C*c^3*d^5 - 15*B*c^3*d^4*e - 18*B*a*c^2*d^2*e^
3 - 3*B*a^2*c*e^5 + 10*(3*C*a*c^2 + A*c^3)*d^3*e^2 + 9*(C*a^2*c + A*a*c^2)*
d*e^4)*x)/e^8 + (28*C*c^3*d^6 - 21*B*c^3*d^5*e - 30*B*a*c^2*d^3*e^3 - 9*B*a
^2*c*d*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 18*(C*a^2*c + A*a*c^2)*d^2*e^
4 + (C*a^3 + 3*A*a^2*c)*e^6)*log(e*x + d)/e^9
```

Fricas [B] time = 1.82771, size = 2198, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^3*e^8*x^8 + 450*C*c^3*d^8 - 390*B*c^3*d^7*e - 810*B*a*c^2*d^5*
e^3 - 450*B*a^2*c*d^3*e^5 - 30*B*a^3*d*e^7 - 30*A*a^3*e^8 + 330*(3*C*a*c^2
+ A*c^3)*d^6*e^2 + 630*(C*a^2*c + A*a*c^2)*d^4*e^4 + 90*(C*a^3 + 3*A*a^2*c)
*d^2*e^6 - 4*(4*C*c^3*d*e^7 - 3*B*c^3*e^8)*x^7 + (28*C*c^3*d^2*e^6 - 21*B*c
^3*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 2*(28*C*c^3*d^3*e^5 - 21*B*c^3
*d^2*e^6 - 30*B*a*c^2*e^8 + 15*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 5*(28*C*c^3
*d^4*e^4 - 21*B*c^3*d^3*e^5 - 30*B*a*c^2*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^2
*e^6 + 18*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 20*(28*C*c^3*d^5*e^3 - 21*B*c^3*d^
4*e^4 - 30*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 15*(3*C*a*c^2 + A*c^3)*d^3*e^5
 + 18*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 - 30*(69*C*c^3*d^6*e^2 - 50*B*c^3*d^5*
e^3 - 63*B*a*c^2*d^3*e^5 - 12*B*a^2*c*d*e^7 + 34*(3*C*a*c^2 + A*c^3)*d^4*e^
4 + 33*(C*a^2*c + A*a*c^2)*d^2*e^6)*x^2 - 60*(13*C*c^3*d^7*e - 8*B*c^3*d^6*
e^2 - 3*B*a*c^2*d^4*e^4 + 6*B*a^2*c*d^2*e^6 + B*a^3*e^8 + 4*(3*C*a*c^2 + A*
c^3)*d^5*e^3 - 3*(C*a^2*c + A*a*c^2)*d^3*e^5 - 2*(C*a^3 + 3*A*a^2*c)*d*e^7)
*x + 60*(28*C*c^3*d^8 - 21*B*c^3*d^7*e - 30*B*a*c^2*d^5*e^3 - 9*B*a^2*c*d^3
*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 18*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C
*a^3 + 3*A*a^2*c)*d^2*e^6 + (28*C*c^3*d^6*e^2 - 21*B*c^3*d^5*e^3 - 30*B*a*c
^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^4 + 18*(C*a^2*c
 + A*a*c^2)*d^2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 2*(28*C*c^3*d^7*e - 21
*B*c^3*d^6*e^2 - 30*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 + 15*(3*C*a*c^2 + A
*c^3)*d^5*e^3 + 18*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)
*x)*log(e*x + d))/(e^11*x^2 + 2*d*e^10*x + d^2*e^9)
```

Sympy [A] time = 36.9157, size = 799, normalized size = 1.71

$$\frac{Cc^3x^6}{6e^3} + \frac{-Aa^3e^8 + 9Aa^2cd^2e^6 + 21Aac^2d^4e^4 + 11Ac^3d^6e^2 - Ba^3de^7 - 15Ba^2cd^3e^5 - 27Bac^2d^5e^3 - 13Bc^3d^7e + 3Ca^3d^2e^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**3*x**6/(6*e**3) + (-A*a**3*e**8 + 9*A*a**2*c*d**2*e**6 + 21*A*a*c**2*d
**4*e**4 + 11*A*c**3*d**6*e**2 - B*a**3*d*e**7 - 15*B*a**2*c*d**3*e**5 - 27
*B*a*c**2*d**5*e**3 - 13*B*c**3*d**7*e + 3*C*a**3*d**2*e**6 + 21*C*a**2*c*d
**4*e**4 + 33*C*a*c**2*d**6*e**2 + 15*C*c**3*d**8 + x*(12*A*a**2*c*d*e**7 +
 24*A*a*c**2*d**3*e**5 + 12*A*c**3*d**5*e**3 - 2*B*a**3*e**8 - 18*B*a**2*c*
d**2*e**6 - 30*B*a*c**2*d**4*e**4 - 14*B*c**3*d**6*e**2 + 4*C*a**3*d*e**7 +
 24*C*a**2*c*d**3*e**5 + 36*C*a*c**2*d**5*e**3 + 16*C*c**3*d**7*e))/(2*d**2
*e**9 + 4*d*e**10*x + 2*e**11*x**2) - x**5*(-B*c**3*e + 3*C*c**3*d)/(5*e**4
) + x**4*(A*c**3*e**2 - 3*B*c**3*d*e + 3*C*a*c**2*e**2 + 6*C*c**3*d**2)/(4*
e**5) - x**3*(3*A*c**3*d*e**2 - 3*B*a*c**2*e**3 - 6*B*c**3*d**2*e + 9*C*a*c
**2*d*e**2 + 10*C*c**3*d**3)/(3*e**6) + x**2*(3*A*a*c**2*e**4 + 6*A*c**3*d*
*2*e**2 - 9*B*a*c**2*d*e**3 - 10*B*c**3*d**3*e + 3*C*a**2*c*e**4 + 18*C*a*c
**2*d**2*e**2 + 15*C*c**3*d**4)/(2*e**7) - x*(9*A*a*c**2*d*e**4 + 10*A*c**3
*d**3*e**2 - 3*B*a**2*c*e**5 - 18*B*a*c**2*d**2*e**3 - 15*B*c**3*d**4*e + 9
*C*a**2*c*d*e**4 + 30*C*a*c**2*d**3*e**2 + 21*C*c**3*d**5)/e**8 + (a*e**2 +
 c*d**2)*(3*A*a*c*e**4 + 15*A*c**2*d**2*e**2 - 9*B*a*c*d*e**3 - 21*B*c**2*d
**3*e + C*a**2*e**4 + 17*C*a*c*d**2*e**2 + 28*C*c**2*d**4)*log(d + e*x)/e**
9
```

Giac [A] time = 1.15108, size = 981, normalized size = 2.11

$$\left(28\,Cc^3d^6 - 21\,Bc^3d^5e + 45\,Cac^2d^4e^2 + 15\,Ac^3d^4e^2 - 30\,Bac^2d^3e^3 + 18\,Ca^2cd^2e^4 + 18\,Aac^2d^2e^4 - 9\,Ba^2cde^5 + Ca^3e^6 + 3\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (28*C*c^3*d^6 - 21*B*c^3*d^5*e + 45*C*a*c^2*d^4*e^2 + 15*A*c^3*d^4*e^2 - 30
*B*a*c^2*d^3*e^3 + 18*C*a^2*c*d^2*e^4 + 18*A*a*c^2*d^2*e^4 - 9*B*a^2*c*d*e^
5 + C*a^3*e^6 + 3*A*a^2*c*e^6)*e^(-9)*log(abs(x*e + d)) + 1/60*(10*C*c^3*x^
6*e^15 - 36*C*c^3*d*x^5*e^14 + 90*C*c^3*d^2*x^4*e^13 - 200*C*c^3*d^3*x^3*e^
12 + 450*C*c^3*d^4*x^2*e^11 - 1260*C*c^3*d^5*x*e^10 + 12*B*c^3*x^5*e^15 - 4
5*B*c^3*d*x^4*e^14 + 120*B*c^3*d^2*x^3*e^13 - 300*B*c^3*d^3*x^2*e^12 + 900*
B*c^3*d^4*x*e^11 + 45*C*a*c^2*x^4*e^15 + 15*A*c^3*x^4*e^15 - 180*C*a*c^2*d*
x^3*e^14 - 60*A*c^3*d*x^3*e^14 + 540*C*a*c^2*d^2*x^2*e^13 + 180*A*c^3*d^2*x
^2*e^13 - 1800*C*a*c^2*d^3*x*e^12 - 600*A*c^3*d^3*x*e^12 + 60*B*a*c^2*x^3*e
^15 - 270*B*a*c^2*d*x^2*e^14 + 1080*B*a*c^2*d^2*x*e^13 + 90*C*a^2*c*x^2*e^1
```

```
5 + 90*A*a*c^2*x^2*e^15 - 540*C*a^2*c*d*x*e^14 - 540*A*a*c^2*d*x*e^14 + 180
*B*a^2*c*x*e^15)*e^(-18) + 1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e + 33*C*a*c^2*
d^6*e^2 + 11*A*c^3*d^6*e^2 - 27*B*a*c^2*d^5*e^3 + 21*C*a^2*c*d^4*e^4 + 21*A
*a*c^2*d^4*e^4 - 15*B*a^2*c*d^3*e^5 + 3*C*a^3*d^2*e^6 + 9*A*a^2*c*d^2*e^6 -
 B*a^3*d*e^7 - A*a^3*e^8 + 2*(8*C*c^3*d^7*e - 7*B*c^3*d^6*e^2 + 18*C*a*c^2*
d^5*e^3 + 6*A*c^3*d^5*e^3 - 15*B*a*c^2*d^4*e^4 + 12*C*a^2*c*d^3*e^5 + 12*A*
a*c^2*d^3*e^5 - 9*B*a^2*c*d^2*e^6 + 2*C*a^3*d*e^7 + 6*A*a^2*c*d*e^7 - B*a^3
*e^8)*x)*e^(-9)/(x*e + d)^2
```

3.39 $\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2} \, dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rubi [A] time = 0.0319914, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {1590}

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2} \, dx = \frac{(a+bx^2)^2}{c+dx}$$

Mathematica [B] time = 0.0263226, size = 62, normalized size = 3.65

$$\frac{a^2d^4+2abd^2\left(c^2+cdx+d^2x^2\right)+b^2\left(c^3dx+c^4+d^4x^4\right)}{d^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

Maple [B] time = 0.049, size = 76, normalized size = 4.5

$$\frac{b\left(bd^2x^3-bcdx^2+2\,ad^2x+bc^2x\right)}{d^3}-\frac{-a^2d^4-2\,abc^2d^2-b^2c^4}{d^4\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x)
```

```
[Out] b/d^3*(b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c
^4)/d^4/(d*x+c)
```

Maxima [B] time = 0.990938, size = 111, normalized size = 6.53

$$\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{d^5x+cd^4}+\frac{b^2d^2x^3-b^2cdx^2+\left(b^2c^2+2\,abd^2\right)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

Fricas [B] time = 1.5962, size = 153, normalized size = 9.

$$\frac{b^2d^4x^4 + 2\,abd^4x^2 + b^2c^4 + 2\,abc^2d^2 + a^2d^4 + \left(b^2c^3d + 2\,abcd^3\right)x}{d^5x + cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

Sympy [B] time = 0.56482, size = 75, normalized size = 4.41

$$-\frac{b^2cx^2}{d^2} + \frac{b^2x^3}{d} + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{cd^4 + d^5x} + \frac{x\left(2abd^2 + b^2c^2\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(3*b*d*x**2+4*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + (a**2*d**4 + 2*a*b*c**2*d**2 + b**2*c**4)
/(c*d**4 + d**5*x) + x*(2*a*b*d**2 + b**2*c**2)/d**3
```

Giac [B] time = 1.17215, size = 150, normalized size = 8.82

$$\frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

3.40 $\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rubi [A] time = 0.0188685, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1590}

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^2}{c+dx}$$

Mathematica [B] time = 0.013127, size = 62, normalized size = 3.65

$$\frac{a^2d^4 + 2abd^2\left(c^2 + cdx + d^2x^2\right) + b^2\left(c^3dx + c^4 + d^4x^4\right)}{d^4(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

Maple [B] time = 0.047, size = 76, normalized size = 4.5

$$\frac{b\left(bd^2x^3 - bcdx^2 + 2\,ad^2x + bc^2x\right)}{d^3} - \frac{-a^2d^4 - 2\,abc^2d^2 - b^2c^4}{d^4\,(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x)
```

```
[Out] b/d^3*(b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c
^4)/d^4/(d*x+c)
```

Maxima [B] time = 0.958837, size = 111, normalized size = 6.53

$$\frac{b^2c^4 + 2\,abc^2d^2 + a^2d^4}{d^5x + cd^4} + \frac{b^2d^2x^3 - b^2cdx^2 + \left(b^2c^2 + 2\,abd^2\right)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

Fricas [B] time = 1.67211, size = 153, normalized size = 9.

$$\frac{b^2d^4x^4 + 2\,abd^4x^2 + b^2c^4 + 2\,abc^2d^2 + a^2d^4 + \left(b^2c^3d + 2\,abcd^3\right)x}{d^5x + cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

Sympy [B] time = 0.545774, size = 75, normalized size = 4.41

$$-\frac{b^2cx^2}{d^2} + \frac{b^2x^3}{d} + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{cd^4 + d^5x} + \frac{x\left(2abd^2 + b^2c^2\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + (a**2*d**4 + 2*a*b*c**2*d**2 + b**2*c**4)
/(c*d**4 + d**5*x) + x*(2*a*b*d**2 + b**2*c**2)/d**3
```

Giac [B] time = 1.16933, size = 150, normalized size = 8.82

$$\frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

3.41 $\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^3}{c+dx}$$

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rubi [A] time = 0.0459761, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1590}

$$\frac{(a+bx^2)^3}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

Mathematica [B] time = 0.0385234, size = 90, normalized size = 5.29

$$\frac{3a^2bd^4\left(c^2+cdx+d^2x^2\right)+a^3d^6+3ab^2d^2\left(c^3dx+c^4+d^4x^4\right)+b^3\left(c^5dx+c^6+d^6x^6\right)}{d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

Maple [B] time = 0.05, size = 157, normalized size = 9.2

$$\frac{b\left(b^2d^4x^5-b^2cd^3x^4+3\,abd^4x^3+b^2c^2d^2x^3-3\,abcd^3x^2-b^2c^3dx^2+3\,a^2d^4x+3\,abc^2d^2x+b^2c^4x\right)}{d^5}-\frac{-a^3d^6-3\,a^2bc^2d^4-}{d^6\,(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x)
```

```
[Out] b/d^5*(b^2*d^4*x^5-b^2*c*d^3*x^4+3*a*b*d^4*x^3+b^2*c^2*d^2*x^3-3*a*b*c*d^3*
x^2-b^2*c^3*d*x^2+3*a^2*d^4*x+3*a*b*c^2*d^2*x+b^2*c^4*x)-(-a^3*d^6-3*a^2*b*
c^2*d^4-3*a*b^2*c^4*d^2-b^3*c^6)/d^6/(d*x+c)
```

Maxima [B] time = 1.00438, size = 216, normalized size = 12.71

$$\frac{b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6}{d^7x+cd^6}+\frac{b^3d^4x^5-b^3cd^3x^4+\left(b^3c^2d^2+3\,ab^2d^4\right)x^3-\left(b^3c^3d+3\,ab^2cd^3\right)x^2+\left(b^3c^4+3\,ab^2c\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxim
a")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

Fricas [B] time = 1.68655, size = 234, normalized size = 13.76

$$\frac{b^3 d^6 x^6 + 3\, a b^2 d^6 x^4 + 3\, a^2 b d^6 x^2 + b^3 c^6 + 3\, a b^2 c^4 d^2 + 3\, a^2 b c^2 d^4 + a^3 d^6 + \left(b^3 c^5 d + 3\, a b^2 c^3 d^3 + 3\, a^2 b c d^5\right) x}{d^7 x + c d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="frica
s")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

Sympy [B] time = 0.777898, size = 155, normalized size = 9.12

$$-\frac{b^3 c x^4}{d^2} + \frac{b^3 x^5}{d} + \frac{a^3 d^6 + 3a^2 b c^2 d^4 + 3ab^2 c^4 d^2 + b^3 c^6}{c d^6 + d^7 x} + \frac{x^3 \left(3ab^2 d^2 + b^3 c^2\right)}{d^3} - \frac{x^2 \left(3ab^2 c d^2 + b^3 c^3\right)}{d^4} + \frac{x \left(3a^2 b d^4 + 3ab^2 c^2 d\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(5*b*d*x**2+6*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**
2*c**4*d**2 + b**3*c**6)/(c*d**6 + d**7*x) + x**3*(3*a*b**2*d**2 + b**3*c**
2)/d**3 - x**2*(3*a*b**2*c*d**2 + b**3*c**3)/d**4 + x*(3*a**2*b*d**4 + 3*a*
b**2*c**2*d**2 + b**3*c**4)/d**5
```

Giac [B] time = 1.16157, size = 292, normalized size = 17.18

$$\frac{\left(b^3 - \frac{6\, b^3 c}{dx+c} + \frac{15\, b^3 c^2}{(dx+c)^2} - \frac{20\, b^3 c^3}{(dx+c)^3} + \frac{15\, b^3 c^4}{(dx+c)^4} + \frac{3\, a b^2 d^2}{(dx+c)^2} - \frac{12\, a b^2 c d^2}{(dx+c)^3} + \frac{18\, a b^2 c^2 d^2}{(dx+c)^4} + \frac{3\, a^2 b d^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{\frac{b^3 c^6 d^5}{dx+c} + \frac{3\, a b^2 c^4 d^7}{dx+c} + \frac{3\, a^2 b c^2 d^9}{dx+c} + \cdots}{d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac"
)
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

3.42 $\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^3}{c+dx}$$

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rubi [A] time = 0.0249113, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {1590}

$$\frac{(a+bx^2)^3}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

Mathematica [B] time = 0.0205465, size = 90, normalized size = 5.29

$$\frac{3a^2bd^4\left(c^2+cdx+d^2x^2\right)+a^3d^6+3ab^2d^2\left(c^3dx+c^4+d^4x^4\right)+b^3\left(c^5dx+c^6+d^6x^6\right)}{d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

Maple [B] time = 0.05, size = 157, normalized size = 9.2

$$\frac{b\left(b^2d^4x^5-b^2cd^3x^4+3\,abd^4x^3+b^2c^2d^2x^3-3\,abcd^3x^2-b^2c^3dx^2+3\,a^2d^4x+3\,abc^2d^2x+b^2c^4x\right)}{d^5}-\frac{-a^3d^6-3\,a^2bc^2d^4-}{d^6\,(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x)
```

```
[Out] b/d^5*(b^2*d^4*x^5-b^2*c*d^3*x^4+3*a*b*d^4*x^3+b^2*c^2*d^2*x^3-3*a*b*c*d^3*
x^2-b^2*c^3*d*x^2+3*a^2*d^4*x+3*a*b*c^2*d^2*x+b^2*c^4*x)-(-a^3*d^6-3*a^2*b*
c^2*d^4-3*a*b^2*c^4*d^2-b^3*c^6)/d^6/(d*x+c)
```

Maxima [B] time = 0.959241, size = 216, normalized size = 12.71

$$\frac{b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6}{d^7x+cd^6}+\frac{b^3d^4x^5-b^3cd^3x^4+\left(b^3c^2d^2+3\,ab^2d^4\right)x^3-\left(b^3c^3d+3\,ab^2cd^3\right)x^2+\left(b^3c^4+3\,ab^2c\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="maxima
")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

Fricas [B] time = 1.60565, size = 234, normalized size = 13.76

$$\frac{b^3d^6x^6 + 3\,ab^2d^6x^4 + 3\,a^2bd^6x^2 + b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6 + \left(b^3c^5d + 3\,ab^2c^3d^3 + 3\,a^2bcd^5\right)x}{d^7x + cd^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="fricas
")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

Sympy [B] time = 0.76113, size = 155, normalized size = 9.12

$$-\frac{b^3cx^4}{d^2} + \frac{b^3x^5}{d} + \frac{a^3d^6 + 3a^2bc^2d^4 + 3ab^2c^4d^2 + b^3c^6}{cd^6 + d^7x} + \frac{x^3\left(3ab^2d^2 + b^3c^2\right)}{d^3} - \frac{x^2\left(3ab^2cd^2 + b^3c^3\right)}{d^4} + \frac{x\left(3a^2bd^4 + 3ab^2c^2d\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**
2*c**4*d**2 + b**3*c**6)/(c*d**6 + d**7*x) + x**3*(3*a*b**2*d**2 + b**3*c**
2)/d**3 - x**2*(3*a*b**2*c*d**2 + b**3*c**3)/d**4 + x*(3*a**2*b*d**4 + 3*a*
b**2*c**2*d**2 + b**3*c**4)/d**5
```

Giac [B] time = 1.17361, size = 292, normalized size = 17.18

$$\frac{\left(b^3 - \frac{6\,b^3c}{dx+c} + \frac{15\,b^3c^2}{(dx+c)^2} - \frac{20\,b^3c^3}{(dx+c)^3} + \frac{15\,b^3c^4}{(dx+c)^4} + \frac{3\,ab^2d^2}{(dx+c)^2} - \frac{12\,ab^2cd^2}{(dx+c)^3} + \frac{18\,ab^2c^2d^2}{(dx+c)^4} + \frac{3\,a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{\frac{b^3c^6d^5}{dx+c} + \frac{3\,ab^2c^4d^7}{dx+c} + \frac{3\,a^2bc^2d^9}{dx+c} + \frac{a}{d}}{d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

3.43 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{a+cx^2}\,dx$

Optimal. Leaf size=240

$$-\frac{ex^2\left(aCe^2-c\left(e(Ae+3Bd)+3Cd^2\right)\right)}{2c^2}+\frac{\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)+Bcd\left(cd^2-3ae^2\right)\right)}{2c^3}-\frac{x\left(ae^2(Be+3Cd)\right.}{}$$

```
[Out] -((((a*e^2*(3*C*d + B*e) - c*d*(C*d^2 + 3*e*(B*d + A*e)))*x)/c^2) - (e*(a*C*
e^2 - c*(3*C*d^2 + e*(3*B*d + A*e)))*x^2)/(2*c^2) + (e^2*(3*C*d + B*e)*x^3)
/(3*c) + (C*e^3*x^4)/(4*c) + ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d +
B*e) - c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2))
 + ((B*c*d*(c*d^2 - 3*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x
^2])/(2*c^3)
```

Rubi [A] time = 0.471217, antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{ex^2\left(-aCe^2+ce(Ae+3Bd)+3cCd^2\right)}{2c^2}+\frac{\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)+Bcd\left(cd^2-3ae^2\right)\right)}{2c^3}+\frac{x\left(-ae^2(Be+3Cd)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^3 + 3*c*d*e*(B*d + A*e) - a*e^2*(3*C*d + B*e))*x)/c^2 + (e*(3*c*C*d
^2 - a*C*e^2 + c*e*(3*B*d + A*e))*x^2)/(2*c^2) + (e^2*(3*C*d + B*e)*x^3)/(3
*c) + (C*e^3*x^4)/(4*c) + ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e
) - c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) +
((B*c*d*(c*d^2 - 3*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2]
)/(2*c^3)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \int \left(\frac{cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+Ae)\right)x}{c^2}+\right.$$

$$=\frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+Ae)\right)x^2}{2c^2}+$$

$$=\frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+Ae)\right)x^2}{2c^2}+$$

$$=\frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+Ae)\right)x^2}{2c^2}+$$

Mathematica [A] time = 0.224381, size = 223, normalized size = 0.93

$$\frac{cx\left(-6ae^2(2Be+6Cd+Cex)+2ce\left(3Ae(6d+ex)+B\left(18d^2+9dex+2e^2x^2\right)\right)+3cC\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)\right)+}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (c*x*(-6*a*e^2*(6*C*d + 2*
B*e + C*e*x) + 3*c*C*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 2*c*e*(3
*A*e*(6*d + e*x) + B*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*(B*c*d*(c*d^2 - 3
```

```
*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(12*c^3)
```

Maple [A] time = 0.052, size = 399, normalized size = 1.7

$$\frac{Ce^3x^4}{4c}+\frac{Bx^3e^3}{3c}+\frac{Cx^3de^2}{c}+\frac{Ax^2e^3}{2c}+\frac{3Bx^2de^2}{2c}-\frac{Cx^2ae^3}{2c^2}+\frac{3Cx^2d^2e}{2c}+3\frac{Ade^2x}{c}-\frac{Bae^3x}{c^2}+3\frac{Bd^2ex}{c}-3\frac{Cade^2x}{c^2}+\frac{Cd^3}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] 1/4*C*e^3*x^4/c+1/3/c*B*x^3*e^3+1/c*C*x^3*d*e^2+1/2/c*A*x^2*e^3+3/2/c*B*x^2
*d*e^2-1/2/c^2*C*x^2*a*e^3+3/2/c*C*x^2*d^2*e+3/c*A*d*e^2*x-1/c^2*B*a*e^3*x+
3/c*B*d^2*e*x-3/c^2*C*a*d*e^2*x+1/c*C*d^3*x-1/2/c^2*ln(c*x^2+a)*a*A*e^3+3/2
/c*ln(c*x^2+a)*A*d^2*e-3/2/c^2*ln(c*x^2+a)*a*B*d*e^2+1/2/c*ln(c*x^2+a)*B*d^
3+1/2/c^3*ln(c*x^2+a)*C*a^2*e^3-3/2/c^2*ln(c*x^2+a)*C*a*d^2*e-3/c/(a*c)^(1/
2)*arctan(x*c/(a*c)^(1/2))*A*d*e^2*a+1/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*
A*d^3+1/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a^2*B*e^3-3/c/(a*c)^(1/2)*a
rctan(x*c/(a*c)^(1/2))*B*d^2*a*e+3/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*
C*a^2*d*e^2-1/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*a*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88152, size = 1239, normalized size = 5.16

$$\left[\frac{3Cac^2e^3x^4+4\left(3Cac^2de^2+Bac^2e^3\right)x^3+6\left(3Cac^2d^2e+3Bac^2de^2-\left(Ca^2c-Aac^2\right)e^3\right)x^2+6\left(3Bacd^2e-Ba^2e^3+\left(Cac\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B*a*c^2*e^3)*x^3 + 6*(3*C*a
*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^2)*e^3)*x^2 + 6*(3*B*a*c*d^
2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^2)*sqrt(-a*c)
*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 12*(C*a*c^2*d^3 + 3*B*a*c^
2*d^2*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d^3 - 3
*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log(c
*x^2 + a))/(a*c^3), 1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B*a*c^2*
e^3)*x^3 + 6*(3*C*a*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^2)*e^3)*
x^2 - 12*(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*
c)*d*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 12*(C*a*c^2*d^3 + 3*B*a*c^2*d^2
*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d^3 - 3*B*a^
2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log(c*x^2
+ a))/(a*c^3)]
```

Sympy [B] time = 6.76913, size = 1000, normalized size = 4.17

$$\frac{Ce^3x^4}{4c} + \left(\frac{-Aace^3 + 3Ac^2d^2e - 3Bacde^2 + Bc^2d^3 + Ca^2e^3 - 3Cacd^2e}{2c^3} - \frac{\sqrt{-ac^7}\left(-3Aacde^2 + Ac^2d^3 + Ba^2e^3 - 3Bacd^2e + \cdots\right.}{2ac^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**3*x**4/(4*c) + ((-A*a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c*
*2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2*c**3) - sqrt(-a*c**7)*(-3*A*a*c*
d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a
*c*d**3)/(2*a*c**6))*log(x + (A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*
c*d*e**2 - B*a*c**2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*
a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3
*C*a*c*d**2*e)/(2*c**3) - sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*
a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3
*A*a*c**2*d*e**2 + A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a*
*2*c*d*e**2 - C*a*c**2*d**3)) + ((-A*a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d
*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2*c**3) + sqrt(-a*c**7
)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*
d*e**2 - C*a*c*d**3)/(2*a*c**6))*log(x + (A*a**2*c*e**3 - 3*A*a*c**2*d**2*e
```

```
 + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*e + 2*
a*c**3*((-A*a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a
**2*e**3 - 3*C*a*c*d**2*e)/(2*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c*
*2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a
*c**6)))/(-3*A*a*c**2*d*e**2 + A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**
2*e + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + x**3*(B*e**3 + 3*C*d*e**2)/(3*c
) - x**2*(-A*c*e**3 - 3*B*c*d*e**2 + C*a*e**3 - 3*C*c*d**2*e)/(2*c**2) - x*
(-3*A*c*d*e**2 + B*a*e**3 - 3*B*c*d**2*e + 3*C*a*d*e**2 - C*c*d**3)/c**2
```

Giac [A] time = 1.14414, size = 377, normalized size = 1.57

$$-\frac{\left(Cacd^3 - Ac^2d^3 + 3\,Bacd^2e - 3\,Ca^2de^2 + 3\,Aacde^2 - Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{\left(Bc^2d^3 - 3\,Cacd^2e + 3\,Ac^2d^2e - 3\,Bacde^2\right.}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*c*d^3 - A*c^2*d^3 + 3*B*a*c*d^2*e - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 - B
*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/2*(B*c^2*d^3 - 3*C*a*c*
d^2*e + 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 + C*a^2*e^3 - A*a*c*e^3)*log(c*x^2 +
a)/c^3 + 1/12*(3*C*c^3*x^4*e^3 + 12*C*c^3*d*x^3*e^2 + 18*C*c^3*d^2*x^2*e +
12*C*c^3*d^3*x + 4*B*c^3*x^3*e^3 + 18*B*c^3*d*x^2*e^2 + 36*B*c^3*d^2*x*e -
6*C*a*c^2*x^2*e^3 + 6*A*c^3*x^2*e^3 - 36*C*a*c^2*d*x*e^2 + 36*A*c^3*d*x*e^2
 - 12*B*a*c^2*x*e^3)/c^4
```

3.44 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$

Optimal. Leaf size=168

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2-2aCde+2Acde+Bcd^2\right)}{2c^2}-\frac{x\left(aCe^2-c\left(e(Ae+2Bd)+Cd^2\right)\right)}{c^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(cd^2-ae^2\right)+a\right.}{\sqrt{a}c^{5/2}}$$

```
[Out] -(((a*C*e^2 - c*(C*d^2 + e*(2*B*d + A*e)))*x)/c^2) + (e*(2*C*d + B*e)*x^2)/
(2*c) + (C*e^2*x^3)/(3*c) + ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d +
 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + ((B*c*d^2 + 2*A*
c*d*e - 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.261574, antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2-2aCde+2Acde+Bcd^2\right)}{2c^2}+\frac{x\left(-aCe^2+ce(Ae+2Bd)+cCd^2\right)}{c^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(cd^2-ae^2\right)+a\right.}{\sqrt{a}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^2 - a*C*e^2 + c*e*(2*B*d + A*e))*x)/c^2 + (e*(2*C*d + B*e)*x^2)/(2*
c) + (C*e^2*x^3)/(3*c) + ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*
B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + ((B*c*d^2 + 2*A*c*d
*e - 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/(2*c^2)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \int \left(\frac{cCd^2-aCe^2+ce(2Bd+Ae)}{c^2} + \frac{e(2Cd+Be)x}{c} + \frac{Ce^2x^2}{c} + \frac{Ac\left(cd^2-ae^2\right)+a\left(aCe^2\right.}{} \right.$$

$$= \frac{\left(cCd^2-aCe^2+ce(2Bd+Ae)\right)x}{c^2} + \frac{e(2Cd+Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{\int \frac{Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(C\right.}{}}{}$$

$$= \frac{\left(cCd^2-aCe^2+ce(2Bd+Ae)\right)x}{c^2} + \frac{e(2Cd+Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{\left(Bcd^2+2Acde-2aCd\right.}{c}$$

$$= \frac{\left(cCd^2-aCe^2+ce(2Bd+Ae)\right)x}{c^2} + \frac{e(2Cd+Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{\left(Ac\left(cd^2-ae^2\right)+a\left(aC\right.\right.}{}$$

Mathematica [A] time = 0.193036, size = 155, normalized size = 0.92

$$\frac{x\left(-6aCe^2+3ce(2Ae+4Bd+Bex)+2cC\left(3d^2+3dex+e^2x^2\right)\right)+3\log\left(a+cx^2\right)\left(-aBe^2-2aCde+2Acde+Bcd^2\right)}{6c^2} + \frac{\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)
/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (x*(-6*a*C*e^2 + 3*c*e*(4*B*d + 2*A*e + B*e*
x) + 2*c*C*(3*d^2 + 3*d*e*x + e^2*x^2)) + 3*(B*c*d^2 + 2*A*c*d*e - 2*a*C*d*
e - a*B*e^2)*Log[a + c*x^2])/(6*c^2)
```

Maple [A] time = 0.069, size = 256, normalized size = 1.5

$$\frac{Ce^2x^3}{3c}+\frac{Bx^2e^2}{2c}+\frac{Cx^2de}{c}+\frac{Ae^2x}{c}+2\,\frac{Bdex}{c}-\frac{aCe^2x}{c^2}+\frac{Cd^2x}{c}+\frac{\ln\left(cx^2+a\right)Ade}{c}-\frac{\ln\left(cx^2+a\right)Bae^2}{2c^2}+\frac{\ln\left(cx^2+a\right)Bd^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] 1/3*C*e^2*x^3/c+1/2/c*B*x^2*e^2+1/c*C*x^2*d*e+1/c*A*e^2*x+2/c*B*d*e*x-1/c^2
*a*C*e^2*x+1/c*C*d^2*x+1/c*ln(c*x^2+a)*A*d*e-1/2/c^2*ln(c*x^2+a)*B*a*e^2+1/
2/c*ln(c*x^2+a)*B*d^2-1/c^2*ln(c*x^2+a)*C*a*d*e-1/c/(a*c)^(1/2)*arctan(x*c/
(a*c)^(1/2))*a*A*e^2+1/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d^2-2/c/(a*c)^
(1/2)*arctan(x*c/(a*c)^(1/2))*B*a*d*e+1/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1
/2))*a^2*C*e^2-1/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*a*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8533, size = 855, normalized size = 5.09

$$\left[\frac{2\,Cac^2e^2x^3+3\left(2\,Cac^2de+Bac^2e^2\right)x^2-3\left(2\,Bacde+\left(Cac-Ac^2\right)d^2-\left(Ca^2-Aac\right)e^2\right)\sqrt{-ac}\,\log\left(\frac{cx^2+2\,\sqrt{-ac}x-a}{cx^2+a}\right)+6\left(C\right.}{6\,ac^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e + B*a*c^2*e^2)*x^2 - 3*(2*B*a*c*
d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*sqrt(-a*c)*log((c*x^2 + 2*
```

```
sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e - (C*a^2*c
- A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*d*
e)*log(c*x^2 + a))/(a*c^3), 1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e + B*a
*c^2*e^2)*x^2 - 6*(2*B*a*c*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)
*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e - (C*a^2*
c - A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*
d*e)*log(c*x^2 + a))/(a*c^3)]
```

Sympy [B] time = 4.46252, size = 638, normalized size = 3.8

$$\frac{Ce^2x^3}{3c} + \left(-\frac{-2Acde + Bae^2 - Bcd^2 + 2Cade}{2c^2} - \frac{\sqrt{-ac^5}\left(-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2\right)}{2ac^5}\right)\log\left(x + \frac{-2Aacde}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**2*x**3/(3*c) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*d*e)/(2*c**
2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 -
 C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*e**2 - B*a*c*d**2 +
 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*d*e)/(
2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e
**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C
*a**2*e**2 - C*a*c*d**2)) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*d*e
)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**
2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*e**2 - B*a*
c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*
a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e +
C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c
*d*e + C*a**2*e**2 - C*a*c*d**2)) + x**2*(B*e**2 + 2*C*d*e)/(2*c) - x*(-A*c
*e**2 - 2*B*c*d*e + C*a*e**2 - C*c*d**2)/c**2
```

Giac [A] time = 1.1366, size = 238, normalized size = 1.42

$$\frac{\left(Bcd^2 - 2\,Cade + 2\,Acde - Bae^2\right)\log\left(cx^2 + a\right)}{2\,c^2} - \frac{\left(Cacd^2 - Ac^2d^2 + 2\,Bacde - Ca^2e^2 + Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{2\,Cc^2x^3e^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d^2 - 2*C*a*d*e + 2*A*c*d*e - B*a*e^2)*log(c*x^2 + a)/c^2 - (C*a*c
*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c
))/(sqrt(a*c)*c^2) + 1/6*(2*C*c^2*x^3*e^2 + 6*C*c^2*d*x^2*e + 6*C*c^2*d^2*x
 + 3*B*c^2*x^2*e^2 + 12*B*c^2*d*x*e - 6*C*a*c*x*e^2 + 6*A*c^2*x*e^2)/c^3
```

3.45 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2} dx$

Optimal. Leaf size=93

$$\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)}{2c^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd-a(Be+Cd))}{\sqrt{a}c^{3/2}}+\frac{x(Be+Cd)}{c}+\frac{Cex^2}{2c}$$

```
[Out] ((C*d + B*e)*x)/c + (C*e*x^2)/(2*c) + ((A*c*d - a*(C*d + B*e))*ArcTan[(Sqrt
[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + ((B*c*d + A*c*e - a*C*e)*Log[a + c*x^2
])/(2*c^2)
```

Rubi [A] time = 0.117307, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)}{2c^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd-a(Be+Cd))}{\sqrt{a}c^{3/2}}+\frac{x(Be+Cd)}{c}+\frac{Cex^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((C*d + B*e)*x)/c + (C*e*x^2)/(2*c) + ((A*c*d - a*(C*d + B*e))*ArcTan[(Sqrt
[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + ((B*c*d + A*c*e - a*C*e)*Log[a + c*x^2
])/(2*c^2)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx &= \int \left(\frac{Cd+Be}{c}+\frac{Cex}{c}+\frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{c\left(a+cx^2\right)}\right)dx\\ &= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{\int \frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{a+cx^2}\,dx}{c}\\ &= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Bcd+Ace-aCe)\int \frac{x}{a+cx^2}\,dx}{c}+\frac{(Acd-a(Cd+Be))\int \frac{1}{a+cx^2}\,dx}{c}\\ &= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Acd-a(Cd+Be))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{(Bcd+Ace-aCe)\log\left(a+\right.}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.109622, size = 86, normalized size = 0.92

$$\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)-\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd-Acd)}{\sqrt{a}}+cx(2Be+2Cd+Cex)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] (c*x*(2*C*d + 2*B*e + C*e*x) - (2*Sqrt[c]*(-(A*c*d) + a*C*d + a*B*e)*ArcTan
[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + (B*c*d + A*c*e - a*C*e)*Log[a + c*x^2])/(2
*c^2)
```

Maple [A] time = 0.05, size = 133, normalized size = 1.4

$$\frac{Cex^2}{2c}+\frac{Bex}{c}+\frac{Cdx}{c}+\frac{\ln\left(cx^2+a\right)Ae}{2c}+\frac{\ln\left(cx^2+a\right)Bd}{2c}-\frac{\ln\left(cx^2+a\right)aCe}{2c^2}+Ad\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{aBe}{c}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] 1/2*C*e*x^2/c+1/c*B*e*x+1/c*C*d*x+1/2/c*ln(c*x^2+a)*A*e+1/2/c*ln(c*x^2+a)*B
*d-1/2/c^2*ln(c*x^2+a)*a*C*e+1/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d-1/c/
(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a*B*e-1/c/(a*c)^(1/2)*arctan(x*c/(a*c)^
(1/2))*C*a*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84328, size = 468, normalized size = 5.03

$$\left[\frac{Cacex^2 - (Bae + (Ca - Ac)d)\sqrt{-ac}\log\left(\frac{cx^2+2\sqrt{-ac}x-a}{cx^2+a}\right) + 2(Cacd + Bace)x + \left(Bacd - \left(Ca^2 - Aac\right)e\right)\log\left(cx^2 + a\right)}{2ac^2}, \frac{Cace}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(C*a*c*e*x^2 - (B*a*e + (C*a - A*c)*d)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(
-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*d - (C*a^2 - A
*a*c)*e)*log(c*x^2 + a))/(a*c^2), 1/2*(C*a*c*e*x^2 - 2*(B*a*e + (C*a - A*c)
*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*d -
(C*a^2 - A*a*c)*e)*log(c*x^2 + a))/(a*c^2)]
```

Sympy [B] time = 2.20166, size = 335, normalized size = 3.6

$$\frac{Cex^2}{2c}+\left(-\frac{-Ace-Bcd+Cae}{2c^2}-\frac{\sqrt{-ac^5}\left(-Acd+Bae+Cad\right)}{2ac^4}\right)\log\left(x+\frac{Aace+Bacd-Ca^2e-2ac^2\left(-\frac{-Ace-Bcd+Cae}{2c^2}-\frac{\sqrt{-}}{}\right.}{-Ac^2d+Bace+Cacd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e*x**2/(2*c) + (-(-A*c*e - B*c*d + C*a*e)/(2*c**2) - sqrt(-a*c**5)*(-A*c*
d + B*a*e + C*a*d)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c*d - C*a**2*e - 2*a*
c**2*(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) - sqrt(-a*c**5)*(-A*c*d + B*a*e +
C*a*d)/(2*a*c**4)))/(-A*c**2*d + B*a*c*e + C*a*c*d)) + (-(-A*c*e - B*c*d +
C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4))*log(x
+ (A*a*c*e + B*a*c*d - C*a**2*e - 2*a*c**2*(-(-A*c*e - B*c*d + C*a*e)/(2*c*
*2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)))/(-A*c**2*d + B*a*
c*e + C*a*c*d)) + x*(B*e + C*d)/c
```

Giac [A] time = 1.16521, size = 123, normalized size = 1.32

$$-\frac{(Cad-Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}+\frac{(Bcd-Cae+Ace)\log\left(cx^2+a\right)}{2c^2}+\frac{Ccx^2e+2\,Ccdx+2\,Bcxe}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*d - A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(B*c*d -
 C*a*e + A*c*e)*log(c*x^2 + a)/c^2 + 1/2*(C*c*x^2*e + 2*C*c*d*x + 2*B*c*x*e
)/c^2
```

3.46 $\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx$

Optimal. Leaf size=55

$$\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

```
[Out] (C*x)/c + ((A*c - a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (B*
Log[a + c*x^2])/(2*c)
```

Rubi [A] time = 0.0525633, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1810, 635, 205, 260}

$$\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2),x]
```

```
[Out] (C*x)/c + ((A*c - a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (B*
Log[a + c*x^2])/(2*c)
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx &= \int \left(\frac{C}{c}+\frac{Ac-aC+Bcx}{c\left(a+cx^2\right)}\right)dx\\ &= \frac{Cx}{c}+\frac{\int \frac{Ac-aC+Bcx}{a+cx^2}\,dx}{c}\\ &= \frac{Cx}{c}+B\int \frac{x}{a+cx^2}\,dx+\frac{(Ac-aC)\int \frac{1}{a+cx^2}\,dx}{c}\\ &= \frac{Cx}{c}+\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.037828, size = 56, normalized size = 1.02

$$-\frac{(aC-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2),x]
```

```
[Out] (C*x)/c - ((-(A*c) + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) +
(B*Log[a + c*x^2])/(2*c)
```

Maple [A] time = 0.046, size = 59, normalized size = 1.1

$$\frac{Cx}{c}+\frac{B\ln\left(cx^2+a\right)}{2\,c}+A\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{aC}{c}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] C*x/c+1/2*B*ln(c*x^2+a)/c+1/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A-1/c/(a*c)
^(1/2)*arctan(x*c/(a*c)^(1/2))*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73884, size = 296, normalized size = 5.38

$$\left[\frac{2\,Cacx + Bac\log\left(cx^2 + a\right) + (Ca - Ac)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)}{2\,ac^2}, \frac{2\,Cacx + Bac\log\left(cx^2 + a\right) - 2\,(Ca - Ac)\sqrt{ac}\,\arctan\left(\right.}{2\,ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*C*a*c*x + B*a*c*log(c*x^2 + a) + (C*a - A*c)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a*c^2), 1/2*(2*C*a*c*x + B*a*c*log(c*x
^2 + a) - 2*(C*a - A*c)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a*c^2)]
```

Sympy [B] time = 0.674107, size = 156, normalized size = 2.84

$$\frac{Cx}{c} + \left(\frac{B}{2c} - \frac{\sqrt{-ac^3}\,(-Ac + Ca)}{2ac^3}\right)\log\left(x + \frac{Ba - 2ac\left(\frac{B}{2c} - \frac{\sqrt{-ac^3}(-Ac+Ca)}{2ac^3}\right)}{-Ac + Ca}\right) + \left(\frac{B}{2c} + \frac{\sqrt{-ac^3}\,(-Ac + Ca)}{2ac^3}\right)\log\left(x + \frac{Ba - 2ac}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*x/c + (B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*
a*c*(B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a)) + (B/(
2*c) + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*a*c*(B/(2*c)
 + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a))
```

Giac [A] time = 1.16636, size = 65, normalized size = 1.18

$$\frac{Cx}{c} + \frac{B\log\left(cx^2 + a\right)}{2\,c} - \frac{(Ca - Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*c)
```

3.47 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=133

$$\frac{\log\left(a+cx^2\right)(aCe-Ace+Bcd)}{2c\left(ae^2+cd^2\right)}+\frac{\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e\left(ae^2+cd^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)}$$

```
[Out] ((A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d
^2 + a*e^2)) + ((C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(e*(c*d^2 + a*e^2)) +
 ((B*c*d - A*c*e + a*C*e)*Log[a + c*x^2])/(2*c*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.162396, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)(aCe-Ace+Bcd)}{2c\left(ae^2+cd^2\right)}+\frac{\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e\left(ae^2+cd^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] ((A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d
^2 + a*e^2)) + ((C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(e*(c*d^2 + a*e^2)) +
 ((B*c*d - A*c*e + a*C*e)*Log[a + c*x^2])/(2*c*(c*d^2 + a*e^2))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)}\,dx = \int \left(\frac{Cd^2-Bde+Ae^2}{\left(cd^2+ae^2\right)(d+ex)}+\frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx$$

$$= \frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)}+\frac{\int \frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{a+cx^2}\,dx}{cd^2+ae^2}$$

$$= \frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)}+\frac{(Acd-aCd+aBe)\int \frac{1}{a+cx^2}\,dx}{cd^2+ae^2}+\frac{(Bcd-Ace+aCe)\int \frac{x}{a+cx^2}\,dx}{cd^2+ae^2}$$

$$= \frac{(Acd-aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)}+\frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)}+\frac{(Bcd-Ace+aCe)\log\left(a+cx^2\right)}{2c\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.115492, size = 120, normalized size = 0.9

$$\frac{\sqrt{a}\left(e\log\left(a+cx^2\right)(aCe-Ace+Bcd)+2c\log(d+ex)\left(Ae^2-Bde+Cd^2\right)\right)+2\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{2\sqrt{a}ce\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] (2*Sqrt[c]*e*(A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[a]*
(2*c*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x] + e*(B*c*d - A*c*e + a*C*e)*Log[a
 + c*x^2]))/(2*Sqrt[a]*c*e*(c*d^2 + a*e^2))
```

Maple [A] time = 0.053, size = 247, normalized size = 1.9

$$-\frac{\ln\left(cx^2+a\right)Ae}{2\,ae^2+2\,cd^2}+\frac{\ln\left(cx^2+a\right)Bd}{2\,ae^2+2\,cd^2}+\frac{\ln\left(cx^2+a\right)aCe}{\left(2\,ae^2+2\,cd^2\right)c}+\frac{Acd}{ae^2+cd^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{aBe}{ae^2+cd^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x)
```

```
[Out] -1/2/(a*e^2+c*d^2)*ln(c*x^2+a)*A*e+1/2/(a*e^2+c*d^2)*ln(c*x^2+a)*B*d+1/2/(a
*e^2+c*d^2)/c*ln(c*x^2+a)*a*C*e+1/(a*e^2+c*d^2)/(a*c)^(1/2)*arctan(x*c/(a*c
)^(1/2))*A*c*d+1/(a*e^2+c*d^2)/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a*B*e-1/
(a*e^2+c*d^2)/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*a*d+1/(a*e^2+c*d^2)*e*l
n(e*x+d)*A-1/(a*e^2+c*d^2)*ln(e*x+d)*B*d+1/(a*e^2+c*d^2)/e*ln(e*x+d)*C*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 47.1002, size = 575, normalized size = 4.32

$$\left[-\frac{\left(Bae^2-(Ca-Ac)de\right)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)-\left(Bacde+\left(Ca^2-Aac\right)e^2\right)\log\left(cx^2+a\right)-2\left(Cacd^2-Bacde+Aace^2\right)\mathrm{l}}{2\left(ac^2d^2e+a^2ce^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*e^2 - (C*a - A*c)*d*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x -
 a)/(c*x^2 + a)) - (B*a*c*d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) - 2*(C*
a*c*d^2 - B*a*c*d*e + A*a*c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3), 1
/2*(2*(B*a*e^2 - (C*a - A*c)*d*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (B*a*c*
d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) + 2*(C*a*c*d^2 - B*a*c*d*e + A*a*
c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13254, size = 169, normalized size = 1.27

$$\frac{(Bcd + Cae - Ace)\log\left(cx^2 + a\right)}{2\left(c^2d^2 + ace^2\right)} + \frac{\left(Cd^2 - Bde + Ae^2\right)\log\left(|xe + d|\right)}{cd^2e + ae^3} - \frac{(Cad - Acd - Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(cd^2 + ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d + C*a*e - A*c*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B
*d*e + A*e^2)*log(abs(x*e + d))/(c*d^2*e + a*e^3) - (C*a*d - A*c*d - B*a*e)
*arctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

3.48 $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)} dx$

Optimal. Leaf size=214

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{Ae^2-Bde+Cd^2}{e(d+ex)\left(ae^2+cd^2\right)}-\frac{\log(d+ex)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{\left(ae^2+cd^2\right)^2}$$

```
[Out] -((C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x))) + ((A*c*(c*d^2 - a
*e^2) + a*(a*C*e^2 - c*d*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt
[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^
2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 + ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*
B*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.355451, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{Ae^2-Bde+Cd^2}{e(d+ex)\left(ae^2+cd^2\right)}-\frac{\log(d+ex)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x))) + ((A*c*(c*d^2 - a
*e^2) + a*(a*C*e^2 - c*d*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt
[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^
2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 + ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*
B*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)}\,dx = \int \left(\frac{Cd^2-Bde+Ae^2}{\left(cd^2+ae^2\right)(d+ex)^2}+\frac{e\left(-Bcd^2+2Acde-2aCde+aBe^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{Ac\left(cd^2-ae^2\right)+a\left(aCe^2\right.}{}\right.$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\left(cd^2+ae^2\right)(d+ex)}-\frac{\left(Bcd^2-2Acde+2aCde-aBe^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2}+\frac{\int \frac{Ac\left(cd^2-ae^2\right)+a\left(aCe^2\right.}{}}{}$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\left(cd^2+ae^2\right)(d+ex)}-\frac{\left(Bcd^2-2Acde+2aCde-aBe^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2}+\frac{\left(c\left(Bcd^2-2Acde\right.\right.}{\left(\right.}$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\left(cd^2+ae^2\right)(d+ex)}+\frac{\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)^2}-\frac{\left(Bcd^2\right.}{}$$

Mathematica [A] time = 0.358212, size = 188, normalized size = 0.88

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-\frac{2\left(ae^2+cd^2\right)\left(e(Ae-Bd)+Cd^2\right)}{e(d+ex)}+\log(d+ex)\left(2aBe^2-4aCde+4Acde-2Bcd^2\right.}{2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] ((-2*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)) + (2*(A*c*(c
*d^2 - a*e^2) + a*(a*C*e^2 + c*d*(-(C*d) + 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt
[a]])/(Sqrt[a]*Sqrt[c]) + (-2*B*c*d^2 + 4*A*c*d*e - 4*a*C*d*e + 2*a*B*e^2)*
Log[d + e*x] + (B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

Maple [B] time = 0.058, size = 462, normalized size = 2.2

$$-\frac{c\ln\left(cx^2+a\right)Ade}{\left(ae^2+cd^2\right)^2}-\frac{\ln\left(cx^2+a\right)Bae^2}{2\left(ae^2+cd^2\right)^2}+\frac{c\ln\left(cx^2+a\right)Bd^2}{2\left(ae^2+cd^2\right)^2}+\frac{\ln\left(cx^2+a\right)Cade}{\left(ae^2+cd^2\right)^2}-\frac{aAe^2c}{\left(ae^2+cd^2\right)^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x)
```

```
[Out] -1/(a*e^2+c*d^2)^2*c*ln(c*x^2+a)*A*d*e-1/2/(a*e^2+c*d^2)^2*ln(c*x^2+a)*B*a*
e^2+1/2/(a*e^2+c*d^2)^2*c*ln(c*x^2+a)*B*d^2+1/(a*e^2+c*d^2)^2*ln(c*x^2+a)*C
*a*d*e-1/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a*A*e^2*c+1/(a
*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*c^2*d^2+2/(a*e^2+c*d^2)
^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*a*c*d*e+1/(a*e^2+c*d^2)^2/(a*c)^(1
/2)*arctan(x*c/(a*c)^(1/2))*a^2*C*e^2-1/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(
x*c/(a*c)^(1/2))*C*a*c*d^2-1/(a*e^2+c*d^2)*e/(e*x+d)*A+1/(a*e^2+c*d^2)/(e*x
+d)*B*d-1/(a*e^2+c*d^2)/e/(e*x+d)*C*d^2+2/(a*e^2+c*d^2)^2*ln(e*x+d)*A*c*d*e
+1/(a*e^2+c*d^2)^2*ln(e*x+d)*a*B*e^2-1/(a*e^2+c*d^2)^2*ln(e*x+d)*B*c*d^2-2/
(a*e^2+c*d^2)^2*ln(e*x+d)*C*a*d*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15432, size = 365, normalized size = 1.71

$$-\frac{\left(Cacd^2e^2 - Ac^2d^2e^2 - 2\,Bacde^3 - Ca^2e^4 + Aace^4\right)\arctan\left(\frac{\left(cd - \frac{cd^2}{xe+d} - \frac{ae^2}{xe+d}\right)e^{(-1)}}{\sqrt{ac}}\right)e^{(-2)}}{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{ac}} + \frac{\left(Bcd^2 + 2\,Cade - 2\,Acde - Bae^2\right)}{2\left(c^2d^4 + 2\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*c*d^2*e^2 - A*c^2*d^2*e^2 - 2*B*a*c*d*e^3 - C*a^2*e^4 + A*a*c*e^4)*ar
ctan((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((c
^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/2*(B*c*d^2 + 2*C*a*d*e - 2
*A*c*d*e - B*a*e^2)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*
e + d)^2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - (C*d^2*e/(x*e + d) - B*d*e^
2/(x*e + d) + A*e^3/(x*e + d))/(c*d^2*e^2 + a*e^4)
```

3.49 $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)}\,dx$

Optimal. Leaf size=305

$$\frac{\log\left(a+cx^2\right)\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{2\left(ae^2+cd^2\right)^3}-\frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2\left(ae^2+cd^2\right)}+\frac{-aBe^2+2aCde-2Acde+Bcd^2}{(d+ex)\left(ae^2+cd^2\right)^2}$$

```
[Out] -(C*d^2 - B*d*e + A*e^2)/(2*e*(c*d^2 + a*e^2)*(d + e*x)^2) + (B*c*d^2 - 2*A
*c*d*e + 2*a*C*d*e - a*B*e^2)/((c*d^2 + a*e^2)^2*(d + e*x)) + (Sqrt[c]*(A*c
*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^3) - ((B*c*d*(c*d^2 - 3*a*
e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x])/(c*d^2 + a*e^2)^3 + (
(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])
/(2*(c*d^2 + a*e^2)^3)
```

Rubi [A] time = 0.650946, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{2\left(ae^2+cd^2\right)^3}-\frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2\left(ae^2+cd^2\right)}+\frac{-aBe^2+2aCde-2Acde+Bcd^2}{(d+ex)\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] -(C*d^2 - B*d*e + A*e^2)/(2*e*(c*d^2 + a*e^2)*(d + e*x)^2) + (B*c*d^2 - 2*A
*c*d*e + 2*a*C*d*e - a*B*e^2)/((c*d^2 + a*e^2)^2*(d + e*x)) + (Sqrt[c]*(A*c
*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^3) - ((B*c*d*(c*d^2 - 3*a*
e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x])/(c*d^2 + a*e^2)^3 + (
(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])
/(2*(c*d^2 + a*e^2)^3)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)}\,dx = \int\left(\frac{Cd^2-Bde+Ae^2}{\left(cd^2+ae^2\right)(d+ex)^3}+\frac{e\left(-Bcd^2+2Acde-2aCde+aBe^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)^2}+\frac{e\left(-Bcd\left(cd^2-3ae^2\right)+(\ldots\right.}{\left(cd^2+ae^2\right)\ldots}\right.$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)}-\frac{\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC\ldots\right.}{\left(cd^2+\ldots\right.}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)}-\frac{\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC\ldots\right.}{\left(cd^2+\ldots\right.}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{\sqrt{c}\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2\ldots\right.\right.}{\sqrt{\ldots}}$$

Mathematica [A] time = 0.348172, size = 277, normalized size = 0.91

$$\frac{\log\left(a+cx^2\right)\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-\frac{\left(ae^2+cd^2\right)^2\left(e(Ae-Bd)+Cd^2\right)}{e(d+ex)^2}+\frac{2\left(ae^2+cd^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{d+ex}-\ldots}{2\left(ae^2+cd\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] (-(((c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)^2)) + (2*(c*
d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2))/(d + e*x) + (2*Sq
rt[c]*(A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d - B*e) + c*d^2*(-(C*d) + 3
*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] - 2*(B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x] + (B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Maple [B] time = 0.061, size = 754, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x)
```

```
[Out] 3*c^2/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^2*a*e-1/2/(a*
e^2+c*d^2)*e/(e*x+d)^2*A+1/2/(a*e^2+c*d^2)/(e*x+d)^2*B*d+3*c/(a*e^2+c*d^2)^
3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*a^2*d*e^2-3*c^2/(a*e^2+c*d^2)^3/(a*
c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d*e^2*a-1/(a*e^2+c*d^2)^3*ln(e*x+d)*B*c^
2*d^3+1/(a*e^2+c*d^2)^3*ln(e*x+d)*C*a^2*e^3-1/2/(a*e^2+c*d^2)/e/(e*x+d)^2*C
*d^2-1/(a*e^2+c*d^2)^2/(e*x+d)*a*B*e^2+1/(a*e^2+c*d^2)^2/(e*x+d)*B*c*d^2+3/
(a*e^2+c*d^2)^3*ln(e*x+d)*A*c^2*d^2*e+2/(a*e^2+c*d^2)^2/(e*x+d)*C*a*d*e-1/2
/(a*e^2+c*d^2)^3*ln(c*x^2+a)*C*a^2*e^3+1/2*c^2/(a*e^2+c*d^2)^3*ln(c*x^2+a)*
B*d^3+1/2*c/(a*e^2+c*d^2)^3*ln(c*x^2+a)*a*A*e^3-3/2*c^2/(a*e^2+c*d^2)^3*ln(
c*x^2+a)*A*d^2*e+c^3/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*
d^3-2/(a*e^2+c*d^2)^2/(e*x+d)*A*c*d*e-1/(a*e^2+c*d^2)^3*ln(e*x+d)*a*A*e^3*c
-3/(a*e^2+c*d^2)^3*ln(e*x+d)*C*a*c*d^2*e+3/2*c/(a*e^2+c*d^2)^3*ln(c*x^2+a)*
C*a*d^2*e-c/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a^2*B*e^3-c
^2/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*a*d^3-3/2*c/(a*e^2
+c*d^2)^3*ln(c*x^2+a)*a*B*d*e^2+3/(a*e^2+c*d^2)^3*ln(e*x+d)*a*B*d*e^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16342, size = 660, normalized size = 2.16

$$\frac{\left(Bc^2d^3 + 3\,Cacd^2e - 3\,Ac^2d^2e - 3\,Bacde^2 - Ca^2e^3 + Aace^3\right)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} - \frac{\left(Bc^2d^3e + 3\,Cacd^2e^2 - 3\,Ac^2d^2e^2 - 3\,Bacde\right.}{c^3d^6e + 3\,ac^2d^4e^3 + 3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c^2*d^3 + 3*C*a*c*d^2*e - 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 - C*a^2*e^3
+ A*a*c*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
```

```
a^3*e^6) - (B*c^2*d^3*e + 3*C*a*c*d^2*e^2 - 3*A*c^2*d^2*e^2 - 3*B*a*c*d*e^3
 - C*a^2*e^4 + A*a*c*e^4)*log(abs(x*e + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 +
3*a^2*c*d^2*e^5 + a^3*e^7) - (C*a*c^2*d^3 - A*c^3*d^3 - 3*B*a*c^2*d^2*e - 3
*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 + B*a^2*c*e^3)*arctan(c*x/sqrt(a*c))/((c^3
*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c^2
*d^6 - 3*B*c^2*d^5*e - 2*C*a*c*d^4*e^2 + 5*A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^3
- 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 + B*a^2*d*e^5 + A*a^2*e^6 - 2*(B*c^2*d^
4*e^2 + 2*C*a*c*d^3*e^3 - 2*A*c^2*d^3*e^3 + 2*C*a^2*d*e^5 - 2*A*a*c*d*e^5 -
 B*a^2*e^6)*x)*e^(-1)/((c*d^2 + a*e^2)^3*(x*e + d)^2)
```

3.50 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=216

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)-a\left(3ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{2a^{3/2}c^{5/2}}-\frac{e\log\left(a+cx^2\right)\left(2aCe^2-c\left(e(Ae+3Bd)+3Cd\right.\right.}{2c^3}$$

```
[Out] (-3*e^2*(A*c*d - a*(3*C*d + B*e))*x)/(2*a*c^2) - ((A*c - 2*a*C)*e^3*x^2)/(2
*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(2*a*c*(a + c*x^2)) + ((A*c*d
*(c*d^2 + 3*a*e^2) - a*(3*a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) - (e*(2*a*C*e^2 - c*(3*C*d^2 +
e*(3*B*d + A*e)))*Log[a + c*x^2])/(2*c^3)
```

Rubi [A] time = 0.504723, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1645, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)-a\left(3ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{2a^{3/2}c^{5/2}}-\frac{e\log\left(a+cx^2\right)\left(2aCe^2-c\left(e(Ae+3Bd)+3Cd\right.\right.}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (-3*e^2*(A*c*d - a*(3*C*d + B*e))*x)/(2*a*c^2) - ((A*c - 2*a*C)*e^3*x^2)/(2
*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(2*a*c*(a + c*x^2)) + ((A*c*d
*(c*d^2 + 3*a*e^2) - a*(3*a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) - (e*(2*a*C*e^2 - c*(3*C*d^2 +
e*(3*B*d + A*e)))*Log[a + c*x^2])/(2*c^3)
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
```

```
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2} dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^2(-Acd-aCd-3aBe+2(Ac-2aC)ex)}{a+cx^2}dx}{2ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)} - \frac{\int \left(\frac{3e^2(Acd-3aCd-aBe)}{c} + \frac{2(Ac-2aC)e^3x}{c} - \frac{Acd\left(cd^2+3ae^2\right)-a\left(3a\ldots\right.}{\ldots}\right.}{2\ldots}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)} + \frac{\int \frac{Acd \ldots}{\ldots}}{\ldots}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)} - \frac{\left(e\left(2a\ldots\right.\right.}{\ldots}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)} + \frac{\left(Acd\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.21356, size = 233, normalized size = 1.08

$$\frac{\frac{a^2ce(e(Ae+3Bd+Bex)+3Cd(d+ex))-a^3Ce^3-ac^2d\left(3Ae(d+ex)+Bd(d+3ex)+Cd^2x\right)+Ac^3d^3x}{a\left(a+cx^2\right)} + \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2(3Be+Cd)-3ae^2(Be+3Cd)\right)\right)\ldots}{a^{3/2}}\ldots}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*(3*C*d + B*e)*x + c*C*e^3*x^2 + (-(a^3*C*e^3) + A*c^3*d^3*x - a*c^
2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) + a^2*c*e*(3*C*d*(d + e*x
) + e*(3*B*d + A*e + B*e*x)))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c*d^2 + 3*
a*e^2) + a*(-3*a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*
x)/Sqrt[a]])/a^(3/2) + e*(3*c*C*d^2 - 2*a*C*e^2 + c*e*(3*B*d + A*e))*Log[a
+ c*x^2])/(2*c^3)
```

Maple [B] time = 0.056, size = 484, normalized size = 2.2

$$\frac{aAe^3}{2\,c^2\left(cx^2+a\right)} - \frac{3\,Ad^2e}{2\,c\left(cx^2+a\right)} - \frac{Ca^2e^3}{2\,c^3\left(cx^2+a\right)} + \frac{Ad^3}{2\,a}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} + \frac{Cd^3}{2\,c}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} + \frac{3\,\ln\left(cx^2+\ldots\right.}{2\,c^2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] 1/2/c^2/(c*x^2+a)*a*A*e^3-3/2/c/(c*x^2+a)*A*d^2*e-1/2/c^3/(c*x^2+a)*C*a^2*e
^3+1/2/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d^3+1/2/c/(a*c)^(1/2)*arctan
(x*c/(a*c)^(1/2))*C*d^3+3/2/c^2*ln(c*x^2+a)*B*d*e^2-1/c^3*a*ln(c*x^2+a)*C*e
^3+1/2/(c*x^2+a)*x/a*A*d^3+3*e^2/c^2*C*d*x-1/2/c/(c*x^2+a)*C*d^3*x+3/2/c^2/
(c*x^2+a)*C*a*d*e^2*x-9/2/c^2*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*d*e^2
+3/2/c^2/(c*x^2+a)*d^2*e*a*C-3/2/c^2*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*
B*e^3+3/2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^2*e+3/2/c/(a*c)^(1/2)*a
rctan(x*c/(a*c)^(1/2))*A*d*e^2+3/2/c^2*ln(c*x^2+a)*C*d^2*e-3/2/c/(c*x^2+a)*
A*d*e^2*x+1/2/c^2/(c*x^2+a)*B*a*e^3*x-3/2/c/(c*x^2+a)*B*d^2*e*x+3/2/c^2/(c*
x^2+a)*a*B*d*e^2+1/2*e^3/c^2*C*x^2+e^3/c^2*B*x-1/2/c/(c*x^2+a)*B*d^3+1/2/c^
2*ln(c*x^2+a)*A*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08102, size = 1881, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*C*a^2*c^2*e^3*x^4 + 2*C*a^3*c*e^3*x^2 - 2*B*a^2*c^2*d^3 + 6*B*a^3*c
*d*e^2 + 6*(C*a^3*c - A*a^2*c^2)*d^2*e - 2*(C*a^4 - A*a^3*c)*e^3 + 4*(3*C*a
^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*c*d^2*e - 3*B*a^3*e^3 + (C*a^2
*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e^2 + (3*B*a*c^2*d^2*e - 3*B*a^
2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*c - A*a*c^2)*d*e^2)*x^2)*sqrt(
```

```
-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(3*B*a^2*c^2*d^2*e
- 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*d*e
^2)*x + 2*(3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (3
*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*lo
g(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(C*a^2*c^2*e^3*x^4 + C*a^3*c*e^3
*x^2 - B*a^2*c^2*d^3 + 3*B*a^3*c*d*e^2 + 3*(C*a^3*c - A*a^2*c^2)*d^2*e - (C
*a^4 - A*a^3*c)*e^3 + 2*(3*C*a^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*
c*d^2*e - 3*B*a^3*e^3 + (C*a^2*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e
^2 + (3*B*a*c^2*d^2*e - 3*B*a^2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*
c - A*a*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (3*B*a^2*c^2*d^2
*e - 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*
d*e^2)*x + (3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (
3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*l
og(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] time = 39.3074, size = 949, normalized size = 4.39

$$\frac{Ce^3x^2}{2c^2} + \left(-\frac{e\left(-Ace^2-3Bcde+2Cae^2-3Ccd^2\right)}{2c^3} - \frac{\sqrt{-a^3c^7}\left(-3Aacde^2-Ac^2d^3+3Ba^2e^3-3Bacd^2e+9Ca^2de^2-Cacd\right.}{4a^3c^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**3*x**2/(2*c**2) + (-e*(-A*c*e**2 - 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2
)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**
3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/(4*a**3*c**6))*log(x + (
2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2 - 4*C*a**3*e**3 + 6*C*a**2*c*d**2*e - 4
*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) -
 sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*
d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/(4*a**3*c**6)))/(-3*A*a*c**2*d*e**2
- A*c**3*d**3 + 3*B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 9*C*a**2*c*d*e**2 - C
*a*c**2*d**3)) + (-e*(-A*c*e**2 - 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c
**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*
B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/(4*a**3*c**6))*log(x + (2*A*a*
*2*c*e**3 + 6*B*a**2*c*d*e**2 - 4*C*a**3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*
c**3*(-e*(-A*c*e**2 - 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(
-a**3*c**7)*(-3*A*a*c*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e
 + 9*C*a**2*d*e**2 - C*a*c*d**3)/(4*a**3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c*
*3*d**3 + 3*B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**
```

```
2*d**3)) + (A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**
2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*e + x*(-3*A*a*c**2*d*e**2 + A*c**3*d
**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3
))/(2*a**2*c**3 + 2*a*c**4*x**2) + x*(B*e**3 + 3*C*d*e**2)/c**2
```

Giac [A] time = 1.17587, size = 390, normalized size = 1.81

$$\frac{\left(3\,Ccd^2e+3\,Bcde^2-2\,Cae^3+Ace^3\right)\log\left(cx^2+a\right)}{2\,c^3}+\frac{\left(Cacd^3+Ac^2d^3+3\,Bacd^2e-9\,Ca^2de^2+3\,Aacde^2-3\,Ba^2e^3\right)\text{arcta}}{2\,\sqrt{ac}ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - 2*C*a*e^3 + A*c*e^3)*log(c*x^2 + a)/c^3 +
1/2*(C*a*c*d^3 + A*c^2*d^3 + 3*B*a*c*d^2*e - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2
- 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) + 1/2*(C*c^2*x^2*e^3
 + 6*C*c^2*d*x*e^2 + 2*B*c^2*x*e^3)/c^4 - 1/2*(B*a*c^2*d^3 - 3*C*a^2*c*d^2*
e + 3*A*a*c^2*d^2*e - 3*B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*
d^3 - A*c^3*d^3 + 3*B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - B*a
^2*c*e^3)*x)/((c*x^2 + a)*a*c^3)
```

3.51 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=146

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+Acd)+ae^2(Ac-3aC)\right)}{2a^{3/2}c^{5/2}}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}-\frac{e^2x(Ac-3aC)}{2ac^2}+\frac{e\log\left(a+cx^2\right.}{2c}$$

```
[Out] -((A*c - 3*a*C)*e^2*x)/(2*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(2*a
*c*(a + c*x^2)) + ((a*(A*c - 3*a*C)*e^2 + c*d*(A*c*d + a*C*d + 2*a*B*e))*Ar
cTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (e*(2*C*d + B*e)*Log[a + c
*x^2])/(2*c^2)
```

Rubi [A] time = 0.245414, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1645, 774, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+Acd)+ae^2(Ac-3aC)\right)}{2a^{3/2}c^{5/2}}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}-\frac{e^2x(Ac-3aC)}{2ac^2}+\frac{e\log\left(a+cx^2\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] -((A*c - 3*a*C)*e^2*x)/(2*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(2*a
*c*(a + c*x^2)) + ((a*(A*c - 3*a*C)*e^2 + c*d*(A*c*d + a*C*d + 2*a*B*e))*Ar
cTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (e*(2*C*d + B*e)*Log[a + c
*x^2])/(2*c^2)
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
```

```
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)}-\frac{\int \frac{(d+ex)(-Acd-aCd-2aBe+(Ac-3aC)ex)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2}-\frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)}-\frac{\int \frac{-a(Ac-3aC)e^2+cd(-Acd-aCd-2aBe)+c((Ac-}{a+cx^2}}{2ac^2}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2}-\frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)}+\frac{(e(2Cd+Be))\int \frac{x}{a+cx^2}\,dx}{c}+\frac{\Big(a(Ac-}{}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2}-\frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)}+\frac{\Big(a(Ac-3aC)e^2+cd(Acd+aCd+2}{2a^{3/2}c^{5/2}}$$

Mathematica [A] time = 0.150622, size = 175, normalized size = 1.2

$$\frac{\frac{\sqrt{c}\left(a^2e(Be+2Cd+Cex)-ac\left(Ae(2d+ex)+Bd(d+2ex)+Cd^2x\right)+Ac^2d^2x\right)}{a\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(ae^2+cd^2\right)+a\left(cd(2Be+Cd)-3aCe^2\right)\right)}{a^{3/2}}+\sqrt{c}e\log\left(a+cx^2\right)(Be+}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*C*e^2*x + (Sqrt[c]*(A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*x) -
a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x))))/(a*(a + c*x^2)) + ((A*c
*(c*d^2 + a*e^2) + a*(-3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/a^(3/2) + Sqrt[c]*e*(2*C*d + B*e)*Log[a + c*x^2])/(2*c^(5/2))
```

Maple [B] time = 0.051, size = 323, normalized size = 2.2

$$\frac{Ce^2x}{c^2}-\frac{Ae^2x}{2c\left(cx^2+a\right)}+\frac{xAd^2}{\left(2cx^2+2a\right)a}-\frac{Bdex}{c\left(cx^2+a\right)}+\frac{aCe^2x}{2c^2\left(cx^2+a\right)}-\frac{Cd^2x}{2c\left(cx^2+a\right)}-\frac{Ade}{c\left(cx^2+a\right)}+\frac{aBe^2}{2c^2\left(cx^2+a\right)}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] C*e^2/c^2*x-1/2/c/(c*x^2+a)*A*e^2*x+1/2/(c*x^2+a)*x/a*A*d^2-1/c/(c*x^2+a)*B
*d*e*x+1/2/c^2/(c*x^2+a)*a*C*e^2*x-1/2/c/(c*x^2+a)*C*d^2*x-1/c/(c*x^2+a)*A*
d*e+1/2/c^2/(c*x^2+a)*a*B*e^2-1/2/c/(c*x^2+a)*B*d^2+1/c^2/(c*x^2+a)*C*a*d*e
+1/2/c^2*ln(c*x^2+a)*B*e^2+1/c^2*ln(c*x^2+a)*C*d*e+1/2/c/(a*c)^(1/2)*arctan
(x*c/(a*c)^(1/2))*A*e^2+1/2/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d^2+1/c
/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d*e-3/2/c^2*a/(a*c)^(1/2)*arctan(x*c
/(a*c)^(1/2))*C*e^2+1/2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1014, size = 1276, normalized size = 8.74

$$\left[\frac{4\,Ca^2c^2e^2x^3 - 2\,Ba^2c^2d^2 + 2\,Ba^3ce^2 + 4\left(Ca^3c - Aa^2c^2\right)de - \left(2\,Ba^2cde + \left(Ca^2c + Aac^2\right)d^2 - \left(3\,Ca^3 - Aa^2c\right)e^2 + \left(2\,Bac^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*C*a^2*c^2*e^2*x^3 - 2*B*a^2*c^2*d^2 + 2*B*a^3*c*e^2 + 4*(C*a^3*c -
A*a^2*c^2)*d*e - (2*B*a^2*c*d*e + (C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A*a^
2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e
^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(2*B*
a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2)*x +
2*(2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)*log
(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(2*C*a^2*c^2*e^2*x^3 - B*a^2*c^2*
d^2 + B*a^3*c*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d*e + (2*B*a^2*c*d*e + (C*a^2*c
 + A*a*c^2)*d^2 - (3*C*a^3 - A*a^2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c
^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) -
 (2*B*a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2
)*x + (2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)
*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] time = 19.2175, size = 593, normalized size = 4.06

$$\frac{Ce^2x}{c^2} + \left(\frac{e\left(Be + 2Cd\right)}{2c^2} - \frac{\sqrt{-a^3c^5}\left(-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2\right)}{4a^3c^5}\right)\log\left(x + \frac{2Ba^2e^2 + 4Ca^2de - 4a^2c^2\left(\frac{e(Be}{}\right.}{-Aace^2 - Ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**2*x/c**2 + (e*(B*e + 2*C*d)/(2*c**2) - sqrt(-a**3*c**5)*(-A*a*c*e**2 -
 A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5))*log
```

```
(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*C*d)/(2*c**2)
- sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2
 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3
*C*a**2*e**2 - C*a*c*d**2)) + (e*(B*e + 2*C*d)/(2*c**2) + sqrt(-a**3*c**5)*
(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a
**3*c**5))*log(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*
C*d)/(2*c**2) + sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e +
 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2
*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)) + (-2*A*a*c*d*e + B*a**2*e**2 - B
*a*c*d**2 + 2*C*a**2*d*e + x*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a
**2*e**2 - C*a*c*d**2))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Giac [A] time = 1.16413, size = 248, normalized size = 1.7

$$\frac{Cxe^2}{c^2} + \frac{\left(2\,Cde + Be^2\right)\log\left(cx^2 + a\right)}{2\,c^2} + \frac{\left(Cacd^2 + Ac^2d^2 + 2\,Bacde - 3\,Ca^2e^2 + Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2} - \frac{Bacd^2 - 2\,Ca^2de}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] C*x*e^2/c^2 + 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(C*a*c*d^2 + A
*c^2*d^2 + 2*B*a*c*d*e - 3*C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c))/(sq
rt(a*c)*a*c^2) - 1/2*(B*a*c*d^2 - 2*C*a^2*d*e + 2*A*a*c*d*e - B*a^2*e^2 + (
C*a*c*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*x)/((c*x^2 + a
)*a*c^2)
```

3.52 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2} dx$

Optimal. Leaf size=97

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(2*a*c*(a + c*x^2)) + ((A*c*d + a*C*d +
a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2)) + (C*e*Log[a + c*x^
2])/(2*c^2)
```

Rubi [A] time = 0.0824126, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1645, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(2*a*c*(a + c*x^2)) + ((A*c*d + a*C*d +
a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2)) + (C*e*Log[a + c*x^
2])/(2*c^2)
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx &= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)}-\frac{\int \frac{-Acd-a(Cd+Be)-2aCex}{a+cx^2}\,dx}{2ac}\\ &= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)}+\frac{(Ce)\int \frac{x}{a+cx^2}\,dx}{c}+\frac{(Acd+aCd+aBe)\int \frac{1}{a+cx^2}\,dx}{2ac}\\ &= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)}+\frac{(Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.104366, size = 102, normalized size = 1.05

$$\frac{\frac{a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx}{a\left(a+cx^2\right)}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{a^{3/2}}+Ce\log\left(a+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] ((a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x)))/(a*(a + c*x^2)) +
(Sqrt[c]*(A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + C*e
*Log[a + c*x^2])/(2*c^2)
```

Maple [A] time = 0.052, size = 134, normalized size = 1.4

$$\frac{1}{cx^2+a}\left(\frac{(Acd-aBe-Cad)\,x}{2\,ac}-\frac{Ace+Bcd-aCe}{2\,c^2}\right)+\frac{Ce\ln\left(cx^2+a\right)}{2\,c^2}+\frac{Ad}{2\,a}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{Be}{2\,c}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] (1/2*(A*c*d-B*a*e-C*a*d)/a/c*x-1/2*(A*c*e+B*c*d-C*a*e)/c^2)/(c*x^2+a)+1/2*C
*e*ln(c*x^2+a)/c^2+1/2/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d+1/2/c/(a*c
)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*e+1/2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2
))*C*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76906, size = 717, normalized size = 7.39

$$\left[-\frac{2\,Ba^2cd+\left(Ba^2e+\left(Bace+\left(Cac+Ac^2\right)d\right)x^2+\left(Ca^2+Aac\right)d\right)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)-2\left(Ca^3-Aa^2c\right)e+2\left(Ba^2ce+\right.}{4\left(a^2c^3x^2+a^3c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c*d + (B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2
+ A*a*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(C
*a^3 - A*a^2*c)*e + 2*(B*a^2*c*e + (C*a^2*c - A*a*c^2)*d)*x - 2*(C*a^2*c*e*
```

```
x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^3*c^2), -1/2*(B*a^2*c*d - (
B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2 + A*a*c)*d)*sqrt(a*c)*
arctan(sqrt(a*c)*x/a) - (C*a^3 - A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c - A*a*c
^2)*d)*x - (C*a^2*c*e*x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^3*c^2
)]
```

Sympy [B] time = 6.4949, size = 318, normalized size = 3.28

$$\left(\frac{Ce}{2c^2} - \frac{\sqrt{-a^3c^5}\,(Acd + Bae + Cad)}{4a^3c^4}\right)\log\left(x + \frac{-2Ca^2e + 4a^2c^2\left(\frac{Ce}{2c^2} - \frac{\sqrt{-a^3c^5}(Acd+Bae+Cad)}{4a^3c^4}\right)}{Ac^2d + Bace + Cacd}\right) + \left(\frac{Ce}{2c^2} + \frac{\sqrt{-a^3c^5}\,(Acd + Ba}{4a^3c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] (C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log
(x + (-2*C*a**2*e + 4*a**2*c**2*(C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B
*a*e + C*a*d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (C*e/(2*c**
2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log(x + (-2*C*
a**2*e + 4*a**2*c**2*(C*e/(2*c**2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*
d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) - (A*a*c*e + B*a*c*d - C
*a**2*e + x*(-A*c**2*d + B*a*c*e + C*a*c*d))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Giac [A] time = 1.15104, size = 151, normalized size = 1.56

$$\frac{Ce\log\left(cx^2 + a\right)}{2\,c^2} + \frac{(Cad + Acd + Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac} - \frac{(Cad - Acd + Bae)x + \frac{Bacd - Ca^2e + Aace}{c}}{2\left(cx^2 + a\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*C*e*log(c*x^2 + a)/c^2 + 1/2*(C*a*d + A*c*d + B*a*e)*arctan(c*x/sqrt(a*
c))/(sqrt(a*c)*a*c) - 1/2*((C*a*d - A*c*d + B*a*e)*x + (B*a*c*d - C*a^2*e +
 A*a*c*e)/c)/((c*x^2 + a)*a*c)
```

3.53 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=69

$$\frac{(aC+Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{aB-x(Ac-aC)}{2ac\left(a+cx^2\right)}$$

```
[Out] -(a*B - (A*c - a*C)*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rubi [A] time = 0.0431747, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1814, 12, 205}

$$\frac{(aC+Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{aB-x(Ac-aC)}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2}\,dx &= -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}-\frac{\int \frac{-A-\frac{aC}{c}}{a+cx^2}\,dx}{2a}\\ &= -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}+\frac{(Ac+aC)\int \frac{1}{a+cx^2}\,dx}{2ac}\\ &= -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}+\frac{(Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0531633, size = 68, normalized size = 0.99

$$\frac{(aC+Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{-aB-aCx+Acx}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

```
[Out] (-(a*B) + A*c*x - a*C*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Maple [A] time = 0.049, size = 76, normalized size = 1.1

$$\frac{1}{cx^2+a}\left(\frac{(Ac-aC)\,x}{2\,ac}-\frac{B}{2\,c}\right)+\frac{A}{2\,a}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{C}{2\,c}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] (1/2*(A*c-C*a)/a/c*x-1/2*B/c)/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(x*c/(a*c)^
(1/2))*A+1/2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77277, size = 412, normalized size = 5.97

$$\left[-\frac{2\,Ba^2c + \left(Ca^2 + Aac + \left(Cac + Ac^2\right)x^2\right)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right) + 2\left(Ca^2c - Aac^2\right)x}{4\left(a^2c^3x^2 + a^3c^2\right)}, -\frac{Ba^2c - \left(Ca^2 + Aac + \left(Cac + A\right.\right.}{2\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c + (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(-a*c)*log((c*
x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^2*c - A*a*c^2)*x)/(a^2*c^3*
x^2 + a^3*c^2), -1/2*(B*a^2*c - (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(
a*c)*arctan(sqrt(a*c)*x/a) + (C*a^2*c - A*a*c^2)*x)/(a^2*c^3*x^2 + a^3*c^2)
]
```

Sympy [A] time = 0.840179, size = 116, normalized size = 1.68

$$-\frac{\sqrt{-\frac{1}{a^3c^3}}\,(Ac + Ca)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3c^3}}\,(Ac + Ca)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}} + x\right)}{4} - \frac{Ba + x\,(-Ac + Ca)}{2a^2c + 2ac^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(-a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 +
 sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 -
(B*a + x*(-A*c + C*a))/(2*a**2*c + 2*a*c**2*x**2)
```

Giac [A] time = 1.15341, size = 81, normalized size = 1.17

$$\frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac}-\frac{Cax-Acx+Ba}{2\left(cx^2+a\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*a + A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c) - 1/2*(C*a*x - A*c*x
+ B*a)/((c*x^2 + a)*a*c)
```

3.54 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=226

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^2}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e\log\left(a+cx^2\right)}{2\left(a\right.}$$

```
[Out] -(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(2*a*c*(c*d^2 +
a*e^2)*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 + 3*a*
e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2 + a*e^2)^2) +
(e*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 - (e*(C*d^2 - B*
d*e + A*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.434996, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^2}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e\log\left(a+cx^2\right)}{2\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```



```
[Out] -(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(2*a*c*(c*d^2 +
a*e^2)*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 + 3*a*
e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2 + a*e^2)^2) +
(e*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 - (e*(C*d^2 - B*
d*e + A*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```


Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx = -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(cd^2+2ae^2\right)\right)}{cd^2+ae^2}-\frac{ce(Acd-aCd+aBe)x}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)}\,dx}{2ac}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(-\frac{2ace^2\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} + \frac{c\left(-a(Cd-Be)\left(cd^2-ae^2\right)-A\right.}{\left(cd\right.}\right.}{2ac}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} - \frac{\int \frac{-a(Cd-B}{}}{}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} - \frac{\left(ce\left(Cd^2\right.\right.}{}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd\left(cd^2+3ae^2\right)\right)}{2a^{3/2}\sqrt{c}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.261031, size = 195, normalized size = 0.86

$$\frac{\frac{\left(ae^2+cd^2\right)\left(a^2(-C)e+ac(Ae-Bd+Bex-Cdx)+Ac^2dx\right)}{ac\left(a+cx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{a^{3/2}\sqrt{c}} - e\log\left(a+cx^2\right)\left(e(Ae-Bd)+Cd^2\right)+2}{2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x + B*e
*x)))/(a*c*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 +
3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(C*d^2 + e*(
-(B*d) + A*e))*Log[d + e*x] - e*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

Maple [B] time = 0.06, size = 742, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x)
```

```
[Out] 1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*A*c*d*e^2*x+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*x/a
*A*d^3*c^2+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*B*a*e^3*x+1/2/(a*e^2+c*d^2)^2/(c*x
^2+a)*B*c*d^2*e*x-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*C*a*d*e^2*x-1/2/(a*e^2+c*d^
2)^2/(c*x^2+a)*C*c*d^3*x+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*a*A*e^3+1/2/(a*e^2+c
*d^2)^2/(c*x^2+a)*A*c*d^2*e-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*a*B*d*e^2-1/2/(a*
e^2+c*d^2)^2/(c*x^2+a)*B*c*d^3-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)/c*C*a^2*e^3-1/
2/(a*e^2+c*d^2)^2/(c*x^2+a)*d^2*e*a*C-1/2/(a*e^2+c*d^2)^2*ln(c*x^2+a)*A*e^3
+1/2/(a*e^2+c*d^2)^2*ln(c*x^2+a)*B*d*e^2-1/2/(a*e^2+c*d^2)^2*ln(c*x^2+a)*C*
d^2*e+3/2/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*c*d*e^2+1/2
/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d^3*c^2+1/2/(a*e^2
+c*d^2)^2*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*e^3-1/2/(a*e^2+c*d^2)^2/(
a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*c*d^2*e-1/2/(a*e^2+c*d^2)^2*a/(a*c)^(1
/2)*arctan(x*c/(a*c)^(1/2))*C*d*e^2+1/2/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(
x*c/(a*c)^(1/2))*C*c*d^3+e^3/(a*e^2+c*d^2)^2*ln(e*x+d)*A-e^2/(a*e^2+c*d^2)^
2*ln(e*x+d)*B*d+e/(a*e^2+c*d^2)^2*ln(e*x+d)*C*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20387, size = 473, normalized size = 2.09

$$-\frac{\left(Cd^2e - Bde^2 + Ae^3\right)\log\left(cx^2 + a\right)}{2\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)} + \frac{\left(Cd^2e^2 - Bde^3 + Ae^4\right)\log\left(|xe + d|\right)}{c^2d^4e + 2\,acd^2e^3 + a^2e^5} + \frac{\left(Cacd^3 + Ac^2d^3 - Bacd^2e - Ca^2de^2 + 3\,Aa\right.}{2\left(ac^2d^4 + 2\,a^2cd^2e^2 + a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4) + (C*d^2*e^2 - B*d*e^3 + A*e^4)*log(abs(x*e + d))/(c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5) + 1/2*(C*a*c*d^3 + A*c^2*d^3 - B*a*c*d^2*e - C*a^2*d*
e^2 + 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*c^2*d^3 + C*a^2*c*d^2*e - A*a*c^
2*d^2*e + B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*d^3 - A*c^3*d^
3 - B*a*c^2*d^2*e + C*a^2*c*d*e^2 - A*a*c^2*d*e^2 - B*a^2*c*e^3)*x)/((c*d^2
 + a*e^2)^2*(c*x^2 + a)*a*c)
```

3.55 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=374

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4-6acde^2(Cd-Be)+c^2d^3(Cd-2Be)\right)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^3}-\frac{a\left(-aBe^2+2aCde-2A\right.}{}$$

```
[Out] -((e*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^2*(d + e*x))) - (a*(B*c*d^2
- 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*
d*(C*d - 2*B*e)))*x)/(2*a*(c*d^2 + a*e^2)^2*(a + c*x^2)) + ((A*c*(c^2*d^4 +
 6*a*c*d^2*e^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) - 6*a*c*
d*e^2*(C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2
+ a*e^2)^3) - (e*(a*e^2*(2*C*d - B*e) - c*d*(2*C*d^2 - e*(3*B*d - 4*A*e)))*
Log[d + e*x])/(c*d^2 + a*e^2)^3 + (e*(a*e^2*(2*C*d - B*e) - c*d*(2*C*d^2 -
e*(3*B*d - 4*A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rubi [A] time = 0.950404, antiderivative size = 371, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4-6acde^2(Cd-Be)+c^2d^3(Cd-2Be)\right)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^3}-\frac{a\left(-aBe^2+2aCde-2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] -((e*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^2*(d + e*x))) - (a*(B*c*d^2
- 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*
d*(C*d - 2*B*e)))*x)/(2*a*(c*d^2 + a*e^2)^2*(a + c*x^2)) + ((A*c*(c^2*d^4 +
 6*a*c*d^2*e^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) - 6*a*c*
d*e^2*(C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2
+ a*e^2)^3) + (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e))*
Log[d + e*x])/(c*d^2 + a*e^2)^3 - (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a
*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} - \frac{\int -}{}$$

$$= -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} - \frac{\int \Big(}{}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-\right.\right.}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-\right.\right.}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-\right.\right.}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

Mathematica [A] time = 0.467226, size = 320, normalized size = 0.86

$$\frac{\frac{\left(ae^2+cd^2\right)\left(a^2e(Be-2Cd+Cex)-ac\left(Ae(ex-2d)+Bd(d-2ex)+Cd^2x\right)+Ac^2d^2x\right)}{a\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4+6acde^2(Be-Cd)+c^2d^3(Cd-2B\right.\right.}{a^{3/2}\sqrt{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] ((-2*e*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + ((c*d^2 + a*
e^2)*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d*(d -
2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)) + ((A*c*(c^2*d^4 + 6*a*c*d^2*e
^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) + 6*a*c*d*e^2*(-(C*d
) + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(2*c*C*d^3
+ c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - e*(2*c*C*d^
3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(2*(c*d^
2 + a*e^2)^3)
```

Maple [B] time = 0.067, size = 1036, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x)
```

```
[Out] e^4/(a*e^2+c*d^2)^3*ln(e*x+d)*B*a+e^2/(a*e^2+c*d^2)^2/(e*x+d)*B*d-e/(a*e^2+
c*d^2)^2/(e*x+d)*C*d^2+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*B*a^2*e^4-1/2/(a*e^2+c
*d^2)^3/(c*x^2+a)*B*c^2*d^4-1/2/(a*e^2+c*d^2)^3*a*ln(c*x^2+a)*B*e^4+1/(a*e^
2+c*d^2)^3/(c*x^2+a)*B*c^2*d^3*e*x+1/(a*e^2+c*d^2)^3/(c*x^2+a)*A*a*c*d*e^3-
1/(a*e^2+c*d^2)^3/(c*x^2+a)*C*a*c*d^3*e-1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*A*a*c
*e^4*x-e^3/(a*e^2+c*d^2)^2/(e*x+d)*A+3/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x
*c/(a*c)^(1/2))*A*c^2*d^2*e^2+1/2/(a*e^2+c*d^2)^3/a/(a*c)^(1/2)*arctan(x*c/
(a*c)^(1/2))*A*c^3*d^4-3/(a*e^2+c*d^2)^3*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/
2))*C*c*d^2*e^2+3/(a*e^2+c*d^2)^3*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d
*e^3*c+1/(a*e^2+c*d^2)^3/(c*x^2+a)*B*a*c*d*e^3*x+1/2/(a*e^2+c*d^2)^3/(c*x^2
+a)*x/a*A*c^3*d^4-1/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*c
^2*d^3*e-3/2/(a*e^2+c*d^2)^3*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*e^4*c+
1/(a*e^2+c*d^2)^3/(c*x^2+a)*A*c^2*d^3*e-1/(a*e^2+c*d^2)^3/(c*x^2+a)*C*a^2*d
*e^3+1/2/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*c^2*d^4-1/(a
*e^2+c*d^2)^3*c*ln(c*x^2+a)*C*d^3*e+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*a^2*C*e^4
*x-2*e^3/(a*e^2+c*d^2)^3*ln(e*x+d)*C*a*d+2*e/(a*e^2+c*d^2)^3*ln(e*x+d)*C*c*
d^3+4*e^3/(a*e^2+c*d^2)^3*ln(e*x+d)*A*c*d+1/2/(a*e^2+c*d^2)^3*a^2/(a*c)^(1/
2)*arctan(x*c/(a*c)^(1/2))*C*e^4+3/2/(a*e^2+c*d^2)^3*c*ln(c*x^2+a)*B*d^2*e^
2+1/(a*e^2+c*d^2)^3*a*ln(c*x^2+a)*C*d*e^3-2/(a*e^2+c*d^2)^3*c*ln(c*x^2+a)*A
*d*e^3-1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*C*c^2*d^4*x-3*e^2/(a*e^2+c*d^2)^3*ln(e
*x+d)*B*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20344, size = 821, normalized size = 2.2

$$\frac{\left(Cac^2d^4e^2 + Ac^3d^4e^2 - 2\,Bac^2d^3e^3 - 6\,Ca^2cd^2e^4 + 6\,Aac^2d^2e^4 + 6\,Ba^2cde^5 + Ca^3e^6 - 3\,Aa^2ce^6\right)\arctan\left(\frac{\left(cd - \frac{cd^2}{xe+d} - \frac{ae^2}{xe+d}\right)e^{(-}}{\sqrt{ac}}\right.}{2\left(ac^3d^6 + 3\,a^2c^2d^4e^2 + 3\,a^3cd^2e^4 + a^4e^6\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*a*c^2*d^4*e^2 + A*c^3*d^4*e^2 - 2*B*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^
4 + 6*A*a*c^2*d^2*e^4 + 6*B*a^2*c*d*e^5 + C*a^3*e^6 - 3*A*a^2*c*e^6)*arctan
((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a*c^3
*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(2*C
*c*d^3*e - 3*B*c*d^2*e^2 - 2*C*a*d*e^3 + 4*A*c*d*e^3 + B*a*e^4)*log(c - 2*c
*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^3*d^6 + 3*a*c^2*d^
4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - (C*d^2*e^5/(x*e + d) - B*d*e^6/(x*e +
```

```
d) + A*e^7/(x*e + d))/(c^2*d^4*e^4 + 2*a*c*d^2*e^6 + a^2*e^8) - 1/2*((C*a*c
^2*d^3*e - A*c^3*d^3*e - 3*B*a*c^2*d^2*e^2 - 3*C*a^2*c*d*e^3 + 3*A*a*c^2*d*
e^3 + B*a^2*c*e^4)/(c*d^2 + a*e^2) - (C*a*c^2*d^4*e^2 - A*c^3*d^4*e^2 - 4*B
*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^4 + 6*A*a*c^2*d^2*e^4 + 4*B*a^2*c*d*e^5 +
C*a^3*e^6 - A*a^2*c*e^6)*e^(-1)/((c*d^2 + a*e^2)*(x*e + d)))/((c*d^2 + a*e^
2)^2*a*(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2))
```

3.56 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=524

$$-\frac{e\log\left(a+cx^2\right)\left(a^2Ce^4-2ace^2\left(4Cd^2-e(3Bd-Ae)\right)+c^2d^2\left(3Cd^2-2e(3Bd-5Ae)\right)\right)}{2\left(ae^2+cd^2\right)^4}+\frac{e\log(d+ex)\left(a^2Ce^4-2ace^2\left(\right.\right.}{}$$

```
[Out] -(e*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^2*(d + e*x)^2) + (e*(a*e^2*
(2*C*d - B*e) - c*d*(2*C*d^2 - e*(3*B*d - 4*A*e))))/((c*d^2 + a*e^2)^3*(d +
 e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) - c
*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*
x)/(2*a*(c*d^2 + a*e^2)^3*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*
d^2*e^2 - 15*a^2*e^4) - a*(2*a*c*d^2*e^2*(7*C*d - 9*B*e) - c^2*d^4*(C*d - 3
*B*e) - 3*a^2*e^4*(3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*(
c*d^2 + a*e^2)^4) + (e*(a^2*C*e^4 + c^2*d^2*(3*C*d^2 - 2*e*(3*B*d - 5*A*e))
 - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^4 -
 (e*(a^2*C*e^4 + c^2*d^2*(3*C*d^2 - 2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d
^2 - e*(3*B*d - A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rubi [A] time = 1.55223, antiderivative size = 524, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$-\frac{e\log\left(a+cx^2\right)\left(a^2Ce^4-2ace^2\left(4Cd^2-e(3Bd-Ae)\right)+c^2\left(3Cd^4-2d^2e(3Bd-5Ae)\right)\right)}{2\left(ae^2+cd^2\right)^4}+\frac{e\log(d+ex)\left(a^2Ce^4-2ace^2\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] -(e*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^2*(d + e*x)^2) - (e*(2*c*C*
d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^3*(d +
 e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) - c
*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*
x)/(2*a*(c*d^2 + a*e^2)^3*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*
d^2*e^2 - 15*a^2*e^4) - a*(2*a*c*d^2*e^2*(7*C*d - 9*B*e) - c^2*d^4*(C*d - 3
*B*e) - 3*a^2*e^4*(3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*(
c*d^2 + a*e^2)^4) + (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e))
```

```
 - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^4 -
 (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d
^2 - e*(3*B*d - A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx = -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)\right.\right.}{2a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$= -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)\right.\right.}{2a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^3(d+ex)}-\frac{a\left(Bcd\left(cd^2-3\right.\right.}{}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^3(d+ex)}-\frac{a\left(Bcd\left(cd^2-3\right.\right.}{}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^3(d+ex)}-\frac{a\left(Bcd\left(cd^2-3\right.\right.}{}$$

Mathematica [A] time = 0.706129, size = 466, normalized size = 0.89

$$\frac{\frac{\left(ae^2+cd^2\right)\left(-a^2ce(e(Ae-3Bd+Bex)+3Cd(d-ex))+a^3Ce^3-ac^2d\left(3Ae(ex-d)+Bd(d-3ex)+Cd^2x\right)+Ac^3d^3x\right)}{a\left(a+cx^2\right)}-\log\left(a+cx^2\right)\left(a^2Ce^5-2ace^3\left(e(Ae-3B\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] (-((e*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2) - (2*e*(c*
d^2 + a*e^2)*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e)))/(
d + e*x) + ((c*d^2 + a*e^2)*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d^2*x + B
*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(-3*B*d +
 A*e + B*e*x))))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*d^2*e^
2 - 15*a^2*e^4) + a*(-2*a*c*d^2*e^2*(7*C*d - 9*B*e) + c^2*d^4*(C*d - 3*B*e)
 - 3*a^2*e^4*(-3*C*d + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 2*(a^2
*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*
(-3*B*d + 5*A*e)))*Log[d + e*x] - (a^2*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B
*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*(-3*B*d + 5*A*e)))*Log[a + c*x^2])/(2
*(c*d^2 + a*e^2)^4)
```

Maple [B] time = 0.074, size = 1588, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x)
```

```
[Out] -3/2*c^2/(a*e^2+c*d^2)^4/(c*x^2+a)*C*a*d^4*e-3*c/(a*e^2+c*d^2)^4*a*ln(c*x^2
+a)*B*d*e^4-1/2*c/(a*e^2+c*d^2)^4/(c*x^2+a)*a^2*e^5*B*x+3/2*c/(a*e^2+c*d^2)
^4/(c*x^2+a)*B*a^2*d*e^4+6*e^4/(a*e^2+c*d^2)^4*ln(e*x+d)*B*a*c*d-8*e^3/(a*e
^2+c*d^2)^4*ln(e*x+d)*C*a*c*d^2+1/2*e^2/(a*e^2+c*d^2)^2/(e*x+d)^2*B*d-1/2*e
/(a*e^2+c*d^2)^2/(e*x+d)^2*C*d^2+e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*a^2*C-1/2*c^
3/(a*e^2+c*d^2)^4/(c*x^2+a)*B*d^5-1/2/(a*e^2+c*d^2)^4*a^2*ln(c*x^2+a)*C*e^5
-e^4/(a*e^2+c*d^2)^3/(e*x+d)*B*a+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)*C*a^3*e^5+3*
c^2/(a*e^2+c*d^2)^4*ln(c*x^2+a)*B*d^3*e^2+c/(a*e^2+c*d^2)^4*a*ln(c*x^2+a)*A
*e^5+1/2*c^3/(a*e^2+c*d^2)^4/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*d^5-3/2*
c^2/(a*e^2+c*d^2)^4*ln(c*x^2+a)*C*d^4*e-5*c^2/(a*e^2+c*d^2)^4*ln(c*x^2+a)*A
*d^2*e^3-4*e^3/(a*e^2+c*d^2)^3/(e*x+d)*A*c*d-3/2*c^3/(a*e^2+c*d^2)^4/(a*c)^
(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^4*e+4*c/(a*e^2+c*d^2)^4*a*ln(c*x^2+a)*C*d
^2*e^3+1/2*c^4/(a*e^2+c*d^2)^4/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d^5-
3/2*c/(a*e^2+c*d^2)^4*a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*e^5-3/2*c^2
/(a*e^2+c*d^2)^4/(c*x^2+a)*A*a*d*e^4*x+c^2/(a*e^2+c*d^2)^4/(c*x^2+a)*B*a*d^
2*e^3*x+c^2/(a*e^2+c*d^2)^4/(c*x^2+a)*C*a*d^3*e^2*x+9*c^2/(a*e^2+c*d^2)^4*a
/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^2*e^3-15/2*c^2/(a*e^2+c*d^2)^4*a/(
a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d*e^4+c^2/(a*e^2+c*d^2)^4/(c*x^2+a)*A*
a*d^2*e^3+c^2/(a*e^2+c*d^2)^4/(c*x^2+a)*B*a*d^3*e^2+3/2*c/(a*e^2+c*d^2)^4/(
c*x^2+a)*C*a^2*d*e^4*x+9/2*c/(a*e^2+c*d^2)^4*a^2/(a*c)^(1/2)*arctan(x*c/(a*
c)^(1/2))*C*d*e^4+1/2*c^4/(a*e^2+c*d^2)^4/(c*x^2+a)*x/a*A*d^5+3/2*c^3/(a*e^
2+c*d^2)^4/(c*x^2+a)*B*d^4*e*x+5*c^3/(a*e^2+c*d^2)^4/(a*c)^(1/2)*arctan(x*c
/(a*c)^(1/2))*A*d^3*e^2-7*c^2/(a*e^2+c*d^2)^4*a/(a*c)^(1/2)*arctan(x*c/(a*c
)^(1/2))*C*d^3*e^2-1/2*e^3/(a*e^2+c*d^2)^2/(e*x+d)^2*A-c/(a*e^2+c*d^2)^4/(c
*x^2+a)*C*a^2*d^2*e^3-c^3/(a*e^2+c*d^2)^4/(c*x^2+a)*A*d^3*e^2*x+3*e^2/(a*e^
2+c*d^2)^3/(e*x+d)*B*c*d^2+2*e^3/(a*e^2+c*d^2)^3/(e*x+d)*C*a*d-2*e/(a*e^2+c
*d^2)^3/(e*x+d)*C*c*d^3-2*e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*A*a*c+10*e^3/(a*e^2
+c*d^2)^4*ln(e*x+d)*A*c^2*d^2-6*e^2/(a*e^2+c*d^2)^4*ln(e*x+d)*B*c^2*d^3+3*e
/(a*e^2+c*d^2)^4*ln(e*x+d)*C*c^2*d^4-1/2*c^3/(a*e^2+c*d^2)^4/(c*x^2+a)*C*d^
5*x+3/2*c^3/(a*e^2+c*d^2)^4/(c*x^2+a)*A*d^4*e-1/2*c/(a*e^2+c*d^2)^4/(c*x^2+
a)*A*a^2*e^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18891, size = 1292, normalized size = 2.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 - 8*C*a*c*d^2*e^3 + 10*A*c^2*d^2*e^3
+ 6*B*a*c*d*e^4 + C*a^2*e^5 - 2*A*a*c*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^
3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e
^2 - 6*B*c^2*d^3*e^3 - 8*C*a*c*d^2*e^4 + 10*A*c^2*d^2*e^4 + 6*B*a*c*d*e^5 +
 C*a^2*e^6 - 2*A*a*c*e^6)*log(abs(x*e + d))/(c^4*d^8*e + 4*a*c^3*d^6*e^3 +
6*a^2*c^2*d^4*e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9) + 1/2*(C*a*c^3*d^5 + A*c^4*d
^5 - 3*B*a*c^3*d^4*e - 14*C*a^2*c^2*d^3*e^2 + 10*A*a*c^3*d^3*e^2 + 18*B*a^2
*c^2*d^2*e^3 + 9*C*a^3*c*d*e^4 - 15*A*a^2*c^2*d*e^4 - 3*B*a^3*c*e^5)*arctan
(c*x/sqrt(a*c))/((a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4
*c*d^2*e^6 + a^5*e^8)*sqrt(a*c)) - 1/2*(B*a*c^3*d^7 + 8*C*a^2*c^2*d^6*e - 3
*A*a*c^3*d^6*e - 9*B*a^2*c^2*d^5*e^2 + 4*C*a^3*c*d^4*e^3 + 7*A*a^2*c^2*d^4*
e^3 - 9*B*a^3*c*d^3*e^4 - 4*C*a^4*d^2*e^5 + 11*A*a^3*c*d^2*e^5 + B*a^4*d*e^
6 + A*a^4*e^7 + (5*C*a*c^3*d^5*e^2 - A*c^4*d^5*e^2 - 9*B*a*c^3*d^4*e^3 - 2*
C*a^2*c^2*d^3*e^4 + 10*A*a*c^3*d^3*e^4 - 6*B*a^2*c^2*d^2*e^5 - 7*C*a^3*c*d*
e^6 + 11*A*a^2*c^2*d*e^6 + 3*B*a^3*c*e^7)*x^3 + (7*C*a*c^3*d^6*e - 2*A*c^4*
d^6*e - 12*B*a*c^3*d^5*e^2 + C*a^2*c^2*d^4*e^3 + 10*A*a*c^3*d^4*e^3 - 12*B*
a^2*c^2*d^3*e^4 - 7*C*a^3*c*d^2*e^5 + 14*A*a^2*c^2*d^2*e^5 - C*a^4*e^7 + 2*
A*a^3*c*e^7)*x^2 + (C*a*c^3*d^7 - A*c^4*d^7 - B*a*c^3*d^6*e + 8*C*a^2*c^2*d
^5*e^2 - 4*A*a*c^3*d^5*e^2 - 12*B*a^2*c^2*d^4*e^3 + C*a^3*c*d^3*e^4 + 7*A*a
^2*c^2*d^3*e^4 - 9*B*a^3*c*d^2*e^5 - 6*C*a^4*d*e^6 + 10*A*a^3*c*d*e^6 + 2*B
*a^4*e^7)*x)/((c*d^2 + a*e^2)^4*(c*x^2 + a)*(x*e + d)^2*a)
```

3.57 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=209

$$-\frac{(d+ex)\left(ae(3aBe+5aCd+3Acd)-x\left(3Ac^2d^2-a\left(4aCe^2-cd(3Be+Cd)\right)\right)\right)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd^2(3aBe+aCd+3Acd\right.}{8a^{5/2}c^{5/}}$$

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(4*a*c*(a + c*x^2)^2) - ((d + e*x)*(a*
e*(3*A*c*d + 5*a*C*d + 3*a*B*e) - (3*A*c^2*d^2 - a*(4*a*C*e^2 - c*d*(C*d +
3*B*e)))*x))/(8*a^2*c^2*(a + c*x^2)) + ((3*a*e^2*(A*c*d + 3*a*C*d + a*B*e)
+ c*d^2*(3*A*c*d + a*C*d + 3*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2
)*c^(5/2)) + (C*e^3*Log[a + c*x^2])/(2*c^3)
```

Rubi [A] time = 0.304423, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1645, 819, 635, 205, 260}

$$-\frac{(d+ex)\left(ae(3aBe+5aCd+3Acd)-x\left(3Ac^2d^2-a\left(4aCe^2-cd(3Be+Cd)\right)\right)\right)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd^2(3aBe+aCd+3Acd\right.}{8a^{5/2}c^{5/}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```


```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(4*a*c*(a + c*x^2)^2) - ((d + e*x)*(a*
e*(3*A*c*d + 5*a*C*d + 3*a*B*e) - (3*A*c^2*d^2 - a*(4*a*C*e^2 - c*d*(C*d +
3*B*e)))*x))/(8*a^2*c^2*(a + c*x^2)) + ((3*a*e^2*(A*c*d + 3*a*C*d + a*B*e)
+ c*d^2*(3*A*c*d + a*C*d + 3*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2
)*c^(5/2)) + (C*e^3*Log[a + c*x^2])/(2*c^3)
```


Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
```

```
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)^2(-3Acd-aCd-3aBe-4aCex)}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2-a\left(4a\right.\right.\right.}{8a^2c^2\left(a+cx^2\right)}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2-a\left(4a\right.\right.\right.}{8a^2c^2\left(a+cx^2\right)}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2-a\left(4a\right.\right.\right.}{8a^2c^2\left(a+cx^2\right)}$$

Mathematica [A] time = 0.324269, size = 281, normalized size = 1.34

$$\frac{\frac{2a^2ce(e(Ae+3Bd+Bex)+3Cd(d+ex))-2a^3Ce^3-2ac^2d\left(3Ae(d+ex)+Bd(d+3ex)+Cd^2x\right)+2Ac^3d^3x}{a\left(a+cx^2\right)^2}+\frac{-a^2ce(e(4Ae+12Bd+5Bex)+3Cd(4d+5ex))+8a^3Ce^3+ac^2dx\left(3e\right.}{a^2\left(a+cx^2\right)}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] ((-2*a^3*C*e^3 + 2*A*c^3*d^3*x - 2*a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d
*(d + 3*e*x)) + 2*a^2*c*e*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x)))/(a*(
a + c*x^2)^2) + (8*a^3*C*e^3 + 3*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d +
A*e))*x - a^2*c*e*(3*C*d*(4*d + 5*e*x) + e*(12*B*d + 4*A*e + 5*B*e*x)))/(a^
2*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c*d^2 + a*e^2) + a*(3*a*e^2*(3*C*d + B*
e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*C*e^3*L
og[a + c*x^2])/(8*c^3)
```

Maple [B] time = 0.056, size = 402, normalized size = 1.9

$$\frac{1}{\left(cx^2+a\right)^2}\left(\frac{\left(3\,Acde^2a+3\,Ad^3c^2-5\,a^2Be^3+3\,Bcd^2ae-15\,Ca^2de^2+Cacd^3\right)x^3}{8\,a^2c}-\frac{e\left(Ace^2+3\,Bcde-2\,aCe^2+3\,Ccd^2\right)}{2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*a*c*d*e^2+3*A*c^2*d^3-5*B*a^2*e^3+3*B*a*c*d^2*e-15*C*a^2*d*e^2+C*
a*c*d^3)/a^2/c*x^3-1/2*e*(A*c*e^2+3*B*c*d*e-2*C*a*e^2+3*C*c*d^2)/c^2*x^2-1/
8*(3*A*a*c*d*e^2-5*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^2*e+9*C*a^2*d*e^2+C*a*c*
d^3)/a/c^2*x-1/4*(A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2+B*c^2*d^3-3*C*a^2*e
^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^2+1/2*C*e^3*ln(c*x^2+a)/c^3+3/8/a/c/(a*c)^
(1/2)*arctan(x*c/(a*c)^(1/2))*A*d*e^2+3/8/a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^
(1/2))*A*d^3+3/8/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*e^3+3/8/a/c/(a*c
)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^2*e+9/8/c^2/(a*c)^(1/2)*arctan(x*c/(a*c
)^(1/2))*C*d*e^2+1/8/a/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66185, size = 2325, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 + 12*(C*a^4*c + A*a^3*c^2)*d^2*e
 - 4*(3*C*a^5 - A*a^4*c)*e^3 - 2*(3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (C*
a^2*c^3 + 3*A*a*c^4)*d^3 - 3*(5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 8*(3*C*
a^3*c^2*d^2*e + 3*B*a^3*c^2*d*e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 + (3*B
*a^3*c*d^2*e + 3*B*a^4*e^3 + (3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a*c^3
+ 3*A*c^4)*d^3 + 3*(3*C*a^2*c^2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*A*a^2*
c^2)*d^3 + 3*(3*C*a^4 + A*a^3*c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e
^3 + (C*a^2*c^2 + 3*A*a*c^3)*d^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x^2)*sq
rt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(3*B*a^3*c^2*d^2
```

```
*e + 3*B*a^4*c*e^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3 + 3*(3*C*a^4*c + A*a^3*c
^2)*d*e^2)*x - 8*(C*a^3*c^2*e^3*x^4 + 2*C*a^4*c*e^3*x^2 + C*a^5*e^3)*log(c*
x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^3 +
6*B*a^4*c*d*e^2 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e - 2*(3*C*a^5 - A*a^4*c)*e^3
 - (3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (C*a^2*c^3 + 3*A*a*c^4)*d^3 - 3*(
5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 4*(3*C*a^3*c^2*d^2*e + 3*B*a^3*c^2*d*
e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 - (3*B*a^3*c*d^2*e + 3*B*a^4*e^3 + (
3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^3 + 3*(3*C*a^2*c^
2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^3 + 3*(3*C*a^4 + A*a^3*
c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2*c^2 + 3*A*a*c^3)*d
^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a)
+ (3*B*a^3*c^2*d^2*e + 3*B*a^4*c*e^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3 + 3*(3
*C*a^4*c + A*a^3*c^2)*d*e^2)*x - 4*(C*a^3*c^2*e^3*x^4 + 2*C*a^4*c*e^3*x^2 +
 C*a^5*e^3)*log(c*x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16451, size = 470, normalized size = 2.25

$$\frac{Ce^3\log\left(cx^2+a\right)}{2\,c^3}+\frac{\left(Cacd^3+3\,Ac^2d^3+3\,Bacd^2e+9\,Ca^2de^2+3\,Aacde^2+3\,Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2}+\frac{\left(Cac^2d^3+3\,Ac^3d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*C*e^3*log(c*x^2 + a)/c^3 + 1/8*(C*a*c*d^3 + 3*A*c^2*d^3 + 3*B*a*c*d^2*e
 + 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt
(a*c)*a^2*c^2) + 1/8*((C*a*c^2*d^3 + 3*A*c^3*d^3 + 3*B*a*c^2*d^2*e - 15*C*a
^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - 5*B*a^2*c*e^3)*x^3 - 4*(3*C*a^2*c*d^2*e + 3*
```

```
B*a^2*c*d*e^2 - 2*C*a^3*e^3 + A*a^2*c*e^3)*x^2 - (C*a^2*c*d^3 - 5*A*a*c^2*d
^3 + 3*B*a^2*c*d^2*e + 9*C*a^3*d*e^2 + 3*A*a^2*c*d*e^2 + 3*B*a^3*e^3)*x - 2
*(B*a^2*c^2*d^3 + 3*C*a^3*c*d^2*e + 3*A*a^2*c^2*d^2*e + 3*B*a^3*c*d*e^2 - 3
*C*a^4*e^3 + A*a^3*c*e^3)/c)/((c*x^2 + a)^2*a^2*c^2)
```

3.58 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=156

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+3Acd)+ae^2(3aC+Ac)\right)}{8a^{5/2}c^{5/2}}-\frac{(d+ex)(ae(3aC+Ac)-cx(2aBe+aCd+3Acd))}{8a^2c^2\left(a+cx^2\right)}-\frac{(d+ex}{4}$$

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(4*a*c*(a + c*x^2)^2) - ((d + e*x)*(a*
(A*c + 3*a*C)*e - c*(3*A*c*d + a*C*d + 2*a*B*e)*x))/(8*a^2*c^2*(a + c*x^2))
 + ((a*(A*c + 3*a*C)*e^2 + c*d*(3*A*c*d + a*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Rubi [A] time = 0.230675, antiderivative size = 175, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1645, 778, 205}

$$-\frac{x\left(ae^2(3aC+Ac)-cd(2aBe+aCd+3Acd)\right)+2ae(aBe+2aCd+2Acd)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+3Acd)+a\right.}{8a^{5/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(4*a*c*(a + c*x^2)^2) - (2*a*e*(2*A*c*
d + 2*a*C*d + a*B*e) + (a*(A*c + 3*a*C)*e^2 - c*d*(3*A*c*d + a*C*d + 2*a*B*
e))*x)/(8*a^2*c^2*(a + c*x^2)) + ((a*(A*c + 3*a*C)*e^2 + c*d*(3*A*c*d + a*C
*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
```

```
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2}-\frac{\int \frac{(d+ex)(-3Acd-aCd-2aBe-(Ac+3aC)ex)}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2}-\frac{2ae(2Acd+2aCd+aBe)+\left(a(Ac+3aC)e^2-cd(3Acd\right.}{8a^2c^2\left(a+cx^2\right)}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2}-\frac{2ae(2Acd+2aCd+aBe)+\left(a(Ac+3aC)e^2-cd(3Acd\right.}{8a^2c^2\left(a+cx^2\right)}$$

Mathematica [A] time = 0.153778, size = 211, normalized size = 1.35

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(ae^2+3cd^2\right)+a\left(3aCe^2+cd(2Be+Cd)\right)\right)}{8a^{5/2}c^{5/2}}+\frac{a^2(-e)(4Be+8Cd+5Cex)+acx\left(e(Ae+2Bd)+Cd^2\right)+3}{8a^2c^2\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(8*C*d + 4*B*e + 5
*C*e*x))/(8*a^2*c^2*(a + c*x^2)) + (A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*
x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(4*a*c^2*(a + c*x^2
)^2) + ((A*c*(3*c*d^2 + a*e^2) + a*(3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[
```

```
(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Maple [A] time = 0.053, size = 283, normalized size = 1.8

$$\frac{1}{\left(cx^2+a\right)^2}\left(\frac{\left(aAe^2c+3\,Ac^2d^2+2\,Bacde-5\,a^2Ce^2+Cacd^2\right)x^3}{8\,a^2c}-\frac{e\,(Be+2\,Cd)\,x^2}{2\,c}-\frac{\left(aAe^2c-5\,Ac^2d^2+2\,Bacde+3\,a^2\right.}{8\,ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e-5*C*a^2*e^2+C*a*c*d^2)/a^2/c*x^3-1/
2*e*(B*e+2*C*d)*x^2/c-1/8*(A*a*c*e^2-5*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+C*
a*c*d^2)/a/c^2*x-1/4*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^2
+1/8/a/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*e^2+3/8/a^2/(a*c)^(1/2)*arct
an(x*c/(a*c)^(1/2))*A*d^2+1/4/a/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d*e
+3/8/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*e^2+1/8/a/c/(a*c)^(1/2)*arct
an(x*c/(a*c)^(1/2))*C*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.48735, size = 1644, normalized size = 10.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^2 + 4*B*a^4*c*e^2 - 2*(2*B*a^2*c^3*d*e + (C*a^2*c^3 +
 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 8*(C*a^4*c + A*a^3*c
^2)*d*e + 8*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 + (2*B*a^3*c*d*e + (2*B*a
*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*c^3)*e^2)*x^4 + (C*
a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(2*B*a^2*c^2*d*e + (
C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2)*x^2)*sqrt(-a*c)*l
og((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(2*B*a^3*c^2*d*e + (C*a^3*
c^2 - 5*A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x^4 + 2*a
^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^2 + 2*B*a^4*c*e^2 - (2*B*a^2*c^3
*d*e + (C*a^2*c^3 + 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 4
*(C*a^4*c + A*a^3*c^2)*d*e + 4*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 - (2*B
*a^3*c*d*e + (2*B*a*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*
c^3)*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(
2*B*a^2*c^2*d*e + (C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2
)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (2*B*a^3*c^2*d*e + (C*a^3*c^2 - 5*
A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x^4 + 2*a^4*c^4*x
^2 + a^5*c^3)]
```

Sympy [B] time = 163.039, size = 389, normalized size = 2.49

$$-\frac{\sqrt{-\frac{1}{a^5c^5}}\left(Aace^2+3Ac^2d^2+2Bacde+3Ca^2e^2+Cacd^2\right)\log\left(-a^3c^2\sqrt{-\frac{1}{a^5c^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^5}}\left(Aace^2+3Ac^2d^2+2Bacde+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**5))*(A*a*c*e**2 + 3*A*c**2*d**2 + 2*B*a*c*d*e + 3*C*a**2*
e**2 + C*a*c*d**2)*log(-a**3*c**2*sqrt(-1/(a**5*c**5)) + x)/16 + sqrt(-1/(a
**5*c**5))*(A*a*c*e**2 + 3*A*c**2*d**2 + 2*B*a*c*d*e + 3*C*a**2*e**2 + C*a*
c*d**2)*log(a**3*c**2*sqrt(-1/(a**5*c**5)) + x)/16 - (4*A*a**2*c*d*e + 2*B*
a**3*e**2 + 2*B*a**2*c*d**2 + 4*C*a**3*d*e + x**3*(-A*a*c**2*e**2 - 3*A*c**
3*d**2 - 2*B*a*c**2*d*e + 5*C*a**2*c*e**2 - C*a*c**2*d**2) + x**2*(4*B*a**2
*c*e**2 + 8*C*a**2*c*d*e) + x*(A*a**2*c*e**2 - 5*A*a*c**2*d**2 + 2*B*a**2*c
*d*e + 3*C*a**3*e**2 + C*a**2*c*d**2))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8
*a**2*c**4*x**4)
```

Giac [A] time = 1.15984, size = 343, normalized size = 2.2

$$\frac{\left(Cacd^2 + 3\,Ac^2d^2 + 2\,Bacde + 3\,Ca^2e^2 + Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2} + \frac{Cac^2d^2x^3 + 3\,Ac^3d^2x^3 + 2\,Bac^2dx^3e - 5\,Ca^2cx^3e^2 + A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*c*d^2 + 3*A*c^2*d^2 + 2*B*a*c*d*e + 3*C*a^2*e^2 + A*a*c*e^2)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2) + 1/8*(C*a*c^2*d^2*x^3 + 3*A*c^3*d^2*x
^3 + 2*B*a*c^2*d*x^3*e - 5*C*a^2*c*x^3*e^2 + A*a*c^2*x^3*e^2 - 8*C*a^2*c*d*
x^2*e - C*a^2*c*d^2*x + 5*A*a*c^2*d^2*x - 4*B*a^2*c*x^2*e^2 - 2*B*a^2*c*d*x
*e - 2*B*a^2*c*d^2 - 3*C*a^3*x*e^2 - A*a^2*c*x*e^2 - 4*C*a^3*d*e - 4*A*a^2*
c*d*e - 2*B*a^3*e^2)/((c*x^2 + a)^2*a^2*c^2)
```

3.59 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3} dx$

Optimal. Leaf size=130

$$-\frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(4*a*c*(a + c*x^2)^2) - (2*a*(A*c + a*C)
*e - c*(3*A*c*d + a*C*d + a*B*e)*x)/(8*a^2*c^2*(a + c*x^2)) + ((3*A*c*d + a
*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

Rubi [A] time = 0.107909, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1645, 639, 205}

$$-\frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(4*a*c*(a + c*x^2)^2) - (2*a*(A*c + a*C)
*e - c*(3*A*c*d + a*C*d + a*B*e)*x)/(8*a^2*c^2*(a + c*x^2)) + ((3*A*c*d + a
*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{\int \frac{-3Acd-a(Cd+Be)-2(Ac+aC)ex}{\left(a+cx^2\right)^2}dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)}+\frac{(3Acd+aC\dots}{\dots}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)}+\frac{(3Acd+aC\dots}{\dots}$$

Mathematica [A] time = 0.112773, size = 137, normalized size = 1.05

$$\frac{\frac{2a^{3/2}\left(a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx\right)}{\left(a+cx^2\right)^2}+\frac{\sqrt{a}\left(-4a^2Ce+acx(Be+Cd)+3Ac^2dx\right)}{a+cx^2}+\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-4*a^2*C*e + 3*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2) + (2*
a^(3/2)*(a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2
)^2 + Sqrt[c]*(3*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(
5/2)*c^2)
```

Maple [A] time = 0.052, size = 157, normalized size = 1.2

$$\frac{1}{\left(cx^2+a\right)^2}\left(\frac{(3\,Acd+aBe+Cad)\,x^3}{8\,a^2}-\frac{Cex^2}{2\,c}+\frac{(5\,Acd-aBe-Cad)\,x}{8\,ac}-\frac{Ace+Bcd+aCe}{4\,c^2}\right)+\frac{3\,Ad}{8\,a^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*c*d+B*a*e+C*a*d)/a^2*x^3-1/2*C*e*x^2/c+1/8*(5*A*c*d-B*a*e-C*a*d)/
a/c*x-1/4*(A*c*e+B*c*d+C*a*e)/c^2)/(c*x^2+a)^2+3/8/a^2/(a*c)^(1/2)*arctan(x
*c/(a*c)^(1/2))*A*d+1/8/a/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*e+1/8/a/c
/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00029, size = 991, normalized size = 7.62

$$\left[-\frac{8\,Ca^3cex^2+4\,Ba^3cd-2\left(Ba^2c^2e+\left(Ca^2c^2+3\,Aac^3\right)d\right)x^3+\left(Ba^3e+\left(Bac^2e+\left(Cac^2+3\,Ac^3\right)d\right)x^4+2\left(Ba^2ce+\left(Ca^2c\right.\right.\right.}{16\left(a^3c^4x^4+2\,a^4c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*C*a^3*c*e*x^2 + 4*B*a^3*c*d - 2*(B*a^2*c^2*e + (C*a^2*c^2 + 3*A*a
*c^3)*d)*x^3 + (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4 + 2*(B*a^
2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqrt(-a*c)*lo
g((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 4*(C*a^4 + A*a^3*c)*e + 2*(B*
```

```
a^3*c*e + (C*a^3*c - 5*A*a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*
c^2), -1/8*(4*C*a^3*c*e*x^2 + 2*B*a^3*c*d - (B*a^2*c^2*e + (C*a^2*c^2 + 3*A
*a*c^3)*d)*x^3 - (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4 + 2*(B*
a^2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqrt(a*c)*a
rctan(sqrt(a*c)*x/a) + 2*(C*a^4 + A*a^3*c)*e + (B*a^3*c*e + (C*a^3*c - 5*A*
a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

Sympy [A] time = 34.2871, size = 240, normalized size = 1.85

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Acd+Bae+Cad)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Acd+Bae+Cad)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{-2Aa^2ce-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(-a**3*c*sqrt(-1/(a**5*c
**3)) + x)/16 + sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(a**3*c*s
qrt(-1/(a**5*c**3)) + x)/16 + (-2*A*a**2*c*e - 2*B*a**2*c*d - 2*C*a**3*e -
4*C*a**2*c*e*x**2 + x**3*(3*A*c**3*d + B*a*c**2*e + C*a*c**2*d) + x*(5*A*a*
c**2*d - B*a**2*c*e - C*a**2*c*d))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**
2*c**4*x**4)
```

Giac [A] time = 1.14763, size = 205, normalized size = 1.58

$$\frac{(Cad+3\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}+\frac{Cac^2dx^3+3\,Ac^3dx^3+Bac^2x^3e-4\,Ca^2cx^2e-Ca^2cdx+5\,Aac^2dx-Ba^2cxe-2\,B}{8\left(cx^2+a\right)^2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*d + 3*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8
*(C*a*c^2*d*x^3 + 3*A*c^3*d*x^3 + B*a*c^2*x^3*e - 4*C*a^2*c*x^2*e - C*a^2*c
*d*x + 5*A*a*c^2*d*x - B*a^2*c*x*e - 2*B*a^2*c*d - 2*C*a^3*e - 2*A*a^2*c*e)
/((c*x^2 + a)^2*a^2*c^2)
```

3.60 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^3} dx$

Optimal. Leaf size=98

$$\frac{(aC+3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}+\frac{x(aC+3Ac)}{8a^2c\left(a+cx^2\right)}-\frac{aB-x(Ac-aC)}{4ac\left(a+cx^2\right)^2}$$

```
[Out] -(a*B - (A*c - a*C)*x)/(4*a*c*(a + c*x^2)^2) + ((3*A*c + a*C)*x)/(8*a^2*c*(
a + c*x^2)) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2
))
```

Rubi [A] time = 0.0630782, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1814, 12, 199, 205}

$$\frac{(aC+3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}+\frac{x(aC+3Ac)}{8a^2c\left(a+cx^2\right)}-\frac{aB-x(Ac-aC)}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(4*a*c*(a + c*x^2)^2) + ((3*A*c + a*C)*x)/(8*a^2*c*(
a + c*x^2)) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2
))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx &= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}-\frac{\int \frac{-3A-\frac{aC}{c}}{\left(a+cx^2\right)^2}\,dx}{4a}\\
&= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{4ac}\\
&= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)}+\frac{(3Ac+aC)\int \frac{1}{a+cx^2}\,dx}{8a^2c}\\
&= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)}+\frac{(3Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0742712, size = 90, normalized size = 0.92

$$\frac{-a^2(2B+Cx)+acx\left(5A+Cx^2\right)+3Ac^2x^3}{8a^2c\left(a+cx^2\right)^2}+\frac{(aC+3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*x^3 - a^2*(2*B + C*x) + a*c*x*(5*A + C*x^2))/(8*a^2*c*(a + c*x^2)^
2) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

Maple [A] time = 0.049, size = 96, normalized size = 1.

$$\frac{1}{\left(cx^2+a\right)^2}\left(\frac{(3\,Ac+aC)\,x^3}{8\,a^2}+\frac{(5\,Ac-aC)\,x}{8\,ac}-\frac{B}{4\,c}\right)+\frac{3\,A}{8\,a^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{C}{8\,ac}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*c+C*a)/a^2*x^3+1/8*(5*A*c-C*a)/a/c*x-1/4*B/c)/(c*x^2+a)^2+3/8/a^2
/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A+1/8/a/c/(a*c)^(1/2)*arctan(x*c/(a*c)
^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76369, size = 656, normalized size = 6.69

$$\left[-\frac{4\,Ba^3c-2\left(Ca^2c^2+3\,Aac^3\right)x^3+\left(\left(Cac^2+3\,Ac^3\right)x^4+Ca^3+3\,Aa^2c+2\left(Ca^2c+3\,Aac^2\right)x^2\right)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)+}{16\left(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c - 2*(C*a^2*c^2 + 3*A*a*c^3)*x^3 + ((C*a*c^2 + 3*A*c^3)*x^
4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)*x^2)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4*
x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/8*(2*B*a^3*c - (C*a^2*c^2 + 3*A*a*c^3)*x
^3 - ((C*a*c^2 + 3*A*c^3)*x^4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)
*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4
*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

Sympy [A] time = 1.56703, size = 156, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Ac+Ca)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Ac+Ca)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{-2Ba^2+x^3\left(3Ac^2+Cac\right)+\dots}{8a^4c+16a^3c^2x^2+8\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(-a**3*c*sqrt(-1/(a**5*c**3)) + x)/1
6 + sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(a**3*c*sqrt(-1/(a**5*c**3)) + x)
/16 + (-2*B*a**2 + x**3*(3*A*c**2 + C*a*c) + x*(5*A*a*c - C*a**2))/(8*a**4*
c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

Giac [A] time = 1.15251, size = 113, normalized size = 1.15

$$\frac{(Ca+3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}+\frac{Cacx^3+3\,Ac^2x^3-Ca^2x+5\,Aacx-2\,Ba^2}{8\left(cx^2+a\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(C*a*c*x^3
+ 3*A*c^2*x^3 - C*a^2*x + 5*A*a*c*x - 2*B*a^2)/((c*x^2 + a)^2*a^2*c)
```

3.61 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=353

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)+a\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)(Cd-Be)\right)}{8a^{5/2}\sqrt{c}\left(ae^2+cd^2\right)^3}+\frac{4a^2e\left(Ae^2-Bde+Cd^2\right)+x\left(\right.}{8a^2\left(\right.}$$

```
[Out] -(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(4*a*c*(c*d^2 +
a*e^2)*(a + c*x^2)^2) + (4*a^2*e*(C*d^2 - B*d*e + A*e^2) + (a*(C*d - B*e)*(
c*d^2 - 3*a*e^2) + A*c*d*(3*c*d^2 + 7*a*e^2))*x)/(8*a^2*(c*d^2 + a*e^2)^2*(
a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + A*c*d
*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8
*a^(5/2)*Sqrt[c]*(c*d^2 + a*e^2)^3) + (e^3*(C*d^2 - B*d*e + A*e^2)*Log[d +
e*x])/(c*d^2 + a*e^2)^3 - (e^3*(C*d^2 - B*d*e + A*e^2)*Log[a + c*x^2])/(2*(
c*d^2 + a*e^2)^3)
```

Rubi [A] time = 0.734348, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1647, 823, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)+a\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)(Cd-Be)\right)}{8a^{5/2}\sqrt{c}\left(ae^2+cd^2\right)^3}+\frac{4a^2e\left(Ae^2-Bde+Cd^2\right)+x\left(\right.}{8a^2\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```


```
[Out] -(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(4*a*c*(c*d^2 +
a*e^2)*(a + c*x^2)^2) + (4*a^2*e*(C*d^2 - B*d*e + A*e^2) + (a*(C*d - B*e)*(
c*d^2 - 3*a*e^2) + A*c*d*(3*c*d^2 + 7*a*e^2))*x)/(8*a^2*(c*d^2 + a*e^2)^2*(
a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + A*c*d
*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8
*a^(5/2)*Sqrt[c]*(c*d^2 + a*e^2)^3) + (e^3*(C*d^2 - B*d*e + A*e^2)*Log[d +
e*x])/(c*d^2 + a*e^2)^3 - (e^3*(C*d^2 - B*d*e + A*e^2)*Log[a + c*x^2])/(2*(
c*d^2 + a*e^2)^3)
```


Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} - \frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(3cd^2+4ae^2\right)\right)}{cd^2+ae^2}-\frac{3ce(Acd-aCd+aBe)x}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)^2}dx}{4ac}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-Be)\left(cd^2 \cdots\right.\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a \cdots\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-Be)\left(cd^2 \cdots\right.\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a \cdots\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-Be)\left(cd^2 \cdots\right.\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a \cdots\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-Be)\left(cd^2 \cdots\right.\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a \cdots\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-Be)\left(cd^2 \cdots\right.\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a \cdots\right.}$$

Mathematica [A] time = 0.486328, size = 321, normalized size = 0.91

$$\frac{\frac{2\left(ae^2+cd^2\right)^2\left(a^2(-C)e+ac(Ae-Bd+Bex-Cdx)+Ac^2dx\right)}{ac\left(a+cx^2\right)^2}+\frac{\left(ae^2+cd^2\right)\left(a^2e(e(4Ae-4Bd+3Bex)+Cd(4d-3ex))+acdx\left(e(7Ae-Bd)+Cd^2\right)+3Ac^2d^3x\right)}{a^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(A \cdots}{\cdots}}{8\left(a \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] ((2*(c*d^2 + a*e^2)^2*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x +
 B*e*x))))/(a*c*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*A*c^2*d^3*x + a*c*d*(C*
d^2 + e*(-(B*d) + 7*A*e))*x + a^2*e*(C*d*(4*d - 3*e*x) + e*(-4*B*d + 4*A*e
+ 3*B*e*x))))/(a^2*(a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2
- 3*a^2*e^4) + A*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqr
t[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[
d + e*x] - 4*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/(8*(c*d^2 + a*e
```

```
^2)^3)
```

Maple [B] time = 0.069, size = 1598, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x)

[Out] 1/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c^2*A*d^4*e-1/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2
/c*C*a^3*e^5+3/8/(a*e^2+c*d^2)^3*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*e^
5-3/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*a^2*d*e^4+5/8/(a*e^2+c*d^2)^3/(c*x^2+a)
^2*a^2*e^5*B*x-1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*c^2*d^5*x+1/2/(a*e^2+c*d^2
)^3/(c*x^2+a)^2*C*x^2*a*c*d^2*e^3-3/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*a*c*d^3
*e^2*x-1/2/(a*e^2+c*d^2)^3*ln(c*x^2+a)*C*d^2*e^3-e^4/(a*e^2+c*d^2)^3*ln(e*x
+d)*B*d+e^3/(a*e^2+c*d^2)^3*ln(e*x+d)*C*d^2-1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2
*B*x^2*a*c*d*e^4-3/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*x^3*a*c*d*e^4+3/4/(a*e^2
+c*d^2)^3/(c*x^2+a)^2*B*a*c*d^2*e^3*x-1/8/(a*e^2+c*d^2)^3/a/(a*c)^(1/2)*arc
tan(x*c/(a*c)^(1/2))*B*c^2*d^4*e+5/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c^3/a*x^3*
A*d^3*e^2-1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c^3/a*x^3*B*d^4*e+9/8/(a*e^2+c*d^
2)^3/(c*x^2+a)^2*A*a*c*d*e^4*x+5/4/(a*e^2+c*d^2)^3/a/(a*c)^(1/2)*arctan(x*c
/(a*c)^(1/2))*A*c^2*d^3*e^2+1/2/(a*e^2+c*d^2)^3*ln(c*x^2+a)*B*d*e^4+3/4/(a*
e^2+c*d^2)^3/(c*x^2+a)^2*A*a^2*e^5-1/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c^2*B*d^
5+1/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^3*c^2*d^2*e^3+3/8/(a*e^2+c*d^2)^3/(c*
x^2+a)^2*c^4/a^2*x^3*A*d^5+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*x^2*a*c*e^5-1/
4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*x^3*c^2*d^3*e^2-1/2/(a*e^2+c*d^2)^3*ln(c*x^
2+a)*A*e^5+e^5/(a*e^2+c*d^2)^3*ln(e*x+d)*A+3/4/(a*e^2+c*d^2)^3/(a*c)^(1/2)*
arctan(x*c/(a*c)^(1/2))*C*c*d^3*e^2+3/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^3*a
*c*e^5+7/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*x^3*c^2*d*e^4+1/8/(a*e^2+c*d^2)^3/
(c*x^2+a)^2*c^3/a*x^3*C*d^5-1/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c*B*a*d^3*e^2+1/4
/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c*C*a*d^4*e+3/8/(a*e^2+c*d^2)^3/a^2/(a*c)^(1/2
)*arctan(x*c/(a*c)^(1/2))*A*c^3*d^5-3/4/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(
x*c/(a*c)^(1/2))*B*c*d^2*e^3-3/8/(a*e^2+c*d^2)^3*a/(a*c)^(1/2)*arctan(x*c/(
a*c)^(1/2))*C*d*e^4+1/8/(a*e^2+c*d^2)^3/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2
))*C*c^2*d^5+15/8/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*c*d
*e^4+1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*c^2*d^4*e*x-1/2/(a*e^2+c*d^2)^3/(c*x
^2+a)^2*B*x^2*c^2*d^3*e^2+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*x^2*c^2*d^2*e^3
+7/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*c^2*d^3*e^2*x+5/8/(a*e^2+c*d^2)^3/(c*x^2
+a)^2*x/a*A*c^3*d^5+1/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c*A*a*d^2*e^3-5/8/(a*e^2+
c*d^2)^3/(c*x^2+a)^2*C*a^2*d*e^4*x+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*x^2*c^
2*d^4*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18124, size = 965, normalized size = 2.73

$$-\frac{\left(Cd^2e^3 - Bde^4 + Ae^5\right)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} + \frac{\left(Cd^2e^4 - Bde^5 + Ae^6\right)\log\left(|xe + d|\right)}{c^3d^6e + 3\,ac^2d^4e^3 + 3\,a^2cd^2e^5 + a^3e^7} + \frac{\left(Cac^2d^5 + 3\,Ac^3d^5 - Bac^2d^4e + 6\,Ca^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^4 - B*d*e^5 + A*e^6)*log(abs(x*e
+ d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + 1/8*(C*a*
c^2*d^5 + 3*A*c^3*d^5 - B*a*c^2*d^4*e + 6*C*a^2*c*d^3*e^2 + 10*A*a*c^2*d^3*
e^2 - 6*B*a^2*c*d^2*e^3 - 3*C*a^3*d*e^4 + 15*A*a^2*c*d*e^4 + 3*B*a^3*e^5)*a
rctan(c*x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 +
a^5*e^6)*sqrt(a*c)) - 1/8*(2*B*a^2*c^3*d^5 - 2*C*a^3*c^2*d^4*e - 2*A*a^2*c^
3*d^4*e + 8*B*a^3*c^2*d^3*e^2 - 8*A*a^3*c^2*d^2*e^3 + 6*B*a^4*c*d*e^4 + 2*C
*a^5*e^5 - 6*A*a^4*c*e^5 - (C*a*c^4*d^5 + 3*A*c^5*d^5 - B*a*c^4*d^4*e - 2*C
*a^2*c^3*d^3*e^2 + 10*A*a*c^4*d^3*e^2 + 2*B*a^2*c^3*d^2*e^3 - 3*C*a^3*c^2*d
*e^4 + 7*A*a^2*c^3*d*e^4 + 3*B*a^3*c^2*e^5)*x^3 - 4*(C*a^2*c^3*d^4*e - B*a^
2*c^3*d^3*e^2 + C*a^3*c^2*d^2*e^3 + A*a^2*c^3*d^2*e^3 - B*a^3*c^2*d*e^4 + A
*a^3*c^2*e^5)*x^2 + (C*a^2*c^3*d^5 - 5*A*a*c^4*d^5 - B*a^2*c^3*d^4*e + 6*C*
a^3*c^2*d^3*e^2 - 14*A*a^2*c^3*d^3*e^2 - 6*B*a^3*c^2*d^2*e^3 + 5*C*a^4*c*d*
e^4 - 9*A*a^3*c^2*d*e^4 - 5*B*a^4*c*e^5)*x)/((c*d^2 + a*e^2)^3*(c*x^2 + a)^
2*a^2*c)
```

3.62 $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^3}\,dx$

Optimal. Leaf size=571

$$-\frac{4a^2e\left(ae^2(2Cd-Be)-cd\left(2Cd^2-e(3Bd-4Ae)\right)\right)-x\left(Ac\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+a\left(3a^2Ce^4-2acde^2(6Cd-7B\right.\right.}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

```
[Out] -((e^3*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^3*(d + e*x))) - (a*(B*c*d^
2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 -
c*d*(C*d - 2*B*e)))*x)/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^2)^2) - (4*a^2*e*(a*
e^2*(2*C*d - B*e) - c*d*(2*C*d^2 - e*(3*B*d - 4*A*e))) - (A*c*(3*c^2*d^4 +
12*a*c*d^2*e^2 - 7*a^2*e^4) + a*(3*a^2*C*e^4 - 2*a*c*d*e^2*(6*C*d - 7*B*e)
+ c^2*d^3*(C*d - 2*B*e)))*x)/(8*a^2*(c*d^2 + a*e^2)^3*(a + c*x^2)) + ((3*A*
c*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e
^6 + a*c^2*d^3*e^2*(13*C*d - 20*B*e) - 3*a^2*c*d*e^4*(11*C*d - 10*B*e) + c^
3*d^5*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c]*(c*d^
2 + a*e^2)^4) - (e^3*(a*e^2*(2*C*d - B*e) - c*d*(4*C*d^2 - e*(5*B*d - 6*A*e
)))*Log[d + e*x])/(c*d^2 + a*e^2)^4 + (e^3*(a*e^2*(2*C*d - B*e) - c*d*(4*C*
d^2 - e*(5*B*d - 6*A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rubi [A] time = 1.92477, antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$\frac{x\left(Ac\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+a\left(3a^2Ce^4-2acde^2(6Cd-7Be)+c^2d^3(Cd-2Be)\right)\right)+4a^2e\left(-ae^2(2Cd-Be)-cde(3\right.}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] -((e^3*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^3*(d + e*x))) - (a*(B*c*d^
2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 -
c*d*(C*d - 2*B*e)))*x)/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^2)^2) + (4*a^2*e*(2*
c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)) + (A*c*(3*c^2*d^4 +
12*a*c*d^2*e^2 - 7*a^2*e^4) + a*(3*a^2*C*e^4 - 2*a*c*d*e^2*(6*C*d - 7*B*e)
+ c^2*d^3*(C*d - 2*B*e)))*x)/(8*a^2*(c*d^2 + a*e^2)^3*(a + c*x^2)) + ((3*A*
c*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e
```

```
^6 + a*c^2*d^3*e^2*(13*C*d - 20*B*e) - 3*a^2*c*d*e^4*(11*C*d - 10*B*e) + c^
3*d^5*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c]*(c*d^
2 + a*e^2)^4) + (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*
e))*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*
e) - a*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3}\,dx = -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} - \int$$

$$= -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} + \frac{4a^2e}{}$$

$$= -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} + \frac{4a^2e}{}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-c\right.\right.}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-c\right.\right.}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-c\right.\right.}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

Mathematica [A] time = 0.817795, size = 498, normalized size = 0.87

$$\frac{\frac{2\left(ae^2+cd^2\right)^2\left(a^2e(Be-2Cd+Cex)-ac\left(Ae(ex-2d)+Bd(d-2ex)+Cd^2x\right)+Ac^2d^2x\right)}{a\left(a+cx^2\right)^2}+\frac{\left(ae^2+cd^2\right)\left(a^2ce\left(e(Ae(16d-7ex)-2Bd(6d-7ex))+4Cd^2(2d-3ex)\right)+a^3e^3(4Be-8Cd+\right.}{a^2\left(a+cx^2\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] ((-8*e^3*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + (2*(c*d^2
+ a*e^2)^2*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d
*(d - 2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*
A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 2*e*(-(B*d) + 6*A*e))*x + a^3*e^3*(-8*C*d
+ 4*B*e + 3*C*e*x) + a^2*c*e*(4*C*d^2*(2*d - 3*e*x) + e*(-2*B*d*(6*d - 7*e*
x) + A*e*(16*d - 7*e*x)))))/(a^2*(a + c*x^2)) + ((3*A*c*(c^3*d^6 + 5*a*c^2*
```

```
d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e^6 + a*c^2*d^3*e^2*(1
3*C*d - 20*B*e) + c^3*d^5*(C*d - 2*B*e) + 3*a^2*c*d*e^4*(-11*C*d + 10*B*e))
)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(4*c*C*d^3 + c*d*e
*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - 4*e^3*(4*c*C*d^3 +
 c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(8*(c*d^2 +
 a*e^2)^4)
```

Maple [B] time = 0.076, size = 2159, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x)
```

```
[Out] -e^5/(a*e^2+c*d^2)^3/(e*x+d)*A+3/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a^3*e^6-1/
4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*c^3*d^6-1/2/(a*e^2+c*d^2)^4*a*ln(c*x^2+a)*B
*e^6+e^6/(a*e^2+c*d^2)^4*ln(e*x+d)*B*a+e^4/(a*e^2+c*d^2)^3/(e*x+d)*B*d-e^3/
(a*e^2+c*d^2)^3/(e*x+d)*C*d^2+3/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^5/a^2*x^3*A
*d^6+1/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^4/a*x^3*C*d^6+13/8/(a*e^2+c*d^2)^4/(
a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*c^2*d^4*e^2+3/8/(a*e^2+c*d^2)^4/a^2/(a
*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*c^4*d^6-15/8/(a*e^2+c*d^2)^4*a/(a*c)^(1
/2)*arctan(x*c/(a*c)^(1/2))*A*c*e^6-1/4/(a*e^2+c*d^2)^4/a/(a*c)^(1/2)*arcta
n(x*c/(a*c)^(1/2))*B*c^3*d^5*e-33/8/(a*e^2+c*d^2)^4*a/(a*c)^(1/2)*arctan(x*
c/(a*c)^(1/2))*C*c*d^2*e^4-1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^2*a^2*c*d*e^5+
5/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a*c^2*d^3*e^3*x+15/8/(a*e^2+c*d^2)^4/a/(a
*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*c^3*d^4*e^2+15/4/(a*e^2+c*d^2)^4*a/(a*c
)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*c*d*e^5+9/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B
*a^2*c*d*e^5*x+2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*x^2*a*c^2*d*e^5-9/8/(a*e^2+c
*d^2)^4/(c*x^2+a)^2*C*x^3*a*c^2*d^2*e^4-1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^2
*a*c^2*d^2*e^4+7/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^3*a*c^2*d*e^5+3/8/(a*e^2
+c*d^2)^4/(c*x^2+a)^2*A*a*c^2*d^2*e^4*x-1/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^4
/a*x^3*B*d^5*e-13/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a*c^2*d^4*e^2*x+15/8/(a*e
^2+c*d^2)^4/(c*x^2+a)^2*c^4/a*x^3*A*d^4*e^2-7/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2
*C*a^2*c*d^2*e^4*x+6*e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*A*c*d-5*e^4/(a*e^2+c*d^2
)^4*ln(e*x+d)*B*c*d^2-2*e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*C*a*d+4*e^3/(a*e^2+c*
d^2)^4*ln(e*x+d)*C*c*d^3+3/8/(a*e^2+c*d^2)^4*a^2/(a*c)^(1/2)*arctan(x*c/(a*
c)^(1/2))*C*e^6-3/(a*e^2+c*d^2)^4*c*ln(c*x^2+a)*A*d*e^5+5/2/(a*e^2+c*d^2)^4
*c*ln(c*x^2+a)*B*d^2*e^4-2/(a*e^2+c*d^2)^4*c*ln(c*x^2+a)*C*d^3*e^3+5/8/(a*e
^2+c*d^2)^4/(c*x^2+a)^2*a^3*C*e^6*x-1/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*c^3*d
^6*x-3/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^3*d*e^5+1/2/(a*e^2+c*d^2)^4/(c*x^2
+a)^2*A*c^3*d^5*e+1/(a*e^2+c*d^2)^4*a*ln(c*x^2+a)*C*d*e^5+1/8/(a*e^2+c*d^2)
```

```
^4/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*c^3*d^6+5/8/(a*e^2+c*d^2)^4/(c*x
^2+a)^2*A*x^3*c^3*d^2*e^4-1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^2*c*d^3*e^3-3/4
/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a^2*c*d^2*e^4-7/4/(a*e^2+c*d^2)^4/(c*x^2+a)^
2*B*a*c^2*d^4*e^2-11/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^3*c^3*d^4*e^2+5/2/(a
*e^2+c*d^2)^4/(c*x^2+a)^2*A*a^2*c*d*e^5+1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^2
*c^3*d^5*e-9/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*a^2*A*c*e^6*x+1/4/(a*e^2+c*d^2)^
4/(c*x^2+a)^2*B*c^3*d^5*e*x-3/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^2*c^3*d^4*e
^2+3/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^3*a^2*c*e^6-7/8/(a*e^2+c*d^2)^4/(c*x
^2+a)^2*A*x^3*a*c^2*e^6+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a*c^2*d^5*e+3/(a*
e^2+c*d^2)^4/(c*x^2+a)^2*A*a*c^2*d^3*e^3+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*
x^2*a^2*c*e^6+5/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*x/a*A*c^4*d^6+3/2/(a*e^2+c*d^
2)^4/(c*x^2+a)^2*B*x^3*c^3*d^3*e^3+2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*x^2*c^3*
d^3*e^3+17/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*c^3*d^4*e^2*x+45/8/(a*e^2+c*d^2)
^4/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*c^2*d^2*e^4-5/2/(a*e^2+c*d^2)^4/(a
*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*c^2*d^3*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21416, size = 1494, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*c^3*d^6*e^2 + 3*A*c^4*d^6*e^2 - 2*B*a*c^3*d^5*e^3 + 13*C*a^2*c^2*d
^4*e^4 + 15*A*a*c^3*d^4*e^4 - 20*B*a^2*c^2*d^3*e^5 - 33*C*a^3*c*d^2*e^6 + 4
5*A*a^2*c^2*d^2*e^6 + 30*B*a^3*c*d*e^7 + 3*C*a^4*e^8 - 15*A*a^3*c*e^8)*arct
an((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a^2
*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^
8)*sqrt(a*c)) - 1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 - 2*C*a*d*e^5 + 6*A*c*d*
e^5 + B*a*e^6)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d
)^2)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4
*e^8) - (C*d^2*e^9/(x*e + d) - B*d*e^10/(x*e + d) + A*e^11/(x*e + d))/(c^3*
d^6*e^6 + 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10 + a^3*e^12) + 1/8*(C*a*c^4*d^5
*e + 3*A*c^5*d^5*e - 2*B*a*c^4*d^4*e^2 - 22*C*a^2*c^3*d^3*e^3 + 14*A*a*c^4*
d^3*e^3 + 32*B*a^2*c^3*d^2*e^4 + 17*C*a^3*c^2*d*e^5 - 29*A*a^2*c^3*d*e^5 -
6*B*a^3*c^2*e^6 - (3*C*a*c^4*d^6*e^2 + 9*A*c^5*d^6*e^2 - 6*B*a*c^4*d^5*e^3
- 77*C*a^2*c^3*d^4*e^4 + 41*A*a*c^4*d^4*e^4 + 116*B*a^2*c^3*d^3*e^5 + 77*C*
a^3*c^2*d^2*e^6 - 121*A*a^2*c^3*d^2*e^6 - 38*B*a^3*c^2*d*e^7 - 3*C*a^4*c*e^
8 + 7*A*a^3*c^2*e^8)*e^(-1)/(x*e + d) + (3*C*a*c^4*d^7*e^3 + 9*A*c^5*d^7*e^
3 - 6*B*a*c^4*d^6*e^4 - 89*C*a^2*c^3*d^5*e^5 + 45*A*a*c^4*d^5*e^5 + 140*B*a
^2*c^3*d^4*e^6 + 85*C*a^3*c^2*d^3*e^7 - 145*A*a^2*c^3*d^3*e^7 - 22*B*a^3*c^
2*d^2*e^8 + 17*C*a^4*c*d*e^9 - 21*A*a^3*c^2*d*e^9 - 8*B*a^4*c*e^10)*e^(-2)/
(x*e + d)^2 - (C*a*c^4*d^8*e^4 + 3*A*c^5*d^8*e^4 - 2*B*a*c^4*d^7*e^5 - 34*C
*a^2*c^3*d^6*e^6 + 18*A*a*c^4*d^6*e^6 + 58*B*a^2*c^3*d^5*e^7 + 20*C*a^3*c^2
*d^4*e^8 - 60*A*a^2*c^3*d^4*e^8 + 26*B*a^3*c^2*d^3*e^9 + 50*C*a^4*c*d^2*e^1
0 - 66*A*a^3*c^2*d^2*e^10 - 34*B*a^4*c*d*e^11 - 5*C*a^5*e^12 + 9*A*a^4*c*e^
12)*e^(-3)/(x*e + d)^3)/((c*d^2 + a*e^2)^4*a^2*(c - 2*c*d/(x*e + d) + c*d^2
/(x*e + d)^2 + a*e^2/(x*e + d)^2)^2)
```

3.63 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=753

$$\frac{cx\left(3Acd\left(-11a^2e^4+6acd^2e^2+c^2d^4\right)-a\left(-7a^2e^4(3Cd-Be)+2acd^2e^2(13Cd-19Be)-c^2d^4(Cd-3Be)\right)\right)+4a^2e\left(a^2Ce^4\right.}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^4}$$

```
[Out] -(e^3*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^3*(d + e*x)^2) + (e^3*(a*
e^2*(2*C*d - B*e) - c*d*(4*C*d^2 - e*(5*B*d - 6*A*e))))/((c*d^2 + a*e^2)^4*
(d + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))
 - c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e
)))*x)/(4*a*(c*d^2 + a*e^2)^3*(a + c*x^2)^2) + (4*a^2*e*(a^2*C*e^4 + c^2*d^
2*(3*C*d^2 - 2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))
+ c*(3*A*c*d*(c^2*d^4 + 6*a*c*d^2*e^2 - 11*a^2*e^4) - a*(2*a*c*d^2*e^2*(13*
C*d - 19*B*e) - c^2*d^4*(C*d - 3*B*e) - 7*a^2*e^4*(3*C*d - B*e)))*x)/(8*a^2
*(c*d^2 + a*e^2)^4*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) -
 5*a^2*c*d^2*e^4*(25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) + 15*a^3*e^6*(3*
C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*(c*d^2 + a*e^2)^5) + (
e^3*(a^2*C*e^4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*d^2*(10*C*d^2
 - 3*e*(5*B*d - 7*A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^5 - (e^3*(a^2*C*e^4
- a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*d^2*(10*C*d^2 - 3*e*(5*B*d -
 7*A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^5)
```

Rubi [A] time = 3.14256, antiderivative size = 753, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$\frac{cx\left(3Acd\left(-11a^2e^4+6acd^2e^2+c^2d^4\right)-a\left(-7a^2e^4(3Cd-Be)+2acd^2e^2(13Cd-19Be)-c^2d^4(Cd-3Be)\right)\right)+4a^2e\left(a^2Ce^4\right.}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] -(e^3*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^3*(d + e*x)^2) - (e^3*(4*
c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^4*
(d + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))
```

```
 - c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e
)))*x)/(4*a*(c*d^2 + a*e^2)^3*(a + c*x^2)^2) + (4*a^2*e*(a^2*C*e^4 + c^2*(3
*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))
+ c*(3*A*c*d*(c^2*d^4 + 6*a*c*d^2*e^2 - 11*a^2*e^4) - a*(2*a*c*d^2*e^2*(13*
C*d - 19*B*e) - c^2*d^4*(C*d - 3*B*e) - 7*a^2*e^4*(3*C*d - B*e)))*x)/(8*a^2
*(c*d^2 + a*e^2)^4*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) -
 5*a^2*c*d^2*e^4*(25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) + 15*a^3*e^6*(3*
C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*(c*d^2 + a*e^2)^5) + (
e^3*(a^2*C*e^4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3
*d^2*e*(5*B*d - 7*A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^5 - (e^3*(a^2*C*e^4
- a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3*d^2*e*(5*B*d -
 7*A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^5)
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
```

```
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx = -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-\right.\right.}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-\right.\right.}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-\right.\right.}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{e^3\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a\left(Bcd\left(cd^2-3\right.\right.}{}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{e^3\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a\left(Bcd\left(cd^2-3\right.\right.}{}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{e^3\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a\left(Bcd\left(cd^2-3\right.\right.}{}$$

Mathematica [A] time = 1.22617, size = 672, normalized size = 0.89

$$\frac{\frac{2\left(ae^2+cd^2\right)^2\left(-a^2ce(e(Ae-3Bd+Bex)+3Cd(d-ex))+a^3Ce^3-ac^2d\left(3Ae(ex-d)+Bd(d-3ex)+Cd^2x\right)+Ac^3d^3x\right)}{a\left(a+cx^2\right)^2}+\frac{\left(ae^2+cd^2\right)\left(a^2c^2de\left(e\left(40Ade-33Ae^2x-24Bd^2+38Bdex\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] ((-4*e^3*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 - (8*e^3
*(c*d^2 + a*e^2)*(4*c*C*d^3 + c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e)
```

```
))/(d + e*x) + (2*(c*d^2 + a*e^2)^2*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d
^2*x + B*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(
-3*B*d + A*e + B*e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(4*a^4*C*e^5
+ 3*A*c^4*d^5*x + a*c^3*d^3*(C*d^2 + 3*e*(-(B*d) + 6*A*e))*x + a^3*c*e^3*(C
*d*(-32*d + 21*e*x) + e*(24*B*d - 8*A*e - 7*B*e*x)) + a^2*c^2*d*e*(2*C*d^2*
(6*d - 13*e*x) + e*(-24*B*d^2 + 40*A*d*e + 38*B*d*e*x - 33*A*e^2*x))))/(a^2
*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 35*a^2*c*d^2
*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) - 5*a^2*c*d^2*e^4*(
25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) - 15*a^3*e^6*(-3*C*d + B*e)))*ArcT
an[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 8*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*
d*e - 3*A*e^2) + c^2*d^2*e^3*(10*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[d + e*x
] - 4*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*d*e - 3*A*e^2) + c^2*d^2*e^3*(1
0*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[a + c*x^2])/(8*(c*d^2 + a*e^2)^5)
```

Maple [B] time = 0.081, size = 2737, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x)
```

```
[Out] -e^6/(a*e^2+c*d^2)^4/(e*x+d)*B*a+1/2*e^4/(a*e^2+c*d^2)^3/(e*x+d)^2*B*d-1/2*
e^3/(a*e^2+c*d^2)^3/(e*x+d)^2*C*d^2+e^7/(a*e^2+c*d^2)^5*ln(e*x+d)*a^2*C+3/4
/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*a^4*e^7+9*e^6/(a*e^2+c*d^2)^5*ln(e*x+d)*B*a*
c*d+1/2*c/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*x^2*a^3*e^7+15/4*c/(a*e^2+c*d^2)^5/
(c*x^2+a)^2*B*a^3*d*e^6+3/8*c^6/(a*e^2+c*d^2)^5/(c*x^2+a)^2/a^2*x^3*A*d^7+1
/8*c^5/(a*e^2+c*d^2)^5/(c*x^2+a)^2/a*x^3*C*d^7+3/2*c^4/(a*e^2+c*d^2)^5/(c*x
^2+a)^2*C*x^2*d^6*e+19/8*c^4/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A*d^5*e^2*x+3/8*c^
4/(a*e^2+c*d^2)^5/(c*x^2+a)^2*B*d^6*e*x+5*c^4/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A
*x^2*d^4*e^3-3*c^4/(a*e^2+c*d^2)^5/(c*x^2+a)^2*B*x^2*d^5*e^2-25/8*c^4/(a*e^
2+c*d^2)^5/(c*x^2+a)^2*C*x^3*d^5*e^2+5/8*c^5/(a*e^2+c*d^2)^5/(c*x^2+a)^2*x/
a*A*d^7+17/4*c^2/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A*a^2*d^2*e^5+25/4*c^3/(a*e^2+
c*d^2)^5/(c*x^2+a)^2*A*a*d^4*e^3+5/4*c^2/(a*e^2+c*d^2)^5/(c*x^2+a)^2*B*a^2*
d^3*e^4-11/4*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^2*B*a*d^5*e^2-15/4*c^2/(a*e^2+c*
d^2)^5/(c*x^2+a)^2*C*a^2*d^4*e^3+3/4*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*a*d^
6*e-9/2*c/(a*e^2+c*d^2)^5*a*ln(c*x^2+a)*B*d*e^6+13/2*c/(a*e^2+c*d^2)^5*a*ln
(c*x^2+a)*C*d^2*e^5-15/4*c/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*a^3*d^2*e^5-15/8*c
/(a*e^2+c*d^2)^5*a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*e^7+105/8*c^3/(a
*e^2+c*d^2)^5/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d^3*e^4+23/8*c^3/(a*e^2
+c*d^2)^5/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*d^5*e^2-45/8*c^3/(a*e^2+c*d
^2)^5/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^4*e^3+3/8*c^5/(a*e^2+c*d^2)^5
```

```
/a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d^7+1/8*c^4/(a*e^2+c*d^2)^5/a/(a
*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*d^7-13*e^5/(a*e^2+c*d^2)^5*ln(e*x+d)*C*
a*c*d^2+21/8*c^2/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*x^3*a^2*d*e^6+5/8*c^2/(a*e^2
+c*d^2)^5/(c*x^2+a)^2*C*a^2*d^3*e^4*x+31/8*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^2*
B*x^3*a*d^2*e^5-7/2*c^2/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*x^2*a^2*d^2*e^5-5/8*c
^3/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*x^3*a*d^3*e^4-39/8*c^2/(a*e^2+c*d^2)^5/(c*
x^2+a)^2*A*a^2*d*e^6*x+3*c^2/(a*e^2+c*d^2)^5/(c*x^2+a)^2*B*x^2*a^2*d*e^6-33
/8*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A*x^3*a*d*e^6-23/8*c^3/(a*e^2+c*d^2)^5/(
c*x^2+a)^2*C*a*d^5*e^2*x+21/8*c^5/(a*e^2+c*d^2)^5/(c*x^2+a)^2/a*x^3*A*d^5*e
^2-25/8*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A*a*d^3*e^4*x+33/8*c^2/(a*e^2+c*d^2
)^5/(c*x^2+a)^2*B*a^2*d^2*e^5*x+4*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A*x^2*a*d
^2*e^5-5/2*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*x^2*a*d^4*e^3+21/8*c^4/(a*e^2+
c*d^2)^5/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d^5*e^2+27/8*c/(a*e^2+c*d^
2)^5/(c*x^2+a)^2*C*a^3*d*e^6*x+45/8*c/(a*e^2+c*d^2)^5*a^2/(a*c)^(1/2)*arcta
n(x*c/(a*c)^(1/2))*C*d*e^6-3/8*c^5/(a*e^2+c*d^2)^5/(c*x^2+a)^2/a*x^3*B*d^6*
e+45/8*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^2*B*a*d^4*e^3*x-125/8*c^2/(a*e^2+c*d^2
)^5*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*d^3*e^4-105/8*c^2/(a*e^2+c*d^2)
^5*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d*e^6+135/8*c^2/(a*e^2+c*d^2)^5*
a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^2*e^5-3/8*c^4/(a*e^2+c*d^2)^5/a/(
a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^6*e-1/2*e^5/(a*e^2+c*d^2)^3/(e*x+d)^
2*A-c^2/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A*x^2*a^2*e^7-1/4*c^4/(a*e^2+c*d^2)^5/(
c*x^2+a)^2*B*d^7-1/2/(a*e^2+c*d^2)^5*a^2*ln(c*x^2+a)*C*e^7+35/8*c^4/(a*e^2+
c*d^2)^5/(c*x^2+a)^2*B*x^3*d^4*e^3-15/8*c^4/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A*x
^3*d^3*e^4-7/8*c^2/(a*e^2+c*d^2)^5/(c*x^2+a)^2*B*x^3*a^2*e^7-9/8*c/(a*e^2+c
*d^2)^5/(c*x^2+a)^2*a^3*e^7*B*x+21*e^5/(a*e^2+c*d^2)^5*ln(e*x+d)*A*c^2*d^2-
15*e^4/(a*e^2+c*d^2)^5*ln(e*x+d)*B*c^2*d^3+10*e^3/(a*e^2+c*d^2)^5*ln(e*x+d)
*C*c^2*d^4-4*e^3/(a*e^2+c*d^2)^4/(e*x+d)*C*c*d^3-3*e^7/(a*e^2+c*d^2)^5*ln(e
*x+d)*A*a*c+2*e^5/(a*e^2+c*d^2)^4/(e*x+d)*C*a*d+15/2*c^2/(a*e^2+c*d^2)^5*ln
(c*x^2+a)*B*d^3*e^4-5*c^2/(a*e^2+c*d^2)^5*ln(c*x^2+a)*C*d^4*e^3-6*e^5/(a*e^
2+c*d^2)^4/(e*x+d)*A*c*d+5*e^4/(a*e^2+c*d^2)^4/(e*x+d)*B*c*d^2-21/2*c^2/(a*
e^2+c*d^2)^5*ln(c*x^2+a)*A*d^2*e^5-5/4*c/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A*a^3*
e^7+3/2*c/(a*e^2+c*d^2)^5*a*ln(c*x^2+a)*A*e^7-1/8*c^4/(a*e^2+c*d^2)^5/(c*x^
2+a)^2*C*d^7*x+3/4*c^4/(a*e^2+c*d^2)^5/(c*x^2+a)^2*A*d^6*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2198, size = 2068, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 - 13*C*a*c*d^2*e^5 + 21*A*c^2*d^2
*e^5 + 9*B*a*c*d*e^6 + C*a^2*e^7 - 3*A*a*c*e^7)*log(c*x^2 + a)/(c^5*d^10 +
5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8
 + a^5*e^10) + (10*C*c^2*d^4*e^4 - 15*B*c^2*d^3*e^5 - 13*C*a*c*d^2*e^6 + 21
*A*c^2*d^2*e^6 + 9*B*a*c*d*e^7 + C*a^2*e^8 - 3*A*a*c*e^8)*log(abs(x*e + d))
/(c^5*d^10*e + 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 + 10*a^3*c^2*d^4*e^7 +
5*a^4*c*d^2*e^9 + a^5*e^11) + 1/8*(C*a*c^4*d^7 + 3*A*c^5*d^7 - 3*B*a*c^4*d^
```

```
6*e + 23*C*a^2*c^3*d^5*e^2 + 21*A*a*c^4*d^5*e^2 - 45*B*a^2*c^3*d^4*e^3 - 12
5*C*a^3*c^2*d^3*e^4 + 105*A*a^2*c^3*d^3*e^4 + 135*B*a^3*c^2*d^2*e^5 + 45*C*
a^4*c*d*e^6 - 105*A*a^3*c^2*d*e^6 - 15*B*a^4*c*e^7)*arctan(c*x/sqrt(a*c))/(
(a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6
 + 5*a^6*c*d^2*e^8 + a^7*e^10)*sqrt(a*c)) + 1/8*(C*a*c^4*d^5*x^5*e^2 + 3*A*
c^5*d^5*x^5*e^2 + 2*C*a*c^4*d^6*x^4*e + 6*A*c^5*d^6*x^4*e + C*a*c^4*d^7*x^3
 + 3*A*c^5*d^7*x^3 - 3*B*a*c^4*d^4*x^5*e^3 - 6*B*a*c^4*d^5*x^4*e^2 - 3*B*a*
c^4*d^6*x^3*e - 58*C*a^2*c^3*d^3*x^5*e^4 + 18*A*a*c^4*d^3*x^5*e^4 - 76*C*a^
2*c^3*d^4*x^4*e^3 + 36*A*a*c^4*d^4*x^4*e^3 - 3*C*a^2*c^3*d^5*x^3*e^2 + 23*A
*a*c^4*d^5*x^3*e^2 + 10*C*a^2*c^3*d^6*x^2*e + 10*A*a*c^4*d^6*x^2*e - C*a^2*
c^3*d^7*x + 5*A*a*c^4*d^7*x + 78*B*a^2*c^3*d^2*x^5*e^5 + 96*B*a^2*c^3*d^3*x
^4*e^4 - 7*B*a^2*c^3*d^4*x^3*e^3 - 20*B*a^2*c^3*d^5*x^2*e^2 - B*a^2*c^3*d^6
*x*e - 2*B*a^2*c^3*d^7 + 37*C*a^3*c^2*d*x^5*e^6 - 81*A*a^2*c^3*d*x^5*e^6 +
22*C*a^3*c^2*d^2*x^4*e^5 - 78*A*a^2*c^3*d^2*x^4*e^5 - 129*C*a^3*c^2*d^3*x^3
*e^4 + 61*A*a^2*c^3*d^3*x^3*e^4 - 142*C*a^3*c^2*d^4*x^2*e^3 + 74*A*a^2*c^3*
d^4*x^2*e^3 - 10*C*a^3*c^2*d^5*x*e^2 + 26*A*a^2*c^3*d^5*x*e^2 + 6*C*a^3*c^2
*d^6*e + 6*A*a^2*c^3*d^6*e - 15*B*a^3*c^2*x^5*e^7 + 6*B*a^3*c^2*d*x^4*e^6 +
 163*B*a^3*c^2*d^2*x^3*e^5 + 176*B*a^3*c^2*d^3*x^2*e^4 + 2*B*a^3*c^2*d^4*x*
e^3 - 20*B*a^3*c^2*d^5*e^2 + 4*C*a^4*c*x^4*e^7 - 12*A*a^3*c^2*x^4*e^7 + 67*
C*a^4*c*d*x^3*e^6 - 151*A*a^3*c^2*d*x^3*e^6 + 46*C*a^4*c*d^2*x^2*e^5 - 146*
A*a^3*c^2*d^2*x^2*e^5 - 77*C*a^4*c*d^3*x*e^4 + 49*A*a^3*c^2*d^3*x*e^4 - 72*
C*a^4*c*d^4*e^3 + 44*A*a^3*c^2*d^4*e^3 - 25*B*a^4*c*x^3*e^7 + 4*B*a^4*c*d*x
^2*e^6 + 91*B*a^4*c*d^2*x*e^5 + 74*B*a^4*c*d^3*e^4 + 6*C*a^5*x^2*e^7 - 18*A
*a^4*c*x^2*e^7 + 28*C*a^5*d*x*e^6 - 68*A*a^4*c*d*x*e^6 + 18*C*a^5*d^2*e^5 -
 62*A*a^4*c*d^2*e^5 - 8*B*a^5*x*e^7 - 4*B*a^5*d*e^6 - 4*A*a^5*e^7)/((a^2*c^
4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*
(c*x^3*e + c*d*x^2 + a*x*e + a*d)^2)
```

3.64 $\int \frac{(d+ex)^4\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$

Optimal. Leaf size=234

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac)\right)}{16a^{7/2}c^{7/2}}-\frac{(d+ex)(ae-cdx)\left(cd(4aBe+aCd+5Acd)+ae^2\right.}{16a^3c^3\left(a+cx^2\right)}$$

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^4)/(6*a*c*(a + c*x^2)^3) - ((d + e*x)^3*(
a*(A*c + 5*a*C)*e - c*(5*A*c*d + a*C*d + 4*a*B*e)*x))/(24*a^2*c^2*(a + c*x^
2)^2) - ((a*(A*c + 5*a*C)*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*(a*e - c*d
*x)*(d + e*x))/(16*a^3*c^3*(a + c*x^2)) + ((c*d^2 + a*e^2)*(a*(A*c + 5*a*C)
*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^
(7/2)*c^(7/2))
```

Rubi [A] time = 0.291445, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 805, 723, 205}

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac)\right)}{16a^{7/2}c^{7/2}}-\frac{(d+ex)(ae-cdx)\left(cd(4aBe+aCd+5Acd)+ae^2\right.}{16a^3c^3\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^4)/(6*a*c*(a + c*x^2)^3) - ((d + e*x)^3*(
a*(A*c + 5*a*C)*e - c*(5*A*c*d + a*C*d + 4*a*B*e)*x))/(24*a^2*c^2*(a + c*x^
2)^2) - ((a*(A*c + 5*a*C)*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*(a*e - c*d
*x)*(d + e*x))/(16*a^3*c^3*(a + c*x^2)) + ((c*d^2 + a*e^2)*(a*(A*c + 5*a*C)
*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^
(7/2)*c^(7/2))
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
```

```
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^4\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)^3(-5Acd-aCd-4aBe-(Ac+5aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}$$

Mathematica [A] time = 0.305285, size = 437, normalized size = 1.87

$$\frac{a^2ce^2x\left(e(Ae+4Bd)+6Cd^2\right)-a^3e^3(8Be+32Cd+11Cex)+ac^2d^2x\left(6Ae^2+4Bde+Cd^2\right)+5Ac^3d^4x}{16a^3c^3\left(a+cx^2\right)} + \frac{a^2ce\left(e(Ae(4d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^2*c*e^2*(6*C*d
^2 + e*(4*B*d + A*e))*x - a^3*e^3*(32*C*d + 8*B*e + 11*C*e*x))/(16*a^3*c^3*
(a + c*x^2)) + (A*c^3*d^4*x - a^3*e^3*(4*C*d + B*e + C*e*x) - a*c^2*d^2*(4*
A*d*e + C*d^2*x + 6*A*e^2*x + B*d*(d + 4*e*x)) + a^2*c*e*(2*C*d^2*(2*d + 3*
e*x) + e*(A*e*(4*d + e*x) + 2*B*d*(3*d + 2*e*x))))/(6*a*c^3*(a + c*x^2)^3)
+ (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^3*e^3*(48*C*
d + 12*B*e + 13*C*e*x) - a^2*c*e*(6*C*d^2*(4*d + 7*e*x) + e*(4*B*d*(9*d + 7
*e*x) + A*e*(24*d + 7*e*x))))/(24*a^2*c^3*(a + c*x^2)^2) + ((c*d^2 + a*e^2)
*(A*c*(5*c*d^2 + a*e^2) + a*(5*a*C*e^2 + c*d*(C*d + 4*B*e)))*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/(16*a^(7/2)*c^(7/2))
```

Maple [B] time = 0.056, size = 647, normalized size = 2.8

$$\frac{1}{\left(cx^2+a\right)^3}\left(\frac{\left(Ae^4a^2c+6\,Aac^2d^2e^2+5\,Ac^3d^4+4\,Bda^2e^3c+4\,Bac^2d^3e-11\,a^3Ce^4+6\,Ca^2cd^2e^2+Cac^2d^4\right)x^5}{16\,a^3c} - \frac{e^3\left(Be+\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*
d^3*e-11*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c*x^5-1/2*e^3*(B*e+4*
C*d)/c*x^4-1/6*(A*a^2*c*e^4-6*A*a*c^2*d^2*e^2-5*A*c^3*d^4+4*B*a^2*c*d*e^3-4
*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2-C*a*c^2*d^4)/a^2/c^2*x^3-1/2*e
*(2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e+4*C*a*d*e^2+2*C*c*d^3)/c^2*x^2-1/16*(A*a^
2*c*e^4+6*A*a*c^2*d^2*e^2-11*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*d^3*e+5*C*
a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a/c^3*x-1/6*(2*A*a*c*d*e^3+4*A*c^2*d
^3*e+B*a^2*e^4+3*B*a*c*d^2*e^2+B*c^2*d^4+4*C*a^2*d*e^3+2*C*a*c*d^3*e)/c^3)/
(c*x^2+a)^3+1/16/a/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*e^4+3/8/a^2/c/
(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d^2*e^2+5/16/a^3/(a*c)^(1/2)*arctan(x
*c/(a*c)^(1/2))*A*d^4+1/4/a/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d*e^3
+1/4/a^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^3*e+5/16/c^3/(a*c)^(1/2)
*arctan(x*c/(a*c)^(1/2))*C*e^4+3/8/a/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2)
)*C*d^2*e^2+1/16/a^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71336, size = 3753, normalized size = 16.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^3*d^4 + 48*B*a^5*c^2*d^2*e^2 + 16*B*a^6*c*e^4 - 6*(4*B*a
^2*c^5*d^3*e + 4*B*a^3*c^4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3*c
```

```
^4 + A*a^2*c^5)*d^2*e^2 - (11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 32*(C*a^5*c
^2 + 2*A*a^4*c^3)*d^3*e + 32*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 48*(4*C*a^4*c^
3*d*e^3 + B*a^4*c^3*e^4)*x^4 - 16*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e^3 +
(C*a^3*c^4 + 5*A*a^2*c^5)*d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (5*C*a^
5*c^2 + A*a^4*c^3)*e^4)*x^3 + 48*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^2*e^2 +
 B*a^5*c^2*e^4 + 2*(2*C*a^5*c^2 + A*a^4*c^3)*d*e^3)*x^2 + 3*(4*B*a^4*c^2*d^
3*e + 4*B*a^5*c*d*e^3 + (4*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*a*c^5 + 5
*A*c^6)*d^4 + 6*(C*a^2*c^4 + A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*a^2*c^4)*e
^4)*x^6 + (C*a^4*c^2 + 5*A*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2)*d^2*e^2 +
 (5*C*a^6 + A*a^5*c)*e^4 + 3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d*e^3 + (C*a^
2*c^4 + 5*A*a*c^5)*d^4 + 6*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (5*C*a^4*c^2 +
 A*a^3*c^3)*e^4)*x^4 + 3*(4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e^3 + (C*a^3*c^
3 + 5*A*a^2*c^4)*d^4 + 6*(C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (5*C*a^5*c + A*a
^4*c^2)*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))
+ 6*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4*c^3 - 11*A*a^3*c^4)*d^4
 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c + A*a^5*c^2)*e^4)*x)/(a^4
*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4), -1/48*(8*B*a^4*c^3*d^4
 + 24*B*a^5*c^2*d^2*e^2 + 8*B*a^6*c*e^4 - 3*(4*B*a^2*c^5*d^3*e + 4*B*a^3*c^
4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3*c^4 + A*a^2*c^5)*d^2*e^2 -
 (11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 16*(C*a^5*c^2 + 2*A*a^4*c^3)*d^3*e +
 16*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 24*(4*C*a^4*c^3*d*e^3 + B*a^4*c^3*e^4)*
x^4 - 8*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e^3 + (C*a^3*c^4 + 5*A*a^2*c^5)*
d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (5*C*a^5*c^2 + A*a^4*c^3)*e^4)*x^
3 + 24*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^2*e^2 + B*a^5*c^2*e^4 + 2*(2*C*a^
5*c^2 + A*a^4*c^3)*d*e^3)*x^2 - 3*(4*B*a^4*c^2*d^3*e + 4*B*a^5*c*d*e^3 + (4
*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*a*c^5 + 5*A*c^6)*d^4 + 6*(C*a^2*c^4
 + A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*a^2*c^4)*e^4)*x^6 + (C*a^4*c^2 + 5*A
*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2)*d^2*e^2 + (5*C*a^6 + A*a^5*c)*e^4 +
 3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^4 + 6
*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (5*C*a^4*c^2 + A*a^3*c^3)*e^4)*x^4 + 3*(
4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e^3 + (C*a^3*c^3 + 5*A*a^2*c^4)*d^4 + 6*(
C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (5*C*a^5*c + A*a^4*c^2)*e^4)*x^2)*sqrt(a*c
)*arctan(sqrt(a*c)*x/a) + 3*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4
*c^3 - 11*A*a^3*c^4)*d^4 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c +
 A*a^5*c^2)*e^4)*x)/(a^4*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16659, size = 859, normalized size = 3.67

$$\frac{\left(Cac^2d^4 + 5\,Ac^3d^4 + 4\,Bac^2d^3e + 6\,Ca^2cd^2e^2 + 6\,Aac^2d^2e^2 + 4\,Ba^2cde^3 + 5\,Ca^3e^4 + Aa^2ce^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^3} + \frac{3\,Cac^4d^4x^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c^2*d^4 + 5*A*c^3*d^4 + 4*B*a*c^2*d^3*e + 6*C*a^2*c*d^2*e^2 + 6*A
*a*c^2*d^2*e^2 + 4*B*a^2*c*d*e^3 + 5*C*a^3*e^4 + A*a^2*c*e^4)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^3) + 1/48*(3*C*a*c^4*d^4*x^5 + 15*A*c^5*d^4*x^5 +
 12*B*a*c^4*d^3*x^5*e + 18*C*a^2*c^3*d^2*x^5*e^2 + 18*A*a*c^4*d^2*x^5*e^2 +
 8*C*a^2*c^3*d^4*x^3 + 40*A*a*c^4*d^4*x^3 + 12*B*a^2*c^3*d*x^5*e^3 + 32*B*a
^2*c^3*d^3*x^3*e - 33*C*a^3*c^2*x^5*e^4 + 3*A*a^2*c^3*x^5*e^4 - 96*C*a^3*c^
2*d*x^4*e^3 - 48*C*a^3*c^2*d^2*x^3*e^2 + 48*A*a^2*c^3*d^2*x^3*e^2 - 48*C*a^
3*c^2*d^3*x^2*e - 3*C*a^3*c^2*d^4*x + 33*A*a^2*c^3*d^4*x - 24*B*a^3*c^2*x^4
*e^4 - 32*B*a^3*c^2*d*x^3*e^3 - 72*B*a^3*c^2*d^2*x^2*e^2 - 12*B*a^3*c^2*d^3
*x*e - 8*B*a^3*c^2*d^4 - 40*C*a^4*c*x^3*e^4 - 8*A*a^3*c^2*x^3*e^4 - 96*C*a^
4*c*d*x^2*e^3 - 48*A*a^3*c^2*d*x^2*e^3 - 18*C*a^4*c*d^2*x*e^2 - 18*A*a^3*c^
2*d^2*x*e^2 - 16*C*a^4*c*d^3*e - 32*A*a^3*c^2*d^3*e - 24*B*a^4*c*x^2*e^4 -
12*B*a^4*c*d*x*e^3 - 24*B*a^4*c*d^2*e^2 - 15*C*a^5*x*e^4 - 3*A*a^4*c*x*e^4
- 32*C*a^5*d*e^3 - 16*A*a^4*c*d*e^3 - 8*B*a^5*e^4)/((c*x^2 + a)^3*a^3*c^3)
```

3.65 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$

Optimal. Leaf size=254

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+5cd^2\right)+a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)}{16a^{7/2}c^{5/2}}-\frac{(d+ex)\left(ae(3aBe+aCd+5Acd)-x\left(3cd(3a\right.\right.}{48a^3c^2\left(a+\right.}$$

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(6*a*c*(a + c*x^2)^3) - ((d + e*x)^2*(
2*a*(A*c + 2*a*C)*e - c*(5*A*c*d + a*C*d + 3*a*B*e)*x))/(24*a^2*c^2*(a + c*
x^2)^2) - ((d + e*x)*(a*e*(5*A*c*d + a*C*d + 3*a*B*e) - (4*a*(A*c + 2*a*C)*
e^2 + 3*c*d*(5*A*c*d + a*C*d + 3*a*B*e))*x))/(48*a^3*c^2*(a + c*x^2)) + ((A
*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*A
rcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

Rubi [A] time = 0.541702, antiderivative size = 288, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 821, 778, 205}

$$-\frac{4ae\left(Ac\left(ae^2+5cd^2\right)+a\left(2aCe^2+cd(3Be+Cd)\right)\right)-cx\left(Acd\left(15cd^2-ae^2\right)+a\left(ae^2(7Cd-3Be)+3cd^2(3Be+Cd)\right)\right)}{48a^3c^3\left(a+cx^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(6*a*c*(a + c*x^2)^3) - ((d + e*x)^2*(
2*a*(A*c + 2*a*C)*e - c*(5*A*c*d + a*C*d + 3*a*B*e)*x))/(24*a^2*c^2*(a + c*
x^2)^2) - (4*a*e*(A*c*(5*c*d^2 + a*e^2) + a*(2*a*C*e^2 + c*d*(C*d + 3*B*e))
) - c*(A*c*d*(15*c*d^2 - a*e^2) + a*(a*e^2*(7*C*d - 3*B*e) + 3*c*d^2*(C*d +
 3*B*e)))*x)/(48*a^3*c^3*(a + c*x^2)) + ((A*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*
e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*
a^(7/2)*c^(5/2))
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
```

```
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)^2(-5Acd-aCd-3aBe-2(Ac+2aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}$$

Mathematica [A] time = 0.321771, size = 350, normalized size = 1.38

$$-\frac{8a^{5/2}\left(-a^2ce(e(Ae+3Bd+Bex)+3Cd(d+ex))+a^3Ce^3+ac^2d\left(3Ae(d+ex)+Bd(d+3ex)+Cd^2x\right)-Ac^3d^3x\right)}{\left(a+cx^2\right)^3} + \frac{2a^{3/2}\left(-a^2ce(e(6Ae+18Bd+7Bex)+3Cd(6d+7ex))+12a^3C\right.}{\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((-3*Sqrt[a]*(8*a^3*C*e^3 - 5*A*c^3*d^3*x - a^2*c*e^2*(3*C*d + B*e)*x - a*c
^2*d*(C*d^2 + 3*e*(B*d + A*e))*x))/(a + c*x^2) - (8*a^(5/2)*(a^3*C*e^3 - A*
c^3*d^3*x + a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) - a^2*c*e
*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x))))/(a + c*x^2)^3 + (2*a^(3/2)*(
12*a^3*C*e^3 + 5*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d + A*e))*x - a^2*c*
e*(3*C*d*(6*d + 7*e*x) + e*(18*B*d + 6*A*e + 7*B*e*x))))/(a + c*x^2)^2 + 3*
Sqrt[c]*(A*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d +
3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^3)
```

Maple [A] time = 0.053, size = 464, normalized size = 1.8

$$\frac{1}{\left(cx^2+a\right)^3}\left(\frac{\left(3\,Acde^2a+5\,Ad^3c^2+a^2Be^3+3\,Bcd^2ae+3\,Ca^2de^2+Cacd^3\right)x^5}{16\,a^3} - \frac{Ce^3x^4}{2\,c} + \frac{\left(3\,Acde^2a+5\,Ad^3c^2-a^2Be^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(3*A*a*c*d*e^2+5*A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a*
c*d^3)/a^3*x^5-1/2*C*e^3*x^4/c+1/6*(3*A*a*c*d*e^2+5*A*c^2*d^3-B*a^2*e^3+3*B
*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/a^2/c*x^3-1/4*e*(A*c*e^2+3*B*c*d*e+2*C*
a*e^2+3*C*c*d^2)/c^2*x^2-1/16*(3*A*a*c*d*e^2-11*A*c^2*d^3+B*a^2*e^3+3*B*a*c
*d^2*e+3*C*a^2*d*e^2+C*a*c*d^3)/a/c^2*x-1/12*(A*a*c*e^3+6*A*c^2*d^2*e+3*B*a
*c*d*e^2+2*B*c^2*d^3+2*C*a^2*e^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^3+3/16/a^2/c
/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d*e^2+5/16/a^3/(a*c)^(1/2)*arctan(x*
c/(a*c)^(1/2))*A*d^3+1/16/a/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*e^3+3
/16/a^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d^2*e+3/16/a/c^2/(a*c)^(1/2
)*arctan(x*c/(a*c)^(1/2))*C*d*e^2+1/16/a^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(
1/2))*C*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.6375, size = 2807, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(48*C*a^4*c^2*e^3*x^4 + 16*B*a^4*c^2*d^3 + 24*B*a^5*c*d*e^2 - 6*(3*B
*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^3 + 3*(C*a^3*c^3
 + A*a^2*c^4)*d*e^2)*x^5 + 24*(C*a^5*c + 2*A*a^4*c^2)*d^2*e + 8*(2*C*a^6 +
A*a^5*c)*e^3 - 16*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*a^3*c^3 + 5*A*a^2
*c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 24*(3*C*a^4*c^2*d^2*e +
```

```
3*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 + 3*(3*B*a^4*c*d^2*e +
 B*a^5*e^3 + (3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3
*(C*a^2*c^3 + A*a*c^4)*d*e^2)*x^6 + 3*(3*B*a^2*c^3*d^2*e + B*a^3*c^2*e^3 +
(C*a^2*c^3 + 5*A*a*c^4)*d^3 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x^4 + (C*a^4
*c + 5*A*a^3*c^2)*d^3 + 3*(C*a^5 + A*a^4*c)*d*e^2 + 3*(3*B*a^3*c^2*d^2*e +
B*a^4*c*e^3 + (C*a^3*c^2 + 5*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2
)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(3*B*a^
4*c^2*d^2*e + B*a^5*c*e^3 + (C*a^4*c^2 - 11*A*a^3*c^3)*d^3 + 3*(C*a^5*c + A
*a^4*c^2)*d*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)
, -1/48*(24*C*a^4*c^2*e^3*x^4 + 8*B*a^4*c^2*d^3 + 12*B*a^5*c*d*e^2 - 3*(3*B
*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^3 + 3*(C*a^3*c^3
 + A*a^2*c^4)*d*e^2)*x^5 + 12*(C*a^5*c + 2*A*a^4*c^2)*d^2*e + 4*(2*C*a^6 +
A*a^5*c)*e^3 - 8*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*a^3*c^3 + 5*A*a^2*
c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 12*(3*C*a^4*c^2*d^2*e + 3
*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 - 3*(3*B*a^4*c*d^2*e +
B*a^5*e^3 + (3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3*
(C*a^2*c^3 + A*a*c^4)*d*e^2)*x^6 + 3*(3*B*a^2*c^3*d^2*e + B*a^3*c^2*e^3 + (
C*a^2*c^3 + 5*A*a*c^4)*d^3 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x^4 + (C*a^4*
c + 5*A*a^3*c^2)*d^3 + 3*(C*a^5 + A*a^4*c)*d*e^2 + 3*(3*B*a^3*c^2*d^2*e + B
*a^4*c*e^3 + (C*a^3*c^2 + 5*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2)
*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(3*B*a^4*c^2*d^2*e + B*a^5*c*e^3
+ (C*a^4*c^2 - 11*A*a^3*c^3)*d^3 + 3*(C*a^5*c + A*a^4*c^2)*d*e^2)*x)/(a^4*c
^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18846, size = 641, normalized size = 2.52

$$\frac{\left(Cacd^3 + 5\,Ac^2d^3 + 3\,Bacd^2e + 3\,Ca^2de^2 + 3\,Aacde^2 + Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2} + \frac{3\,Cac^4d^3x^5 + 15\,Ac^5d^3x^5 + 9\,Bac^4d^2x^5e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^3 + 5*A*c^2*d^3 + 3*B*a*c*d^2*e + 3*C*a^2*d*e^2 + 3*A*a*c*d*e
^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^4
*d^3*x^5 + 15*A*c^5*d^3*x^5 + 9*B*a*c^4*d^2*x^5*e + 9*C*a^2*c^3*d*x^5*e^2 +
 9*A*a*c^4*d*x^5*e^2 + 8*C*a^2*c^3*d^3*x^3 + 40*A*a*c^4*d^3*x^3 + 3*B*a^2*c
^3*x^5*e^3 + 24*B*a^2*c^3*d^2*x^3*e - 24*C*a^3*c^2*x^4*e^3 - 24*C*a^3*c^2*d
*x^3*e^2 + 24*A*a^2*c^3*d*x^3*e^2 - 36*C*a^3*c^2*d^2*x^2*e - 3*C*a^3*c^2*d^
3*x + 33*A*a^2*c^3*d^3*x - 8*B*a^3*c^2*x^3*e^3 - 36*B*a^3*c^2*d*x^2*e^2 - 9
*B*a^3*c^2*d^2*x*e - 8*B*a^3*c^2*d^3 - 24*C*a^4*c*x^2*e^3 - 12*A*a^3*c^2*x^
2*e^3 - 9*C*a^4*c*d*x*e^2 - 9*A*a^3*c^2*d*x*e^2 - 12*C*a^4*c*d^2*e - 24*A*a
^3*c^2*d^2*e - 3*B*a^4*c*x*e^3 - 12*B*a^4*c*d*e^2 - 8*C*a^5*e^3 - 4*A*a^4*c
*e^3)/((c*x^2 + a)^3*a^3*c^3)
```

3.66 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$

Optimal. Leaf size=225

$$\frac{x\left(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac)\right)}{16a^3c^2\left(a+cx^2\right)}-\frac{x\left(3ae^2(aC+Ac)-cd(2aBe+aCd+5Acd)\right)+2ae(aBe+2aCd+4Acd}{24a^2c^2\left(a+cx^2\right)^2}$$

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(6*a*c*(a + c*x^2)^3) - (2*a*e*(4*A*c*
d + 2*a*C*d + a*B*e) + (3*a*(A*c + a*C)*e^2 - c*d*(5*A*c*d + a*C*d + 2*a*B*
e))*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((a*(A*c + a*C)*e^2 + c*d*(5*A*c*d + a*
C*d + 2*a*B*e))*x)/(16*a^3*c^2*(a + c*x^2)) + ((a*(A*c + a*C)*e^2 + c*d*(5*
A*c*d + a*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

Rubi [A] time = 0.397508, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 778, 199, 205}

$$\frac{x\left(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac)\right)}{16a^3c^2\left(a+cx^2\right)}-\frac{x\left(3ae^2(aC+Ac)-cd(2aBe+aCd+5Acd)\right)+2ae(aBe+2aCd+4Acd}{24a^2c^2\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(6*a*c*(a + c*x^2)^3) - (2*a*e*(4*A*c*
d + 2*a*C*d + a*B*e) + (3*a*(A*c + a*C)*e^2 - c*d*(5*A*c*d + a*C*d + 2*a*B*
e))*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((a*(A*c + a*C)*e^2 + c*d*(5*A*c*d + a*
C*d + 2*a*B*e))*x)/(16*a^3*c^2*(a + c*x^2)) + ((a*(A*c + a*C)*e^2 + c*d*(5*
A*c*d + a*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
```

```
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)(-5Acd-aCd-2aBe-3(Ac+aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2-cd(5Acd\right.}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2-cd(5Acd\right.}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2-cd(5Acd\right.}{24a^2c^2\left(a+cx^2\right)^2}$$

Mathematica [A] time = 0.169364, size = 266, normalized size = 1.18

$$\frac{x\left(Ac\left(ae^2+5cd^2\right)+a\left(aCe^2+cd(2Be+Cd)\right)\right)}{16a^3c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(ae^2+5cd^2\right)+a\left(aCe^2+cd(2Be+Cd)\right)\right)}{16a^{7/2}c^{5/2}}+\frac{a^2(-e)(6$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x)/(16*a^3*c^2*(
a + c*x^2)) + (5*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(12*
C*d + 6*B*e + 7*C*e*x))/(24*a^2*c^2*(a + c*x^2)^2) + (A*c^2*d^2*x + a^2*e*(
2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(
6*a*c^2*(a + c*x^2)^3) + ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d +
2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

Maple [A] time = 0.053, size = 333, normalized size = 1.5

$$\frac{1}{\left(cx^2+a\right)^3}\left(\frac{\left(aAe^2c+5\,Ac^2d^2+2\,Bacde+a^2Ce^2+Cacd^2\right)x^5}{16\,a^3}+\frac{\left(aAe^2c+5\,Ac^2d^2+2\,Bacde-a^2Ce^2+Cacd^2\right)x^3}{6\,a^2c}-\frac{e(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)/a^3*x^5+1/6*(
A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e-C*a^2*e^2+C*a*c*d^2)/a^2/c*x^3-1/4*e*(B*e
+2*C*d)*x^2/c-1/16*(A*a*c*e^2-11*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)
/a/c^2*x-1/12*(4*A*c*d*e+B*a*e^2+2*B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^3+1/16
/a^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*e^2+5/16/a^3/(a*c)^(1/2)*arcta
n(x*c/(a*c)^(1/2))*A*d^2+1/8/a^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*d*
e+1/16/a/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*e^2+1/16/a^2/c/(a*c)^(1/
2)*arctan(x*c/(a*c)^(1/2))*C*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.55657, size = 2163, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^2*d^2 + 8*B*a^5*c*e^2 - 6*(2*B*a^2*c^4*d*e + (C*a^2*c^4
+ 5*A*a*c^5)*d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 16*(2*B*a^3*c^3*d*e +
 (C*a^3*c^3 + 5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 16*(C*a
^5*c + 2*A*a^4*c^2)*d*e + 24*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 + 3*(2*B
*a^4*c*d*e + (2*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A*a*c^
4)*e^2)*x^6 + 3*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a^3*c^2
 + A*a^2*c^3)*e^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^4*c)*e^
2 + 3*(2*B*a^3*c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c + A*a^3*c
^2)*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*
(2*B*a^4*c^2*d*e + (C*a^4*c^2 - 11*A*a^3*c^3)*d^2 + (C*a^5*c + A*a^4*c^2)*e
^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3), -1/48*(8*B*
a^4*c^2*d^2 + 4*B*a^5*c*e^2 - 3*(2*B*a^2*c^4*d*e + (C*a^2*c^4 + 5*A*a*c^5)*
d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 8*(2*B*a^3*c^3*d*e + (C*a^3*c^3 +
5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 8*(C*a^5*c + 2*A*a^4*
c^2)*d*e + 12*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 - 3*(2*B*a^4*c*d*e + (2
*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A*a*c^4)*e^2)*x^6 + 3
*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a^3*c^2 + A*a^2*c^3)*e
^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^4*c)*e^2 + 3*(2*B*a^3*
c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c + A*a^3*c^2)*e^2)*x^2)*s
qrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(2*B*a^4*c^2*d*e + (C*a^4*c^2 - 11*A*a^3
*c^3)*d^2 + (C*a^5*c + A*a^4*c^2)*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*
a^6*c^4*x^2 + a^7*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15398, size = 443, normalized size = 1.97

$$\frac{\left(Cacd^2 + 5\,Ac^2d^2 + 2\,Bacde + Ca^2e^2 + Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2} + \frac{3\,Cac^3d^2x^5 + 15\,Ac^4d^2x^5 + 6\,Bac^3dx^5e + 3\,Ca^2c^2x^5e^2 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^2 + 5*A*c^2*d^2 + 2*B*a*c*d*e + C*a^2*e^2 + A*a*c*e^2)*arctan
(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^3*d^2*x^5 + 15*A*c^4*d^
2*x^5 + 6*B*a*c^3*d*x^5*e + 3*C*a^2*c^2*x^5*e^2 + 3*A*a*c^3*x^5*e^2 + 8*C*a
^2*c^2*d^2*x^3 + 40*A*a*c^3*d^2*x^3 + 16*B*a^2*c^2*d*x^3*e - 8*C*a^3*c*x^3*
e^2 + 8*A*a^2*c^2*x^3*e^2 - 24*C*a^3*c*d*x^2*e - 3*C*a^3*c*d^2*x + 33*A*a^2
*c^2*d^2*x - 12*B*a^3*c*x^2*e^2 - 6*B*a^3*c*d*x*e - 8*B*a^3*c*d^2 - 3*C*a^4
*x*e^2 - 3*A*a^3*c*x*e^2 - 8*C*a^4*d*e - 16*A*a^3*c*d*e - 4*B*a^4*e^2)/((c*
x^2 + a)^3*a^3*c^2)
```

3.67 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4} \, dx$

Optimal. Leaf size=165

$$-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{24a^2c^2\left(a+cx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+5Acd)}{16a^{7/2}c^{3/2}}+\frac{x(aBe+aCd+5Acd)}{16a^3c\left(a+cx^2\right)}-\frac{(d+ex)(aB-x}{6ac\left(a+\right.}$$

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(6*a*c*(a + c*x^2)^3) - (2*a*(2*A*c + a*
C)*e - c*(5*A*c*d + a*C*d + a*B*e)*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((5*A*c*
d + a*C*d + a*B*e)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c*d + a*C*d + a*B*e)*A
rcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rubi [A] time = 0.135809, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1645, 639, 199, 205}

$$-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{24a^2c^2\left(a+cx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+5Acd)}{16a^{7/2}c^{3/2}}+\frac{x(aBe+aCd+5Acd)}{16a^3c\left(a+cx^2\right)}-\frac{(d+ex)(aB-x}{6ac\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(6*a*c*(a + c*x^2)^3) - (2*a*(2*A*c + a*
C)*e - c*(5*A*c*d + a*C*d + a*B*e)*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((5*A*c*
d + a*C*d + a*B*e)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c*d + a*C*d + a*B*e)*A
rcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
```

```
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{-5Acd-a(Cd+Be)-2(2Ac+aC)ex}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5Acd+a\cdots}{\cdots}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5Acd+a\cdots}{16a^3c\left(\cdots\right.}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5Acd+a\cdots}{16a^3c\left(\cdots\right.}$$

Mathematica [A] time = 0.161578, size = 171, normalized size = 1.04

$$\frac{\frac{8a^{5/2}\left(a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx\right)}{\left(a+cx^2\right)^3}+\frac{2a^{3/2}\left(-6a^2Ce+acx(Be+Cd)+5Ac^2dx\right)}{\left(a+cx^2\right)^2}+\frac{3\sqrt{a}cx(aBe+aCd+5Acd)}{a+cx^2}+3\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+5}{48a^{7/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((2*a^(3/2)*(-6*a^2*C*e + 5*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2)^2 +
 (3*Sqrt[a]*c*(5*A*c*d + a*C*d + a*B*e)*x)/(a + c*x^2) + (8*a^(5/2)*(a^2*C*
e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2)^3 + 3*Sqrt[c]
*(5*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^2)
```

Maple [A] time = 0.049, size = 182, normalized size = 1.1

$$\frac{1}{\left(cx^2+a\right)^3}\left(\frac{(5\,Acd+aBe+Cad)\,cx^5}{16\,a^3}+\frac{(5\,Acd+aBe+Cad)\,x^3}{6\,a^2}-\frac{Cex^2}{4\,c}+\frac{(11\,Acd-aBe-Cad)\,x}{16\,ac}-\frac{2\,Ace+2\,Bcd+aC}{12\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(5*A*c*d+B*a*e+C*a*d)/a^3*c*x^5+1/6/a^2*(5*A*c*d+B*a*e+C*a*d)*x^3-1/4
*C*e*x^2/c+1/16*(11*A*c*d-B*a*e-C*a*d)/a/c*x-1/12*(2*A*c*e+2*B*c*d+C*a*e)/c
^2)/(c*x^2+a)^3+5/16/a^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A*d+1/16/a^2/c
/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*e+1/16/a^2/c/(a*c)^(1/2)*arctan(x*c/
(a*c)^(1/2))*C*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.4554, size = 1341, normalized size = 8.13

$$\left[-\frac{24\,Ca^4cex^2+16\,Ba^4cd-6\left(Ba^2c^3e+\left(Ca^2c^3+5\,Aac^4\right)d\right)x^5-16\left(Ba^3c^2e+\left(Ca^3c^2+5\,Aa^2c^3\right)d\right)x^3+3\left(\left(Bac^3e+\left(Ca\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(24*C*a^4*c*e*x^2 + 16*B*a^4*c*d - 6*(B*a^2*c^3*e + (C*a^2*c^3 + 5*A
*a*c^4)*d)*x^5 - 16*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 + 3*((B
*a*c^3*e + (C*a*c^3 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^2*c
^2 + 5*A*a*c^3)*d)*x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2 + (C
*a^4 + 5*A*a^3*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a
)) + 8*(C*a^5 + 2*A*a^4*c)*e + 6*(B*a^4*c*e + (C*a^4*c - 11*A*a^3*c^2)*d)*x
)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(12*C*a^4*
c*e*x^2 + 8*B*a^4*c*d - 3*(B*a^2*c^3*e + (C*a^2*c^3 + 5*A*a*c^4)*d)*x^5 - 8
*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 - 3*((B*a*c^3*e + (C*a*c^3
 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*a*c^3)*d)*
x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2 + (C*a^4 + 5*A*a^3*c)*d
)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 4*(C*a^5 + 2*A*a^4*c)*e + 3*(B*a^4*c*e
+ (C*a^4*c - 11*A*a^3*c^2)*d)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x
^2 + a^7*c^2)]
```

Sympy [A] time = 127.102, size = 298, normalized size = 1.81

$$-\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Acd+Bae+Cad)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Acd+Bae+Cad)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{-8Aa^3ce-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**3))*(5*A*c*d + B*a*e + C*a*d)*log(-a**4*c*sqrt(-1/(a**7*c
**3)) + x)/32 + sqrt(-1/(a**7*c**3))*(5*A*c*d + B*a*e + C*a*d)*log(a**4*c*s
qrt(-1/(a**7*c**3)) + x)/32 + (-8*A*a**3*c*e - 8*B*a**3*c*d - 4*C*a**4*e -
```

```
12*C*a**3*c*e*x**2 + x**5*(15*A*c**4*d + 3*B*a*c**3*e + 3*C*a*c**3*d) + x**
3*(40*A*a*c**3*d + 8*B*a**2*c**2*e + 8*C*a**2*c**2*d) + x*(33*A*a**2*c**2*d
 - 3*B*a**3*c*e - 3*C*a**3*c*d))/(48*a**6*c**2 + 144*a**5*c**3*x**2 + 144*a
**4*c**4*x**4 + 48*a**3*c**5*x**6)
```

Giac [A] time = 1.15865, size = 262, normalized size = 1.59

$$\frac{(Cad + 5\,Acd + Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c} + \frac{3\,Cac^3dx^5 + 15\,Ac^4dx^5 + 3\,Bac^3x^5e + 8\,Ca^2c^2dx^3 + 40\,Aac^3dx^3 + 8\,Ba^2c^2x^3e - 12\,C}{48\,\left(cx^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*d + 5*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/
48*(3*C*a*c^3*d*x^5 + 15*A*c^4*d*x^5 + 3*B*a*c^3*x^5*e + 8*C*a^2*c^2*d*x^3
+ 40*A*a*c^3*d*x^3 + 8*B*a^2*c^2*x^3*e - 12*C*a^3*c*x^2*e - 3*C*a^3*c*d*x +
 33*A*a^2*c^2*d*x - 3*B*a^3*c*x*e - 8*B*a^3*c*d - 4*C*a^4*e - 8*A*a^3*c*e)/
((c*x^2 + a)^3*a^3*c^2)
```

3.68 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4}\,dx$

Optimal. Leaf size=126

$$\frac{(aC+5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{x(aC+5Ac)}{16a^3c\left(a+cx^2\right)}+\frac{x(aC+5Ac)}{24a^2c\left(a+cx^2\right)^2}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

```
[Out] -(a*B - (A*c - a*C)*x)/(6*a*c*(a + c*x^2)^3) + ((5*A*c + a*C)*x)/(24*a^2*c*
(a + c*x^2)^2) + ((5*A*c + a*C)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c + a*C)*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rubi [A] time = 0.078232, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1814, 12, 199, 205}

$$\frac{(aC+5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{x(aC+5Ac)}{16a^3c\left(a+cx^2\right)}+\frac{x(aC+5Ac)}{24a^2c\left(a+cx^2\right)^2}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(6*a*c*(a + c*x^2)^3) + ((5*A*c + a*C)*x)/(24*a^2*c*
(a + c*x^2)^2) + ((5*A*c + a*C)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c + a*C)*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4}\,dx &= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}-\frac{\int \frac{-5A-\frac{aC}{c}}{\left(a+cx^2\right)^3}\,dx}{6a}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)\int \frac{1}{\left(a+cx^2\right)^3}\,dx}{6ac}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2}+\frac{(5Ac+aC)\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{8a^2c}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2}+\frac{(5Ac+aC)x}{16a^3c\left(a+cx^2\right)}+\frac{(5Ac+aC)\int \frac{1}{a+cx^2}\,dx}{16a^3c}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2}+\frac{(5Ac+aC)x}{16a^3c\left(a+cx^2\right)}+\frac{(5Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0885953, size = 112, normalized size = 0.89

$$\frac{a^2cx\left(33A+8Cx^2\right)-a^3(8B+3Cx)+ac^2x^3\left(40A+3Cx^2\right)+15Ac^3x^5}{48a^3c\left(a+cx^2\right)^3}+\frac{(aC+5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] (15*A*c^3*x^5 - a^3*(8*B + 3*C*x) + a*c^2*x^3*(40*A + 3*C*x^2) + a^2*c*x*(3
3*A + 8*C*x^2))/(48*a^3*c*(a + c*x^2)^3) + ((5*A*c + a*C)*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Maple [A] time = 0.05, size = 113, normalized size = 0.9

$$\frac{1}{\left(cx^2+a\right)^3}\left(\frac{(5\,Ac+aC)\,cx^5}{16\,a^3}+\frac{(5\,Ac+aC)\,x^3}{6\,a^2}+\frac{(11\,Ac-aC)\,x}{16\,ac}-\frac{B}{6\,c}\right)+\frac{5\,A}{16\,a^3}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{C}{16\,a^2c}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(5*A*c+C*a)/a^3*c*x^5+1/6/a^2*(5*A*c+C*a)*x^3+1/16*(11*A*c-C*a)/a/c*x
-1/6*B/c)/(c*x^2+a)^3+5/16/a^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*A+1/16/a
^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.00482, size = 900, normalized size = 7.14

$$\left[-\frac{16\,Ba^4c-6\left(Ca^2c^3+5\,Aac^4\right)x^5-16\left(Ca^3c^2+5\,Aa^2c^3\right)x^3+3\left(\left(Cac^3+5\,Ac^4\right)x^6+Ca^4+5\,Aa^3c+3\left(Ca^2c^2+5\,Aac\right.\right.}{96\left(a^4c^5x^6+3\,a^5c^4x^4+3\,a^6c^3x^2+a^7c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c - 6*(C*a^2*c^3 + 5*A*a*c^4)*x^5 - 16*(C*a^3*c^2 + 5*A*a^
2*c^3)*x^3 + 3*((C*a*c^3 + 5*A*c^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2
+ 5*A*a*c^3)*x^4 + 3*(C*a^3*c + 5*A*a^2*c^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2
*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a^4*c - 11*A*a^3*c^2)*x)/(a^4*c^5*x^
6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(8*B*a^4*c - 3*(C*a^2*c
^3 + 5*A*a*c^4)*x^5 - 8*(C*a^3*c^2 + 5*A*a^2*c^3)*x^3 - 3*((C*a*c^3 + 5*A*c
^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2 + 5*A*a*c^3)*x^4 + 3*(C*a^3*c +
5*A*a^2*c^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(C*a^4*c - 11*A*a^3*c
^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

Sympy [A] time = 2.22933, size = 196, normalized size = 1.56

$$-\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Ac+Ca)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Ac+Ca)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{-8Ba^3+x^5\left(15Ac^3+3Cac^2\right)}{48a^6c+144a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(-a**4*c*sqrt(-1/(a**7*c**3)) + x)/3
2 + sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(a**4*c*sqrt(-1/(a**7*c**3)) + x)
/32 + (-8*B*a**3 + x**5*(15*A*c**3 + 3*C*a*c**2) + x**3*(40*A*a*c**2 + 8*C*
a**2*c) + x*(33*A*a**2*c - 3*C*a**3))/(48*a**6*c + 144*a**5*c**2*x**2 + 144
*a**4*c**3*x**4 + 48*a**3*c**4*x**6)
```

Giac [A] time = 1.15811, size = 147, normalized size = 1.17

$$\frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c}+\frac{3\,Cac^2x^5+15\,Ac^3x^5+8\,Ca^2cx^3+40\,Aac^2x^3-3\,Ca^3x+33\,Aa^2cx-8\,Ba^3}{48\left(cx^2+a\right)^3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*(3*C*a*c^
2*x^5 + 15*A*c^3*x^5 + 8*C*a^2*c*x^3 + 40*A*a*c^2*x^3 - 3*C*a^3*x + 33*A*a^
2*c*x - 8*B*a^3)/((c*x^2 + a)^3*a^3*c)
```

3.69 $\int \frac{x^3(1+x+x^2)}{(1+x^2)^2} dx$

Optimal. Leaf size=43

$$-\frac{x^3}{2(x^2+1)}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)$$

```
[Out] (3*x)/2 + x^2/2 - x^3/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rubi [A] time = 0.0500244, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1804, 801, 635, 203, 260}

$$-\frac{x^3}{2(x^2+1)}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] (3*x)/2 + x^2/2 - x^3/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(1+x+x^2\right)}{\left(1+x^2\right)^2}\,dx &= -\frac{x^3}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{(-3-2x)x^2}{1+x^2}\,dx\\
&= -\frac{x^3}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-3-2x+\frac{3+2x}{1+x^2}\right)dx\\
&= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{3+2x}{1+x^2}\,dx\\
&= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{3}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0178352, size = 29, normalized size = 0.67

$$\frac{1}{2}\left(x\left(\frac{1}{x^2+1}+x+2\right)-\log\left(x^2+1\right)-3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] (x*(2 + x + (1 + x^2)^(-1)) - 3*ArcTan[x] - Log[1 + x^2])/2
```

Maple [A] time = 0.046, size = 30, normalized size = 0.7

$$x+\frac{x^{2}}{2}+\frac{x}{2x^{2}+2}-\frac{\ln\left(x^{2}+1\right)}{2}-\frac{3\arctan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+x+1)/(x^2+1)^2,x)
```

```
[Out] x+1/2*x^2+1/2*x/(x^2+1)-1/2*ln(x^2+1)-3/2*arctan(x)
```

Maxima [A] time = 1.47725, size = 39, normalized size = 0.91

$$\frac{1}{2}x^{2}+x+\frac{x}{2\left(x^{2}+1\right)}-\frac{3}{2}\arctan\left(x\right)-\frac{1}{2}\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.974796, size = 122, normalized size = 2.84

$$\frac{x^{4}+2x^{3}+x^{2}-3\left(x^{2}+1\right)\arctan\left(x\right)-\left(x^{2}+1\right)\log\left(x^{2}+1\right)+3x}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 + 2*x^3 + x^2 - 3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 3
*x)/(x^2 + 1)
```

Sympy [A] time = 0.119279, size = 29, normalized size = 0.67

$$\frac{x^2}{2} + x + \frac{x}{2x^2 + 2} - \frac{\log\left(x^2 + 1\right)}{2} - \frac{3\,\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] x**2/2 + x + x/(2*x**2 + 2) - log(x**2 + 1)/2 - 3*atan(x)/2
```

Giac [A] time = 1.15523, size = 39, normalized size = 0.91

$$\frac{1}{2}\,x^2 + x + \frac{x}{2\left(x^2 + 1\right)} - \frac{3}{2}\,\arctan\,(x) - \frac{1}{2}\,\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

3.70 $\int \frac{x^2(1+x+x^2)}{(1+x^2)^2} dx$

Optimal. Leaf size=30

$$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}\log(x^2+1)+x-\tan^{-1}(x)$$

```
[Out] x - x^2/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.0395859, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1804, 774, 635, 203, 260}

$$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}\log(x^2+1)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] x - x^2/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(1+x+x^2\right)}{\left(1+x^2\right)^2}\,dx &= -\frac{x^2}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{(-2-2x)x}{1+x^2}\,dx\\ &= x-\frac{x^2}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{2-2x}{1+x^2}\,dx\\ &= x-\frac{x^2}{2\left(1+x^2\right)}-\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= x-\frac{x^2}{2\left(1+x^2\right)}-\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0101956, size = 27, normalized size = 0.9

$$\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] x + 1/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Maple [A] time = 0.061, size = 24, normalized size = 0.8

$$x+\frac{1}{2\,x^{2}+2}+\frac{\ln\left(x^{2}+1\right)}{2}-\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)/(x^2+1)^2,x)
```

```
[Out] x+1/2/(x^2+1)+1/2*ln(x^2+1)-arctan(x)
```

Maxima [A] time = 1.52833, size = 31, normalized size = 1.03

$$x+\frac{1}{2\left(x^{2}+1\right)}-\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.984418, size = 111, normalized size = 3.7

$$\frac{2\,x^{3}-2\left(x^{2}+1\right)\arctan\left(x\right)+\left(x^{2}+1\right)\log\left(x^{2}+1\right)+2\,x+1}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^3 - 2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x + 1)/(x^2
 + 1)
```

Sympy [A] time = 0.110712, size = 20, normalized size = 0.67

$$x+\frac{\log\left(x^{2}+1\right)}{2}-\operatorname{atan}\left(x\right)+\frac{1}{2x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] x + log(x**2 + 1)/2 - atan(x) + 1/(2*x**2 + 2)
```

Giac [A] time = 1.15406, size = 31, normalized size = 1.03

$$x + \frac{1}{2\left(x^2 + 1\right)} - \arctan\left(x\right) + \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

3.71 $\int \frac{x(1+x+x^2)}{(1+x^2)^2} dx$

Optimal. Leaf size=29

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[1 + x^2]/2
```

Rubi [A] time = 0.0247806, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1804, 635, 203, 260}

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(1+x+x^2\right)}{\left(1+x^2\right)^2}\,dx &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-1-2x}{1+x^2}\,dx\\ &= -\frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0084116, size = 23, normalized size = 0.79

$$\frac{1}{2}\left(-\frac{x}{x^2+1}+\log\left(x^2+1\right)+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x + x^2))/(1 + x^2)^2,x]

[Out] (-(x/(1 + x^2)) + ArcTan[x] + Log[1 + x^2])/2
```

Maple [A] time = 0.046, size = 24, normalized size = 0.8

$$-\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x+1)/(x^2+1)^2,x)
```

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)+1/2*ln(x^2+1)
```

Maxima [A] time = 1.46719, size = 31, normalized size = 1.07

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.989002, size = 89, normalized size = 3.07

$$\frac{\left(x^2+1\right)\arctan\left(x\right)+\left(x^2+1\right)\log\left(x^2+1\right)-x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - x)/(x^2 + 1)
```

Sympy [A] time = 0.114108, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2+2}+\frac{\log\left(x^2+1\right)}{2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 + atan(x)/2
```

Giac [A] time = 1.1516, size = 31, normalized size = 1.07

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

3.72 $\int \frac{1+x+x^2}{(1+x^2)^2}\,dx$

Optimal. Leaf size=14

$$\tan^{-1}(x) - \frac{1}{2(x^2+1)}$$

[Out] -1/(2*(1 + x^2)) + ArcTan[x]

Rubi [A] time = 0.0109896, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1814, 12, 203}

$$\tan^{-1}(x) - \frac{1}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(1 + x^2)^2,x]

[Out] -1/(2*(1 + x^2)) + ArcTan[x]

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{\left(1+x^2\right)^2}\,dx &= -\frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int -\frac{2}{1+x^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}+\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0067723, size = 14, normalized size = 1.

$$\tan^{-1}(x)-\frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -1/(2*(1 + x^2)) + ArcTan[x]
```

Maple [A] time = 0.045, size = 13, normalized size = 0.9

$$-\frac{1}{2\,x^2+2}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2+1)^2,x)
```

```
[Out] -1/2/(x^2+1)+arctan(x)
```

Maxima [A] time = 1.4843, size = 16, normalized size = 1.14

$$-\frac{1}{2\left(x^2+1\right)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 + 1) + arctan(x)
```

Fricas [A] time = 0.980056, size = 58, normalized size = 4.14

$$\frac{2\left(x^2+1\right)\arctan\left(x\right)-1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

Sympy [A] time = 0.104805, size = 10, normalized size = 0.71

$$\operatorname{atan}\left(x\right)-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] atan(x) - 1/(2*x**2 + 2)
```

Giac [A] time = 1.16207, size = 16, normalized size = 1.14

$$-\frac{1}{2\left(x^2+1\right)}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2/(x^2 + 1) + arctan(x)
```

3.73 $\int \frac{1+x+x^2}{x\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=31

$$\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0389849, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1805, 801, 635, 203, 260}

$$\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[x] - Log[1 + x^2]/2
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx &= \frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-x}{x\left(1+x^2\right)}\,dx\\
&= \frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x}+\frac{-1+2x}{1+x^2}\right)dx\\
&= \frac{x}{2\left(1+x^2\right)}+\log(x)-\frac{1}{2}\int \frac{-1+2x}{1+x^2}\,dx\\
&= \frac{x}{2\left(1+x^2\right)}+\log(x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= \frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0102037, size = 28, normalized size = 0.9

$$\frac{1}{2}\left(\frac{x}{x^2+1}-\log\left(x^2+1\right)+2\log(x)+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

```
[Out] (x/(1 + x^2) + ArcTan[x] + 2*Log[x] - Log[1 + x^2])/2
```

Maple [A] time = 0.049, size = 26, normalized size = 0.8

$$\frac{x}{2\,x^{2}+2}+\frac{\arctan\left(x\right)}{2}+\ln\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x/(x^2+1)^2,x)
```

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.46137, size = 34, normalized size = 1.1

$$\frac{x}{2\left(x^{2}+1\right)}+\frac{1}{2}\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A] time = 0.993583, size = 117, normalized size = 3.77

$$\frac{\left(x^{2}+1\right)\arctan\left(x\right)-\left(x^{2}+1\right)\log\left(x^{2}+1\right)+2\left(x^{2}+1\right)\log\left(x\right)+x}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 2*(x^2 + 1)*log(x) + x)
/(x^2 + 1)
```

Sympy [A] time = 0.135621, size = 24, normalized size = 0.77

$$\frac{x}{2x^2+2}+\log(x)-\frac{\log(x^2+1)}{2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 + atan(x)/2
```

Giac [A] time = 1.15628, size = 35, normalized size = 1.13

$$\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)-\frac{1}{2}\log(x^2+1)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

3.74 $\int \frac{1+x+x^2}{x^2(1+x^2)^2}\,dx$

Optimal. Leaf size=33

$$\frac{1}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)$$

```
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0447255, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1805, 801, 635, 203, 260}

$$\frac{1}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

```
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx &= \frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-2x}{x^2\left(1+x^2\right)}\,dx\\
&= \frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x^2}-\frac{2}{x}+\frac{2(1+x)}{1+x^2}\right)dx\\
&= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}+\log(x)-\int \frac{1+x}{1+x^2}\,dx\\
&= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}+\log(x)-\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0161544, size = 33, normalized size = 1.

$$\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

```
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.053, size = 30, normalized size = 0.9

$$-x^{-1}+\frac{1}{2\,x^{2}+2}-\arctan\left(x\right)+\ln\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^2/(x^2+1)^2,x)
```

```
[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.50181, size = 46, normalized size = 1.39

$$-\frac{2\,x^{2}-x+2}{2\left(x^{3}+x\right)}-\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A] time = 0.986168, size = 138, normalized size = 4.18

$$-\frac{2\,x^{2}+2\left(x^{3}+x\right)\arctan\left(x\right)+\left(x^{3}+x\right)\log\left(x^{2}+1\right)-2\left(x^{3}+x\right)\log\left(x\right)-x+2}{2\left(x^{3}+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 + 2*(x^3 + x)*arctan(x) + (x^3 + x)*log(x^2 + 1) - 2*(x^3 + x)*
log(x) - x + 2)/(x^3 + x)
```

Sympy [A] time = 0.137438, size = 31, normalized size = 0.94

$$\log(x) - \frac{\log(x^2+1)}{2} - \operatorname{atan}(x) - \frac{2x^2 - x + 2}{2x^3 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**2/(x**2+1)**2,x)
```

```
[Out] log(x) - log(x**2 + 1)/2 - atan(x) - (2*x**2 - x + 2)/(2*x**3 + 2*x)
```

Giac [A] time = 1.18142, size = 47, normalized size = 1.42

$$-\frac{2x^2 - x + 2}{2(x^3 + x)} - \arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

3.75 $\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=45

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2x^2}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}-\log(x)-\frac{3}{2}\tan^{-1}(x)$$

```
[Out] -1/(2*x^2) - x^(-1) - x/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[x] + Log[1 +
x^2]/2
```

Rubi [A] time = 0.0633532, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1805, 1802, 635, 203, 260}

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2x^2}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}-\log(x)-\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

```
[Out] -1/(2*x^2) - x^(-1) - x/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[x] + Log[1 +
x^2]/2
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-2x+x^3}{x^3\left(1+x^2\right)}\,dx\\
&= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x^3}-\frac{2}{x^2}+\frac{2}{x}+\frac{3-2x}{1+x^2}\right)dx\\
&= -\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\log(x)-\frac{1}{2}\int \frac{3-2x}{1+x^2}\,dx\\
&= -\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\log(x)-\frac{3}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\frac{3}{2}\tan^{-1}(x)-\log(x)+\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0157861, size = 39, normalized size = 0.87

$$\frac{1}{2}\left(-\frac{x}{x^2+1}-\frac{1}{x^2}+\log\left(x^2+1\right)-\frac{2}{x}-2\log(x)-3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

```
[Out] (-x^(-2) - 2/x - x/(1 + x^2) - 3*ArcTan[x] - 2*Log[x] + Log[1 + x^2])/2
```

<u>Maple</u> [A] time = 0.051, size = 38, normalized size = 0.8

$$-\frac{1}{2\,x^{2}}-x^{-1}-\frac{x}{2\,x^{2}+2}-\frac{3\,\arctan\left(x\right)}{2}-\ln\left(x\right)+\frac{\ln\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^3/(x^2+1)^2,x)
```

```
[Out] -1/2/x^2-1/x-1/2*x/(x^2+1)-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

<u>Maxima</u> [A] time = 1.47377, size = 55, normalized size = 1.22

$$-\frac{3\,x^{3}+x^{2}+2\,x+1}{2\left(x^{4}+x^{2}\right)}-\frac{3}{2}\,\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/(x^4 + x^2) - 3/2*arctan(x) + 1/2*log(x^2 + 1)
 - log(x)
```

<u>Fricas</u> [A] time = 1.03687, size = 159, normalized size = 3.53

$$-\frac{3\,x^{3}+x^{2}+3\left(x^{4}+x^{2}\right)\arctan\left(x\right)-\left(x^{4}+x^{2}\right)\log\left(x^{2}+1\right)+2\left(x^{4}+x^{2}\right)\log\left(x\right)+2\,x+1}{2\left(x^{4}+x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^3 + x^2 + 3*(x^4 + x^2)*arctan(x) - (x^4 + x^2)*log(x^2 + 1) + 2*
(x^4 + x^2)*log(x) + 2*x + 1)/(x^4 + x^2)
```

Sympy [A] time = 0.158863, size = 41, normalized size = 0.91

$$-\log(x)+\frac{\log(x^2+1)}{2}-\frac{3\operatorname{atan}(x)}{2}-\frac{3x^3+x^2+2x+1}{2x^4+2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**3/(x**2+1)**2,x)
```

```
[Out] -log(x) + log(x**2 + 1)/2 - 3*atan(x)/2 - (3*x**3 + x**2 + 2*x + 1)/(2*x**4
 + 2*x**2)
```

Giac [A] time = 1.15525, size = 58, normalized size = 1.29

$$-\frac{3\,x^3+x^2+2\,x+1}{2\left(x^2+1\right)x^2}-\frac{3}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)-\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/((x^2 + 1)*x^2) - 3/2*arctan(x) + 1/2*log(x^2
+ 1) - log(abs(x))
```

3.76 $\int \frac{1+2x+x^2}{\left(1+x^2\right)^2} dx$

Optimal. Leaf size=12

$$\tan^{-1}(x) - \frac{1}{x^2+1}$$

```
[Out] -(1 + x^2)^(-1) + ArcTan[x]
```

Rubi [A] time = 0.0071213, antiderivative size = 22, normalized size of antiderivative = 1.83, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {27, 723, 203}

$$\tan^{-1}(x) - \frac{(1-x)(x+1)}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -((1 - x)*(1 + x))/(2*(1 + x^2)) + ArcTan[x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x+x^2}{\left(1+x^2\right)^2}\,dx = \int \frac{(1+x)^2}{\left(1+x^2\right)^2}\,dx$$
$$= -\frac{(1-x)(1+x)}{2\left(1+x^2\right)} + \int \frac{1}{1+x^2}\,dx$$
$$= -\frac{(1-x)(1+x)}{2\left(1+x^2\right)} + \tan^{-1}(x)$$

Mathematica [A] time = 0.0122203, size = 12, normalized size = 1.

$$\tan^{-1}(x) - \frac{1}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -(1 + x^2)^(-1) + ArcTan[x]
```

Maple [A] time = 0.046, size = 13, normalized size = 1.1

$$-\left(x^2+1\right)^{-1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+1)/(x^2+1)^2,x)
```

```
[Out] -1/(x^2+1)+arctan(x)
```

Maxima [A] time = 1.46191, size = 16, normalized size = 1.33

$$-\frac{1}{x^2+1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/(x^2 + 1) + arctan(x)
```

Fricas [A] time = 0.957922, size = 50, normalized size = 4.17

$$\frac{\left(x^2+1\right)\arctan\left(x\right)-1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

Sympy [A] time = 0.102492, size = 8, normalized size = 0.67

$$\operatorname{atan}\left(x\right)-\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+1)/(x**2+1)**2,x)
```

```
[Out] atan(x) - 1/(x**2 + 1)
```

Giac [A] time = 1.17692, size = 16, normalized size = 1.33

$$-\frac{1}{x^2+1}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/(x^2 + 1) + arctan(x)
```

3.77 $\int \frac{2+12x+3x^2}{\left(4+x^2\right)^2} \, dx$

Optimal. Leaf size=27

$$\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)-\frac{5x+24}{4\left(x^2+4\right)}$$

```
[Out] -(24 + 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

Rubi [A] time = 0.0139091, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1814, 12, 203}

$$\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)-\frac{5x+24}{4\left(x^2+4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

```
[Out] -(24 + 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+12x+3x^2}{\left(4+x^2\right)^2}\,dx &= -\frac{24+5x}{4\left(4+x^2\right)}-\frac{1}{8}\int -\frac{14}{4+x^2}\,dx\\ &= -\frac{24+5x}{4\left(4+x^2\right)}+\frac{7}{4}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{24+5x}{4\left(4+x^2\right)}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.010329, size = 27, normalized size = 1.

$$\frac{-5x-24}{4\left(x^2+4\right)}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

```
[Out] (-24 - 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

Maple [A] time = 0.048, size = 21, normalized size = 0.8

$$\frac{1}{x^2+4}\left(-\frac{5\,x}{4}-6\right)+\frac{7}{8}\arctan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+12*x+2)/(x^2+4)^2,x)
```

```
[Out] (-5/4*x-6)/(x^2+4)+7/8*arctan(1/2*x)
```

Maxima [A] time = 1.49413, size = 28, normalized size = 1.04

$$-\frac{5\,x+24}{4\left(x^2+4\right)}+\frac{7}{8}\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

Fricas [A] time = 1.0288, size = 74, normalized size = 2.74

$$\frac{7\left(x^2+4\right)\arctan\left(\frac{1}{2}\,x\right)-10\,x-48}{8\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(7*(x^2 + 4)*arctan(1/2*x) - 10*x - 48)/(x^2 + 4)
```

Sympy [A] time = 0.11676, size = 19, normalized size = 0.7

$$-\frac{5x+24}{4x^2+16}+\frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+12*x+2)/(x**2+4)**2,x)
```

```
[Out] -(5*x + 24)/(4*x**2 + 16) + 7*atan(x/2)/8
```

Giac [A] time = 1.17825, size = 28, normalized size = 1.04

$$-\frac{5\,x+24}{4\left(x^2+4\right)}+\frac{7}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

3.78 $\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=390

$$\frac{(a+cx^2)^{3/2}\Big(8\big(8a^2fh^4-2ach^2\big(7h(dh+3eg)+15fg^2\big)-c^2g^2\big(3fg^2-7h(12dh+eg)\big)\big)-3chx\big(ah^2(35eh+41fg)+2cg\,(\cdots}{840c^3h}$$

```
[Out] ((8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*
x*Sqrt[a + c*x^2])/(16*c^2) - ((8*a*f*h^2 + c*(3*f*g^2 - 7*h*(e*g + 2*d*h))
)*(g + h*x)^2*(a + c*x^2)^(3/2))/(70*c^2*h) - ((3*f*g - 7*e*h)*(g + h*x)^3*
(a + c*x^2)^(3/2))/(42*c*h) + (f*(g + h*x)^4*(a + c*x^2)^(3/2))/(7*c*h) + (
(8*(8*a^2*f*h^4 - 2*a*c*h^2*(15*f*g^2 + 7*h*(3*e*g + d*h)) - c^2*g^2*(3*f*g
^2 - 7*h*(e*g + 12*d*h))) - 3*c*h*(a*h^2*(41*f*g + 35*e*h) + 2*c*g*(3*f*g^2
 - 7*h*(e*g + 7*d*h)))*x)*(a + c*x^2)^(3/2))/(840*c^3*h) + (a*(8*c^2*d*g^3
+ a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[
c]*x)/Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Rubi [A] time = 0.829677, antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{(a+cx^2)^{3/2}\Big(8\big(8a^2fh^4-2ach^2\big(7h(dh+3eg)+15fg^2\big)-c^2\big(3fg^4-7g^2h(12dh+eg)\big)\big)-3chx\big(ah^2(35eh+41fg)-14cg\cdots}{840c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*
x*Sqrt[a + c*x^2])/(16*c^2) - ((3*c*f*g^2 + 8*a*f*h^2 - 7*c*h*(e*g + 2*d*h)
)*(g + h*x)^2*(a + c*x^2)^(3/2))/(70*c^2*h) - ((3*f*g - 7*e*h)*(g + h*x)^3*
(a + c*x^2)^(3/2))/(42*c*h) + (f*(g + h*x)^4*(a + c*x^2)^(3/2))/(7*c*h) + (
(8*(8*a^2*f*h^4 - 2*a*c*h^2*(15*f*g^2 + 7*h*(3*e*g + d*h)) - c^2*(3*f*g^4 -
 7*g^2*h*(e*g + 12*d*h))) - 3*c*h*(6*c*f*g^3 - 14*c*g*h*(e*g + 7*d*h) + a*h
^2*(41*f*g + 35*e*h))*x)*(a + c*x^2)^(3/2))/(840*c^3*h) + (a*(8*c^2*d*g^3 +
 a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)^3\left((7cd-4af)h^2-ch(3fg-7eh)x\right)\sqrt{a+c}}{7ch^2}$$

$$= -\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)^2\left(3c\right.}{}$$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h} - \frac{(3fg-7eh)(g+hx}{42ch}$$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h} - \frac{(3fg-7eh)(g+hx}{42ch}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{\left(3cfg^2+\right.}{}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{\left(3cfg^2+\right.}{}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{\left(3cfg^2+\right.}{}$$

Mathematica [A] time = 0.470584, size = 362, normalized size = 0.93

$$\frac{\sqrt{a+cx^2}\left(16cx^2\left(-4a^2fh^3+7ach\left(h(dh+3eg)+3fg^2\right)+35c^2g^2(3dh+eg)\right)+105cx\left(-a^2h^2(eh+3fg)+2acg\left(3h(dh+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(16*a*(8*a^2*f*h^3 + 35*c^2*g^2*(e*g + 3*d*h) - 14*a*c*h*(
3*f*g^2 + h*(3*e*g + d*h))) + 105*c*(8*c^2*d*g^3 - a^2*h^2*(3*f*g + e*h) +
2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x + 16*c*(-4*a^2*f*h^3 + 35*c^2*g^2*(e*g
 + 3*d*h) + 7*a*c*h*(3*f*g^2 + h*(3*e*g + d*h)))*x^2 + 70*c^2*(a*h^2*(3*f*g
 + e*h) + 6*c*(f*g^3 + 3*g*h*(e*g + d*h)))*x^3 + 48*c^2*h*(a*f*h^2 + 7*c*(3
*f*g^2 + h*(3*e*g + d*h)))*x^4 + 280*c^3*h^2*(3*f*g + e*h)*x^5 + 240*c^3*f*
h^3*x^6) + 105*a*Sqrt[c]*(8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*
```

```
g^2 + 3*h*(e*g + d*h)))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(1680*c^3)
```

Maple [A] time = 0.058, size = 661, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 8/105*f*h^3*a^2/c^3*(c*x^2+a)^(3/2)+1/7*f*h^3*x^4*(c*x^2+a)^(3/2)/c+(c*x^2+
a)^(3/2)/c*d*g^2*h+1/2*d*g^3*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-2/5*a/
c^2*(c*x^2+a)^(3/2)*e*g*h^2-2/5*a/c^2*(c*x^2+a)^(3/2)*f*g^2*h+3/16*a^3/c^(5
/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*f*g*h^2-4/35*f*h^3*a/c^2*x^2*(c*x^2+a)^(3
/2)+3/4*x*(c*x^2+a)^(3/2)/c*d*g*h^2+3/4*x*(c*x^2+a)^(3/2)/c*e*g^2*h-1/8*a/c
*x*(c*x^2+a)^(1/2)*f*g^3-3/8*a^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*d*g*
h^2+1/2*d*g^3*x*(c*x^2+a)^(1/2)+1/3*(c*x^2+a)^(3/2)/c*e*g^3+3/5*x^2*(c*x^2+
a)^(3/2)/c*f*g^2*h+1/16*a^2/c^2*x*(c*x^2+a)^(1/2)*e*h^3+3/5*x^2*(c*x^2+a)^(
3/2)/c*e*g*h^2-3/8*a^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*e*g^2*h-1/8*a/
c^2*x*(c*x^2+a)^(3/2)*e*h^3+1/2*x^3*(c*x^2+a)^(3/2)/c*f*g*h^2+1/5*x^2*(c*x^
2+a)^(3/2)/c*d*h^3-2/15*a/c^2*(c*x^2+a)^(3/2)*d*h^3+1/16*a^3/c^(5/2)*ln(x*c
^(1/2)+(c*x^2+a)^(1/2))*e*h^3+1/6*x^3*(c*x^2+a)^(3/2)/c*e*h^3+1/4*x*(c*x^2+
a)^(3/2)/c*f*g^3-1/8*a^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*f*g^3-3/8*a/
c^2*x*(c*x^2+a)^(3/2)*f*g*h^2+3/16*a^2/c^2*x*(c*x^2+a)^(1/2)*f*g*h^2-3/8*a/
c*x*(c*x^2+a)^(1/2)*d*g*h^2-3/8*a/c*x*(c*x^2+a)^(1/2)*e*g^2*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64562, size = 1889, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3360*(105*(6*a^2*c*e*g^2*h - a^3*e*h^3 - 2*(4*a*c^2*d - a^2*c*f)*g^3 +
3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(
c)*x - a) - 2*(240*c^3*f*h^3*x^6 + 560*a*c^2*e*g^3 - 672*a^2*c*e*g*h^2 + 28
0*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*f*g^2*h + 21*c^3*e*g*h^2 + (
7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^2*h - 32*(7*a^2
*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^3*e*g^2*h + a*c^2*e*h^3 + 3*(6
*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*g^3 + 21*a*c^2*e*g*h^2 + 21*(5*
c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 + 105*(6*a*c^2*e*
g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2*d - a^2*c*f)*g
*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/1680*(105*(6*a^2*c*e*g^2*h - a^3*e*h^3 - 2
*(4*a*c^2*d - a^2*c*f)*g^3 + 3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(-c)*arctan(s
qrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f*h^3*x^6 + 560*a*c^2*e*g^3 - 672*a^2
*c*e*g*h^2 + 280*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*f*g^2*h + 21*
c^3*e*g*h^2 + (7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^
2*h - 32*(7*a^2*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^3*e*g^2*h + a*c
^2*e*h^3 + 3*(6*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*g^3 + 21*a*c^2*e
*g*h^2 + 21*(5*c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 +
105*(6*a*c^2*e*g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2
*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 25.4176, size = 1088, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*e*h**3*x/(16*c**2*sqrt(1 + c*x**2/a)) - 3*a**(5/2)*f*g*h**2*x/(16
*c**2*sqrt(1 + c*x**2/a)) + 3*a**(3/2)*d*g*h**2*x/(8*c*sqrt(1 + c*x**2/a))
+ 3*a**(3/2)*e*g**2*h*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3/2)*e*h**3*x**3/(48
*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g**3*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3
/2)*f*g*h**2*x**3/(16*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*g**3*x*sqrt(1 + c*x
**2/a)/2 + 9*sqrt(a)*d*g*h**2*x**3/(8*sqrt(1 + c*x**2/a)) + 9*sqrt(a)*e*g**
2*h*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*e*h**3*x**5/(24*sqrt(1 + c*x**2
/a)) + 3*sqrt(a)*f*g**3*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*f*g*h**2*x*
*5/(8*sqrt(1 + c*x**2/a)) + a**3*e*h**3*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/
2)) + 3*a**3*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/2)) - 3*a**2*d*g*h
```

```
**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - 3*a**2*e*g**2*h*asinh(sqrt(c)*x
/sqrt(a))/(8*c**(3/2)) - a**2*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2))
+ a*d*g**3*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 3*d*g**2*h*Piecewise((sqr
t(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + d*h**3*Piecewi
se((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x
**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + e*g**3*Piecewi
se((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 3*e*g*h
**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)
/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*f
*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x
**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) +
 f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g*h**2*x**5/(4*sqrt(a)*sqr
t(1 + c*x**2/a)) + 3*c*e*g**2*h*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*h
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*g**3*x**5/(4*sqrt(a)*sqrt(1 +
 c*x**2/a)) + c*f*g*h**2*x**7/(2*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.18769, size = 641, normalized size = 1.64

$$\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,fh^3x+\frac{7\left(3\,c^5fgh^2+c^5h^3e\right)}{c^5}\right)x+\frac{6\left(21\,c^5fg^2h+7\,c^5dh^3+ac^4fh^3+21\,c^5gh^2e\right)}{c^5}\right)x+\frac{35\left(6\,c^5\right.}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*f*h^3*x + 7*(3*c^5*f*g*h^2 + c^5*h^3*e
)/c^5)*x + 6*(21*c^5*f*g^2*h + 7*c^5*d*h^3 + a*c^4*f*h^3 + 21*c^5*g*h^2*e)/
c^5)*x + 35*(6*c^5*f*g^3 + 18*c^5*d*g*h^2 + 3*a*c^4*f*g*h^2 + 18*c^5*g^2*h*
e + a*c^4*h^3*e)/c^5)*x + 8*(105*c^5*d*g^2*h + 21*a*c^4*f*g^2*h + 7*a*c^4*d
*h^3 - 4*a^2*c^3*f*h^3 + 35*c^5*g^3*e + 21*a*c^4*g*h^2*e)/c^5)*x + 105*(8*c
^5*d*g^3 + 2*a*c^4*f*g^3 + 6*a*c^4*d*g*h^2 - 3*a^2*c^3*f*g*h^2 + 6*a*c^4*g^
2*h*e - a^2*c^3*h^3*e)/c^5)*x + 16*(105*a*c^4*d*g^2*h - 42*a^2*c^3*f*g^2*h
- 14*a^2*c^3*d*h^3 + 8*a^3*c^2*f*h^3 + 35*a*c^4*g^3*e - 42*a^2*c^3*g*h^2*e)
/c^5) - 1/16*(8*a*c^2*d*g^3 - 2*a^2*c*f*g^3 - 6*a^2*c*d*g*h^2 + 3*a^3*f*g*h
^2 - 6*a^2*c*g^2*h*e + a^3*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

3.79 $\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=280

$$\frac{x\sqrt{a+cx^2}\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{16c^{5/2}}$$

```
[Out] ((8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a + c*x
^2])/(16*c^2) - ((f*g - 2*e*h)*(g + h*x)^2*(a + c*x^2)^(3/2))/(10*c*h) + (f
*(g + h*x)^3*(a + c*x^2)^(3/2))/(6*c*h) - ((8*(2*a*h^2*(2*f*g + e*h) + c*g*
(f*g^2 - 2*h*(e*g + 5*d*h))) - 3*h*(5*(2*c*d - a*f)*h^2 - 2*c*g*(f*g - 2*e*
h))*x)*(a + c*x^2)^(3/2))/(120*c^2*h) + (a*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c
*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(5/
2))
```

Rubi [A] time = 0.498083, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{x\sqrt{a+cx^2}\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a + c*x
^2])/(16*c^2) - ((f*g - 2*e*h)*(g + h*x)^2*(a + c*x^2)^(3/2))/(10*c*h) + (f
*(g + h*x)^3*(a + c*x^2)^(3/2))/(6*c*h) - ((8*(c*f*g^3 - 2*c*g*h*(e*g + 5*d
*h) + 2*a*h^2*(2*f*g + e*h)) - 3*h*(5*(2*c*d - a*f)*h^2 - 2*c*g*(f*g - 2*e*
h))*x)*(a + c*x^2)^(3/2))/(120*c^2*h) + (a*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c
*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(5/
2))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
```

```
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+hx)^2\left(3(2cd-af)h^2-3ch(fg-2eh)x\right)\sqrt{a+c}}{6ch^2}$$

$$= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+hx)\left(3ch^2\right.}{}$$

$$= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} - \frac{\left(8\left(cfg^3-2cgh\right.\right.}{}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg-2eh)(g+hx)^2}{10ch}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg-2eh)(g+hx)^2}{10ch}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg-2eh)(g+hx)^2}{10ch}$$

Mathematica [A] time = 0.697218, size = 256, normalized size = 0.91

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\left(a^2(-h)(32eh+64fg+15fhx)+2ac\left(5dh(16g+3hx)+e\left(40g^2+30ghx+8h^2x^2\right)+fx\left(15g^2+16ghx+5\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(-(a^2*h*(64*f*g + 32*e*h + 15*f*h*x)) + 2*a*c*(5
*d*h*(16*g + 3*h*x) + f*x*(15*g^2 + 16*g*h*x + 5*h^2*x^2) + e*(40*g^2 + 30*
g*h*x + 8*h^2*x^2)) + 4*c^2*x*(5*d*(6*g^2 + 8*g*h*x + 3*h^2*x^2) + x*(2*e*(
10*g^2 + 15*g*h*x + 6*h^2*x^2) + f*x*(15*g^2 + 24*g*h*x + 10*h^2*x^2)))) +
(15*Sqrt[a]*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*Arc
Sinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(240*c^(5/2))
```

Maple [A] time = 0.054, size = 446, normalized size = 1.6

$$\frac{fh^2x^3}{6c}\left(cx^2+a\right)^{\frac{3}{2}} - \frac{afh^2x}{8c^2}\left(cx^2+a\right)^{\frac{3}{2}} + \frac{a^2fh^2x}{16c^2}\sqrt{cx^2+a} + \frac{fh^2a^3}{16}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{5}{2}} + \frac{ex^2h^2}{5c}\left(cx^2+a\right)^{\frac{3}{2}} + \frac{2fgx^2}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/6*f*h^2*x^3*(c*x^2+a)^(3/2)/c-1/8*f*h^2*a/c^2*x*(c*x^2+a)^(3/2)+1/16*f*h^
2*a^2/c^2*x*(c*x^2+a)^(1/2)+1/16*f*h^2*a^3/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(
1/2))+1/5*x^2*(c*x^2+a)^(3/2)/c*e*h^2+2/5*x^2*(c*x^2+a)^(3/2)/c*f*g*h-2/15*
a/c^2*(c*x^2+a)^(3/2)*e*h^2-4/15*a/c^2*(c*x^2+a)^(3/2)*f*g*h+1/4*x*(c*x^2+a
)^(3/2)/c*d*h^2+1/2*x*(c*x^2+a)^(3/2)/c*e*g*h+1/4*x*(c*x^2+a)^(3/2)/c*f*g^2
-1/8*a/c*x*(c*x^2+a)^(1/2)*d*h^2-1/4*a/c*x*(c*x^2+a)^(1/2)*e*g*h-1/8*a/c*x*
(c*x^2+a)^(1/2)*f*g^2-1/8*a^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*d*h^2-1
/4*a^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*e*g*h-1/8*a^2/c^(3/2)*ln(x*c^(
1/2)+(c*x^2+a)^(1/2))*f*g^2+2/3*(c*x^2+a)^(3/2)/c*d*g*h+1/3*(c*x^2+a)^(3/2)
/c*e*g^2+1/2*d*g^2*x*(c*x^2+a)^(1/2)+1/2*d*g^2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^
2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60811, size = 1319, normalized size = 4.71

$$\left[-\frac{15\left(4\,a^{2}cegh-2\left(4\,ac^{2}d-a^{2}cf\right)g^{2}+\left(2\,a^{2}cd-a^{3}f\right)h^{2}\right)\sqrt{c}\log\left(-2\,cx^{2}-2\,\sqrt{cx^{2}+a}\sqrt{c}x-a\right)-2\left(40\,c^{3}fh^{2}x^{5}+80\,ac^{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(4*a^2*c*e*g*h - 2*(4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3
*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(40*c^
3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2*c*e*h^2 + 48*(2*c^3*f*g*h + c^3*e*h^2
```

```
)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g*h + (6*c^3*d + a*c^2*f)*h^2)*x^3 + 32*
(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5*c^3*e*g^2 + a*c^2*e*h^2 + 2*(5*c^3*d +
a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*g*h + 2*(4*c^3*d + a*c^2*f)*g^2 + (2*a*c^
2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/240*(15*(4*a^2*c*e*g*h - 2*(
4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3*f)*h^2)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) + (40*c^3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2*c*e*h^2
+ 48*(2*c^3*f*g*h + c^3*e*h^2)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g*h + (6*c^
3*d + a*c^2*f)*h^2)*x^3 + 32*(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5*c^3*e*g^2
+ a*c^2*e*h^2 + 2*(5*c^3*d + a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*g*h + 2*(4*c
^3*d + a*c^2*f)*g^2 + (2*a*c^2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 19.0432, size = 738, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*f*h**2*x/(16*c**2*sqrt(1 + c*x**2/a)) + a**(3/2)*d*h**2*x/(8*c*sq
rt(1 + c*x**2/a)) + a**(3/2)*e*g*h*x/(4*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*
g**2*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3/2)*f*h**2*x**3/(48*c*sqrt(1 + c*x**
2/a)) + sqrt(a)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*d*h**2*x**3/(8*sq
rt(1 + c*x**2/a)) + 3*sqrt(a)*e*g*h*x**3/(4*sqrt(1 + c*x**2/a)) + 3*sqrt(a)
*f*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*f*h**2*x**5/(24*sqrt(1 + c*
x**2/a)) + a**3*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/2)) - a**2*d*h**2
*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*e*g*h*asinh(sqrt(c)*x/sqrt(a)
)/(4*c**(3/2)) - a**2*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) + a*d*g*
*2*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 2*d*g*h*Piecewise((sqrt(a)*x**2/2
, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g**2*Piecewise((sqrt(a)
*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*h**2*Piecewise((
-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*
sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 2*f*g*h*Piecewise(
(-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4
*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*d*h**2*x**5/(4*
sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*g*h*x**5/(2*sqrt(a)*sqrt(1 + c*x**2/a)) +
 c*f*g**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*h**2*x**7/(6*sqrt(a)*sq
rt(1 + c*x**2/a))
```

Giac [A] time = 1.19823, size = 433, normalized size = 1.55

$$\frac{1}{240}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\,fh^2x+\frac{6\left(2\,c^4fgh+c^4h^2e\right)}{c^4}\right)x+\frac{5\left(6\,c^4fg^2+6\,c^4dh^2+ac^3fh^2+12\,c^4ghe\right)}{c^4}\right)x+\frac{8\left(10\,c^4dgh+2\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*f*h^2*x + 6*(2*c^4*f*g*h + c^4*h^2*e)/c^4)
*x + 5*(6*c^4*f*g^2 + 6*c^4*d*h^2 + a*c^3*f*h^2 + 12*c^4*g*h*e)/c^4)*x + 8*
(10*c^4*d*g*h + 2*a*c^3*f*g*h + 5*c^4*g^2*e + a*c^3*h^2*e)/c^4)*x + 15*(8*c
^4*d*g^2 + 2*a*c^3*f*g^2 + 2*a*c^3*d*h^2 - a^2*c^2*f*h^2 + 4*a*c^3*g*h*e)/c
^4)*x + 16*(10*a*c^3*d*g*h - 4*a^2*c^2*f*g*h + 5*a*c^3*g^2*e - 2*a^2*c^2*h^
2*e)/c^4) - 1/16*(8*a*c^2*d*g^2 - 2*a^2*c*f*g^2 - 2*a^2*c*d*h^2 + a^3*f*h^2
 - 4*a^2*c*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.80 $\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=175

$$-\frac{\left(a+cx^2\right)^{3/2}\left(4\left(2afh^2+c\left(3fg^2-5h(dh+eg)\right)\right)+3chx(3fg-5eh)\right)}{60c^2h}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh-afg+4cdg)}{8c^{3/2}}+\frac{x\sqrt{a}}{}$$

```
[Out] ((4*c*d*g - a*(f*g + e*h))*x*Sqrt[a + c*x^2])/(8*c) + (f*(g + h*x)^2*(a + c
*x^2)^(3/2))/(5*c*h) - ((4*(2*a*f*h^2 + c*(3*f*g^2 - 5*h*(e*g + d*h))) + 3*
c*h*(3*f*g - 5*e*h)*x)*(a + c*x^2)^(3/2))/(60*c^2*h) + (a*(4*c*d*g - a*f*g
- a*e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Rubi [A] time = 0.268063, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1654, 780, 195, 217, 206}

$$-\frac{\left(a+cx^2\right)^{3/2}\left(4\left(2afh^2+c\left(3fg^2-5h(dh+eg)\right)\right)+3chx(3fg-5eh)\right)}{60c^2h}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh-afg+4cdg)}{8c^{3/2}}+\frac{x\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d*g - a*(f*g + e*h))*x*Sqrt[a + c*x^2])/(8*c) + (f*(g + h*x)^2*(a + c
*x^2)^(3/2))/(5*c*h) - ((4*(2*a*f*h^2 + c*(3*f*g^2 - 5*h*(e*g + d*h))) + 3*
c*h*(3*f*g - 5*e*h)*x)*(a + c*x^2)^(3/2))/(60*c^2*h) + (a*(4*c*d*g - a*f*g
- a*e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} + \frac{\int (g+hx)\left((5cd-2af)h^2-ch(3fg-5eh)x\right)\sqrt{a+cx}}{5ch^2}$$

$$= \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \frac{\left(4\left(2afh^2+c\left(3fg^2-5h(eg+dh)\right)\right)+3ch(3fg-5e\right.}{60c^2h}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \frac{\left(4\left(2afh^2+c\left(3fg\right.\right.\right.}{}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \frac{\left(4\left(2afh^2+c\left(3fg\right.\right.\right.}{}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \frac{\left(4\left(2afh^2+c\left(3fg\right.\right.\right.}{}$$

Mathematica [A] time = 0.438547, size = 153, normalized size = 0.87

$$\frac{\sqrt{a+cx^2}\left(-16a^2fh-\frac{15\sqrt{a}\sqrt{c}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aeh+afg-4cdg)}{\sqrt{\frac{cx^2}{a}+1}}+ac(40dh+5e(8g+3hx)+fx(15g+8hx))+2c^2x(10d(3g+2hx)+\cdots\right.}{120c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-16*a^2*f*h + a*c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*g
 + 8*h*x)) + 2*c^2*x*(10*d*(3*g + 2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(5*
g + 4*h*x))) - (15*Sqrt[a]*Sqrt[c]*(-4*c*d*g + a*f*g + a*e*h)*ArcSinh[(Sqrt
[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(120*c^2)
```

Maple [A] time = 0.053, size = 230, normalized size = 1.3

$$\frac{fhx^2}{5c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{2afh}{15c^2}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{ehx}{4c}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{fgx}{4c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{aehx}{8c}\sqrt{cx^2+a}-\frac{afgx}{8c}\sqrt{cx^2+a}-\frac{a^2eh}{8}\ln\left(x\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/5*h*f*x^2*(c*x^2+a)^(3/2)/c-2/15*h*f*a/c^2*(c*x^2+a)^(3/2)+1/4*x*(c*x^2+a
)^(3/2)/c*e*h+1/4*x*(c*x^2+a)^(3/2)/c*f*g-1/8*a/c*x*(c*x^2+a)^(1/2)*e*h-1/8
*a/c*x*(c*x^2+a)^(1/2)*f*g-1/8*a^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*e*
h-1/8*a^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*f*g+1/3*(c*x^2+a)^(3/2)/c*d
*h+1/3*(c*x^2+a)^(3/2)/c*e*g+1/2*d*g*x*(c*x^2+a)^(1/2)+1/2*d*g*a/c^(1/2)*ln
(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4262, size = 770, normalized size = 4.4

$$\left[\frac{15\left(a^2eh-\left(4\,acd-a^2f\right)g\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(24\,c^2fhx^4+40\,aceg+30\left(c^2fg+c^2eh\right)x^3+8\left(5\,c\right.\right.}{240\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*
x^2 + a)*sqrt(c)*x - a) + 2*(24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g + c^
2*e*h)*x^3 + 8*(5*c^2*e*g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^2*f
)*h + 15*(a*c*e*h + (4*c^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/120*(15
*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
) + (24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g + c^2*e*h)*x^3 + 8*(5*c^2*e*
g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^2*f)*h + 15*(a*c*e*h + (4*c
^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 10.0936, size = 384, normalized size = 2.19

$$\frac{a^{\frac{3}{2}}ehx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}fgx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}dgx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}ehx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}fgx^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{a^2eh\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}-\frac{a^2fg\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+\frac{adg}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*e*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g*x/(8*c*sqrt(1 + c*x*
*2/a)) + sqrt(a)*d*g*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*e*h*x**3/(8*sqrt(1
+ c*x**2/a)) + 3*sqrt(a)*f*g*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*e*h*asinh(s
qrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*f*g*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3
/2)) + a*d*g*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + d*h*Piecewise((sqrt(a)*
x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g*Piecewise((sqrt
(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + f*h*Piecewise((
-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*
```

```
sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*e*h*x**5/(4*sqrt
(a)*sqrt(1 + c*x**2/a)) + c*f*g*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.19162, size = 243, normalized size = 1.39

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(4\,fhx+\frac{5\left(c^3fg+c^3he\right)}{c^3}\right)x+\frac{4\left(5\,c^3dh+ac^2fh+5\,c^3ge\right)}{c^3}\right)x+\frac{15\left(4\,c^3dg+ac^2fg+ac^2he\right)}{c^3}\right)x+\frac{8\left(5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h*x + 5*(c^3*f*g + c^3*h*e)/c^3)*x + 4*(5
*c^3*d*h + a*c^2*f*h + 5*c^3*g*e)/c^3)*x + 15*(4*c^3*d*g + a*c^2*f*g + a*c^
2*h*e)/c^3)*x + 8*(5*a*c^2*d*h - 2*a^2*c*f*h + 5*a*c^2*g*e)/c^3) - 1/8*(4*a
*c*d*g - a^2*f*g - a^2*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.81 $\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=106

$$\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}(4cd-af)}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

```
[Out] ((4*c*d - a*f)*x*Sqrt[a + c*x^2])/(8*c) + (e*(a + c*x^2)^(3/2))/(3*c) + (f*
x*(a + c*x^2)^(3/2))/(4*c) + (a*(4*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
c*x^2]])/(8*c^(3/2))
```

Rubi [A] time = 0.0642928, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1815, 641, 195, 217, 206}

$$\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}(4cd-af)}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d - a*f)*x*Sqrt[a + c*x^2])/(8*c) + (e*(a + c*x^2)^(3/2))/(3*c) + (f*
x*(a + c*x^2)^(3/2))/(4*c) + (a*(4*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
c*x^2]])/(8*c^(3/2))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)dx &= \frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{\int (4cd-af+4cex)\sqrt{a+cx^2}\,dx}{4c}\\ &= \frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{(4cd-af)\int \sqrt{a+cx^2}\,dx}{4c}\\ &= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{(a(4cd-af))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{8c}\\ &= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{(a(4cd-af))\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\right.}{8c}\\ &= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.199741, size = 98, normalized size = 0.92

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}(a(8e+3fx)+2cx(6d+x(4e+3fx)))-\frac{3\sqrt{a}(af-4cd)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(a*(8*e + 3*f*x) + 2*c*x*(6*d + x*(4*e + 3*f*x)))
 - (3*Sqrt[a]*(-4*c*d + a*f)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)
/a]))/(24*c^(3/2))
```

Maple [A] time = 0.05, size = 111, normalized size = 1.1

$$\frac{fx}{4c}\left(cx^{2}+a\right)^{\frac{3}{2}}-\frac{afx}{8c}\sqrt{cx^{2}+a}-\frac{a^{2}f}{8}\ln\left(x\sqrt{c}+\sqrt{cx^{2}+a}\right)c^{-\frac{3}{2}}+\frac{e}{3c}\left(cx^{2}+a\right)^{\frac{3}{2}}+\frac{dx}{2}\sqrt{cx^{2}+a}+\frac{ad}{2}\ln\left(x\sqrt{c}+\sqrt{cx^{2}+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/4*f*x*(c*x^2+a)^(3/2)/c-1/8*f*a/c*x*(c*x^2+a)^(1/2)-1/8*f*a^2/c^(3/2)*ln(
x*c^(1/2)+(c*x^2+a)^(1/2))+1/3*e*(c*x^2+a)^(3/2)/c+1/2*d*x*(c*x^2+a)^(1/2)+
1/2*d*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35248, size = 448, normalized size = 4.23

$$\left[-\frac{3\left(4\,acd-a^{2}f\right)\sqrt{c}\log\left(-2\,cx^{2}+2\,\sqrt{cx^{2}+a}\sqrt{c}x-a\right)-2\left(6\,c^{2}fx^{3}+8\,c^{2}ex^{2}+8\,ace+3\left(4\,c^{2}d+acf\right)x\right)\sqrt{cx^{2}+a}}{48\,c^{2}},-\frac{3\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*a*c*d - a^2*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c
)*x - a) - 2*(6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d + a*c*f)*x)*
sqrt(c*x^2 + a))/c^2, -1/24*(3*(4*a*c*d - a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x
/sqrt(c*x^2 + a)) - (6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d + a*c
*f)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 6.02397, size = 170, normalized size = 1.6

$$\frac{a^{\frac{3}{2}}fx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}fx^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{a^2f\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{c}}+e\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } c=0\\ \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+\frac{cf x}{4\sqrt{a}\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*f*x/(8*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*x*sqrt(1 + c*x**2/a)/2 +
3*sqrt(a)*f*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*f*asinh(sqrt(c)*x/sqrt(a))/(
8*c**(3/2)) + a*d*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + e*Piecewise((sqrt(
a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + c*f*x**5/(4*sqrt
(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.15261, size = 117, normalized size = 1.1

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(3fx+4e\right)x+\frac{3\left(4c^2d+acf\right)}{c^2}\right)x+\frac{8ae}{c}\right)-\frac{\left(4acd-a^2f\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*x + 4*e)*x + 3*(4*c^2*d + a*c*f)/c^2)*x + 8*a
*e/c) - 1/8*(4*a*c*d - a^2*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2
)
```

3.82 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx$

Optimal. Leaf size=206

$$\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)}{2\sqrt{c}h^4}-\frac{\sqrt{ah^2+cg^2}\left(dh^2\right.}{}$$

```
[Out] ((2*(f*g^2 - e*g*h + d*h^2) - h*(f*g - e*h)*x)*Sqrt[a + c*x^2])/(2*h^3) + (
f*(a + c*x^2)^(3/2))/(3*c*h) - ((2*c*d*g*h^2 + (f*g - e*h)*(2*c*g^2 + a*h^2
))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*h^4) - (Sqrt[c*g^2 + a*
h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/h^4
```

Rubi [A] time = 0.391578, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 815, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)}{2\sqrt{c}h^4}-\frac{\sqrt{ah^2+cg^2}\left(dh^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((2*(f*g^2 - e*g*h + d*h^2) - h*(f*g - e*h)*x)*Sqrt[a + c*x^2])/(2*h^3) + (
f*(a + c*x^2)^(3/2))/(3*c*h) - ((2*c*d*g*h^2 + (f*g - e*h)*(2*c*g^2 + a*h^2
))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*h^4) - (Sqrt[c*g^2 + a*
h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/h^4
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
```

```
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+cx^2\right)^{3/2}}{3ch} + \frac{\int \frac{\left(3cdh^2-3ch(fg-eh)x\right)\sqrt{a+cx^2}}{g+hx}\,dx}{3ch^2}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} + \int \frac{3ac^2h^2\left(fg^2-h(eg-2dh)\right.\cdots}{\cdots}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} + \frac{\left(\left(cg^2+ah^2\right)\left(fg^2-\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} - \frac{\left(\left(cg^2+ah^2\right)\left(fg^2-\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} - \frac{\left(2cdgh^2+(fg-eh)\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.477019, size = 224, normalized size = 1.09

$$\frac{\left(h(dh-eg)+fg^2\right)\left(-\sqrt{ah^2+cg^2}\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)-\sqrt{c}g\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+h\sqrt{a+cx^2}\right)}{h^4} + \frac{\sqrt{a+cx^2}\left(\sqrt{c}x\sqrt{\frac{cx^2}{a}}\cdots\right.}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (f*(a + c*x^2)^(3/2))/(3*c*h) + ((-(f*g) + e*h)*Sqrt[a + c*x^2]*(Sqrt[c]*x*
Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/(2*Sqrt[c]*h^2
*Sqrt[1 + (c*x^2)/a]) + ((f*g^2 + h*(-(e*g) + d*h))*(h*Sqrt[a + c*x^2] - Sq
rt[c]*g*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[c*g^2 + a*h^2]*ArcTanh[
(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])]))/h^4
```

Maple [B] time = 0.301, size = 1265, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x)
```

```
[Out] 1/3*f*(c*x^2+a)^(3/2)/c/h+1/2/h*e*x*(c*x^2+a)^(1/2)+1/2/h*e*a/c^(1/2)*ln(x*
c^(1/2)+(c*x^2+a)^(1/2))-1/2/h^2*f*g*x*(c*x^2+a)^(1/2)-1/2/h^2*f*g*a/c^(1/2
)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+1/h*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g
^2)/h^2)^(1/2)*d-1/h^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2
)*e*g+1/h^3*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2-1/h
^2*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*
h^2+c*g^2)/h^2)^(1/2))*d+1/h^3*c^(1/2)*g^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((
x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*e-1/h^4*c^(1/2)*g^3*ln
((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)
^(1/2))*f-1/h/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+
g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)
/h^2)^(1/2))/(x+g/h))*a*d+1/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^
2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+
g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*e*g-1/h^3/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+
g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g^2-1/h^3/(
(a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+
c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x
+g/h))*c*g^2*d+1/h^4/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*
g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2
+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3*e-1/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*
(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-
2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^4*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Giac [A] time = 1.26604, size = 375, normalized size = 1.82

$$\frac{1}{6}\sqrt{cx^2+a}\left(\left(\frac{2fx}{h}-\frac{3\left(cfgh^8-ch^9e\right)}{ch^{10}}\right)x+\frac{2\left(3cfg^2h^7+3cdh^9+afh^9-3cgh^8e\right)}{ch^{10}}\right)+\frac{2\left(cfg^4+cdg^2h^2+afg^2h^2+adh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*((2*f*x/h - 3*(c*f*g*h^8 - c*h^9*e)/(c*h^10))*x + 2*(3*
c*f*g^2*h^7 + 3*c*d*h^9 + a*f*h^9 - 3*c*g*h^8*e)/(c*h^10)) + 2*(c*f*g^4 + c
*d*g^2*h^2 + a*f*g^2*h^2 + a*d*h^4 - c*g^3*h*e - a*g*h^3*e)*arctan(-((sqrt(
c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 -
 a*h^2)*h^4) + 1/2*(2*c^(3/2)*f*g^3 + 2*c^(3/2)*d*g*h^2 + a*sqrt(c)*f*g*h^2
 - 2*c^(3/2)*g^2*h*e - a*sqrt(c)*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a
)))/(c*h^4)
```

3.83 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}\left(2\left(ah^2(2fg-eh)+cg\left(3fg^2-h(2eg-dh)\right)\right)-hx\left(afh^2+c\left(3fg^2-2h(eg-\right.\right.\right.}{2h^3\left(ah^2+cg^2\right)}$$

```
[Out] -((2*(a*h^2*(2*f*g - e*h) + c*g*(3*f*g^2 - h*(2*e*g - d*h))) - h*(a*f*h^2 +
 c*(3*f*g^2 - 2*h*(e*g - d*h)))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2))
 - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h*x)
) + ((a*f*h^2 + 2*c*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a
 + c*x^2]])/(2*Sqrt[c]*h^4) + ((a*h^2*(2*f*g - e*h) + c*g*(3*f*g^2 - h*(2*e
*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(
h^4*Sqrt[c*g^2 + a*h^2])
```

Rubi [A] time = 0.509402, antiderivative size = 303, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 815, 844, 217, 206, 725}

$$-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}\left(2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)-hx\left(afh^2-2ch(eg-dh)+3cf\right.\right.}{2h^3\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((2*(3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(2*f*g - e*h)) - h*(3*c*f*g^2
 + a*f*h^2 - 2*c*h*(e*g - d*h))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2))
 - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h*x)
) + ((6*c*f*g^2 + a*f*h^2 - 2*c*h*(2*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a
 + c*x^2]])/(2*Sqrt[c]*h^4) + ((3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(2*
f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(
h^4*Sqrt[c*g^2 + a*h^2])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{\left(-cdg+afg-aeh-\left(afh-c\left(2eg-\frac{3fg^2}{h}-2dh\right)\right)x\right)\sqrt{a+cx^2}}{g+hx}\,dx}{cg^2+ah^2}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-dh)\right)x\right)\sqrt{a}}{2h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-dh)\right)x\right)\sqrt{a}}{2h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-dh)\right)x\right)\sqrt{a}}{2h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-dh)\right)x\right)\sqrt{a}}{2h^3\left(cg^2+ah^2\right)}$$

Mathematica [A] time = 0.283715, size = 264, normalized size = 0.86

$$\frac{\frac{h\sqrt{a+cx^2}\left(2h(-dh+2eg+ehx)+f\left(-6g^2-3ghx+h^2x^2\right)\right)}{g+hx}+\frac{\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(afh^2+2ch(dh-2eg)+6cfg^2\right)}{\sqrt{c}}+\frac{2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(ah^2(2fg-eh)+cg\right.}{\sqrt{ah^2+cg^2}}}{2h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(2*h*(2*e*g - d*h + e*h*x) + f*(-6*g^2 - 3*g*h*x + h^2*
x^2)))/(g + h*x) - (2*(3*c*f*g^3 + c*g*h*(-2*e*g + d*h) + a*h^2*(2*f*g - e*
h))*Log[g + h*x])/Sqrt[c*g^2 + a*h^2] + ((6*c*f*g^2 + a*f*h^2 + 2*c*h*(-2*e
*g + d*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + (2*(3*c*f*g^3 + c*
g*h*(-2*e*g + d*h) + a*h^2*(2*f*g - e*h))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*
h^2]*Sqrt[a + c*x^2]])/Sqrt[c*g^2 + a*h^2])/(2*h^4)
```

Maple [B] time = 0.234, size = 2818, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x)
```

```
[Out] -1/h^4*c^2*g^4/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/
h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h
)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+1/2*f/h^2*a/c^(1/2)*ln(x*c^(1/2)+(c*
x^2+a)^(1/2))-1/h^3*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*
c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*e+2/h^4*c^(1/2)*g^2*ln((-c*g/h+(x+g
/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*f-1/h
^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*
h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)
)/(x+g/h))*a*e-1/h*c*g/(a*h^2+c*g^2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*
g^2)/h^2)^(1/2)*d+1/h^2*c*g^2/(a*h^2+c*g^2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a
*h^2+c*g^2)/h^2)^(1/2)*e-1/h^4/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2
)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g
/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^2*e+2/h^3/((a*h^2+c*g^2)/h^2)^(1
/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x
+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g+1/h*c*g/
(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+
g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)
/h^2)^(1/2))/(x+g/h))*a*d+1/h^3*c*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+
g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f-1/h^2*c*g^2
/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x
+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2
)/h^2)^(1/2))/(x+g/h))*a*e-1/(a*h^2+c*g^2)/(x+g/h)*((x+g/h)^2*c-2*c*g/h*(x+
g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*d+1/2*f/h^2*x*(c*x^2+a)^(1/2)-2/h^3*((x+g/h)^
2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g+1/h^2*c^(3/2)*g^2/(a*h^2+c
*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g
^2)/h^2)^(1/2))*d-1/h^3*c^(3/2)*g^3/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(
1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*e+1/h^4*c^(3/2)
*g^4/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/
h)+(a*h^2+c*g^2)/h^2)^(1/2))*f+1/h/(a*h^2+c*g^2)/(x+g/h)*((x+g/h)^2*c-2*c*g
/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)*((x+g/h
)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*f*g^2+c/(a*h^2+c*g^2)*((x+g/
h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*d+c^(1/2)/(a*h^2+c*g^2)*l
n((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2
)^(1/2))*a*d+1/h^2*c/(a*h^2+c*g^2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^
2)/h^2)^(1/2)*x*f*g^2+1/h^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)
^(1/2)*e+1/h^3*c^2*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2
+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/
h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+2/h^5/((a*h^2+c*g^2)/h^2)^(1
/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x
```

```
+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3*f-1/h^3*
c*g^3/(a*h^2+c*g^2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f
+1/h^5*c^2*g^5/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/
h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h
)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-1/h*c/(a*h^2+c*g^2)*((x+g/h)^2*c-2*c
*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*e*g-1/h*c^(1/2)/(a*h^2+c*g^2)*ln((-
c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1
/2))*a*e*g+1/h^2*c^(1/2)/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/
h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*a*f*g^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{\left(g+hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.84 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=296

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)}{2h^4\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}+$$

```
[Out] ((2*(3*f*g - e*h)*(c*g^2 + a*h^2) + h*(2*a*f*h^2 + c*(3*f*g^2 - h*(e*g - d*
h)))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2)*(g + h*x)) - ((f*g^2 - e*g*
h + d*h^2)*(a + c*x^2)^(3/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2) - (Sqrt[c]*
(3*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^4 - ((2*a^2*f*h^4 + 2
*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h
- c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^4*(c*g^2 + a*h^2)^(3/
2))
```

Rubi [A] time = 0.545689, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 813, 844, 217, 206, 725}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)}{2h^4\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*(3*f*g - e*h)*(c*g^2 + a*h^2) + h*(3*c*f*g^2 + 2*a*f*h^2 - c*h*(e*g - d
*h))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2)*(g + h*x)) - ((f*g^2 - e*g*
h + d*h^2)*(a + c*x^2)^(3/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2) - (Sqrt[c]*
(3*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^4 - ((2*a^2*f*h^4 + 2
*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h
- c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^4*(c*g^2 + a*h^2)^(3/
2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(eg-\frac{3fg^2}{h}-dh\right)\right)x\right)\sqrt{a+cx^2}}{(g+hx)^2}\,dx}{2\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \frac{\left(fg^2-egh\ \cdots\right.}{2h\left(cg^2+\cdots\right.}$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \frac{\left(fg^2-egh\ \cdots\right.}{2h\left(cg^2+\cdots\right.}$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \frac{\left(fg^2-egh\ \cdots\right.}{2h\left(cg^2+\cdots\right.}$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \frac{\left(fg^2-egh\ \cdots\right.}{2h\left(cg^2+\cdots\right.}$$

Mathematica [A] time = 0.652985, size = 318, normalized size = 1.07

$$\frac{-\frac{\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(2a^2fh^4+ach^2\left(h(dh-3eg)+9fg^2\right)+2c^2g^3(3fg-eh)\right)}{\left(ah^2+cg^2\right)^{3/2}}+\frac{\log(g+hx)\left(2a^2fh^4+ach^2\left(h(dh-3eg)+9fg^2\right)+2c^2g^3(3fg-eh)\right)}{\left(ah^2+cg^2\right)^{3/2}}+h\sqrt{a\ \cdots}}{2h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (h*Sqrt[a + c*x^2]*(2*f + (-(f*g^2) + h*(e*g - d*h))/(g + h*x)^2 + (5*c*f*g
^3 + c*g*h*(-3*e*g + d*h) - 2*a*h^2*(-2*f*g + e*h))/((c*g^2 + a*h^2)*(g + h
*x))) + ((2*a^2*f*h^4 + 2*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 + h*(-3*
e*g + d*h)))*Log[g + h*x])/(c*g^2 + a*h^2)^(3/2) + 2*Sqrt[c]*(-3*f*g + e*h)
*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - ((2*a^2*f*h^4 + 2*c^2*g^3*(3*f*g - e*
h) + a*c*h^2*(9*f*g^2 + h*(-3*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a
*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(3/2))/(2*h^4)
```

Maple [B] time = 0.239, size = 4432, normalized size = 15.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x)
```

```
[Out] -1/2/h/(a*h^2+c*g^2)/(x+g/h)^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h
^2)^(3/2)*d-f/h^4*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*
g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))-f/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2
*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c
-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a+1/2/h*c/(a*h^2+c*g^2)
*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/2/h^3*c^2*g^4/(a
*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+
g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)
/h^2)^(1/2))/(x+g/h))*a*f+1/2/h*c^2*g^2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)
^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*
((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*d-1/2/h^2
*c^2*g^3/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-
2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a
*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*e-5/2/h^3*c/(a*h^2+c*g^2)/((a*h^2+c*g^2)
/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(
1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g^
2+3/2/h^2*c/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2
-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(
a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*e*g+f/h^3*((x+g/h)^2*c-2*c*g/h*(x+g/h)+
(a*h^2+c*g^2)/h^2)^(1/2)-1/h/(a*h^2+c*g^2)/(x+g/h)*((x+g/h)^2*c-2*c*g/h*(x+
g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*e+1/2/h^5*c^3*g^6/(a*h^2+c*g^2)^2/((a*h^2+c*g
^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2
)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f+1
/2/h*c*g^2/(a*h^2+c*g^2)^2/(x+g/h)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^
2)/h^2)^(3/2)*e+1/2/h^2*c^(3/2)*g^3/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c
^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*a*f-1/2/h*c/(
a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g
/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/
h^2)^(1/2))/(x+g/h))*a*d-1/2/h^3*c^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2
)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g
/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*g^2*d-5/2/h^5*c^
2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(
x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^
2)/h^2)^(1/2))/(x+g/h))*g^4*f-1/2/h*c^(3/2)*g^2/(a*h^2+c*g^2)^2*ln((-c*g/h+
(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*a
*e-1/2/h*c^2*g^2/(a*h^2+c*g^2)^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)
/h^2)^(1/2)*x*e+1/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2*((x+g/h)^2*c-2*c*g/h*(x+g/h
)+(a*h^2+c*g^2)/h^2)^(1/2)*x*f-1/2/h^2*c*g^3/(a*h^2+c*g^2)^2/(x+g/h)*((x+g/
h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*f-2/h^2*c/(a*h^2+c*g^2)*((x
+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*f*g-2/h^2*c^(1/2)/(a*h
```

```
^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2
+c*g^2)/h^2)^(1/2))*a*f*g+3/2/h^4*c^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+
g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*g^3*e+1/2/h^3*c
^3*g^4/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*
c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h
^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d-1/2/h^4*c^3*g^5/(a*h^2+c*g^2)^2/((a*h^2+c*
g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^
2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+
3/2/h^3*c^(3/2)/(a*h^2+c*g^2)*g^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*
c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*e-5/2/h^4*c^(3/2)/(a*h^2+c*g^2)
*g^3*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^
2)/h^2)^(1/2))*f+1/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2*((x+g/h)^2*c-2*c*g/h*(x+g/
h)+(a*h^2+c*g^2)/h^2)^(1/2)*e-1/2/h^3*c^2*g^4/(a*h^2+c*g^2)^2*((x+g/h)^2*c-
2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f+1/2/h^2*c^(5/2)*g^3/(a*h^2+c*g^2
)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2
)/h^2)^(1/2))*d-1/2/h^3*c^(5/2)*g^4/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c
^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*e-1/2/h*c^2*g
^2/(a*h^2+c*g^2)^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*d-
f/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*(
(a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1
/2))/(x+g/h))*c*g^2-1/2/h^3/(a*h^2+c*g^2)/(x+g/h)^2*((x+g/h)^2*c-2*c*g/h*(x
+g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*f*g^2+1/h*c/(a*h^2+c*g^2)*((x+g/h)^2*c-2*c*g
/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*e+1/2/h^2/(a*h^2+c*g^2)/(x+g/h)^2*((x
+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*e*g+1/2*c^(3/2)*g/(a*h^2
+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2
+c*g^2)/h^2)^(1/2))*a*d-1/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)*((x+g/h)^2*c-2*c*g/
h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*d+1/2*c^2*g/(a*h^2+c*g^2)^2*((x+g/h)^2*c
-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*d+2/h^2/(a*h^2+c*g^2)/(x+g/h)*(
(x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*f*g+5/2/h^3*c/(a*h^2+c
*g^2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2-1/2/h^2*c
^(3/2)/(a*h^2+c*g^2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(
x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*d+1/2/h^4*c^(5/2)*g^5/(a*h^2+c*g^2)^2*ln((
-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(
1/2))*f-3/2/h^2*c/(a*h^2+c*g^2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/
h^2)^(1/2)*e*g+1/h*c^(1/2)/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+
g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*a*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}\left(d + ex + fx^2\right)}{\left(g + hx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**3,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Giac [B] time = 1.42267, size = 1246, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] -(6*c^2*f*g^4 + 9*a*c*f*g^2*h^2 + a*c*d*h^4 + 2*a^2*f*h^4 - 2*c^2*g^3*h*e -
 3*a*c*g*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-
c*g^2 - a*h^2))/((c*g^2*h^4 + a*h^6)*sqrt(-c*g^2 - a*h^2)) + sqrt(c*x^2 + a
```

```
)*f/h^3 + (6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h + 2*(sqrt(c)*x - s
qrt(c*x^2 + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g
^2*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 4*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*c^2*g^3*h^2*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e +
 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 2*(sqrt(c)*x - sqrt(c*x
^2 + a))^2*c^(5/2)*d*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*
f*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*
x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 6*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*c^(5/2)*g^4*h*e - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e +
 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 14*(sqrt(c)*x - sqrt
(c*x^2 + a))*a*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^
3 - 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*
x^2 + a))*a^2*c*d*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e + 5
*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 5*a^2*c^(3/2)*f*g^3*h^2 + a^
2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 3*a^2*c^(3/2)*g^2*h^3*e - 2*a^3
*sqrt(c)*h^5*e)/((c*g^2*h^4 + a*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2
*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2) + (3*sqrt(c)*f*g - sqrt(
c)*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4
```

3.85 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=314

$$-\frac{\sqrt{a+cx^2}\left(hx\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-dg^2h^2\right)\right)+a^2eh^5+acgh^2\left(dh^2+3fg^2\right)+2c^2fg^5\right)}{2h^3(g+hx)^2\left(ah^2+cg^2\right)^2}+\frac{c\tanh^{-1}\big(\ldots}{\ldots}$$

```
[Out] -((2*c^2*f*g^5 + a^2*e*h^5 + a*c*g*h^2*(3*f*g^2 + d*h^2) + h*(2*a^2*f*h^4 +
 a*c*g*h^2*(6*f*g - e*h) + c^2*(3*f*g^4 - d*g^2*h^2))*x)*Sqrt[a + c*x^2])/(
2*h^3*(c*g^2 + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)
^(3/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*ArcTanh[(Sqrt[c]*x)/
Sqrt[a + c*x^2]])/h^4 + (c*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2
*(5*f*g^2 - d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x
^2])])/(2*h^4*(c*g^2 + a*h^2)^(5/2))
```

Rubi [A] time = 0.505451, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 811, 844, 217, 206, 725}

$$-\frac{\sqrt{a+cx^2}\left(hx\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-dg^2h^2\right)\right)+a^2eh^5+acgh^2\left(dh^2+3fg^2\right)+2c^2fg^5\right)}{2h^3(g+hx)^2\left(ah^2+cg^2\right)^2}+\frac{c\tanh^{-1}\big(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -((2*c^2*f*g^5 + a^2*e*h^5 + a*c*g*h^2*(3*f*g^2 + d*h^2) + h*(2*a^2*f*h^4 +
 a*c*g*h^2*(6*f*g - e*h) + c^2*(3*f*g^4 - d*g^2*h^2))*x)*Sqrt[a + c*x^2])/(
2*h^3*(c*g^2 + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)
^(3/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*ArcTanh[(Sqrt[c]*x)/
Sqrt[a + c*x^2]])/h^4 + (c*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2
*(5*f*g^2 - d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x
^2])])/(2*h^4*(c*g^2 + a*h^2)^(5/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-3(cdg-afg+aeh)-3f\left(\frac{cg^2}{h}+ah\right)x\right)\sqrt{a+cx^2}}{(g+hx)^3}\,dx}{3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-\right.\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-\right.\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-\right.\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-\right.\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

Mathematica [A] time = 0.984721, size = 382, normalized size = 1.22

$$\frac{\frac{h\sqrt{a+cx^2}\left(-\frac{(g+hx)^2\left(6a^2fh^4+ach^2\left(h(2dh-5eg)+20fg^2\right)+c^2\left(11fg^4-g^2h(dh+2eg)\right)\right)}{\left(ah^2+cg^2\right)^2}+\frac{(g+hx)\left(-3ah^2(eh-2fg)+cgh(dh-4eg)+7cfg^3\right)}{ah^2+cg^2}-2\left(h(dh-eg)+fg^2\right)\right)}{(g+hx)^3}+\frac{3c\log\left(\sqrt{a+cx^2}\sqrt{ah^2+c\right.}{}}{6h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(-2*(f*g^2 + h*(-(e*g) + d*h)) + ((7*c*f*g^3 + c*g*h*(-
4*e*g + d*h) - 3*a*h^2*(-2*f*g + e*h))*(g + h*x))/(c*g^2 + a*h^2) - ((6*a^2
*f*h^4 + c^2*(11*f*g^4 - g^2*h*(2*e*g + d*h)) + a*c*h^2*(20*f*g^2 + h*(-5*e
*g + 2*d*h)))*(g + h*x)^2)/(c*g^2 + a*h^2)^2))/(g + h*x)^3 - (3*c*(2*c^2*f*
g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[g + h*x])/(c
*g^2 + a*h^2)^(5/2) + 6*Sqrt[c]*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c
*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[a*
h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/(6
*h^4)
```

Maple [B] time = 0.244, size = 5565, normalized size = 17.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}\left(d + ex + fx^2\right)}{\left(g + hx\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Giac [B] time = 1.47842, size = 2321, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] -(2*c^3*f*g^5 + 5*a*c^2*f*g^3*h^2 - a*c^2*d*g*h^4 + 4*a^2*c*f*g*h^4 - a^2*c
*h^5*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 -
 a*h^2))/((c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8)*sqrt(-c*g^2 - a*h^2)) - s
qrt(c)*f*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4 - 1/3*(18*(sqrt(c)*x -
sqrt(c*x^2 + a))^5*c^3*f*g^5*h^2 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2
*f*g^3*h^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*
x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^
3*g^4*h^3*e - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*g^2*h^5*e - 3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a^2*c*h^7*e + 54*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
c^(7/2)*f*g^6*h - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 87*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*f*g^4*h^3 + 3*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c
^(3/2)*f*g^2*h^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*f*h^7 - 12*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*c^(7/2)*g^5*h^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2
)*g^3*h^4*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*g*h^6*e + 44*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*c^4*f*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*c^4*d*g^5*h^2 + 14*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 + 14*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*d*g^3*h^4 - 96*(sqrt(c)*x - sqrt(c*x^2
+ a))^3*a^2*c^2*f*g^3*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*
h^6 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c*f*g*h^6 - 8*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*c^4*g^6*h*e - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*g^4*h
^3*e + 30*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*g^2*h^5*e - 78*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a*c^(7/2)*f*g^6*h + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a*c^(7/2)*d*g^4*h^3 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*f*g^
4*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*d*g^2*h^5 + 12*(sqrt
(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*f*h^7 + 12*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a*c^(7/2)*g^5*h^2*e + 30*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)
```

*g^3*h^4*e - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*g*h^6*e + 48*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*d*g^3*h^4 + 87*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*f*g^3*h^4 + 9*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*f*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*g^4*h^3*e - 18*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*g^2*h^5*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*h^7*e - 11*a^3*c^(5/2)*f*g^4*h^3 + a^3*c^(5/2)*d*g^2*h^5 - 20*a^4*c^(3/2)*f*g^2*h^5 - 2*a^4*c^(3/2)*d*h^7 - 6*a^5*sqrt(c)*f*h^7 + 2*a^3*c^(5/2)*g^3*h^4*e + 5*a^4*c^(3/2)*g*h^6*e)/((c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^3)

3.86 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5} dx$

Optimal. Leaf size=313

$$-\frac{\sqrt{a+cx^2}(ah-cgx)\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^3}-\frac{ac\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(4a^2fh^2-ac\left(fg^2-h(5\right.\right.}{8\left(ah^2+cg^2\right)^{7/2}}$$

```
[Out] -((4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*(5*e*g - d*h)))*(a*h - c*g*x)
*Sqrt[a + c*x^2])/(8*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h
^2)*(a + c*x^2)^(3/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) + ((4*a*h^2*(2*f*g
 - e*h) + c*g*(3*f*g^2 + h*(e*g - 5*d*h)))*(a + c*x^2)^(3/2))/(12*h*(c*g^2
+ a*h^2)^2*(g + h*x)^3) - (a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*
(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
])/(8*(c*g^2 + a*h^2)^(7/2))
```

Rubi [A] time = 0.42881, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 807, 721, 725, 206}

$$-\frac{\sqrt{a+cx^2}(ah-cgx)\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^3}-\frac{ac\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(4a^2fh^2-ac\left(fg^2-h(5\right.\right.}{8\left(ah^2+cg^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] -((4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*(5*e*g - d*h)))*(a*h - c*g*x)
*Sqrt[a + c*x^2])/(8*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h
^2)*(a + c*x^2)^(3/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) + ((3*c*f*g^3 + c*
g*h*(e*g - 5*d*h) + 4*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(3/2))/(12*h*(c*g^2
+ a*h^2)^2*(g + h*x)^3) - (a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*
(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
])/(8*(c*g^2 + a*h^2)^(7/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh+c\left(eg+\frac{3fg^2}{h}-dh\right)\right)x\right)\sqrt{a+cx^2}}{(g+hx)^4}dx}{4\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} + \frac{\left(3cfg^3+cgh(eg-5dh)+4ah^2(2fg-eh)\right)\left(a+cx^2\right.}{12h\left(cg^2+ah^2\right)^2(g+hx)^3}$$

$$= -\frac{\left(4c^2dg^2+4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} - \frac{\left(fg^2-egh+dh^2\right.}{4h\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(4c^2dg^2+4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} - \frac{\left(fg^2-egh+dh^2\right.}{4h\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(4c^2dg^2+4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} - \frac{\left(fg^2-egh+dh^2\right.}{4h\left(cg^2+ah^2\right)}$$

Mathematica [A] time = 1.31562, size = 439, normalized size = 1.4

$$\frac{1}{24}\left(-\frac{\sqrt{a+cx^2}\left((g+hx)^2\left(ah^2+cg^2\right)\left(12a^2fh^4+ach^2\left(h(3dh-7eg)+35fg^2\right)+2c^2\left(9fg^4-g^2h(dh+eg)\right)\right)-c(g+hx\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) - 2*(c*
g^2 + a*h^2)^2*(9*c*f*g^3 + c*g*h*(-5*e*g + d*h) - 4*a*h^2*(-2*f*g + e*h))*
(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(9*f*g^4 - g^2*h*(e*g + d
*h)) + a*c*h^2*(35*f*g^2 + h*(-7*e*g + 3*d*h)))*(g + h*x)^2 - c*(4*a^2*h^4*
(7*f*g - 2*e*h) + a*c*g*h^2*(19*f*g^2 + h*(9*e*g - 13*d*h)) + 2*c^2*(3*f*g^
5 + g^3*h*(e*g + d*h)))*(g + h*x)^3))/((c*g^2*h + a*h^3)^3*(g + h*x)^4)) +
(3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 + h*(-5*e*g + d*h)))*Log[g +
 h*x])/(c*g^2 + a*h^2)^(7/2) - (3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g
^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^
2]])/(c*g^2 + a*h^2)^(7/2))/24
```

Maple [B] time = 0.25, size = 7237, normalized size = 23.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}\left(d + ex + fx^2\right)}{\left(g + hx\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**5,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Giac [B] time = 3.61287, size = 2869, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] -1/24*((6*c^(9/2)*f*g^7*h^9*abs(h) + 2*c^(9/2)*d*g^5*h^11*abs(h) + 25*a*c^(
7/2)*f*g^5*h^11*abs(h) - 11*a*c^(7/2)*d*g^3*h^13*abs(h) + 47*a^2*c^(5/2)*f*
g^3*h^13*abs(h) - 13*a^2*c^(5/2)*d*g*h^15*abs(h) + 28*a^3*c^(3/2)*f*g*h^15*
abs(h) + 2*c^(9/2)*g^6*h^10*abs(h)*e + 11*a*c^(7/2)*g^4*h^12*abs(h)*e + a^2
*c^(5/2)*g^2*h^14*abs(h)*e - 8*a^3*c^(3/2)*h^16*abs(h)*e - 12*sqrt(c*g^2 +
a*h^2)*a*c^3*d*g^2*h^14*log(abs(c^(3/2)*g^2*abs(h) + a*sqrt(c)*h^2*abs(h) -
 sqrt(c*g^2 + a*h^2)*c*g*h)) + 3*sqrt(c*g^2 + a*h^2)*a^2*c^2*f*g^2*h^14*log
(abs(c^(3/2)*g^2*abs(h) + a*sqrt(c)*h^2*abs(h) - sqrt(c*g^2 + a*h^2)*c*g*h)
) + 3*sqrt(c*g^2 + a*h^2)*a^2*c^2*d*h^16*log(abs(c^(3/2)*g^2*abs(h) + a*sqr
t(c)*h^2*abs(h) - sqrt(c*g^2 + a*h^2)*c*g*h)) - 12*sqrt(c*g^2 + a*h^2)*a^3*
c*f*h^16*log(abs(c^(3/2)*g^2*abs(h) + a*sqrt(c)*h^2*abs(h) - sqrt(c*g^2 + a
*h^2)*c*g*h)) - 15*sqrt(c*g^2 + a*h^2)*a^2*c^2*g*h^15*e*log(abs(c^(3/2)*g^2
*abs(h) + a*sqrt(c)*h^2*abs(h) - sqrt(c*g^2 + a*h^2)*c*g*h)))*sgn(1/(h*x +
g))*sgn(h)/(c^5*g^10*abs(h) + 5*a*c^4*g^8*h^2*abs(h) + 10*a^2*c^3*g^6*h^4*a
bs(h) + 10*a^3*c^2*g^4*h^6*abs(h) + 5*a^4*c*g^2*h^8*abs(h) + a^5*h^10*abs(h
)) - sqrt(c - 2*c*g/(h*x + g) + c*g^2/(h*x + g)^2 + a*h^2/(h*x + g)^2)*((6*
c^3*f*g^5*h^17*sgn(1/(h*x + g))*sgn(h) + 2*c^3*d*g^3*h^19*sgn(1/(h*x + g))*
sgn(h) + 19*a*c^2*f*g^3*h^19*sgn(1/(h*x + g))*sgn(h) - 13*a*c^2*d*g*h^21*sg
n(1/(h*x + g))*sgn(h) + 28*a^2*c*f*g*h^21*sgn(1/(h*x + g))*sgn(h) + 2*c^3*g
^4*h^18*e*sgn(1/(h*x + g))*sgn(h) + 9*a*c^2*g^2*h^20*e*sgn(1/(h*x + g))*sgn
(h) - 8*a^2*c*h^22*e*sgn(1/(h*x + g))*sgn(h))/(c^4*g^8*h^8 + 4*a*c^3*g^6*h^
10 + 6*a^2*c^2*g^4*h^12 + 4*a^3*c*g^2*h^14 + a^4*h^16) - ((18*c^3*f*g^6*h^1
8*sgn(1/(h*x + g))*sgn(h) - 2*c^3*d*g^4*h^20*sgn(1/(h*x + g))*sgn(h) + 53*a
*c^2*f*g^4*h^20*sgn(1/(h*x + g))*sgn(h) + a*c^2*d*g^2*h^22*sgn(1/(h*x + g))
*sgn(h) + 47*a^2*c*f*g^2*h^22*sgn(1/(h*x + g))*sgn(h) + 3*a^2*c*d*h^24*sgn(
1/(h*x + g))*sgn(h) + 12*a^3*f*h^24*sgn(1/(h*x + g))*sgn(h) - 2*c^3*g^5*h^1
9*e*sgn(1/(h*x + g))*sgn(h) - 9*a*c^2*g^3*h^21*e*sgn(1/(h*x + g))*sgn(h) -
7*a^2*c*g*h^23*e*sgn(1/(h*x + g))*sgn(h))/(c^4*g^8*h^8 + 4*a*c^3*g^6*h^10 +
 6*a^2*c^2*g^4*h^12 + 4*a^3*c*g^2*h^14 + a^4*h^16) - 2*((9*c^3*f*g^7*h^19*s
gn(1/(h*x + g))*sgn(h) + c^3*d*g^5*h^21*sgn(1/(h*x + g))*sgn(h) + 26*a*c^2*
```

```
f*g^5*h^21*sgn(1/(h*x + g))*sgn(h) + 2*a*c^2*d*g^3*h^23*sgn(1/(h*x + g))*sg
n(h) + 25*a^2*c*f*g^3*h^23*sgn(1/(h*x + g))*sgn(h) + a^2*c*d*g*h^25*sgn(1/(
h*x + g))*sgn(h) + 8*a^3*f*g*h^25*sgn(1/(h*x + g))*sgn(h) - 5*c^3*g^6*h^20*
e*sgn(1/(h*x + g))*sgn(h) - 14*a*c^2*g^4*h^22*e*sgn(1/(h*x + g))*sgn(h) - 1
3*a^2*c*g^2*h^24*e*sgn(1/(h*x + g))*sgn(h) - 4*a^3*h^26*e*sgn(1/(h*x + g))*
sgn(h))/(c^4*g^8*h^8 + 4*a*c^3*g^6*h^10 + 6*a^2*c^2*g^4*h^12 + 4*a^3*c*g^2*
h^14 + a^4*h^16) - 3*(c^3*f*g^8*h^20*sgn(1/(h*x + g))*sgn(h) + c^3*d*g^6*h^
22*sgn(1/(h*x + g))*sgn(h) + 3*a*c^2*f*g^6*h^22*sgn(1/(h*x + g))*sgn(h) + 3
*a*c^2*d*g^4*h^24*sgn(1/(h*x + g))*sgn(h) + 3*a^2*c*f*g^4*h^24*sgn(1/(h*x +
 g))*sgn(h) + 3*a^2*c*d*g^2*h^26*sgn(1/(h*x + g))*sgn(h) + a^3*f*g^2*h^26*s
gn(1/(h*x + g))*sgn(h) + a^3*d*h^28*sgn(1/(h*x + g))*sgn(h) - c^3*g^7*h^21*
e*sgn(1/(h*x + g))*sgn(h) - 3*a*c^2*g^5*h^23*e*sgn(1/(h*x + g))*sgn(h) - 3*
a^2*c*g^3*h^25*e*sgn(1/(h*x + g))*sgn(h) - a^3*g*h^27*e*sgn(1/(h*x + g))*sg
n(h))/((c^4*g^8*h^8 + 4*a*c^3*g^6*h^10 + 6*a^2*c^2*g^4*h^12 + 4*a^3*c*g^2*h
^14 + a^4*h^16)*(h*x + g)*h))/((h*x + g)*h))/((h*x + g)*h)) + 3*(4*a*c^3*d*
g^2*h^14*sgn(1/(h*x + g))*sgn(h) - a^2*c^2*f*g^2*h^14*sgn(1/(h*x + g))*sgn(
h) - a^2*c^2*d*h^16*sgn(1/(h*x + g))*sgn(h) + 4*a^3*c*f*h^16*sgn(1/(h*x + g
))*sgn(h) + 5*a^2*c^2*g*h^15*e*sgn(1/(h*x + g))*sgn(h))*sqrt(c*g^2 + a*h^2)
*log(abs(-sqrt(c*g^2 + a*h^2)*c*g*h + (c*g^2 + a*h^2)*(sqrt(c - 2*c*g/(h*x
+ g) + c*g^2/(h*x + g)^2 + a*h^2/(h*x + g)^2) + sqrt(c*g^2*h^2 + a*h^4)/((h
*x + g)*h))*abs(h)))/((c^5*g^10 + 5*a*c^4*g^8*h^2 + 10*a^2*c^3*g^6*h^4 + 10
*a^3*c^2*g^4*h^6 + 5*a^4*c*g^2*h^8 + a^5*h^10)*abs(h)))*h^2
```

3.87 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

Optimal. Leaf size=433

$$-\frac{c\sqrt{a+cx^2}(ah-cgx)\left(a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)+4c^2dg^3\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^4}-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18fg^2-h\right.\right.}{60h(g+h}$$

```
[Out] -(c*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)
))*(a*h - c*g*x)*Sqrt[a + c*x^2])/(8*(c*g^2 + a*h^2)^4*(g + h*x)^2) - ((f*g
^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(5*h*(c*g^2 + a*h^2)*(g + h*x)^5) +
((5*a*h^2*(2*f*g - e*h) + c*g*(3*f*g^2 + h*(2*e*g - 7*d*h)))*(a + c*x^2)^(3
/2))/(20*h*(c*g^2 + a*h^2)^2*(g + h*x)^4) - ((20*a^2*f*h^4 - c^2*g^2*(3*f*g
^2 + h*(2*e*g - 27*d*h)) - a*c*h^2*(18*f*g^2 - h*(33*e*g - 8*d*h)))*(a + c*
x^2)^(3/2))/(60*h*(c*g^2 + a*h^2)^3*(g + h*x)^3) - (a*c^2*(4*c^2*d*g^3 + a^
2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)))*ArcTanh[(a*h - c*g
*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(8*(c*g^2 + a*h^2)^(9/2))
```

Rubi [A] time = 0.742644, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 835, 807, 721, 725, 206}

$$-\frac{c\sqrt{a+cx^2}(ah-cgx)\left(a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)+4c^2dg^3\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^4}-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18fg^2-h\right.\right.}{60h(g+h}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(c*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)
))*(a*h - c*g*x)*Sqrt[a + c*x^2])/(8*(c*g^2 + a*h^2)^4*(g + h*x)^2) - ((f*g
^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(5*h*(c*g^2 + a*h^2)*(g + h*x)^5) +
((3*c*f*g^3 + c*g*h*(2*e*g - 7*d*h) + 5*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(3
/2))/(20*h*(c*g^2 + a*h^2)^2*(g + h*x)^4) - ((20*a^2*f*h^4 - c^2*(3*f*g^4 +
 g^2*h*(2*e*g - 27*d*h)) - a*c*h^2*(18*f*g^2 - h*(33*e*g - 8*d*h)))*(a + c*
x^2)^(3/2))/(60*h*(c*g^2 + a*h^2)^3*(g + h*x)^3) - (a*c^2*(4*c^2*d*g^3 + a^
2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)))*ArcTanh[(a*h - c*g
*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(8*(c*g^2 + a*h^2)^(9/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-5(cdg-afg+aeh)-\left(5afh+c\left(2eg+\frac{3fg^2}{h}-2dh\right)\right)x\right)\sqrt{a+cx^2}}{(g+hx)^5}\,dx}{5\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-eh)\right)\left(a+c\right.}{20h\left(cg^2+ah^2\right)^2(g+hx)^4}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-eh)\right)\left(a+c\right.}{20h\left(cg^2+ah^2\right)^2(g+hx)^4}$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(fg^2\right.}{5h}$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(fg^2\right.}{5h}$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(fg^2\right.}{5h}$$

Mathematica [A] time = 1.52842, size = 583, normalized size = 1.35

$$-\frac{\sqrt{a+cx^2}\left(2(g+hx)^2\left(ah^2+cg^2\right)^2\left(20a^2fh^4+ach^2\left(h(4dh-9eg)+54fg^2\right)+c^2\left(27fg^4-g^2h(3dh+2eg)\right)\right)-c(g+hx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) - 6*(c*g
^2 + a*h^2)^3*(11*c*f*g^3 + c*g*h*(-6*e*g + d*h) - 5*a*h^2*(-2*f*g + e*h))*
(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(27*f*g^4 - g^2*h*(2*e*
```

```
g + 3*d*h)) + a*c*h^2*(54*f*g^2 + h*(-9*e*g + 4*d*h)))*(g + h*x)^2 - c*(c*g
^2 + a*h^2)*(5*a^2*h^4*(10*f*g - 3*e*h) + a*c*g*h^2*(21*f*g^2 + h*(24*e*g -
 29*d*h)) + c^2*(6*f*g^5 + 2*g^3*h*(2*e*g + 3*d*h)))*(g + h*x)^3 - c*(-40*a
^3*f*h^6 + a*c^2*g^2*h^2*(27*f*g^2 + h*(28*e*g - 83*d*h)) + c^3*(6*f*g^6 +
2*g^4*h*(2*e*g + 3*d*h)) + a^2*c*h^4*(86*f*g^2 + h*(-81*e*g + 16*d*h)))*(g
+ h*x)^4))/(120*h^3*(c*g^2 + a*h^2)^4*(g + h*x)^5) + (a*c^2*(4*c^2*d*g^3 +
a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[g + h*x])/(
8*(c*g^2 + a*h^2)^(9/2)) - (a*c^2*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) - a*
c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2]])/(8*(c*g^2 + a*h^2)^(9/2))
```

Maple [B] time = 0.25, size = 8546, normalized size = 19.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x)

[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.62264, size = 5686, normalized size = 13.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(4*a*c^4*d*g^3 - a^2*c^3*f*g^3 - 3*a^2*c^3*d*g*h^2 + 6*a^3*c^2*f*g*h^2
 + 6*a^2*c^3*g^2*h*e - a^3*c^2*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))
*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c
^2*g^4*h^4 + 4*a^3*c*g^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/60*(60*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*d*g^3*h^8 - 15*(sqrt(c)*x - sqrt(c*x^2
+ a))^9*a^2*c^3*f*g^3*h^8 - 45*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*g*
h^10 + 90*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*f*g*h^10 + 90*(sqrt(c)*x
- sqrt(c*x^2 + a))^9*a^2*c^3*g^2*h^9*e - 15*(sqrt(c)*x - sqrt(c*x^2 + a))^9
*a^3*c^2*h^11*e - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 -
480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*f*g^6*h^5 + 540*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a*c^(9/2)*d*g^4*h^7 - 855*(sqrt(c)*x - sqrt(c*x^2 + a))^
8*a^2*c^(7/2)*f*g^4*h^7 - 405*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d
*g^2*h^9 + 330*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 120*
(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(3/2)*f*h^11 + 810*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(7/2)*g^3*h^8*e - 135*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a
^3*c^(5/2)*g*h^10*e - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 9
60*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*f*g^7*h^4 + 1880*(sqrt(c)*x - sqrt
```

(c*x^2 + a))^7*a*c^5*d*g^5*h^6 - 1910*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c
^4*f*g^5*h^6 - 1690*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 + 193
0*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*f*g^3*h^8 + 210*(sqrt(c)*x - sqrt
(c*x^2 + a))^7*a^3*c^3*d*g*h^10 - 660*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c
^2*f*g*h^10 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*g^8*h^3*e - 640*(sqrt
(c)*x - sqrt(c*x^2 + a))^7*a*c^5*g^6*h^5*e + 1860*(sqrt(c)*x - sqrt(c*x^2 +
 a))^7*a^2*c^4*g^4*h^7*e - 1530*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*g^2
*h^9*e - 90*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*h^11*e - 240*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*c^(13/2)*f*g^10*h - 240*(sqrt(c)*x - sqrt(c*x^2 + a))
^6*c^(13/2)*d*g^8*h^3 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^
8*h^3 + 2120*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 1250*(s
qrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h^5 - 5710*(sqrt(c)*x - sqr
t(c*x^2 + a))^6*a^2*c^(9/2)*d*g^4*h^7 + 5590*(sqrt(c)*x - sqrt(c*x^2 + a))^
6*a^3*c^(7/2)*f*g^4*h^7 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d
*g^2*h^9 - 2220*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 - 240
*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*d*h^11 + 240*(sqrt(c)*x - sqrt
(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(
13/2)*g^9*h^2*e - 640*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*g^7*h^4*e
 + 3660*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*g^5*h^6*e - 4350*(sqrt(c
)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*g^3*h^8*e + 330*(sqrt(c)*x - sqrt(c*x^
2 + a))^6*a^4*c^(5/2)*g*h^10*e - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*f*g
^11 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d*g^9*h^2 + 48*(sqrt(c)*x - sq
rt(c*x^2 + a))^5*a*c^6*f*g^9*h^2 + 1808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c
^6*d*g^7*h^4 + 604*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*f*g^7*h^4 - 7076
*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 + 6710*(sqrt(c)*x - sqrt
(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 + 3770*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3
*c^4*d*g^3*h^8 - 5780*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^3*h^8 - 4
80*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*g*h^10 + 1200*(sqrt(c)*x - sqr
t(c*x^2 + a))^5*a^5*c^2*f*g*h^10 - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*g
^10*h*e - 128*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*g^8*h^3*e + 3416*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*g^6*h^5*e - 7320*(sqrt(c)*x - sqrt(c*x^2
 + a))^5*a^3*c^4*g^4*h^7*e + 2430*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*g^
2*h^9*e + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10*h + 240*(sq
rt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 + 720*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^2*c^(11/2)*f*g^8*h^3 - 5240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
a^2*c^(11/2)*d*g^6*h^5 + 2450*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*f
*g^6*h^5 + 5590*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*d*g^4*h^7 - 766
0*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*f*g^4*h^7 - 2240*(sqrt(c)*x -
 sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 + 3440*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a^5*c^(5/2)*f*g^2*h^9 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2
)*d*h^11 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)*f*h^11 + 160*(sq
rt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*g^9*h^2*e + 1120*(sqrt(c)*x - sqrt(
c*x^2 + a))^4*a^2*c^(11/2)*g^7*h^4*e - 6140*(sqrt(c)*x - sqrt(c*x^2 + a))^4
*a^3*c^(9/2)*g^5*h^6*e + 5650*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*g
^3*h^8*e - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*g*h^10*e - 240*(

sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*d*g^7*h^4 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*f*g^7*h^4 + 5000*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*d*g^5*h^6 - 3890*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 - 2910*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 + 4710*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*f*g^3*h^8 + 430*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*d*g*h^10 - 940*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*f*g*h^10 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*g^8*h^3*e - 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*g^6*h^5*e + 5740*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*g^4*h^7*e - 1710*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*g^2*h^9*e + 90*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*h^11*e + 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*f*g^8*h^3 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*d*g^6*h^5 + 570*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*f*g^6*h^5 - 2810*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*d*g^4*h^7 + 2450*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*f*g^4*h^7 + 650*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*d*g^2*h^9 - 1700*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*f*g^2*h^9 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*d*h^11 + 80*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(3/2)*f*h^11 + 160*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*g^7*h^4*e + 1100*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*g^5*h^6*e - 2570*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*g^3*h^8*e + 270*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*g*h^10*e - 60*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*f*g^7*h^4 - 60*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*d*g^5*h^6 - 270*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*f*g^5*h^6 + 770*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*d*g^3*h^8 - 845*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*f*g^3*h^8 - 115*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*d*g*h^10 + 310*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^2*f*g*h^10 - 40*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*g^6*h^5*e - 280*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*g^4*h^7*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*g^2*h^9*e + 15*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^2*h^11*e + 6*a^5*c^(9/2)*f*g^6*h^5 + 6*a^5*c^(9/2)*d*g^4*h^7 + 27*a^6*c^(7/2)*f*g^4*h^7 - 83*a^6*c^(7/2)*d*g^2*h^9 + 86*a^7*c^(5/2)*f*g^2*h^9 + 16*a^7*c^(5/2)*d*h^11 - 40*a^8*c^(3/2)*f*h^11 + 4*a^5*c^(9/2)*g^5*h^6*e + 28*a^6*c^(7/2)*g^3*h^8*e - 81*a^7*c^(5/2)*g*h^10*e)/((c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^5)

3.88 $\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=462

$$\frac{\left(a+cx^2\right)^{5/2} \left(4 \left(32a^2fh^4-8ach^2\left(9h(dh+3eg)+17fg^2\right)-3c^2g^2\left(5fg^2-3h(64dh+3eg)\right)\right)-5chx\left(ah^2(63eh+61fg)+2\right.\right.}{5040c^3h}$$

```
[Out] (a*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*
h)))*x*Sqrt[a + c*x^2])/(128*c^2) + ((48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h
) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) + ((8
*(9*c*d - 4*a*f)*h^2 - 3*c*g*(5*f*g - 9*e*h))*(g + h*x)^2*(a + c*x^2)^(5/2)
)/(504*c^2*h) - ((5*f*g - 9*e*h)*(g + h*x)^3*(a + c*x^2)^(5/2))/(72*c*h) +
(f*(g + h*x)^4*(a + c*x^2)^(5/2))/(9*c*h) + ((4*(32*a^2*f*h^4 - 8*a*c*h^2*(
17*f*g^2 + 9*h*(3*e*g + d*h)) - 3*c^2*g^2*(5*f*g^2 - 3*h*(3*e*g + 64*d*h)))
 - 5*c*h*(a*h^2*(61*f*g + 63*e*h) + 2*c*g*(5*f*g^2 - 9*h*(e*g + 12*d*h)))*x
)*(a + c*x^2)^(5/2))/(5040*c^3*h) + (a^2*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g +
 e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/(128*c^(5/2))
```

Rubi [A] time = 1.13443, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{\left(a+cx^2\right)^{5/2} \left(4 \left(32a^2fh^4-8ach^2\left(9h(dh+3eg)+17fg^2\right)-c^2\left(15fg^4-9g^2h(64dh+3eg)\right)\right)-5chx\left(ah^2(63eh+61fg)+2\right.\right.}{5040c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*
h)))*x*Sqrt[a + c*x^2])/(128*c^2) + ((48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h
) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) + ((8
*(9*c*d - 4*a*f)*h^2 - 3*c*g*(5*f*g - 9*e*h))*(g + h*x)^2*(a + c*x^2)^(5/2)
)/(504*c^2*h) - ((5*f*g - 9*e*h)*(g + h*x)^3*(a + c*x^2)^(5/2))/(72*c*h) +
(f*(g + h*x)^4*(a + c*x^2)^(5/2))/(9*c*h) + ((4*(32*a^2*f*h^4 - 8*a*c*h^2*(
17*f*g^2 + 9*h*(3*e*g + d*h)) - c^2*(15*f*g^4 - 9*g^2*h*(3*e*g + 64*d*h)))
- 5*c*h*(a*h^2*(61*f*g + 63*e*h) + 2*c*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)))*x)
*(a + c*x^2)^(5/2))/(5040*c^3*h) + (a^2*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g +
e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
```

```
2]])/(128*c^(5/2))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^4 \left(a+cx^2\right)^{5/2}}{9ch} + \frac{\int (g+hx)^3 \left((9cd-4af)h^2 - ch(5fg-9eh)x\right) \left(a+\right.}{9ch^2}$$

$$= -\frac{(5fg-9eh)(g+hx)^3 \left(a+cx^2\right)^{5/2}}{72ch} + \frac{f(g+hx)^4 \left(a+cx^2\right)^{5/2}}{9ch} + \frac{\int (g+hx)^2}{}$$

$$= \frac{\left(8(9cd-4af)h^2 - 3cg(5fg-9eh)\right)(g+hx)^2 \left(a+cx^2\right)^{5/2}}{504c^2h} - \frac{(5fg-9eh)(g+}{7}$$

$$= \frac{\left(8(9cd-4af)h^2 - 3cg(5fg-9eh)\right)(g+hx)^2 \left(a+cx^2\right)^{5/2}}{504c^2h} - \frac{(5fg-9eh)(g+}{7}$$

$$= \frac{\left(48c^2dg^3 + 3a^2h^2(3fg+eh) - 8acg\left(fg^2+3h(eg+dh)\right)\right)x\left(a+cx^2\right)^{3/2}}{192c^2} + \frac{(}{}$$

$$= \frac{a\left(48c^2dg^3 + 3a^2h^2(3fg+eh) - 8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} + \frac{(4}{}$$

$$= \frac{a\left(48c^2dg^3 + 3a^2h^2(3fg+eh) - 8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} + \frac{(4}{}$$

$$= \frac{a\left(48c^2dg^3 + 3a^2h^2(3fg+eh) - 8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} + \frac{(4}{}$$

Mathematica [A] time = 0.573367, size = 481, normalized size = 1.04

$$\frac{\sqrt{a+cx^2}\left(384c^2x^4\left(a^2fh^3+24ach\left(h(dh+3eg)+3fg^2\right)+21c^2g^2(3dh+eg)\right)+210c^2x^3\left(3a^2h^2(eh+3fg)+56acg\left(3h(d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(128*a^2*(8*a^2*f*h^3 + 63*c^2*g^2*(e*g + 3*d*h) - 18*a*c*
h*(3*f*g^2 + h*(3*e*g + d*h))) + 315*a*c*(80*c^2*d*g^3 - 3*a^2*h^2*(3*f*g +
```

```
 e*h) + 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x + 128*a*c*(-4*a^2*f*h^3 + 126*
c^2*g^2*(e*g + 3*d*h) + 9*a*c*h*(3*f*g^2 + h*(3*e*g + d*h)))*x^2 + 210*c^2*
(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) + 56*a*c*g*(f*g^2 + 3*h*(e*g + d*h)
))*x^3 + 384*c^2*(a^2*f*h^3 + 21*c^2*g^2*(e*g + 3*d*h) + 24*a*c*h*(3*f*g^2
+ h*(3*e*g + d*h)))*x^4 + 840*c^3*(9*a*h^2*(3*f*g + e*h) + 8*c*(f*g^3 + 3*g
*h*(e*g + d*h)))*x^5 + 640*c^3*h*(10*a*f*h^2 + 9*c*(3*f*g^2 + h*(3*e*g + d*
h)))*x^6 + 5040*c^4*h^2*(3*f*g + e*h)*x^7 + 4480*c^4*f*h^3*x^8) + 315*a^2*S
qrt[c]*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g
+ d*h)))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(40320*c^3)
```

Maple [A] time = 0.061, size = 794, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] -1/8*a/c*x*(c*x^2+a)^(3/2)*e*g^2*h-3/16*a^2/c*x*(c*x^2+a)^(1/2)*d*g*h^2-3/1
6*a^2/c*x*(c*x^2+a)^(1/2)*e*g^2*h-3/16*a/c^2*x*(c*x^2+a)^(5/2)*f*g*h^2+3/64
*a^2/c^2*x*(c*x^2+a)^(3/2)*f*g*h^2-1/16*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^
(1/2))*f*g^3+1/8*x^3*(c*x^2+a)^(5/2)/c*e*h^3+3/128*a^4/c^(5/2)*ln(x*c^(1/2)
+(c*x^2+a)^(1/2))*e*h^3+1/9*f*h^3*x^4*(c*x^2+a)^(5/2)/c+8/315*f*h^3*a^2/c^3
*(c*x^2+a)^(5/2)+3/5*(c*x^2+a)^(5/2)/c*d*g^2*h+1/7*x^2*(c*x^2+a)^(5/2)/c*d*
h^3-2/35*a/c^2*(c*x^2+a)^(5/2)*d*h^3+1/6*x*(c*x^2+a)^(5/2)/c*f*g^3+3/8*d*g^
3*a*x*(c*x^2+a)^(1/2)+3/8*d*g^3*a^2/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+1
/5*(c*x^2+a)^(5/2)/c*e*g^3+1/4*d*g^3*x*(c*x^2+a)^(3/2)+9/128*a^3/c^2*x*(c*x
^2+a)^(1/2)*f*g*h^2-1/8*a/c*x*(c*x^2+a)^(3/2)*d*g*h^2-4/63*f*h^3*a/c^2*x^2*
(c*x^2+a)^(5/2)+3/8*x^3*(c*x^2+a)^(5/2)/c*f*g*h^2-1/16*a/c^2*x*(c*x^2+a)^(5
/2)*e*h^3+1/64*a^2/c^2*x*(c*x^2+a)^(3/2)*e*h^3+3/128*a^3/c^2*x*(c*x^2+a)^(1
/2)*e*h^3+9/128*a^4/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*f*g*h^2+3/7*x^2*(
c*x^2+a)^(5/2)/c*f*g^2*h-6/35*a/c^2*(c*x^2+a)^(5/2)*e*g*h^2-6/35*a/c^2*(c*x
^2+a)^(5/2)*f*g^2*h+1/2*x*(c*x^2+a)^(5/2)/c*d*g*h^2+1/2*x*(c*x^2+a)^(5/2)/c
*e*g^2*h-1/24*a/c*x*(c*x^2+a)^(3/2)*f*g^3-1/16*a^2/c*x*(c*x^2+a)^(1/2)*f*g^
3-3/16*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*d*g*h^2-3/16*a^3/c^(3/2)*l
n(x*c^(1/2)+(c*x^2+a)^(1/2))*e*g^2*h+3/7*x^2*(c*x^2+a)^(5/2)/c*e*g*h^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.57994, size = 2634, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/80640*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^2*c^2*d - a^3*c*f)*
g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 +
a)*sqrt(c)*x - a) - 2*(4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^3*c
*e*g*h^2 + 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h + 27*
c^4*e*g*h^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*c^4*e
*g^2*h + 9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21*c^4*e
*g^3 + 72*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*d + a^2
*c^2*f)*h^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3*c*d -
4*a^4*f)*h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4*d + 7*a*
c^3*f)*g^3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a*c^3*e*g^3
 + 27*a^2*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2*d -
4*a^3*c*f)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 + 8*(10*a*c^3
*d + a^2*c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*sqrt(c*x^2 + a)
)/c^3, 1/40320*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^2*c^2*d - a^3*
c*f)*g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^3*c*e*g*h^2
+ 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h + 27*c^4*e*g*h
^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*c^4*e*g^2*h +
9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21*c^4*e*g^3 + 72
*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*d + a^2*c^2*f)*h
^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3*c*d - 4*a^4*f)*
h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4*d + 7*a*c^3*f)*g^
3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a*c^3*e*g^3 + 27*a^2
*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2*d - 4*a^3*c*f
)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 + 8*(10*a*c^3*d + a^2*
c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 67.6183, size = 1916, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] -3*a**(7/2)*e*h**3*x/(128*c**2*sqrt(1 + c*x**2/a)) - 9*a**(7/2)*f*g*h**2*x/
(128*c**2*sqrt(1 + c*x**2/a)) + 3*a**(5/2)*d*g*h**2*x/(16*c*sqrt(1 + c*x**2
/a)) + 3*a**(5/2)*e*g**2*h*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*e*h**3*x*
*3/(128*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g**3*x/(16*c*sqrt(1 + c*x**2/a))
 - 3*a**(5/2)*f*g*h**2*x**3/(128*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*g**3*x*
sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**3*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/
2)*d*g*h**2*x**3/(16*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*e*g**2*h*x**3/(16*sq
rt(1 + c*x**2/a)) + 13*a**(3/2)*e*h**3*x**5/(64*sqrt(1 + c*x**2/a)) + 17*a*
*(3/2)*f*g**3*x**3/(48*sqrt(1 + c*x**2/a)) + 39*a**(3/2)*f*g*h**2*x**5/(64*
sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g**3*x**3/(8*sqrt(1 + c*x**2/a)) + 11*s
qrt(a)*c*d*g*h**2*x**5/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g**2*h*x**5/
(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*e*h**3*x**7/(16*sqrt(1 + c*x**2/a)) +
11*sqrt(a)*c*f*g**3*x**5/(24*sqrt(1 + c*x**2/a)) + 15*sqrt(a)*c*f*g*h**2*x*
*7/(16*sqrt(1 + c*x**2/a)) + 3*a**4*e*h**3*asinh(sqrt(c)*x/sqrt(a))/(128*c*
*(5/2)) + 9*a**4*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)) - 3*a**3*
d*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - 3*a**3*e*g**2*h*asinh(sqr
t(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(16*c*
*(3/2)) + 3*a**2*d*g**3*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 3*a*d*g**2*h
*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) +
 a*d*h**3*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c
*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True))
 + a*e*g**3*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c
), True)) + 3*a*e*g*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*
x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)
*x**4/4, True)) + 3*a*f*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2
) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (s
qrt(a)*x**4/4, True)) + a*f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c*
*3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35
*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g
**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**
2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c
*d*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*e*g**3*Piecewise((-2*a**2*sqrt
(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x
**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*c*e*g*h**2*Piecewise((8*a**3
```

```
*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*
x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)
*x**6/6, True)) + 3*c*f*g**2*h*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3
) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c
) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*f*h**3*
Piecewise((-16*a**4*sqrt(a + c*x**2)/(315*c**4) + 8*a**3*x**2*sqrt(a + c*x*
*2)/(315*c**3) - 2*a**2*x**4*sqrt(a + c*x**2)/(105*c**2) + a*x**6*sqrt(a +
c*x**2)/(63*c) + x**8*sqrt(a + c*x**2)/9, Ne(c, 0)), (sqrt(a)*x**8/8, True)
) + c**2*d*g**3*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*d*g*h**2*x**7/(2
*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*g**2*h*x**7/(2*sqrt(a)*sqrt(1 + c*x**
2/a)) + c**2*e*h**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*g**3*x**7/
(6*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*c**2*f*g*h**2*x**9/(8*sqrt(a)*sqrt(1 + c
*x**2/a))
```

Giac [A] time = 1.22935, size = 880, normalized size = 1.9

$$\frac{1}{40320}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(2\left(7\left(8\,cfh^3x+\frac{9\left(3\,c^8fgh^2+c^8h^3e\right)}{c^7}\right)x+\frac{8\left(27\,c^8fg^2h+9\,c^8dh^3+10\,ac^7fh^3+27\,c^8gh^2e\right)}{c^7}\right)x\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/40320*sqrt(c*x^2 + a)*((2*((4*(5*(2*(7*(8*c*f*h^3*x + 9*(3*c^8*f*g*h^2 +
c^8*h^3*e)/c^7)*x + 8*(27*c^8*f*g^2*h + 9*c^8*d*h^3 + 10*a*c^7*f*h^3 + 27*c
^8*g*h^2*e)/c^7)*x + 21*(8*c^8*f*g^3 + 24*c^8*d*g*h^2 + 27*a*c^7*f*g*h^2 +
24*c^8*g^2*h*e + 9*a*c^7*h^3*e)/c^7)*x + 48*(63*c^8*d*g^2*h + 72*a*c^7*f*g^
2*h + 24*a*c^7*d*h^3 + a^2*c^6*f*h^3 + 21*c^8*g^3*e + 72*a*c^7*g*h^2*e)/c^7
)*x + 105*(48*c^8*d*g^3 + 56*a*c^7*f*g^3 + 168*a*c^7*d*g*h^2 + 9*a^2*c^6*f*
g*h^2 + 168*a*c^7*g^2*h*e + 3*a^2*c^6*h^3*e)/c^7)*x + 64*(378*a*c^7*d*g^2*h
 + 27*a^2*c^6*f*g^2*h + 9*a^2*c^6*d*h^3 - 4*a^3*c^5*f*h^3 + 126*a*c^7*g^3*e
 + 27*a^2*c^6*g*h^2*e)/c^7)*x + 315*(80*a*c^7*d*g^3 + 8*a^2*c^6*f*g^3 + 24*
a^2*c^6*d*g*h^2 - 9*a^3*c^5*f*g*h^2 + 24*a^2*c^6*g^2*h*e - 3*a^3*c^5*h^3*e)
/c^7)*x + 128*(189*a^2*c^6*d*g^2*h - 54*a^3*c^5*f*g^2*h - 18*a^3*c^5*d*h^3
+ 8*a^4*c^4*f*h^3 + 63*a^2*c^6*g^3*e - 54*a^3*c^5*g*h^2*e)/c^7) - 1/128*(48
*a^2*c^2*d*g^3 - 8*a^3*c*f*g^3 - 24*a^3*c*d*g*h^2 + 9*a^4*f*g*h^2 - 24*a^3*
c*g^2*h*e + 3*a^4*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.89 $\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=346

$$\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2dg^2\right)}{192c^2}+\frac{ax\sqrt{a+cx^2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2d\right.}{128c^2}$$

```
[Out] (a*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a
+ c*x^2])/(128*c^2) + ((48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*
g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) - ((5*f*g - 8*e*h)*(g + h*x)^2*(a
 + c*x^2)^(5/2))/(56*c*h) + (f*(g + h*x)^3*(a + c*x^2)^(5/2))/(8*c*h) - ((1
2*(8*a*h^2*(2*f*g + e*h) + c*g*(5*f*g^2 - 8*h*(e*g + 7*d*h))) - 5*h*(7*(8*c
*d - 3*a*f)*h^2 - 2*c*g*(5*f*g - 8*e*h))*x)*(a + c*x^2)^(5/2))/(1680*c^2*h)
 + (a^2*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(5/2))
```

Rubi [A] time = 0.524336, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2dg^2\right)}{192c^2}+\frac{ax\sqrt{a+cx^2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2d\right.}{128c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a
+ c*x^2])/(128*c^2) + ((48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*
g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) - ((5*f*g - 8*e*h)*(g + h*x)^2*(a
 + c*x^2)^(5/2))/(56*c*h) + (f*(g + h*x)^3*(a + c*x^2)^(5/2))/(8*c*h) - ((1
2*(5*c*f*g^3 - 8*c*g*h*(e*g + 7*d*h) + 8*a*h^2*(2*f*g + e*h)) - 5*h*(7*(8*c
*d - 3*a*f)*h^2 - 2*c*g*(5*f*g - 8*e*h))*x)*(a + c*x^2)^(5/2))/(1680*c^2*h)
 + (a^2*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(5/2))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch}+\frac{\int (g+hx)^2\left((8cd-3af)h^2-ch(5fg-8eh)x\right)\left(a\right.}{8ch^2}$$

$$= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch}+\frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch}+\frac{\int (g+hx)}{}$$

$$= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch}+\frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch}-\frac{\left(12\left(5cfg\right.\right.}{}$$

$$= \frac{\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\left(a+cx^2\right)^{3/2}}{192c^2}-\frac{(5fg-8eh}{}$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2}+\frac{\left(48c^2dg^2+\right.}{}$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2}+\frac{\left(48c^2dg^2+\right.}{}$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2}+\frac{\left(48c^2dg^2+\right.}{}$$

Mathematica [A] time = 1.14949, size = 346, normalized size = 1.

$$\frac{\sqrt{a+cx^2}\left(-\frac{280a\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+\sqrt{c}x\left(5a+2cx^2\right)\sqrt{\frac{cx^2}{a}+1}\right)\left(h(dh+2eg)+fg^2\right)}{c^{3/2}\sqrt{\frac{cx^2}{a}+1}}+\frac{105afh^2\left(\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a}+1}}-\sqrt{c}x\left(3a^2+14acx^2+8c^2x^4\right)\right)}{c^{5/2}}+1680dg^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*((2688*g*(e*g + 2*d*h)*(a + c*x^2)^2)/c + (2240*(f*g^2 + h
*(2*e*g + d*h))*x*(a + c*x^2)^2)/c + (1680*f*h^2*x^3*(a + c*x^2)^2)/c + (38
4*h*(2*f*g + e*h)*(a + c*x^2)^2*(-2*a + 5*c*x^2))/c^2 - (280*a*(f*g^2 + h*(
2*e*g + d*h))*(Sqrt[c]*x*(5*a + 2*c*x^2)*Sqrt[1 + (c*x^2)/a] + 3*a^(3/2)*Ar
cSinh[(Sqrt[c]*x)/Sqrt[a]]))/(c^(3/2)*Sqrt[1 + (c*x^2)/a]) + (105*a*f*h^2*(
```

```
-(Sqrt[c]*x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4)) + (3*a^(5/2)*ArcSinh[(Sqrt[c]
*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/c^(5/2) + 1680*d*g^2*(5*a*x + 2*c*x^3 +
 (3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/a]))))/
13440
```

Maple [A] time = 0.057, size = 552, normalized size = 1.6

$$-\frac{a^3egh}{8}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+\frac{2fgx^2h}{7c}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{4afgh}{35c^2}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{afh^2x}{16c^2}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{aexgh}{12c}\left(cx^2+a\right)^{\frac{3}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] -1/8*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*e*g*h+2/7*x^2*(c*x^2+a)^(5/2
)/c*f*g*h-4/35*a/c^2*(c*x^2+a)^(5/2)*f*g*h-1/16*f*h^2*a/c^2*x*(c*x^2+a)^(5/
2)-1/12*a/c*x*(c*x^2+a)^(3/2)*e*g*h-1/8*a^2/c*x*(c*x^2+a)^(1/2)*e*g*h+3/8*d
*g^2*a^2/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+1/6*x*(c*x^2+a)^(5/2)/c*d*h^
2+1/6*x*(c*x^2+a)^(5/2)/c*f*g^2-1/16*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/
2))*d*h^2-1/16*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*f*g^2+2/5*(c*x^2+a
)^(5/2)/c*d*g*h+3/8*d*g^2*a*x*(c*x^2+a)^(1/2)+1/8*f*h^2*x^3*(c*x^2+a)^(5/2)
/c+3/128*f*h^2*a^4/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+1/7*x^2*(c*x^2+a)^
(5/2)/c*e*h^2-2/35*a/c^2*(c*x^2+a)^(5/2)*e*h^2+1/5*(c*x^2+a)^(5/2)/c*e*g^2+
1/4*d*g^2*x*(c*x^2+a)^(3/2)-1/16*a^2/c*x*(c*x^2+a)^(1/2)*f*g^2+1/64*f*h^2*a
^2/c^2*x*(c*x^2+a)^(3/2)+3/128*f*h^2*a^3/c^2*x*(c*x^2+a)^(1/2)+1/3*x*(c*x^2
+a)^(5/2)/c*e*g*h-1/24*a/c*x*(c*x^2+a)^(3/2)*d*h^2-1/16*a^2/c*x*(c*x^2+a)^(
1/2)*d*h^2-1/24*a/c*x*(c*x^2+a)^(3/2)*f*g^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11568, size = 1878, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/26880*(105*(16*a^3*c*e*g*h - 8*(6*a^2*c^2*d - a^3*c*f)*g^2 + (8*a^3*c*d
 - 3*a^4*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) -
2*(1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3*c*e*h^2 + 1920*(2*c^4*
f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*e*g*h + (8*c^4*d + 9*a*c
^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2 + 2*(7*c^4*d + 8*a*c^3*f)
*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*a*c^3*f)*g^2 + (56*a*c^3*d
 + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 384*(14*a*c^
3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c^2*f)*g*h)*x^2 + 105*(16*a^2
*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a^3*c*f)*h^2
)*x)*sqrt(c*x^2 + a))/c^3, 1/13440*(105*(16*a^3*c*e*g*h - 8*(6*a^2*c^2*d -
a^3*c*f)*g^2 + (8*a^3*c*d - 3*a^4*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3*c*e*h^2 + 1
920*(2*c^4*f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*e*g*h + (8*c^
4*d + 9*a*c^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2 + 2*(7*c^4*d +
 8*a*c^3*f)*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*a*c^3*f)*g^2 +
(56*a*c^3*d + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 3
84*(14*a*c^3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c^2*f)*g*h)*x^2 +
105*(16*a^2*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a
^3*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 50.0334, size = 1304, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] -3*a**(7/2)*f*h**2*x/(128*c**2*sqrt(1 + c*x**2/a)) + a**(5/2)*d*h**2*x/(16*
c*sqrt(1 + c*x**2/a)) + a**(5/2)*e*g*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(5/2
)*f*g**2*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*f*h**2*x**3/(128*c*sqrt(1 +
 c*x**2/a)) + a**(3/2)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**2*x/(8
*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*d*h**2*x**3/(48*sqrt(1 + c*x**2/a)) + 17
*a**(3/2)*e*g*h*x**3/(24*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*g**2*x**3/(48*
```

```
sqrt(1 + c*x**2/a)) + 13*a**(3/2)*f*h**2*x**5/(64*sqrt(1 + c*x**2/a)) + 3*s
qrt(a)*c*d*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*d*h**2*x**5/(24*
sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g*h*x**5/(12*sqrt(1 + c*x**2/a)) + 11*
sqrt(a)*c*f*g**2*x**5/(24*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*f*h**2*x**7/(16
*sqrt(1 + c*x**2/a)) + 3*a**4*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)
) - a**3*d*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*e*g*h*asinh(s
qrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**3*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(16*c
**(3/2)) + 3*a**2*d*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 2*a*d*g*h*P
iecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + a
*e*g**2*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), T
rue)) + a*e*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqr
t(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4,
 True)) + 2*a*f*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*
sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4
/4, True)) + 2*c*d*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x*
*2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x
**4/4, True)) + c*e*g**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*
x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)
*x**4/4, True)) + c*e*h**2*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) -
4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) +
x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 2*c*f*g*h*Pie
cewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(
105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c,
 0)), (sqrt(a)*x**6/6, True)) + c**2*d*g**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2
/a)) + c**2*d*h**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*g*h*x**7/(3
*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*g**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/
a)) + c**2*f*h**2*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.18857, size = 610, normalized size = 1.76

$$\frac{1}{13440}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\left(7\,cfh^2x+\frac{8\left(2\,c^7fgh+c^7h^2e\right)}{c^6}\right)x+\frac{7\left(8\,c^7fg^2+8\,c^7dh^2+9\,ac^6fh^2+16\,c^7ghe\right)}{c^6}\right)x+\frac{48\left(14\right.}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/13440*sqrt(c*x^2 + a)*((2*((4*(5*(6*(7*c*f*h^2*x + 8*(2*c^7*f*g*h + c^7*h
^2*e)/c^6)*x + 7*(8*c^7*f*g^2 + 8*c^7*d*h^2 + 9*a*c^6*f*h^2 + 16*c^7*g*h*e)
/c^6)*x + 48*(14*c^7*d*g*h + 16*a*c^6*f*g*h + 7*c^7*g^2*e + 8*a*c^6*h^2*e)/
c^6)*x + 35*(48*c^7*d*g^2 + 56*a*c^6*f*g^2 + 56*a*c^6*d*h^2 + 3*a^2*c^5*f*h
```

```
^2 + 112*a*c^6*g*h*e)/c^6)*x + 192*(28*a*c^6*d*g*h + 2*a^2*c^5*f*g*h + 14*a
*c^6*g^2*e + a^2*c^5*h^2*e)/c^6)*x + 105*(80*a*c^6*d*g^2 + 8*a^2*c^5*f*g^2
+ 8*a^2*c^5*d*h^2 - 3*a^3*c^4*f*h^2 + 16*a^2*c^5*g*h*e)/c^6)*x + 384*(14*a^
2*c^5*d*g*h - 4*a^3*c^4*f*g*h + 7*a^2*c^5*g^2*e - 2*a^3*c^4*h^2*e)/c^6) - 1
/128*(48*a^2*c^2*d*g^2 - 8*a^3*c*f*g^2 - 8*a^3*c*d*h^2 + 3*a^4*f*h^2 - 16*a
^3*c*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.90 $\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=213

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh-afg+6cdg)}{16c^{3/2}}-\frac{\left(a+cx^2\right)^{5/2}\left(6\left(2afh^2+c\left(5fg^2-7h(dh+eg)\right)\right)+5chx(5fg-7eh)\right)}{210c^2h}+\frac{x\left(a+\right.}{}$$

```
[Out] (a*(6*c*d*g - a*f*g - a*e*h)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d*g - a*(f*g
 + e*h))*x*(a + c*x^2)^(3/2))/(24*c) + (f*(g + h*x)^2*(a + c*x^2)^(5/2))/(7
*c*h) - ((6*(2*a*f*h^2 + c*(5*f*g^2 - 7*h*(e*g + d*h))) + 5*c*h*(5*f*g - 7*
e*h)*x)*(a + c*x^2)^(5/2))/(210*c^2*h) + (a^2*(6*c*d*g - a*f*g - a*e*h)*Arc
Tanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rubi [A] time = 0.270914, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1654, 780, 195, 217, 206}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh-afg+6cdg)}{16c^{3/2}}-\frac{\left(a+cx^2\right)^{5/2}\left(6\left(2afh^2-7ch(dh+eg)+5cfg^2\right)+5chx(5fg-7eh)\right)}{210c^2h}+\frac{x\left(a+c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(6*c*d*g - a*f*g - a*e*h)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d*g - a*(f*g
 + e*h))*x*(a + c*x^2)^(3/2))/(24*c) + (f*(g + h*x)^2*(a + c*x^2)^(5/2))/(7
*c*h) - ((6*(5*c*f*g^2 + 2*a*f*h^2 - 7*c*h*(e*g + d*h)) + 5*c*h*(5*f*g - 7*
e*h)*x)*(a + c*x^2)^(5/2))/(210*c^2*h) + (a^2*(6*c*d*g - a*f*g - a*e*h)*Arc
Tanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
```

```
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} + \frac{\int (g+hx)\left((7cd-2af)h^2-ch(5fg-7eh)x\right)\left(a+c\right.}{7ch^2}$$

$$= \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} - \frac{\left(6\left(5cfg^2+2afh^2-7ch(eg+dh)\right)+5ch(5fg-7e\right.}{210c^2h}$$

$$= \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} - \frac{\left(6\left(5cfg^2+2a\right.\right.}{}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c} + \frac{f(g+}{}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c} + \frac{f(g+}{}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c} + \frac{f(g+}{}$$

Mathematica [A] time = 0.688368, size = 209, normalized size = 0.98

$$\frac{\sqrt{a+cx^2}\left(-\frac{105a^{5/2}\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aeh+afg-6cdg)}{c^{3/2}\left(a+cx^2\right)} - \frac{96a^3fh}{c^2} + \frac{3a^2(112dh+7e(16g+5hx)+fx(35g+16hx))}{c} + 2ax(21d(25g+16hx)+x(7\right.}{1680}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*((-96*a^3*f*h)/c^2 + (3*a^2*(112*d*h + 7*e*(16*g + 5*h*x)
+ f*x*(35*g + 16*h*x)))/c + 4*c*x^3*(21*d*(5*g + 4*h*x) + 2*x*(7*e*(6*g + 5
*h*x) + 5*f*x*(7*g + 6*h*x))) + 2*a*x*(21*d*(25*g + 16*h*x) + x*(7*e*(48*g
+ 35*h*x) + f*x*(245*g + 192*h*x))) - (105*a^(5/2)*(-6*c*d*g + a*f*g + a*e*
h)*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(c^(3/2)*(a + c*x^2)))
)/1680
```

Maple [A] time = 0.052, size = 287, normalized size = 1.4

$$\frac{fhx^2}{7c}\left(cx^2+a\right)^{\frac{5}{2}} - \frac{2afh}{35c^2}\left(cx^2+a\right)^{\frac{5}{2}} + \frac{ehx}{6c}\left(cx^2+a\right)^{\frac{5}{2}} + \frac{fgx}{6c}\left(cx^2+a\right)^{\frac{5}{2}} - \frac{aehx}{24c}\left(cx^2+a\right)^{\frac{3}{2}} - \frac{afgx}{24c}\left(cx^2+a\right)^{\frac{3}{2}} - \frac{a^2xeh}{16c}\surd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/7*h*f*x^2*(c*x^2+a)^(5/2)/c-2/35*h*f*a/c^2*(c*x^2+a)^(5/2)+1/6*x*(c*x^2+a
)^(5/2)/c*e*h+1/6*x*(c*x^2+a)^(5/2)/c*f*g-1/24*a/c*x*(c*x^2+a)^(3/2)*e*h-1/
24*a/c*x*(c*x^2+a)^(3/2)*f*g-1/16*a^2/c*x*(c*x^2+a)^(1/2)*e*h-1/16*a^2/c*x*
(c*x^2+a)^(1/2)*f*g-1/16*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*e*h-1/16
*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*f*g+1/5*(c*x^2+a)^(5/2)/c*d*h+1/
5*(c*x^2+a)^(5/2)/c*e*g+1/4*d*g*x*(c*x^2+a)^(3/2)+3/8*d*g*a*x*(c*x^2+a)^(1/
2)+3/8*d*g*a^2/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47333, size = 1107, normalized size = 5.2

$$\left[\frac{105\left(a^3eh-\left(6\,a^2cd-a^3f\right)g\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(240\,c^3fhx^6+280\left(c^3fg+c^3eh\right)x^5+336\,a^2ceg\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*(a^3*e*h - (6*a^2*c*d - a^3*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqr
t(c*x^2 + a)*sqrt(c)*x - a) + 2*(240*c^3*f*h*x^6 + 280*(c^3*f*g + c^3*e*h)*
x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*c^3*d + 8*a*c^2*f)*h)*x^4 + 70*(7*
a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3 + 48*(14*a*c^2*e*g + (14*a*c^2*d +
 a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^3*f)*h + 105*(a^2*c*e*h + (10*a*c^2*
d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/1680*(105*(a^3*e*h - (6*a^2*c*d
- a^3*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f*h*x^6
```

```
+ 280*(c^3*f*g + c^3*e*h)*x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*c^3*d +
8*a*c^2*f)*h)*x^4 + 70*(7*a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3 + 48*(14
*a*c^2*e*g + (14*a*c^2*d + a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^3*f)*h + 1
05*(a^2*c*e*h + (10*a*c^2*d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 25.5837, size = 768, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] a**(5/2)*e*h*x/(16*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g*x/(16*c*sqrt(1 + c*
x**2/a)) + a**(3/2)*d*g*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g*x/(8*sqrt(1 +
 c*x**2/a)) + 17*a**(3/2)*e*h*x**3/(48*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*
g*x**3/(48*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g*x**3/(8*sqrt(1 + c*x**2/a)
) + 11*sqrt(a)*c*e*h*x**5/(24*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*g*x**5/(
24*sqrt(1 + c*x**2/a)) - a**3*e*h*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) -
a**3*f*g*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) + 3*a**2*d*g*asinh(sqrt(c)*
x/sqrt(a))/(8*sqrt(c)) + a*d*h*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a +
c*x**2)**(3/2)/(3*c), True)) + a*e*g*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)),
((a + c*x**2)**(3/2)/(3*c), True)) + a*f*h*Piecewise((-2*a**2*sqrt(a + c*x*
*2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, N
e(c, 0)), (sqrt(a)*x**4/4, True)) + c*d*h*Piecewise((-2*a**2*sqrt(a + c*x**
2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne
(c, 0)), (sqrt(a)*x**4/4, True)) + c*e*g*Piecewise((-2*a**2*sqrt(a + c*x**2
)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(
c, 0)), (sqrt(a)*x**4/4, True)) + c*f*h*Piecewise((8*a**3*sqrt(a + c*x**2)/
(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x*
*2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) +
c**2*d*g*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*h*x**7/(6*sqrt(a)*sqr
t(1 + c*x**2/a)) + c**2*f*g*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.17146, size = 356, normalized size = 1.67

$$\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,cfhx+\frac{7\left(c^6fg+c^6he\right)}{c^5}\right)x+\frac{6\left(7\,c^6dh+8\,ac^5fh+7\,c^6ge\right)}{c^5}\right)x+\frac{35\left(6\,c^6dg+7\,ac^5fg+7\,ac^5h\right.}{c^5}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*c*f*h*x + 7*(c^6*f*g + c^6*h*e)/c^5)*x
 + 6*(7*c^6*d*h + 8*a*c^5*f*h + 7*c^6*g*e)/c^5)*x + 35*(6*c^6*d*g + 7*a*c^5
*f*g + 7*a*c^5*h*e)/c^5)*x + 24*(14*a*c^5*d*h + a^2*c^4*f*h + 14*a*c^5*g*e)
/c^5)*x + 105*(10*a*c^5*d*g + a^2*c^4*f*g + a^2*c^4*h*e)/c^5)*x + 48*(7*a^2
*c^4*d*h - 2*a^3*c^3*f*h + 7*a^2*c^4*g*e)/c^5) - 1/16*(6*a^2*c*d*g - a^3*f*
g - a^3*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.91 $\int \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=137

$$\frac{a^2(6cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{x\left(a+cx^2\right)^{3/2}(6cd-af)}{24c}+\frac{ax\sqrt{a+cx^2}(6cd-af)}{16c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

```
[Out] (a*(6*c*d - a*f)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d - a*f)*x*(a + c*x^2)^(
3/2))/(24*c) + (e*(a + c*x^2)^(5/2))/(5*c) + (f*x*(a + c*x^2)^(5/2))/(6*c)
+ (a^2*(6*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rubi [A] time = 0.0834329, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1815, 641, 195, 217, 206}

$$\frac{a^2(6cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{x\left(a+cx^2\right)^{3/2}(6cd-af)}{24c}+\frac{ax\sqrt{a+cx^2}(6cd-af)}{16c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```


```
[Out] (a*(6*c*d - a*f)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d - a*f)*x*(a + c*x^2)^(
3/2))/(24*c) + (e*(a + c*x^2)^(5/2))/(5*c) + (f*x*(a + c*x^2)^(5/2))/(6*c)
+ (a^2*(6*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```


Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx &= \frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{\int (6cd-af+6cex)\left(a+cx^2\right)^{3/2}dx}{6c}\\
&= \frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{(6cd-af)\int \left(a+cx^2\right)^{3/2}dx}{6c}\\
&= \frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{(a(6cd-af))\int \sqrt{a+}}{8c}\\
&= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}\\
&= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}\\
&= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}
\end{aligned}
$$

Mathematica [A] time = 0.268972, size = 125, normalized size = 0.91

$$
\frac{\sqrt{a+cx^2}\left(\sqrt{c}\left(3a^2(16e+5fx)+2acx(75d+x(48e+35fx))+4c^2x^3(15d+2x(6e+5fx))\right)-\frac{15a^{3/2}(af-6cd)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{240c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(3*a^2*(16*e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*
e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*e + 35*f*x))) - (15*a^(3/2)*(-6*c*d + a
*f)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(240*c^(3/2))
```

Maple [A] time = 0.049, size = 146, normalized size = 1.1

$$\frac{fx}{6c}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{afx}{24c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{a^2fx}{16c}\sqrt{cx^2+a}-\frac{a^3f}{16}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+\frac{e}{5c}\left(cx^2+a\right)^{\frac{5}{2}}+\frac{dx}{4}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{3a}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/6*f*x*(c*x^2+a)^(5/2)/c-1/24*f*a/c*x*(c*x^2+a)^(3/2)-1/16*f*a^2/c*x*(c*x^
2+a)^(1/2)-1/16*f*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+1/5*e*(c*x^2+a)
^(5/2)/c+1/4*d*x*(c*x^2+a)^(3/2)+3/8*d*a*x*(c*x^2+a)^(1/2)+3/8*d*a^2/c^(1/2
)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39463, size = 618, normalized size = 4.51

$$\left[-\frac{15\left(6a^2cd-a^3f\right)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(40c^3fx^5+48c^3ex^4+96ac^2ex^2+48a^2ce+10\left(6c^3d+7a\right.\right.}{480c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(6*a^2*c*d - a^3*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c*e
 + 10*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^2 +
 a))/c^2, -1/240*(15*(6*a^2*c*d - a^3*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*
x^2 + a)) - (40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c*e + 10
*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^2 + a))/
c^2]
```

Sympy [A] time = 14.9926, size = 348, normalized size = 2.54

$$\frac{a^{\frac{5}{2}}fx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}dx}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}fx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}cdx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{11\sqrt{a}cfx^5}{24\sqrt{1+\frac{cx^2}{a}}}-\frac{a^3f\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2d\operatorname{asin}}{8\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] a**(5/2)*f*x/(16*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*x*sqrt(1 + c*x**2/a)/2
+ a**(3/2)*d*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*x**3/(48*sqrt(1 + c*x
**2/a)) + 3*sqrt(a)*c*d*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*x**5/(
24*sqrt(1 + c*x**2/a)) - a**3*f*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) + 3*
a**2*d*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + a*e*Piecewise((sqrt(a)*x**2/2
, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + c*e*Piecewise((-2*a**2*sq
rt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c
*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c**2*d*x**5/(4*sqrt(a)*sqrt(
1 + c*x**2/a)) + c**2*f*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.19338, size = 174, normalized size = 1.27

$$\frac{1}{240}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5cfx+6ce\right)x+\frac{5\left(6c^5d+7ac^4f\right)}{c^4}\right)x+48ae\right)x+\frac{15\left(10ac^4d+a^2c^3f\right)}{c^4}\right)x+\frac{48a^2e}{c}\right)-\frac{\left(6a^2cd-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*c*f*x + 6*c*e)*x + 5*(6*c^5*d + 7*a*c^4*f)
/c^4)*x + 48*a*e)*x + 15*(10*a*c^4*d + a^2*c^3*f)/c^4)*x + 48*a^2*e/c) - 1/
16*(6*a^2*c*d - a^3*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.92 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$

Optimal. Leaf size=326

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2g^3\left(fg^2-h(eg-dh)\right)\right)}{8\sqrt{c}h^6}+\frac{(a+cx^2)^{3/2}\left(4\left(dh^2-eg\right.\right.}{1}$$

```
[Out] ((8*(c*g^2 + a*h^2)*(f*g^2 - e*g*h + d*h^2) - h*(4*c*d*g*h^2 + (f*g - e*h)*
(4*c*g^2 + 3*a*h^2))*x)*Sqrt[a + c*x^2])/(8*h^5) + ((4*(f*g^2 - e*g*h + d*h
^2) - 3*h*(f*g - e*h)*x)*(a + c*x^2)^(3/2))/(12*h^3) + (f*(a + c*x^2)^(5/2)
)/(5*c*h) - ((3*a^2*h^4*(f*g - e*h) + 8*c^2*g^3*(f*g^2 - h*(e*g - d*h)) + 1
2*a*c*g*h^2*(f*g^2 - h*(e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(8*Sqrt[c]*h^6) - ((c*g^2 + a*h^2)^(3/2)*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a
*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/h^6
```

Rubi [A] time = 0.76619, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 815, 844, 217, 206, 725}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2\left(fg^5-g^3h(eg-dh)\right)\right)}{8\sqrt{c}h^6}+\frac{(a+cx^2)^{3/2}\left(4\left(dh^2-eg\right.\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((8*(c*g^2 + a*h^2)*(f*g^2 - e*g*h + d*h^2) - h*(4*c*d*g*h^2 + (f*g - e*h)*
(4*c*g^2 + 3*a*h^2))*x)*Sqrt[a + c*x^2])/(8*h^5) + ((4*(f*g^2 - e*g*h + d*h
^2) - 3*h*(f*g - e*h)*x)*(a + c*x^2)^(3/2))/(12*h^3) + (f*(a + c*x^2)^(5/2)
)/(5*c*h) - ((3*a^2*h^4*(f*g - e*h) + 12*a*c*g*h^2*(f*g^2 - h*(e*g - d*h))
+ 8*c^2*(f*g^5 - g^3*h*(e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(8*Sqrt[c]*h^6) - ((c*g^2 + a*h^2)^(3/2)*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a
*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/h^6
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+cx^2\right)^{5/2}}{5ch} + \frac{\int \frac{\left(5cdh^2-5ch(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{g+hx}\,dx}{5ch^2}$$

$$= \frac{\left(4\left(fg^2-egh+dh^2\right)-3h(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{12h^3} + \frac{f\left(a+cx^2\right)^{5/2}}{5ch} + \frac{\int \frac{\left(5ac^2h^2\left(fg^2-h\right.\right.}{}}{}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{a+cx}}{8h^5}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{a+cx}}{8h^5}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{a+cx}}{8h^5}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{a+cx}}{8h^5}$$

Mathematica [A] time = 1.42591, size = 348, normalized size = 1.07

$$\frac{\sqrt{a+cx^2}\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+\sqrt{c}x\left(5a+2cx^2\right)\sqrt{\frac{cx^2}{a}+1}\right)(eh-fg)}{8\sqrt{c}h^2\sqrt{\frac{cx^2}{a}+1}} - \frac{\left(h(dh-eg)+fg^2\right)\left(\sqrt{\frac{cx^2}{a}+1}\left(-h\sqrt{a+cx^2}\left(8ah\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (f*(a + c*x^2)^(5/2))/(5*c*h) + ((-(f*g) + e*h)*Sqrt[a + c*x^2]*(Sqrt[c]*x*
(5*a + 2*c*x^2)*Sqrt[1 + (c*x^2)/a] + 3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]
]))/(8*Sqrt[c]*h^2*Sqrt[1 + (c*x^2)/a]) - ((f*g^2 + h*(-(e*g) + d*h))*(3*Sq
rt[a]*Sqrt[c]*g*h^2*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[1 +
 (c*x^2)/a]*(-(h*Sqrt[a + c*x^2]*(6*c*g^2 + 8*a*h^2 - 3*c*g*h*x + 2*c*h^2*x
^2)) + 6*Sqrt[c]*g*(c*g^2 + a*h^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 6
*(c*g^2 + a*h^2)^(3/2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a +
c*x^2])])))/(6*h^6*Sqrt[1 + (c*x^2)/a])
```

Maple [B] time = 0.228, size = 2420, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x)
```

```
[Out] -3/2/h^2*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/
h)+(a*h^2+c*g^2)/h^2)^(1/2))*a*d-1/2/h^2*c*g*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(
a*h^2+c*g^2)/h^2)^(1/2)*x*d+3/2/h^3*c^(1/2)*g^2*ln((-c*g/h+(x+g/h)*c)/c^(1/
2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*a*e+1/h*((x+g/h)^
2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*a*d+3/8/h*e*a^2/c^(1/2)*ln(x*c
^(1/2)+(c*x^2+a)^(1/2))-1/4/h^2*f*g*x*(c*x^2+a)^(3/2)+1/h^5*c^(3/2)*g^4*ln(
(-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^
(1/2))*e-1/h^6*c^(3/2)*g^5*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g
/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*f-1/h^4*c^(3/2)*g^3*ln((-c*g/h+(x+g/h)
*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*d-2/h^3/
((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2
+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(
x+g/h))*a*c*g^2*d+2/h^4/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2
*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*
h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*c*g^3*e-2/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*l
n((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)
^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*c*g^4*f+3/8/h*e*a
*x*(c*x^2+a)^(1/2)+1/h^3*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1
/2)*c*g^2*d+1/h^6/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h
*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*
g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^5*e-3/2/h^4*c^(1/2)*g^3*ln((-c*g/h+(x+g/h)*
c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*a*f+1/2/h
^3*c*g^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*e-1/2/h^4*
c*g^3*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*f-1/h^7/((a*h
^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^
2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h
))*c^2*g^6*f+1/h^3*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*a*
f*g^2-1/h/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)
+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2
)^(1/2))/(x+g/h))*a^2*d+1/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)
/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/
h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a^2*e*g-1/h^5/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+
g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^4*d-1/h^2
*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*a*e*g+1/h^5*((x+g/h)
^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*c*g^4*f-1/h^4*((x+g/h)^2*c-2*
```

```
c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*c*g^3*e+1/3/h*((x+g/h)^2*c-2*c*g/h*(
x+g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*d-1/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*
h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c
*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a^2*f*g^2-3/8/h^2*f*g*a*x*(
c*x^2+a)^(1/2)-1/3/h^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(3/2
)*e*g+1/3/h^3*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(3/2)*f*g^2+1
/4/h*e*x*(c*x^2+a)^(3/2)-3/8/h^2*f*g*a^2/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/
2))+1/5*f*(c*x^2+a)^(5/2)/c/h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Giac [A] time = 1.22853, size = 744, normalized size = 2.28

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(\frac{4\,cfx}{h}-\frac{5\left(c^4fgh^{19}-c^4h^{20}e\right)}{c^3h^{21}}\right)x+\frac{4\left(5\,c^4fg^2h^{18}+5\,c^4dh^{20}+6\,ac^3fh^{20}-5\,c^4gh^{19}e\right)}{c^3h^{21}}\right)x-\frac{15\left(4\,c^4fg\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*c*f*x/h - 5*(c^4*f*g*h^19 - c^4*h^20*e)/(c^
3*h^21))*x + 4*(5*c^4*f*g^2*h^18 + 5*c^4*d*h^20 + 6*a*c^3*f*h^20 - 5*c^4*g*
h^19*e)/(c^3*h^21))*x - 15*(4*c^4*f*g^3*h^17 + 4*c^4*d*g*h^19 + 5*a*c^3*f*g
*h^19 - 4*c^4*g^2*h^18*e - 5*a*c^3*h^20*e)/(c^3*h^21))*x + 8*(15*c^4*f*g^4*
h^16 + 15*c^4*d*g^2*h^18 + 20*a*c^3*f*g^2*h^18 + 20*a*c^3*d*h^20 + 3*a^2*c^
2*f*h^20 - 15*c^4*g^3*h^17*e - 20*a*c^3*g*h^19*e)/(c^3*h^21)) + 2*(c^2*f*g^
6 + c^2*d*g^4*h^2 + 2*a*c*f*g^4*h^2 + 2*a*c*d*g^2*h^4 + a^2*f*g^2*h^4 + a^2
*d*h^6 - c^2*g^5*h*e - 2*a*c*g^3*h^3*e - a^2*g*h^5*e)*arctan(-((sqrt(c)*x -
 sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2
)*h^6) + 1/8*(8*c^(5/2)*f*g^5 + 8*c^(5/2)*d*g^3*h^2 + 12*a*c^(3/2)*f*g^3*h^
2 + 12*a*c^(3/2)*d*g*h^4 + 3*a^2*sqrt(c)*f*g*h^4 - 8*c^(5/2)*g^4*h*e - 12*a
*c^(3/2)*g^2*h^3*e - 3*a^2*sqrt(c)*h^5*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/(c*h^6)
```

3.93 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

Optimal. Leaf size=432

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2g^2\left(5fg^2-h(4eg-3dh)\right)\right)}{8\sqrt{c}h^6}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg\right.}{h(g+hx)\left(ah^2+cg^2\right)}$$

```
[Out] -((8*(a*h^2*(2*f*g - e*h) + c*g*(5*f*g^2 - h*(4*e*g - 3*d*h))) - h*(20*c*f*
g^2 - 16*c*e*g*h + 12*c*d*h^2 + 3*a*f*h^2)*x)*Sqrt[a + c*x^2])/(8*h^5) - ((
4*(a*h^2*(2*f*g - e*h) + c*g*(5*f*g^2 - h*(4*e*g - 3*d*h))) - 3*h*(a*f*h^2
+ c*(5*f*g^2 - 4*h*(e*g - d*h)))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*h
^2)) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g +
h*x)) + ((3*a^2*f*h^4 + 8*c^2*g^2*(5*f*g^2 - h*(4*e*g - 3*d*h)) + 12*a*c*h^
2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqr
t[c]*h^6) + (Sqrt[c*g^2 + a*h^2]*(a*h^2*(2*f*g - e*h) + c*g*(5*f*g^2 - h*(4
*e*g - 3*d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
])/h^6
```

Rubi [A] time = 0.90139, antiderivative size = 428, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 815, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2\left(5fg^4-g^2h(4eg-3dh)\right)\right)}{8\sqrt{c}h^6}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg\right.}{h(g+hx)\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((8*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - h*(20*c*f*
g^2 - 16*c*e*g*h + 12*c*d*h^2 + 3*a*f*h^2)*x)*Sqrt[a + c*x^2])/(8*h^5) - ((
4*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - 3*h*(5*c*f*g^
2 + a*f*h^2 - 4*c*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*h
^2)) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g +
h*x)) + ((3*a^2*f*h^4 + 8*c^2*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h)) + 12*a*c*h^
2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqr
t[c]*h^6) + (Sqrt[c*g^2 + a*h^2]*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2
*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
```

```
])/h^6
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{\left(-cdg+afg-aeh-\left(afh-c\left(4eg-\frac{5fg^2}{h}-4dh\right)\right)x\right)\left(a+cx^2\right)^{3/2}}{g+hx}\,dx}{cg^2+ah^2}$$

$$= -\frac{\left(4\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-3h\left(5cfg^2+afh^2-4ch(eg-dh)\right)}{12h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12cdh^2+3a\right.\right.}{8h^5}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12cdh^2+3a\right.\right.}{8h^5}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12cdh^2+3a\right.\right.}{8h^5}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12cdh^2+3a\right.\right.}{8h^5}$$

Mathematica [A] time = 0.582258, size = 392, normalized size = 0.91

$$\frac{\frac{3\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(3a^2fh^4+12ach^2\left(h(dh-2eg)+3fg^2\right)+8c^2\left(g^2h(3dh-4eg)+5fg^4\right)\right)}{\sqrt{c}}+h\sqrt{a+cx^2}\left(3hx\left(5afh^2+4c\left(h(dh-2eg)+3fg^2\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] (h*Sqrt[a + c*x^2]*(8*(4*a*h^2*(-2*f*g + e*h) - 3*c*(4*f*g^3 + g*h*(-3*e*g
+ 2*d*h))) + 3*h*(5*a*f*h^2 + 4*c*(3*f*g^2 + h*(-2*e*g + d*h)))*x + 8*c*h^2
*(-2*f*g + e*h)*x^2 + 6*c*f*h^3*x^3 - (24*(c*g^2 + a*h^2)*(f*g^2 + h*(-(e*g
```

```
) + d*h)))/(g + h*x)) - 24*Sqrt[c*g^2 + a*h^2]*(5*c*f*g^3 + c*g*h*(-4*e*g +
 3*d*h) + a*h^2*(2*f*g - e*h))*Log[g + h*x] + (3*(3*a^2*f*h^4 + 12*a*c*h^2*
(3*f*g^2 + h*(-2*e*g + d*h)) + 8*c^2*(5*f*g^4 + g^2*h*(-4*e*g + 3*d*h)))*Lo
g[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + 24*Sqrt[c*g^2 + a*h^2]*(5*c*f*g
^3 + c*g*h*(-4*e*g + 3*d*h) + a*h^2*(2*f*g - e*h))*Log[a*h - c*g*x + Sqrt[c
*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(24*h^6)
```

Maple [B] time = 0.226, size = 5121, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x)

[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.94 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=488

$$\frac{\sqrt{a+cx^2}\left(2a^2fh^4-chx\left(ah^2(7fg-3eh)+cg\left(10fg^2-3h(2eg-dh)\right)\right)+ach^2\left(19fg^2-3h(3eg-dh)\right)+2c^2g^2\left(10fg^2-3\right.\right.}{2h^5\left(ah^2+cg^2\right)}$$

```
[Out] ((2*a^2*f*h^4 + 2*c^2*g^2*(10*f*g^2 - 3*h*(2*e*g - d*h)) + a*c*h^2*(19*f*g^
2 - 3*h*(3*e*g - d*h)) - c*h*(a*h^2*(7*f*g - 3*e*h) + c*g*(10*f*g^2 - 3*h*(
2*e*g - d*h)))*x)*Sqrt[a + c*x^2])/(2*h^5*(c*g^2 + a*h^2)) - ((2*(c*g*(6*e*
g - (10*f*g^2)/h - 3*d*h) - a*h*(7*f*g - 3*e*h)) - (2*a*f*h^2 + c*(5*f*g^2
- 3*h*(e*g - d*h)))*x)*(a + c*x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x))
 - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(2*h*(c*g^2 + a*h^2)*(g + h*
x)^2) - (Sqrt[c]*(3*a*h^2*(3*f*g - e*h) + 2*c*g*(10*f*g^2 - 3*h*(2*e*g - d*
h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*h^6) - ((2*a^2*f*h^4 + 2*c^2*
g^2*(10*f*g^2 - 3*h*(2*e*g - d*h)) + a*c*h^2*(19*f*g^2 - 3*h*(3*e*g - d*h))
)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*Sqrt
[c*g^2 + a*h^2])
```

Rubi [A] time = 0.921043, antiderivative size = 480, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1651, 813, 815, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(2a^2fh^3-cx\left(ah^2(7fg-3eh)-3cgh(2eg-dh)+10cfg^3\right)+ach\left(19fg^2-3h(3eg-dh)\right)-2c^2g^2\left(-3dh+6eg-\right.\right.}{2h^4\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*a^2*f*h^3 - 2*c^2*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) + a*c*h*(19*f*g^2
- 3*h*(3*e*g - d*h)) - c*(10*c*f*g^3 - 3*c*g*h*(2*e*g - d*h) + a*h^2*(7*f*g
 - 3*e*h))*x)*Sqrt[a + c*x^2])/(2*h^4*(c*g^2 + a*h^2)) - ((2*(c*g*(6*e*g -
(10*f*g^2)/h - 3*d*h) - a*h*(7*f*g - 3*e*h)) - (5*c*f*g^2 + 2*a*f*h^2 - 3*c
*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)) - (
(f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2
) - (Sqrt[c]*(20*c*f*g^3 - 6*c*g*h*(2*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*h^6) - ((2*a^2*f*h^4 + 2*c^2*(10*f*
```

```
g^4 - 3*g^2*h*(2*e*g - d*h)) + a*c*h^2*(19*f*g^2 - 3*h*(3*e*g - d*h)))*ArcT
anh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*Sqrt[c*g^2
 + a*h^2])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(3eg-\frac{5fg^2}{h}-3dh\right)\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^2}\,dx}{2\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-ah(7fg-3eh)\right)-\left(5cfg^2+2afh^2-3ch(eg-dh)\right)x\right)\left(\ldots\right.}{6h^2\left(cg^2+ah^2\right)(g+hx)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10cfg^3-3cg\ldots\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10cfg^3-3cg\ldots\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10cfg^3-3cg\ldots\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10cfg^3-3cg\ldots\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

Mathematica [A] time = 0.706477, size = 435, normalized size = 0.89

$$-\frac{3\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(2a^2fh^4+ach^2\left(3h(dh-3eg)+19fg^2\right)+2c^2\left(3g^2h(dh-2eg)+10fg^4\right)\right)}{\sqrt{ah^2+cg^2}}+\frac{3\log(g+hx)\left(2a^2fh^4+ach^2\left(3h(dh-3eg)+19fg^2\right)+2c^2(3\right.}{\sqrt{ah^2+cg^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(a*h^2*(-3*h*(e*g + d*h + 2*e*h*x) + f*(17*g^2 + 28*g*h
*x + 8*h^2*x^2)) + c*(f*(60*g^4 + 90*g^3*h*x + 20*g^2*h^2*x^2 - 5*g*h^3*x^3
 + 2*h^4*x^4) + 3*h*(d*h*(6*g^2 + 9*g*h*x + 2*h^2*x^2) + e*(-12*g^3 - 18*g^
2*h*x - 4*g*h^2*x^2 + h^3*x^3)))))/(g + h*x)^2 + (3*(2*a^2*f*h^4 + a*c*h^2*
(19*f*g^2 + 3*h*(-3*e*g + d*h)) + 2*c^2*(10*f*g^4 + 3*g^2*h*(-2*e*g + d*h))
)*Log[g + h*x])/Sqrt[c*g^2 + a*h^2] - 3*Sqrt[c]*(20*c*f*g^3 + 6*c*g*h*(-2*e
*g + d*h) - 3*a*h^2*(-3*f*g + e*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - (3
*(2*a^2*f*h^4 + a*c*h^2*(19*f*g^2 + 3*h*(-3*e*g + d*h)) + 2*c^2*(10*f*g^4 +
 3*g^2*h*(-2*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*
x^2]])/Sqrt[c*g^2 + a*h^2])/(6*h^6)
```

Maple [B] time = 0.239, size = 7817, normalized size = 16.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}\left(d + ex + fx^2\right)}{\left(g + hx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Giac [B] time = 1.4696, size = 1399, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*(x*(2*c*f*x/h^3 - 3*(3*c^2*f*g*h^14 - c^2*h^15*e)/(c*h^
18)) + 2*(18*c^2*f*g^2*h^13 + 3*c^2*d*h^15 + 4*a*c*f*h^15 - 9*c^2*g*h^14*e)
/(c*h^18)) + 1/2*(20*c^(3/2)*f*g^3 + 6*c^(3/2)*d*g*h^2 + 9*a*sqrt(c)*f*g*h^
2 - 12*c^(3/2)*g^2*h*e - 3*a*sqrt(c)*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + a)))/h^6 + (20*c^2*f*g^4 + 6*c^2*d*g^2*h^2 + 19*a*c*f*g^2*h^2 + 3*a*c*d*
```

```
h^4 + 2*a^2*f*h^4 - 12*c^2*g^3*h*e - 9*a*c*g*h^3*e)*arctan(-((sqrt(c)*x - s
qrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2)*
h^6) + (10*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h + 6*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2
*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 8*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*c^2*g^3*h^2*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e + 1
8*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 10*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*c^(5/2)*d*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g
^3*h^2 - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 14*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*c^(5/2)*g^4*h*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e
 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 26*(sqrt(c)*x - sq
rt(c*x^2 + a))*a*c^2*f*g^4*h - 14*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2
*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt
(c*x^2 + a))*a^2*c*d*h^5 + 20*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e
 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 9*a^2*c^(3/2)*f*g^3*h^2
+ 5*a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 7*a^2*c^(3/2)*g^2*h^3*e -
 2*a^3*sqrt(c)*h^5*e)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x -
sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2*h^6)
```

3.95 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=475

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2g^3\left(10fg^2-h(4eg-dh)\right)\right)}{2h^6\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{5/}}{3h(g+}$$

```
[Out] -((((c*g^2 + a*h^2)*(3*a*f*h^2 + 2*c*(10*f*g^2 - h*(4*e*g - d*h))) + c*h*(3*
a*h^2*(3*f*g - e*h) + c*g*(10*f*g^2 - h*(4*e*g - d*h)))*x)*Sqrt[a + c*x^2])
/(2*h^5*(c*g^2 + a*h^2)*(g + h*x)) - ((c*g*(4*e*g - (10*f*g^2)/h - d*h) - 3
*a*h*(3*f*g - e*h) - (3*a*f*h^2 + c*(5*f*g^2 - 2*h*(e*g - d*h)))*x)*(a + c*
x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*
(a + c*x^2)^(5/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*(3*a*f*h^2
+ 2*c*(10*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(
2*h^6) + (c*(3*a^2*h^4*(4*f*g - e*h) + 2*c^2*g^3*(10*f*g^2 - h*(4*e*g - d*h
)) + 3*a*c*g*h^2*(11*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[
c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*(c*g^2 + a*h^2)^(3/2))
```

Rubi [A] time = 0.844642, antiderivative size = 469, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 813, 844, 217, 206, 725}

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2\left(10fg^5-g^3h(4eg-dh)\right)\right)}{2h^6\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{5/}}{3h(g+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -((((c*g^2 + a*h^2)*(20*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h)) + c*h*(10
*c*f*g^3 - c*g*h*(4*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*x)*Sqrt[a + c*x^2])
/(2*h^5*(c*g^2 + a*h^2)*(g + h*x)) - ((c*g*(4*e*g - (10*f*g^2)/h - d*h) - 3
*a*h*(3*f*g - e*h) - (5*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(e*g - d*h))*x)*(a + c*
x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*
(a + c*x^2)^(5/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*(20*c*f*g^2
 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(
2*h^6) + (c*(3*a^2*h^4*(4*f*g - e*h) + 3*a*c*g*h^2*(11*f*g^2 - h*(4*e*g - d
```

```
*h)) + 2*c^2*(10*f*g^5 - g^3*h*(4*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[
c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*(c*g^2 + a*h^2)^(3/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-3(cdg-afg+aeh)-\left(3afh-c\left(2eg-\frac{5fg^2}{h}-2dh\right)\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^3}\,dx}{3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-\left(5cfg^2+3afh^2-2ch(eg-dh)\right)x\right)\left(a+c\ldots\right.}{6h^2\left(cg^2+ah^2\right)(g+hx)^2}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg-dh)+3a\ldots\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg-dh)+3a\ldots\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg-dh)+3a\ldots\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg-dh)+3a\ldots\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

Mathematica [A] time = 1.40119, size = 517, normalized size = 1.09

$$-\frac{\frac{h\sqrt{a+cx^2}\left((g+hx)^2\left(6a^2fh^4+ach^2\left(h(8dh-23eg)+50fg^2\right)+c^2\left(g^2h(11dh-26eg)+47fg^4\right)\right)-(g+hx)\left(ah^2+cg^2\right)\left(-3ah^2(eh-2fg)+cgh(7dh-10eg)+13cfg^3\right)+2\left(ah^2+cg^2\right.\ldots\right.}{(g+hx)^3\left(ah^2+cg^2\right)}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(2*(c*g^2 + a*h^2)^2*(f*g^2 + h*(-(e*g) + d*h)) - (c*
g^2 + a*h^2)*(13*c*f*g^3 + c*g*h*(-10*e*g + 7*d*h) - 3*a*h^2*(-2*f*g + e*h)
```

```
)*(g + h*x) + (6*a^2*f*h^4 + a*c*h^2*(50*f*g^2 + h*(-23*e*g + 8*d*h)) + c^2
*(47*f*g^4 + g^2*h*(-26*e*g + 11*d*h)))*(g + h*x)^2 + 6*c*(4*f*g - e*h)*(c*
g^2 + a*h^2)*(g + h*x)^3 - 3*c*f*h*(c*g^2 + a*h^2)*x*(g + h*x)^3))/((c*g^2
+ a*h^2)*(g + h*x)^3)) - (3*c*(-3*a^2*h^4*(-4*f*g + e*h) + 3*a*c*g*h^2*(11*
f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^3*h*(-4*e*g + d*h)))*Log[g
+ h*x])/(c*g^2 + a*h^2)^(3/2) + 3*Sqrt[c]*(20*c*f*g^2 + 3*a*f*h^2 + 2*c*h*(
-4*e*g + d*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(-3*a^2*h^4*(-4*f*
g + e*h) + 3*a*c*g*h^2*(11*f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^
3*h*(-4*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]]
)/(c*g^2 + a*h^2)^(3/2))/(6*h^6)
```

Maple [B] time = 0.252, size = 9835, normalized size = 20.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x)

[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}} \left(d + ex + fx^2\right)}{\left(g + hx\right)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Giac [B] time = 1.68426, size = 2565, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(c*f*x/h^4 - 2*(4*c*f*g*h^10 - c*h^11*e)/h^15) - (20*c^
3*f*g^5 + 2*c^3*d*g^3*h^2 + 33*a*c^2*f*g^3*h^2 + 3*a*c^2*d*g*h^4 + 12*a^2*c
*f*g*h^4 - 8*c^3*g^4*h*e - 12*a*c^2*g^2*h^3*e - 3*a^2*c*h^5*e)*arctan(-((sq
rt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c*g^2*h^6
 + a*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/3*(60*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^3*f*g^5*h^2 + 18*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*d*g^3*h^4 + 69*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a*c^2*f*g^3*h^4 + 15*(sqrt(c)*x - sqrt(c*x^2 + a)
)^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 36*(
sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*g^4*h^3*e - 36*(sqrt(c)*x - sqrt(c*x^2 +
 a))^5*a*c^2*g^2*h^5*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*h^7*e + 21
0*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*f*g^6*h + 54*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 183*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/
2)*f*g^4*h^3 + 27*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 - 18*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*f*g^2*h^5 - 12*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*s
```

qrt(c)*f*h^7 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*g^5*h^2*e - 84*(
sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*g^3*h^4*e + 21*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^2*c^(3/2)*g*h^6*e + 188*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*
f*g^7 + 44*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*d*g^5*h^2 - 82*(sqrt(c)*x -
sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 - 34*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c
^3*d*g^3*h^4 - 276*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*f*g^3*h^4 - 48*(
sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*h^6 - 36*(sqrt(c)*x - sqrt(c*x^2
 + a))^3*a^3*c*f*g*h^6 - 104*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*g^6*h*e +
64*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*g^4*h^3*e + 138*(sqrt(c)*x - sqrt(
c*x^2 + a))^3*a^2*c^2*g^2*h^5*e - 354*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(
7/2)*f*g^6*h - 78*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(7/2)*d*g^4*h^3 - 276
*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*f*g^4*h^3 - 36*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^2*c^(5/2)*d*g^2*h^5 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a^3*c^(3/2)*f*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*d*h
^7 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*f*h^7 + 192*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a*c^(7/2)*g^5*h^2*e + 114*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*a^2*c^(5/2)*g^3*h^4*e - 48*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*
g*h^6*e + 222*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 + 48*(sqrt(c)
*x - sqrt(c*x^2 + a))*a^2*c^3*d*g^3*h^4 + 231*(sqrt(c)*x - sqrt(c*x^2 + a))
*a^3*c^2*f*g^3*h^4 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*
(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*f*g*h^6 - 120*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^2*c^3*g^4*h^3*e - 102*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*g^2*h^5*
e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*h^7*e - 47*a^3*c^(5/2)*f*g^4*h^3
- 11*a^3*c^(5/2)*d*g^2*h^5 - 50*a^4*c^(3/2)*f*g^2*h^5 - 8*a^4*c^(3/2)*d*h^7
 - 6*a^5*sqrt(c)*f*h^7 + 26*a^3*c^(5/2)*g^3*h^4*e + 23*a^4*c^(3/2)*g*h^6*e)
/((c*g^2*h^6 + a*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - s
qrt(c*x^2 + a))*sqrt(c)*g - a*h)^3) - 1/2*(20*c^(3/2)*f*g^2 + 2*c^(3/2)*d*h
^2 + 3*a*sqrt(c)*f*h^2 - 8*c^(3/2)*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/h^6

3.96 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal. Leaf size=511

$$\frac{(a+cx^2)^{3/2}\left(-3hx\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2g^2\left(5fg^2-h(dh+eg)\right)\right)+4a^2h^4(fg-2eh)-acgh^2\left(25fg^2\right.\right.}{24h^3(g+hx)^3\left(ah^2+cg^2\right)^2}$$

```
[Out] (c*(8*(5*f*g - e*h)*(c*g^2 + a*h^2)^2 + h*(12*a^2*f*h^4 + 4*c^2*g^3*(5*f*g
- e*h) + a*c*h^2*(35*f*g^2 - h*(7*e*g - 3*d*h)))*x)*Sqrt[a + c*x^2])/(8*h^5
*(c*g^2 + a*h^2)^2*(g + h*x)) + ((4*a^2*h^4*(f*g - 2*e*h) - 4*c^2*g^4*(5*f*
g - e*h) - a*c*g*h^2*(25*f*g^2 - h*(5*e*g - 9*d*h)) - 3*h*(4*a^2*f*h^4 + a*
c*h^2*(17*f*g^2 - h*(5*e*g - d*h)) + 2*c^2*g^2*(5*f*g^2 - h*(e*g + d*h)))*x
)*(a + c*x^2)^(3/2))/(24*h^3*(c*g^2 + a*h^2)^2*(g + h*x)^3) - ((f*g^2 - e*g
*h + d*h^2)*(a + c*x^2)^(5/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) - (c^(3/2)
*(5*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^6 - (c*(12*a^3*f*h^6
 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*c*h^4*(
25*f*g^2 - h*(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(5/2))
```

Rubi [A] time = 1.0919, antiderivative size = 511, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1651, 811, 813, 844, 217, 206, 725}

$$\frac{(a+cx^2)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-\right.\right.}{24h^2(g+hx)^3\left(ah^2+cg^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (c*(8*(5*f*g - e*h)*(c*g^2 + a*h^2)^2 + h*(12*a^2*f*h^4 + 4*c^2*g^3*(5*f*g
- e*h) + a*c*h^2*(35*f*g^2 - h*(7*e*g - 3*d*h)))*x)*Sqrt[a + c*x^2])/(8*h^5
*(c*g^2 + a*h^2)^2*(g + h*x)) + ((4*a^2*h^3*(f*g - 2*e*h) - (4*c^2*g^4*(5*f
*g - e*h))/h - a*c*g*h*(25*f*g^2 - h*(5*e*g - 9*d*h)) - 3*(4*a^2*f*h^4 + a*
c*h^2*(17*f*g^2 - h*(5*e*g - d*h)) + 2*c^2*(5*f*g^4 - g^2*h*(e*g + d*h)))*x
)*(a + c*x^2)^(3/2))/(24*h^2*(c*g^2 + a*h^2)^2*(g + h*x)^3) - ((f*g^2 - e*g
*h + d*h^2)*(a + c*x^2)^(5/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) - (c^(3/2)
```

```
*(5*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^6 - (c*(12*a^3*f*h^6
 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*c*h^4*(
25*f*g^2 - h*(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(5/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{4h(cg^2+ah^2)(g+hx)^4} - \frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh-c\left(eg-\frac{5fg^2}{h}-dh\right)\right)x\right)(a+cx^2)^{3/2}}{(g+hx)^4}\,dx}{4(cg^2+ah^2)}$$

$$= \frac{(4a^2h^3(fg-2eh) - \frac{4c^2g^4(5fg-eh)}{h} - acgh(25fg^2-h(5eg-9dh)) - 3(4a^2fh^4+ach^2}{24h^2(cg^2+ah^2)^2(g+}$$

$$= \frac{c(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35fg^2-h(7eg}{8h^5(cg^2+ah^2)^2(g+hx)}$$

$$= \frac{c(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35fg^2-h(7eg}{8h^5(cg^2+ah^2)^2(g+hx)}$$

$$= \frac{c(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35fg^2-h(7eg}{8h^5(cg^2+ah^2)^2(g+hx)}$$

$$= \frac{c(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35fg^2-h(7eg}{8h^5(cg^2+ah^2)^2(g+hx)}$$

Mathematica [A] time = 2.52245, size = 575, normalized size = 1.13

$$-\frac{\frac{h\sqrt{a+cx^2}(-c(g+hx)^3(4a^2h^4(31fg-8eh)+acgh^2(h(15dh-91eg)+287fg^2)+2c^2(g^3h(3dh-25eg)+77fg^5))+(g+hx)^2(ah^2+cg^2)(12a^2fh^4+ach^2(h(15dh-43eg)+95f}{(g+hx)^4(ah}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] -((h*Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) - 2*(c
*g^2 + a*h^2)^2*(17*c*f*g^3 + c*g*h*(-13*e*g + 9*d*h) - 4*a*h^2*(-2*f*g + e
*h))*(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(43*f*g^4 + g^2*h*(-
23*e*g + 9*d*h)) + a*c*h^2*(95*f*g^2 + h*(-43*e*g + 15*d*h)))*(g + h*x)^2 -
 c*(4*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(77*f*g^5 + g^3*h*(-25*e*g + 3*d*h))
 + a*c*g*h^2*(287*f*g^2 + h*(-91*e*g + 15*d*h)))*(g + h*x)^3 - 24*c*f*(c*g^
2 + a*h^2)^2*(g + h*x)^4))/((c*g^2 + a*h^2)^2*(g + h*x)^4) - (3*c*(12*a^3*f
```

```
*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*c*h
^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[g + h*x])/(c*g^2 + a*h^2)^(5/2) + 24*
c^(3/2)*(5*f*g - e*h)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(12*a^3*f*h
^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*c*h^4
*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[
a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/(24*h^6)
```

Maple [B] time = 0.254, size = 12481, normalized size = 24.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

Maple **[F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}} \left(d + ex + fx^2\right)}{\left(g + hx\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.97 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

Optimal. Leaf size=507

$$-\frac{\left(a+cx^2\right)^{3/2}\left(hx\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^4(2fg-3eh)+acgh^2\left(3dh^2+5fg^2\right)+4c^2fg\right.}{12h^3(g+hx)^4\left(ah^2+cg^2\right)^2}$$

```
[Out] -(c*(8*c^3*f*g^7 + 20*a*c^2*f*g^5*h^2 - a^3*h^6*(2*f*g - 3*e*h) + a^2*c*g*h
^4*(13*f*g^2 + 3*d*h^2) + h*(12*c^3*f*g^6 + 8*a^3*f*h^6 + a^2*c*g*h^4*(34*f
*g - 3*e*h) + a*c^2*g^2*h^2*(35*f*g^2 - 3*d*h^2))*x)*Sqrt[a + c*x^2])/(8*h^
5*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((4*c^2*f*g^5 - a^2*h^4*(2*f*g - 3*e*h)
+ a*c*g*h^2*(5*f*g^2 + 3*d*h^2) + h*(4*a^2*f*h^4 + a*c*g*h^2*(14*f*g - 3*e*
h) + c^2*(7*f*g^4 - 3*d*g^2*h^2))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*
h^2)^2*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(5*h*(c*g
^2 + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])
/h^6 + (c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 + 3*a^3*h^6*(6*f*g - e*h) + a
^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]
*Sqrt[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(7/2))
```

Rubi [A] time = 0.859968, antiderivative size = 507, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 811, 844, 217, 206, 725}

$$-\frac{\left(a+cx^2\right)^{3/2}\left(hx\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^4(2fg-3eh)+acgh^2\left(3dh^2+5fg^2\right)+4c^2fg\right.}{12h^3(g+hx)^4\left(ah^2+cg^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(c*(8*c^3*f*g^7 + 20*a*c^2*f*g^5*h^2 - a^3*h^6*(2*f*g - 3*e*h) + a^2*c*g*h
^4*(13*f*g^2 + 3*d*h^2) + h*(12*c^3*f*g^6 + 8*a^3*f*h^6 + a^2*c*g*h^4*(34*f
*g - 3*e*h) + a*c^2*g^2*h^2*(35*f*g^2 - 3*d*h^2))*x)*Sqrt[a + c*x^2])/(8*h^
5*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((4*c^2*f*g^5 - a^2*h^4*(2*f*g - 3*e*h)
+ a*c*g*h^2*(5*f*g^2 + 3*d*h^2) + h*(4*a^2*f*h^4 + a*c*g*h^2*(14*f*g - 3*e*
h) + c^2*(7*f*g^4 - 3*d*g^2*h^2))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*
h^2)^2*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(5*h*(c*g
```

```
^2 + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])
/h^6 + (c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 + 3*a^3*h^6*(6*f*g - e*h) + a
^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]
*Sqrt[a + c*x^2]])])/(8*h^6*(c*g^2 + a*h^2)^(7/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-5(cdg-afg+aeh)-5f\left(\frac{cg^2}{h}+ah\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^5}\,dx}{5\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2\left(5fg^2+3dh^2\right)+h\left(4a^2fh^4+acgh^2(14fg-3eh\right.\right.}{12h^3\left(cg^2+ah^2\right)^2(g+hx)^4}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\left(12c^3fg^6+\right.\right.}{8h^5\left(cg^2+ah^2\right)^3(g}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\left(12c^3fg^6+\right.\right.}{8h^5\left(cg^2+ah^2\right)^3(g}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\left(12c^3fg^6+\right.\right.}{8h^5\left(cg^2+ah^2\right)^3(g}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\left(12c^3fg^6+\right.\right.}{8h^5\left(cg^2+ah^2\right)^3(g}$$

Mathematica [A] time = 2.60789, size = 639, normalized size = 1.26

$$-\frac{h\sqrt{a+cx^2}\left(2(g+hx)^2\left(ah^2+cg^2\right)^2\left(20a^2fh^4+ach^2\left(3h(8dh-23eg)+154fg^2\right)+c^2\left(9g^2h(3dh-8eg)+137fg^4\right)\right)-c(g+hx)^3\left(ah^2+cg^2\right)\left(5a^2h^4(58fg-15eh)+acgh^2\left(3h(7dh-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) - 6*
(c*g^2 + a*h^2)^3*(21*c*f*g^3 + c*g*h*(-16*e*g + 11*d*h) - 5*a*h^2*(-2*f*g
+ e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(137*f*g^4 + 9*
g^2*h*(-8*e*g + 3*d*h)) + a*c*h^2*(154*f*g^2 + 3*h*(-23*e*g + 8*d*h)))*(g +
 h*x)^2 - c*(c*g^2 + a*h^2)*(5*a^2*h^4*(58*f*g - 15*e*h) + c^2*(326*f*g^5 +
 6*g^3*h*(-16*e*g + d*h)) + a*c*g*h^2*(631*f*g^2 + 3*h*(-62*e*g + 7*d*h)))*
(g + h*x)^3 + c*(160*a^3*f*h^6 + c^3*(274*f*g^6 - 6*g^4*h*(4*e*g + d*h)) +
3*a^2*c*h^4*(238*f*g^2 + h*(-33*e*g + 8*d*h)) + 3*a*c^2*g^2*h^2*(261*f*g^2
- h*(26*e*g + 9*d*h)))*(g + h*x)^4))/((c*g^2 + a*h^2)^3*(g + h*x)^5)) - (15
*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g
*h^4*(35*f*g^2 - 3*d*h^2))*Log[g + h*x])/(c*g^2 + a*h^2)^(7/2) + 120*c^(3/2
)*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (15*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*
g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*Log[
a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(7/2))/
(120*h^6)
```

Maple [B] time = 0.26, size = 14169, normalized size = 28.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.13869, size = 5951, normalized size = 11.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(8*c^5*f*g^7 + 28*a*c^4*f*g^5*h^2 + 35*a^2*c^3*f*g^3*h^4 - 3*a^2*c^3*d
*g*h^6 + 18*a^3*c^2*f*g*h^6 - 3*a^3*c^2*h^7*e)*arctan(-((sqrt(c)*x - sqrt(c
*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^3*g^6*h^6 + 3*a*c^2*g^4
*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*sqrt(-c*g^2 - a*h^2)) - c^(3/2)*f*log(a
bs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^6 - 1/60*(600*(sqrt(c)*x - sqrt(c*x^2 +
 a))^9*c^5*f*g^7*h^4 + 1740*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*f*g^5*h^6
 + 1635*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*f*g^3*h^8 + 45*(sqrt(c)*x -
 sqrt(c*x^2 + a))^9*a^2*c^3*d*g*h^10 + 450*(sqrt(c)*x - sqrt(c*x^2 + a))^9*
a^3*c^2*f*g*h^10 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^5*g^6*h^5*e - 360*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*g^4*h^7*e - 360*(sqrt(c)*x - sqrt(c*x
^2 + a))^9*a^2*c^3*g^2*h^9*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*h
^11*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 - 120*(sqrt
(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*d*g^6*h^5 + 10020*(sqrt(c)*x - sqrt(c*x
^2 + a))^8*a*c^(9/2)*f*g^6*h^5 - 360*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9
/2)*d*g^4*h^7 + 8595*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*f*g^4*h^7
```

+ 45*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d*g^2*h^9 + 1530*(sqrt(c)*
x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 120*(sqrt(c)*x - sqrt(c*x^2
+ a))^8*a^3*c^(5/2)*d*h^11 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(3/2
)*f*h^11 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*g^7*h^4*e - 1440*(s
qrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*g^5*h^6*e - 1440*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(7/2)*g^3*h^8*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^
3*c^(5/2)*g*h^10*e + 8800*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 2
40*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*d*g^7*h^4 + 21240*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a*c^5*f*g^7*h^4 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*d
*g^5*h^6 + 11670*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*f*g^5*h^6 + 690*(s
qrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 - 4970*(sqrt(c)*x - sqrt(c*
x^2 + a))^7*a^3*c^3*f*g^3*h^8 - 450*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3
*d*g*h^10 - 2580*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*f*g*h^10 - 960*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*c^6*g^8*h^3*e - 2640*(sqrt(c)*x - sqrt(c*x^2 +
a))^7*a*c^5*g^6*h^5*e - 2160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*g^4*h
^7*e + 1170*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*g^2*h^9*e + 30*(sqrt(c)
*x - sqrt(c*x^2 + a))^7*a^4*c^2*h^11*e + 10000*(sqrt(c)*x - sqrt(c*x^2 + a)
)^6*c^(13/2)*f*g^10*h - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*d*g^8*
h^3 + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^8*h^3 - 720*(sqr
t(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 14430*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h^5 + 1590*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
a^2*c^(9/2)*d*g^4*h^7 - 28790*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*f
*g^4*h^7 - 1710*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d*g^2*h^9 - 582
0*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 + 720*(sqrt(c)*x -
sqrt(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^6
*c^(13/2)*g^9*h^2*e - 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*g^7*h
^4*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*g^5*h^6*e + 4950*(sq
rt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*g^3*h^8*e - 270*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^4*c^(5/2)*g*h^10*e + 4384*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^7*f*g^11 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d*g^9*h^2 - 9392*(sqrt(c
)*x - sqrt(c*x^2 + a))^5*a*c^6*f*g^9*h^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))
^5*a*c^6*d*g^7*h^4 - 42996*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*f*g^7*h^
4 + 2364*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 - 31070*(sqrt(c)
*x - sqrt(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 - 2730*(sqrt(c)*x - sqrt(c*x^2 +
a))^5*a^3*c^4*d*g^3*h^8 + 8620*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^
3*h^8 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*g*h^10 + 4800*(sqrt(c
)*x - sqrt(c*x^2 + a))^5*a^5*c^2*f*g*h^10 - 384*(sqrt(c)*x - sqrt(c*x^2 + a
))^5*c^7*g^10*h*e + 672*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*g^8*h^3*e + 3
936*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*g^6*h^5*e + 5580*(sqrt(c)*x - s
qrt(c*x^2 + a))^5*a^3*c^4*g^4*h^7*e - 2970*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a^4*c^3*g^2*h^9*e - 11920*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10
h + 240(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 - 15720*(sqrt
(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*f*g^8*h^3 + 720*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^2*c^(11/2)*d*g^6*h^5 + 19670*(sqrt(c)*x - sqrt(c*x^2 + a))^4
*a^3*c^(9/2)*f*g^6*h^5 - 3510*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*d

*g^4*h^7 + 36260*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*f*g^4*h^7 + 14
40*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 + 6240*(sqrt(c)*x
- sqrt(c*x^2 + a))^4*a^5*c^(5/2)*f*g^2*h^9 - 240*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a^5*c^(5/2)*d*h^11 - 880*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)*
f*h^11 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*g^9*h^2*e + 1680*(s
qrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*g^7*h^4*e - 480*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^3*c^(9/2)*g^5*h^6*e - 6150*(sqrt(c)*x - sqrt(c*x^2 + a))^
4*a^4*c^(7/2)*g^3*h^8*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*g
*h^10*e + 13120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 240*(sq
rt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*d*g^7*h^4 + 30440*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*a^3*c^5*f*g^7*h^4 - 1200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^
5*d*g^5*h^6 + 14130*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 + 231
0*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 - 10790*(sqrt(c)*x - sq
rt(c*x^2 + a))^3*a^5*c^3*f*g^3*h^8 - 510*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
5*c^3*d*g*h^10 - 3820*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*f*g*h^10 - 96
0*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*g^8*h^3*e - 2640*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*a^3*c^5*g^6*h^5*e - 2640*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
4*c^4*g^4*h^7*e + 2790*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*g^2*h^9*e -
30*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*h^11*e - 7360*(sqrt(c)*x - sqrt(
c*x^2 + a))^2*a^3*c^(11/2)*f*g^8*h^3 + 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*
a^3*c^(11/2)*d*g^6*h^5 - 19930*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*
f*g^6*h^5 + 690*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*d*g^4*h^7 - 160
50*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*f*g^4*h^7 - 1050*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a^5*c^(7/2)*d*g^2*h^9 - 1300*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*a^6*c^(5/2)*f*g^2*h^9 + 560*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(3
/2)*f*h^11 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*g^7*h^4*e + 1
560*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*g^5*h^6*e + 2130*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a^5*c^(7/2)*g^3*h^8*e - 570*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*a^6*c^(5/2)*g*h^10*e + 2140*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*f*
g^7*h^4 - 60*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*d*g^5*h^6 + 6090*(sqrt(c
)*x - sqrt(c*x^2 + a))*a^5*c^4*f*g^5*h^6 - 270*(sqrt(c)*x - sqrt(c*x^2 + a)
)*a^5*c^4*d*g^3*h^8 + 5505*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*f*g^3*h^8
+ 195*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*d*g*h^10 + 1150*(sqrt(c)*x - sq
rt(c*x^2 + a))*a^7*c^2*f*g*h^10 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5
*g^6*h^5*e - 420*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*g^4*h^7*e - 630*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^6*c^3*g^2*h^9*e + 75*(sqrt(c)*x - sqrt(c*x^2 +
a))*a^7*c^2*h^11*e - 274*a^5*c^(9/2)*f*g^6*h^5 + 6*a^5*c^(9/2)*d*g^4*h^7 -
783*a^6*c^(7/2)*f*g^4*h^7 + 27*a^6*c^(7/2)*d*g^2*h^9 - 714*a^7*c^(5/2)*f*g^
2*h^9 - 24*a^7*c^(5/2)*d*h^11 - 160*a^8*c^(3/2)*f*h^11 + 24*a^5*c^(9/2)*g^5
*h^6*e + 78*a^6*c^(7/2)*g^3*h^8*e + 99*a^7*c^(5/2)*g*h^10*e)/((c^3*g^6*h^6
+ 3*a*c^2*g^4*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*((sqrt(c)*x - sqrt(c*x^2 +
a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^5)

3.98 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx$

Optimal. Leaf size=404

$$-\frac{\left(a+cx^2\right)^{3/2}(ah-cgx)\left(6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)+6c^2dg^2\right)}{24(g+hx)^4\left(ah^2+cg^2\right)^3}-\frac{ac\sqrt{a+cx^2}(ah-cgx)\left(6a^2fh^2-ac\left(fg^2-h(7eg\right.\right.}{16(g+hx)^2\left(ah^2+cg^2\right)^4}$$

```
[Out] -(a*c*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*
g*x)*Sqrt[a + c*x^2])/(16*(c*g^2 + a*h^2)^4*(g + h*x)^2) - ((6*c^2*d*g^2 +
6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*g*x)*(a + c*x^2)^(3/2
))/(24*(c*g^2 + a*h^2)^3*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2
)^(5/2))/(6*h*(c*g^2 + a*h^2)*(g + h*x)^6) + ((6*a*h^2*(2*f*g - e*h) + c*g*
(5*f*g^2 + h*(e*g - 7*d*h)))*(a + c*x^2)^(5/2))/(30*h*(c*g^2 + a*h^2)^2*(g
+ h*x)^5) - (a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d
*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*
g^2 + a*h^2)^(9/2))
```

Rubi [A] time = 0.552242, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 807, 721, 725, 206}

$$-\frac{\left(a+cx^2\right)^{3/2}(ah-cgx)\left(6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)+6c^2dg^2\right)}{24(g+hx)^4\left(ah^2+cg^2\right)^3}-\frac{ac\sqrt{a+cx^2}(ah-cgx)\left(6a^2fh^2-ac\left(fg^2-h(7eg\right.\right.}{16(g+hx)^2\left(ah^2+cg^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -(a*c*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*
g*x)*Sqrt[a + c*x^2])/(16*(c*g^2 + a*h^2)^4*(g + h*x)^2) - ((6*c^2*d*g^2 +
6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*g*x)*(a + c*x^2)^(3/2
))/(24*(c*g^2 + a*h^2)^3*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2
)^(5/2))/(6*h*(c*g^2 + a*h^2)*(g + h*x)^6) + ((5*c*f*g^3 + c*g*h*(e*g - 7*d
*h) + 6*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(5/2))/(30*h*(c*g^2 + a*h^2)^2*(g
+ h*x)^5) - (a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d
*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*
g^2 + a*h^2)^(9/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} - \frac{\int \frac{\left(-6(cdg-afg+aeh)-\left(6afh+c\left(eg+\frac{5fg^2}{h}-dh\right)\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^6}\,dx}{6\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} + \frac{\left(5cfg^3+cgh(eg-7dh)+6ah^2(2fg-eh)\right)\left(a+\right.}{30h\left(cg^2+ah^2\right)^2(g+hx)^5}$$

$$= -\frac{\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\left(a+cx^2\right)^{3/2}}{24\left(cg^2+ah^2\right)^3(g+hx)^4} - \frac{\left(fg^2-egh\right.}{6h\left(cg^2+\right.}$$

$$= -\frac{ac\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(6c^2dg^2+6a\right.}{}$$

$$= -\frac{ac\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(6c^2dg^2+6a\right.}{}$$

$$= -\frac{ac\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(6c^2dg^2+6a\right.}{}$$

Mathematica [A] time = 2.47528, size = 696, normalized size = 1.72

$$\frac{1}{240}\left(-\frac{\sqrt{a+cx^2}\left(2(g+hx)^2\left(ah^2+cg^2\right)^3\left(30a^2fh^4+ach^2\left(h(35dh-101eg)+227fg^2\right)+2c^2\left(g^2h(19dh-52eg)+100f\right.\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(40*(c*g^2 + a*h^2)^5*(f*g^2 + h*(-(e*g) + d*h)) - 8*(c
*g^2 + a*h^2)^4*(25*c*f*g^3 + c*g*h*(-19*e*g + 13*d*h) - 6*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^3*(30*a^2*f*h^4 + 2*c^2*(100*f*g^4 + g^
2*h*(-52*e*g + 19*d*h)) + a*c*h^2*(227*f*g^2 + h*(-101*e*g + 35*d*h)))*(g +
 h*x)^2 - 2*c*(c*g^2 + a*h^2)^2*(6*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(100*f*
g^5 + g^3*h*(-28*e*g + d*h)) + 3*a*c*g*h^2*(131*f*g^2 + h*(-37*e*g + 3*d*h)
))*(g + h*x)^3 + c*(c*g^2 + a*h^2)*(150*a^3*f*h^6 + 4*c^3*(50*f*g^6 - g^4*h
*(2*e*g + d*h)) + 6*a*c^2*g^2*h^2*(99*f*g^2 - h*(5*e*g + 4*d*h)) + 3*a^2*c*
```

```
h^4*(193*f*g^2 + h*(-19*e*g + 5*d*h)))*(g + h*x)^4 - c^2*(6*a^3*h^6*(41*f*g
 - 8*e*h) + 3*a^2*c*g*h^4*(89*f*g^2 + h*(29*e*g - 27*d*h)) + 4*c^3*(10*f*g^
7 + g^5*h*(2*e*g + d*h)) + 2*a*c^2*g^3*h^2*(83*f*g^2 + h*(19*e*g + 14*d*h))
)*(g + h*x)^5))/(h^5*(c*g^2 + a*h^2)^4*(g + h*x)^6)) + (15*a^2*c^2*(6*c^2*d
*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7*e*g + d*h)))*Log[g + h*x])/(c*g^2 +
 a*h^2)^(9/2) - (15*a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7
*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^
2 + a*h^2)^(9/2))/240
```

Maple [B] time = 0.27, size = 17026, normalized size = 42.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.11242, size = 8265, normalized size = 20.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] 1/8*(6*a^2*c^4*d*g^2 - a^3*c^3*f*g^2 - a^3*c^3*d*h^2 + 6*a^4*c^2*f*h^2 + 7*
a^3*c^3*g*h*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-
c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c^2*g^4*h^4 + 4*a^3*c*g
^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) + 1/120*(240*(sqrt(c)*x - sqrt(c*x^
2 + a))^11*c^6*f*g^8*h^5 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a*c^5*f*g^6
*h^7 + 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^4*f*g^4*h^9 - 90*(sqrt(c
)*x - sqrt(c*x^2 + a))^11*a^2*c^4*d*g^2*h^11 + 975*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a^3*c^3*f*g^2*h^11 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^3*d
*h^13 + 150*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^2*f*h^13 - 105*(sqrt(c)*
x - sqrt(c*x^2 + a))^11*a^3*c^3*g*h^12*e + 1200*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(13/2)*f*g^9*h^4 + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)
*f*g^7*h^6 + 7200*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*f*g^5*h^8 -
990*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*d*g^3*h^10 + 4965*(sqrt(c)
*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*f*g^3*h^10 + 165*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(7/2)*d*g*h^12 + 210*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4
*c^(5/2)*f*g*h^12 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*c^(13/2)*g^8*h^5*e
 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)*g^6*h^7*e + 1440*(sqrt(c
)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*g^4*h^9*e - 195*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(7/2)*g^2*h^11*e + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a
```

^4*c^(5/2)*h^13*e + 3200*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*f*g^10*h^3 + 3
20*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*d*g^8*h^5 + 12080*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a*c^6*f*g^8*h^5 + 1280*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*
d*g^6*h^7 + 16320*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*f*g^6*h^7 - 2520*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*d*g^4*h^9 + 9220*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a^3*c^4*f*g^4*h^9 + 2530*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*
c^4*d*g^2*h^11 - 4205*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*f*g^2*h^11 +
235*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*d*h^13 - 210*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a^5*c^2*f*h^13 + 640*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*g^9*
h^4*e + 2560*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*g^7*h^6*e + 3840*(sqrt(c
)*x - sqrt(c*x^2 + a))^9*a^2*c^5*g^5*h^8*e - 2620*(sqrt(c)*x - sqrt(c*x^2 +
a))^9*a^3*c^4*g^3*h^10*e + 1235*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*g*
h^12*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*f*g^11*h^2 + 480*(sq
rt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*d*g^9*h^4 + 15120*(sqrt(c)*x - sqrt(c
*x^2 + a))^8*a*c^(13/2)*f*g^9*h^4 + 1920*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*
c^(13/2)*d*g^7*h^6 + 12480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*f*g
^7*h^6 - 7380*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*d*g^5*h^8 - 3570
*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9/2)*f*g^5*h^8 + 8220*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a^3*c^(9/2)*d*g^3*h^10 - 22545*(sqrt(c)*x - sqrt(c*x^2 +
a))^8*a^4*c^(7/2)*f*g^3*h^10 - 285*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(
7/2)*d*g*h^12 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*f*g*h^12 +
960*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*g^10*h^3*e + 3600*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a*c^(13/2)*g^8*h^5*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a
))^8*a^2*c^(11/2)*g^6*h^7*e - 9570*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9
/2)*g^4*h^9*e + 5355*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(7/2)*g^2*h^11*e
- 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*h^13*e + 3840*(sqrt(c)*x
- sqrt(c*x^2 + a))^7*c^8*f*g^12*h + 384*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^
8*d*g^10*h^3 + 6336*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*f*g^10*h^3 + 1728
*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*d*g^8*h^5 - 11808*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a^2*c^6*f*g^8*h^5 - 9456*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*
c^6*d*g^6*h^7 - 31704*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*f*g^6*h^7 + 2
0760*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*d*g^4*h^9 - 39960*(sqrt(c)*x -
sqrt(c*x^2 + a))^7*a^4*c^4*f*g^4*h^9 - 2700*(sqrt(c)*x - sqrt(c*x^2 + a))^
7*a^4*c^4*d*g^2*h^11 + 12150*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*f*g^2*
h^11 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*d*h^13 + 60*(sqrt(c)*x -
sqrt(c*x^2 + a))^7*a^6*c^2*f*h^13 + 768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^
8*g^11*h^2*e + 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*g^9*h^4*e - 768*(
sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^6*g^7*h^6*e - 19608*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a^3*c^5*g^5*h^8*e + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4
*c^4*g^3*h^10*e - 2730*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*g*h^12*e + 1
280*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(17/2)*f*g^13 + 128*(sqrt(c)*x - sqrt
(c*x^2 + a))^6*c^(17/2)*d*g^11*h^2 - 4288*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a
*c^(15/2)*f*g^11*h^2 - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*d*g^9*
h^4 - 24096*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*f*g^9*h^4 - 8592*(
sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*d*g^7*h^6 - 26728*(sqrt(c)*x -

sqrt(c*x^2 + a))^6*a^3*c^(11/2)*f*g^7*h^6 + 24440*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(11/2)*d*g^5*h^8 - 12640*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(9/2)*f*g^5*h^8 - 14860*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(9/2)*d*g^3*h^10 + 41610*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*f*g^3*h^10 + 810*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*d*g*h^12 - 2460*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(5/2)*f*g*h^12 + 256*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(17/2)*g^12*h*e - 704*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*g^10*h^3*e - 4896*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*g^8*h^5*e - 15656*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(11/2)*g^6*h^7*e + 26800*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(9/2)*g^4*h^9*e - 9510*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*g^2*h^11*e + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(5/2)*h^13*e - 3840*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*f*g^12*h - 384*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*d*g^10*h^3 - 6336*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*f*g^10*h^3 - 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*d*g^8*h^5 + 11808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*f*g^8*h^5 + 19056*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*d*g^6*h^7 + 29304*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^5*f*g^6*h^7 - 21480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^5*d*g^4*h^9 + 46080*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*f*g^4*h^9 + 7020*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*d*g^2*h^11 - 17370*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^3*f*g^2*h^11 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^3*d*h^13 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^2*f*h^13 - 768*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*g^11*h^2*e - 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*g^9*h^4*e - 192*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*g^7*h^6*e + 26808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^5*g^5*h^8*e - 19440*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*g^3*h^10*e + 3030*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^3*g*h^12*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(15/2)*f*g^11*h^2 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(15/2)*d*g^9*h^4 + 15120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*f*g^9*h^4 + 3840*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*d*g^7*h^6 + 12360*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*f*g^7*h^6 - 18720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*d*g^5*h^8 + 1020*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*f*g^5*h^8 + 11640*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*d*g^3*h^10 - 32490*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*f*g^3*h^10 - 930*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*d*g*h^12 + 3180*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(5/2)*f*g*h^12 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(15/2)*g^10*h^3*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*g^8*h^5*e + 7080*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*g^6*h^7*e - 22260*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*g^4*h^9*e + 7470*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*g^2*h^11*e - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(5/2)*h^13*e - 3200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^7*f*g^10*h^3 - 320*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^7*d*g^8*h^5 - 12080*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^6*f*g^8*h^5 - 2960*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^6*d*g^6*h^7 - 16440*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^5*f*g^6*h^7 + 12120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^5*d*g^4*h^9 - 14120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*f*g^4*h^9 - 2330*(sqrt(

c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*d*g^2*h^11 + 10555*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*f*g^2*h^11 + 235*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*d*h^13 - 210*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^2*f*h^13 - 640*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^7*g^9*h^4*e - 3040*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^6*g^7*h^6*e - 7800*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^5*g^5*h^8*e + 10280*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*g^3*h^10*e - 1645*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*g*h^12*e + 1200*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(13/2)*f*g^9*h^4 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(13/2)*d*g^7*h^6 + 4920*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*f*g^7*h^6 + 1656*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*d*g^5*h^8 + 7824*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(9/2)*f*g^5*h^8 - 4038*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(9/2)*d*g^3*h^10 + 8193*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(7/2)*f*g^3*h^10 + 321*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(7/2)*d*g*h^12 - 1686*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(5/2)*f*g*h^12 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(13/2)*g^8*h^5*e + 1272*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*g^6*h^7*e + 3552*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(9/2)*g^4*h^9*e - 3207*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(7/2)*g^2*h^11*e + 48*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(5/2)*h^13*e - 240*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*f*g^8*h^5 - 48*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*d*g^6*h^7 - 1032*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*f*g^6*h^7 - 336*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*d*g^4*h^9 - 1764*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^4*f*g^4*h^9 + 882*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^4*d*g^2*h^11 - 1977*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*f*g^2*h^11 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*d*h^13 + 150*(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^2*f*h^13 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*g^7*h^6*e - 456*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*g^5*h^8*e - 1044*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^4*g^3*h^10*e + 471*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*g*h^12*e + 40*a^6*c^(11/2)*f*g^7*h^6 + 4*a^6*c^(11/2)*d*g^5*h^8 + 166*a^7*c^(9/2)*f*g^5*h^8 + 28*a^7*c^(9/2)*d*g^3*h^10 + 267*a^8*c^(7/2)*f*g^3*h^10 - 81*a^8*c^(7/2)*d*g*h^12 + 246*a^9*c^(5/2)*f*g*h^12 + 8*a^6*c^(11/2)*g^6*h^7*e + 38*a^7*c^(9/2)*g^4*h^9*e + 87*a^8*c^(7/2)*g^2*h^11*e - 48*a^9*c^(5/2)*h^13*e)/((c^4*g^8*h^6 + 4*a*c^3*g^6*h^8 + 6*a^2*c^2*g^4*h^10 + 4*a^3*c*g^2*h^12 + a^4*h^14)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^6)

3.99 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8} dx$

Optimal. Leaf size=532

$$-\frac{ac^2\sqrt{a+cx^2}(ah-cgx)\left(a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)+6c^2dg^3\right)}{16(g+hx)^2\left(ah^2+cg^2\right)^5}-\frac{c\left(a+cx^2\right)^{3/2}(ah-cgx)\left(a^2h^2(8fg-e\right.}{24(g+hx)}$$

```
[Out] -(a*c^2*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*
d*h)))*(a*h - c*g*x)*Sqrt[a + c*x^2])/(16*(c*g^2 + a*h^2)^5*(g + h*x)^2) -
(c*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h))
)*(a*h - c*g*x)*(a + c*x^2)^(3/2))/(24*(c*g^2 + a*h^2)^4*(g + h*x)^4) - ((f
*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(7*h*(c*g^2 + a*h^2)*(g + h*x)^7)
+ ((7*a*h^2*(2*f*g - e*h) + c*g*(5*f*g^2 + h*(2*e*g - 9*d*h)))*(a + c*x^2)^
(5/2))/(42*h*(c*g^2 + a*h^2)^2*(g + h*x)^6) - ((42*a^2*f*h^4 - c^2*g^2*(5*f
*g^2 + h*(2*e*g - 51*d*h)) - a*c*h^2*(26*f*g^2 - h*(61*e*g - 12*d*h)))*(a +
 c*x^2)^(5/2))/(210*h*(c*g^2 + a*h^2)^3*(g + h*x)^5) - (a^2*c^3*(6*c^2*d*g^
3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h)))*ArcTanh[(a*h
 - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*g^2 + a*h^2)^(11/2
))
```

Rubi [A] time = 0.888746, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 835, 807, 721, 725, 206}

$$-\frac{ac^2\sqrt{a+cx^2}(ah-cgx)\left(a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)+6c^2dg^3\right)}{16(g+hx)^2\left(ah^2+cg^2\right)^5}-\frac{c\left(a+cx^2\right)^{3/2}(ah-cgx)\left(a^2h^2(8fg-e\right.}{24(g+hx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -(a*c^2*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*
d*h)))*(a*h - c*g*x)*Sqrt[a + c*x^2])/(16*(c*g^2 + a*h^2)^5*(g + h*x)^2) -
(c*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h))
)*(a*h - c*g*x)*(a + c*x^2)^(3/2))/(24*(c*g^2 + a*h^2)^4*(g + h*x)^4) - ((f
*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(7*h*(c*g^2 + a*h^2)*(g + h*x)^7)
+ ((5*c*f*g^3 + c*g*h*(2*e*g - 9*d*h) + 7*a*h^2*(2*f*g - e*h))*(a + c*x^2)^
```

```
(5/2))/(42*h*(c*g^2 + a*h^2)^2*(g + h*x)^6) - ((42*a^2*f*h^4 - c^2*(5*f*g^4
 + g^2*h*(2*e*g - 51*d*h)) - a*c*h^2*(26*f*g^2 - h*(61*e*g - 12*d*h)))*(a +
 c*x^2)^(5/2))/(210*h*(c*g^2 + a*h^2)^3*(g + h*x)^5) - (a^2*c^3*(6*c^2*d*g^
3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h)))*ArcTanh[(a*h
 - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*g^2 + a*h^2)^(11/2
))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} - \frac{\int \frac{\left(-7(cdg-afg+aeh)-\left(7afh+c\left(2eg+\frac{5fg^2}{h}-2dh\right)\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^7}}{7\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} + \frac{\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh)\right)\left(a+\right.}{42h\left(cg^2+ah^2\right)^2(g+hx)^6}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} + \frac{\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh)\right)\left(a+\right.}{42h\left(cg^2+ah^2\right)^2(g+hx)^6}$$

$$= -\frac{c\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\left(a+cx^2\right)^{3/2}}{24\left(cg^2+ah^2\right)^4(g+hx)^4} -$$

$$= -\frac{ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^5(g+hx)^2} -$$

$$= -\frac{ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^5(g+hx)^2} -$$

$$= -\frac{ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^5(g+hx)^2} -$$

Mathematica [A] time = 2.63601, size = 863, normalized size = 1.62

$$\frac{a^2\left(6c^2dg^3-ac\left(fg^2+h(3dh-8eg)\right)g+a^2h^2(8fg-eh)\right)\log(g+hx)c^3}{16\left(cg^2+ah^2\right)^{11/2}}-\frac{a^2\left(6c^2dg^3-ac\left(fg^2+h(3dh-8eg)\right)g+a^2h^2(\right.}{16\left(cg^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -(Sqrt[a + c*x^2]*(240*(c*g^2 + a*h^2)^6*(f*g^2 + h*(-(e*g) + d*h)) - 40*(c
*g^2 + a*h^2)^5*(29*c*f*g^3 + c*g*h*(-22*e*g + 15*d*h) - 7*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 8*(c*g^2 + a*h^2)^4*(42*a^2*f*h^4 + a*c*h^2*(314*f*g^2 +
h*(-139*e*g + 48*d*h)) + c^2*(275*f*g^4 + g^2*h*(-142*e*g + 51*d*h)))*(g +
h*x)^2 - 2*c*(c*g^2 + a*h^2)^3*(7*a^2*h^4*(136*f*g - 35*e*h) + 2*c^2*(500*f
*g^5 + g^3*h*(-136*e*g + 3*d*h)) + a*c*g*h^2*(1979*f*g^2 + h*(-544*e*g + 33
*d*h)))*(g + h*x)^3 + 2*c*(c*g^2 + a*h^2)^2*(336*a^3*f*h^6 + c^3*(400*f*g^6
 - 2*g^4*h*(4*e*g + 3*d*h)) + 3*a^2*c*h^4*(400*f*g^2 + h*(-29*e*g + 8*d*h))
 + a*c^2*g^2*h^2*(1201*f*g^2 - h*(32*e*g + 45*d*h)))*(g + h*x)^4 - c^2*(c*g
^2 + a*h^2)*(21*a^3*h^6*(24*f*g - 5*e*h) + 2*a*c^2*g^3*h^2*(89*f*g^2 + 44*e
*g*h + 54*d*h^2) + 3*a^2*c*g*h^4*(109*f*g^2 + h*(94*e*g - 73*d*h)) + 4*c^3*
(10*f*g^7 + g^5*h*(4*e*g + 3*d*h)))*(g + h*x)^5 - c^2*(-336*a^4*f*h^8 + 2*a
*c^3*g^4*h^2*(109*f*g^2 + 52*e*g*h + 60*d*h^2) + a^2*c^2*g^2*h^4*(505*f*g^2
 + h*(370*e*g - 741*d*h)) + 4*c^4*(10*f*g^8 + g^6*h*(4*e*g + 3*d*h)) + 3*a^
3*c*h^6*(312*f*g^2 + h*(-221*e*g + 32*d*h)))*(g + h*x)^6))/(1680*(c*g^2*h +
 a*h^3)^5*(g + h*x)^7) + (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*
c*g*(f*g^2 + h*(-8*e*g + 3*d*h)))*Log[g + h*x])/(16*(c*g^2 + a*h^2)^(11/2))
 - (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 + h*(-8*e*g
 + 3*d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(16*(c*
g^2 + a*h^2)^(11/2))
```

Maple [B] time = 0.307, size = 19093, normalized size = 35.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.14222, size = 10714, normalized size = 20.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] -1/8*(6*a^2*c^5*d*g^3 - a^3*c^4*f*g^3 - 3*a^3*c^4*d*g*h^2 + 8*a^4*c^3*f*g*h
^2 + 8*a^3*c^4*g^2*h*e - a^4*c^3*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a
))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^5*g^10 + 5*a*c^4*g^8*h^2 + 10*a
^2*c^3*g^6*h^4 + 10*a^3*c^2*g^4*h^6 + 5*a^4*c*g^2*h^8 + a^5*h^10)*sqrt(-c*g
^2 - a*h^2)) - 1/840*(630*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^2*c^5*d*g^3*h^
12 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*f*g^3*h^12 - 315*(sqrt(c)
*x - sqrt(c*x^2 + a))^13*a^3*c^4*d*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a
))^13*a^4*c^3*f*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*g^2*h
^13*e - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^4*c^3*h^15*e - 1680*(sqrt(c)
*x - sqrt(c*x^2 + a))^12*c^(15/2)*f*g^10*h^5 - 8400*(sqrt(c)*x - sqrt(c*x^2
 + a))^12*a*c^(13/2)*f*g^8*h^7 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2
*c^(11/2)*f*g^6*h^9 + 8190*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2*c^(11/2)*d*
g^4*h^11 - 18165*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*f*g^4*h^11 -
4095*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*d*g^2*h^13 + 2520*(sqrt(c
)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*f*g^2*h^13 - 1680*(sqrt(c)*x - sqrt(c
*x^2 + a))^12*a^5*c^(5/2)*f*h^15 + 10920*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a
^3*c^(9/2)*g^3*h^12*e - 1365*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*g
*h^14*e - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*f*g^11*h^4 - 28000*(sqr
t(c)*x - sqrt(c*x^2 + a))^11*a*c^7*f*g^9*h^6 - 56000*(sqrt(c)*x - sqrt(c*x^
2 + a))^11*a^2*c^6*f*g^7*h^8 + 44940*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c
^6*d*g^5*h^10 - 63490*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*f*g^5*h^10 -
 26670*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*d*g^3*h^12 + 32620*(sqrt(c)
*x - sqrt(c*x^2 + a))^11*a^4*c^4*f*g^3*h^12 + 2100*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a^4*c^4*d*g*h^14 - 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*
f*g*h^14 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*g^10*h^5*e - 11200*(sq
rt(c)*x - sqrt(c*x^2 + a))^11*a*c^7*g^8*h^7*e - 22400*(sqrt(c)*x - sqrt(c*x
^2 + a))^11*a^2*c^6*g^6*h^9*e + 37520*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*
c^5*g^4*h^11*e - 24290*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^4*g^2*h^13*e
- 1540*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*h^15*e - 11200*(sqrt(c)*x -
 sqrt(c*x^2 + a))^10*c^(17/2)*f*g^12*h^3 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(17/2)*d*g^10*h^5 - 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/
2)*f*g^10*h^5 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/2)*d*g^8*h^7
 - 95200*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*f*g^8*h^7 + 100380*(
sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*d*g^6*h^9 - 100730*(sqrt(c)*x
- sqrt(c*x^2 + a))^10*a^3*c^(11/2)*f*g^6*h^9 - 146790*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(11/2)*d*g^4*h^11 + 163940*(sqrt(c)*x - sqrt(c*x^2 + a))^
10*a^4*c^(9/2)*f*g^4*h^11 + 6300*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/
2)*d*g^2*h^13 - 56000*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*f*g^2*h^
13 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*d*h^15 + 3360*(sqrt(
c)*x - sqrt(c*x^2 + a))^10*a^6*c^(5/2)*f*h^15 - 4480*(sqrt(c)*x - sqrt(c*x^
2 + a))^10*c^(17/2)*g^11*h^4*e - 22400*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c
^(15/2)*g^9*h^6*e - 44800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*g^7
```

*h^8*e + 133840*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(11/2)*g^5*h^10*e - 106330*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/2)*g^3*h^12*e + 3220*(sqrt (c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*g*h^14*e - 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*f*g^13*h^2 - 4032*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*d*g ^11*h^4 - 50848*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*f*g^11*h^4 - 20160*(s qrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*d*g^9*h^6 - 52640*(sqrt(c)*x - sqrt(c*x ^2 + a))^9*a^2*c^7*f*g^9*h^6 + 191016*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c ^7*d*g^7*h^8 - 9436*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*f*g^7*h^8 - 363 216*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*d*g^5*h^10 + 439306*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^5*f*g^5*h^10 + 95340*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^5*d*g^3*h^12 - 209965*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*f* g^3*h^12 - 9975*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*d*g*h^14 + 32200*(s qrt(c)*x - sqrt(c*x^2 + a))^9*a^6*c^3*f*g*h^14 - 5376*(sqrt(c)*x - sqrt(c*x ^2 + a))^9*c^9*g^12*h^3*e - 25984*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*g^1 0*h^5*e - 49280*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^7*g^8*h^7*e + 263648* (sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*g^6*h^9*e - 332780*(sqrt(c)*x - sqr t(c*x^2 + a))^9*a^4*c^5*g^4*h^11*e + 49490*(sqrt(c)*x - sqrt(c*x^2 + a))^9* a^5*c^4*g^2*h^13*e - 1085*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^6*c^3*h^15*e - 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/2)*f*g^14*h - 2688*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/2)*d*g^12*h^3 - 15232*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*f*g^12*h^3 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*d*g^10*h^5 + 53200*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*f*g^10*h^ 5 + 181104*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*d*g^8*h^7 + 143416* (sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*f*g^8*h^7 - 651924*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*d*g^6*h^9 + 580034*(sqrt(c)*x - sqrt(c*x^ 2 + a))^8*a^4*c^(11/2)*f*g^6*h^9 + 299460*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a ^4*c^(11/2)*d*g^4*h^11 - 568085*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2) *f*g^4*h^11 - 72975*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)*d*g^2*h^13 + 147000*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*f*g^2*h^13 - 3360*(sqr t(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*d*h^15 - 5040*(sqrt(c)*x - sqrt(c*x ^2 + a))^8*a^7*c^(5/2)*f*h^15 - 3584*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/ 2)*g^13*h^2*e - 9856*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*g^11*h^4*e + 4480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*g^9*h^6*e + 344512*(sqr t(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*g^7*h^8*e - 613480*(sqrt(c)*x - sq rt(c*x^2 + a))^8*a^4*c^(11/2)*g^5*h^10*e + 259210*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)*g^3*h^12*e - 9765*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^6*c^ (7/2)*g*h^14*e - 2560*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^10*f*g^15 - 768*(sq rt(c)*x - sqrt(c*x^2 + a))^7*c^10*d*g^13*h^2 + 12928*(sqrt(c)*x - sqrt(c*x^ 2 + a))^7*a*c^9*f*g^13*h^2 + 384*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^9*d*g^ 11*h^4 + 80576*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*f*g^11*h^4 + 117984* (sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*d*g^9*h^6 + 101936*(sqrt(c)*x - sqr t(c*x^2 + a))^7*a^3*c^7*f*g^9*h^6 - 603216*(sqrt(c)*x - sqrt(c*x^2 + a))^7* a^3*c^7*d*g^7*h^8 + 256816*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*f*g^7*h^ 8 + 703752*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*d*g^5*h^10 - 941332*(sqr t(c)*x - sqrt(c*x^2 + a))^7*a^5*c^5*f*g^5*h^10 - 184380*(sqrt(c)*x - sqrt(c *x^2 + a))^7*a^5*c^5*d*g^3*h^12 + 413280*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*f*g^3*h^12 + 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*d*g*h^14 - 47040*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^7*c^3*f*g*h^14 - 1024*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^10*g^14*h*e + 4096*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^9*g^12*h^3*e + 32768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*g^10*h^5*e + 205952*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^7*g^8*h^7*e - 741776*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*g^6*h^9*e + 608720*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^5*g^4*h^11*e - 92820*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*g^2*h^13*e + 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*f*g^14*h + 2688*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*d*g^12*h^3 + 15232*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*f*g^12*h^3 + 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*d*g^10*h^5 - 53200*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*f*g^10*h^5 - 342384*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*d*g^8*h^7 - 103936*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*f*g^8*h^7 + 736344*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*d*g^6*h^9 - 726404*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*f*g^6*h^9 - 488460*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*d*g^4*h^11 + 764960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*f*g^4*h^11 + 33600*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*d*g^2*h^13 - 168000*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*f*g^2*h^13 - 6720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*d*h^15 + 6720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^8*c^(5/2)*f*h^15 + 3584*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*g^13*h^2*e + 9856*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*g^11*h^4*e + 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*g^9*h^6*e - 487312*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*g^7*h^8*e + 807520*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*g^5*h^10*e - 310660*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*g^3*h^12*e + 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*g*h^14*e - 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*f*g^13*h^2 - 4032*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*d*g^11*h^4 - 50848*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*f*g^11*h^4 - 47040*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*d*g^9*h^6 - 50960*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*f*g^9*h^6 + 438816*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*d*g^7*h^8 - 99736*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*f*g^7*h^8 - 556416*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*d*g^5*h^10 + 728756*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*f*g^5*h^10 + 167790*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*d*g^3*h^12 - 362915*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*f*g^3*h^12 - 10185*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*d*g*h^14 + 38360*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^8*c^3*f*g*h^14 - 5376*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*g^12*h^3*e - 25984*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*g^10*h^5*e - 86240*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*g^8*h^7*e + 574448*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*g^6*h^9*e - 487480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*g^4*h^11*e + 89740*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*g^2*h^13*e + 1085*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^8*c^3*h^15*e + 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*f*g^12*h^3 + 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*d*g^10*h^5 + 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(15/2)*f*g^10*h^5 + 45360*(sqrt(c)*x - sq

rt(c*x^2 + a))^4*a^4*c^(15/2)*d*g^8*h^7 + 96880*(sqrt(c)*x - sqrt(c*x^2 + a
))^4*a^5*c^(13/2)*f*g^8*h^7 - 364728*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^
(13/2)*d*g^6*h^9 + 215908*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*f*g^
6*h^9 + 220710*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*d*g^4*h^11 - 40
6735*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*f*g^4*h^11 - 49581*(sqrt(c
)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*d*g^2*h^13 + 104776*(sqrt(c)*x - sqrt(
c*x^2 + a))^4*a^8*c^(7/2)*f*g^2*h^13 - 1344*(sqrt(c)*x - sqrt(c*x^2 + a))^4
*a^8*c^(7/2)*d*h^15 - 3696*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^9*c^(5/2)*f*h^
15 + 4480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*g^11*h^4*e + 29120*(
sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(15/2)*g^9*h^6*e + 119056*(sqrt(c)*x -
 sqrt(c*x^2 + a))^4*a^5*c^(13/2)*g^7*h^8*e - 390656*(sqrt(c)*x - sqrt(c*x^2
 + a))^4*a^6*c^(11/2)*g^5*h^10*e + 179900*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a
^7*c^(9/2)*g^3*h^12*e - 10703*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*g
*h^14*e - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*f*g^11*h^4 - 3360*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*d*g^9*h^6 - 29680*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^5*c^7*f*g^9*h^6 - 32592*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*
c^7*d*g^7*h^8 - 67088*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*f*g^7*h^8 + 1
72620*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*d*g^5*h^10 - 156170*(sqrt(c)*
x - sqrt(c*x^2 + a))^3*a^7*c^5*f*g^5*h^10 - 62454*(sqrt(c)*x - sqrt(c*x^2 +
 a))^3*a^7*c^5*d*g^3*h^12 + 140084*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*
f*g^3*h^12 + 5964*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*d*g*h^14 - 17024*
(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^9*c^3*f*g*h^14 - 2240*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^4*c^8*g^10*h^5*e - 16576*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5
*c^7*g^8*h^7*e - 72464*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*g^6*h^9*e +
179200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*g^4*h^11*e - 31402*(sqrt(c)*
x - sqrt(c*x^2 + a))^3*a^8*c^4*g^2*h^13*e + 1540*(sqrt(c)*x - sqrt(c*x^2 +
a))^3*a^9*c^3*h^15*e + 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*f*
g^10*h^5 + 1008*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*d*g^8*h^7 + 96
32*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*f*g^8*h^7 + 9996*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*d*g^6*h^9 + 24094*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*a^7*c^(11/2)*f*g^6*h^9 - 54894*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^
7*c^(11/2)*d*g^4*h^11 + 56924*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*f
*g^4*h^11 + 9156*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*d*g^2*h^13 - 3
2256*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*f*g^2*h^13 - 672*(sqrt(c)*
x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*d*h^15 + 672*(sqrt(c)*x - sqrt(c*x^2 + a
))^2*a^10*c^(5/2)*f*h^15 + 1344*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2
)*g^9*h^6*e + 8624*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*g^7*h^8*e +
 30352*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(11/2)*g^5*h^10*e - 47362*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*g^3*h^12*e + 3276*(sqrt(c)*x - sqrt
(c*x^2 + a))^2*a^9*c^(7/2)*g*h^14*e - 560*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6
*c^7*f*g^9*h^6 - 168*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*d*g^7*h^8 - 3052
*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^6*f*g^7*h^8 - 1680*(sqrt(c)*x - sqrt(c
*x^2 + a))*a^7*c^6*d*g^5*h^10 - 7070*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^5*
f*g^5*h^10 + 9744*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^5*d*g^3*h^12 - 12999*
(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^4*f*g^3*h^12 - 1029*(sqrt(c)*x - sqrt(c

*x^2 + a))*a^9*c^4*d*g*h^14 + 3864*(sqrt(c)*x - sqrt(c*x^2 + a))*a^10*c^3*f
*g*h^14 - 224*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*g^8*h^7*e - 1456*(sqrt(
c)*x - sqrt(c*x^2 + a))*a^7*c^6*g^6*h^9*e - 5180*(sqrt(c)*x - sqrt(c*x^2 +
a))*a^8*c^5*g^4*h^11*e + 8442*(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^4*g^2*h^1
3*e + 105*(sqrt(c)*x - sqrt(c*x^2 + a))*a^10*c^3*h^15*e + 40*a^7*c^(13/2)*f
*g^8*h^7 + 12*a^7*c^(13/2)*d*g^6*h^9 + 218*a^8*c^(11/2)*f*g^6*h^9 + 120*a^8
*c^(11/2)*d*g^4*h^11 + 505*a^9*c^(9/2)*f*g^4*h^11 - 741*a^9*c^(9/2)*d*g^2*h
^13 + 936*a^10*c^(7/2)*f*g^2*h^13 + 96*a^10*c^(7/2)*d*h^15 - 336*a^11*c^(5/
2)*f*h^15 + 16*a^7*c^(13/2)*g^7*h^8*e + 104*a^8*c^(11/2)*g^5*h^10*e + 370*a
^9*c^(9/2)*g^3*h^12*e - 663*a^10*c^(7/2)*g*h^14*e)/((c^5*g^10*h^6 + 5*a*c^4
*g^8*h^8 + 10*a^2*c^3*g^6*h^10 + 10*a^3*c^2*g^4*h^12 + 5*a^4*c*g^2*h^14 + a
^5*h^16)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a
))*sqrt(c)*g - a*h)^7)

3.100 $\int \left(a+cx^2\right)^{5/2} \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=168

$$\frac{5a^3(8Ac-aC)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}(8Ac-aC)}{128c}+\frac{x\left(a+cx^2\right)^{5/2}(8Ac-aC)}{48c}+\frac{5ax\left(a+cx^2\right)^{3/2}(8Ac-aC)}{192c}$$

```
[Out] (5*a^2*(8*A*c - a*C)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*(8*A*c - a*C)*x*(a +
 c*x^2)^(3/2))/(192*c) + ((8*A*c - a*C)*x*(a + c*x^2)^(5/2))/(48*c) + (B*(a
 + c*x^2)^(7/2))/(7*c) + (C*x*(a + c*x^2)^(7/2))/(8*c) + (5*a^3*(8*A*c - a*
C)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(3/2))
```

Rubi [A] time = 0.101682, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1815, 641, 195, 217, 206}

$$\frac{5a^3(8Ac-aC)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}(8Ac-aC)}{128c}+\frac{x\left(a+cx^2\right)^{5/2}(8Ac-aC)}{48c}+\frac{5ax\left(a+cx^2\right)^{3/2}(8Ac-aC)}{192c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (5*a^2*(8*A*c - a*C)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*(8*A*c - a*C)*x*(a +
 c*x^2)^(3/2))/(192*c) + ((8*A*c - a*C)*x*(a + c*x^2)^(5/2))/(48*c) + (B*(a
 + c*x^2)^(7/2))/(7*c) + (C*x*(a + c*x^2)^(7/2))/(8*c) + (5*a^3*(8*A*c - a*
C)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(3/2))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)dx = \frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{\int (8Ac-aC+8Bcx)\left(a+cx^2\right)^{5/2}dx}{8c}$$

$$= \frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{(8Ac-aC)\int\left(a+cx^2\right)^{5/2}dx}{8c}$$

$$= \frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{(5a(8Ac-aC))\int\left(a\right.}{48c}$$

$$= \frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+\right.}{8}$$

$$= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}$$

$$= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}$$

$$= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}$$

Mathematica [A] time = 0.334406, size = 150, normalized size = 0.89

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\left(2a^2cx(924A+x(576B+413Cx))+3a^3(128B+35Cx)+8ac^2x^3(182A+x(144B+119Cx))+16c^3x^5(28\right.\right.}{2688c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(3*a^3*(128*B + 35*C*x) + 16*c^3*x^5*(28*A + 3*x*
(8*B + 7*C*x)) + 8*a*c^2*x^3*(182*A + x*(144*B + 119*C*x)) + 2*a^2*c*x*(924
*A + x*(576*B + 413*C*x))) - (105*a^(5/2)*(-8*A*c + a*C)*ArcSinh[(Sqrt[c]*x
)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(2688*c^(3/2))
```

Maple [A] time = 0.05, size = 181, normalized size = 1.1

$$\frac{Cx}{8c}\left(cx^2+a\right)^{\frac{7}{2}}-\frac{Cax}{48c}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{5a^2Cx}{192c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{5Ca^3x}{128c}\sqrt{cx^2+a}-\frac{5Ca^4}{128}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+\frac{B}{7c}\left(cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x)
```

```
[Out] 1/8*C*x*(c*x^2+a)^(7/2)/c-1/48*C*a/c*x*(c*x^2+a)^(5/2)-5/192*C*a^2/c*x*(c*x
^2+a)^(3/2)-5/128*C*a^3/c*x*(c*x^2+a)^(1/2)-5/128*C*a^4/c^(3/2)*ln(x*c^(1/2
)+(c*x^2+a)^(1/2))+1/7*B*(c*x^2+a)^(7/2)/c+1/6*A*x*(c*x^2+a)^(5/2)+5/24*A*a
*x*(c*x^2+a)^(3/2)+5/16*A*a^2*x*(c*x^2+a)^(1/2)+5/16*A*a^3/c^(1/2)*ln(x*c^(
1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16134, size = 809, normalized size = 4.82

$$\left[-\frac{105\left(Ca^4-8\,Aa^3c\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(336\,Cc^4x^7+384\,Bc^4x^6+1152\,Bac^3x^4+1152\,Ba^2c^2x^2+\right.}{5376\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] [-1/5376*(105*(C*a^4 - 8*A*a^3*c)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*
sqrt(c)*x - a) - 2*(336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152
*B*a^2*c^2*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2
*c^2 + 104*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))
/c^2, 1/2688*(105*(C*a^4 - 8*A*a^3*c)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) + (336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152*B*a^2*c^2
*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2*c^2 + 104
*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 30.5604, size = 510, normalized size = 3.04

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{cx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}cx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{17A\sqrt{a}c^2x^5}{24\sqrt{1+\frac{cx^2}{a}}}+\frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16\sqrt{c}}+\frac{Ac^3x^7}{6\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+Ba^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\\ \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{3c} & \text{oth}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)*(C*x**2+B*x+A),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + c*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + c*x**2/a))
+ 35*A*a**(3/2)*c*x**3/(48*sqrt(1 + c*x**2/a)) + 17*A*sqrt(a)*c**2*x**5/(24
*sqrt(1 + c*x**2/a)) + 5*A*a**3*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c)) + A*c
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2,
 Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 2*B*a*c*Piecewise((-2*a**2
*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a
+ c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + B*c**2*Piecewise((8*a**3*
sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x
```

```
**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*
x**6/6, True)) + 5*C*a**(7/2)*x/(128*c*sqrt(1 + c*x**2/a)) + 133*C*a**(5/2)
*x**3/(384*sqrt(1 + c*x**2/a)) + 127*C*a**(3/2)*c*x**5/(192*sqrt(1 + c*x**2
/a)) + 23*C*sqrt(a)*c**2*x**7/(48*sqrt(1 + c*x**2/a)) - 5*C*a**4*asinh(sqrt
(c)*x/sqrt(a))/(128*c**(3/2)) + C*c**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.1761, size = 227, normalized size = 1.35

$$\frac{1}{2688}\left(\frac{384\,Ba^3}{c}+\left(2\left(576\,Ba^2+\left(4\left(144\,Bac+\left(6\left(7\,Cc^2x+8\,Bc^2\right)x+\frac{7\left(17\,Cac^7+8\,Ac^8\right)}{c^6}\right)x\right)x+\frac{7\left(59\,Ca^2c^6+104\,Aa\right.}{c^6}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/2688*(384*B*a^3/c + (2*(576*B*a^2 + (4*(144*B*a*c + (6*(7*C*c^2*x + 8*B*c
^2)*x + 7*(17*C*a*c^7 + 8*A*c^8)/c^6)*x)*x + 7*(59*C*a^2*c^6 + 104*A*a*c^7)
/c^6)*x)*x + 21*(5*C*a^3*c^5 + 88*A*a^2*c^6)/c^6)*x)*sqrt(c*x^2 + a) + 5/12
8*(C*a^4 - 8*A*a^3*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.101 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=325

$$\frac{\sqrt{a+cx^2}\left(4\left(16a^2fh^4-4ach^2\left(5h(dh+3eg)+13fg^2\right)-c^2g^2\left(3fg^2-5h(16dh+3eg)\right)\right)-chx\left(ah^2(45eh+71fg)+2cg\left(3\right.\right.\right.}{120c^3h}$$

```
[Out] ((4*(5*c*d - 4*a*f)*h^2 - 3*c*g*(f*g - 5*e*h))*(g + h*x)^2*Sqrt[a + c*x^2])
/(60*c^2*h) - ((f*g - 5*e*h)*(g + h*x)^3*Sqrt[a + c*x^2])/(20*c*h) + (f*(g
+ h*x)^4*Sqrt[a + c*x^2])/(5*c*h) + ((4*(16*a^2*f*h^4 - 4*a*c*h^2*(13*f*g^2
 + 5*h*(3*e*g + d*h)) - c^2*g^2*(3*f*g^2 - 5*h*(3*e*g + 16*d*h))) - c*h*(a*
h^2*(71*f*g + 45*e*h) + 2*c*g*(3*f*g^2 - 5*h*(3*e*g + 10*d*h)))*x)*Sqrt[a +
 c*x^2])/(120*c^3*h) + ((8*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f
*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rubi [A] time = 0.664388, antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 833, 780, 217, 206}

$$\frac{\sqrt{a+cx^2}\left(4\left(16a^2fh^4-4ach^2\left(5h(dh+3eg)+13fg^2\right)-c^2g^2\left(3fg^2-5h(16dh+3eg)\right)\right)-chx\left(ah^2(45eh+71fg)-10cgh\right.\right.}{120c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] ((4*(5*c*d - 4*a*f)*h^2 - 3*c*g*(f*g - 5*e*h))*(g + h*x)^2*Sqrt[a + c*x^2])
/(60*c^2*h) - ((f*g - 5*e*h)*(g + h*x)^3*Sqrt[a + c*x^2])/(20*c*h) + (f*(g
+ h*x)^4*Sqrt[a + c*x^2])/(5*c*h) + ((4*(16*a^2*f*h^4 - 4*a*c*h^2*(13*f*g^2
 + 5*h*(3*e*g + d*h)) - c^2*g^2*(3*f*g^2 - 5*h*(3*e*g + 16*d*h))) - c*h*(6*
c*f*g^3 - 10*c*g*h*(3*e*g + 10*d*h) + a*h^2*(71*f*g + 45*e*h))*x)*Sqrt[a +
c*x^2])/(120*c^3*h) + ((8*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*
g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
```

```
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}dx = \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^3\left((5cd-4af)h^2-ch(fg-5eh)x\right)}{\sqrt{a+cx^2}}dx}{5ch^2}$$

$$= -\frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} + \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^2\left(ch^2(20cdg-13afg-15aeh)+c\right.}{\sqrt{a+c}}}{20c^2}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} +$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} +$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} +$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} +$$

Mathematica [A] time = 0.393918, size = 252, normalized size = 0.78

$$\sqrt{a+cx^2}\left(8\left(8a^2fh^3-10ach\left(h(dh+3eg)+3fg^2\right)+15c^2g^2(3dh+eg)\right)+8chx^2\left(5c\left(h(dh+3eg)+3fg^2\right)-4afh^2\right)+15c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(8*(8*a^2*f*h^3 + 15*c^2*g^2*(e*g + 3*d*h) - 10*a*c*h*(3*f
*g^2 + h*(3*e*g + d*h))) + 15*c*(-3*a*h^2*(3*f*g + e*h) + 4*c*(f*g^3 + 3*g*
h*(e*g + d*h)))*x + 8*c*h*(-4*a*f*h^2 + 5*c*(3*f*g^2 + h*(3*e*g + d*h)))*x^
2 + 30*c^2*h^2*(3*f*g + e*h)*x^3 + 24*c^2*f*h^3*x^4) + 15*Sqrt[c]*(8*c^2*d*
g^3 + 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[c*x
+ Sqrt[c]*Sqrt[a + c*x^2]])/(120*c^3)
```

Maple [A] time = 0.062, size = 528, normalized size = 1.6

$$\frac{h^3fx^4}{5c}\sqrt{cx^2+a} - \frac{4ah^3fx^2}{15c^2}\sqrt{cx^2+a} + \frac{8a^2fh^3}{15c^3}\sqrt{cx^2+a} + \frac{x^3h^3e}{4c}\sqrt{cx^2+a} + \frac{3x^3gh^2f}{4c}\sqrt{cx^2+a} - \frac{3axh^3e}{8c^2}\sqrt{cx^2+a} - \frac{9a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/5*h^3*f*x^4/c*(c*x^2+a)^(1/2)-4/15*h^3*f*a/c^2*x^2*(c*x^2+a)^(1/2)+8/15*h
^3*f*a^2/c^3*(c*x^2+a)^(1/2)+1/4*x^3/c*(c*x^2+a)^(1/2)*h^3*e+3/4*x^3/c*(c*x
^2+a)^(1/2)*g*h^2*f-3/8*a/c^2*x*(c*x^2+a)^(1/2)*h^3*e-9/8*a/c^2*x*(c*x^2+a)
^(1/2)*g*h^2*f+3/8*a^2/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*h^3*e+9/8*a^2/
c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*g*h^2*f+1/3*x^2/c*(c*x^2+a)^(1/2)*h^3
*d+x^2/c*(c*x^2+a)^(1/2)*g*h^2*e+x^2/c*(c*x^2+a)^(1/2)*g^2*h*f-2/3*a/c^2*(c
*x^2+a)^(1/2)*h^3*d-2*a/c^2*(c*x^2+a)^(1/2)*g*h^2*e-2*a/c^2*(c*x^2+a)^(1/2)
*g^2*h*f+3/2*x/c*(c*x^2+a)^(1/2)*g*h^2*d+3/2*x/c*(c*x^2+a)^(1/2)*g^2*h*e+1/
2*x/c*(c*x^2+a)^(1/2)*g^3*f-3/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*g*h
^2*d-3/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*g^2*h*e-1/2*a/c^(3/2)*ln(x
*c^(1/2)+(c*x^2+a)^(1/2))*g^3*f+3/c*(c*x^2+a)^(1/2)*g^2*h*d+1/c*(c*x^2+a)^(
1/2)*g^3*e+g^3*d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08434, size = 1284, normalized size = 3.95

$$\left[-\frac{15\left(12\,aceg^2h - 3\,a^2eh^3 - 4\left(2\,c^2d - acf\right)g^3 + 3\left(4\,acd - 3\,a^2f\right)gh^2\right)\sqrt{c}\log\left(-2\,cx^2 - 2\,\sqrt{cx^2 + a}\sqrt{c}x - a\right) - 2\left(24\,c^2fh\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/240*(15*(12*a*c*e*g^2*h - 3*a^2*e*h^3 - 4*(2*c^2*d - a*c*f)*g^3 + 3*(4*
a*c*d - 3*a^2*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
```

```
- a) - 2*(24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a*c*e*g*h^2 + 120*(3*c^2*d
 - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 30*(3*c^2*f*g*h^2 + c^2*e*
h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 + (5*c^2*d - 4*a*c*f)*h^3)*x^
2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*h^3 + 3*(4*c^2*d - 3*a*c*f)*
g*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/120*(15*(12*a*c*e*g^2*h - 3*a^2*e*h^3 - 4
*(2*c^2*d - a*c*f)*g^3 + 3*(4*a*c*d - 3*a^2*f)*g*h^2)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) + (24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a*c*e*g*h^
2 + 120*(3*c^2*d - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 30*(3*c^2*
f*g*h^2 + c^2*e*h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 + (5*c^2*d -
4*a*c*f)*h^3)*x^2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*h^3 + 3*(4*c
^2*d - 3*a*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 19.6929, size = 796, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*e*h**3*x/(8*c**2*sqrt(1 + c*x**2/a)) - 9*a**(3/2)*f*g*h**2*x/(8
*c**2*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*d*g*h**2*x*sqrt(1 + c*x**2/a)/(2*c) +
 3*sqrt(a)*e*g**2*h*x*sqrt(1 + c*x**2/a)/(2*c) - sqrt(a)*e*h**3*x**3/(8*c*s
qrt(1 + c*x**2/a)) + sqrt(a)*f*g**3*x*sqrt(1 + c*x**2/a)/(2*c) - 3*sqrt(a)*
f*g*h**2*x**3/(8*c*sqrt(1 + c*x**2/a)) + 3*a**2*e*h**3*asinh(sqrt(c)*x/sqrt
(a))/(8*c**(5/2)) + 9*a**2*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(5/2)) -
 3*a*d*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) - 3*a*e*g**2*h*asinh(sq
rt(c)*x/sqrt(a))/(2*c**(3/2)) - a*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/
2)) + d*g**3*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c
 < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a
/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + 3*d*g**2*h*Piecewis
e((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + d*h**3*Piecew
ise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)
), (x**4/(4*sqrt(a)), True)) + e*g**3*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)
), (sqrt(a + c*x**2)/c, True)) + 3*e*g*h**2*Piecewise((-2*a*sqrt(a + c*x**2
)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), Tru
e)) + 3*f*g**2*h*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a +
c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + f*h**3*Piecewise((8*a
**2*sqrt(a + c*x**2)/(15*c**3) - 4*a*x**2*sqrt(a + c*x**2)/(15*c**2) + x**4
*sqrt(a + c*x**2)/(5*c), Ne(c, 0)), (x**6/(6*sqrt(a)), True)) + e*h**3*x**5
/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*f*g*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**
2/a))
```

Giac [A] time = 1.17259, size = 424, normalized size = 1.3

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(\frac{4\,fh^3x}{c}+\frac{5\left(3\,c^4fgh^2+c^4h^3e\right)}{c^5}\right)x+\frac{4\left(15\,c^4fg^2h+5\,c^4dh^3-4\,ac^3fh^3+15\,c^4gh^2e\right)}{c^5}\right)x+\frac{15\left(4\,c^4fg\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h^3*x/c + 5*(3*c^4*f*g*h^2 + c^4*h^3*e)/c
^5)*x + 4*(15*c^4*f*g^2*h + 5*c^4*d*h^3 - 4*a*c^3*f*h^3 + 15*c^4*g*h^2*e)/c
^5)*x + 15*(4*c^4*f*g^3 + 12*c^4*d*g*h^2 - 9*a*c^3*f*g*h^2 + 12*c^4*g^2*h*e
 - 3*a*c^3*h^3*e)/c^5)*x + 8*(45*c^4*d*g^2*h - 30*a*c^3*f*g^2*h - 10*a*c^3*
d*h^3 + 8*a^2*c^2*f*h^3 + 15*c^4*g^3*e - 30*a*c^3*g*h^2*e)/c^5) - 1/8*(8*c^
2*d*g^3 - 4*a*c*f*g^3 - 12*a*c*d*g*h^2 + 9*a^2*f*g*h^2 - 12*a*c*g^2*h*e + 3
*a^2*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.102 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=223

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{8c^{5/2}}-\frac{\sqrt{a+cx^2}\left(4\left(4ah^2(eh+2fg)+cg\left(fg^2-4h(3dh+eg)\right)\right)\right.}{24c^2h}$$

```
[Out] -((f*g - 4*e*h)*(g + h*x)^2*Sqrt[a + c*x^2])/(12*c*h) + (f*(g + h*x)^3*Sqrt
[a + c*x^2])/(4*c*h) - ((4*(4*a*h^2*(2*f*g + e*h) + c*g*(f*g^2 - 4*h*(e*g +
 3*d*h))) - h*(3*(4*c*d - 3*a*f)*h^2 - 2*c*g*(f*g - 4*e*h))*x)*Sqrt[a + c*x
^2])/(24*c^2*h) + ((8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d
*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rubi [A] time = 0.372031, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 833, 780, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{8c^{5/2}}-\frac{\sqrt{a+cx^2}\left(4\left(4ah^2(eh+2fg)-4cgh(3dh+eg)+cfg^3\right)\right.}{24c^2h}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] -((f*g - 4*e*h)*(g + h*x)^2*Sqrt[a + c*x^2])/(12*c*h) + (f*(g + h*x)^3*Sqrt
[a + c*x^2])/(4*c*h) - ((4*(c*f*g^3 - 4*c*g*h*(e*g + 3*d*h) + 4*a*h^2*(2*f*
g + e*h)) - h*(3*(4*c*d - 3*a*f)*h^2 - 2*c*g*(f*g - 4*e*h))*x)*Sqrt[a + c*x
^2])/(24*c^2*h) + ((8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d
*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
```

```
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left((4cd-3af)h^2-ch(fg-4eh)x\right)}{\sqrt{a+cx^2}}dx}{4ch^2}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)\left(ch^2(12cdg-7afg-8aeh)+ch(3\right.}{\sqrt{a+cx^2}}}{12c^2h}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{\left(4\left(cfg^3-4cgh(eg+3dh)+4\right.\right.}{}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{\left(4\left(cfg^3-4cgh(eg+3dh)+4\right.\right.}{}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{\left(4\left(cfg^3-4cgh(eg+3dh)+4\right.\right.}{}$$

Mathematica [A] time = 0.249788, size = 164, normalized size = 0.74

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)+\sqrt{c}\sqrt{a+cx^2}\left(2c\left(6dh(4g+hx)+4e\left(3g^2+3ghx+h^2x\right.\right.\right.}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(-(a*h*(32*f*g + 16*e*h + 9*f*h*x)) + 2*c*(6*d*h*(
4*g + h*x) + 4*e*(3*g^2 + 3*g*h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2
*x^2))) + 3*(8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d*h)))*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(24*c^(5/2))
```

Maple [A] time = 0.061, size = 339, normalized size = 1.5

$$\frac{h^2fx^3}{4c}\sqrt{cx^2+a} - \frac{3afh^2x}{8c^2}\sqrt{cx^2+a} + \frac{3a^2fh^2}{8}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{5}{2}} + \frac{h^2x^2e}{3c}\sqrt{cx^2+a} + \frac{2x^2ghf}{3c}\sqrt{cx^2+a} - \frac{2ah^2e}{3c^2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/4*h^2*f*x^3/c*(c*x^2+a)^(1/2)-3/8*h^2*f*a/c^2*x*(c*x^2+a)^(1/2)+3/8*h^2*f
*a^2/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+1/3*x^2/c*(c*x^2+a)^(1/2)*h^2*e+
2/3*x^2/c*(c*x^2+a)^(1/2)*g*h*f-2/3*a/c^2*(c*x^2+a)^(1/2)*h^2*e-4/3*a/c^2*(
c*x^2+a)^(1/2)*g*h*f+1/2*x/c*(c*x^2+a)^(1/2)*d*h^2+x/c*(c*x^2+a)^(1/2)*e*g*
h+1/2*x/c*(c*x^2+a)^(1/2)*f*g^2-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
*d*h^2-a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*e*g*h-1/2*a/c^(3/2)*ln(x*c^(
1/2)+(c*x^2+a)^(1/2))*f*g^2+2/c*(c*x^2+a)^(1/2)*g*h*d+1/c*(c*x^2+a)^(1/2)*e
*g^2+g^2*d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18368, size = 871, normalized size = 3.91

$$\left[-\frac{3\left(8acegh-4\left(2c^2d-acf\right)g^2+\left(4acd-3a^2f\right)h^2\right)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(6c^2fh^2x^3+24c^2eg^2-\cdots\right.}{48c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f)*g^2 + (4*a*c*d - 3*a^2*f)*h^2)
*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(6*c^2*f*h^2*x
^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*f)*g*h + 8*(2*c^2*f*
g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h + (4*c^2*d - 3*a*c*f)*h
^2)*x)*sqrt(c*x^2 + a))/c^3, 1/24*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f)*g^2
 + (4*a*c*d - 3*a^2*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (
6*c^2*f*h^2*x^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*f)*g*h
+ 8*(2*c^2*f*g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h + (4*c^2*d
 - 3*a*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 13.7865, size = 518, normalized size = 2.32

$$-\frac{3a^{\frac{3}{2}}fh^2x}{8c^2\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}dh^2x\sqrt{1+\frac{cx^2}{a}}}{2c}+\frac{\sqrt{a}eghx\sqrt{1+\frac{cx^2}{a}}}{c}+\frac{\sqrt{a}fg^2x\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{\sqrt{a}fh^2x^3}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{3a^2fh^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8c^{\frac{5}{2}}}-\frac{adh^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*f*h**2*x/(8*c**2*sqrt(1 + c*x**2/a)) + sqrt(a)*d*h**2*x*sqrt(1
+ c*x**2/a)/(2*c) + sqrt(a)*e*g*h*x*sqrt(1 + c*x**2/a)/c + sqrt(a)*f*g**2*x
*sqrt(1 + c*x**2/a)/(2*c) - sqrt(a)*f*h**2*x**3/(8*c*sqrt(1 + c*x**2/a)) +
3*a**2*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(5/2)) - a*d*h**2*asinh(sqrt(c
)*x/sqrt(a))/(2*c**(3/2)) - a*e*g*h*asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - a*f
*g**2*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) + d*g**2*Piecewise((sqrt(-a/c)*
asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a
))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (
c > 0) & (a < 0))) + 2*d*g*h*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(
a + c*x**2)/c, True)) + e*g**2*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqr
t(a + c*x**2)/c, True)) + e*h**2*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2)
+ x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + 2*f*g
*h*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c),
 Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + f*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x
**2/a))
```

Giac [A] time = 1.17298, size = 278, normalized size = 1.25

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(\frac{3\,fh^2x}{c}+\frac{4\left(2\,c^3fgh+c^3h^2e\right)}{c^4}\right)x+\frac{3\left(4\,c^3fg^2+4\,c^3dh^2-3\,ac^2fh^2+8\,c^3ghe\right)}{c^4}\right)x+\frac{8\left(6\,c^3dgh-4\,ac^2fg\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*h^2*x/c + 4*(2*c^3*f*g*h + c^3*h^2*e)/c^4)*x
+ 3*(4*c^3*f*g^2 + 4*c^3*d*h^2 - 3*a*c^2*f*h^2 + 8*c^3*g*h*e)/c^4)*x + 8*(6
```

```
*c^3*d*g*h - 4*a*c^2*f*g*h + 3*c^3*g^2*e - 2*a*c^2*h^2*e)/c^4) - 1/8*(8*c^2
*d*g^2 - 4*a*c*f*g^2 - 4*a*c*d*h^2 + 3*a^2*f*h^2 - 8*a*c*g*h*e)*log(abs(-sq
rt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.103 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{a+cx^2}\left(2\left(2afh^2+c\left(fg^2-3h(dh+eg)\right)\right)+chx(fg-3eh)\right)}{6c^2h}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{2c^{3/2}}+\frac{f\sqrt{a+cx^2}(g+$$

```
[Out] (f*(g + h*x)^2*Sqrt[a + c*x^2])/(3*c*h) - ((2*(2*a*f*h^2 + c*(f*g^2 - 3*h*(
e*g + d*h))) + c*h*(f*g - 3*e*h)*x)*Sqrt[a + c*x^2])/(6*c^2*h) + ((2*c*d*g
- a*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rubi [A] time = 0.178773, antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1654, 780, 217, 206}

$$-\frac{\sqrt{a+cx^2}\left(2\left(2afh^2-3ch(dh+eg)+cfg^2\right)+chx(fg-3eh)\right)}{6c^2h}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{2c^{3/2}}+\frac{f\sqrt{a+cx^2}(g+$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + c*x^2])/(3*c*h) - ((2*(c*f*g^2 + 2*a*f*h^2 - 3*c*h*
(e*g + d*h)) + c*h*(f*g - 3*e*h)*x)*Sqrt[a + c*x^2])/(6*c^2*h) + ((2*c*d*g
- a*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx &= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch}+\frac{\int \frac{(g+hx)\left((3cd-2af)h^2-ch(fg-3eh)x\right)}{\sqrt{a+cx^2}}\,dx}{3ch^2}\\
&= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch}-\frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{a+cx^2}}{6c^2h}+\\
&= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch}-\frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{a+cx^2}}{6c^2h}+\\
&= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch}-\frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{a+cx^2}}{6c^2h}+\end{aligned}$$

Mathematica [A] time = 0.113045, size = 96, normalized size = 0.71

$$\frac{\sqrt{a+cx^2}\left(c\left(6dh+6eg+3ehx+3fgx+2fhx^2\right)-4afh\right)+3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(-4*a*f*h + c*(6*e*g + 6*d*h + 3*f*g*x + 3*e*h*x + 2*f*h*x
^2)) + 3*Sqrt[c]*(2*c*d*g - a*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/(6*c^2)
```

Maple [A] time = 0.055, size = 172, normalized size = 1.3

$$\frac{fhx^2}{3c}\sqrt{cx^2+a}-\frac{2afh}{3c^2}\sqrt{cx^2+a}+\frac{ehx}{2c}\sqrt{cx^2+a}+\frac{fgx}{2c}\sqrt{cx^2+a}-\frac{aeh}{2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}-\frac{afg}{2}\ln\left(x\sqrt{c}+\sqrt{cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3*h*f*x^2/c*(c*x^2+a)^(1/2)-2/3*h*f*a/c^2*(c*x^2+a)^(1/2)+1/2*x/c*(c*x^2+
a)^(1/2)*e*h+1/2*x/c*(c*x^2+a)^(1/2)*f*g-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+
a)^(1/2))*e*h-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*f*g+1/c*(c*x^2+a)
^(1/2)*d*h+1/c*(c*x^2+a)^(1/2)*e*g+d*g*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20611, size = 482, normalized size = 3.54

$$\left[\frac{3\left(aeh-\left(2\,cd-af\right)g\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(2\,cfhx^2+6\,ceg+2\left(3\,cd-2\,af\right)h+3\left(cfg+ceh\right)x\right)\sqrt{c}}{12\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) + 2*(2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*d - 2*a*f)*h + 3*(c*f*g
 + c*e*h)*x)*sqrt(c*x^2 + a))/c^2, 1/6*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt(-c
)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*d -
2*a*f)*h + 3*(c*f*g + c*e*h)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 7.04748, size = 282, normalized size = 2.07

$$\frac{\sqrt{a}ehx\sqrt{1+\frac{cx^2}{a}}}{2c}+\frac{\sqrt{a}fgx\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{aeh\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}-\frac{afg\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+dg\left\{\begin{array}{ll}\left(\frac{\sqrt{-\frac{a}{c}}\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}}\right. & \text{for } a>0\wedge c<0 \\ \frac{\sqrt{\frac{a}{c}}\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c>0 \\ \frac{\sqrt{-\frac{a}{c}}\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for } c>0\wedge a<0\end{array}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*e*h*x*sqrt(1 + c*x**2/a)/(2*c) + sqrt(a)*f*g*x*sqrt(1 + c*x**2/a)/(
2*c) - a*e*h*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) - a*f*g*asinh(sqrt(c)*x/
sqrt(a))/(2*c**(3/2)) + d*g*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a
), (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c
> 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + d*h*
Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + e*g*P
iecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + f*h*Pi
ecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c
, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A] time = 1.2111, size = 149, normalized size = 1.1

$$\frac{1}{6}\sqrt{cx^2+a}\left(\left(\frac{2\,fhx}{c}+\frac{3\left(c^2fg+c^2he\right)}{c^3}\right)x+\frac{2\left(3\,c^2dh-2\,acfh+3\,c^2ge\right)}{c^3}\right)-\frac{\left(2\,cdg-afg-ahe\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right.}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*((2*f*h*x/c + 3*(c^2*f*g + c^2*h*e)/c^3)*x + 2*(3*c^2*d
*h - 2*a*c*f*h + 3*c^2*g*e)/c^3) - 1/2*(2*c*d*g - a*f*g - a*h*e)*log(abs(-s
qrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.104 $\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

```
[Out] (e*Sqrt[a + c*x^2])/c + (f*x*Sqrt[a + c*x^2])/(2*c) + ((2*c*d - a*f)*ArcTan
h[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rubi [A] time = 0.0483364, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1815, 641, 217, 206}

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

```
[Out] (e*Sqrt[a + c*x^2])/c + (f*x*Sqrt[a + c*x^2])/(2*c) + ((2*c*d - a*f)*ArcTan
h[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx &= \frac{fx\sqrt{a+cx^2}}{2c}+\frac{\int \frac{2cd-af+2cex}{\sqrt{a+cx^2}}\,dx}{2c}\\
&= \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2c}\\
&= \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{2c}\\
&= \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0406388, size = 63, normalized size = 0.85

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+\sqrt{c}\sqrt{a+cx^2}(2e+fx)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*(2*e + f*x)*Sqrt[a + c*x^2] + (2*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sq
rt[a + c*x^2]])/(2*c^(3/2))
```

Maple [A] time = 0.052, size = 76, normalized size = 1.

$$\frac{fx}{2\,c}\sqrt{cx^2+a}-\frac{af}{2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+\frac{e}{c}\sqrt{cx^2+a}+d\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(c*x^2+a)^(1/2)/c-1/2*f*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+e*(
c*x^2+a)^(1/2)/c+d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00256, size = 305, normalized size = 4.12

$$\left[-\frac{\left(2\,cd-af\right)\sqrt{c}\log\left(-2\,cx^{2}+2\,\sqrt{cx^{2}+a}\sqrt{c}x-a\right)-2\left(cfx+2\,ce\right)\sqrt{cx^{2}+a}}{4\,c^{2}},-\frac{\left(2\,cd-af\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^{2}+a}}\right)-\left(cfx\right.}{2\,c^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*c*d - a*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) - 2*(c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2, -1/2*((2*c*d - a*f)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 3.07121, size = 150, normalized size = 2.03

$$\frac{\sqrt{a}fx\sqrt{1+\frac{cx^{2}}{a}}}{2c}-\frac{af\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge c<0\\ \frac{\sqrt{\frac{a}{c}}\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge c>0\\ \frac{\sqrt{-\frac{a}{c}}\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for}\ c>0\wedge a<0\end{cases}\right)+e\left(\begin{cases}\frac{x^{2}}{2\sqrt{a}} & \text{for}\ c=0\\ \frac{\sqrt{a+cx^{2}}}{c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x*sqrt(1 + c*x**2/a)/(2*c) - a*f*asinh(sqrt(c)*x/sqrt(a))/(2*c**(
3/2)) + d*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c <
0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a/c)
*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + e*Piecewise((x**2/(2*s
qrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True))
```

Giac [A] time = 1.17328, size = 78, normalized size = 1.05

$$\frac{1}{2}\sqrt{cx^2+a}\left(\frac{fx}{c}+\frac{2e}{c}\right)-\frac{\left(2cd-af\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(f*x/c + 2*e/c) - 1/2*(2*c*d - a*f)*log(abs(-sqrt(c)*x
+ sqrt(c*x^2 + a)))/c^(3/2)
```

3.105 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=130

$$-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^2\sqrt{ah^2+cg^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}h^2}+\frac{f\sqrt{a+cx^2}}{ch}$$

```
[Out] (f*Sqrt[a + c*x^2])/(c*h) - ((f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]])/(Sqrt[c]*h^2) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*
g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h^2*Sqrt[c*g^2 + a*h^2])
```

Rubi [A] time = 0.174346, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 844, 217, 206, 725}

$$-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^2\sqrt{ah^2+cg^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}h^2}+\frac{f\sqrt{a+cx^2}}{ch}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (f*Sqrt[a + c*x^2])/(c*h) - ((f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]])/(Sqrt[c]*h^2) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*
g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h^2*Sqrt[c*g^2 + a*h^2])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx &= \frac{f\sqrt{a+cx^2}}{ch}+\frac{\int \frac{cdh^2-ch(fg-eh)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{ch^2}\\
&= \frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2}+\frac{\left(fg^2-egh+dh^2\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{h^2}\\
&= \frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{h^2}-\frac{\left(fg^2-egh+dh^2\right)\mathrm{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\right.}{h^2}\\
&= \frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}-\frac{\left(fg^2-egh+dh^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^2\sqrt{cg^2+ah^2}}
\end{aligned}
$$

Mathematica [A] time = 0.230856, size = 125, normalized size = 0.96

$$\frac{-\frac{\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\sqrt{ah^2+cg^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(eh-fg)}{\sqrt{c}}+\frac{fh\sqrt{a+cx^2}}{c}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((f*h*Sqrt[a + c*x^2])/c + ((-(f*g) + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/Sqrt[c] - ((f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g
^2 + a*h^2]*Sqrt[a + c*x^2])])/Sqrt[c*g^2 + a*h^2])/h^2
```

Maple [B] time = 0.236, size = 453, normalized size = 3.5

$$\frac{f}{ch}\sqrt{cx^2+a}+\frac{e}{h}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{fg}{h^2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{d}{h}\ln\left(\left(2\frac{ah^2+cg^2}{h^2}-2\frac{cg}{h}\left(x+\frac{g}{h}\right)+2\sqrt{\frac{ah^2}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x)
```

```
[Out] f*(c*x^2+a)^(1/2)/c/h+1/h*e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/h^2*f*g
*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/h/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*
(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-
2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/h^2/((a*h^2+c*g^2)/h
^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/
2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e*g-1/h^
3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h
^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))
/(x+g/h))*f*g^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\left(g+hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)), x)
```

Giac [A] time = 1.21738, size = 186, normalized size = 1.43

$$\frac{\sqrt{cx^2+a}f}{ch}+\frac{2\left(fg^2+dh^2-ghe\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}h^2}+\frac{\left(\sqrt{c}fg-\sqrt{c}he\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{ch^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + a)*f/(c*h) + 2*(f*g^2 + d*h^2 - g*h*e)*arctan(-((sqrt(c)*x - s
qrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2)*
h^2) + (sqrt(c)*f*g - sqrt(c)*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/(
c*h^2)
```

3.106 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=168

$$-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)}{h^2\left(ah^2+cg^2\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}$$

```
[Out] -(((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)))
+ (f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h^2) + ((a*h^2*(2*f*g -
 e*h) + c*(f*g^3 - d*g*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/(h^2*(c*g^2 + a*h^2)^(3/2))
```

Rubi [A] time = 0.234886, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 844, 217, 206, 725}

$$-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)}{h^2\left(ah^2+cg^2\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)))
+ (f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h^2) + ((a*h^2*(2*f*g -
 e*h) + c*(f*g^3 - d*g*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/(h^2*(c*g^2 + a*h^2)^(3/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{-cdg+afg-aeh-f\left(\frac{cg^2}{h}+ah\right)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2} + \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,d}{cg^2+ah^2}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{h^2} - \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right.}{}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2} - \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{\left(cg^2+ah^2\right)^{3/2}}$$

Mathematica [A] time = 0.418786, size = 218, normalized size = 1.3

$$\frac{-\frac{h\sqrt{a+cx^2}\left(h(dh-eg)+fg^2\right)}{(g+hx)\left(ah^2+cg^2\right)}+\frac{\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)}{\left(ah^2+cg^2\right)^{3/2}}+\frac{\log(g+hx)\left(ah^2(eh-2fg)+c\left(dgh^2-fg^3\right)\right)}{\left(ah^2+cg^2\right)^{3/2}}+\frac{f\log\left(\sqrt{c}\sqrt{a+cx^2}+c\right.}{\sqrt{c}}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)*(g + h*x
))) + ((a*h^2*(-2*f*g + e*h) + c*(-(f*g^3) + d*g*h^2))*Log[g + h*x])/(c*g^2
 + a*h^2)^(3/2) + (f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + ((a*h^2*
(2*f*g - e*h) + c*(f*g^3 - d*g*h^2))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(3/2))/h^2
```

Maple [B] time = 0.249, size = 923, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] f/h^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/h^2/((a*h^2+c*g^2)/h^2)^(1/2)
*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/
h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h^3/((a*h^2+c
*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h
^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f
*g-1/(a*h^2+c*g^2)/(x+g/h)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^
(1/2)*d+1/h/(a*h^2+c*g^2)/(x+g/h)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2
)/h^2)^(1/2)*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(
a*h^2+c*g^2)/h^2)^(1/2)*f*g^2-1/h*c*g/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+
g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/h^2*c*g^2/(
a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g
/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/
h^2)^(1/2))/(x+g/h))*e-1/h^3*c*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*
ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h
)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2}{\sqrt{a + cx^2}\left(g + hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.107 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=225

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)}{2\left(ah^2+cg^2\right)^{5/2}}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}+\frac{\sqrt{a+cx^2}\left(2ah^2\right.}{2}$$

```
[Out] -((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2
) + ((2*a*h^2*(2*f*g - e*h) + c*g*(f*g^2 + h*(e*g - 3*d*h)))*Sqrt[a + c*x^2
])/(2*h*(c*g^2 + a*h^2)^2*(g + h*x)) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f
*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a
+ c*x^2])])/(2*(c*g^2 + a*h^2)^(5/2))
```

Rubi [A] time = 0.292261, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1651, 807, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)}{2\left(ah^2+cg^2\right)^{5/2}}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}+\frac{\sqrt{a+cx^2}\left(2ah^2\right.}{2h}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] -((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2
) + ((c*f*g^3 + c*g*h*(e*g - 3*d*h) + 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x^2
])/(2*h*(c*g^2 + a*h^2)^2*(g + h*x)) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f
*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a
+ c*x^2])])/(2*(c*g^2 + a*h^2)^(5/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} - \frac{\int \frac{-2(cdg-afg+aeh)-\left(2afh+c\left(eg+\frac{fg^2}{h}-dh\right)\right)x}{(g+hx)^2\sqrt{a+cx^2}}\,dx}{2\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} + \frac{\left(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh)\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)^2(g+hx)} + \frac{\left(2c^2dg\right.}{}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} + \frac{\left(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh)\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)^2(g+hx)} - \frac{\left(2c^2dg\right.}{}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} + \frac{\left(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh)\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)^2(g+hx)} - \frac{\left(2c^2dg\right.}{}$$

Mathematica [A] time = 0.503486, size = 254, normalized size = 1.13

$$\frac{(g+hx)^2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(-2a^2fh^2+ac\left(h(dh-3eg)+fg^2\right)-2c^2dg^2\right)+(g+hx)^2\log(g+hx)\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]*(c*g*(f*g^2*x + e*g*(2*g + h*x) - d*h*
(4*g + 3*h*x)) - a*h*(-(f*g*(3*g + 4*h*x)) + h*(d*h + e*(g + 2*h*x)))) + (2
*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f*g^2 + h*(-3*e*g + d*h)))*(g + h*x)^2*Log[
g + h*x] + (-2*c^2*d*g^2 - 2*a^2*f*h^2 + a*c*(f*g^2 + h*(-3*e*g + d*h)))*(g
 + h*x)^2*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(2*(c*g^2
 + a*h^2)^(5/2)*(g + h*x)^2)
```

Maple [B] time = 0.246, size = 1574, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/2/h/(a*h^2+c*g^2)/(x+g/h)^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h
^2)^(1/2)*d+1/2/h^2/(a*h^2+c*g^2)/(x+g/h)^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a
*h^2+c*g^2)/h^2)^(1/2)*e*g-1/2/h^3/(a*h^2+c*g^2)/(x+g/h)^2*((x+g/h)^2*c-2*c
*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2-3/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)
*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*d+3/2/h*c*g^2/(a*h^2
+c*g^2)^2/(x+g/h)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*e-3
/2/h^2*c*g^3/(a*h^2+c*g^2)^2/(x+g/h)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*
g^2)/h^2)^(1/2)*f-3/2/h*c^2*g^2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*l
n((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)
^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+3/2/h^2*c^2*g^3/(
a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x
+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2
)/h^2)^(1/2))/(x+g/h))*e-3/2/h^3*c^2*g^4/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2
)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)
*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f+1/2/h*c/
(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+
```

```
g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)
/h^2)^(1/2))/(x+g/h))*d-3/2/h^2*c/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*l
n((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)
^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e*g+5/2/h^3*c/(a*h^
2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+
2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)
^(1/2))/(x+g/h))*f*g^2-f/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/
h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h
)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))-1/h/(a*h^2+c*g^2)/(x+g/h)*((x+g/h)^2*c
-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*e+2/h^2/(a*h^2+c*g^2)/(x+g/h)*((x
+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 62.2531, size = 2209, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2*f)*g^2*h^2 +
(3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f)*h^4)*x^2 + 2
*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*f)*g*h^3)*x)*s
qrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*
c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2
+ 2*g*h*x + g^2)) + 2*(2*c^2*e*g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^
5 - (4*c^2*d - 3*a*c*f)*g^4*h - (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 +
c^2*e*g^4*h - a*c*e*g^2*h^3 - 2*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (
3*a*c*d - 4*a^2*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 +
3*a^2*c*g^4*h^4 + a^3*g^2*h^6 + (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^
```

```
2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a
^3*g*h^7)*x), -1/2*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2
*f)*g^2*h^2 + (3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f
)*h^4)*x^2 + 2*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*
f)*g*h^3)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)
*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (2*c^2*e*
g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^5 - (4*c^2*d - 3*a*c*f)*g^4*h -
 (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 + c^2*e*g^4*h - a*c*e*g^2*h^3 - 2
*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (3*a*c*d - 4*a^2*f)*g*h^4)*x)*sq
rt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6 +
 (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g
^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\left(g+hx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**3), x)
```

Giac [B] time = 1.25171, size = 1145, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*c^2*d*g^2 - a*c*f*g^2 - a*c*d*h^2 + 2*a^2*f*h^2 + 3*a*c*g*h*e)*arctan((
(sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^2*g^
4 + 2*a*c*g^2*h^2 + a^2*h^4)*sqrt(-c*g^2 - a*h^2)) + (2*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3
+ 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2
 + a))^3*a*c*d*h^5 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e + 2*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))
```

```
^2*c^(5/2)*d*g^3*h^2 + 7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^
2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^
(5/2)*g^4*h*e - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e + 2*(
sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 2*(sqrt(c)*x - sqrt(c*x^
2 + a))*a*c^2*f*g^4*h + 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 -
11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2
+ a))*a^2*c*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e + 5*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + a^2*c^(3/2)*f*g^3*h^2 - 3*a^2*c^
(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 + a^2*c^(3/2)*g^2*h^3*e - 2*a^3*sqrt(
c)*h^5*e)/((c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2
 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

3.108 $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{h\sqrt{a+cx^2}\left(4\left(4a^2fh^2-ac\left(3h(dh+3eg)+7fg^2\right)+3c^2dg^2\right)+chx(-9aeh-11afg+6cdg)\right)}{6ac^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3ah^2(\right.}{}$$

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^3)/(a*c*Sqrt[a + c*x^2])) - ((3*c*d - 4*
a*f)*h*(g + h*x)^2*Sqrt[a + c*x^2])/(3*a*c^2) - (h*(4*(3*c^2*d*g^2 + 4*a^2*
f*h^2 - a*c*(7*f*g^2 + 3*h*(3*e*g + d*h))) + c*h*(6*c*d*g - 11*a*f*g - 9*a*
e*h)*x)*Sqrt[a + c*x^2])/(6*a*c^3) - ((3*a*h^2*(3*f*g + e*h) - 2*c*g*(f*g^2
 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(5/2))
```

Rubi [A] time = 0.324773, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1645, 833, 780, 217, 206}

$$-\frac{h\sqrt{a+cx^2}\left(4\left(4a^2fh^2-ac\left(3h(dh+3eg)+7fg^2\right)+3c^2dg^2\right)+chx(-9aeh-11afg+6cdg)\right)}{6ac^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(-3ah\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^3)/(a*c*Sqrt[a + c*x^2])) - ((3*c*d - 4*
a*f)*h*(g + h*x)^2*Sqrt[a + c*x^2])/(3*a*c^2) - (h*(4*(3*c^2*d*g^2 + 4*a^2*
f*h^2 - a*c*(7*f*g^2 + 3*h*(3*e*g + d*h))) + c*h*(6*c*d*g - 11*a*f*g - 9*a*
e*h)*x)*Sqrt[a + c*x^2])/(6*a*c^3) + ((2*c*f*g^3 + 6*c*g*h*(e*g + d*h) - 3*
a*h^2*(3*f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(5/2))
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{\int \frac{(g+hx)^2(-a(fg+3eh)+(3cd-4af)hx)}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{\int \frac{(g+hx)\left(-a\left(2(3cd-4af)h \right.\right.}{}}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h\left(4\left(3c^2dg^2+4a^2f\right.\right.}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h\left(4\left(3c^2dg^2+4a^2f\right.\right.}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h\left(4\left(3c^2dg^2+4a^2f\right.\right.}{}$$

Mathematica [A] time = 0.503031, size = 246, normalized size = 1.07

$$\frac{\frac{a^2ch\left(3h(4dh+3e(4g+hx))+f\left(36g^2+27ghx-8h^2x^2\right)\right)-16a^3fh^3+ac^2\left(6dh\left(-3g^2-3ghx+h^2x^2\right)-3e\left(6g^2hx+2g^3-6gh^2x^2-h^3x^3\right)+fx\left(18g^2hx-6g^3+9gh^2x^2+2h^3x^3\right)\right)+6}{a\sqrt{a+cx^2}}}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] ((-16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2)
 - 3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h
*x + 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2)
 + 3*h*(4*d*h + 3*e*(4*g + h*x))))/(a*Sqrt[a + c*x^2]) + 3*Sqrt[c]*(2*c*f*g
^3 + 6*c*g*h*(e*g + d*h) - 3*a*h^2*(3*f*g + e*h))*Log[c*x + Sqrt[c]*Sqrt[a
+ c*x^2]])/(6*c^3)
```

Maple [B] time = 0.06, size = 516, normalized size = 2.3

$$\frac{h^3fx^4}{3c}\frac{1}{\sqrt{cx^2+a}} - \frac{4ah^3fx^2}{3c^2}\frac{1}{\sqrt{cx^2+a}} - \frac{8a^2fh^3}{3c^3}\frac{1}{\sqrt{cx^2+a}} + \frac{x^3h^3e}{2c}\frac{1}{\sqrt{cx^2+a}} + \frac{3x^3gh^2f}{2c}\frac{1}{\sqrt{cx^2+a}} + \frac{3axh^3e}{2c^2}\frac{1}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/3*h^3*f*x^4/c/(c*x^2+a)^(1/2)-4/3*h^3*f*a/c^2*x^2/(c*x^2+a)^(1/2)-8/3*h^3
*f*a^2/c^3/(c*x^2+a)^(1/2)+1/2*x^3/c/(c*x^2+a)^(1/2)*h^3*e+3/2*x^3/c/(c*x^2
+a)^(1/2)*g*h^2*f+3/2*a/c^2*x/(c*x^2+a)^(1/2)*h^3*e+9/2*a/c^2*x/(c*x^2+a)^(
1/2)*g*h^2*f-3/2*a/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*h^3*e-9/2*a/c^(5/2
)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*g*h^2*f+x^2/c/(c*x^2+a)^(1/2)*h^3*d+3*x^2/c
/(c*x^2+a)^(1/2)*g*h^2*e+3*x^2/c/(c*x^2+a)^(1/2)*g^2*h*f+2*a/c^2/(c*x^2+a)^
(1/2)*h^3*d+6*a/c^2/(c*x^2+a)^(1/2)*g*h^2*e+6*a/c^2/(c*x^2+a)^(1/2)*g^2*h*f
-3*x/c/(c*x^2+a)^(1/2)*g*h^2*d-3*x/c/(c*x^2+a)^(1/2)*g^2*h*e-x/c/(c*x^2+a)^
(1/2)*g^3*f+3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*g*h^2*d+3/c^(3/2)*ln(x*
c^(1/2)+(c*x^2+a)^(1/2))*g^2*h*e+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*g^
3*f-3/c/(c*x^2+a)^(1/2)*g^2*h*d-1/c/(c*x^2+a)^(1/2)*g^3*e+g^3*d*x/a/(c*x^2+
a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20619, size = 1635, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a^2*c*d - 3
*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 + 3*(2*a*c
^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt
(c)*x - a) - 2*(2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(
a*c^2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h
^2 + a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d -
```

```
 4*a^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^
2*f)*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2
+ a^2*c^3), -1/6*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a
^2*c*d - 3*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3
+ 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x
^2 + a)) - (2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(a*c^
2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h^2 +
 a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d - 4*a
^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^2*f)
*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^
2*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g+hx\right)^3\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

Giac [A] time = 1.20149, size = 458, normalized size = 2.

$$\frac{\left(\left(\left(\left(\frac{2\,fh^3x}{c}+\frac{3\left(3\,ac^4fgh^2+ac^4h^3e\right)}{ac^5}\right)x+\frac{2\left(9\,ac^4fg^2h+3\,ac^4dh^3-4\,a^2c^3fh^3+9\,ac^4gh^2e\right)}{ac^5}\right)x+\frac{3\left(2\,c^5dg^3-2\,ac^4fg^3-6\,ac^4dgh^2+9\,a^2c^3fgh^2-6\,ac^4g^2he+3\,a^2c^3\right.}{ac^5}}{6\,\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*((((2*f*h^3*x/c + 3*(3*a*c^4*f*g*h^2 + a*c^4*h^3*e)/(a*c^5))*x + 2*(9*a
*c^4*f*g^2*h + 3*a*c^4*d*h^3 - 4*a^2*c^3*f*h^3 + 9*a*c^4*g*h^2*e)/(a*c^5))*
x + 3*(2*c^5*d*g^3 - 2*a*c^4*f*g^3 - 6*a*c^4*d*g*h^2 + 9*a^2*c^3*f*g*h^2 -
6*a*c^4*g^2*h*e + 3*a^2*c^3*h^3*e)/(a*c^5))*x - 2*(9*a*c^4*d*g^2*h - 18*a^2
*c^3*f*g^2*h - 6*a^2*c^3*d*h^3 + 8*a^3*c^2*f*h^3 + 3*a*c^4*g^3*e - 18*a^2*c
```

```
^3*g*h^2*e)/(a*c^5))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^3 + 6*c*d*g*h^2 - 9*a*f
*g*h^2 + 6*c*g^2*h*e - 3*a*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

3.109 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(h^2(2cd-3af)+2cg(2eh+fg)\right)}{2c^{5/2}}-\frac{h\sqrt{a+cx^2}(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2}-\frac{(g+hx)^2(ae-}{ac\sqrt{a}}$$

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^2)/(a*c*Sqrt[a + c*x^2])) - (h*(4*(c*d*g
 - a*(2*f*g + e*h)) + (2*c*d - 3*a*f)*h*x)*Sqrt[a + c*x^2])/(2*a*c^2) + (((
2*c*d - 3*a*f)*h^2 + 2*c*g*(f*g + 2*e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(2*c^(5/2))
```

Rubi [A] time = 0.183902, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1645, 780, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(h^2(2cd-3af)+2cg(2eh+fg)\right)}{2c^{5/2}}-\frac{h\sqrt{a+cx^2}(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2}-\frac{(g+hx)^2(ae-}{ac\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```


```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^2)/(a*c*Sqrt[a + c*x^2])) - (h*(4*(c*d*g
 - a*(2*f*g + e*h)) + (2*c*d - 3*a*f)*h*x)*Sqrt[a + c*x^2])/(2*a*c^2) + (((
2*c*d - 3*a*f)*h^2 + 2*c*g*(f*g + 2*e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(2*c^(5/2))
```


Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
```

```
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{\int \frac{(g+hx)(-a(fg+2eh)+(2cd-3af)hx)}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx^2}}{2ac^2} + \frac{((2\cdots}{\cdots}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx^2}}{2ac^2} + \frac{((2\cdots}{\cdots}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx^2}}{2ac^2} + \frac{((2\cdots}{\cdots}$$

Mathematica [A] time = 0.301613, size = 177, normalized size = 1.19

$$\frac{\sqrt{c}\left(a^2h(4eh+8fg+3fhx)+ac\left(-2dh(2g+hx)-2e\left(g^2+2ghx-h^2x^2\right)+fx\left(-2g^2+4ghx+h^2x^2\right)\right)+2c^2dg^2x\right)-a^{3/2}\cdots}{2ac^{5/2}\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h + 3*f*h*x) + a*c*(-2*d*h*(2*
g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(-2*g^2 + 4*g*h*x + h^2*x^2)
)) - a^(3/2)*(3*a*f*h^2 - 2*c*(f*g^2 + h*(2*e*g + d*h)))*Sqrt[1 + (c*x^2)/a
]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(2*a*c^(5/2)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.058, size = 327, normalized size = 2.2

$$\frac{h^2fx^3}{2c}\frac{1}{\sqrt{cx^2+a}}+\frac{3afh^2x}{2c^2}\frac{1}{\sqrt{cx^2+a}}-\frac{3afh^2}{2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{5}{2}}+\frac{h^2x^2e}{c}\frac{1}{\sqrt{cx^2+a}}+2\frac{x^2ghf}{c\sqrt{cx^2+a}}+2\frac{ah^2e}{c^2\sqrt{cx^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/2*h^2*f*x^3/c/(c*x^2+a)^(1/2)+3/2*h^2*f*a/c^2*x/(c*x^2+a)^(1/2)-3/2*h^2*f
*a/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+x^2/c/(c*x^2+a)^(1/2)*h^2*e+2*x^2/
c/(c*x^2+a)^(1/2)*g*h*f+2*a/c^2/(c*x^2+a)^(1/2)*h^2*e+4*a/c^2/(c*x^2+a)^(1/
2)*g*h*f-x/c/(c*x^2+a)^(1/2)*d*h^2-2*x/c/(c*x^2+a)^(1/2)*e*g*h-x/c/(c*x^2+a
)^(1/2)*f*g^2+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*d*h^2+2/c^(3/2)*ln(x*
c^(1/2)+(c*x^2+a)^(1/2))*e*g*h+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*f*g^
2-2/c/(c*x^2+a)^(1/2)*g*h*d-1/c/(c*x^2+a)^(1/2)*e*g^2+g^2*d*x/a/(c*x^2+a)^(
1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05239, size = 1142, normalized size = 7.66

$$\left[-\frac{\left(2\,a^2cfg^2+4\,a^2cegh+\left(2\,a^2cd-3\,a^3f\right)h^2+\left(2\,ac^2fg^2+4\,ac^2egh+\left(2\,ac^2d-3\,a^2cf\right)h^2\right)x^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a^2*c*f*g^2 + 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^
2*f*g^2 + 4*a*c^2*e*g*h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(c)*log(-2*
c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(a*c^2*f*h^2*x^3 - 2*a*c^2*e*g
^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2
*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2
*c*f)*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^2*c^3), -1/2*((2*a^2*c*f*g^2
+ 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^2*f*g^2 + 4*a*c^2*e*g*
h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) - (a*c^2*f*h^2*x^3 - 2*a*c^2*e*g^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*
a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^
3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/(a*c^
4*x^2 + a^2*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g+hx\right)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

Giac [A] time = 1.188, size = 296, normalized size = 1.99

$$\frac{\left(\left(\left(\frac{fh^2x}{c}+\frac{2\left(2\,ac^3fgh+ac^3h^2e\right)}{ac^4}\right)x+\frac{2\,c^4dg^2-2\,ac^3fg^2-2\,ac^3dh^2+3\,a^2c^2fh^2-4\,ac^3ghe}{ac^4}\right)x-\frac{2\left(2\,ac^3dgh-4\,a^2c^2fgh+ac^3g^2e-2\,a^2c^2h^2e\right)}{ac^4}\right.}{2\,\sqrt{cx^2+a}}-\frac{\left(2\,cfg^2+2\,ca\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((f*h^2*x/c + 2*(2*a*c^3*f*g*h + a*c^3*h^2*e)/(a*c^4))*x + (2*c^4*d*g^
2 - 2*a*c^3*f*g^2 - 2*a*c^3*d*h^2 + 3*a^2*c^2*f*h^2 - 4*a*c^3*g*h*e)/(a*c^4
))*x - 2*(2*a*c^3*d*g*h - 4*a^2*c^2*f*g*h + a*c^3*g^2*e - 2*a^2*c^2*h^2*e)/
(a*c^4))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^2 + 2*c*d*h^2 - 3*a*f*h^2 + 4*c*g*h
*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.110 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{h\sqrt{a+cx^2}(cd-2af)}{ac^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x))/(a*c*Sqrt[a + c*x^2])) - ((c*d - 2*a*f)
*h*Sqrt[a + c*x^2])/(a*c^2) + ((f*g + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/c^(3/2)
```

Rubi [A] time = 0.0869532, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 641, 217, 206}

$$-\frac{h\sqrt{a+cx^2}(cd-2af)}{ac^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x))/(a*c*Sqrt[a + c*x^2])) - ((c*d - 2*a*f)
*h*Sqrt[a + c*x^2])/(a*c^2) + ((f*g + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/c^(3/2)
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{\int \frac{-a(fg+eh)+(cd-2af)hx}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \ldots\right)}{c}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.151096, size = 102, normalized size = 1.02

$$\frac{a^{3/2}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(eh+fg)+2a^2fh-ac(dh+e(g+hx)+fx(g-hx))+c^2dgx}{ac^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (2*a^2*f*h + c^2*d*g*x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x)) + a^(3/2)*
Sqrt[c]*(f*g + e*h)*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(a*c^
2*Sqrt[a + c*x^2])
```

Maple [A] time = 0.056, size = 163, normalized size = 1.6

$$\frac{fhx^2}{c}\frac{1}{\sqrt{cx^2+a}}+2\frac{afh}{c^2\sqrt{cx^2+a}}-\frac{ehx}{c}\frac{1}{\sqrt{cx^2+a}}-\frac{fgx}{c}\frac{1}{\sqrt{cx^2+a}}+eh\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+fg\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] h*f*x^2/c/(c*x^2+a)^(1/2)+2*h*f*a/c^2/(c*x^2+a)^(1/2)-x/c/(c*x^2+a)^(1/2)*e
*h-x/c/(c*x^2+a)^(1/2)*f*g+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*e*h+1/c^
(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))*f*g-1/c/(c*x^2+a)^(1/2)*d*h-1/c/(c*x^2+
a)^(1/2)*e*g+d*g*x/a/(c*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9, size = 606, normalized size = 6.06

$$\left[\frac{\left(a^2fg+a^2eh+\left(acfg+aceh\right)x^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(acfhx^2-aceg-\left(acd-2\,a^2f\right)h-\left(aceh-(c\right.\right.}{2\left(ac^3x^2+a^2c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(c)*log(-2*c*x^2 -
2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^
2*f)*h - (a*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2
*c^2), -((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(-c)*arctan(sqrt
(-c)*x/sqrt(c*x^2 + a)) - (a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^2*f)*h - (a
*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

Sympy [A] time = 12.7682, size = 209, normalized size = 2.09

$$dh\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+eg\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+eh\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}c\sqrt{1+\frac{cx^2}{a}}}\right)+fg\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] d*h*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True
)) + e*g*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)),
 True)) + e*h*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*
x**2/a))) + f*g*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 +
c*x**2/a))) + f*h*Piecewise((2*a/(c**2*sqrt(a + c*x**2)) + x**2/(c*sqrt(a +
 c*x**2)), Ne(c, 0)), (x**4/(4*a**(3/2)), True)) + d*g*x/(a**(3/2)*sqrt(1 +
 c*x**2/a))
```

Giac [A] time = 1.16895, size = 157, normalized size = 1.57

$$\frac{\left(\frac{fhx}{c}+\frac{c^3dg-ac^2fg-ac^2he}{ac^3}\right)x-\frac{ac^2dh-2\,a^2cfh+ac^2ge}{ac^3}}{\sqrt{cx^2+a}}-\frac{(fg+he)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((f*h*x/c + (c^3*d*g - a*c^2*f*g - a*c^2*h*e)/(a*c^3))*x - (a*c^2*d*h - 2*a
^2*c*f*h + a*c^2*g*e)/(a*c^3))/sqrt(c*x^2 + a) - (f*g + h*e)*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.111 $\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

```
[Out] -((a*e - (c*d - a*f)*x)/(a*c*Sqrt[a + c*x^2])) + (f*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]])/c^(3/2)
```

Rubi [A] time = 0.0357731, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 217, 206}

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

```
[Out] -((a*e - (c*d - a*f)*x)/(a*c*Sqrt[a + c*x^2])) + (f*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]])/c^(3/2)
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{\int \frac{af}{c\sqrt{a+cx^2}}\,dx}{a}\\ &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}\\ &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{c}\\ &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0625536, size = 74, normalized size = 1.21

$$\frac{a^{3/2}f\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+\sqrt{c}(cdx-a(e+fx))}{ac^{3/2}\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*(c*d*x - a*(e + f*x)) + a^(3/2)*f*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqr
t[c]*x)/Sqrt[a]])/(a*c^(3/2)*Sqrt[a + c*x^2])
```

Maple [A] time = 0.052, size = 69, normalized size = 1.1

$$-\frac{fx}{c}\frac{1}{\sqrt{cx^2+a}}+f\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}-\frac{e}{c}\frac{1}{\sqrt{cx^2+a}}+\frac{dx}{a}\frac{1}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -f*x/c/(c*x^2+a)^(1/2)+f/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-e/c/(c*x^2+a
)^(1/2)+d*x/a/(c*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7177, size = 396, normalized size = 6.49

$$\left[\frac{\left(acfx^2+a^2f\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(ace-\left(c^2d-acf\right)x\right)\sqrt{cx^2+a}}{2\left(ac^3x^2+a^2c^2\right)},-\frac{\left(acfx^2+a^2f\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-}}{\sqrt{cx^2}}\right.}{ac^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*f*x^2 + a^2*f)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*(a*c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^
2), -((a*c*f*x^2 + a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (a*
c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

Sympy [A] time = 5.60967, size = 87, normalized size = 1.43

$$e\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for}\ c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}c\sqrt{1+\frac{cx^2}{a}}}\right)+\frac{dx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True))
 + f*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*x**2/a)))
 + d*x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.18497, size = 85, normalized size = 1.39

$$-\frac{\frac{e}{c}-\frac{(c^2d-acf)x}{ac^2}}{\sqrt{cx^2+a}}-\frac{f\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(e/c - (c^2*d - a*c*f)*x/(a*c^2))/sqrt(c*x^2 + a) - f*log(abs(-sqrt(c)*x +
 sqrt(c*x^2 + a)))/c^(3/2)
```

3.112 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)}-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

```
[Out] -((a*(c*e*g - c*d*h + a*f*h) - c*(c*d*g - a*f*g + a*e*h)*x)/(a*c*(c*g^2 + a
*h^2)*Sqrt[a + c*x^2])) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(S
qrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

Rubi [A] time = 0.141356, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 12, 725, 206}

$$-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)}-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*e*g - c*d*h + a*f*h) - c*(c*d*g - a*f*g + a*e*h)*x)/(a*c*(c*g^2 + a
*h^2)*Sqrt[a + c*x^2])) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(S
qrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{ac\left(fg^2-egh+dh^2\right)}{\left(cg^2+ah^2\right)(g+hx)\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}+\frac{\left(fg^2-egh+dh^2\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}-\frac{\left(fg^2-egh+dh^2\right)\text{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\,dx,x\right.}{cg^2+ah^2}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}-\frac{\left(fg^2-egh+dh^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{\left(cg^2+ah^2\right)^{3/2}}$$

Mathematica [A] time = 0.203466, size = 137, normalized size = 0.99

$$\frac{a^2(-f)h+ac(dh-eg+ehx-fgx)+c^2dgx}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)}-\frac{\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*f*h) + c^2*d*g*x + a*c*(-(e*g) + d*h - f*g*x + e*h*x))/(a*c*(c*g^2 +
 a*h^2)*Sqrt[a + c*x^2]) - ((f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(a*h - c*g*x
)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

Maple [B] time = 0.256, size = 862, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/h*f/c/(c*x^2+a)^(1/2)+1/h*e*x/a/(c*x^2+a)^(1/2)-1/h^2*f*g*x/a/(c*x^2+a)^
(1/2)+h/(a*h^2+c*g^2)/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)
*d-1/(a*h^2+c*g^2)/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*e*
g+1/h/(a*h^2+c*g^2)/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f
*g^2+g/(a*h^2+c*g^2)/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2
)*x*c*d-1/h*g^2/(a*h^2+c*g^2)/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/
h^2)^(1/2)*x*c*e+1/h^2*g^3/(a*h^2+c*g^2)/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*
h^2+c*g^2)/h^2)^(1/2)*x*c*f-h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2
*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c
-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/(a*h^2+c*g^2)/((a*h
^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^
2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h
))*e*g-1/h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-
2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a
*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.2165, size = 1431, normalized size = 10.37

$$\left[\frac{\left(a^2cfg^2 - a^2cegh + a^2cdh^2 + \left(ac^2fg^2 - ac^2egh + ac^2dh^2\right)x^2\right)\sqrt{cg^2 + ah^2}\log\left(\frac{2\,acghx - acg^2 - 2\,a^2h^2 - \left(2\,c^2g^2 + ach^2\right)x^2 - 2\,\sqrt{cg^2 + ah^2}\left(c\right.}{h^2x^2 + 2\,ghx + g^2}\right.}{2\left(a^2c^3g^4 + 2\,a^3c^2g^2h\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*h^2 + (a*c^2*f*g^2 - a*c^2*e*g*h
 + a*c^2*d*h^2)*x^2)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2
*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt
(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(a*c^2*e*g^3 + a^2*c*e*g*h^2 -
(a*c^2*d - a^2*c*f)*g^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a^2*c*
e*h^3 + (c^3*d - a*c^2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 +
a))/(a^2*c^3*g^4 + 2*a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g
^2*h^2 + a^3*c^2*h^4)*x^2), -((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*h^2 + (a
*c^2*f*g^2 - a*c^2*e*g*h + a*c^2*d*h^2)*x^2)*sqrt(-c*g^2 - a*h^2)*arctan(sq
rt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*
g^2 + a*c*h^2)*x^2)) + (a*c^2*e*g^3 + a^2*c*e*g*h^2 - (a*c^2*d - a^2*c*f)*g
^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a^2*c*e*h^3 + (c^3*d - a*c^
2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a^2*c^3*g^4 + 2*
a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g^2*h^2 + a^3*c^2*h^4)
*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2}{\left(a + cx^2\right)^{\frac{3}{2}}\left(g + hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((a + c*x**2)**(3/2)*(g + h*x)), x)
```

Giac [B] time = 1.18535, size = 397, normalized size = 2.88

$$\frac{\frac{\left(c^3dg^3-ac^2fg^3+ac^2dgh^2-a^2cfgh^2+ac^2g^2he+a^2ch^3e\right)x}{ac^3g^4+2\,a^2c^2g^2h^2+a^3ch^4}+\frac{ac^2dg^2h-a^2cfg^2h+a^2cdh^3-a^3fh^3-ac^2g^3e-a^2cgh^2e}{ac^3g^4+2\,a^2c^2g^2h^2+a^3ch^4}}{\sqrt{cx^2+a}}-\frac{2\left(fg^2+dh^2-ghe\right)\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{}\right.}{\sqrt{-}}\right.}{\left(cg^2+ah^2\right)\sqrt{-cg^2-ah^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^3*d*g^3 - a*c^2*f*g^3 + a*c^2*d*g*h^2 - a^2*c*f*g*h^2 + a*c^2*g^2*h*e +
 a^2*c*h^3*e)*x/(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4) + (a*c^2*d*g^2*
h - a^2*c*f*g^2*h + a^2*c*d*h^3 - a^3*f*h^3 - a*c^2*g^3*e - a^2*c*g*h^2*e)/
(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4))/sqrt(c*x^2 + a) - 2*(f*g^2 + d
*h^2 - g*h*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*
g^2 - a*h^2))/((c*g^2 + a*h^2)*sqrt(-c*g^2 - a*h^2))
```

3.113 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x\left(a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)+c^2dg^2\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^2}-\frac{h\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{(g+hx)\left(ah^2+cg^2\right)^2}+\frac{\tanh^{-1}}{}$$

```
[Out] -((a*(c*g*(e*g - 2*d*h) + a*h*(2*f*g - e*h)) - (c^2*d*g^2 + a^2*f*h^2 - a*c
*(f*g^2 - h*(2*e*g - d*h)))*x)/(a*(c*g^2 + a*h^2)^2*Sqrt[a + c*x^2])) - (h*
(f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^2*(g + h*x)) + ((
a*h^2*(2*f*g - e*h) - c*g*(f*g^2 - h*(2*e*g - 3*d*h)))*ArcTanh[(a*h - c*g*x
)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(5/2)
```

Rubi [A] time = 0.418131, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 807, 725, 206}

$$-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x\left(a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)+c^2dg^2\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^2}-\frac{h\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{(g+hx)\left(ah^2+cg^2\right)^2}-\frac{\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*g*(e*g - 2*d*h) + a*h*(2*f*g - e*h)) - (c^2*d*g^2 + a^2*f*h^2 - a*c
*(f*g^2 - h*(2*e*g - d*h)))*x)/(a*(c*g^2 + a*h^2)^2*Sqrt[a + c*x^2])) - (h*
(f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^2*(g + h*x)) - ((
c*f*g^3 - c*g*h*(2*e*g - 3*d*h) - a*h^2*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x
)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(5/2)
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \frac{\int \frac{ac\left(ah^2\left(fg\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \frac{h\left(fg^2-\cdots\right.}{\left(cg^2\cdots\right.}$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \frac{h\left(fg^2-\cdots\right.}{\left(cg^2\cdots\right.}$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \frac{h\left(fg^2-\cdots\right.}{\left(cg^2\cdots\right.}$$

Mathematica [A] time = 0.726963, size = 285, normalized size = 1.19

$$\frac{2h\left(h\left(a+cx^2\right)\sqrt{ah^2+cg^2}\left(a^2fh^2+ac\left(h(3eg-2dh)-2fg^2\right)+c^2dg^2\right)-ac\sqrt{a+cx^2}(g+hx)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(\right.}{2ach\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a*f*(c*g^2 + a*h^2)^(5/2)) + (c*g^2 + a*h^2)^(3/2)*(-(a^2*f*h^2) + 2*c^2
*d*g*h*x + a*c*(f*g*(g - 2*h*x) + 2*h*(-(e*g) + d*h + e*h*x))) + 2*h*(h*Sqr
t[c*g^2 + a*h^2]*(c^2*d*g^2 + a^2*f*h^2 + a*c*(-2*f*g^2 + h*(3*e*g - 2*d*h)
))*(a + c*x^2) - a*c*(c*f*g^3 + c*g*h*(-2*e*g + 3*d*h) + a*h^2*(-2*f*g + e*
h))*(g + h*x)*Sqrt[a + c*x^2]*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sq
rt[a + c*x^2])]))/(2*a*c*h*(c*g^2 + a*h^2)^(5/2)*(g + h*x)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.234, size = 1663, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x)
```

```
[Out] f/h^2*x/a/(c*x^2+a)^(1/2)+1/(a*h^2+c*g^2)/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h
^2+c*g^2)/h^2)^(1/2)*e-2/h/(a*h^2+c*g^2)/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^
2+c*g^2)/h^2)^(1/2)*f*g+3/h*g/(a*h^2+c*g^2)/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+
(a*h^2+c*g^2)/h^2)^(1/2)*x*c*e-4/h^2*g^2/(a*h^2+c*g^2)/a/((x+g/h)^2*c-2*c*g
/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*c*f-1/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^
2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2
)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h/(a*
h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h
)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^
2)^(1/2))/(x+g/h))*f*g-1/(a*h^2+c*g^2)/(x+g/h)/((x+g/h)^2*c-2*c*g/h*(x+g/h)
+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/h/(a*h^2+c*g^2)/(x+g/h)/((x+g/h)^2*c-2*c*g/h*
(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)/((x+g/h)^2
*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2+3*h*c*g/(a*h^2+c*g^2)^2/(
(x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*d-3*c*g^2/(a*h^2+c*g^2
)^2/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*e+3/h*c*g^3/(a*h^
2+c*g^2)^2/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f+3*c^2*g^
```

```
2/(a*h^2+c*g^2)^2/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x
*d-3/h*c^2*g^3/(a*h^2+c*g^2)^2/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)
/h^2)^(1/2)*x*e+3/h^2*c^2*g^4/(a*h^2+c*g^2)^2/a/((x+g/h)^2*c-2*c*g/h*(x+g/h
)+(a*h^2+c*g^2)/h^2)^(1/2)*x*f-3*h*c*g/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^
(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*(
(x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+3*c*g^2/(a
*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+
g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)
/h^2)^(1/2))/(x+g/h))*e-3/h*c*g^3/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)
*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/
h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-2/(a*h^2+c*g^2)
/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 21.0906, size = 3160, normalized size = 13.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f)
*g^2*h^2 + (a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d -
2*a^2*c*f)*g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 + (3
*a*c^2*d - 2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2 + a
^3*e*h^4 + (3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g
*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(c*g^2 + a*h
^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(a*c^2*e*
g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a*c^2*d - 3*a^2*c*f)
*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^2 + 3*a^2*c*e*g*h^4
```

```
 + (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h^3 - (2*a^2*c*d - a
^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^3*e*h^5 + (c^3*d -
a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d - a^3*f)*g*h^4)*x)*
sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^4*c*g^3*h^4 + a^5*g
*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*h^5 + a^4*c*h^7)*x^
3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 + a^4*c*g*h^6)*x^2 +
 (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 + a^5*h^7)*x), -((a^2
*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f)*g^2*h^2 +
(a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d - 2*a^2*c*f)*
g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 + (3*a*c^2*d -
2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2 + a^3*e*h^4 +
(3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a
*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2
)*x^2)) + (a*c^2*e*g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a
*c^2*d - 3*a^2*c*f)*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^
2 + 3*a^2*c*e*g*h^4 + (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h
^3 - (2*a^2*c*d - a^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^
3*e*h^5 + (c^3*d - a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d
- a^3*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^
4*c*g^3*h^4 + a^5*g*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*
h^5 + a^4*c*h^7)*x^3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 +
 a^4*c*g*h^6)*x^2 + (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 +
a^5*h^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + ex + d}{\left(cx^2 + a\right)^{\frac{3}{2}} \left(hx + g\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="giac")

[Out] integrate((f*x^2 + e*x + d)/((c*x^2 + a)^(3/2)*(h*x + g)^2), x)
```

3.114 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=374

$$\frac{cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\right)+a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}-\text{tanh}$$

```
[Out] (a*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*c*h*(3*f*g^2 - h*(3*e*g - d*h)))
+ c*(c^2*d*g^3 + a^2*h^2*(3*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(e*g - d*h)))*x
)/(a*(c*g^2 + a*h^2)^3*Sqrt[a + c*x^2]) - (h*(f*g^2 - e*g*h + d*h^2)*Sqrt[a
 + c*x^2])/(2*(c*g^2 + a*h^2)^2*(g + h*x)^2) + (h*(2*a*h^2*(2*f*g - e*h) -
c*g*(3*f*g^2 - h*(5*e*g - 7*d*h)))*Sqrt[a + c*x^2])/(2*(c*g^2 + a*h^2)^3*(g
 + h*x)) - ((2*a^2*f*h^4 - a*c*h^2*(11*f*g^2 - 9*e*g*h + 3*d*h^2) + 2*c^2*g
^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2])])/(2*(c*g^2 + a*h^2)^(7/2))
```

Rubi [A] time = 1.02786, antiderivative size = 372, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 1651, 807, 725, 206}

$$\frac{cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\right)+a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}-\text{tanh}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*c*h*(3*f*g^2 - h*(3*e*g - d*h)))
+ c*(c^2*d*g^3 + a^2*h^2*(3*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(e*g - d*h)))*x
)/(a*(c*g^2 + a*h^2)^3*Sqrt[a + c*x^2]) - (h*(f*g^2 - e*g*h + d*h^2)*Sqrt[a
 + c*x^2])/(2*(c*g^2 + a*h^2)^2*(g + h*x)^2) - (h*(3*c*f*g^3 - c*g*h*(5*e*g
 - 7*d*h) - 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x^2])/(2*(c*g^2 + a*h^2)^3*(g
 + h*x)) - ((2*a^2*f*h^4 - a*c*h^2*(11*f*g^2 - 9*e*g*h + 3*d*h^2) + 2*c^2*g
^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2])])/(2*(c*g^2 + a*h^2)^(7/2))
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx = \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-eh)-acg\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-eh)-acg\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-eh)-acg\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-eh)-acg\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-eh)-acg\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

Mathematica [A] time = 1.27153, size = 404, normalized size = 1.08

$$\frac{1}{2}\left(-\frac{\sqrt{a+cx^2}\left(\frac{2\left(a^2ch(h(dh-3eg+ehx)+3fg(g-hx))-a^3fh^3+ac^2g\left(3dh(hx-g)+eg(g-3hx)+fg^2x\right)-c^3dg^3x\right)}{a\left(a+cx^2\right)}+\frac{h\left(ah^2+cg^2\right)\left(h(dh-eg)+fg^2\right)}{(g+hx)^2}+\frac{h\left(2ah^2(eh-2\right.}{}\right.}{\left(ah^2+cg^2\right)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((Sqrt[a + c*x^2]*((h*(c*g^2 + a*h^2)*(f*g^2 + h*(-(e*g) + d*h)))/(g + h*
x)^2 + (h*(3*c*f*g^3 + c*g*h*(-5*e*g + 7*d*h) + 2*a*h^2*(-2*f*g + e*h)))/(g
 + h*x) + (2*(-(a^3*f*h^3) - c^3*d*g^3*x + a*c^2*g*(f*g^2*x + e*g*(g - 3*h*
x) + 3*d*h*(-g + h*x)) + a^2*c*h*(3*f*g*(g - h*x) + h*(-3*e*g + d*h + e*h*x
))))/(a*(a + c*x^2))))/(c*g^2 + a*h^2)^3) + ((2*a^2*f*h^4 + a*c*h^2*(-11*f*
g^2 + 9*e*g*h - 3*d*h^2) + 2*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*Log[g + h
*x])/(c*g^2 + a*h^2)^(7/2) - ((2*a^2*f*h^4 + a*c*h^2*(-11*f*g^2 + 9*e*g*h -
 3*d*h^2) + 2*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*Log[a*h - c*g*x + Sqrt[c
*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(7/2))/2
```

Maple [B] time = 0.258, size = 2584, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x)
```

```
[Out] 2/h^2/(a*h^2+c*g^2)/(x+g/h)/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)
^(1/2)*f*g+1/2/h^2/(a*h^2+c*g^2)/(x+g/h)^2/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*
h^2+c*g^2)/h^2)^(1/2)*e*g-1/2/h^3/(a*h^2+c*g^2)/(x+g/h)^2/((x+g/h)^2*c-2*c*
g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2-5/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)/
((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*d+15/2*h*c^2*g^2/(a*h
^2+c*g^2)^3/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*d+15/2/h*
c^2*g^4/(a*h^2+c*g^2)^3/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/
2)*f+15/2*c^2*g^3/(a*h^2+c*g^2)^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*
g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(
x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e-15/2/h*c/(a*h^2+c*g^2)^2/((x+g/
h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2+3/2*h*c/(a*h^2+c*g^2)
^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*
h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)
)/(x+g/h))*d-9/2*c/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c
*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*
(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e*g-15/2*c^2*g^3/(a*h^2+c*g^2)^3
/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*e+19/2/h*c^2*g^2/(a*
h^2+c*g^2)^2/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*e-15
/2/h*c^3*g^4/(a*h^2+c*g^2)^3/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h
^2)^(1/2)*x*e+15/2/h^2*c^3*g^5/(a*h^2+c*g^2)^3/a/((x+g/h)^2*c-2*c*g/h*(x+g/
h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*f+5*f/h^2*g/(a*h^2+c*g^2)/a/((x+g/h)^2*c-2*c*
g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*c-25/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2/a
/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*f+9/2*c/(a*h^2+c*g
^2)^2/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-f/h/(a*h^2+
c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*
((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(
1/2))/(x+g/h))-1/2/h/(a*h^2+c*g^2)/(x+g/h)^2/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(
a*h^2+c*g^2)/h^2)^(1/2)*d-1/h/(a*h^2+c*g^2)/(x+g/h)/((x+g/h)^2*c-2*c*g/h*(x
+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*e-3/2*h*c/(a*h^2+c*g^2)^2/((x+g/h)^2*c-2*c*g
/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*d+f/h/(a*h^2+c*g^2)/((x+g/h)^2*c-2*c*g/
h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)-2/h/(a*h^2+c*g^2)/a/((x+g/h)^2*c-2*c*g/h
*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*c*e+15/2/h*c/(a*h^2+c*g^2)^2/((a*h^2+c*
g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^
2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f*
g^2+5/2/h*c*g^2/(a*h^2+c*g^2)^2/(x+g/h)/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2
```

```
+c*g^2)/h^2)^(1/2)*e-5/2/h^2*c*g^3/(a*h^2+c*g^2)^2/(x+g/h)/((x+g/h)^2*c-2*c
*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f+15/2*c^3*g^3/(a*h^2+c*g^2)^3/a/((x+
g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*d-15/2*h*c^2*g^2/(a*h^2
+c*g^2)^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)
+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2
)^(1/2))/(x+g/h))*d-15/2/h*c^2*g^4/(a*h^2+c*g^2)^3/((a*h^2+c*g^2)/h^2)^(1/2
)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g
/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-13/2*c^2*g/(a*
h^2+c*g^2)^2/a/((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*x*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 110.411, size = 5783, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^2*c^2*f*g^6 - 6*a^2*c^2*e*g^5*h + 9*a^3*c*e*g^3*h^3 + (12*a^2*c^
2*d - 11*a^3*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*
h^2 - 6*a*c^3*e*g^3*h^3 + 9*a^2*c^2*e*g*h^5 + (12*a*c^3*d - 11*a^2*c^2*f)*g
^2*h^4 - (3*a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h - 6*a*c^3*
e*g^4*h^2 + 9*a^2*c^2*e*g^2*h^4 + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*
a^2*c^2*d - 2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 - 6*a*c^3*e*g^5*h + 3*a^
2*c^2*e*g^3*h^3 + 9*a^3*c*e*g*h^5 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9
*(a^2*c^2*d - a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*
c^2*f*g^5*h - 6*a^2*c^2*e*g^4*h^2 + 9*a^3*c*e*g^2*h^4 + (12*a^2*c^2*d - 11*
a^3*c*f)*g^3*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x)*sqrt(c*g^2 + a*h^2)*log(
(2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g
^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(
```

2*a*c^3*e*g^7 - 10*a^2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4 + a^4*e*g*h^6 + a
^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2*d + 5*a^3*c*f)*g^
4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - (11*a*c^3*e*g^4*h^3 + 7*a^2*c^2*e*
g^2*h^5 - 4*a^3*c*e*h^7 + (2*c^4*d - 5*a*c^3*f)*g^5*h^2 - (11*a*c^3*d - 5*a
^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^3 - (16*a*c^3*e*g^
5*h^2 + 17*a^2*c^2*e*g^3*h^4 + a^3*c*e*g*h^6 + 4*(c^4*d - 2*a*c^3*f)*g^6*h
- (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a^3*c*f)*g^2*h^5 -
(3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*a*c^3*e*g^6*h + 17*a^2*c^2*e*g^4*h^3 +
13*a^3*c*e*g^2*h^5 - 2*a^4*e*h^7 + 2*(c^4*d - a*c^3*f)*g^7 + (8*a*c^3*d - 1
1*a^2*c^2*f)*g^5*h^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 - (11*a^3*c*d - 8*a^
4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3*g^8*h^2 + 6*a^4*c
^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h^2 + 4*a^2*c^4*g^6
*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10)*x^4 + 2*(a*c^5*g
^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*g^3*h^7 + a^5*c*g*
h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g^6*h^4 + 10*a^4*c^
2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*g^9*h + 4*a^3*c^3*
g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9)*x), -1/2*((2*a^2
*c^2*f*g^6 - 6*a^2*c^2*e*g^5*h + 9*a^3*c*e*g^3*h^3 + (12*a^2*c^2*d - 11*a^3
*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*h^2 - 6*a*c^
3*e*g^3*h^3 + 9*a^2*c^2*e*g*h^5 + (12*a*c^3*d - 11*a^2*c^2*f)*g^2*h^4 - (3*
a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h - 6*a*c^3*e*g^4*h^2 +
9*a^2*c^2*e*g^2*h^4 + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*a^2*c^2*d -
2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 - 6*a*c^3*e*g^5*h + 3*a^2*c^2*e*g^3*
h^3 + 9*a^3*c*e*g*h^5 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9*(a^2*c^2*d
- a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*c^2*f*g^5*h
- 6*a^2*c^2*e*g^4*h^2 + 9*a^3*c*e*g^2*h^4 + (12*a^2*c^2*d - 11*a^3*c*f)*g^3
*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*
g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 +
a*c*h^2)*x^2)) + (2*a*c^3*e*g^7 - 10*a^2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4
 + a^4*e*g*h^6 + a^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2
*d + 5*a^3*c*f)*g^4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - (11*a*c^3*e*g^4*
h^3 + 7*a^2*c^2*e*g^2*h^5 - 4*a^3*c*e*h^7 + (2*c^4*d - 5*a*c^3*f)*g^5*h^2 -
 (11*a*c^3*d - 5*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^
3 - (16*a*c^3*e*g^5*h^2 + 17*a^2*c^2*e*g^3*h^4 + a^3*c*e*g*h^6 + 4*(c^4*d -
 2*a*c^3*f)*g^6*h - (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a
^3*c*f)*g^2*h^5 - (3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*a*c^3*e*g^6*h + 17*a^
2*c^2*e*g^4*h^3 + 13*a^3*c*e*g^2*h^5 - 2*a^4*e*h^7 + 2*(c^4*d - a*c^3*f)*g^
7 + (8*a*c^3*d - 11*a^2*c^2*f)*g^5*h^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 -
(11*a^3*c*d - 8*a^4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3
*g^8*h^2 + 6*a^4*c^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h
^2 + 4*a^2*c^4*g^6*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10
)*x^4 + 2*(a*c^5*g^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*
g^3*h^7 + a^5*c*g*h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g
^6*h^4 + 10*a^4*c^2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*
g^9*h + 4*a^3*c^3*g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9

```
)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(3/2),x)

[Out] Timed out
```

Giac [B] time = 1.38954, size = 1944, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="giac")

[Out] ((c^6*d*g^9 - a*c^5*f*g^9 - 6*a^2*c^4*d*g^5*h^4 + 6*a^3*c^3*f*g^5*h^4 - 8*a
^3*c^3*d*g^3*h^6 + 8*a^4*c^2*f*g^3*h^6 - 3*a^4*c^2*d*g*h^8 + 3*a^5*c*f*g*h^
8 + 3*a*c^5*g^8*h*e + 8*a^2*c^4*g^6*h^3*e + 6*a^3*c^3*g^4*h^5*e - a^5*c*h^9
*e)*x/(a*c^6*g^12 + 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^
6*h^6 + 15*a^5*c^2*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12) + (3*a*c^5*d*g^8*
h - 3*a^2*c^4*f*g^8*h + 8*a^2*c^4*d*g^6*h^3 - 8*a^3*c^3*f*g^6*h^3 + 6*a^3*c
^3*d*g^4*h^5 - 6*a^4*c^2*f*g^4*h^5 - a^5*c*d*h^9 + a^6*f*h^9 - a*c^5*g^9*e
+ 6*a^3*c^3*g^5*h^4*e + 8*a^4*c^2*g^3*h^6*e + 3*a^5*c*g*h^8*e)/(a*c^6*g^12
+ 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^6*h^6 + 15*a^5*c^2
*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12))/sqrt(c*x^2 + a) - (2*c^2*f*g^4 + 1
2*c^2*d*g^2*h^2 - 11*a*c*f*g^2*h^2 - 3*a*c*d*h^4 + 2*a^2*f*h^4 - 6*c^2*g^3*
h*e + 9*a*c*g*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/s
qrt(-c*g^2 - a*h^2))/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*a^2*c*g^2*h^4 + a^3*h^
6)*sqrt(-c*g^2 - a*h^2)) - (2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h +
 6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3 - 5*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*a*c*f*g^2*h^3 - (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 4*(sq
rt(c)*x - sqrt(c*x^2 + a))^3*c^2*g^3*h^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a)
)^3*a*c*g*h^4*e + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 14*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^3*h^2 - 11*(sqrt(c)*x - sqrt(c*x^2
```

```
+ a))^2*a*c^(3/2)*f*g^3*h^2 - 7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d
*g*h^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 10*(sqrt(c
)*x - sqrt(c*x^2 + a))^2*c^(5/2)*g^4*h*e + 9*(sqrt(c)*x - sqrt(c*x^2 + a))^
2*a*c^(3/2)*g^2*h^3*e - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e
 - 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*f*g^4*h - 22*(sqrt(c)*x - sqrt(c*
x^2 + a))*a*c^2*d*g^2*h^3 + 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^
3 - (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 16*(sqrt(c)*x - sqrt(c*x^2
+ a))*a*c^2*g^3*h^2*e - 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 3*a
^2*c^(3/2)*f*g^3*h^2 + 7*a^2*c^(3/2)*d*g*h^4 - 4*a^3*sqrt(c)*f*g*h^4 - 5*a^
2*c^(3/2)*g^2*h^3*e + 2*a^3*sqrt(c)*h^5*e)/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*
a^2*c*g^2*h^4 + a^3*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x
- sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

3.115 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x(aC+2Ac)}{3a^2c\sqrt{a+cx^2}}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

```
[Out] -(a*B - (A*c - a*C)*x)/(3*a*c*(a + c*x^2)^(3/2)) + ((2*A*c + a*C)*x)/(3*a^2
*c*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0422141, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1814, 12, 191}

$$\frac{x(aC+2Ac)}{3a^2c\sqrt{a+cx^2}}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(3*a*c*(a + c*x^2)^(3/2)) + ((2*A*c + a*C)*x)/(3*a^2
*c*Sqrt[a + c*x^2])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx &= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}-\frac{\int \frac{-2A-\frac{aC}{c}}{\left(a+cx^2\right)^{3/2}}\,dx}{3a}\\ &= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}+\frac{(2Ac+aC)\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{3ac}\\ &= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}+\frac{(2Ac+aC)x}{3a^2c\sqrt{a+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.033789, size = 50, normalized size = 0.75

$$\frac{-a^2B+acx\left(3A+Cx^2\right)+2Ac^2x^3}{3a^2c\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

```
[Out] (-(a^2*B) + 2*A*c^2*x^3 + a*c*x*(3*A + C*x^2))/(3*a^2*c*(a + c*x^2)^(3/2))
```

Maple [A] time = 0.052, size = 47, normalized size = 0.7

$$\frac{2\,Ax^3c^2+Cacx^3+3\,Axac-Ba^2}{3\,a^2c}\left(cx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(2*A*c^2*x^3+C*a*c*x^3+3*A*a*c*x-B*a^2)/(c*x^2+a)^(3/2)/a^2/c
```

Maxima [A] time = 0.979341, size = 112, normalized size = 1.67

$$\frac{2\,Ax}{3\,\sqrt{cx^2+a}a^2}+\frac{Ax}{3\,\left(cx^2+a\right)^{\frac{3}{2}}a}-\frac{Cx}{3\,\left(cx^2+a\right)^{\frac{3}{2}}c}+\frac{Cx}{3\,\sqrt{cx^2+a}ac}-\frac{B}{3\,\left(cx^2+a\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(c*x^2 + a)*a^2) + 1/3*A*x/((c*x^2 + a)^(3/2)*a) - 1/3*C*x/((c
*x^2 + a)^(3/2)*c) + 1/3*C*x/(sqrt(c*x^2 + a)*a*c) - 1/3*B/((c*x^2 + a)^(3/
2)*c)
```

Fricas [A] time = 1.59628, size = 139, normalized size = 2.07

$$\frac{\left(3\,Aacx+\left(Cac+2\,Ac^2\right)x^3-Ba^2\right)\sqrt{cx^2+a}}{3\,\left(a^2c^3x^4+2\,a^3c^2x^2+a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*A*a*c*x + (C*a*c + 2*A*c^2)*x^3 - B*a^2)*sqrt(c*x^2 + a)/(a^2*c^3*x^
4 + 2*a^3*c^2*x^2 + a^4*c)
```

Sympy [A] time = 13.8247, size = 194, normalized size = 2.9

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}+\frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}\right)+B\left(\begin{cases}-\frac{1}{3ac\sqrt{a+cx^2}+3c^2x^2\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2
/a)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 +
```

```
c*x**2/a))) + B*Piecewise((-1/(3*a*c*sqrt(a + c*x**2) + 3*c**2*x**2*sqrt(a
 + c*x**2)), Ne(c, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)*sqr
t(1 + c*x**2/a) + 3*a**(3/2)*c*x**2*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.16968, size = 65, normalized size = 0.97

$$\frac{x\left(\frac{3\,A}{a}+\frac{\left(Cac+2\,Ac^2\right)x^2}{a^2c}\right)-\frac{B}{c}}{3\,\left(cx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x*(3*A/a + (C*a*c + 2*A*c^2)*x^2/(a^2*c)) - B/c)/(c*x^2 + a)^(3/2)
```

3.116 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2x(aC+4Ac)}{15a^3c\sqrt{a+cx^2}}+\frac{x(aC+4Ac)}{15a^2c\left(a+cx^2\right)^{3/2}}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

```
[Out] -(a*B - (A*c - a*C)*x)/(5*a*c*(a + c*x^2)^(5/2)) + ((4*A*c + a*C)*x)/(15*a^
2*c*(a + c*x^2)^(3/2)) + (2*(4*A*c + a*C)*x)/(15*a^3*c*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0573023, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{2x(aC+4Ac)}{15a^3c\sqrt{a+cx^2}}+\frac{x(aC+4Ac)}{15a^2c\left(a+cx^2\right)^{3/2}}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(5*a*c*(a + c*x^2)^(5/2)) + ((4*A*c + a*C)*x)/(15*a^
2*c*(a + c*x^2)^(3/2)) + (2*(4*A*c + a*C)*x)/(15*a^3*c*Sqrt[a + c*x^2])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx &= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}-\frac{\int \frac{-4A-\frac{aC}{c}}{\left(a+cx^2\right)^{5/2}}\,dx}{5a}\\
&= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}+\frac{(4Ac+aC)\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{5ac}\\
&= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\left(a+cx^2\right)^{3/2}}+\frac{(2(4Ac+aC))\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{15a^2c}\\
&= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\left(a+cx^2\right)^{3/2}}+\frac{2(4Ac+aC)x}{15a^3c\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0517778, size = 71, normalized size = 0.73

$$\frac{5a^2cx\left(3A+Cx^2\right)-3a^3B+2ac^2x^3\left(10A+Cx^2\right)+8Ac^3x^5}{15a^3c\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

```
[Out] (-3*a^3*B + 8*A*c^3*x^5 + 5*a^2*c*x*(3*A + C*x^2) + 2*a*c^2*x^3*(10*A + C*x
^2))/(15*a^3*c*(a + c*x^2)^(5/2))
```

Maple [A] time = 0.048, size = 72, normalized size = 0.7

$$\frac{8\,A c^{3} x^{5}+2\,C a c^{2} x^{5}+20\,A a c^{2} x^{3}+5\,C a^{2} c x^{3}+15\,A x a^{2} c-3\,B a^{3}}{15\,a^{3} c}\left(c x^{2}+a\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x)
```

```
[Out] 1/15*(8*A*c^3*x^5+2*C*a*c^2*x^5+20*A*a*c^2*x^3+5*C*a^2*c*x^3+15*A*a^2*c*x-3
*B*a^3)/(c*x^2+a)^(5/2)/a^3/c
```

Maxima [A] time = 0.992292, size = 159, normalized size = 1.64

$$\frac{8\,A x}{15\,\sqrt{c x^{2}+a} a^{3}}+\frac{4\,A x}{15\,\left(c x^{2}+a\right)^{\frac{3}{2}} a^{2}}+\frac{A x}{5\,\left(c x^{2}+a\right)^{\frac{5}{2}} a}-\frac{C x}{5\,\left(c x^{2}+a\right)^{\frac{5}{2}} c}+\frac{2\,C x}{15\,\sqrt{c x^{2}+a} a^{2} c}+\frac{C x}{15\,\left(c x^{2}+a\right)^{\frac{3}{2}} a c}-\frac{B}{5\,\left(c x^{2}+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*A*x/(sqrt(c*x^2 + a)*a^3) + 4/15*A*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*A*x
/((c*x^2 + a)^(5/2)*a) - 1/5*C*x/((c*x^2 + a)^(5/2)*c) + 2/15*C*x/(sqrt(c*x
^2 + a)*a^2*c) + 1/15*C*x/((c*x^2 + a)^(3/2)*a*c) - 1/5*B/((c*x^2 + a)^(5/2
)*c)
```

Fricas [A] time = 1.62208, size = 215, normalized size = 2.22

$$\frac{\left(2\left(C a c^{2}+4\,A c^{3}\right)x^{5}+15\,A a^{2} c x-3\,B a^{3}+5\left(C a^{2} c+4\,A a c^{2}\right)x^{3}\right)\sqrt{c x^{2}+a}}{15\left(a^{3} c^{4} x^{6}+3\,a^{4} c^{3} x^{4}+3\,a^{5} c^{2} x^{2}+a^{6} c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(C*a*c^2 + 4*A*c^3)*x^5 + 15*A*a^2*c*x - 3*B*a^3 + 5*(C*a^2*c + 4*A
*a*c^2)*x^3)*sqrt(c*x^2 + a)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 +
```

```
a^6*c)
```

Sympy [B] time = 39.5587, size = 638, normalized size = 6.58

$$A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx}{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(7/2),x)
```

```
[Out] A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1
+ c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3
*x**6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2/a)
 + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 +
 c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x**5
/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a)
+ 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1
 + c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(
15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a)
 + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))) + B*Piecewise((-1/(5*a**2*c*
sqrt(a + c*x**2) + 10*a*c**2*x**2*sqrt(a + c*x**2) + 5*c**3*x**4*sqrt(a + c
*x**2)), Ne(c, 0)), (x**2/(2*a**(7/2)), True)) + C*(5*a*x**3/(15*a**(9/2)*s
qrt(1 + c*x**2/a) + 30*a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**
2*x**4*sqrt(1 + c*x**2/a)) + 2*c*x**5/(15*a**(9/2)*sqrt(1 + c*x**2/a) + 30*
a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**2*x**4*sqrt(1 + c*x**2/
a)))
```

Giac [A] time = 1.20136, size = 108, normalized size = 1.11

$$\frac{\left(x^2\left(\frac{2\left(Cac^3+4\,Ac^4\right)x^2}{a^3c^2}+\frac{5\left(Ca^2c^2+4\,Aac^3\right)}{a^3c^2}\right)+\frac{15\,A}{a}\right)x-\frac{3\,B}{c}}{15\left(cx^2+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((x^2*(2*(C*a*c^3 + 4*A*c^4)*x^2/(a^3*c^2) + 5*(C*a^2*c^2 + 4*A*a*c^3)
/(a^3*c^2)) + 15*A/a)*x - 3*B/c)/(c*x^2 + a)^(5/2)
```

3.117 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}} \, dx$

Optimal. Leaf size=127

$$\frac{8x(aC+6Ac)}{105a^4c\sqrt{a+cx^2}}+\frac{4x(aC+6Ac)}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{x(aC+6Ac)}{35a^2c\left(a+cx^2\right)^{5/2}}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

```
[Out] -(a*B - (A*c - a*C)*x)/(7*a*c*(a + c*x^2)^(7/2)) + ((6*A*c + a*C)*x)/(35*a^
2*c*(a + c*x^2)^(5/2)) + (4*(6*A*c + a*C)*x)/(105*a^3*c*(a + c*x^2)^(3/2))
+ (8*(6*A*c + a*C)*x)/(105*a^4*c*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0871571, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{8x(aC+6Ac)}{105a^4c\sqrt{a+cx^2}}+\frac{4x(aC+6Ac)}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{x(aC+6Ac)}{35a^2c\left(a+cx^2\right)^{5/2}}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(7*a*c*(a + c*x^2)^(7/2)) + ((6*A*c + a*C)*x)/(35*a^
2*c*(a + c*x^2)^(5/2)) + (4*(6*A*c + a*C)*x)/(105*a^3*c*(a + c*x^2)^(3/2))
+ (8*(6*A*c + a*C)*x)/(105*a^4*c*Sqrt[a + c*x^2])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx &= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}-\frac{\int \frac{-6A-\frac{aC}{c}}{\left(a+cx^2\right)^{7/2}}\,dx}{7a}\\
&= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}+\frac{(6Ac+aC)\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx}{7ac}\\
&= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}}+\frac{(4(6Ac+aC))\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{35a^2c}\\
&= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}}+\frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{(8(6Ac+aC))\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{105a^3c}\\
&= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}}+\frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{8(6Ac+aC)x}{105a^4c\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0670119, size = 92, normalized size = 0.72

$$\frac{14a^2c^2x^3\left(15A+2Cx^2\right)+35a^3cx\left(3A+Cx^2\right)-15a^4B+8ac^3x^5\left(21A+Cx^2\right)+48Ac^4x^7}{105a^4c\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*c^4*x^7 + 35*a^3*c*x*(3*A + C*x^2) + 8*a*c^3*x^5*(21*A +
C*x^2) + 14*a^2*c^2*x^3*(15*A + 2*C*x^2))/(105*a^4*c*(a + c*x^2)^(7/2))
```

Maple [A] time = 0.052, size = 96, normalized size = 0.8

$$\frac{48\,A c^4 x^7+8\,C a c^3 x^7+168\,A a c^3 x^5+28\,C a^2 c^2 x^5+210\,A a^2 c^2 x^3+35\,C a^3 c x^3+105\,A x a^3 c-15\,B a^4}{105\,a^4 c}\left(c x^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(48*A*c^4*x^7+8*C*a*c^3*x^7+168*A*a*c^3*x^5+28*C*a^2*c^2*x^5+210*A*a^
2*c^2*x^3+35*C*a^3*c*x^3+105*A*a^3*c*x-15*B*a^4)/(c*x^2+a)^(7/2)/a^4/c
```

Maxima [A] time = 0.99926, size = 207, normalized size = 1.63

$$\frac{16\,A x}{35\,\sqrt{c x^2+a}a^4}+\frac{8\,A x}{35\,\left(c x^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,A x}{35\,\left(c x^2+a\right)^{\frac{5}{2}}a^2}+\frac{A x}{7\,\left(c x^2+a\right)^{\frac{7}{2}}a}-\frac{C x}{7\,\left(c x^2+a\right)^{\frac{7}{2}}c}+\frac{8\,C x}{105\,\sqrt{c x^2+a}a^3 c}+\frac{4\,C x}{105\,\left(c x^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(c*x^2 + a)*a^4) + 8/35*A*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((c*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((c*x^2 + a)^(7/2)*a) - 1/7*C*x/((c*x^
2 + a)^(7/2)*c) + 8/105*C*x/(sqrt(c*x^2 + a)*a^3*c) + 4/105*C*x/((c*x^2 + a
)^(3/2)*a^2*c) + 1/35*C*x/((c*x^2 + a)^(5/2)*a*c) - 1/7*B/((c*x^2 + a)^(7/2
)*c)
```

Fricas [A] time = 1.71249, size = 289, normalized size = 2.28

$$\frac{\left(8\left(C a c^3+6\,A c^4\right)x^7+105\,A a^3 c x+28\left(C a^2 c^2+6\,A a c^3\right)x^5-15\,B a^4+35\left(C a^3 c+6\,A a^2 c^2\right)x^3\right)\sqrt{c x^2+a}}{105\left(a^4 c^5 x^8+4\,a^5 c^4 x^6+6\,a^6 c^3 x^4+4\,a^7 c^2 x^2+a^8 c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*c^3 + 6*A*c^4)*x^7 + 105*A*a^3*c*x + 28*(C*a^2*c^2 + 6*A*a*c^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*c + 6*A*a^2*c^2)*x^3)*sqrt(c*x^2 + a)/(a^4*c^
5*x^8 + 4*a^5*c^4*x^6 + 6*a^6*c^3*x^4 + 4*a^7*c^2*x^2 + a^8*c)
```

Sympy [B] time = 122.907, size = 1880, normalized size = 14.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(
1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*
c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) +
 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt
(1 + c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a
**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**
2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8
*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(
25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**(37/2)*sq
rt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*
c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) +
 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt
(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 429*a**11*c*
*3*x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*
x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x
**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a
**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c
*x**2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(
35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a
) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sq
rt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/
2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 104*a**9*c**5*x**11/(35*a**(37/2)*sqrt(
1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**
2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 52
5*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1
+ c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 16*a**8*c**6*x*
*13/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2
/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*
```

```
sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(2
7/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**
2/a))) + B*Piecewise((-1/(7*a**3*c*sqrt(a + c*x**2) + 21*a**2*c**2*x**2*sqr
t(a + c*x**2) + 21*a*c**3*x**4*sqrt(a + c*x**2) + 7*c**4*x**6*sqrt(a + c*x*
*2)), Ne(c, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a**(15
/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x**2/
a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 63*a**4*c*x**5/(105*a**(
19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a*
*(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x
**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 36*a**3*c**2*x**7/(1
05*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) +
 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(
1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 8*a**2*c**3*x
**9/(105*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**
2/a) + 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6
*sqrt(1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)))
```

Giac [A] time = 1.21873, size = 151, normalized size = 1.19

$$\frac{\left(\left(4\,x^2\left(\frac{2\left(Cac^5+6\,Ac^6\right)x^2}{a^4c^3}+\frac{7\left(Ca^2c^4+6\,Aac^5\right)}{a^4c^3}\right)+\frac{35\left(Ca^3c^3+6\,Aa^2c^4\right)}{a^4c^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{c}}{105\left(cx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*c^5 + 6*A*c^6)*x^2/(a^4*c^3) + 7*(C*a^2*c^4 + 6*A*a*
c^5)/(a^4*c^3)) + 35*(C*a^3*c^3 + 6*A*a^2*c^4)/(a^4*c^3))*x^2 + 105*A/a)*x
- 15*B/c)/(c*x^2 + a)^(7/2)
```

3.118 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=106

$$\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4+\frac{13}{60}\sqrt{3x^2+2}(2x+1)^3-\frac{19}{540}\sqrt{3x^2+2}(2x+1)^2-\frac{1}{810}(2073x+3937)\sqrt{3x^2+2}+\frac{5\sinh^{-1}\left(\sqrt{\frac{3}{2}}\right.}{3\sqrt{3}}$$

```
[Out] (-19*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/540 + (13*(1 + 2*x)^3*Sqrt[2 + 3*x^2])/60
 + (2*(1 + 2*x)^4*Sqrt[2 + 3*x^2])/15 - ((3937 + 2073*x)*Sqrt[2 + 3*x^2])/8
10 + (5*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rubi [A] time = 0.114004, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1654, 833, 780, 215}

$$\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4+\frac{13}{60}\sqrt{3x^2+2}(2x+1)^3-\frac{19}{540}\sqrt{3x^2+2}(2x+1)^2-\frac{1}{810}(2073x+3937)\sqrt{3x^2+2}+\frac{5\sinh^{-1}\left(\sqrt{\frac{3}{2}}\right.}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (-19*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/540 + (13*(1 + 2*x)^3*Sqrt[2 + 3*x^2])/60
 + (2*(1 + 2*x)^4*Sqrt[2 + 3*x^2])/15 - ((3937 + 2073*x)*Sqrt[2 + 3*x^2])/8
10 + (5*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{2}{15}(1+2x)^4\sqrt{2+3x^2}+\frac{1}{60}\int \frac{(1+2x)^3(-68+156x)}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{13}{60}(1+2x)^3\sqrt{2+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2+3x^2}+\frac{1}{720}\int \frac{(-2688-228x)(1+2x)^2}{\sqrt{2+3x^2}}\,dx$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2}+\frac{13}{60}(1+2x)^3\sqrt{2+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2+3x^2}+\frac{\int \frac{(-22368-49}{\sqrt{2+}}}{64}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2}+\frac{13}{60}(1+2x)^3\sqrt{2+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2+3x^2}-\frac{1}{810}(3937+$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2}+\frac{13}{60}(1+2x)^3\sqrt{2+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2+3x^2}-\frac{1}{810}(3937+$$

Mathematica [A] time = 0.0677779, size = 54, normalized size = 0.51

$$\frac{1}{405}\left(\sqrt{3x^2+2}\left(864x^4+2430x^3+2292x^2-135x-1841\right)+225\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-1841 - 135*x + 2292*x^2 + 2430*x^3 + 864*x^4) + 225*Sqrt
[3]*ArcSinh[Sqrt[3/2]*x])/405
```

Maple [A] time = 0.062, size = 79, normalized size = 0.8

$$\frac{32x^4}{15}\sqrt{3x^2+2}+\frac{764x^2}{135}\sqrt{3x^2+2}-\frac{1841}{405}\sqrt{3x^2+2}+6x^3\sqrt{3x^2+2}-\frac{x}{3}\sqrt{3x^2+2}+\frac{5\sqrt{3}}{9}\mathrm{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

```
[Out] 32/15*x^4*(3*x^2+2)^(1/2)+764/135*x^2*(3*x^2+2)^(1/2)-1841/405*(3*x^2+2)^(1
/2)+6*x^3*(3*x^2+2)^(1/2)-1/3*x*(3*x^2+2)^(1/2)+5/9*arcsinh(1/2*x*6^(1/2))*
3^(1/2)
```

Maxima [A] time = 1.44823, size = 105, normalized size = 0.99

$$\frac{32}{15}\sqrt{3x^2+2}x^4+6\sqrt{3x^2+2}x^3+\frac{764}{135}\sqrt{3x^2+2}x^2-\frac{1}{3}\sqrt{3x^2+2}x+\frac{5}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)-\frac{1841}{405}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 + 2)*x^4 + 6*sqrt(3*x^2 + 2)*x^3 + 764/135*sqrt(3*x^2 + 2)
*x^2 - 1/3*sqrt(3*x^2 + 2)*x + 5/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 1841/40
5*sqrt(3*x^2 + 2)
```

Fricas [A] time = 1.5656, size = 174, normalized size = 1.64

$$\frac{1}{405}\left(864x^4+2430x^3+2292x^2-135x-1841\right)\sqrt{3x^2+2}+\frac{5}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/405*(864*x^4 + 2430*x^3 + 2292*x^2 - 135*x - 1841)*sqrt(3*x^2 + 2) + 5/18
*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] time = 2.18135, size = 94, normalized size = 0.89

$$\frac{32x^4\sqrt{3x^2+2}}{15}+6x^3\sqrt{3x^2+2}+\frac{764x^2\sqrt{3x^2+2}}{135}-\frac{x\sqrt{3x^2+2}}{3}-\frac{1841\sqrt{3x^2+2}}{405}+\frac{5\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 32*x**4*sqrt(3*x**2 + 2)/15 + 6*x**3*sqrt(3*x**2 + 2) + 764*x**2*sqrt(3*x**
2 + 2)/135 - x*sqrt(3*x**2 + 2)/3 - 1841*sqrt(3*x**2 + 2)/405 + 5*sqrt(3)*a
sinh(sqrt(6)*x/2)/9
```

Giac [A] time = 1.21552, size = 73, normalized size = 0.69

$$\frac{1}{405}\left(3\left(2\left(9\left(16x+45\right)x+382\right)x-45\right)x-1841\right)\sqrt{3x^2+2}-\frac{5}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/405*(3*(2*(9*(16*x + 45)*x + 382)*x - 45)*x - 1841)*sqrt(3*x^2 + 2) - 5/9
*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

3.119 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=82

$$\frac{1}{6}\sqrt{3x^2+2}(2x+1)^3+\frac{5}{18}\sqrt{3x^2+2}(2x+1)^2-\frac{1}{27}(3x+61)\sqrt{3x^2+2}-\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

```
[Out] (5*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/18 + ((1 + 2*x)^3*Sqrt[2 + 3*x^2])/6 - ((61
 + 3*x)*Sqrt[2 + 3*x^2])/27 - Sqrt[3]*ArcSinh[Sqrt[3/2]*x]
```

Rubi [A] time = 0.0884072, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1654, 833, 780, 215}

$$\frac{1}{6}\sqrt{3x^2+2}(2x+1)^3+\frac{5}{18}\sqrt{3x^2+2}(2x+1)^2-\frac{1}{27}(3x+61)\sqrt{3x^2+2}-\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```


```
[Out] (5*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/18 + ((1 + 2*x)^3*Sqrt[2 + 3*x^2])/6 - ((61
 + 3*x)*Sqrt[2 + 3*x^2])/27 - Sqrt[3]*ArcSinh[Sqrt[3/2]*x]
```


Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx &= \frac{1}{6}(1+2x)^3\sqrt{2+3x^2}+\frac{1}{48}\int \frac{(1+2x)^2(-48+120x)}{\sqrt{2+3x^2}}\,dx\\ &= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}+\frac{1}{432}\int \frac{(-1392-144x)(1+2x)}{\sqrt{2+3x^2}}\,dx\\ &= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}-\frac{1}{27}(61+3x)\sqrt{2+3x^2}-3\int \frac{1}{\sqrt{2+3x^2}}\,d\\ &= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}-\frac{1}{27}(61+3x)\sqrt{2+3x^2}-\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}\right.\end{aligned}$$

Mathematica [A] time = 0.0426069, size = 48, normalized size = 0.59

$$\frac{1}{27}\sqrt{3x^2+2}\left(36x^3+84x^2+54x-49\right)-\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-49 + 54*x + 84*x^2 + 36*x^3))/27 - Sqrt[3]*ArcSinh[Sqrt[
3/2]*x]
```

Maple [A] time = 0.055, size = 65, normalized size = 0.8

$$\frac{4x^3}{3}\sqrt{3x^2+2}+2x\sqrt{3x^2+2}-\mathrm{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}+\frac{28x^2}{9}\sqrt{3x^2+2}-\frac{49}{27}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

```
[Out] 4/3*x^3*(3*x^2+2)^(1/2)+2*x*(3*x^2+2)^(1/2)-arcsinh(1/2*x*6^(1/2))*3^(1/2)+
28/9*x^2*(3*x^2+2)^(1/2)-49/27*(3*x^2+2)^(1/2)
```

Maxima [A] time = 1.49621, size = 86, normalized size = 1.05

$$\frac{4}{3}\sqrt{3x^2+2}x^3+\frac{28}{9}\sqrt{3x^2+2}x^2+2\sqrt{3x^2+2}x-\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)-\frac{49}{27}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 + 2)*x^3 + 28/9*sqrt(3*x^2 + 2)*x^2 + 2*sqrt(3*x^2 + 2)*x -
sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 49/27*sqrt(3*x^2 + 2)
```

Fricas [A] time = 1.52905, size = 147, normalized size = 1.79

$$\frac{1}{27}\left(36x^3+84x^2+54x-49\right)\sqrt{3x^2+2}+\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(36*x^3 + 84*x^2 + 54*x - 49)*sqrt(3*x^2 + 2) + 1/2*sqrt(3)*log(sqrt(3
)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] time = 1.15238, size = 75, normalized size = 0.91

$$\frac{4x^3\sqrt{3x^2+2}}{3}+\frac{28x^2\sqrt{3x^2+2}}{9}+2x\sqrt{3x^2+2}-\frac{49\sqrt{3x^2+2}}{27}-\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 4*x**3*sqrt(3*x**2 + 2)/3 + 28*x**2*sqrt(3*x**2 + 2)/9 + 2*x*sqrt(3*x**2 +
2) - 49*sqrt(3*x**2 + 2)/27 - sqrt(3)*asinh(sqrt(6)*x/2)
```

Giac [A] time = 1.1942, size = 65, normalized size = 0.79

$$\frac{1}{27}\left(6\left(2\left(3x+7\right)x+9\right)x-49\right)\sqrt{3x^2+2}+\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(6*(2*(3*x + 7)*x + 9)*x - 49)*sqrt(3*x^2 + 2) + sqrt(3)*log(-sqrt(3)*
x + sqrt(3*x^2 + 2))
```

3.120 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=62

$$\frac{2}{9}\sqrt{3x^2+2}(2x+1)^2+\frac{7}{27}(3x+1)\sqrt{3x^2+2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 + (7*(1 + 3*x)*Sqrt[2 + 3*x^2])/27 - (7*A
rcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rubi [A] time = 0.0516871, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1654, 780, 215}

$$\frac{2}{9}\sqrt{3x^2+2}(2x+1)^2+\frac{7}{27}(3x+1)\sqrt{3x^2+2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 + (7*(1 + 3*x)*Sqrt[2 + 3*x^2])/27 - (7*A
rcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{36}\int \frac{(1+2x)(-28+84x)}{\sqrt{2+3x^2}}\,dx\\ &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\ &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0283797, size = 44, normalized size = 0.71

$$\frac{1}{27}\left(\sqrt{3x^2+2}\left(24x^2+45x+13\right)-21\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(13 + 45*x + 24*x^2) - 21*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/27
```

Maple [A] time = 0.049, size = 51, normalized size = 0.8

$$\frac{8\,x^2}{9}\sqrt{3\,x^2+2}+\frac{13}{27}\sqrt{3\,x^2+2}+\frac{5\,x}{3}\sqrt{3\,x^2+2}-\frac{7\,\sqrt{3}}{9}\mathrm{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

```
[Out] 8/9*x^2*(3*x^2+2)^(1/2)+13/27*(3*x^2+2)^(1/2)+5/3*x*(3*x^2+2)^(1/2)-7/9*arc
sinh(1/2*x*6^(1/2))*3^(1/2)
```

Maxima [A] time = 1.46932, size = 68, normalized size = 1.1

$$\frac{8}{9}\sqrt{3x^2+2}x^2+\frac{5}{3}\sqrt{3x^2+2}x-\frac{7}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)+\frac{13}{27}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 + 2)*x^2 + 5/3*sqrt(3*x^2 + 2)*x - 7/9*sqrt(3)*arcsinh(1/2*s
qrt(6)*x) + 13/27*sqrt(3*x^2 + 2)
```

Fricas [A] time = 1.5361, size = 136, normalized size = 2.19

$$\frac{1}{27}\left(24x^2+45x+13\right)\sqrt{3x^2+2}+\frac{7}{18}\sqrt{3}\log\left(\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(24*x^2 + 45*x + 13)*sqrt(3*x^2 + 2) + 7/18*sqrt(3)*log(sqrt(3)*sqrt(3
*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] time = 0.569814, size = 63, normalized size = 1.02

$$\frac{8x^2\sqrt{3x^2+2}}{9}+\frac{5x\sqrt{3x^2+2}}{3}+\frac{13\sqrt{3x^2+2}}{27}-\frac{7\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 8*x**2*sqrt(3*x**2 + 2)/9 + 5*x*sqrt(3*x**2 + 2)/3 + 13*sqrt(3*x**2 + 2)/27
 - 7*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

Giac [A] time = 1.21317, size = 59, normalized size = 0.95

$$\frac{1}{27}\left(3\left(8x+15\right)x+13\right)\sqrt{3x^2+2}+\frac{7}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(3*(8*x + 15)*x + 13)*sqrt(3*x^2 + 2) + 7/9*sqrt(3)*log(-sqrt(3)*x + s
qrt(3*x^2 + 2))
```

3.121 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=67

$$\frac{2}{3}\sqrt{3x^2+2}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{2\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}$$

```
[Out] (2*Sqrt[2 + 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/(2*Sqrt[3]) - ArcTanh[(4 - 3*x
)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/(2*Sqrt[11])
```

Rubi [A] time = 0.0806947, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 844, 215, 725, 206}

$$\frac{2}{3}\sqrt{3x^2+2}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{2\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 + 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/(2*Sqrt[3]) - ArcTanh[(4 - 3*x
)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/(2*Sqrt[11])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx &= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{12}\int \frac{12+12x}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{2}\int \frac{1}{\sqrt{2+3x^2}}\,dx+\frac{1}{2}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{2\sqrt{11}}
\end{aligned}$$

Mathematica [A] time = 0.0303362, size = 60, normalized size = 0.9

$$\frac{1}{66}\left(44\sqrt{3x^2+2}-3\sqrt{11}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)+11\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (44*Sqrt[2 + 3*x^2] + 11*Sqrt[3]*ArcSinh[Sqrt[3/2]*x] - 3*Sqrt[11]*ArcTanh[
(4 - 3*x)/Sqrt[22 + 33*x^2]])/66
```

Maple [A] time = 0.052, size = 55, normalized size = 0.8

$$\frac{2}{3}\sqrt{3x^2+2}+\frac{\sqrt{3}}{6}\mathrm{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)-\frac{\sqrt{11}}{22}\mathrm{Artanh}\left(\frac{(8-6x)\sqrt{11}}{11}\frac{1}{\sqrt{12\,(x+1/2)^2-12\,x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x)
```

```
[Out] 2/3*(3*x^2+2)^(1/2)+1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)-1/22*11^(1/2)*arctan
h(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Maxima [A] time = 1.49652, size = 78, normalized size = 1.16

$$\frac{1}{6}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{22}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{2}{3}\sqrt{3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 1/22*sqrt(11)*arcsinh(1/2*sqrt(6)*x/ab
s(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) + 2/3*sqrt(3*x^2 + 2)
```

Fricas [A] time = 1.57985, size = 240, normalized size = 3.58

$$\frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+\frac{1}{44}\sqrt{11}\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+\frac{2}{3}\sqrt{3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 1/44*sqrt(11)*lo
g(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x +
 1)) + 2/3*sqrt(3*x^2 + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)\sqrt{3x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 + 2)), x)
```

Giac [B] time = 1.32843, size = 134, normalized size = 2.

$$-\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}x + \sqrt{3x^2+2}\right) + \frac{1}{22}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x - \sqrt{11} - \sqrt{3} + 2\sqrt{3x^2+2}\right|}{2\sqrt{3}x - \sqrt{11} + \sqrt{3} - 2\sqrt{3x^2+2}}\right) + \frac{2}{3}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/22*sqrt(11)*log(-abs(-2*
sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(11)
 + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 2/3*sqrt(3*x^2 + 2)
```

3.122 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

```
[Out] -Sqrt[2 + 3*x^2]/(11*(1 + 2*x)) + ArcSinh[Sqrt[3/2]*x]/Sqrt[3] + (4*ArcTanh
[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11])
```

Rubi [A] time = 0.069962, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 844, 215, 725, 206}

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -Sqrt[2 + 3*x^2]/(11*(1 + 2*x)) + ArcSinh[Sqrt[3/2]*x]/Sqrt[3] + (4*ArcTanh
[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx &= -\frac{\sqrt{2+3x^2}}{11(1+2x)} - \frac{1}{11}\int \frac{-7-22x}{(1+2x)\sqrt{2+3x^2}}\,dx \\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)} - \frac{4}{11}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx + \int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{4}{11}\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right) \\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}
\end{aligned}$$

Mathematica [A] time = 0.106519, size = 64, normalized size = 0.9

$$-\frac{\sqrt{3x^2+2}}{22x+11} + \frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{11\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -(Sqrt[2 + 3*x^2]/(11 + 22*x)) + ArcSinh[Sqrt[3/2]*x]/Sqrt[3] + (4*ArcTanh[
(4 - 3*x)/Sqrt[22 + 33*x^2]])/(11*Sqrt[11])
```

Maple [A] time = 0.058, size = 65, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\mathrm{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)-\frac{1}{22}\sqrt{3\,(x+1/2)^2-3\,x+\frac{5}{4}}\left(x+\frac{1}{2}\right)^{-1}+\frac{4\,\sqrt{11}}{121}\mathrm{Artanh}\left(\frac{(8-6\,x)\,\sqrt{11}}{11}\frac{1}{\sqrt{12\,(x+1/2)^2-12\,x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)-1/22/(x+1/2)*(3*(x+1/2)^2-3*x+5/4)^(1/2)
+4/121*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Maxima [A] time = 1.49284, size = 88, normalized size = 1.24

$$\frac{1}{3}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right)-\frac{4}{121}\,\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2+2}}{11\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 4/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/a
bs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) - 1/11*sqrt(3*x^2 + 2)/(2*x + 1)
```

Fricas [A] time = 1.65382, size = 285, normalized size = 4.01

$$\frac{121\,\sqrt{3}(2\,x+1)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+12\,\sqrt{11}(2\,x+1)\log\left(\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)-21\,x^2+12\,x-19}{4\,x^2+4\,x+1}\right)-66\,\sqrt{3\,x^2+2}}{726\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/726*(121*sqrt(3)*(2*x + 1)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
12*sqrt(11)*(2*x + 1)*log((sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) - 21*x^2 + 12
*x - 19)/(4*x^2 + 4*x + 1)) - 66*sqrt(3*x^2 + 2))/(2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2 + 3x + 1}{(2x+1)^2 \sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4\,x^2 + 3\,x + 1}{\sqrt{3\,x^2+2}(2\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*x + 1)/(sqrt(3*x^2 + 2)*(2*x + 1)^2), x)
```

3.123 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=77

$$\frac{13\sqrt{3x^2+2}}{242(2x+1)}-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

```
[Out] -Sqrt[2 + 3*x^2]/(22*(1 + 2*x)^2) + (13*Sqrt[2 + 3*x^2])/(242*(1 + 2*x)) -
(103*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rubi [A] time = 0.066702, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1651, 807, 725, 206}

$$\frac{13\sqrt{3x^2+2}}{242(2x+1)}-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -Sqrt[2 + 3*x^2]/(22*(1 + 2*x)^2) + (13*Sqrt[2 + 3*x^2])/(242*(1 + 2*x)) -
(103*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx &= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}-\frac{1}{22}\int \frac{-14-41x}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\
&= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}+\frac{103}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103}{121}\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

Mathematica [A] time = 0.0677386, size = 55, normalized size = 0.71

$$\frac{\frac{11(13x+1)\sqrt{3x^2+2}}{(2x+1)^2}-103\sqrt{11}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{1331}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((11*(1 + 13*x)*Sqrt[2 + 3*x^2])/(1 + 2*x)^2 - 103*Sqrt[11]*ArcTanh[(4 - 3*
x)/Sqrt[22 + 33*x^2]])/1331
```

Maple [A] time = 0.058, size = 74, normalized size = 1.

$$\frac{13}{484}\sqrt{3\ (x+1/2)^2-3x+\frac{5}{4}}\left(x+\frac{1}{2}\right)^{-1}-\frac{103\sqrt{11}}{1331}\text{Artanh}\left(\frac{(8-6x)\sqrt{11}}{11}\frac{1}{\sqrt{12\ (x+1/2)^2-12x+5}}\right)-\frac{1}{88}\sqrt{3\ (x+1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x)
```

```
[Out] 13/484/(x+1/2)*(3*(x+1/2)^2-3*x+5/4)^(1/2)-103/1331*11^(1/2)*arctanh(2/11*(
4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))-1/88/(x+1/2)^2*(3*(x+1/2)^2-3*
x+5/4)^(1/2)
```

Maxima [A] time = 1.47774, size = 103, normalized size = 1.34

$$\frac{103}{1331}\sqrt{11}\,\text{arsinh}\left(\frac{\sqrt{6}x}{2\,|2x+1|}-\frac{2\sqrt{6}}{3\,|2x+1|}\right)-\frac{\sqrt{3x^2+2}}{22\left(4x^2+4x+1\right)}+\frac{13\sqrt{3x^2+2}}{242\,(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 103/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) - 1/22*sqrt(3*x^2 + 2)/(4*x^2 + 4*x + 1) + 13/242*sqrt(3*x^2 + 2)/(2*
x + 1)
```

Fricas [A] time = 1.57685, size = 234, normalized size = 3.04

$$\frac{103\sqrt{11}\left(4x^2+4x+1\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+22\sqrt{3x^2+2}(13x+1)}{2662\left(4x^2+4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(103*sqrt(11)*(4*x^2 + 4*x + 1)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x
- 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 22*sqrt(3*x^2 + 2)*(13*x +
1))/(4*x^2 + 4*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26592, size = 243, normalized size = 3.16

$$\frac{103}{1331}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{72\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3-13\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2-168\sqrt{3}x+}{484\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 103/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2
 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/484*(72*
(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 13*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^
2 - 168*sqrt(3)*x + 104*sqrt(3) + 168*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3
*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

3.124 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{32}{27}\sqrt{3x^2+2}x^2+4\sqrt{3x^2+2}x+\frac{292}{81}\sqrt{3x^2+2}+\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] (398 + 279*x)/(54*Sqrt[2 + 3*x^2]) + (292*Sqrt[2 + 3*x^2])/81 + 4*x*Sqrt[2
+ 3*x^2] + (32*x^2*Sqrt[2 + 3*x^2])/27 - (38*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[
3])
```

Rubi [A] time = 0.103597, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1814, 1815, 641, 215}

$$\frac{32}{27}\sqrt{3x^2+2}x^2+4\sqrt{3x^2+2}x+\frac{292}{81}\sqrt{3x^2+2}+\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (398 + 279*x)/(54*Sqrt[2 + 3*x^2]) + (292*Sqrt[2 + 3*x^2])/81 + 4*x*Sqrt[2
+ 3*x^2] + (32*x^2*Sqrt[2 + 3*x^2])/27 - (38*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[
3])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
```

```
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx &= \frac{398+279x}{54\sqrt{2+3x^2}}-\frac{1}{2}\int \frac{\frac{28}{3}-\frac{280x}{9}-48x^2-\frac{64x^3}{3}}{\sqrt{2+3x^2}}\,dx \\
&= \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{1}{18}\int \frac{84-\frac{584x}{3}-432x^2}{\sqrt{2+3x^2}}\,dx \\
&= \frac{398+279x}{54\sqrt{2+3x^2}}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{1}{108}\int \frac{1368-1168x}{\sqrt{2+3x^2}}\,dx \\
&= \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{292}{81}\sqrt{2+3x^2}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{38}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{292}{81}\sqrt{2+3x^2}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0572413, size = 58, normalized size = 0.67

$$\frac{576x^4+1944x^3+2136x^2-684\sqrt{9x^2+6}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)+2133x+2362}{162\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2362 + 2133*x + 2136*x^2 + 1944*x^3 + 576*x^4 - 684*Sqrt[6 + 9*x^2]*ArcSin
h[Sqrt[3/2]*x])/(162*Sqrt[2 + 3*x^2])
```

Maple [A] time = 0.056, size = 79, normalized size = 0.9

$$\frac{32x^4}{9}\frac{1}{\sqrt{3x^2+2}}+\frac{356x^2}{27}\frac{1}{\sqrt{3x^2+2}}+\frac{1181}{81}\frac{1}{\sqrt{3x^2+2}}+12\frac{x^3}{\sqrt{3x^2+2}}+\frac{79x}{6}\frac{1}{\sqrt{3x^2+2}}-\frac{38\sqrt{3}}{9}\mathrm{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

```
[Out] 32/9*x^4/(3*x^2+2)^(1/2)+356/27*x^2/(3*x^2+2)^(1/2)+1181/81/(3*x^2+2)^(1/2)
+12*x^3/(3*x^2+2)^(1/2)+79/6*x/(3*x^2+2)^(1/2)-38/9*arcsinh(1/2*x*6^(1/2))*
3^(1/2)
```

Maxima [A] time = 1.50175, size = 105, normalized size = 1.21

$$\frac{32x^4}{9\sqrt{3x^2+2}}+\frac{12x^3}{\sqrt{3x^2+2}}+\frac{356x^2}{27\sqrt{3x^2+2}}-\frac{38}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)+\frac{79x}{6\sqrt{3x^2+2}}+\frac{1181}{81\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 32/9*x^4/sqrt(3*x^2 + 2) + 12*x^3/sqrt(3*x^2 + 2) + 356/27*x^2/sqrt(3*x^2 +
 2) - 38/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 79/6*x/sqrt(3*x^2 + 2) + 1181/8
1/sqrt(3*x^2 + 2)
```

Fricas [A] time = 1.60615, size = 208, normalized size = 2.39

$$\frac{342\sqrt{3}\left(3x^2+2\right)\log\left(\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+\left(576x^4+1944x^3+2136x^2+2133x+2362\right)\sqrt{3x^2+2}}{162\left(3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/162*(342*sqrt(3)*(3*x^2 + 2)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 (576*x^4 + 1944*x^3 + 2136*x^2 + 2133*x + 2362)*sqrt(3*x^2 + 2))/(3*x^2 +
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^3 (4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

Giac [A] time = 1.31974, size = 73, normalized size = 0.84

$$\frac{38}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)+\frac{3\,(8\,(3\,(8\,x+27)x+89)x+711)x+2362}{162\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 38/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(3*(8*(3*(8*x + 27)*
x + 89)*x + 711)*x + 2362)/sqrt(3*x^2 + 2)
```

3.125 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{70-47x}{18\sqrt{3x^2+2}}+\frac{8}{9}x\sqrt{3x^2+2}+\frac{28}{9}\sqrt{3x^2+2}+\frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] (70 - 47*x)/(18*Sqrt[2 + 3*x^2]) + (28*Sqrt[2 + 3*x^2])/9 + (8*x*Sqrt[2 + 3
*x^2])/9 + (4*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rubi [A] time = 0.0840017, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1814, 1815, 641, 215}

$$\frac{70-47x}{18\sqrt{3x^2+2}}+\frac{8}{9}x\sqrt{3x^2+2}+\frac{28}{9}\sqrt{3x^2+2}+\frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (70 - 47*x)/(18*Sqrt[2 + 3*x^2]) + (28*Sqrt[2 + 3*x^2])/9 + (8*x*Sqrt[2 + 3
*x^2])/9 + (4*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
```

```
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx &= \frac{70-47x}{18\sqrt{2+3x^2}}-\frac{1}{2}\int \frac{-\frac{56}{9}-\frac{56x}{3}-\frac{32x^2}{3}}{\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{18\sqrt{2+3x^2}}+\frac{8}{9}x\sqrt{2+3x^2}-\frac{1}{12}\int \frac{-16-112x}{\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{18\sqrt{2+3x^2}}+\frac{28}{9}\sqrt{2+3x^2}+\frac{8}{9}x\sqrt{2+3x^2}+\frac{4}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{18\sqrt{2+3x^2}}+\frac{28}{9}\sqrt{2+3x^2}+\frac{8}{9}x\sqrt{2+3x^2}+\frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.0525299, size = 53, normalized size = 0.75

$$
\frac{48x^3+168x^2+8\sqrt{9x^2+6}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)-15x+182}{18\sqrt{3x^2+2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (182 - 15*x + 168*x^2 + 48*x^3 + 8*Sqrt[6 + 9*x^2]*ArcSinh[Sqrt[3/2]*x])/(1
8*Sqrt[2 + 3*x^2])
```

Maple [A] time = 0.054, size = 65, normalized size = 0.9

$$\frac{8x^3}{3}\frac{1}{\sqrt{3x^2+2}}-\frac{5x}{6}\frac{1}{\sqrt{3x^2+2}}+\frac{4\sqrt{3}}{9}\operatorname{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)+\frac{28x^2}{3}\frac{1}{\sqrt{3x^2+2}}+\frac{91}{9}\frac{1}{\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

```
[Out] 8/3*x^3/(3*x^2+2)^(1/2)-5/6*x/(3*x^2+2)^(1/2)+4/9*arcsinh(1/2*x*6^(1/2))*3^
(1/2)+28/3*x^2/(3*x^2+2)^(1/2)+91/9/(3*x^2+2)^(1/2)
```

Maxima [A] time = 1.53049, size = 86, normalized size = 1.21

$$\frac{8x^3}{3\sqrt{3x^2+2}}+\frac{28x^2}{3\sqrt{3x^2+2}}+\frac{4}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)-\frac{5x}{6\sqrt{3x^2+2}}+\frac{91}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^3/sqrt(3*x^2 + 2) + 28/3*x^2/sqrt(3*x^2 + 2) + 4/9*sqrt(3)*arcsinh(1/
2*sqrt(6)*x) - 5/6*x/sqrt(3*x^2 + 2) + 91/9/sqrt(3*x^2 + 2)
```

Fricas [A] time = 1.55454, size = 184, normalized size = 2.59

$$\frac{4\sqrt{3}\left(3x^2+2\right)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+\left(48x^3+168x^2-15x+182\right)\sqrt{3x^2+2}}{18\left(3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*(4*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + (
48*x^3 + 168*x^2 - 15*x + 182)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^2 \left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

Giac [A] time = 1.26012, size = 66, normalized size = 0.93

$$-\frac{4}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)+\frac{3\,(8\,(2\,x+7)x-5)x+182}{18\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -4/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(8*(2*x + 7)*x - 5
)*x + 182)/sqrt(3*x^2 + 2)
```

3.126 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{2-51x}{18\sqrt{3x^2+2}}+\frac{8}{9}\sqrt{3x^2+2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] (2 - 51*x)/(18*Sqrt[2 + 3*x^2]) + (8*Sqrt[2 + 3*x^2])/9 + (10*ArcSinh[Sqrt[
3/2]*x])/(3*Sqrt[3])
```

Rubi [A] time = 0.0444929, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1814, 641, 215}

$$\frac{2-51x}{18\sqrt{3x^2+2}}+\frac{8}{9}\sqrt{3x^2+2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2 - 51*x)/(18*Sqrt[2 + 3*x^2]) + (8*Sqrt[2 + 3*x^2])/9 + (10*ArcSinh[Sqrt[
3/2]*x])/(3*Sqrt[3])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx &= \frac{2-51x}{18\sqrt{2+3x^2}}-\frac{1}{2}\int \frac{-\frac{20}{3}-\frac{16x}{3}}{\sqrt{2+3x^2}}\,dx\\ &= \frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\ &= \frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0304869, size = 48, normalized size = 0.87

$$\frac{48x^2+20\sqrt{9x^2+6}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)-51x+34}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (34 - 51*x + 48*x^2 + 20*Sqrt[6 + 9*x^2]*ArcSinh[Sqrt[3/2]*x])/(18*Sqrt[2 +
 3*x^2])
```

Maple [A] time = 0.052, size = 51, normalized size = 0.9

$$\frac{8\,x^2}{3}\frac{1}{\sqrt{3\,x^2+2}}+\frac{17}{9}\frac{1}{\sqrt{3\,x^2+2}}-\frac{17\,x}{6}\frac{1}{\sqrt{3\,x^2+2}}+\frac{10\,\sqrt{3}}{9}\mathrm{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

```
[Out] 8/3*x^2/(3*x^2+2)^(1/2)+17/9/(3*x^2+2)^(1/2)-17/6*x/(3*x^2+2)^(1/2)+10/9*ar
csinh(1/2*x*6^(1/2))*3^(1/2)
```

Maxima [A] time = 1.47274, size = 68, normalized size = 1.24

$$\frac{8\,x^2}{3\,\sqrt{3\,x^2+2}}+\frac{10}{9}\,\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right)-\frac{17\,x}{6\,\sqrt{3\,x^2+2}}+\frac{17}{9\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 + 2) + 10/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 17/6*x/sqrt
(3*x^2 + 2) + 17/9/sqrt(3*x^2 + 2)
```

Fricas [A] time = 1.85049, size = 170, normalized size = 3.09

$$\frac{10\,\sqrt{3}\left(3\,x^2+2\right)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+\left(48\,x^2-51\,x+34\right)\sqrt{3\,x^2+2}}{18\left(3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*(10*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
(48*x^2 - 51*x + 34)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [B] time = 12.906, size = 114, normalized size = 2.07

$$\frac{30\sqrt{3}x^2\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18}+\frac{8x^2}{3\sqrt{3x^2+2}}-\frac{30x\sqrt{3x^2+2}}{27x^2+18}+\frac{x}{2\sqrt{3x^2+2}}+\frac{20\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18}+\frac{17}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] 30*sqrt(3)*x**2*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 8*x**2/(3*sqrt(3*x**2 +
 2)) - 30*x*sqrt(3*x**2 + 2)/(27*x**2 + 18) + x/(2*sqrt(3*x**2 + 2)) + 20*s
qrt(3)*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 17/(9*sqrt(3*x**2 + 2))
```

Giac [A] time = 1.20877, size = 59, normalized size = 1.07

$$-\frac{10}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)+\frac{3\,(16\,x-17)x+34}{18\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(16*x - 17)*x + 3
4)/sqrt(3*x^2 + 2)
```

3.127 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{38-21x}{66\sqrt{3x^2+2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}$$

```
[Out] -(38 - 21*x)/(66*Sqrt[2 + 3*x^2]) - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 +
 3*x^2])])/(11*Sqrt[11])
```

Rubi [A] time = 0.0599846, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 12, 725, 206}

$$-\frac{38-21x}{66\sqrt{3x^2+2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] -(38 - 21*x)/(66*Sqrt[2 + 3*x^2]) - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 +
 3*x^2])])/(11*Sqrt[11])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx &= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{1}{6}\int -\frac{12}{11(1+2x)\sqrt{2+3x^2}}\,dx\\ &= -\frac{38-21x}{66\sqrt{2+3x^2}}+\frac{2}{11}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\ &= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2}{11}\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right)\\ &= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}\end{aligned}$$

Mathematica [A] time = 0.0238863, size = 51, normalized size = 0.96

$$\frac{-12\sqrt{33x^2+22}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)+231x-418}{726\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (-418 + 231*x - 12*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/
(726*Sqrt[2 + 3*x^2])
```

Maple [B] time = 0.053, size = 88, normalized size = 1.7

$$-\frac{2}{3}\frac{1}{\sqrt{3x^{2}+2}}+\frac{x}{4}\frac{1}{\sqrt{3x^{2}+2}}+\frac{1}{11}\frac{1}{\sqrt{3\left(x+1/2\right)^{2}-3x+\frac{5}{4}}}+\frac{3x}{44}\frac{1}{\sqrt{3\left(x+1/2\right)^{2}-3x+\frac{5}{4}}}-\frac{2\sqrt{11}}{121}\text{Artanh}\left(\frac{(8-6x)\sqrt{1}}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x)
```

```
[Out] -2/3/(3*x^2+2)^(1/2)+1/4*x/(3*x^2+2)^(1/2)+1/11/(3*(x+1/2)^2-3*x+5/4)^(1/2)
+3/44*x/(3*(x+1/2)^2-3*x+5/4)^(1/2)-2/121*11^(1/2)*arctanh(2/11*(4-3*x)*11^
(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Maxima [A] time = 1.50389, size = 78, normalized size = 1.47

$$\frac{2}{121}\sqrt{11}\,\text{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{7\,x}{22\sqrt{3\,x^{2}+2}}-\frac{19}{33\sqrt{3\,x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1
)) + 7/22*x/sqrt(3*x^2 + 2) - 19/33/sqrt(3*x^2 + 2)
```

Fricas [A] time = 1.81235, size = 215, normalized size = 4.06

$$\frac{6\sqrt{11}\left(3\,x^{2}+2\right)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^{2}+2}(3\,x-4)+21\,x^{2}-12\,x+19}{4\,x^{2}+4\,x+1}\right)+11\sqrt{3\,x^{2}+2}(21\,x-38)}{726\left(3\,x^{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/726*(6*sqrt(11)*(3*x^2 + 2)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21
*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*sqrt(3*x^2 + 2)*(21*x - 38))/(3*x
```

```
^2 + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 + 2)**(3/2)), x)
```

Giac [A] time = 1.20817, size = 111, normalized size = 2.09

$$\frac{2}{121}\,\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right)+\frac{21\,x-38}{66\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/121*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/66*(21*x -
38)/sqrt(3*x^2 + 2)
```

3.128 $\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{10-97x}{242\sqrt{3x^2+2}}-\frac{4\sqrt{3x^2+2}}{121(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

```
[Out] -(10 - 97*x)/(242*Sqrt[2 + 3*x^2]) - (4*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)) +
(4*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rubi [A] time = 0.0772505, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 807, 725, 206}

$$-\frac{10-97x}{242\sqrt{3x^2+2}}-\frac{4\sqrt{3x^2+2}}{121(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```


```
[Out] -(10 - 97*x)/(242*Sqrt[2 + 3*x^2]) - (4*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)) +
(4*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```


Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx &= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{1}{6}\int \frac{-\frac{72}{121}+\frac{120x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}-\frac{4}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4}{121}\operatorname{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

Mathematica [A] time = 0.0419811, size = 71, normalized size = 0.95

$$\frac{11\left(170x^2+77x-26\right)+8(2x+1)\sqrt{33x^2+22}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{2662(2x+1)\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (11*(-26 + 77*x + 170*x^2) + 8*(1 + 2*x)*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x
)/Sqrt[22 + 33*x^2]])/(2662*(1 + 2*x)*Sqrt[2 + 3*x^2])
```

Maple [A] time = 0.057, size = 98, normalized size = 1.3

$$\frac{x}{2}\frac{1}{\sqrt{3x^2+2}}-\frac{1}{22}\left(x+\frac{1}{2}\right)^{-1}\frac{1}{\sqrt{3\left(x+1/2\right)^2-3x+\frac{5}{4}}}-\frac{2}{121}\frac{1}{\sqrt{3\left(x+1/2\right)^2-3x+\frac{5}{4}}}-\frac{18x}{121}\frac{1}{\sqrt{3\left(x+1/2\right)^2-3x+\frac{5}{4}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x)
```

```
[Out] 1/2*x/(3*x^2+2)^(1/2)-1/22/(x+1/2)/(3*(x+1/2)^2-3*x+5/4)^(1/2)-2/121/(3*(x+
1/2)^2-3*x+5/4)^(1/2)-18/121*x/(3*(x+1/2)^2-3*x+5/4)^(1/2)+4/1331*11^(1/2)*
arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Maxima [A] time = 1.48201, size = 113, normalized size = 1.51

$$-\frac{4}{1331}\sqrt{11}\,\text{arsinh}\left(\frac{\sqrt{6}x}{2\,|2x+1|}-\frac{2\sqrt{6}}{3\,|2x+1|}\right)+\frac{85x}{242\sqrt{3x^2+2}}-\frac{2}{121\sqrt{3x^2+2}}-\frac{1}{11\left(2\sqrt{3x^2+2}x+\sqrt{3x^2+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
 1)) + 85/242*x/sqrt(3*x^2 + 2) - 2/121/sqrt(3*x^2 + 2) - 1/11/(2*sqrt(3*x^
2 + 2)*x + sqrt(3*x^2 + 2))
```

Fricas [A] time = 1.70027, size = 266, normalized size = 3.55

$$\frac{4\sqrt{11}\left(6x^3+3x^2+4x+2\right)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)+11\left(170x^2+77x-26\right)\sqrt{3x^2+2}}{2662\left(6x^3+3x^2+4x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(4*sqrt(11)*(6*x^3 + 3*x^2 + 4*x + 2)*log((sqrt(11)*sqrt(3*x^2 + 2)*
(3*x - 4) - 21*x^2 + 12*x - 19)/(4*x^2 + 4*x + 1)) + 11*(170*x^2 + 77*x - 2
6)*sqrt(3*x^2 + 2))/(6*x^3 + 3*x^2 + 4*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2+3x+1}{\left(3x^2+2\right)^{\frac{3}{2}}(2x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*x + 1)/((3*x^2 + 2)^(3/2)*(2*x + 1)^2), x)
```

3.129 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=97

$$\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{2\sqrt{3x^2+2}}{1331(2x+1)}-\frac{2\sqrt{3x^2+2}}{121(2x+1)^2}-\frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

```
[Out] (358 + 351*x)/(2662*Sqrt[2 + 3*x^2]) - (2*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)^2
) + (2*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (322*ArcTanh[(4 - 3*x)/(Sqrt[11]
*Sqrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rubi [A] time = 0.122966, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 1651, 807, 725, 206}

$$\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{2\sqrt{3x^2+2}}{1331(2x+1)}-\frac{2\sqrt{3x^2+2}}{121(2x+1)^2}-\frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (358 + 351*x)/(2662*Sqrt[2 + 3*x^2]) - (2*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)^2
) + (2*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (322*ArcTanh[(4 - 3*x)/(Sqrt[11]
*Sqrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{1}{6}\int \frac{-\frac{2940}{1331}-\frac{7272x}{1331}-\frac{8592x^2}{1331}}{(1+2x)^3\sqrt{2+3x^2}}\,dx$$

$$= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{1}{132}\int \frac{\frac{3768}{121}+\frac{7800x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx$$

$$= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} + \frac{322\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{1331}$$

$$= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} - \frac{322\,\mathrm{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right)}{1331}$$

$$= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} - \frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

Mathematica [A] time = 0.088082, size = 78, normalized size = 0.8

$$\frac{11\left(1428x^3+2716x^2+1799x+278\right)-644(2x+1)^2\sqrt{33x^2+22}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{29282(2x+1)^2\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (11*(278 + 1799*x + 2716*x^2 + 1428*x^3) - 644*(1 + 2*x)^2*Sqrt[22 + 33*x^2
]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(29282*(1 + 2*x)^2*Sqrt[2 + 3*x^2])
```

Maple [A] time = 0.059, size = 107, normalized size = 1.1

$$\frac{7}{484}\left(x+\frac{1}{2}\right)^{-1}\frac{1}{\sqrt{3\,(x+1/2)^2-3\,x+\frac{5}{4}}}+\frac{161}{1331}\frac{1}{\sqrt{3\,(x+1/2)^2-3\,x+\frac{5}{4}}}+\frac{357\,x}{2662}\frac{1}{\sqrt{3\,(x+1/2)^2-3\,x+\frac{5}{4}}}-\frac{322\sqrt{11}}{14641}\mathrm{Ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x)
```

```
[Out] 7/484/(x+1/2)/(3*(x+1/2)^2-3*x+5/4)^(1/2)+161/1331/(3*(x+1/2)^2-3*x+5/4)^(1
/2)+357/2662*x/(3*(x+1/2)^2-3*x+5/4)^(1/2)-322/14641*11^(1/2)*arctanh(2/11*
(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))-1/88/(x+1/2)^2/(3*(x+1/2)^2-3
*x+5/4)^(1/2)
```

Maxima [A] time = 1.53963, size = 167, normalized size = 1.72

$$\frac{322}{14641}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{357\,x}{2662\,\sqrt{3\,x^2+2}}+\frac{161}{1331\,\sqrt{3\,x^2+2}}-\frac{1}{22\left(4\,\sqrt{3\,x^2+2}x^2+4\,\sqrt{3\,x^2+2}x+\sqrt{\ }\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 322/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
 + 1)) + 357/2662*x/sqrt(3*x^2 + 2) + 161/1331/sqrt(3*x^2 + 2) - 1/22/(4*sq
rt(3*x^2 + 2)*x^2 + 4*sqrt(3*x^2 + 2)*x + sqrt(3*x^2 + 2)) + 7/242/(2*sqrt(
3*x^2 + 2)*x + sqrt(3*x^2 + 2))
```

Fricas [A] time = 1.65576, size = 321, normalized size = 3.31

$$\frac{322\,\sqrt{11}\left(12\,x^4+12\,x^3+11\,x^2+8\,x+2\right)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\left(1428\,x^3+2716\,x^2+1799\,x+278\right)\sqrt{3}}{29282\left(12\,x^4+12\,x^3+11\,x^2+8\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/29282*(322*sqrt(11)*(12*x^4 + 12*x^3 + 11*x^2 + 8*x + 2)*log(-(sqrt(11)*s
qrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(142
8*x^3 + 2716*x^2 + 1799*x + 278)*sqrt(3*x^2 + 2))/(12*x^4 + 12*x^3 + 11*x^2
 + 8*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30109, size = 265, normalized size = 2.73

$$\frac{322}{14641}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{36\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2\right.}{1331\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 322/14641*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^
2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2662*(3
51*x + 358)/sqrt(3*x^2 + 2) + 1/1331*(36*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 -
sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 48*sqrt(3)*x + 8*sqrt(3) - 48*sqr
t(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2)) - 2)^2
```

3.130 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{279x+398}{162\left(3x^2+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2+2}-\frac{465x+152}{54\sqrt{3x^2+2}}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

```
[Out] (398 + 279*x)/(162*(2 + 3*x^2)^(3/2)) - (152 + 465*x)/(54*Sqrt[2 + 3*x^2])
+ (32*Sqrt[2 + 3*x^2])/27 + (8*ArcSinh[Sqrt[3/2]*x])/Sqrt[3]
```

Rubi [A] time = 0.0849701, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1814, 641, 215}

$$\frac{279x+398}{162\left(3x^2+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2+2}-\frac{465x+152}{54\sqrt{3x^2+2}}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```


```
[Out] (398 + 279*x)/(162*(2 + 3*x^2)^(3/2)) - (152 + 465*x)/(54*Sqrt[2 + 3*x^2])
+ (32*Sqrt[2 + 3*x^2])/27 + (8*ArcSinh[Sqrt[3/2]*x])/Sqrt[3]
```


Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx &= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{1}{6}\int \frac{\frac{22}{3}-\frac{280x}{3}-144x^2-64x^3}{\left(2+3x^2\right)^{3/2}}\,dx\\ &= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{1}{12}\int \frac{96+\frac{128x}{3}}{\sqrt{2+3x^2}}\,dx\\ &= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{32}{27}\sqrt{2+3x^2}+8\int \frac{1}{\sqrt{2+3x^2}}\,dx\\ &= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{32}{27}\sqrt{2+3x^2}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0738641, size = 63, normalized size = 0.86

$$\frac{1728x^4-4185x^3+936x^2+432\sqrt{3}\left(3x^2+2\right)^{3/2}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)-2511x+254}{162\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (254 - 2511*x + 936*x^2 - 4185*x^3 + 1728*x^4 + 432*Sqrt[3]*(2 + 3*x^2)^(3/
2)*ArcSinh[Sqrt[3/2]*x])/(162*(2 + 3*x^2)^(3/2))
```

Maple [A] time = 0.062, size = 91, normalized size = 1.3

$$\frac{32\,x^4}{3}\left(3\,x^2+2\right)^{-\frac{3}{2}}+\frac{52\,x^2}{9}\left(3\,x^2+2\right)^{-\frac{3}{2}}+\frac{127}{81}\left(3\,x^2+2\right)^{-\frac{3}{2}}-8\,\frac{x^3}{\left(3\,x^2+2\right)^{3/2}}-\frac{107\,x}{18}\frac{1}{\sqrt{3\,x^2+2}}+\frac{8\,\sqrt{3}}{3}\mathrm{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

```
[Out] 32/3*x^4/(3*x^2+2)^(3/2)+52/9*x^2/(3*x^2+2)^(3/2)+127/81/(3*x^2+2)^(3/2)-8*
x^3/(3*x^2+2)^(3/2)-107/18*x/(3*x^2+2)^(1/2)+8/3*arcsinh(1/2*x*6^(1/2))*3^(
1/2)-65/18*x/(3*x^2+2)^(3/2)
```

Maxima [A] time = 1.50444, size = 142, normalized size = 1.95

$$\frac{32\,x^4}{3\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{8}{3}\,x\left(\frac{9\,x^2}{\left(3\,x^2+2\right)^{\frac{3}{2}}}+\frac{4}{\left(3\,x^2+2\right)^{\frac{3}{2}}}\right)+\frac{8}{3}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right)-\frac{11\,x}{18\,\sqrt{3\,x^2+2}}+\frac{52\,x^2}{9\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{65\,x}{18\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 + 2)^(3/2) - 8/3*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)
^(3/2)) + 8/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 11/18*x/sqrt(3*x^2 + 2) + 52
/9*x^2/(3*x^2 + 2)^(3/2) - 65/18*x/(3*x^2 + 2)^(3/2) + 127/81/(3*x^2 + 2)^(
3/2)
```

Fricas [A] time = 1.54365, size = 232, normalized size = 3.18

$$\frac{216\,\sqrt{3}\left(9\,x^4+12\,x^2+4\right)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+\left(1728\,x^4-4185\,x^3+936\,x^2-2511\,x+254\right)\sqrt{3\,x^2+2}}{162\left(9\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/162*(216*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*
x^2 - 1) + (1728*x^4 - 4185*x^3 + 936*x^2 - 2511*x + 254)*sqrt(3*x^2 + 2))/
(9*x^4 + 12*x^2 + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^3 \left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

Giac [A] time = 1.21445, size = 72, normalized size = 0.99

$$-\frac{8}{3}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)+\frac{9\,((3\,(64\,x-155)x+104)x-279)x+254}{162\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -8/3*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(9*((3*(64*x - 155)*
x + 104)*x - 279)*x + 254)/(3*x^2 + 2)^(3/2)
```

3.131 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=60

$$\frac{70-47x}{54\left(3x^2+2\right)^{3/2}}-\frac{59x+168}{54\sqrt{3x^2+2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

```
[Out] (70 - 47*x)/(54*(2 + 3*x^2)^(3/2)) - (168 + 59*x)/(54*Sqrt[2 + 3*x^2]) + (1
6*ArcSinh[Sqrt[3/2]*x])/(9*Sqrt[3])
```

Rubi [A] time = 0.0752021, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1814, 12, 215}

$$\frac{70-47x}{54\left(3x^2+2\right)^{3/2}}-\frac{59x+168}{54\sqrt{3x^2+2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (70 - 47*x)/(54*(2 + 3*x^2)^(3/2)) - (168 + 59*x)/(54*Sqrt[2 + 3*x^2]) + (1
6*ArcSinh[Sqrt[3/2]*x])/(9*Sqrt[3])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{1}{6}\int \frac{-\frac{74}{9}-56x-32x^2}{\left(2+3x^2\right)^{3/2}}\,dx\\ &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{1}{12}\int \frac{64}{3\sqrt{2+3x^2}}\,dx\\ &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16}{9}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\ &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0527414, size = 58, normalized size = 0.97

$$\frac{-177x^3-504x^2+32\sqrt{3}\left(3x^2+2\right)^{3/2}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)-165x-266}{54\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (-266 - 165*x - 504*x^2 - 177*x^3 + 32*Sqrt[3]*(2 + 3*x^2)^(3/2)*ArcSinh[Sq
rt[3/2]*x])/(54*(2 + 3*x^2)^(3/2))
```

Maple [A] time = 0.057, size = 77, normalized size = 1.3

$$-\frac{16x^3}{9}\left(3x^2+2\right)^{-\frac{3}{2}}-\frac{x}{2}\frac{1}{\sqrt{3x^2+2}}+\frac{16\sqrt{3}}{27}\mathrm{Arcsinh}\left(\frac{x\sqrt{6}}{2}\right)-\frac{28x^2}{3}\left(3x^2+2\right)^{-\frac{3}{2}}-\frac{133}{27}\left(3x^2+2\right)^{-\frac{3}{2}}-\frac{37x}{18}\left(3x^2+2\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

```
[Out] -16/9*x^3/(3*x^2+2)^(3/2)-1/2*x/(3*x^2+2)^(1/2)+16/27*arcsinh(1/2*x*6^(1/2)
)*3^(1/2)-28/3*x^2/(3*x^2+2)^(3/2)-133/27/(3*x^2+2)^(3/2)-37/18*x/(3*x^2+2)
^(3/2)
```

Maxima [B] time = 1.46374, size = 123, normalized size = 2.05

$$-\frac{16}{27}x\left(\frac{9x^2}{(3x^2+2)^{\frac{3}{2}}}+\frac{4}{(3x^2+2)^{\frac{3}{2}}}\right)+\frac{16}{27}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)+\frac{37x}{54\sqrt{3x^2+2}}-\frac{28x^2}{3(3x^2+2)^{\frac{3}{2}}}-\frac{37x}{18(3x^2+2)^{\frac{3}{2}}}-\frac{13}{27(3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -16/27*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)) + 16/27*sqrt(3)*ar
csinh(1/2*sqrt(6)*x) + 37/54*x/sqrt(3*x^2 + 2) - 28/3*x^2/(3*x^2 + 2)^(3/2)
 - 37/18*x/(3*x^2 + 2)^(3/2) - 133/27/(3*x^2 + 2)^(3/2)
```

Fricas [A] time = 1.58438, size = 212, normalized size = 3.53

$$\frac{16\sqrt{3}(9x^4+12x^2+4)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)-(177x^3+504x^2+165x+266)\sqrt{3x^2+2}}{54(9x^4+12x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(16*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^
2 - 1) - (177*x^3 + 504*x^2 + 165*x + 266)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2
 + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2x+1)^2\left(4x^2+3x+1\right)}{(3x^2+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

Giac [A] time = 1.27473, size = 65, normalized size = 1.08

$$-\frac{16}{27}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)-\frac{3\left(\left(59x+168\right)x+55\right)x+266}{54\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -16/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/54*(3*((59*x + 168)*x
+ 55)*x + 266)/(3*x^2 + 2)^(3/2)
```

3.132 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}} \, dx$

Optimal. Leaf size=41

$$\frac{2-51x}{54\left(3x^2+2\right)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

```
[Out] (2 - 51*x)/(54*(2 + 3*x^2)^(3/2)) - (16 - 13*x)/(18*Sqrt[2 + 3*x^2])
```

Rubi [A] time = 0.0533089, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1814, 637}

$$\frac{2-51x}{54\left(3x^2+2\right)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (2 - 51*x)/(54*(2 + 3*x^2)^(3/2)) - (16 - 13*x)/(18*Sqrt[2 + 3*x^2])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{2-51x}{54\left(2+3x^2\right)^{3/2}} - \frac{1}{6}\int \frac{-\frac{26}{3}-16x}{\left(2+3x^2\right)^{3/2}}\,dx$$
$$= \frac{2-51x}{54\left(2+3x^2\right)^{3/2}} - \frac{16-13x}{18\sqrt{2+3x^2}}$$

Mathematica [A] time = 0.018033, size = 30, normalized size = 0.73

$$\frac{117x^3-144x^2+27x-94}{54\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (-94 + 27*x - 144*x^2 + 117*x^3)/(54*(2 + 3*x^2)^(3/2))
```

Maple [A] time = 0.049, size = 27, normalized size = 0.7

$$\frac{117\,x^3-144\,x^2+27\,x-94}{54}\left(3\,x^2+2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

```
[Out] 1/54*(117*x^3-144*x^2+27*x-94)/(3*x^2+2)^(3/2)
```

Maxima [A] time = 0.970827, size = 68, normalized size = 1.66

$$\frac{13\,x}{18\,\sqrt{3\,x^2+2}} - \frac{8\,x^2}{3\left(3\,x^2+2\right)^{\frac{3}{2}}} - \frac{17\,x}{18\left(3\,x^2+2\right)^{\frac{3}{2}}} - \frac{47}{27\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 13/18*x/sqrt(3*x^2 + 2) - 8/3*x^2/(3*x^2 + 2)^(3/2) - 17/18*x/(3*x^2 + 2)^(
3/2) - 47/27/(3*x^2 + 2)^(3/2)
```

Fricas [A] time = 1.5486, size = 101, normalized size = 2.46

$$\frac{\left(117\,x^3-144\,x^2+27\,x-94\right)\sqrt{3\,x^2+2}}{54\left(9\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(117*x^3 - 144*x^2 + 27*x - 94)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 + 4)
```

Sympy [B] time = 111.998, size = 180, normalized size = 4.39

$$\frac{10x^3}{18x^2\sqrt{3x^2+2}+12\sqrt{3x^2+2}}+\frac{x^3}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}-\frac{72x^2}{81x^2\sqrt{3x^2+2}+54\sqrt{3x^2+2}}+\frac{x}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] 10*x**3/(18*x**2*sqrt(3*x**2 + 2) + 12*sqrt(3*x**2 + 2)) + x**3/(6*x**2*sqr
t(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 72*x**2/(81*x**2*sqrt(3*x**2 + 2) + 5
4*sqrt(3*x**2 + 2)) + x/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 32
/(81*x**2*sqrt(3*x**2 + 2) + 54*sqrt(3*x**2 + 2)) - 5/(27*x**2*sqrt(3*x**2
+ 2) + 18*sqrt(3*x**2 + 2))
```

Giac [A] time = 1.25106, size = 34, normalized size = 0.83

$$\frac{9\,((13\,x-16)x+3)x-94}{54\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/54*(9*((13*x - 16)*x + 3)*x - 94)/(3*x^2 + 2)^(3/2)
```

3.133 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}+\frac{95x+24}{726\sqrt{3x^2+2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

```
[Out] -(38 - 21*x)/(198*(2 + 3*x^2)^(3/2)) + (24 + 95*x)/(726*Sqrt[2 + 3*x^2]) -
(8*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rubi [A] time = 0.0850621, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 823, 12, 725, 206}

$$-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}+\frac{95x+24}{726\sqrt{3x^2+2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] -(38 - 21*x)/(198*(2 + 3*x^2)^(3/2)) + (24 + 95*x)/(726*Sqrt[2 + 3*x^2]) -
(8*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx &= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int \frac{-\frac{78}{11}-\frac{84x}{11}}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{\int \frac{864}{11(1+2x)\sqrt{2+3x^2}}\,dx}{1188}\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{8}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8}{121}\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}
$$

Mathematica [A] time = 0.0514794, size = 58, normalized size = 0.79

$$\frac{855x^3+216x^2+801x-274}{2178\left(3x^2+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (-274 + 801*x + 216*x^2 + 855*x^3)/(2178*(2 + 3*x^2)^(3/2)) - (8*ArcTanh[(4
 - 3*x)/Sqrt[22 + 33*x^2]])/(121*Sqrt[11])
```

Maple [B] time = 0.056, size = 133, normalized size = 1.8

$$-\frac{2}{9}\left(3x^2+2\right)^{-\frac{3}{2}}+\frac{x}{12}\left(3x^2+2\right)^{-\frac{3}{2}}+\frac{x}{12}\frac{1}{\sqrt{3x^2+2}}+\frac{1}{33}\left(3\left(x+1/2\right)^2-3x+\frac{5}{4}\right)^{-\frac{3}{2}}+\frac{x}{44}\left(3\left(x+1/2\right)^2-3x+\frac{5}{4}\right)^{-\frac{3}{2}}+\frac{2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x)
```

```
[Out] -2/9/(3*x^2+2)^(3/2)+1/12*x/(3*x^2+2)^(3/2)+1/12*x/(3*x^2+2)^(1/2)+1/33/(3*
(x+1/2)^2-3*x+5/4)^(3/2)+1/44*x/(3*(x+1/2)^2-3*x+5/4)^(3/2)+23/484*x/(3*(x+
1/2)^2-3*x+5/4)^(1/2)+4/121/(3*(x+1/2)^2-3*x+5/4)^(1/2)-8/1331*11^(1/2)*arc
tanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Maxima [A] time = 1.49518, size = 109, normalized size = 1.49

$$\frac{8}{1331}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{95\,x}{726\sqrt{3\,x^2+2}}+\frac{4}{121\sqrt{3\,x^2+2}}+\frac{7\,x}{66\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{19}{99\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
1)) + 95/726*x/sqrt(3*x^2 + 2) + 4/121/sqrt(3*x^2 + 2) + 7/66*x/(3*x^2 + 2)
```

```
^(3/2) - 19/99/(3*x^2 + 2)^(3/2)
```

Fricas [A] time = 1.56274, size = 273, normalized size = 3.74

$$\frac{72\sqrt{11}\left(9x^4+12x^2+4\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(855x^3+216x^2+801x-274\right)\sqrt{3x^2+2}}{23958\left(9x^4+12x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23958*(72*sqrt(11)*(9*x^4 + 12*x^2 + 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3
*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(855*x^3 + 216*x^2 +
801*x - 274)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27288, size = 123, normalized size = 1.68

$$\frac{8}{1331}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{9((95x+24)x+89)x-274}{2178\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2178*(9*((
95*x + 24)*x + 89)*x - 274)/(3*x^2 + 2)^(3/2)
```

3.134 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{10-97x}{726\left(3x^2+2\right)^{3/2}}-\frac{16\sqrt{3x^2+2}}{1331(2x+1)}+\frac{887x+24}{7986\sqrt{3x^2+2}}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

```
[Out] -(10 - 97*x)/(726*(2 + 3*x^2)^(3/2)) + (24 + 887*x)/(7986*Sqrt[2 + 3*x^2])
- (16*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (32*ArcTanh[(4 - 3*x)/(Sqrt[11]*S
qrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rubi [A] time = 0.169063, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 807, 725, 206}

$$-\frac{10-97x}{726\left(3x^2+2\right)^{3/2}}-\frac{16\sqrt{3x^2+2}}{1331(2x+1)}+\frac{887x+24}{7986\sqrt{3x^2+2}}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```


```
[Out] -(10 - 97*x)/(726*(2 + 3*x^2)^(3/2)) + (24 + 887*x)/(7986*Sqrt[2 + 3*x^2])
- (16*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (32*ArcTanh[(4 - 3*x)/(Sqrt[11]*S
qrt[2 + 3*x^2])])/(1331*Sqrt[11])
```


Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx &= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int \frac{-\frac{798}{121}-\frac{1968x}{121}-\frac{2328x^2}{121}}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}+\frac{1}{108}\int \frac{\frac{10368}{1331}+\frac{1728x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}+\frac{32\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{1331}\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)}{1331}\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}
\end{aligned}$$

Mathematica [A] time = 0.0632395, size = 91, normalized size = 0.96

$$\frac{11\left(4458x^4+2805x^3+4602x^2+2717x-446\right)-192\sqrt{33x^2+22}\left(6x^3+3x^2+4x+2\right)\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{87846(2x+1)\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (11*(-446 + 2717*x + 4602*x^2 + 2805*x^3 + 4458*x^4) - 192*Sqrt[22 + 33*x^2
]*(2 + 4*x + 3*x^2 + 6*x^3)*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(87846*(1
 + 2*x)*(2 + 3*x^2)^(3/2))
```

Maple [A] time = 0.059, size = 143, normalized size = 1.5

$$\frac{x}{6}\left(3x^2+2\right)^{-\frac{3}{2}}+\frac{x}{6}\frac{1}{\sqrt{3x^2+2}}-\frac{1}{22}\left(x+\frac{1}{2}\right)^{-1}\left(3\left(x+1/2\right)^2-3x+\frac{5}{4}\right)^{-\frac{3}{2}}+\frac{4}{363}\left(3\left(x+1/2\right)^2-3x+\frac{5}{4}\right)^{-\frac{3}{2}}-\frac{10x}{121}\left(3\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x)
```

```
[Out] 1/6*x/(3*x^2+2)^(3/2)+1/6*x/(3*x^2+2)^(1/2)-1/22/(x+1/2)/(3*(x+1/2)^2-3*x+5
/4)^(3/2)+4/363/(3*(x+1/2)^2-3*x+5/4)^(3/2)-10/121*x/(3*(x+1/2)^2-3*x+5/4)^
(3/2)-98/1331*x/(3*(x+1/2)^2-3*x+5/4)^(1/2)+16/1331/(3*(x+1/2)^2-3*x+5/4)^(
1/2)-32/14641*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^
(1/2))
```

Maxima [A] time = 1.50504, size = 144, normalized size = 1.52

$$\frac{32}{14641}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\left|2x+1\right|}-\frac{2\sqrt{6}}{3\left|2x+1\right|}\right)+\frac{743x}{7986\sqrt{3x^2+2}}+\frac{16}{1331\sqrt{3x^2+2}}+\frac{61x}{726\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{1}{11\left(2\left(3x^2+2\right)^{\frac{3}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) + 743/7986*x/sqrt(3*x^2 + 2) + 16/1331/sqrt(3*x^2 + 2) + 61/726*x/(3*
x^2 + 2)^(3/2) - 1/11/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)) + 4/363/(
3*x^2 + 2)^(3/2)
```

Fricas [A] time = 1.65366, size = 356, normalized size = 3.75

$$\frac{96\sqrt{11}\left(18x^5+9x^4+24x^3+12x^2+8x+4\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(4458x^4+2805x^3+4602x^2+2\right.}{87846\left(18x^5+9x^4+24x^3+12x^2+8x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/87846*(96*sqrt(11)*(18*x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)*log(-(sqr
t(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) +
11*(4458*x^4 + 2805*x^3 + 4602*x^2 + 2717*x - 446)*sqrt(3*x^2 + 2))/(18*x^5
 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2+3x+1}{\left(3x^2+2\right)^{\frac{5}{2}}(2x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*x + 1)/((3*x^2 + 2)^(5/2)*(2*x + 1)^2), x)
```

3.135 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{351x+358}{7986\left(3x^2+2\right)^{3/2}}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)^2}+\frac{2133x+1216}{29282\sqrt{3x^2+2}}-\frac{1216\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{14641\sqrt{11}}$$

```
[Out] (358 + 351*x)/(7986*(2 + 3*x^2)^(3/2)) + (1216 + 2133*x)/(29282*Sqrt[2 + 3*
x^2]) - (8*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)^2) - (8*Sqrt[2 + 3*x^2])/(1331*
(1 + 2*x)) - (1216*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(14641*Sq
rt[11])
```

Rubi [A] time = 0.20628, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 1651, 807, 725, 206}

$$\frac{351x+358}{7986\left(3x^2+2\right)^{3/2}}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)^2}+\frac{2133x+1216}{29282\sqrt{3x^2+2}}-\frac{1216\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{14641\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (358 + 351*x)/(7986*(2 + 3*x^2)^(3/2)) + (1216 + 2133*x)/(29282*Sqrt[2 + 3*
x^2]) - (8*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)^2) - (8*Sqrt[2 + 3*x^2])/(1331*
(1 + 2*x)) - (1216*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(14641*Sq
rt[11])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} - \frac{1}{18}\int \frac{-\frac{10926}{1331}-\frac{3132x}{121}-\frac{51048x^2}{1331}-\frac{16848x^3}{1331}}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} + \frac{1}{108}\int \frac{\frac{245376}{14641}+\frac{544320x}{14641}+\frac{525312x^2}{14641}}{(1+2x)^3\sqrt{2+3x^2}}\,dx$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{\int \frac{-\frac{338688}{1331}-\frac{468288x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}\,dx}{2376}$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} + \frac{1216\int \frac{1}{(1+2x)\sqrt{2+}}}{14641}$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216\,\mathrm{Subst}\left(\int \frac{}{1}\right.}{14}$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{14641\sqrt{1}}$$

Mathematica [A] time = 0.105104, size = 75, normalized size = 0.64

$$\frac{\frac{11\left(67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010\right)}{(2x+1)^2\left(3x^2+2\right)^{3/2}} - 7296\sqrt{11}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{966306}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] ((11*(7010 + 57371*x + 109844*x^2 + 116937*x^3 + 111060*x^4 + 67284*x^5))/(
(1 + 2*x)^2*(2 + 3*x^2)^(3/2)) - 7296*Sqrt[11]*ArcTanh[(4 - 3*x)/Sqrt[22 +
33*x^2]])/966306
```

Maple [A] time = 0.063, size = 140, normalized size = 1.2

$$\frac{1}{484}\left(x+\frac{1}{2}\right)^{-1}\left(3\,(x+1/2)^2-3\,x+\frac{5}{4}\right)^{-\frac{3}{2}} + \frac{152}{3993}\left(3\,(x+1/2)^2-3\,x+\frac{5}{4}\right)^{-\frac{3}{2}} + \frac{87\,x}{2662}\left(3\,(x+1/2)^2-3\,x+\frac{5}{4}\right)^{-\frac{3}{2}} + \frac{186}{292}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x)
```

```
[Out] 1/484/(x+1/2)/(3*(x+1/2)^2-3*x+5/4)^(3/2)+152/3993/(3*(x+1/2)^2-3*x+5/4)^(3
/2)+87/2662*x/(3*(x+1/2)^2-3*x+5/4)^(3/2)+1869/29282*x/(3*(x+1/2)^2-3*x+5/4
)^(1/2)+608/14641/(3*(x+1/2)^2-3*x+5/4)^(1/2)-1216/161051*11^(1/2)*arctanh(
2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))-1/88/(x+1/2)^2/(3*(x+1/2
)^2-3*x+5/4)^(3/2)
```

Maxima [A] time = 1.52314, size = 198, normalized size = 1.69

$$\frac{1216}{161051}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{1869\,x}{29282\,\sqrt{3\,x^2+2}}+\frac{608}{14641\,\sqrt{3\,x^2+2}}+\frac{87\,x}{2662\,\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{}{22\left(4\left(3\,x^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1216/161051*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2
*x + 1)) + 1869/29282*x/sqrt(3*x^2 + 2) + 608/14641/sqrt(3*x^2 + 2) + 87/26
62*x/(3*x^2 + 2)^(3/2) - 1/22/(4*(3*x^2 + 2)^(3/2)*x^2 + 4*(3*x^2 + 2)^(3/2
)*x + (3*x^2 + 2)^(3/2)) + 1/242/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)
) + 152/3993/(3*x^2 + 2)^(3/2)
```

Fricas [A] time = 1.70003, size = 417, normalized size = 3.56

$$\frac{3648\,\sqrt{11}\left(36\,x^6+36\,x^5+57\,x^4+48\,x^3+28\,x^2+16\,x+4\right)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\left(67284\,x^5+111060\right.}{966306\left(36\,x^6+36\,x^5+57\,x^4+48\,x^3+28\,x^2+16\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/966306*(3648*sqrt(11)*(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16*x
+ 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2
+ 4*x + 1)) + 11*(67284*x^5 + 111060*x^4 + 116937*x^3 + 109844*x^2 + 57371*
```

```
x + 7010)*sqrt(3*x^2 + 2))/(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16
*x + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32507, size = 247, normalized size = 2.11

$$\frac{1216}{161051}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{9\left(\left(2133x+1216\right)x+1851\right)x+11234}{87846\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{4\left(\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2\right.\right.}{1331\left(\left(\sqrt{3}x-\sqrt{3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1216/161051*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*
x^2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/87846
*(9*((2133*x + 1216)*x + 1851)*x + 11234)/(3*x^2 + 2)^(3/2) + 4/1331*(sqrt(
3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 24*sqrt(3)*x - 8*sqrt(3) - 24*sqrt(3*x
^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2
 + 2)) - 2)^2
```

3.136 $\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=420

$$-\frac{\left(a+cx^2\right)^p (g+hx)^{m+1}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(afh^2(m+1)-c\left(2fg^2(p\right.\right.}{ch^3(m+1)(m+2p+3)}$$

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) - ((a*f*h^2*(
1 + m) - c*(2*f*g^2*(1 + p) - h*(e*g - d*h)*(3 + m + 2*p)))*(g + h*x)^(1 +
m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)
/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(3 + m + 2
*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[
-a]*h)/Sqrt[c]))^p) - ((2*f*g*(1 + p) - e*h*(3 + m + 2*p))*(g + h*x)^(2 + m
)*(a + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/
Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(3 + m + 2*p)
*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[-a]
*h)/Sqrt[c]))^p)
```

Rubi [A] time = 0.604235, antiderivative size = 417, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1654, 844, 760, 133}

$$-\frac{\left(a+cx^2\right)^p (g+hx)^{m+1}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(afh^2(m+1)+ch(m+2\right.}{ch^3(m+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) - ((a*f*h^2*(
1 + m) - 2*c*f*g^2*(1 + p) + c*h*(e*g - d*h)*(3 + m + 2*p))*(g + h*x)^(1 +
m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)
/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(3 + m + 2
*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[
-a]*h)/Sqrt[c]))^p) - ((2*f*g*(1 + p) - e*h*(3 + m + 2*p))*(g + h*x)^(2 + m
)*(a + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/
Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(3 + m + 2*p)
*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[-a]
```

```
*h)/Sqrt[c]))^p)
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\int (g+hx)^m \left(-h^2(af(1+m)-cd(3+m+2p))\right.}{ch^2(3}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\int (g+hx)^{1+m}\left(a+cx^2\right)^p dx}{h^2} -$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\right.\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{\left(afh^2(1+m)-2cfg^2(1+p)+ch(eg-dh)(3+\right.}{}$$

Mathematica [F] time = 1.19881, size = 0, normalized size = 0.

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F] time = 0.796, size = 0, normalized size = 0.

$$\int \left(hx+g\right)^m \left(cx^2+a\right)^p \left(fx^2+ex+d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

```
[Out] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^2 + ex + d)(cx^2 + a)^p (hx + g)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((fx^2 + ex + d)(cx^2 + a)^p (hx + g)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(c*x**2+a)**p*(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^2 + ex + d)(cx^2 + a)^p (hx + g)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")

[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

3.137 $\int (g+hx)^m \sqrt{a+cx^2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=403

$$-\frac{\sqrt{a+cx^2}(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(afh^2(m+1)-c\left(3fg^2-h(m+4)(eg-dh)\right)\right)}{ch^3(m+1)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}}} - \sqrt{a+$$

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(3/2))/(c*h*(4 + m)) - ((a*f*h^2*(1 + m) -
 c*(3*f*g^2 - h*(e*g - d*h)*(4 + m)))*(g + h*x)^(1 + m)*Sqrt[a + c*x^2]*App
ellF1[1 + m, -1/2, -1/2, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g +
h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (g + h*x)
/(g - (Sqrt[-a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
) - ((3*f*g - e*h*(4 + m))*(g + h*x)^(2 + m)*Sqrt[a + c*x^2]*AppellF1[2 + m
, -1/2, -1/2, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (
Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(4 + m)*Sqrt[1 - (g + h*x)/(g - (Sqrt[-
a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])
```

Rubi [A] time = 0.453214, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1654, 844, 760, 133}

$$\frac{\sqrt{a+cx^2}(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(-afh^2(m+1)-ch(m+4)(eg-dh)+3cfg^2\right)}{ch^3(m+1)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}}} - \sqrt{a+c}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 - a*f*h
^2*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[a + c*x^2]*App
ellF1[1 + m, -1/2, -1/2, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g +
h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (g + h*x)
/(g - (Sqrt[-a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
) - ((3*f*g - e*h*(4 + m))*(g + h*x)^(2 + m)*Sqrt[a + c*x^2]*AppellF1[2 + m
, -1/2, -1/2, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (
```

```
Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(4 + m)*Sqrt[1 - (g + h*x)/(g - (Sqrt[-
a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\int (g+hx)^m\left(-h^2(af(1+m)-cd(4+m))-ch(3\right.}{ch^2(4+m)}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{(3fg-eh(4+m))\int (g+hx)^{1+m}\sqrt{a+cx^2}\,dx}{h^2(4+m)} + $$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{\left((3fg-eh(4+m))\sqrt{a+cx^2}\right)\text{Subst}\left(\int x^{1+m}\sqrt{1\ldots}\right.}{h^3(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\ldots}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(3cfg^2-afh^2(1+m)-ch(eg-dh)(4+m)\right)(g\ldots}{ch^3(1+m)(4+\ldots}$$

Mathematica [F] time = 0.725941, size = 0, normalized size = 0.

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2), x]
```

Maple [F] time = 0.742, size = 0, normalized size = 0.

$$\int \left(hx+g\right)^m\left(fx^2+ex+d\right)\sqrt{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+cx^2}\left(g+hx\right)^m\left(d+ex+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(g + h*x)**m*(d + e*x + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

3.138 $\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=474

$$-\frac{f\left(a+cx^2\right)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{2h^3p}+\frac{\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)}{}$$

```
[Out] -((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(1 + p))/(2*h*(c*g^2 + a*h^2)*(1 + p)
*(g + h*x)^(2*(1 + p))) - (f*(a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p,
(g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
)/(2*h^3*p*(g + h*x)^(2*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1
- (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c]))^p) + ((a*h^2*(2*f*g - e*h) + c*(f*g
^3 - d*g*h^2))*(Sqrt[-a] - Sqrt[c]*x)*(g + h*x)^(-1 - 2*p)*(a + c*x^2)^p*Hy
pergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(g + h*x))/((Sqrt[c
]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))])/(h^2*(Sqrt[c]*g + Sqrt[-a]*h)*(
c*g^2 + a*h^2)*(1 + 2*p)*(-(((Sqrt[c]*g + Sqrt[-a]*h)*(Sqrt[-a] + Sqrt[c]*x
))/((Sqrt[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rubi [A] time = 0.519098, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1656, 760, 133, 807, 727}

$$-\frac{f\left(a+cx^2\right)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{2h^3p}+\frac{\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(1 + p))/(2*h*(c*g^2 + a*h^2)*(1 + p)
*(g + h*x)^(2*(1 + p))) - (f*(a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p,
(g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
)/(2*h^3*p*(g + h*x)^(2*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1
- (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c]))^p) + ((a*h^2*(2*f*g - e*h) + c*(f*g
^3 - d*g*h^2))*(Sqrt[-a] - Sqrt[c]*x)*(g + h*x)^(-1 - 2*p)*(a + c*x^2)^p*Hy
pergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(g + h*x))/((Sqrt[c
]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))])/(h^2*(Sqrt[c]*g + Sqrt[-a]*h)*(
```

```
c*g^2 + a*h^2)*(1 + 2*p)*(-(((Sqrt[c]*g + Sqrt[-a]*h)*(Sqrt[-a] + Sqrt[c]*x
))/((Sqrt[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rule 1656

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d + e*x)^(m + q)
*(a + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(a + c*x^2)^p*ExpandTo
Sum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x], x]] /; FreeQ[{a, c, d, e,
 m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(IGtQ[m, 0] && Rat
ionalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 727

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((Rt[-(a*c), 2] - c*x)*(d + e*x)^(m + 1)*(a + c*x^2)^p*Hypergeometric2F1[m
+ 1, -p, m + 2, (2*c*Rt[-(a*c), 2]*(d + e*x))/((c*d - e*Rt[-(a*c), 2])*(Rt[
-(a*c), 2] - c*x))])/((m + 1)*(c*d + e*Rt[-(a*c), 2])*(((c*d + e*Rt[-(a*c),
 2])*(Rt[-(a*c), 2] + c*x))/((c*d - e*Rt[-(a*c), 2])*(-Rt[-(a*c), 2] + c*x)
))^p), x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx = \frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)\left(a+cx^2\right)^p dx}{h^2}+\frac{f\int(g+hx$$

$$= -\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)}-\frac{\left(ah^2(2fg-eh)+c\left(fg\right.\right.}{}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)}-\frac{f(g+hx)^{-2p}\left(a+cx^2\right)}{}$$

Mathematica [F] time = 2.91037, size = 0, normalized size = 0.

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]

[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F] time = 0.754, size = 0, normalized size = 0.

$$\int \left(hx+g\right)^{-3-2p}\left(cx^2+a\right)^p\left(fx^2+ex+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)

[Out] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^{-2p-3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^{-2p-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+a)**p*(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

3.139 $\int (d+ex)^m \left(-cd^2 + bde + be^2x + ce^2x^2\right)^p \left(-(cd - be)f + (ce$

Optimal. Leaf size=222

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)} - \frac{(d+ex)^m(-be+cd-cex)^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(b}{c^2e^2(p}$$

```
[Out] (g*(d + e*x)^(-1 + m)*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^(2 + p))/(c*
e^2*(3 + m + 2*p)) - ((b*e*g*(1 + m + p) + c*(d*g*(1 - m) - e*f*(3 + m + 2*
p)))*(d + e*x)^m*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*(c*d - b*e - c*e*x)
^2*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^p*Hypergeometric2F1[-m - p, 2 +
 p, 3 + p, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(c^2*e^2*(2 + p)*(3 + m + 2*
p))
```

Rubi [A] time = 0.40739, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 70, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1632, 794, 679, 677, 70, 69}

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)} - \frac{(d+ex)^m(-be+cd-cex)^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(b}{c^2e^2(p}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*(-((c*d - b*e)*f
) + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```


```
[Out] (g*(d + e*x)^(-1 + m)*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^(2 + p))/(c*
e^2*(3 + m + 2*p)) - ((b*e*g*(1 + m + p) + c*(d*g*(1 - m) - e*f*(3 + m + 2*
p)))*(d + e*x)^m*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*(c*d - b*e - c*e*x)
^2*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^p*Hypergeometric2F1[-m - p, 2 +
 p, 3 + p, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(c^2*e^2*(2 + p)*(3 + m + 2*
p))
```


Rule 1632

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e
 + c*d*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
```

```
0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m
], Int[(1 + (e*x)/d)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !Intege
rQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left(-(cd-be)f+(cef-cdg+beg)x+cegx^2\right)\,dx=(de)\int (d+ex)^{-1+m}\left(\frac{f}{de}+\frac{gx}{de}\right.$$

$$=\frac{g(d+ex)^{-1+m}\left(-d(cd-be)+\right.}{ce^2(3+m+2}$$

$$=\frac{g(d+ex)^{-1+m}\left(-d(cd-be)+\right.}{ce^2(3+m+2}$$

$$=\frac{g(d+ex)^{-1+m}\left(-d(cd-be)+\right.}{ce^2(3+m+2}$$

$$=\frac{g(d+ex)^{-1+m}\left(-d(cd-be)+\right.}{ce^2(3+m+2}$$

$$=\frac{g(d+ex)^{-1+m}\left(-d(cd-be)+\right.}{ce^2(3+m+2}$$

Mathematica [A] time = 0.361065, size = 165, normalized size = 0.74

$$\frac{(d+ex)^m(be-cd+cex)^2(-(d+ex)(c(d-ex)-be))^p\left(\frac{e\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(-beg(m+p+1)+cdg(m-1)+cef(m+2p+3))\,{}_2F_1\left(-m-p,p+2;p+3;\frac{-cd+be+cex}{be-2cd}\right.}{p+2}\right.}{c^2e^3(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*(-((c*d -
b*e)*f) + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

```
[Out] ((d + e*x)^m*(-(c*d) + b*e + c*e*x)^2*(-((d + e*x)*(-(b*e) + c*(d - e*x))))
^p*(c*e*g*(d + e*x) + (e*(c*d*g*(-1 + m) - b*e*g*(1 + m + p) + c*e*f*(3 + m
 + 2*p))*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*Hypergeometric2F1[-m - p, 2
 + p, 3 + p, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)])/(2 + p)))/(c^2*e^3*(3
+ m + 2*p))
```

Maple [F] time = 1.615, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(ce^2x^2+xbe^2+bde-cd^2\right)^p \left(-(-be+cd)\, f+\left(beg-cdg+cef\right) x+cegx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

```
[Out] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cegx^2-(cd-be)f+\left(cef-cdg+beg\right)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p (ex+d)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="maxima")
```

```
[Out] integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cegx^2-(cd-be)f+\left(cef-(cd-be)g\right)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="fricas")
```

```
[Out] integral((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - (c*d - b*e)*g)*x)*(c*e^2*x^2
 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+b*e**2*x+b*d*e-c*d**2)**p*(-(-b*e+c*d)*f+
(b*e*g-c*d*g+c*e*f)*x+c*e*g*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cegx^2 - (cd - be)f + \left(cef - cdg + beg\right)x\right)\left(ce^2x^2 + be^2x - cd^2 + bde\right)^p (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="giac")
```

```
[Out] integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

3.140 $\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx$

Optimal. Leaf size=254

$$\frac{1}{7}x^7\left(C\left(6a^2c^2+12ab^2c+b^4\right)+2Ac^2\left(2ac+3b^2\right)\right)+\frac{1}{5}x^5\left(A\left(6a^2c^2+12ab^2c+b^4\right)+2a^2C\left(2ac+3b^2\right)\right)+abx^4\left(a^2C+\right.$$

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
(b^2 + 3*a*c) + a^2*C)*x^4 + ((A*(b^4 + 12*a*b^2*c + 6*a^2*c^2) + 2*a^2*(3*
b^2 + 2*a*c)*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((2*A*c^2*
(3*b^2 + 2*a*c) + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*C)*x^7)/7 + (b*c*(A*c^2 +
(b^2 + 3*a*c)*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*b*c^
3*C*x^10)/5 + (c^4*C*x^11)/11
```

Rubi [A] time = 0.325565, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1657}

$$\frac{1}{7}x^7\left(C\left(6a^2c^2+12ab^2c+b^4\right)+2Ac^2\left(2ac+3b^2\right)\right)+\frac{1}{5}x^5\left(A\left(6a^2c^2+12ab^2c+b^4\right)+2a^2C\left(2ac+3b^2\right)\right)+abx^4\left(a^2C+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
(b^2 + 3*a*c) + a^2*C)*x^4 + ((A*(b^4 + 12*a*b^2*c + 6*a^2*c^2) + 2*a^2*(3*
b^2 + 2*a*c)*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((2*A*c^2*
(3*b^2 + 2*a*c) + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*C)*x^7)/7 + (b*c*(A*c^2 +
(b^2 + 3*a*c)*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*b*c^
3*C*x^10)/5 + (c^4*C*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right) dx = \int \left(a^4A+4a^3Abx+a^2\left(6Ab^2+4aAc+a^2C\right)x^2+4ab\left(A\left(b^2+3ac\right)+a^2C\right)x^3+\left(A\right.\right.$$
$$= a^4Ax+2a^3Abx^2+\frac{1}{3}a^2\left(6Ab^2+4aAc+a^2C\right)x^3+ab\left(A\left(b^2+3ac\right)+a^2C\right)x^4+\frac{1}{5}\left(\right.$$

Mathematica [A] time = 0.0970406, size = 256, normalized size = 1.01

$$\frac{1}{7}x^7\left(6a^2c^2C+4aAc^3+12ab^2cC+6Ab^2c^2+b^4C\right)+\frac{1}{5}x^5\left(6a^2Ac^2+6a^2b^2C+4a^3cC+12aAb^2c+Ab^4\right)+abx^4\left(a^2C+3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
b^2 + 3*a*A*c + a^2*C)*x^4 + ((A*b^4 + 12*a*A*b^2*c + 6*a^2*A*c^2 + 6*a^2*b
^2*C + 4*a^3*c*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((6*A*b^
2*c^2 + 4*a*A*c^3 + b^4*C + 12*a*b^2*c*C + 6*a^2*c^2*C)*x^7)/7 + (b*c*(A*c^
2 + b^2*C + 3*a*c*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*
b*c^3*C*x^10)/5 + (c^4*C*x^11)/11
```

Maple [A] time = 0.046, size = 343, normalized size = 1.4

$$\frac{c^4Cx^{11}}{11}+\frac{2\,bc^3Cx^{10}}{5}+\frac{\left(\left(2\,\left(2\,ac+b^2\right)c^2+4\,b^2c^2\right)C+c^4A\right)x^9}{9}+\frac{\left(\left(4\,bac^2+4\,\left(2\,ac+b^2\right)bc\right)C+4\,bc^3A\right)x^8}{8}+\frac{\left(\left(2\,a^2c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4*(C*x^2+A),x)
```

```
[Out] 1/11*c^4*C*x^11+2/5*b*c^3*C*x^10+1/9*((2*(2*a*c+b^2)*c^2+4*b^2*c^2)*C+c^4*A
)*x^9+1/8*((4*b*a*c^2+4*(2*a*c+b^2)*b*c)*C+4*b*c^3*A)*x^8+1/7*((2*a^2*c^2+8
*a*c*b^2+(2*a*c+b^2)^2)*C+(2*(2*a*c+b^2)*c^2+4*b^2*c^2)*A)*x^7+1/6*((4*a^2*
b*c+4*a*b*(2*a*c+b^2))*C+(4*b*a*c^2+4*(2*a*c+b^2)*b*c)*A)*x^6+1/5*((2*a^2*(
2*a*c+b^2)+4*b^2*a^2)*C+(2*a^2*c^2+8*a*c*b^2+(2*a*c+b^2)^2)*A)*x^5+1/4*(4*a
^3*b*C+(4*a^2*b*c+4*a*b*(2*a*c+b^2))*A)*x^4+1/3*(a^4*C+(2*a^2*(2*a*c+b^2)+4
*b^2*a^2)*A)*x^3+2*a^3*A*b*x^2+a^4*A*x
```

Maxima [A] time = 1.00883, size = 355, normalized size = 1.4

$$\frac{1}{11}\,Cc^4x^{11} + \frac{2}{5}\,Cbc^3x^{10} + \frac{1}{9}\left(6\,Cb^2c^2 + 4\,Cac^3 + Ac^4\right)x^9 + \frac{1}{2}\left(Cb^3c + 3\,Cabc^2 + Abc^3\right)x^8 + \frac{1}{7}\left(Cb^4 + 12\,Cab^2c + 4\,Aac^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)*
x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2*c
 + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3 +
3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 + A*b
^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 + 3*A*
a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3
```

Fricas [A] time = 1.22246, size = 718, normalized size = 2.83

$$\frac{1}{11}x^{11}c^4C + \frac{2}{5}x^{10}c^3bC + \frac{2}{3}x^9c^2b^2C + \frac{4}{9}x^9c^3aC + \frac{1}{9}x^9c^4A + \frac{1}{2}x^8cb^3C + \frac{3}{2}x^8c^2baC + \frac{1}{2}x^8c^3bA + \frac{1}{7}x^7b^4C + \frac{12}{7}x^7cb^2aC$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^4*C + 2/5*x^10*c^3*b*C + 2/3*x^9*c^2*b^2*C + 4/9*x^9*c^3*a*C +
1/9*x^9*c^4*A + 1/2*x^8*c*b^3*C + 3/2*x^8*c^2*b*a*C + 1/2*x^8*c^3*b*A + 1/7
*x^7*b^4*C + 12/7*x^7*c*b^2*a*C + 6/7*x^7*c^2*a^2*C + 6/7*x^7*c^2*b^2*A + 4
/7*x^7*c^3*a*A + 2/3*x^6*b^3*a*C + 2*x^6*c*b*a^2*C + 2/3*x^6*c*b^3*A + 2*x^
6*c^2*b*a*A + 6/5*x^5*b^2*a^2*C + 4/5*x^5*c*a^3*C + 1/5*x^5*b^4*A + 12/5*x^
5*c*b^2*a*A + 6/5*x^5*c^2*a^2*A + x^4*b*a^3*C + x^4*b^3*a*A + 3*x^4*c*b*a^2
*A + 1/3*x^3*a^4*C + 2*x^3*b^2*a^2*A + 4/3*x^3*c*a^3*A + 2*x^2*b*a^3*A + x*
a^4*A
```

Sympy [A] time = 0.112548, size = 320, normalized size = 1.26

$$Aa^4x + 2Aa^3bx^2 + \frac{2Cbc^3x^{10}}{5} + \frac{Cc^4x^{11}}{11} + x^9\left(\frac{Ac^4}{9} + \frac{4Cac^3}{9} + \frac{2Cb^2c^2}{3}\right) + x^8\left(\frac{Abc^3}{2} + \frac{3Cabc^2}{2} + \frac{Cb^3c}{2}\right) + x^7\left(\frac{4Aac^3}{7} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4*(C*x**2+A),x)
```

```
[Out] A*a**4*x + 2*A*a**3*b*x**2 + 2*C*b*c**3*x**10/5 + C*c**4*x**11/11 + x**9*(A
*c**4/9 + 4*C*a*c**3/9 + 2*C*b**2*c**2/3) + x**8*(A*b*c**3/2 + 3*C*a*b*c**2
/2 + C*b**3*c/2) + x**7*(4*A*a*c**3/7 + 6*A*b**2*c**2/7 + 6*C*a**2*c**2/7 +
 12*C*a*b**2*c/7 + C*b**4/7) + x**6*(2*A*a*b*c**2 + 2*A*b**3*c/3 + 2*C*a**2
*b*c + 2*C*a*b**3/3) + x**5*(6*A*a**2*c**2/5 + 12*A*a*b**2*c/5 + A*b**4/5 +
 4*C*a**3*c/5 + 6*C*a**2*b**2/5) + x**4*(3*A*a**2*b*c + A*a*b**3 + C*a**3*b
) + x**3*(4*A*a**3*c/3 + 2*A*a**2*b**2 + C*a**4/3)
```

<u>Giac</u> [A] time = 1.1739, size = 416, normalized size = 1.64

$$\frac{1}{11}\,Cc^4x^{11} + \frac{2}{5}\,Cbc^3x^{10} + \frac{2}{3}\,Cb^2c^2x^9 + \frac{4}{9}\,Cac^3x^9 + \frac{1}{9}\,Ac^4x^9 + \frac{1}{2}\,Cb^3cx^8 + \frac{3}{2}\,Cabc^2x^8 + \frac{1}{2}\,Abc^3x^8 + \frac{1}{7}\,Cb^4x^7 + \frac{12}{7}\,Cab^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 2/3*C*b^2*c^2*x^9 + 4/9*C*a*c^3*x^9 +
1/9*A*c^4*x^9 + 1/2*C*b^3*c*x^8 + 3/2*C*a*b*c^2*x^8 + 1/2*A*b*c^3*x^8 + 1/7
*C*b^4*x^7 + 12/7*C*a*b^2*c*x^7 + 6/7*C*a^2*c^2*x^7 + 6/7*A*b^2*c^2*x^7 + 4
/7*A*a*c^3*x^7 + 2/3*C*a*b^3*x^6 + 2*C*a^2*b*c*x^6 + 2/3*A*b^3*c*x^6 + 2*A*
a*b*c^2*x^6 + 6/5*C*a^2*b^2*x^5 + 1/5*A*b^4*x^5 + 4/5*C*a^3*c*x^5 + 12/5*A*
a*b^2*c*x^5 + 6/5*A*a^2*c^2*x^5 + C*a^3*b*x^4 + A*a*b^3*x^4 + 3*A*a^2*b*c*x
^4 + 1/3*C*a^4*x^3 + 2*A*a^2*b^2*x^3 + 4/3*A*a^3*c*x^3 + 2*A*a^3*b*x^2 + A*
a^4*x
```

3.141 $\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) dx$

Optimal. Leaf size=161

$$\frac{1}{4}bx^4\left(3a^2C+A\left(6ac+b^2\right)\right)+\frac{1}{3}ax^3\left(a^2C+3A\left(ac+b^2\right)\right)+\frac{3}{2}a^2Abx^2+a^3Ax+\frac{1}{7}cx^7\left(3C\left(ac+b^2\right)+Ac^2\right)+\frac{1}{6}bx^6\left(C\right.$$

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*(b^2 + a*c) + a^2*C)*x^3)/3 + (b*(A*(
b^2 + 6*a*c) + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(3*
A*c^2 + (b^2 + 6*a*c)*C)*x^6)/6 + (c*(A*c^2 + 3*(b^2 + a*c)*C)*x^7)/7 + (3*
b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

Rubi [A] time = 0.193015, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1657}

$$\frac{1}{4}bx^4\left(3a^2C+A\left(6ac+b^2\right)\right)+\frac{1}{3}ax^3\left(a^2C+3A\left(ac+b^2\right)\right)+\frac{3}{2}a^2Abx^2+a^3Ax+\frac{1}{7}cx^7\left(3C\left(ac+b^2\right)+Ac^2\right)+\frac{1}{6}bx^6\left(C\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*(b^2 + a*c) + a^2*C)*x^3)/3 + (b*(A*(
b^2 + 6*a*c) + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(3*
A*c^2 + (b^2 + 6*a*c)*C)*x^6)/6 + (c*(A*c^2 + 3*(b^2 + a*c)*C)*x^7)/7 + (3*
b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) dx &= \int \left(a^3A+3a^2Abx+a\left(3A\left(b^2+ac\right)+a^2C\right)x^2+b\left(A\left(b^2+6ac\right)+3a^2C\right)x^3+3\left(b^2\right.\right.\\ &= a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a\left(3A\left(b^2+ac\right)+a^2C\right)x^3+\frac{1}{4}b\left(A\left(b^2+6ac\right)+3a^2C\right)x^4+\frac{3}{5}\end{aligned}$$

Mathematica [A] time = 0.0444664, size = 163, normalized size = 1.01

$$\frac{1}{4}bx^4\left(3a^2C+6aAc+Ab^2\right)+\frac{1}{3}ax^3\left(a^2C+3aAc+3Ab^2\right)+\frac{3}{2}a^2Abx^2+a^3Ax+\frac{1}{7}cx^7\left(3acC+Ac^2+3b^2C\right)+\frac{1}{6}bx^6\left(6ac\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*b^2 + 3*a*A*c + a^2*C)*x^3)/3 + (b*(A
*b^2 + 6*a*A*c + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(
3*A*c^2 + b^2*C + 6*a*c*C)*x^6)/6 + (c*(A*c^2 + 3*b^2*C + 3*a*c*C)*x^7)/7 +
 (3*b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

Maple [A] time = 0.046, size = 223, normalized size = 1.4

$$\frac{c^3Cx^9}{9}+\frac{3\,bc^2Cx^8}{8}+\frac{\left(\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)C+Ac^3\right)x^7}{7}+\frac{\left(\left(4\,abc+b\left(2\,ac+b^2\right)\right)C+3\,bc^2A\right)x^6}{6}+\frac{\left(\left(a\left(2\,ac+b^2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3*(C*x^2+A),x)
```

```
[Out] 1/9*c^3*C*x^9+3/8*b*c^2*C*x^8+1/7*((a*c^2+2*b^2*c+c*(2*a*c+b^2))*C+A*c^3)*x
^7+1/6*((4*a*b*c+b*(2*a*c+b^2))*C+3*b*c^2*A)*x^6+1/5*((a*(2*a*c+b^2)+2*b^2*
a+a^2*c)*C+(a*c^2+2*b^2*c+c*(2*a*c+b^2))*A)*x^5+1/4*(3*b*a^2*C+(4*a*b*c+b*(
2*a*c+b^2))*A)*x^4+1/3*(a^3*C+(a*(2*a*c+b^2)+2*b^2*a+a^2*c)*A)*x^3+3/2*a^2*
A*b*x^2+a^3*A*x
```

Maxima [A] time = 0.976297, size = 223, normalized size = 1.39

$$\frac{1}{9}\,Cc^3x^9+\frac{3}{8}\,Cbc^2x^8+\frac{1}{7}\left(3\,Cb^2c+3\,Cac^2+Ac^3\right)x^7+\frac{1}{6}\left(Cb^3+6\,Cabc+3\,Abc^2\right)x^6+\frac{3}{2}\,Aa^2bx^2+\frac{3}{5}\left(Cab^2+Aac^2+\left(C\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7 +
 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^2 + 3/5*(C*a*b^2 +
```

```
 A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6*A*
a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3
```

Fricas [A] time = 1.30706, size = 444, normalized size = 2.76

$$\frac{1}{9}x^9c^3C + \frac{3}{8}x^8c^2bC + \frac{3}{7}x^7cb^2C + \frac{3}{7}x^7c^2aC + \frac{1}{7}x^7c^3A + \frac{1}{6}x^6b^3C + x^6cbaC + \frac{1}{2}x^6c^2bA + \frac{3}{5}x^5b^2aC + \frac{3}{5}x^5ca^2C + \frac{3}{5}x^5cb^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^3*C + 3/8*x^8*c^2*b*C + 3/7*x^7*c*b^2*C + 3/7*x^7*c^2*a*C + 1/7*x
^7*c^3*A + 1/6*x^6*b^3*C + x^6*c*b*a*C + 1/2*x^6*c^2*b*A + 3/5*x^5*b^2*a*C
+ 3/5*x^5*c*a^2*C + 3/5*x^5*c*b^2*A + 3/5*x^5*c^2*a*A + 3/4*x^4*b*a^2*C + 1
/4*x^4*b^3*A + 3/2*x^4*c*b*a*A + 1/3*x^3*a^3*C + x^3*b^2*a*A + x^3*c*a^2*A
+ 3/2*x^2*b*a^2*A + x*a^3*A
```

Sympy [A] time = 0.09245, size = 197, normalized size = 1.22

$$Aa^3x + \frac{3Aa^2bx^2}{2} + \frac{3Cbc^2x^8}{8} + \frac{Cc^3x^9}{9} + x^7\left(\frac{Ac^3}{7} + \frac{3Cac^2}{7} + \frac{3Cb^2c}{7}\right) + x^6\left(\frac{Abc^2}{2} + Cabc + \frac{Cb^3}{6}\right) + x^5\left(\frac{3Aac^2}{5} + \frac{3Ab^2}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3*(C*x**2+A),x)
```

```
[Out] A*a**3*x + 3*A*a**2*b*x**2/2 + 3*C*b*c**2*x**8/8 + C*c**3*x**9/9 + x**7*(A*
c**3/7 + 3*C*a*c**2/7 + 3*C*b**2*c/7) + x**6*(A*b*c**2/2 + C*a*b*c + C*b**3
/6) + x**5*(3*A*a*c**2/5 + 3*A*b**2*c/5 + 3*C*a**2*c/5 + 3*C*a*b**2/5) + x*
*4*(3*A*a*b*c/2 + A*b**3/4 + 3*C*a**2*b/4) + x**3*(A*a**2*c + A*a*b**2 + C*
a**3/3)
```

Giac [A] time = 1.21464, size = 252, normalized size = 1.57

$$\frac{1}{9}Cc^3x^9 + \frac{3}{8}Cbc^2x^8 + \frac{3}{7}Cb^2cx^7 + \frac{3}{7}Cac^2x^7 + \frac{1}{7}Ac^3x^7 + \frac{1}{6}Cb^3x^6 + Cabcx^6 + \frac{1}{2}Abc^2x^6 + \frac{3}{5}Cab^2x^5 + \frac{3}{5}Ca^2cx^5 + \frac{3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 3/7*C*b^2*c*x^7 + 3/7*C*a*c^2*x^7 + 1/7*A
*c^3*x^7 + 1/6*C*b^3*x^6 + C*a*b*c*x^6 + 1/2*A*b*c^2*x^6 + 3/5*C*a*b^2*x^5
+ 3/5*C*a^2*c*x^5 + 3/5*A*b^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*C*a^2*b*x^4 + 1
/4*A*b^3*x^4 + 3/2*A*a*b*c*x^4 + 1/3*C*a^3*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3
+ 3/2*A*a^2*b*x^2 + A*a^3*x
```

3.142 $\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) \, dx$

Optimal. Leaf size=96

$$\frac{1}{3}x^3\left(a^2C+A\left(2ac+b^2\right)\right)+a^2Ax+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+Ac^2\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*(b^2 + 2*a*c) + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + (b^2 + 2*a*c)*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

Rubi [A] time = 0.0994486, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1657}

$$\frac{1}{3}x^3\left(a^2C+A\left(2ac+b^2\right)\right)+a^2Ax+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+Ac^2\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*(b^2 + 2*a*c) + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + (b^2 + 2*a*c)*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) dx = \int \left(a^2A+2aAbx+\left(A\left(b^2+2ac\right)+a^2C\right)x^2+2b(Ac+aC)x^3+\left(Ac^2+\left(b^2+2ac\right)\right.\right.$$
$$= a^2Ax+aAbx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4+\frac{1}{5}\left(Ac^2+\left(b^2+2ac\right.\right.$$

Mathematica [A] time = 0.0214049, size = 96, normalized size = 1.

$$\frac{1}{3}x^3\left(a^2C+2aAc+Ab^2\right)+a^2Ax+\frac{1}{5}x^5\left(2acC+Ac^2+b^2C\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*b^2 + 2*a*A*c + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + b^2*C + 2*a*c*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

Maple [A] time = 0.048, size = 90, normalized size = 0.9

$$\frac{c^2Cx^7}{7}+\frac{bcCx^6}{3}+\frac{\left(Ac^2+\left(2\,ac+b^2\right)C\right)x^5}{5}+\frac{(2\,Abc+2\,abC)\,x^4}{4}+\frac{\left(A\left(2\,ac+b^2\right)+a^2C\right)x^3}{3}+aAbx^2+a^2Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2*(C*x^2+A),x)
```

```
[Out] 1/7*c^2*C*x^7+1/3*b*c*C*x^6+1/5*(A*c^2+(2*a*c+b^2)*C)*x^5+1/4*(2*A*b*c+2*C*
a*b)*x^4+1/3*(A*(2*a*c+b^2)+a^2*C)*x^3+a*A*b*x^2+a^2*A*x
```

Maxima [A] time = 0.964131, size = 117, normalized size = 1.22

$$\frac{1}{7}\,Cc^2x^7+\frac{1}{3}\,Cbcx^6+\frac{1}{5}\left(Cb^2+2\,Cac+Ac^2\right)x^5+Aabx^2+\frac{1}{2}\,(Cab+Abc)x^4+Aa^2x+\frac{1}{3}\left(Ca^2+Ab^2+2\,Aac\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*(C*b^2 + 2*C*a*c + A*c^2)*x^5 + A*a*b*x
^2 + 1/2*(C*a*b + A*b*c)*x^4 + A*a^2*x + 1/3*(C*a^2 + A*b^2 + 2*A*a*c)*x^3
```

Fricas [A] time = 1.31042, size = 244, normalized size = 2.54

$$\frac{1}{7}x^7c^2C+\frac{1}{3}x^6cbC+\frac{1}{5}x^5b^2C+\frac{2}{5}x^5caC+\frac{1}{5}x^5c^2A+\frac{1}{2}x^4baC+\frac{1}{2}x^4cbA+\frac{1}{3}x^3a^2C+\frac{1}{3}x^3b^2A+\frac{2}{3}x^3caA+x^2baA+xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^2*C + 1/3*x^6*c*b*C + 1/5*x^5*b^2*C + 2/5*x^5*c*a*C + 1/5*x^5*c^2
*A + 1/2*x^4*b*a*C + 1/2*x^4*c*b*A + 1/3*x^3*a^2*C + 1/3*x^3*b^2*A + 2/3*x^
3*c*a*A + x^2*b*a*A + x*a^2*A
```

Sympy [A] time = 0.07848, size = 102, normalized size = 1.06

$$Aa^2x + Aabx^2 + \frac{Cbcx^6}{3} + \frac{Cc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Cac}{5} + \frac{Cb^2}{5}\right) + x^4\left(\frac{Abc}{2} + \frac{Cab}{2}\right) + x^3\left(\frac{2Aac}{3} + \frac{Ab^2}{3} + \frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2*(C*x**2+A),x)
```

```
[Out] A*a**2*x + A*a*b*x**2 + C*b*c*x**6/3 + C*c**2*x**7/7 + x**5*(A*c**2/5 + 2*C
*a*c/5 + C*b**2/5) + x**4*(A*b*c/2 + C*a*b/2) + x**3*(2*A*a*c/3 + A*b**2/3
+ C*a**2/3)
```

Giac [A] time = 1.24712, size = 134, normalized size = 1.4

$$\frac{1}{7}Cc^2x^7 + \frac{1}{3}Cbcx^6 + \frac{1}{5}Cb^2x^5 + \frac{2}{5}Cacx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{2}Cabx^4 + \frac{1}{2}Abcx^4 + \frac{1}{3}Ca^2x^3 + \frac{1}{3}Ab^2x^3 + \frac{2}{3}Aacx^3 + Aabx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x
^5 + 1/2*C*a*b*x^4 + 1/2*A*b*c*x^4 + 1/3*C*a^2*x^3 + 1/3*A*b^2*x^3 + 2/3*A*
a*c*x^3 + A*a*b*x^2 + A*a^2*x
```

3.143 $\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

Rubi [A] time = 0.0295376, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1657}

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(A + C*x^2),x]
```

```
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx &= \int \left(aA+Abx+(Ac+aC)x^2+bCx^3+cCx^4\right)\,dx\\ &= aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5\end{aligned}$$

Mathematica [A] time = 0.0093153, size = 46, normalized size = 1.

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(A + C*x^2),x]
```

```
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

Maple [A] time = 0.05, size = 39, normalized size = 0.9

$$aAx + \frac{Abx^2}{2} + \frac{(Ac + aC)\,x^3}{3} + \frac{bCx^4}{4} + \frac{cCx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(C*x^2+A),x)
```

```
[Out] a*A*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*c*C*x^5
```

Maxima [A] time = 0.983255, size = 51, normalized size = 1.11

$$\frac{1}{5}\,Ccx^5 + \frac{1}{4}\,Cbx^4 + \frac{1}{2}\,Abx^2 + \frac{1}{3}\,(Ca + Ac)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/2*A*b*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Fricas [A] time = 1.31628, size = 104, normalized size = 2.26

$$\frac{1}{5}x^5cC + \frac{1}{4}x^4bC + \frac{1}{3}x^3aC + \frac{1}{3}x^3cA + \frac{1}{2}x^2bA + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c*C + 1/4*x^4*b*C + 1/3*x^3*a*C + 1/3*x^3*c*A + 1/2*x^2*b*A + x*a*A
```

Sympy [A] time = 0.063846, size = 42, normalized size = 0.91

$$Aax + \frac{Abx^2}{2} + \frac{Cbx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(C*x**2+A),x)
```

```
[Out] A*a*x + A*b*x**2/2 + C*b*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

Giac [A] time = 1.16908, size = 54, normalized size = 1.17

$$\frac{1}{5}\,Ccx^5 + \frac{1}{4}\,Cbx^4 + \frac{1}{3}\,Cax^3 + \frac{1}{3}\,Acx^3 + \frac{1}{2}\,Abx^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*A*b*x^2 + A*a*x
```

3.144 $\int \frac{A+Cx^2}{a+bx+cx^2} \, dx$

Optimal. Leaf size=81

$$-\frac{\left(C\left(b^2-2ac\right)+2Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

```
[Out] (C*x)/c - ((2*A*c^2 + (b^2 - 2*a*c)*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.100283, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\left(C\left(b^2-2ac\right)+2Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c - ((2*A*c^2 + (b^2 - 2*a*c)*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Cx^2}{a+bx+cx^2}\,dx &= \int \left(\frac{C}{c}+\frac{Ac-aC-bCx}{c\left(a+bx+cx^2\right)}\right)dx\\
&= \frac{Cx}{c}+\frac{\int \frac{Ac-aC-bCx}{a+bx+cx^2}\,dx}{c}\\
&= \frac{Cx}{c}-\frac{(bC)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{1}{2}\left(2A+\frac{\left(b^2-2ac\right)C}{c^2}\right)\int \frac{1}{a+bx+cx^2}\,dx\\
&= \frac{Cx}{c}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\left(-2A-\frac{\left(b^2-2ac\right)C}{c^2}\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)\\
&= \frac{Cx}{c}-\frac{\left(2A+\frac{\left(b^2-2ac\right)C}{c^2}\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.092001, size = 84, normalized size = 1.04

$$\frac{\left(-2acC+2Ac^2+b^2C\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c + ((2*A*c^2 + b^2*C - 2*a*c*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/(c^2*Sqrt[-b^2 + 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A] time = 0.184, size = 140, normalized size = 1.7

$$\frac{Cx}{c} - \frac{Cb\ln\left(cx^2+bx+a\right)}{2c^2} + 2\frac{A}{\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) - 2\frac{aC}{c\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) + \frac{Cb^2}{c^2}\arctan\Big((2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a),x)
```

```
[Out] C*x/c-1/2*b*C*ln(c*x^2+b*x+a)/c^2+2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*A-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
a*C+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57218, size = 595, normalized size = 7.35

$$\left[\frac{\left(Cb^2-2Cac+2Ac^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+2\left(Cb^2c-4Cac^2\right)x-\left(Cb^3-4Cabc\right)\log\left(cx^2\right.}{2\left(b^2c^2-4ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(C*
b^2*c - 4*C*a*c^2)*x - (C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 -
 4*a*c^3), -1/2*(2*(C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(C*b^2*c - 4*C*a*c^2)*x +
(C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 1.22134, size = 413, normalized size = 5.1

$$\frac{Cx}{c}+\left(-\frac{Cb}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(-2Ac^2+2Cac-Cb^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-Abc-Cab-4ac^2\left(-\frac{Cb}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(-2Ac^2+2Cac-Cb^2\right)}{2c^2\left(4ac-b^2\right)}\right)+b^2c}{-2Ac^2+2Cac-Cb^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a),x)
```

```
[Out] C*x/c + (-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)
/(2*c**2*(4*a*c - b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2)
 - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2
))) + b**2*c*(-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*
b**2)/(2*c**2*(4*a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2)) + (-C*b/(2*
c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c -
 b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2) + sqrt(-4*a*c +
b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-C*
b/(2*c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*
a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2))
```

Giac [A] time = 1.31595, size = 105, normalized size = 1.3

$$\frac{Cx}{c}-\frac{Cb\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(Cb^2-2\,Cac+2\,Ac^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] C*x/c - 1/2*C*b*log(c*x^2 + b*x + a)/c^2 + (C*b^2 - 2*C*a*c + 2*A*c^2)*arct
an((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.145 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2} dx$

Optimal. Leaf size=100

$$\frac{4(aC+Ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] -((b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(a +
 b*x + c*x^2))) + (4*(A*c + a*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0682772, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1660, 12, 618, 206}

$$\frac{4(aC+Ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(a +
 b*x + c*x^2))) + (4*(A*c + a*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(3/2)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx &= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{2(Ac+aC)}{a+bx+cx^2}\,dx}{-b^2+4ac}\\ &= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2(Ac+aC))\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\ &= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4(Ac+aC))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\ &= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4(Ac+aC)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0839401, size = 98, normalized size = 0.98

$$\frac{aC(b-2cx)+Ac(b+2cx)+b^2Cx}{c\left(4ac-b^2\right)(a+x(b+cx))}+\frac{4(aC+Ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b +
 c*x))) + (4*(A*c + a*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*
a*c)^(3/2)
```

Maple [A] time = 0.19, size = 146, normalized size = 1.5

$$\frac{1}{cx^2+bx+a}\left(\frac{\left(2\,Ac^2-2\,Cac+Cb^2\right)x}{c\left(4\,ac-b^2\right)}+\frac{b\,(Ac+aC)}{c\left(4\,ac-b^2\right)}\right)+4\,\frac{Ac}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+4\,\frac{aC}{\left(4\,ac-b^2\right)^{3/2}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^2,x)
```

```
[Out] ((2*A*c^2-2*C*a*c+C*b^2)/c/(4*a*c-b^2)*x+b/c*(A*c+C*a)/(4*a*c-b^2))/(c*x^2+
b*x+a)+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c+4/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60237, size = 1085, normalized size = 10.85

$$\left[-\frac{Cab^3-4\,Aabc^2+2\left(Ca^2c+Aac^2+\left(Cac^2+Ac^3\right)x^2+\left(Cabc+Abc^2\right)x\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b}{cx^2+bx+a}\right.}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^2+\left(b^5c-8\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-(C*a*b^3 - 4*A*a*b*c^2 + 2*(C*a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (
C*a*b*c + A*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (4*C*a^2*b - A*b^
3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x)/(a*b^
4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2
 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(C*a*b^3 - 4*A*a*b*c^2 - 4*(C*
a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (C*a*b*c + A*b*c^2)*x)*sqrt(-b^2
+ 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (4*C*a^2*b
 - A*b^3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x
)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]
```

Sympy [B] time = 1.28657, size = 376, normalized size = 3.76

$$-2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(Ac+Ca\right)\log\left(x+\frac{2Abc+2Cab-32a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(Ac+Ca\right)+16ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(Ac+Ca\right)-2}{4Ac^2+4Cac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2*A*b*c + 2*C*a*b - 32*a
**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 16*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(A*c + C*a) - 2*b**4*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a))/
(4*A*c**2 + 4*C*a*c)) + 2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2
*A*b*c + 2*C*a*b + 32*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) - 16
*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 2*b**4*sqrt(-1/(4*a*c -
b**2)**3)*(A*c + C*a))/(4*A*c**2 + 4*C*a*c)) - (-A*b*c - C*a*b + x*(-2*A*c*
*2 + 2*C*a*c - C*b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**
2) + x*(4*a*b*c**2 - b**3*c))
```

Giac [A] time = 1.24393, size = 146, normalized size = 1.46

$$-\frac{4\,(Ca+Ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{Cb^2x-2\,Cacx+2\,Ac^2x+Cab+Abc}{\left(b^2c-4\,ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*(C*a + A*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-
b^2 + 4*a*c)) - (C*b^2*x - 2*C*a*c*x + 2*A*c^2*x + C*a*b + A*b*c)/((b^2*c -
 4*a*c^2)*(c*x^2 + b*x + a))
```

3.146 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^3} \, dx$

Optimal. Leaf size=161

$$-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{2\left(C\left(2ac+b^2\right)+6Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{(b+2cx)\left(2aC+6Ac+\frac{b^2C}{c}\right)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

```
[Out] -(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(2*c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)^2) + ((6*A*c + 2*a*C + (b^2*C)/c)*(b + 2*c*x))/(2*(b^2 - 4*a
*c)^2*(a + b*x + c*x^2)) - (2*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.113923, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1660, 12, 614, 618, 206}

$$-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{2\left(C\left(2ac+b^2\right)+6Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{(b+2cx)\left(2aC+6Ac+\frac{b^2C}{c}\right)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(2*c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)^2) + ((6*A*c + 2*a*C + (b^2*C)/c)*(b + 2*c*x))/(2*(b^2 - 4*a
*c)^2*(a + b*x + c*x^2)) - (2*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{6Ac+2aC+\frac{b^2C}{c}}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(6Ac^2+\left(b^2+2ac\right.\right.}{\left(b^2-\right.}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(2\left(6Ac^2+\left(b^2+2\right.\right.\right.}{}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{2\left(6Ac^2+\left(b^2+2a\right.\right.}{\left(b^2\right.}$$

Mathematica [A] time = 0.220095, size = 160, normalized size = 0.99

$$\frac{1}{2}\left(\frac{(b+2cx)\left(C\left(2ac+b^2\right)+6Ac^2\right)}{c\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{4\left(C\left(2ac+b^2\right)+6Ac^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{aC(b-2cx)+Ac(b+2cx)+b^2Cx}{c\left(4ac-b^2\right)(a+x(b+cx))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (((6*A*c^2 + (b^2 + 2*a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c
*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) + (4*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTan[(b + 2*c*x)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [B] time = 0.183, size = 373, normalized size = 2.3

$$\frac{1}{\left(cx^2+bx+a\right)^2}\left(\frac{c\left(6\,Ac^2+2\,Cac+Cb^2\right)x^3}{16\,a^2c^2-8\,acb^2+b^4} + \frac{3\,b\left(6\,Ac^2+2\,Cac+Cb^2\right)x^2}{32\,a^2c^2-16\,acb^2+2\,b^4} + \frac{\left(10\,aAc^2+2\,Ab^2c-2\,Ca^2c+5\,Cab^2\right)x}{16\,a^2c^2-8\,acb^2+b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^3,x)
```

```
[Out] (c*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(6*A*c^2+2*
C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+(10*A*a*c^2+2*A*b^2*c-2*C*a^2*c
+5*C*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*(10*A*a*c-A*b^2+6*C*a^2)/(16
*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c^2+4/(16*a^2*c^2-8*a*b
^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*C*a*c+2/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*C*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.73505, size = 2592, normalized size = 16.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 - 2
4*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A*
a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 + 2*(C*a^2*b^2 + 2*C*a^3*c + 6*A
*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^2
 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b^
2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) - 2*(12*C*a^3*b - 7*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^
```

```
3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)
, 1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 -
24*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A
*a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 - 4*(C*a^2*b^2 + 2*C*a^3*c + 6*
A*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^
2 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b
^2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(-b^2 + 4*a*c
)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(12*C*a^3*b - 7
*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(
11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] time = 2.6655, size = 774, normalized size = 4.81

$$-\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(6Ac^2+2Cac+Cb^2\right)\log\left(x+\frac{6Abc^2+2Cabc+Cb^3-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(6Ac^2+2Cac+Cb^2\right)+48}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c*
*2 + 2*C*a*b*c + C*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2
 + 2*C*a*c + C*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c*
*2 + 2*C*a*c + C*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 +
 2*C*a*c + C*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c +
C*b**2))/(12*A*c**3 + 4*C*a*c**2 + 2*C*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5
)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c**2 + 2*C*a*b*c + C*b**3 +
64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - 48*
a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) + 1
2*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - b**6*
sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2))/(12*A*c**3 + 4*C*
a*c**2 + 2*C*b**2*c)) + (10*A*a*b*c - A*b**3 + 6*C*a**2*b + x**3*(12*A*c**3
```

```
 + 4*C*a*c**2 + 2*C*b**2*c) + x**2*(18*A*b*c**2 + 6*C*a*b*c + 3*C*b**3) + x
*(20*A*a*c**2 + 4*A*b**2*c - 4*C*a**2*c + 10*C*a*b**2))/(32*a**4*c**2 - 16*
a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c*
*2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**
3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [A] time = 1.27177, size = 293, normalized size = 1.82

$$\frac{2\left(Cb^2+2\,Cac+6\,Ac^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{2\,Cb^2cx^3+4\,Cac^2x^3+12\,Ac^3x^3+3\,Cb^3x^2+6\,Cabcx^2+18\,Abc^2x^2+10\,C}{2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*(C*b^2 + 2*C*a*c + 6*A*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*C*b^2*c*x^3 + 4*C*a*
c^2*x^3 + 12*A*c^3*x^3 + 3*C*b^3*x^2 + 6*C*a*b*c*x^2 + 18*A*b*c^2*x^2 + 10*
C*a*b^2*x - 4*C*a^2*c*x + 4*A*b^2*c*x + 20*A*a*c^2*x + 6*C*a^2*b - A*b^3 +
10*A*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.147 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4}\,dx$

Optimal. Leaf size=206

$$-\frac{2(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{8c\left(C\left(ac+b^2\right)+5Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{7/2}}$$

```
[Out] -(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(3*c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)^3) + ((5*A*c + (a + b^2/c)*C)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^
2*(a + b*x + c*x^2)^2) - (2*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 -
4*a*c)^3*(a + b*x + c*x^2)) + (8*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rubi [A] time = 0.190133, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1660, 12, 614, 618, 206}

$$-\frac{2(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{8c\left(C\left(ac+b^2\right)+5Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] -(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(3*c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)^3) + ((5*A*c + (a + b^2/c)*C)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^
2*(a + b*x + c*x^2)^2) - (2*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 -
4*a*c)^3*(a + b*x + c*x^2)) + (8*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
```

```
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\int \frac{2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\left(2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)\right)\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{\left(2\left(5Ac^2+\left(b^2+a\right.\right.\right.}{\left(b^2\right.}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2\left(5Ac^2+\left(b^2+ac\right.\right.}{\left(b^2-4ac\right)^3\left(a+\right.}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2\left(5Ac^2+\left(b^2+ac\right.\right.}{\left(b^2-4ac\right)^3\left(a+\right.}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2\left(5Ac^2+\left(b^2+ac\right.\right.}{\left(b^2-4ac\right)^3\left(a+\right.}$$

Mathematica [A] time = 0.399707, size = 204, normalized size = 0.99

$$\frac{1}{3}\left(-\frac{6(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3(a+x(b+cx))} + \frac{(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{c\left(b^2-4ac\right)^2(a+x(b+cx))^2} + \frac{24c\left(C\left(ac+b^2\right)+5Ac^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{7/2}} + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] (((5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x
))^2) - (6*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + x*(
b + c*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c
)*(a + x*(b + c*x))^3) + (24*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/3
```

Maple [B] time = 0.187, size = 643, normalized size = 3.1

$$\frac{1}{\left(cx^{2}+bx+a\right)^{3}}\left(4\,\frac{c^{3}\left(5\,Ac^{2}+Cac+Cb^{2}\right)x^{5}}{64\,c^{3}a^{3}-48\,a^{2}b^{2}c^{2}+12\,ab^{4}c-b^{6}}+10\,\frac{c^{2}\left(5\,Ac^{2}+Cac+Cb^{2}\right)bx^{4}}{64\,c^{3}a^{3}-48\,a^{2}b^{2}c^{2}+12\,ab^{4}c-b^{6}}+\frac{\left(32\,ac+22\,b^{2}\right)c\left(5\,Ac^{2}\right.}{192\,c^{3}a^{3}-144\,a^{2}b^{2}c^{2}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^4,x)
```

```
[Out] (4*c^3*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5
+10*c^2*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*b*
x^4+2/3*(16*a*c+11*b^2)*c*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+
12*a*b^4*c-b^6)*x^3+b*(16*a*c+b^2)*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2
*b^2*c^2+12*a*b^4*c-b^6)*x^2+(44*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-4*C*a^3*c
^2+22*C*a^2*b^2*c+C*a*b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x+1/3
*(66*A*a^2*c^2-13*A*a*b^2*c+A*b^4+26*C*a^3*c+C*a^2*b^2)*b/(64*a^3*c^3-48*a^
2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+40*c^3/(64*a^3*c^3-48*a^2*b^2*c^
2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A+8
*c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*C*a+8*c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*C*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75595, size = 4547, normalized size = 22.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(C*a^2*b^5 + A*b^7 - 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4
- 20*A*a*c^6 - (4*C*a^2 - 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3
 - 20*A*a*b*c^5 - (4*C*a^2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b
^4*c^2 - 320*A*a^2*c^5 - 4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55
*A*b^4)*c^3)*x^3 - 2*(52*C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^
5*c - 320*A*a^2*b*c^4 - 4*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5
*A*b^5)*c^2)*x^2 + 12*(C*a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 +
 C*a*c^5 + 5*A*c^6)*x^6 + 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*
b^4*c^2 + 2*C*a*b^2*c^3 + 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c
 + 7*C*a*b^3*c^2 + 30*A*a*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b
^4*c + 2*C*a^2*b^2*c^2 + 5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*
a^2*b^3*c + C*a^3*b*c^2 + 5*A*a^2*b*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c^4 + 4*(4*C*a^
4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (18*C*a^2*b^4 -
A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 2
56*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 2
56*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*
c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*
a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 128
0*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6
*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*
a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/3*(C*a^2*b^5 + A*b^7
- 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4 - 20*A*a*c^6 - (4*C*a^2 -
 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3 - 20*A*a*b*c^5 - (4*C*a^
2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b^4*c^2 - 320*A*a^2*c^5 -
4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55*A*b^4)*c^3)*x^3 - 2*(52*
C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^5*c - 320*A*a^2*b*c^4 - 4
*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5*A*b^5)*c^2)*x^2 - 24*(C*
a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 + C*a*c^5 + 5*A*c^6)*x^6 +
 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*b^4*c^2 + 2*C*a*b^2*c^3 +
 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c + 7*C*a*b^3*c^2 + 30*A*a
*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b^4*c + 2*C*a^2*b^2*c^2 +
5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*a^2*b^3*c + C*a^3*b*c^2 +
 5*A*a^2*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c
^4 + 4*(4*C*a^4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (1
8*C*a^2*b^4 - A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4
- 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*
b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3
*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*
c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a
^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x)]
```

Sympy [B] time = 5.19637, size = 1224, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**4,x)
```

```
[Out] -4*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b
*c**3 + 4*C*a*b*c**2 + 4*C*b**3*c - 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) + 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)
**7)*(5*A*c**2 + C*a*c + C*b**2) - 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2
)**7)*(5*A*c**2 + C*a*c + C*b**2) + 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) - 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + 4*c*sqrt
(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b*c**3 + 4
*C*a*b*c**2 + 4*C*b**3*c + 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) - 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)**7)*(5*A
*c**2 + C*a*c + C*b**2) + 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2)**7)*(5*
A*c**2 + C*a*c + C*b**2) - 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) + 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a
*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + (66*A*a**2*b*c**2
 - 13*A*a*b**3*c + A*b**5 + 26*C*a**3*b*c + C*a**2*b**3 + x**5*(60*A*c**5 +
 12*C*a*c**4 + 12*C*b**2*c**3) + x**4*(150*A*b*c**4 + 30*C*a*b*c**3 + 30*C*
b**3*c**2) + x**3*(160*A*a*c**4 + 110*A*b**2*c**3 + 32*C*a**2*c**3 + 54*C*a
*b**2*c**2 + 22*C*b**4*c) + x**2*(240*A*a*b*c**3 + 15*A*b**3*c**2 + 48*C*a*
*2*b*c**2 + 51*C*a*b**3*c + 3*C*b**5) + x*(132*A*a**2*c**3 + 54*A*a*b**2*c*
*2 - 3*A*b**4*c - 12*C*a**3*c**2 + 66*C*a**2*b**2*c + 3*C*a*b**4))/(192*a**
6*c**3 - 144*a**5*b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**3
*c**6 - 144*a**2*b**2*c**5 + 36*a*b**4*c**4 - 3*b**6*c**3) + x**5*(576*a**3
*b*c**5 - 432*a**2*b**3*c**4 + 108*a*b**5*c**3 - 9*b**7*c**2) + x**4*(576*a
**4*c**5 + 144*a**3*b**2*c**4 - 324*a**2*b**4*c**3 + 99*a*b**6*c**2 - 9*b**
8*c) + x**3*(1152*a**4*b*c**4 - 672*a**3*b**3*c**3 + 72*a**2*b**5*c**2 + 18
*a*b**7*c - 3*b**9) + x**2*(576*a**5*c**4 + 144*a**4*b**2*c**3 - 324*a**3*b
**4*c**2 + 99*a**2*b**6*c - 9*a*b**8) + x*(576*a**5*b*c**3 - 432*a**4*b**3*
c**2 + 108*a**3*b**5*c - 9*a**2*b**7))
```

Giac [B] time = 1.17805, size = 549, normalized size = 2.67

$$-\frac{8\left(Cb^{2}c+Cac^{2}+5\,Ac^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{6}-12\,ab^{4}c+48\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}\right)\sqrt{-b^{2}+4\,ac}}-\frac{12\,Cb^{2}c^{3}x^{5}+12\,Cac^{4}x^{5}+60\,Ac^{5}x^{5}+30\,Cb^{3}c^{2}x^{4}+30\,Cabc^{3}x^{4}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -8*(C*b^2*c + C*a*c^2 + 5*A*c^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(1
2*C*b^2*c^3*x^5 + 12*C*a*c^4*x^5 + 60*A*c^5*x^5 + 30*C*b^3*c^2*x^4 + 30*C*a
*b*c^3*x^4 + 150*A*b*c^4*x^4 + 22*C*b^4*c*x^3 + 54*C*a*b^2*c^2*x^3 + 32*C*a
^2*c^3*x^3 + 110*A*b^2*c^3*x^3 + 160*A*a*c^4*x^3 + 3*C*b^5*x^2 + 51*C*a*b^3
*c*x^2 + 48*C*a^2*b*c^2*x^2 + 15*A*b^3*c^2*x^2 + 240*A*a*b*c^3*x^2 + 3*C*a*
b^4*x + 66*C*a^2*b^2*c*x - 3*A*b^4*c*x - 12*C*a^3*c^2*x + 54*A*a*b^2*c^2*x
+ 132*A*a^2*c^3*x + C*a^2*b^3 + A*b^5 + 26*C*a^3*b*c - 13*A*a*b^3*c + 66*A*
a^2*b*c^2)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x +
 a)^3)
```

3.148 $\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=591

$$\frac{\log\left(a+bx+cx^2\right)\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\left(3d^2h+3\right.\right.\right.}{2c^5}$$

```
[Out] -(((b^3*e^3*h - c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) - b*c*e^2*(b*e*g + 3*b*d*
h + 2*a*e*h) + c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*h)))*x
)/c^4) + (e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g + 3*b
*d*h + a*e*h))*x^2)/(2*c^3) + (e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3)/(3*c^2) +
 (e^3*h*x^4)/(4*c) - ((2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h
 + 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) -
 b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^
2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) +
3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(c^5*Sqrt[b^2 - 4*a*c]) + ((c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^
2*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) +
b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a
*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + b*x + c*x^2])/(2*c^5)
```

Rubi [A] time = 1.42811, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log\left(a+bx+cx^2\right)\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\left(3d^2h+3\right.\right.\right.}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] -(((b^3*e^3*h - c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) - b*c*e^2*(b*e*g + 3*b*d*
h + 2*a*e*h) + c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*h)))*x
)/c^4) + (e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g + 3*b
*d*h + a*e*h))*x^2)/(2*c^3) + (e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3)/(3*c^2) +
 (e^3*h*x^4)/(4*c) - ((2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h
 + 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) -
 b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^
2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) +
```

```
3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(c^5*Sqrt[b^2 - 4*a*c]) + ((c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^
2*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) +
b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a
*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + b*x + c*x^2])/(2*c^5)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int\Bigg(-\frac{b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+3dh)+\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+3dh)+b\left(\right.\right.\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+3dh)+b\left(\right.\right.\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+3dh)+b\left(\right.\right.\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+3dh)+b\left(\right.\right.\right.}{c^4}$$

Mathematica [A] time = 0.630873, size = 585, normalized size = 0.99

$$\frac{6\log(a+x(b+cx))\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\left(3d^2h+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (12*c*(-(b^3*e^3*h) + c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) + b*c*e^2*(b*e*g +
3*b*d*h + 2*a*e*h) - c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*
h)))*x + 6*c^2*e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g
+ 3*b*d*h + a*e*h))*x^2 + 4*c^3*e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3 + 3*c^4*e
^3*h*x^4 + (12*(2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h + 5*a*
e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) - b*c^2*
e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) +
 c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) + 3*a*b*e
*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + 6*(c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^2*(b*e*g +
3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f
+ 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a*e*(e^2*f +
 3*d*e*g + 3*d^2*h)))*Log[a + x*(b + c*x)])/(12*c^5)
```

Maple [B] time = 0.181, size = 1738, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] -1/3/c^2*x^3*b*e^3*h+1/c*x^3*d*e^2*h-1/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^5*e^3*h-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b^3*e^3*f-3/2/c^2*ln(c*x^2+b*x+a)*b*d*e^2*f-2/c/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d^3*h+3/2/c^3*ln(c*x^2+b*x+a)
*b^2*d^2*e*h+3/2/c^3*ln(c*x^2+b*x+a)*b^2*d*e^2*g-3/2/c^2*ln(c*x^2+b*x+a)*b*
d^2*e*g+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^3
*g-3/2/c^4*ln(c*x^2+b*x+a)*b^3*d*e^2*h-3/2/c^2*ln(c*x^2+b*x+a)*a*d^2*e*h-3/
2/c^2*ln(c*x^2+b*x+a)*a*d*e^2*g-3/c^2*b*d*e^2*g*x+3/c^3*b^2*d*e^2*h*x-3/c^2
*b*d^2*e*h*x+3/c*d*e^2*f*x+1/c^3*b^2*e^3*g*x-1/c^2*b*e^3*f*x+3/c*d^2*e*g*x-
1/c^4*b^3*e^3*h*x+1/2/c^3*x^2*b^2*e^3*h-1/2/c^2*x^2*b*e^3*g+3/2/c*x^2*d^2*e
*h+3/2/c*x^2*d*e^2*g-1/c^2*a*e^3*g*x-1/2/c^2*x^2*a*e^3*h-1/2/c^2*ln(c*x^2+b
*x+a)*a*e^3*f+1/2/c^5*ln(c*x^2+b*x+a)*b^4*e^3*h+1/2/c^3*ln(c*x^2+b*x+a)*b^2
*e^3*f-1/2/c^2*ln(c*x^2+b*x+a)*b*d^3*h-1/2/c^4*ln(c*x^2+b*x+a)*b^3*e^3*g+1/
2/c^3*ln(c*x^2+b*x+a)*a^2*e^3*h+3/2/c*ln(c*x^2+b*x+a)*d^2*e*f+1/3/c*x^3*e^3
*g+1/2/c*x^2*e^3*f+1/c*d^3*h*x+2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*d^3*f+1/2/c*ln(c*x^2+b*x+a)*d^3*g-12/c^3/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*d*e^2*h+9/c^2/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d^2*e*h+9/c^2/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e^2*g-3/2/c^2*x^2*b*d*e^2*h+2/c^3*a*b*e^3
*h*x-3/c^2*a*d*e^2*h*x+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*b^2*d^3*h-3/2/c^4*ln(c*x^2+b*x+a)*a*b^2*e^3*h+1/c^4/(4*a*c-b^2)^(1/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^3*g+1/c^3*ln(c*x^2+b*x+a)*a*b*
e^3*g-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^3*g+3/c
^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*d*e^2*h+3/c^2/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e^3*f-6/c/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d^2*e*g-6/c/(4*a*c-b^2)^(1
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*e^2*f-3/c/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^2*e*f-3/c^3/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d^2*e*h-3/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e^2*g-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*a*b^2*e^3*g+3/c^3*ln(c*x^2+b*x+a)*a*b*d*e^2*h-5/c^3/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*e^3*h+6/c^2/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*d*e^2*h+3/c^2/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^2*e*g+3/c^2/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d*e^2*f+5/c^4/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^3*h+1/4*e^3*h*x^4/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.13653, size = 4321, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3*
(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c^4
 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^
3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*h)*x^2 - 6*sqrt(b^2 - 4*a*c)*((2*c^5*d^
3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 - 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^3)*e^3
)*f - (b*c^4*d^3 - 3*(b^2*c^3 - 2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*c^3)*d*
e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^4)*d^3 -
 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2
- (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 12*((3*(b^2*c^4
 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4 - 4*a*c^5)*d
^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*
e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*
e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2
 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4 - 4*a*c^5)*d^3 -
3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2
 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^3 - 4*a*b*c^4)*d^3
- 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b^5*c - 6*a*b^3*c^2 + 8*
a^2*b*c^3)*d*e^2 - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*h)*l
og(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h
```

```
*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c
^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c^4 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 -
4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*
e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3
)*h)*x^2 - 12*sqrt(-b^2 + 4*a*c)*((2*c^5*d^3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 -
 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^3)*e^3)*f - (b*c^4*d^3 - 3*(b^2*c^3 -
2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*c^3)*d*e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a
^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e +
 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*e^3)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 12*((3*(b
^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4 - 4*a*
c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2
*c^4)*e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3
*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b
*c^3)*e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)
*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4 - 4*a*c^5)*
d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)
*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^3 - 4*a*b*c^4
)*d^3 - 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d*e^2 - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3
)*h)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]
```

Sympy [B] time = 83.9893, size = 4962, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2
*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*
e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*
a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**
4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g
+ b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*
d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*
c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h +
2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f +
 b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3
*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g
- 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5))*log(x + (2*a
```

```
**3*c**2*e**3*h - 4*a**2*b**2*c*e**3*h + 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*
c**2*e**3*g - 6*a**2*c**3*d**2*e*h - 6*a**2*c**3*d*e**2*g - 2*a**2*c**3*e**
3*f + a*b**4*e**3*h - 3*a*b**3*c*d*e**2*h - a*b**3*c*e**3*g + 3*a*b**2*c**2
*d**2*e*h + 3*a*b**2*c**2*d*e**2*g + a*b**2*c**2*e**3*f - a*b*c**3*d**3*h -
 3*a*b*c**3*d**2*e*g - 3*a*b*c**3*d*e**2*f - 4*a*c**5*(-sqrt(-4*a*c + b**2)
*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b*
*3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d
**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c
**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c
*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f -
b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**
3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c*
*2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3
*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3
*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g +
 b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f +
 c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)) + 2*a*c**4*d**3*g + 6*a*c**4*d**2
*e*f + b**2*c**4*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*
d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h
 + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b
*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b*
*5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b*
*3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e
*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3
*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b
*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g
- a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*
c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3
*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c*
*5)) - b*c**4*d**3*f)/(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2
*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*
e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*
a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**
4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g
+ b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*
d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)) + (sqrt(-4*a
*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*
g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*
a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3
*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*
h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2
*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f +
b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2))
+ (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*
e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*
```

```
h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*
d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*
d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e*
*3*h - 4*a**2*b**2*c*e**3*h + 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3*g
 - 6*a**2*c**3*d**2*e*h - 6*a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f + a*b**
4*e**3*h - 3*a*b**3*c*d*e**2*h - a*b**3*c*e**3*g + 3*a*b**2*c**2*d**2*e*h +
 3*a*b**2*c**2*d*e**2*g + a*b**2*c**2*e**3*f - a*b*c**3*d**3*h - 3*a*b*c**3
*d**2*e*g - 3*a*b*c**3*d*e**2*f - 4*a*c**5*(sqrt(-4*a*c + b**2)*(5*a**2*b*c
**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h
+ 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*
a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g
 + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*
b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d*
*3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c*
*4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h -
3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2
*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h
- b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e
**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g
 + 3*c**4*d**2*e*f)/(2*c**5)) + 2*a*c**4*d**3*g + 6*a*c**4*d**2*e*f + b**2*
c**4*(sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*
a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c
**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f
+ 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3
*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**
2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c
**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(
4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*
h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3
*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h
 + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*
e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)) - b*c**4
*d**3*f)/(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g
- 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*
b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h
 + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h
- b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e
**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*
c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)) + e**3*h*x**4/(4*c) - x**
3*(b*e**3*h - 3*c*d*e**2*h - c*e**3*g)/(3*c**2) - x**2*(a*c*e**3*h - b**2*e
**3*h + 3*b*c*d*e**2*h + b*c*e**3*g - 3*c**2*d**2*e*h - 3*c**2*d*e**2*g - c
**2*e**3*f)/(2*c**3) + x*(2*a*b*c*e**3*h - 3*a*c**2*d*e**2*h - a*c**2*e**3*
g - b**3*e**3*h + 3*b**2*c*d*e**2*h + b**2*c*e**3*g - 3*b*c**2*d**2*e*h - 3
*b*c**2*d*e**2*g - b*c**2*e**3*f + c**3*d**3*h + 3*c**3*d**2*e*g + 3*c**3*d
*e**2*f)/c**4
```

Giac [A] time = 1.28457, size = 1041, normalized size = 1.76

$$\frac{3\,c^3hx^4e^3+12\,c^3dhx^3e^2+18\,c^3d^2hx^2e+12\,c^3d^3hx+4\,c^3gx^3e^3-4\,bc^2hx^3e^3+18\,c^3dgx^2e^2-18\,bc^2dhx^2e^2+36\,c^3d^2gxe-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*h*x^4*e^3 + 12*c^3*d*h*x^3*e^2 + 18*c^3*d^2*h*x^2*e + 12*c^3*d^
3*h*x + 4*c^3*g*x^3*e^3 - 4*b*c^2*h*x^3*e^3 + 18*c^3*d*g*x^2*e^2 - 18*b*c^2
*d*h*x^2*e^2 + 36*c^3*d^2*g*x*e - 36*b*c^2*d^2*h*x*e + 6*c^3*f*x^2*e^3 - 6*
b*c^2*g*x^2*e^3 + 6*b^2*c*h*x^2*e^3 - 6*a*c^2*h*x^2*e^3 + 36*c^3*d*f*x*e^2
- 36*b*c^2*d*g*x*e^2 + 36*b^2*c*d*h*x*e^2 - 36*a*c^2*d*h*x*e^2 - 12*b*c^2*f
*x*e^3 + 12*b^2*c*g*x*e^3 - 12*a*c^2*g*x*e^3 - 12*b^3*h*x*e^3 + 24*a*b*c*h*
x*e^3)/c^4 + 1/2*(c^4*d^3*g - b*c^3*d^3*h + 3*c^4*d^2*f*e - 3*b*c^3*d^2*g*e
 + 3*b^2*c^2*d^2*h*e - 3*a*c^3*d^2*h*e - 3*b*c^3*d*f*e^2 + 3*b^2*c^2*d*g*e^
2 - 3*a*c^3*d*g*e^2 - 3*b^3*c*d*h*e^2 + 6*a*b*c^2*d*h*e^2 + b^2*c^2*f*e^3 -
 a*c^3*f*e^3 - b^3*c*g*e^3 + 2*a*b*c^2*g*e^3 + b^4*h*e^3 - 3*a*b^2*c*h*e^3
+ a^2*c^2*h*e^3)*log(c*x^2 + b*x + a)/c^5 + (2*c^5*d^3*f - b*c^4*d^3*g + b^
2*c^3*d^3*h - 2*a*c^4*d^3*h - 3*b*c^4*d^2*f*e + 3*b^2*c^3*d^2*g*e - 6*a*c^4
*d^2*g*e - 3*b^3*c^2*d^2*h*e + 9*a*b*c^3*d^2*h*e + 3*b^2*c^3*d*f*e^2 - 6*a*
c^4*d*f*e^2 - 3*b^3*c^2*d*g*e^2 + 9*a*b*c^3*d*g*e^2 + 3*b^4*c*d*h*e^2 - 12*
a*b^2*c^2*d*h*e^2 + 6*a^2*c^3*d*h*e^2 - b^3*c^2*f*e^3 + 3*a*b*c^3*f*e^3 + b
^4*c*g*e^3 - 4*a*b^2*c^2*g*e^3 + 2*a^2*c^3*g*e^3 - b^5*h*e^3 + 5*a*b^3*c*h*
e^3 - 5*a^2*b*c^2*h*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*c^5)
```

3.149 $\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=348

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\left(d^2h+2d\right.\right.\right.}{c^4\sqrt{b^2-4ac}}$$

```
[Out] ((b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*e*h)
)*x)/c^3 + (e*(c*e*g + 2*c*d*h - b*e*h)*x^2)/(2*c^2) + (e^2*h*x^3)/(3*c) -
((2*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*
(2*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e
*(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + ((c^3*d*(2*e*f + d*g) - b^3*e^2*h +
 b*c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*
d*e*g + d^2*h)))*Log[a + b*x + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.676836, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\left(d^2h+2d\right.\right.\right.}{c^4\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*e*h)
)*x)/c^3 + (e*(c*e*g + 2*c*d*h - b*e*h)*x^2)/(2*c^2) + (e^2*h*x^3)/(3*c) -
((2*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*
(2*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e
*(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + ((c^3*d*(2*e*f + d*g) - b^3*e^2*h +
 b*c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*
d*e*g + d^2*h)))*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int \left(\frac{b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)}{c^3}+\frac{e(ceg+2cdh-beh)x}{c^2}+\frac{e^2h\ldots}{\ldots}\right.$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh)x^2}{2c^2}+\frac{e^2\ldots}{\ldots}$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh)x^2}{2c^2}+\frac{e^2\ldots}{\ldots}$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh)x^2}{2c^2}+\frac{e^2\ldots}{\ldots}$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh)x^2}{2c^2}+\frac{e^2\ldots}{\ldots}$$

Mathematica [A] time = 0.364076, size = 345, normalized size = 0.99

$$\frac{\frac{6\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\left(d^2h+2deg+e^2f\right)+bd(dg+2ef)\right)+b^4e^2h+2c^4d^2f\right)}{\sqrt{4ac-b^2}}+6cx\left(-ce\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (6*c*(b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*
e*h))*x + 3*c^2*e*(c*e*g + 2*c*d*h - b*e*h)*x^2 + 2*c^3*e^2*h*x^3 + (6*(2*c
^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*(2*e*
f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e*(e*g
 + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h))))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + 3*(c^3*d*(2*e*f + d*g) - b^3*e^2*h + b*c*e*(b
*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*d*e*g + d
^2*h)))*Log[a + x*(b + c*x)])/(6*c^4)
```

Maple [B] time = 0.196, size = 1028, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/c^3*ln(c*x^2+b*x+a)*b^2*d*e*h-1/c^2*ln(c*x^2+b*x+a)*b*d*e*g+6/c^2/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e*h+2/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^2*f+1/2/c*x^2*e^2*g+1/c*d^2*h*x+1/c
*e^2*f*x-1/c^2*a*e^2*h*x+1/2/c*ln(c*x^2+b*x+a)*d^2*g+1/c^4/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^2*h-2/c^2*b*d*e*h*x-1/c^3/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2*g+1/c^3*ln(c*x^2+b
*x+a)*a*b*e^2*h-1/c^2*ln(c*x^2+b*x+a)*a*d*e*h-2/c/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*e*f-2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^3*d*e*h-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*a*b^2*e^2*h+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*a*b*e^2*g-4/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*a*d*e*g+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
b^2*d*e*g-1/2/c^2*ln(c*x^2+b*x+a)*a*e^2*g-1/2/c^2*ln(c*x^2+b*x+a)*b*e^2*f+1
/c*ln(c*x^2+b*x+a)*d*e*f+1/c^3*b^2*e^2*h*x+1/c*x^2*d*e*h-1/2/c^2*x^2*b*e^2*
h-1/c^2*b*e^2*g*x+2/c*d*e*g*x-1/2/c^4*ln(c*x^2+b*x+a)*b^3*e^2*h+1/2/c^3*ln(
```

```
c*x^2+b*x+a)*b^2*e^2*g-1/2/c^2*ln(c*x^2+b*x+a)*b*d^2*h+2/c^2/(4*a*c-b^2)^(1
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^2*h+1/c^2/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^2*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2*f-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*b*d^2*g-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*a*e^2*f-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*a*d^2*h+1/3*e^2*h*x^3/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.85181, size = 2600, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g + (2*(
b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 + 3*sqrt(b^2 - 4
*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*d^2 -
 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 - 2*a*c
^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^2)*h)
*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a)) + 6*((b^2*c^3 - 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*
e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 -
4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*((2*(b^2*c
^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*a*c^4)*d^2
 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*g -
 ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^
5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*h)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c
^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g +
```

```
(2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 - 6*sqrt(-b^
2 + 4*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*
d^2 - 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 -
2*a*c^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^
2)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*((b^2*c^3 -
 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g
 + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^
2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*((2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4
*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e +
 (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*g - ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*
(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2
)*h)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5)]
```

Sympy [B] time = 37.3905, size = 2839, normalized size = 8.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*
d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**
2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b
**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4
*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**
2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2
*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4))*log(x
 + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g + a*b**3*e*
*2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*a*b*c**2*d*
e*g + a*b*c**2*e**2*f + 4*a*c**4*(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h
- 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*
h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*
c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3
*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*
c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h +
b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*
g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f - b**2*c**3*
(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*
d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**
2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b
**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4
```

```
*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**
2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2
*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) + b*c
**3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*
a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4
*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d
*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f))
+ (sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2
*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e*
*2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*
b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**
4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c*
*2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h -
2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4))*log(
x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g + a*b**3*e
**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*a*b*c**2*d
*e*g + a*b*c**2*e**2*f + 4*a*c**4*(sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h
- 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*
h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*
c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3
*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*
c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h +
b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*
g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f - b**2*c**3*
(sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d
*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2
*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b*
*2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*
d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2
*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*
b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) + b*c*
*3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a
*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*
e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*
e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)) +
 e**2*h*x**3/(3*c) - x**2*(b*e**2*h - 2*c*d*e*h - c*e**2*g)/(2*c**2) - x*(a
*c*e**2*h - b**2*e**2*h + 2*b*c*d*e*h + b*c*e**2*g - c**2*d**2*h - 2*c**2*d
*e*g - c**2*e**2*f)/c**3
```

Giac [A] time = 1.25937, size = 575, normalized size = 1.65

$$\frac{2\,c^2hx^3e^2+6\,c^2dhx^2e+6\,c^2d^2hx+3\,c^2gx^2e^2-3\,bchx^2e^2+12\,c^2dgxe-12\,bcdhxe+6\,c^2fxe^2-6\,bcgxe^2+6\,b^2hxe^2-6\,a}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*h*x^3*e^2 + 6*c^2*d*h*x^2*e + 6*c^2*d^2*h*x + 3*c^2*g*x^2*e^2 -
3*b*c*h*x^2*e^2 + 12*c^2*d*g*x*e - 12*b*c*d*h*x*e + 6*c^2*f*x*e^2 - 6*b*c*g
*x*e^2 + 6*b^2*h*x*e^2 - 6*a*c*h*x*e^2)/c^3 + 1/2*(c^3*d^2*g - b*c^2*d^2*h
+ 2*c^3*d*f*e - 2*b*c^2*d*g*e + 2*b^2*c*d*h*e - 2*a*c^2*d*h*e - b*c^2*f*e^2
 + b^2*c*g*e^2 - a*c^2*g*e^2 - b^3*h*e^2 + 2*a*b*c*h*e^2)*log(c*x^2 + b*x +
 a)/c^4 + (2*c^4*d^2*f - b*c^3*d^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b*
c^3*d*f*e + 2*b^2*c^2*d*g*e - 4*a*c^3*d*g*e - 2*b^3*c*d*h*e + 6*a*b*c^2*d*h
*e + b^2*c^2*f*e^2 - 2*a*c^3*f*e^2 - b^3*c*g*e^2 + 3*a*b*c^2*g*e^2 + b^4*h*
e^2 - 4*a*b^2*c*h*e^2 + 2*a^2*c^2*h*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a
*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

3.150 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=177

$$\frac{\log\left(a+bx+cx^2\right)\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)}{2c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c^2(2adh+2aeg+bdg+bef)+bc(3\right.}{c^3\sqrt{b^2-4ac}}$$

```
[Out] ((c*e*g + c*d*h - b*e*h)*x)/c^2 + (e*h*x^2)/(2*c) - ((2*c^3*d*f - b^3*e*h -
 c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) + b*c*(b*e*g + b*d*h + 3*a*e*h))*A
rcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*(e*f
 + d*g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + b*x + c*x^2])/(2*c^3
)
```

Rubi [A] time = 0.349583, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log\left(a+bx+cx^2\right)\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)}{2c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c^2(2adh+2aeg+bdg+bef)+bc(3\right.}{c^3\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((c*e*g + c*d*h - b*e*h)*x)/c^2 + (e*h*x^2)/(2*c) - ((2*c^3*d*f - b^3*e*h -
 c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) + b*c*(b*e*g + b*d*h + 3*a*e*h))*A
rcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*(e*f
 + d*g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + b*x + c*x^2])/(2*c^3
)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int\left(\frac{ceg+cdh-beh}{c^2}+\frac{ehx}{c}+\frac{c^2df+abeh-ac(eg+dh)+\left(c^2(ef+dg)+b^2eh-c(beg\right.}{c^2\left(a+bx+cx^2\right)}\right.$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\int\frac{c^2df+abeh-ac(eg+dh)+\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)x}{a+bx+cx^2}\,dx}{c^2}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\log\left(a+bx+c\right.}{2c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}-\frac{\left(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+bc(beg+\right.}{c^3\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.210293, size = 173, normalized size = 0.98

$$\frac{\log(a+x(b+cx))\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)-\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c^2(2adh+2aeg+bdg+bef)-bc(3aeh+bdh+beg)+b^3eh-2\right.}{\sqrt{4ac-b^2}}}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*(c*e*g + c*d*h - b*e*h)*x + c^2*e*h*x^2 - (2*(-2*c^3*d*f + b^3*e*h + c
^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) - b*c*(b*e*g + b*d*h + 3*a*e*h))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*(e*f + d*g)
+ b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + x*(b + c*x)])/(2*c^3)
```

Maple [B] time = 0.18, size = 510, normalized size = 2.9

$$\frac{ehx^2}{2c} - \frac{behx}{c^2} + \frac{dhx}{c} + \frac{egx}{c} - \frac{\ln\left(cx^2+bx+a\right)aeh}{2c^2} + \frac{\ln\left(cx^2+bx+a\right)b^2eh}{2c^3} - \frac{\ln\left(cx^2+bx+a\right)bdh}{2c^2} - \frac{\ln\left(cx^2+bx+a\right)b}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2*e*h*x^2/c-1/c^2*b*e*h*x+1/c*d*h*x+1/c*e*g*x-1/2/c^2*ln(c*x^2+b*x+a)*a*e
*h+1/2/c^3*ln(c*x^2+b*x+a)*b^2*e*h-1/2/c^2*ln(c*x^2+b*x+a)*b*d*h-1/2/c^2*ln
(c*x^2+b*x+a)*b*e*g+1/2/c*ln(c*x^2+b*x+a)*d*g+1/2/c*ln(c*x^2+b*x+a)*e*f+3/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e*h-2/c/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*h-2/c/(4*a*c-b^2)^(1/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e*g+2/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*d*f-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b^3*e*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b^2*d*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b
^2*e*g-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*g-1/c/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02193, size = 1361, normalized size = 7.69

$$\left[\frac{\left(b^{2}c^{2}-4\,ac^{3}\right)ehx^{2}+\sqrt{b^{2}-4\,ac}\left(\left(2\,c^{3}d-bc^{2}e\right)f-\left(bc^{2}d-\left(b^{2}c-2\,ac^{2}\right)e\right)g+\left(\left(b^{2}c-2\,ac^{2}\right)d-\left(b^{3}-3\,abc\right)e\right)h\right)\log\left(\frac{2\,c}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*e*h*x^2 + sqrt(b^2 - 4*a*c)*((2*c^3*d - b*c^2*e)*
f - (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b
*c)*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x
 + b))/(c*x^2 + b*x + a)) + 2*((b^2*c^2 - 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^
3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 -
 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*
a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*(
(b^2*c^2 - 4*a*c^3)*e*h*x^2 - 2*sqrt(-b^2 + 4*a*c)*((2*c^3*d - b*c^2*e)*f -
 (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)
*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*((b^2*c^2
- 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b
^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g -
 ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b
*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] time = 12.0705, size = 1265, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h
 + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*
(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2
```

```
*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h + a*b*c*e*g
+ 4*a*c**3*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g
 - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*
f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2
*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g - 2*a*c**2*e*f - b**2*c**2*(-sqrt
(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**
2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c
 - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/
(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e
*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)) + (sq
rt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b
**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a
*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f
)/(2*c**3))*log(x + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*
a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b*
*3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2
*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g
- c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g - 2*a*c**2*e*f - b**2*c**2*(sqrt(-4*a*
c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*
h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**
2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**
3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b
**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)) + e*h*x**2/
(2*c) - x*(b*e*h - c*d*h - c*e*g)/c**2
```

Giac [A] time = 1.23855, size = 271, normalized size = 1.53

$$\frac{chx^2e + 2\,cdhx + 2\,cgxe - 2\,bhxe}{2\,c^2} + \frac{\left(c^2dg - bcdh + c^2fe - bcge + b^2he - ache\right)\log\left(cx^2 + bx + a\right)}{2\,c^3} + \frac{\left(2\,c^3df - bc^2dg + b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*h*x^2*e + 2*c*d*h*x + 2*c*g*x*e - 2*b*h*x*e)/c^2 + 1/2*(c^2*d*g - b*
c*d*h + c^2*f*e - b*c*g*e + b^2*h*e - a*c*h*e)*log(c*x^2 + b*x + a)/c^3 + (
2*c^3*d*f - b*c^2*d*g + b^2*c*d*h - 2*a*c^2*d*h - b*c^2*f*e + b^2*c*g*e - 2
*a*c^2*g*e - b^3*h*e + 3*a*b*c*h*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*c^3)
```

3.151 $\int \frac{f+gx+hx^2}{a+bx+cx^2} \, dx$

Optimal. Leaf size=92

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

```
[Out] (h*x)/c - ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(2
*c^2)
```

Rubi [A] time = 0.156163, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c - ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(2
*c^2)
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx &= \int \left(\frac{h}{c}+\frac{cf-ah+(cg-bh)x}{c\left(a+bx+cx^2\right)}\right)dx\\
&= \frac{hx}{c}+\frac{\int \frac{cf-ah+(cg-bh)x}{a+bx+cx^2}\,dx}{c}\\
&= \frac{hx}{c}+\frac{(cg-bh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(2c^2f-bcg+b^2h-2ach\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}\\
&= \frac{hx}{c}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(2c^2f-bcg+b^2h-2ach\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2}\\
&= \frac{hx}{c}-\frac{\left(2c^2f-bcg+b^2h-2ach\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.0710634, size = 95, normalized size = 1.03

$$\frac{\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{4ac-b^2}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c + ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(
2*c^2)
```

Maple [B] time = 0.175, size = 196, normalized size = 2.1

$$\frac{hx}{c} - \frac{\ln\left(cx^2+bx+a\right)bh}{2\,c^2} + \frac{\ln\left(cx^2+bx+a\right)g}{2\,c} - 2\,\frac{ah}{c\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right) + 2\,\frac{f}{\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] h*x/c-1/2/c^2*ln(c*x^2+b*x+a)*b*h+1/2/c*ln(c*x^2+b*x+a)*g-2/c/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*h+2/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*f+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b^2*h-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b*g
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62718, size = 670, normalized size = 7.28

$$\left[\frac{2\left(b^2c-4\,ac^2\right)hx-\left(2\,c^2f-bcg+\left(b^2-2\,ac\right)h\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+\left(\left(b^2c-4\,ac^2\right)g-\left(b\right.\right.}{2\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - 4*a*c^2)*h*x - (2*c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*log(
c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*h*x - 2*(2*
c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*
log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 2.03855, size = 488, normalized size = 5.3

$$\left(-\frac{\sqrt{-4ac+b^2}\left(2ach-b^2h+bcg-2c^2f\right)}{2c^2\left(4ac-b^2\right)}-\frac{bh-cg}{2c^2}\right)\log\left(x+\frac{-abh-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\left(2ach-b^2h+bcg-2c^2f\right)}{2c^2\left(4ac-b^2\right)}-\frac{bh-cg}{2c^2}\right)+2acg+}{2ach-b^2h+bcg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c
- b**2)) - (b*h - c*g)/(2*c**2))*log(x + (-a*b*h - 4*a*c**2*(-sqrt(-4*a*c +
 b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h
 - c*g)/(2*c**2)) + 2*a*c*g + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*
h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c
*f)/(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)) + (sqrt(-4*a*c + b**2)*(2*a*c*h
- b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)
)*log(x + (-a*b*h - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g
 - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) + 2*a*c*g + b*
*2*c*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*
a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c*f)/(2*a*c*h - b**2*h + b*c*g - 2
*c**2*f)) + h*x/c
```

Giac [A] time = 1.13261, size = 120, normalized size = 1.3

$$\frac{hx}{c}+\frac{\left(cg-bh\right)\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(2\,c^2f-bcg+b^2h-2\,ach\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] h*x/c + 1/2*(c*g - b*h)*log(c*x^2 + b*x + a)/c^2 + (2*c^2*f - b*c*g + b^2*h
 - 2*a*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.152 $\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=196

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg+cef)}{2c\left(ae^2-bde+cd^2\right)}$$

```
[Out] -(((2*c^2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*Ar
cTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
 a*e^2))) + ((e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^
2)) - ((c*e*f - c*d*g + b*d*h - a*e*h)*Log[a + b*x + c*x^2])/(2*c*(c*d^2 -
b*d*e + a*e^2))
```

Rubi [A] time = 0.348852, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg+cef)}{2c\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```


```
[Out] -(((2*c^2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*Ar
cTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
 a*e^2))) + ((e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^
2)) - ((c*e*f - c*d*g + b*d*h - a*e*h)*Log[a + b*x + c*x^2])/(2*c*(c*d^2 -
b*d*e + a*e^2))
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{cdf-bef+aeg-adh-(cef-cdg+bdh-aeh)x}{\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}\right) dx$$

$$= \frac{\left(e^2f-deg+d^2h\right)\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}+\frac{\int \frac{cdf-bef+aeg-adh-(cef-cdg+bdh-aeh)x}{a+bx+cx^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{\left(e^2f-deg+d^2h\right)\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}-\frac{(cef-cdg+bdh-aeh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c\left(cd^2-bde+ae^2\right)}+\frac{\left(2c^2df+b(bd\right.}{}$$

$$= \frac{\left(e^2f-deg+d^2h\right)\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}-\frac{(cef-cdg+bdh-aeh)\log\left(a+bx+cx^2\right)}{2c\left(cd^2-bde+ae^2\right)}-\frac{\left(2c^2df+\right.}{}$$

$$= -\frac{\left(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}+\frac{\left(e^2f-deg+\right.}{e\left(cd^2-\right.}$$

Mathematica [A] time = 0.24331, size = 193, normalized size = 0.98

$$\frac{-2e\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c(2adh-2aeg+bdg+bef)+bh(ae-bd)-2c^2df\right)+2c\sqrt{4ac-b^2}\log(d+ex)\left(d^2h-deg+e^2f\right)-e}{2ce\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*e*(-2*c^2*d*f + b*(-(b*d) + a*e)*h + c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d
*h))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*c*Sqrt[-b^2 + 4*a*c]*(e^2*f
 - d*e*g + d^2*h)*Log[d + e*x] - Sqrt[-b^2 + 4*a*c]*e*(c*e*f - c*d*g + b*d*
h - a*e*h)*Log[a + x*(b + c*x)])/(2*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*
d) + a*e)))
```

Maple [B] time = 0.183, size = 622, normalized size = 3.2

$$\frac{\ln\left(cx^2+bx+a\right)aeh}{\left(2\,ae^2-2\,bde+2\,cd^2\right)c}-\frac{\ln\left(cx^2+bx+a\right)bdh}{\left(2\,ae^2-2\,bde+2\,cd^2\right)c}+\frac{\ln\left(cx^2+bx+a\right)dg}{2\,ae^2-2\,bde+2\,cd^2}-\frac{\ln\left(cx^2+bx+a\right)ef}{2\,ae^2-2\,bde+2\,cd^2}-2\,\frac{adh}{\left(ae^2-bde+cd^2\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)/c*ln(c*x^2+b*x+a)*a*e*h-1/2/(a*e^2-b*d*e+c*d^2)/c*l
n(c*x^2+b*x+a)*b*d*h+1/2/(a*e^2-b*d*e+c*d^2)*ln(c*x^2+b*x+a)*d*g-1/2/(a*e^2
-b*d*e+c*d^2)*ln(c*x^2+b*x+a)*e*f-2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*h+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e*g-1/(a*e^2-b*d*e+c*d^2)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e*f+2/(a*e^2-b*d*e+c*d^2
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*d*f-1/(a*e^2-b*d*
e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b/c*a*e*h+1/
(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b
^2/c*d*h-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b*d*g+1/(a*e^2-b*d*e+c*d^2)/e*ln(e*x+d)*d^2*h-1/(a*e^2-b*d*e+c*d^
2)*ln(e*x+d)*d*g+1/(a*e^2-b*d*e+c*d^2)*e*ln(e*x+d)*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27565, size = 275, normalized size = 1.4

$$\frac{\left(cdg - bdh - cfe + ahe\right)\log\left(cx^2 + bx + a\right)}{2\left(c^2d^2 - bcde + ace^2\right)} + \frac{\left(d^2h - dge + fe^2\right)\log\left(|xe + d|\right)}{cd^2e - bde^2 + ae^3} + \frac{\left(2\,c^2df - bcdg + b^2dh - 2\,acdh - bcfe\right.}{\left(c^2d^2 - bcde + ace\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*d*g - b*d*h - c*f*e + a*h*e)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d*e
 + a*c*e^2) + (d^2*h - d*g*e + f*e^2)*log(abs(x*e + d))/(c*d^2*e - b*d*e^2
+ a*e^3) + (2*c^2*d*f - b*c*d*g + b^2*d*h - 2*a*c*d*h - b*c*f*e + 2*a*c*g*e
 - a*b*h*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a*
c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.153 $\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=316

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}-\frac{\mathrm{lo}}{}$$

```
[Out] -((e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2*
d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*d*(2
*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2) + ((c*d*(2*e*f - d
*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Log[d + e*x])/(c*d^2 - b*d*e +
 a*e^2)^2 - ((c*d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Lo
g[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rubi [A] time = 0.760674, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}-\frac{\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```


```
[Out] -((e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2*
d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*d*(2
*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2) + ((c*d*(2*e*f - d
*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Log[d + e*x])/(c*d^2 - b*d*e +
 a*e^2)^2 - ((c*d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Lo
g[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx = \int\left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{e\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}+\frac{c^2}{}\right.$$

$$= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)\log(d+}{\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)\log(d+}{\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)\log(d+}{\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2\left(e^2f+d^2h\right)-c\right.}{\sqrt{b^2-4ac}\,\left(c\right.}$$

Mathematica [A] time = 0.657377, size = 281, normalized size = 0.89

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)}{\sqrt{4ac-b^2}}-\frac{2\left(e(ae-bd)+cd^2\right)\left(d^2h-deg+e^2f\right)}{e(d+ex)}+2\log(d+ex)\left(\right.}{2\left(e(ae-bd)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*(c*d^2 + e*(-(b*d) + a*e))*(e^2*f - d*e*g + d^2*h))/(e*(d + e*x)) + (2
*(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c
*(b*d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(c*d*(2*e*f - d*g) + a*e*(e*g - 2
*d*h) + b*(-(e^2*f) + d^2*h))*Log[d + e*x] + (c*d*(-2*e*f + d*g) + a*e*(-(e
*g) + 2*d*h) + b*(e^2*f - d^2*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*
d) + a*e))^2)
```

Maple [B] time = 0.192, size = 1125, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x)
```

```
[Out] -1/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*b*e^2*f-1/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d
)*c*d^2*g+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*a*c*d*e*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*e*f-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e*h+1/2/(a*e^2-b*d*e+c*d^2)^2*
ln(c*x^2+b*x+a)*b*e^2*f+1/2/(a*e^2-b*d*e+c*d^2)^2*c*ln(c*x^2+b*x+a)*d^2*g-1
/(a*e^2-b*d*e+c*d^2)/e/(e*x+d)*d^2*h-1/2/(a*e^2-b*d*e+c*d^2)^2*ln(c*x^2+b*x
+a)*a*e^2*g-1/2/(a*e^2-b*d*e+c*d^2)^2*ln(c*x^2+b*x+a)*b*d^2*h+1/(a*e^2-b*d*
e+c*d^2)^2*ln(e*x+d)*a*e^2*g+1/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*b*d^2*h-1/(a
*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
*b*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*a*c*d^2*h-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*a*c*e^2*f-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d^2*g-1/(a*e^2-b*d*e+c*d^2)*e/(e*
x+d)*f+1/(a*e^2-b*d*e+c*d^2)/(e*x+d)*d*g+1/(a*e^2-b*d*e+c*d^2)^2*ln(c*x^2+b
*x+a)*a*d*e*h-1/(a*e^2-b*d*e+c*d^2)^2*c*ln(c*x^2+b*x+a)*d*e*f+2/(a*e^2-b*d*
e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^2*h+
1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*b^2*d^2*h+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))*b^2*e^2*f+2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*d^2*f-2/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*a*
d*e*h+2/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*d*e*c*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31722, size = 606, normalized size = 1.92

$$\frac{\left(2\,c^2d^2fe^2 - bcd^2ge^2 + b^2d^2he^2 - 2\,acd^2he^2 - 2\,bcdfe^3 + 4\,acdge^3 - 2\,abdhe^3 + b^2fe^4 - 2\,acfe^4 - abge^4 + 2\,a^2he^4\right)\text{arcta}}{\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] (2*c^2*d^2*f*e^2 - b*c*d^2*g*e^2 + b^2*d^2*h*e^2 - 2*a*c*d^2*h*e^2 - 2*b*c*
d*f*e^3 + 4*a*c*d*g*e^3 - 2*a*b*d*h*e^3 + b^2*f*e^4 - 2*a*c*f*e^4 - a*b*g*e
^4 + 2*a^2*h*e^4)*arctan((2*c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e +
d) - 2*a*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((c^2*d^4 - 2*b*c
*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4
*a*c)) + 1/2*(c*d^2*g - b*d^2*h - 2*c*d*f*e + 2*a*d*h*e + b*f*e^2 - a*g*e^2
)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e
+ d)^2 + a*e^2/(x*e + d)^2)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^
2*e^2 - 2*a*b*d*e^3 + a^2*e^4) - (d^2*h*e/(x*e + d) - d*g*e^2/(x*e + d) + f
*e^3/(x*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4)
```

3.154 $\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=509

$$-\frac{\log\left(a+bx+cx^2\right)\left(e^3\left(a^2h-abg+b^2f\right)-c\left(ae\left(3d^2h-3deg+e^2f\right)+b\left(3de^2f-d^3h\right)\right)+c^2d^2(3ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^3}+\frac{\log(d+ex}{}$$

```
[Out] -(e^2*f - d*e*g + d^2*h)/(2*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (c*d*(
2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))/((c*d^2 - b*d*e + a*e
^2)^2*(d + e*x)) - ((2*c^3*d^3*f - b*e^3*(b^2*f - a*b*g + a^2*h) - c^2*d*(b
*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(2*a^2*e^2*(e*g - 3
*d*h) - 3*a*b*e*(e^2*f - d*e*g - d^2*h) - b^2*(3*d*e^2*f + d^3*h)))*ArcTanh
[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)
^3) + ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - c*(a*e*(e^2*f
 - 3*d*e*g + 3*d^2*h) + b*(3*d*e^2*f - d^3*h)))*Log[d + e*x])/(c*d^2 - b*d*
e + a*e^2)^3 - ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - c*(a
*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*(3*d*e^2*f - d^3*h)))*Log[a + b*x + c*x^
2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

Rubi [A] time = 1.25078, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\log\left(a+bx+cx^2\right)\left(e^3\left(a^2h-abg+b^2f\right)-ace\left(3d^2h-3deg+e^2f\right)-bc\left(3de^2f-d^3h\right)+c^2d^2(3ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^3}+\frac{\log(d+ex)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -(e^2*f - d*e*g + d^2*h)/(2*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (c*d*(
2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))/((c*d^2 - b*d*e + a*e
^2)^2*(d + e*x)) - ((2*c^3*d^3*f - b*e^3*(b^2*f - a*b*g + a^2*h) - c^2*d*(b
*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(2*a^2*e^2*(e*g - 3
*d*h) - 3*a*b*e*(e^2*f - d*e*g - d^2*h) - b^2*(3*d*e^2*f + d^3*h)))*ArcTanh
[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)
^3) + ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*e*(e^2*f
- 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[d + e*x])/(c*d^2 - b*d*
```

```
e + a*e^2)^3 - ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*
e*(e^2*f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[a + b*x + c*x^
2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx = \int\left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{e\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)^2}+\frac{e\left(c\right.}{}\right.$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}+\frac{\left(c^2d^2(3\right.}{}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}+\frac{\left(c^2d^2(3\right.}{}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}+\frac{\left(c^2d^2(3\right.}{}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}-\frac{\left(2c^3d^3\right.}{}$$

Mathematica [A] time = 0.902485, size = 504, normalized size = 0.99

$$-\frac{\log(d+ex)\left(e^3\left(-\left(a^2h-abg+b^2f\right)\right)+ace\left(3d^2h-3deg+e^2f\right)+bc\left(3de^2f-d^3h\right)+c^2d^2(dg-3ef)\right)}{\left(e(ae-bd)+cd^2\right)^3}+\frac{\log(a+x(b+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -(e^2*f - d*e*g + d^2*h)/(2*e*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (c*
d*(-2*e*f + d*g) + a*e*(-(e*g) + 2*d*h) + b*(e^2*f - d^2*h))/((c*d^2 + e*(-
(b*d) + a*e))^2*(d + e*x)) + ((-2*c^3*d^3*f + b*e^3*(b^2*f - a*b*g + a^2*h)
 + c^2*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(-2*a^2*
e^2*(e*g - 3*d*h) + 3*a*b*e*(e^2*f - d*e*g - d^2*h) + b^2*(3*d*e^2*f + d^3*
h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(-(c*d^2)
+ e*(b*d - a*e))^3) - ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f - a*b*g + a^2*h
) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3*h))*Log[d + e*
x])/(c*d^2 + e*(-(b*d) + a*e))^3 + ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f -
a*b*g + a^2*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3*h
))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Maple [B] time = 0.194, size = 1945, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x)
```

```
[Out] -2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*a*c^2*d^3*h+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*b^2*c*d^3*h-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d^3*g-3/2/(a*e^2-b*d*e+c*d^2)^3*c*l
n(c*x^2+b*x+a)*a*d*e^2*g+3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*c*d*e^2*g-3/(a
*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*c*d*e^2*f-3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)
*a*c*d^2*e*h+3/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)*b*d*e^2*f+3/2/(a*e
^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)*a*d^2*e*h-3/(a*e^2-b*d*e+c*d^2)^3/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*d*e^2*g-3/(a*e^2-b*d
*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*d^2
*e*h+1/2/(a*e^2-b*d*e+c*d^2)^3*c^2*ln(c*x^2+b*x+a)*d^3*g-1/2/(a*e^2-b*d*e+c
*d^2)^3*ln(c*x^2+b*x+a)*a^2*e^3*h-1/2/(a*e^2-b*d*e+c*d^2)^3*ln(c*x^2+b*x+a)
*b^2*e^3*f-1/2/(a*e^2-b*d*e+c*d^2)/e/(e*x+d)^2*d^2*h+1/(a*e^2-b*d*e+c*d^2)^
3*ln(e*x+d)*a^2*e^3*h+1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b^2*e^3*f-1/(a*e^2-
b*d*e+c*d^2)^3*ln(e*x+d)*c^2*d^3*g-1/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*a*e^2*g-
1/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*b*d^2*h+1/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*b*e
^2*f+1/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*c*d^2*g+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d^2*e*g-6/(a*e^2-b*d*
e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d*e^
2*f+3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*b^2*c*d*e^2*f-3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d^2*e*f+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c*d*e^2*h+3/(a*e^2-b*d*e+c*d^2
)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*e^3*f+1/2/(
a*e^2-b*d*e+c*d^2)/(e*x+d)^2*d*g-1/2/(a*e^2-b*d*e+c*d^2)*e/(e*x+d)^2*f-1/(a
*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
^2*b*e^3*h+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*a*b^2*e^3*g-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c*e^3*g+3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*
c^2*d^2*e*f+2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*a*d*e*h-1/(a*e^2-b*d*e+c*d^2)^3
*ln(e*x+d)*a*b*e^3*g-1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*c*e^3*f+1/(a*e^2-b
*d*e+c*d^2)^3*ln(e*x+d)*b*c*d^3*h-2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*d*e*c*f-1
/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^3*e^3*f+2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*c^3*d^3*f+1/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)*a*e^3*
f-1/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)*b*d^3*h-3/2/(a*e^2-b*d*e+c*d^
```

```
2)^3*c^2*ln(c*x^2+b*x+a)*d^2*e*f+1/2/(a*e^2-b*d*e+c*d^2)^3*ln(c*x^2+b*x+a)*
a*b*e^3*g
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**3/(c*x**2+b*x+a),x)

[Out] Timed out
```

Giac [B] time = 1.26058, size = 1353, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c^2*d^3*g - b*c*d^3*h - 3*c^2*d^2*f*e + 3*a*c*d^2*h*e + 3*b*c*d*f*e^2
- 3*a*c*d*g*e^2 - b^2*f*e^3 + a*c*f*e^3 + a*b*g*e^3 - a^2*h*e^3)*log(c*x^2
+ b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b
^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*
d*e^5 + a^3*e^6) - (c^2*d^3*g*e - b*c*d^3*h*e - 3*c^2*d^2*f*e^2 + 3*a*c*d^2
*h*e^2 + 3*b*c*d*f*e^3 - 3*a*c*d*g*e^3 - b^2*f*e^4 + a*c*f*e^4 + a*b*g*e^4
- a^2*h*e^4)*log(abs(x*e + d))/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e
^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*
a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) + (2*c^3*d^3*f - b*c^2*d^3*g + b^2
*c*d^3*h - 2*a*c^2*d^3*h - 3*b*c^2*d^2*f*e + 6*a*c^2*d^2*g*e - 3*a*b*c*d^2*
h*e + 3*b^2*c*d*f*e^2 - 6*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 + 6*a^2*c*d*h*e^2
 - b^3*f*e^3 + 3*a*b*c*f*e^3 + a*b^2*g*e^3 - 2*a^2*c*g*e^3 - a^2*b*h*e^3)*a
rctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d
^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4
+ 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^2 + 4*a*c)) - 1/2*(c^2
*d^6*h - 3*c^2*d^5*g*e + 5*c^2*d^4*f*e^2 + 4*b*c*d^4*g*e^2 - b^2*d^4*h*e^2
- 2*a*c*d^4*h*e^2 - 8*b*c*d^3*f*e^3 - b^2*d^3*g*e^3 - 2*a*c*d^3*g*e^3 + 4*a
*b*d^3*h*e^3 + 3*b^2*d^2*f*e^4 + 6*a*c*d^2*f*e^4 - 3*a^2*d^2*h*e^4 - 4*a*b*
d*f*e^5 + a^2*d*g*e^5 + a^2*f*e^6 - 2*(c^2*d^4*g*e^2 - b*c*d^4*h*e^2 - 2*c^
2*d^3*f*e^3 - b*c*d^3*g*e^3 + b^2*d^3*h*e^3 + 2*a*c*d^3*h*e^3 + 3*b*c*d^2*f
*e^4 - 3*a*b*d^2*h*e^4 - b^2*d*f*e^5 - 2*a*c*d*f*e^5 + a*b*d*g*e^5 + 2*a^2*
d*h*e^5 + a*b*f*e^6 - a^2*g*e^6)*x)*e^(-1)/((c*d^2 - b*d*e + a*e^2)^3*(x*e
+ d)^2)
```

3.155 $\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=288

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(b^2ce(12aeh+2bdh+beg)-c^3\left(2bd(dg+2ef)-4a\left(d^2h+2deg+e^2f\right)\right)-6ac^2e(2aeh+2bdh+beg)-\right.}{c^3\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (e^2*(2*c^2*f - b*c*g + 2*b^2*h - 6*a*c*h)*x)/(c^2*(b^2 - 4*a*c)) + ((d + e
*x)^2*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)
)/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c
^2*e*(b*e*g + 2*b*d*h + 2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) - c
^3*(2*b*d*(2*e*f + d*g) - 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) + (e*(c*e*g + 2*c*d*h - 2*
b*e*h)*Log[a + b*x + c*x^2])/(2*c^3)
```

Rubi [A] time = 0.700316, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1644, 773, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(b^2ce(12aeh+2bdh+beg)-c^3\left(2bd(dg+2ef)-4a\left(d^2h+2deg+e^2f\right)\right)-6ac^2e(2aeh+2bdh+beg)-\right.}{c^3\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```



```
[Out] (e^2*(2*c^2*f - b*c*g + 2*b^2*h - 6*a*c*h)*x)/(c^2*(b^2 - 4*a*c)) + ((d + e
*x)^2*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)
)/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c
^2*e*(b*e*g + 2*b*d*h + 2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) - c
^3*(2*b*d*(2*e*f + d*g) - 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) + (e*(c*e*g + 2*c*d*h - 2*
b*e*h)*Log[a + b*x + c*x^2])/(2*c^3)
```


Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
```

```
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \int \frac{(d+ex)\left(2cdf-2bef-bdg+\right.}{}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)} + \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)} + \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)} + \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)} + \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Mathematica [A] time = 0.949623, size = 398, normalized size = 1.38

$$-\frac{\frac{2\left(bc\left(-3a^2e^2h+ac\left(d^2h+2de(g+3hx)+e^2(f+3gx)\right)+c^2d(d(f-gx)-2efx)\right)+2c^2\left(a^2e(2dh+e(g+hx))-ac\left(d^2(g+hx)+2de(f+gx)+e^2fx\right)+c^2d^2fx\right)+b^2c\left(cx\left(d^2h+2deg+e^2\right.\right.\right.}{\left(b^2-4ac\right)(a+x(b+cx))}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*h*x - (2*(b^4*e^2*h*x + b^3*e*(a*e*h - c*(e*g + 2*d*h)*x) + b^2*c*
(c*(e^2*f + 2*d*e*g + d^2*h)*x - a*e*(e*g + 2*d*h + 4*e*h*x)) + 2*c^2*(c^2*
d^2*f*x - a*c*(e^2*f*x + 2*d*e*(f + g*x) + d^2*(g + h*x)) + a^2*e*(2*d*h +
e*(g + h*x))) + b*c*(-3*a^2*e^2*h + c^2*d*(-2*e*f*x + d*(f - g*x)) + a*c*(d
^2*h + e^2*(f + 3*g*x) + 2*d*e*(g + 3*h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c
*x))) + (2*(4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c^2*e*(b*e*g + 2*b*d*h + 2*a*e*
h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) + c^3*(-2*b*d*(2*e*f + d*g) + 4*a
*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2
+ 4*a*c)^(3/2) + e*(c*e*g + 2*c*d*h - 2*b*e*h)*Log[a + x*(b + c*x)])/(2*c^3
)
```

Maple [B] time = 0.194, size = 1712, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*d*e*g+3/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b
*e^2*g+e^2*h/c^2*x+6/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*d*e*h-2/(4*a*c-b^2)^
(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^2*g+4/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e^2*f-2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*d^2*g+
1/(c*x^2+b*x+a)/(4*a*c-b^2)*b*d^2*f+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*a*d^2*h+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*d^2*f-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b^2*e^2*h-2/c^2/(c*x^2+b*
x+a)/(4*a*c-b^2)*x*b^3*d*e*h-2/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*d*e*h+2/
c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*d*e*g-12/c/(4*a*c-b^2)^(3/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e*h+4/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*d*e*h+1
/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*e^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*
b^2*d^2*h-2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d*e*f-4/(c*x^2+b*x+a)/(4*a*c-b^2)
*x*a*d*e*g-1/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^3*e^2*g-1/c^2/(4*a*c-b^2)*ln
(c*x^2+b*x+a)*b^2*d*e*h+12/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a*b^2*e^2*h+2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*b^3*d*e*h+4/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*d*e*h-6/c/(4*a*c-b^2)^(3
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e^2*g-4/c^2/(4*a*c-b^2)*ln(c*x^
2+b*x+a)*a*b*e^2*h+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2*e^2*h+1/c^3/(c*x^2+b
*x+a)/(4*a*c-b^2)*x*b^4*e^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*d^2*h-3/c^2
/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*e^2*h-1/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^
2*e^2*g+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*e^2*f+1/c/(c*x^2+b*x+a)/(4*a*c-
b^2)*a*b*e^2*f-1/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d^2*g-2/(c*x^2+b*x+a)/(4*a*c
-b^2)*x*a*d^2*h-2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*e^2*f-4/(c*x^2+b*x+a)/(4*a*
c-b^2)*a*d*e*f-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*e^2*g+1/c^2/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2*g-2/c^3/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^2*h+2/c/(4*a*c-b^2)*ln(c*x^2
+b*x+a)*a*e^2*g+8/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d
*e*g-4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*e*f+2*c/(c
*x^2+b*x+a)/(4*a*c-b^2)*x*d^2*f+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*e^2*g+1/c
^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3*e^2*h-12/c/(4*a*c-b^2)^(3/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^2*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.94007, size = 5646, normalized size = 19.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + ((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e^2)*f -
(2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*c^4*d^2
+ (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2)*h)*x^2
 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8*a^2*c^3
*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c - 6*a^2
*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*d^2 - b^
2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*c - 6*a*
b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (b^5 - 6*a
*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
 + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^
3*c^3 - 4*a*b*c^4)*d^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3*c^2 - 4*a^2
*b*c^3)*e^2)*f + 2*(2*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*
c^3)*d*e + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*((a*b^3*c^2 - 4
*a^2*b*c^3)*d^2 - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*e + (a*b^5 - 7*
a^2*b^3*c + 12*a^3*b*c^2)*e^2)*h - 2*((2*(b^2*c^4 - 4*a*c^5)*d^2 - 2*(b^3*c
^3 - 4*a*b*c^4)*d*e + (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*e^2)*f - ((b^3*c^
3 - 4*a*b*c^4)*d^2 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*e + (b^5*c - 7
*a*b^3*c^2 + 12*a^2*b*c^3)*e^2)*g + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^
2 - 2*(b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*d*e + (b^6 - 9*a*b^4*c + 26*a^2*
b^2*c^2 - 24*a^3*c^3)*e^2)*h)*x + ((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e
^2*g + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*g + 2*((b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d*e - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2)*h)*x^2 +
2*((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d*e - (a*b^5 - 8*a^2*b^3*c + 16*a
^3*b*c^2)*e^2)*h + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*g + 2*((b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^2)
*h)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4
*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^
```

```
5)*x), 1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + 2*((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e
^2)*f - (2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*
c^4*d^2 + (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2
)*h)*x^2 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8
*a^2*c^3*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c
 - 6*a^2*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*
d^2 - b^2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*
c - 6*a*b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (b
^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*((b^3*c^3 - 4*a*b*c^4)*d^2 - 4*(
a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3*c^2 - 4*a^2*b*c^3)*e^2)*f + 2*(2*(a*b^2
*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e + (a*b^4*c - 6*a^2*
b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*((a*b^3*c^2 - 4*a^2*b*c^3)*d^2 - 2*(a*b^4*c
 - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*e + (a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e^
2)*h - 2*((2*(b^2*c^4 - 4*a*c^5)*d^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d*e + (b^4*c
^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*e^2)*f - ((b^3*c^3 - 4*a*b*c^4)*d^2 - 2*(b^4*
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*e + (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e
^2)*g + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2 - 2*(b^5*c - 7*a*b^3*c^2 +
 12*a^2*b*c^3)*d*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*e^2)*h
)*x + ((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e^2*g + ((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*e^2*g + 2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e - (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2)*h)*x^2 + 2*((a*b^4*c - 8*a^2*b^2*c^2
+ 16*a^3*c^3)*d*e - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e^2)*h + ((b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*g + 2*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d*e - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^2)*h)*x)*log(c*x^2 + b*x + a))
/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*
c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30768, size = 729, normalized size = 2.53

$$\frac{hxe^2}{c^2} - \frac{\left(4\,c^4d^2f - 2\,bc^3d^2g + 4\,ac^3d^2h - 4\,bc^3dfe + 8\,ac^3dge + 2\,b^3cdhe - 12\,abc^2dhe + 4\,ac^3fe^2 + b^3cge^2 - 6\,abc^2ge^2 - 2\right.}{\left(b^2c^3 - 4\,ac^4\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] h*x*e^2/c^2 - (4*c^4*d^2*f - 2*b*c^3*d^2*g + 4*a*c^3*d^2*h - 4*b*c^3*d*f*e
+ 8*a*c^3*d*g*e + 2*b^3*c*d*h*e - 12*a*b*c^2*d*h*e + 4*a*c^3*f*e^2 + b^3*c*
g*e^2 - 6*a*b*c^2*g*e^2 - 2*b^4*h*e^2 + 12*a*b^2*c*h*e^2 - 12*a^2*c^2*h*e^2
)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4
*a*c)) + 1/2*(2*c*d*h*e + c*g*e^2 - 2*b*h*e^2)*log(c*x^2 + b*x + a)/c^3 - (
(2*c^4*d^2*f - b*c^3*d^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b*c^3*d*f*e
+ 2*b^2*c^2*d*g*e - 4*a*c^3*d*g*e - 2*b^3*c*d*h*e + 6*a*b*c^2*d*h*e + b^2*c
^2*f*e^2 - 2*a*c^3*f*e^2 - b^3*c*g*e^2 + 3*a*b*c^2*g*e^2 + b^4*h*e^2 - 4*a*
b^2*c*h*e^2 + 2*a^2*c^2*h*e^2)*x/c + (b*c^3*d^2*f - 2*a*c^3*d^2*g + a*b*c^2
*d^2*h - 4*a*c^3*d*f*e + 2*a*b*c^2*d*g*e - 2*a*b^2*c*d*h*e + 4*a^2*c^2*d*h*
e + a*b*c^2*f*e^2 - a*b^2*c*g*e^2 + 2*a^2*c^2*g*e^2 + a*b^3*h*e^2 - 3*a^2*b
*c*h*e^2)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

3.156 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=178

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg))-6\right.}{c^2\left(b^2-4ac\right)^{3/2}}$$

```
[Out] ((d + e*x)*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*
h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^3*d*f + b^3*e*h - 6*a*b*
c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + (e*h*Log[a + b*x + c*x^2])/(2*c^2
)
```

Rubi [A] time = 0.265863, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1644, 634, 618, 206, 628}

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg))-6\right.}{c^2\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```



```
[Out] ((d + e*x)*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*
h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^3*d*f + b^3*e*h - 6*a*b*
c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + (e*h*Log[a + b*x + c*x^2])/(2*c^2
)
```


Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{2cdf-b(ef+dg)-\frac{abeh}{c}+2a(}{a+bx+c}}{-b^2+}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(eh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} - \frac{(4}{}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{eh\log\left(a+bx+cx^2\right)}{2c^2}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\left(4c^3df+b^3eh-6abce\right.}{}$$

Mathematica [A] time = 0.539798, size = 225, normalized size = 1.26

$$\frac{-\frac{2\left(2c\left(a^2eh-ac(d(g+hx)+e(f+gx))+c^2dfx\right)+b^2(cx(dh+eg)-aeh)+bc(adh+ae(g+3hx)+cd(f-gx)-cefx)+b^3(-e)hx\right)}{\left(b^2-4ac\right)(a+x(b+cx))} + \frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-2c^2(b(dg+ef)-2a(dh+\right.}{\left(4ac-b^2\right)^{3/2}}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((-2*(-(b^3*e*h*x) + b^2*(-(a*e*h) + c*(e*g + d*h)*x) + b*c*(a*d*h - c*e*f*
x + c*d*(f - g*x) + a*e*(g + 3*h*x)) + 2*c*(a^2*e*h + c^2*d*f*x - a*c*(e*(f
 + g*x) + d*(g + h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(4*c^3*d*f
 + b^3*e*h - 6*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + e*h*Log[a + x*(b +
c*x)])/(2*c^2)
```

Maple [B] time = 0.176, size = 500, normalized size = 2.8

$$\frac{1}{cx^2+bx+a}\left(\frac{\left(3\,abceh-2\,ac^2dh-2\,ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2\,c^3df\right)x}{c^2\left(4\,ac-b^2\right)} + \frac{2\,a^2ceh-ab^2eh+abc}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] ((3*a*b*c*e*h-2*a*c^2*d*h-2*a*c^2*e*g-b^3*e*h+b^2*c*d*h+b^2*c*e*g-b*c^2*d*g
-b*c^2*e*f+2*c^3*d*f)/c^2/(4*a*c-b^2)*x+(2*a^2*c*e*h-a*b^2*e*h+a*b*c*d*h+a*
b*c*e*g-2*a*c^2*d*g-2*a*c^2*e*f+b*c^2*d*f)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a)+2
/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*e*h-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^
2*e*h-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e*h+4/(
4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*h+4/(4*a*c-b^2)^(3
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e*g-2/(4*a*c-b^2)^(3/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*g-2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*b*e*f+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*d*f+1/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.70378, size = 2957, normalized size = 16.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((2*(2*c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d +
(b^3*c - 6*a*b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*d
- 2*a^2*c^2*e)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3*d
- b^2*c^2*e)*f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 - 6*a*
b^2*c)*e)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c +
sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^3*c^2 - 4*a*b*c^3
)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - (a*
b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b*c^2)*d - (a*b^4 - 6*a^2*b
```

```
^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3
)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e)*g
+ ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*e
)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*x^2 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e*h)*log(c*x^2 + b*
x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x), 1/2*(2*((2*(2*
c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d + (b^3*c - 6*a*
b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*d - 2*a^2*c^2*e
)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3*d - b^2*c^2*e)*
f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 - 6*a*b^2*c)*e)*h)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 - 2*((b^3*c^2 - 4*a*b*c^3)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^
2*c^2 - 4*a^2*c^3)*d - (a*b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b
*c^2)*d - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4
)*d - (b^3*c^2 - 4*a*b*c^3)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*
b^2*c^2 + 8*a^2*c^3)*e)*g + ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7
*a*b^3*c + 12*a^2*b*c^2)*e)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*
x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)*e*h)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 +
 (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*x)]
```

Sympy [B] time = 53.1521, size = 1535, normalized size = 8.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] (e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*
c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-16*a**2*c*
*3*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4
*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8*a**2*c*e*h +
8*a*b**2*c**2*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c
**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*
f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b*
*2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2) - sqrt(-(4*a*c -
b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*
```

```
d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c**2*d*f)/(6*a*b*c*
e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f
- 4*c**3*d*f)) + (e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*
a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3
*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log
(x + (-16*a**2*c**3*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h -
 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c
**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))
+ 8*a**2*c*e*h + 8*a*b**2*c**2*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*
a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**
2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6))) - a*b**2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2)
+ sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**
3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c*
*2*d*f)/(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*
g + 2*b*c**2*e*f - 4*c**3*d*f)) + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h +
a*b*c*e*g - 2*a*c**2*d*g - 2*a*c**2*e*f + b*c**2*d*f + x*(3*a*b*c*e*h - 2*a
*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
- b*c**2*e*f + 2*c**3*d*f))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b
**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))
```

Giac [A] time = 1.34085, size = 385, normalized size = 2.16

$$\frac{he\log\left(cx^2+bx+a\right)}{2\,c^2} - \frac{\left(4\,c^3df - 2\,bc^2dg + 4\,ac^2dh - 2\,bc^2fe + 4\,ac^2ge + b^3he - 6\,abche\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}} - \frac{bc^2df - 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*h*e*log(c*x^2 + b*x + a)/c^2 - (4*c^3*d*f - 2*b*c^2*d*g + 4*a*c^2*d*h -
 2*b*c^2*f*e + 4*a*c^2*g*e + b^3*h*e - 6*a*b*c*h*e)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d*f - 2*a
*c^2*d*g + a*b*c*d*h - 2*a*c^2*f*e + a*b*c*g*e - a*b^2*h*e + 2*a^2*c*h*e +
(2*c^3*d*f - b*c^2*d*g + b^2*c*d*h - 2*a*c^2*d*h - b*c^2*f*e + b^2*c*g*e -
2*a*c^2*g*e - b^3*h*e + 3*a*b*c*h*e)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^
2)
```

3.157 $\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2} \, dx$

Optimal. Leaf size=118

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)/(c*(b
^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c*f - b*g + 2*a*h)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0979572, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1660, 12, 618, 206}

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)/(c*(b
^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c*f - b*g + 2*a*h)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx &= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{2cf-bg+2ah}{a+bx+cx^2}\,dx}{-b^2+4ac}\\
&= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2cf-bg+2ah)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\
&= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(2(2cf-bg+2ah))\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx\right.}{b^2-4ac}\\
&= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2(2cf-bg+2ah)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.114411, size = 114, normalized size = 0.97

$$\frac{abh-2ac(g+hx)+b^2hx+bc(f-gx)+2c^2fx}{c\left(4ac-b^2\right)(a+x(b+cx))}-\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)(-2ah+bg-2cf)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (a*b*h + 2*c^2*f*x + b^2*h*x + b*c*(f - g*x) - 2*a*c*(g + h*x))/(c*(-b^2 +
4*a*c)*(a + x*(b + c*x))) - (2*(-2*c*f + b*g - 2*a*h)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.187, size = 194, normalized size = 1.6

$$\frac{1}{cx^2+bx+a}\left(-\frac{\left(2\,ach-b^2h+bcg-2\,c^2f\right)x}{c\left(4\,ac-b^2\right)}+\frac{abh-2\,acg+bcf}{c\left(4\,ac-b^2\right)}\right)+4\,\frac{ah}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-2\,\frac{bg}{\left(4\,ac-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] (-(2*a*c*h-b^2*h+b*c*g-2*c^2*f)/c/(4*a*c-b^2)*x+1/c*(a*b*h-2*a*c*g+b*c*f)/(
4*a*c-b^2))/(c*x^2+b*x+a)+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*a*h-2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*g+4/(4
*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.5575, size = 1335, normalized size = 11.31

$$\left[-\frac{\left(2\,ac^2f-abcg+2\,a^2ch+\left(2\,c^3f-bc^2g+2\,ac^2h\right)x^2+\left(2\,bc^2f-b^2cg+2\,abch\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b}}{cx^2+bx+a}\right.}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+\left(b^4c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-((2*a*c^2*f - a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 +
 (2*b*c^2*f - b^2*c*g + 2*a*b*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*
b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (
b^3*c - 4*a*b*c^2)*f - 2*(a*b^2*c - 4*a^2*c^2)*g + (a*b^3 - 4*a^2*b*c)*h +
(2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g + (b^4 - 6*a*b^2*c + 8*a^2
*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), (2*(2*a*c^2*f
- a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 + (2*b*c^2*f -
b^2*c*g + 2*a*b*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) - (b^3*c - 4*a*b*c^2)*f + 2*(a*b^2*c - 4*a^2*c^2)*g -
 (a*b^3 - 4*a^2*b*c)*h - (2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g +
 (b^4 - 6*a*b^2*c + 8*a^2*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3
+ (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*x)]
```

Sympy [B] time = 2.31425, size = 459, normalized size = 3.89

$$-\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2ah-bg+2cf\right)\log\left(x+\frac{-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2ah-bg+2cf\right)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2ah-bg+2cf\right)+\cdots}{4ach-2bcg+4c^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (-16*a**2*c**2*sq
rt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 8*a*b**2*c*sqrt(-1/(4*a*c
- b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h - b**4*sqrt(-1/(4*a*c - b**2)**
3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*f)
) + sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (16*a**2*c**2*
sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) - 8*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h + b**4*sqrt(-1/(4*a*c - b**2)
**3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*
f)) - (-a*b*h + 2*a*c*g - b*c*f + x*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f))/
(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*b*c**2 - b**
3*c))
```

Giac [A] time = 1.16536, size = 169, normalized size = 1.43

$$-\frac{2\left(2\,cf - bg + 2\,ah\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2 - 4\,ac\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,c^2fx - bcgx + b^2hx - 2\,achx + bcf - 2\,acg + abh}{\left(b^2c - 4\,ac^2\right)\left(cx^2 + bx + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(2*c*f - b*g + 2*a*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a
*c)*sqrt(-b^2 + 4*a*c)) - (2*c^2*f*x - b*c*g*x + b^2*h*x - 2*a*c*h*x + b*c*
f - 2*a*c*g + a*b*h)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

3.158 $\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=407

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)-deg+e^2f\cdots\right.\right.\right.}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}$$

```
[Out] (b^2*e*f - b*(c*d*f + a*e*g + a*d*h) - 2*a*(c*e*f - c*d*g - a*e*h) - (2*c^2
*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*x)/((b^2 -
4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) + ((4*c^3*d^3*f + b*e*(4*
a*b*d*e*h - 2*a^2*e^2*h + b^2*(e^2*f - d*e*g - d^2*h)) - 2*c^2*d*(b*d*(3*e*
f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + 2*c*e*(2*b^2*d^2*g + 2*a^2*e*(e
*g - d*h) - a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^2) + (e*(e^2*f - d*e*
g + d^2*h)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^2 - (e*(e^2*f - d*e*g + d^
2*h)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rubi [A] time = 1.08674, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1646, 800, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)-deg+e^2f\cdots\right.\right.\right.}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (b^2*e*f - b*(c*d*f + a*e*g + a*d*h) - 2*a*(c*e*f - c*d*g - a*e*h) - (2*c^2
*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*x)/((b^2 -
4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) + ((4*c^3*d^3*f + b*e*(4*
a*b*d*e*h - 2*a^2*e^2*h + b^2*(e^2*f - d*e*g - d^2*h)) - 2*c^2*d*(b*d*(3*e*
f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + 2*c*e*(2*b^2*d^2*g + 2*a^2*e*(e
*g - d*h) - a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^2) + (e*(e^2*f - d*e*
g + d^2*h)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^2 - (e*(e^2*f - d*e*g + d^
2*h)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(bef+bdg\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(bef+bdg\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(bef+bdg\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(bef+bdg\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(bef+bdg\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(bef+bdg\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

Mathematica [A] time = 1.14962, size = 405, normalized size = 1.

$$-\frac{\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2ce\left(2a^2e(dh-eg)+ab\left(d^2h+deg+3e^2f\right)-2b^2d^2g\right)+be\left(2a^2e^2h-4abdeh+b^2\left(d^2h+deg-e^2f\right)\right)+\right.}{\left(4ac-b^2\right)^{3/2}\left(e(ae-bd)+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-2*a^2*e*h + 2*c^2*d*f*x + b^2*(-(e*f) + d*h*x) + b*c*(-(e*f*x) + d*(f - g
*x)) + a*b*(d*h + e*(g - h*x)) + 2*a*c*(e*(f + g*x) - d*(g + h*x)))/((b^2 -
 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x))) - ((-4*c^3*d^3*f + 2*
c^2*d*(b*d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + b*e*(-4*a*b*d*e
*h + 2*a^2*e^2*h + b^2*(-(e^2*f) + d*e*g + d^2*h)) + 2*c*e*(-2*b^2*d^2*g +
2*a^2*e*(-(e*g) + d*h) + a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2) +
```

```
(e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^2 - (e
*(e^2*f - d*e*g + d^2*h)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e)
)^2)
```

Maple [B] time = 0.199, size = 3202, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*c*d^2*e*h-3/(a*e^2-
b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*c*d*e^2*g-2/(a*e^2-b*d*e+c*d
^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c*d*e^2*g-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^
2+b*x+a)/(4*a*c-b^2)*a*b^2*d^2*e*h+e^3/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*f-4/
(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*a^2*c*d*e^2*h+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))*a*b^2*d*e^2*h-6/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*e^3*f-4/(a*e^2-b*d*e+c*d^2)^2/(4*a
*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d^2*e*g+12/(a*e^2-b
*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d
*e^2*f-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*c*e^3*f+2/(a
*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*c^2*d^2*e*g+2/(a*e^2-b*d*
e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*c^2*d*e^2*f-1/(a*e^2-b*d*e+c*d^2)^
2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c*d*e^2*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*
x+a)/(4*a*c-b^2)*x*b^2*c*d^2*e*g+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a
*c-b^2)*x*b^2*c*d*e^2*f-3/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x
*b*c^2*d^2*e*f-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2*c*d*
e^2*h+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b^2*d*e^2*h-2/(
a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
a*b*c*d*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*a*b*c*d^2*e*h-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c
-b^2)*b^2*c*d^2*e*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2
*c*d^3*h-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b*c^2*d^3*g+e/
(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*d^2*h-e^2/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*d
*g+3/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c*d^2*e*g+3/(a*e^2
-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*d*e^2*h-2/(a*e^2-b*d*e+c*d^
2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c*d^2*e*h+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2
+b*x+a)/(4*a*c-b^2)*a*c^2*d^2*e*f-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*
a*c-b^2)*x*a^2*b*e^3*h+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*
a^2*c*e^3*g-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*c^2*d^3*h
```

```
-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^3*d^2*e*h+4/(a*e^2-b
*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*d
^2*e*g-6/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b*c^2*d^2*e*f-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+
a)*a*d^2*e*h+2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*a*d*e^2*
g-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*d*e^2*g+1/(a*e^2-
b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c*d^3*h+1/(a*e^2-b*d*e+c*d^2)^
2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*e^3*g+1/2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^
2)*ln(c*x^2+b*x+a)*b^2*e^3*f+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3*d^3*f+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^
2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3*f-2/(a*e^2-b*d*e+c*d^2
)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*e^3*h+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+
a)/(4*a*c-b^2)*b^3*d*e^2*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2
)*b*c^2*d^3*f+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*c^3*d^3*f
-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*a^2*b*e^3*h+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*a^2*c*e^3*g+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*ar
ctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d^3*h-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*
c-b^2)*c*ln(c*x^2+b*x+a)*a*e^3*f+1/2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*ln(c
*x^2+b*x+a)*b^2*d^2*e*h-1/2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*ln(c*x^2+b*x+
a)*b^2*d*e^2*g-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))*b^3*d^2*e*h-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d^3*g+2/(a*e^2-b*d*e+
c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c*e^3*f-1/(a*e^2-b*d*e+c*d^2)^2/(c*x
^2+b*x+a)/(4*a*c-b^2)*a*b^2*e^3*f-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*
a*c-b^2)*a*c^2*d^3*g
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3405, size = 1161, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d^2*h*e - d*g*e^2 + f*e^3)*log(c*x^2 + b*x + a)/(c^2*d^4 - 2*b*c*d^3*
e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) + (d^2*h*e^2 - d*g
*e^3 + f*e^4)*log(abs(x*e + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 +
2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - (4*c^3*d^3*f - 2*b*c^2*d^3*g + 4*a
*c^2*d^3*h - 6*b*c^2*d^2*f*e + 4*b^2*c*d^2*g*e - 4*a*c^2*d^2*g*e - b^3*d^2*
h*e - 2*a*b*c*d^2*h*e + 12*a*c^2*d*f*e^2 - b^3*d*g*e^2 - 2*a*b*c*d*g*e^2 +
4*a*b^2*d*h*e^2 - 4*a^2*c*d*h*e^2 + b^3*f*e^3 - 6*a*b*c*f*e^3 + 4*a^2*c*g*e
^3 - 2*a^2*b*h*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2*d^4 -
4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2
*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 -
4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d^3*f - 2*a*c^2*d^3*g + a*b*c*d^3
*h - 2*b^2*c*d^2*f*e + 2*a*c^2*d^2*f*e + 3*a*b*c*d^2*g*e - a*b^2*d^2*h*e -
2*a^2*c*d^2*h*e + b^3*d*f*e^2 - a*b*c*d*f*e^2 - a*b^2*d*g*e^2 - 2*a^2*c*d*g
*e^2 + 3*a^2*b*d*h*e^2 - a*b^2*f*e^3 + 2*a^2*c*f*e^3 + a^2*b*g*e^3 - 2*a^3*
h*e^3 + (2*c^3*d^3*f - b*c^2*d^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - 3*b*c^2*
```

```
d^2*f*e + b^2*c*d^2*g*e + 2*a*c^2*d^2*g*e - b^3*d^2*h*e + a*b*c*d^2*h*e + b
^2*c*d*f*e^2 + 2*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 + 2*a*b^2*d*h*e^2 - 2*a^2*
c*d*h*e^2 - a*b*c*f*e^3 + 2*a^2*c*g*e^3 - a^2*b*h*e^3)*x)/((c*d^2 - b*d*e +
 a*e^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.159 $\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=673

$$-\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h-ac\,(d^2\right.}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)(ae^2-bd}$$

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x))) - (b^3
*e^2*f - b^2*e*(2*c*d*f + a*e*g) + 2*a*c*(c*d*(2*e*f - d*g) + a*e*(e*g - 2*
d*h)) + b*(c^2*d^2*f + a^2*e^2*h - a*c*(3*e^2*f - 2*d*e*g - d^2*h)) + c*(2*
c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*
d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*x)/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(a + b*x + c*x^2)) + ((4*c^4*d^4*f - b^3*e^3*(2*b*e*f -
b*d*g - a*e*g + 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f - 2*
d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2*d
*e*g + 2*d^2*h)) - c*e*(6*a^2*b*e^3*g - 4*a^3*e^3*h - b^3*d*(4*e^2*f - 3*d*
e*g - 2*d^2*h) - 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3) - (e*
(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h
))*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^3 + (e*(e^2*(2*b*e*f - b*d*g - a*e
*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + b*x + c*x^2])/(2
*(c*d^2 - b*d*e + a*e^2)^3)
```

Rubi [A] time = 2.55858, antiderivative size = 673, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1646, 1628, 634, 618, 206, 628}

$$-\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h-ac\,(d^2\right.}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)(ae^2-bd}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x))) - (b^3
*e^2*f - b^2*e*(2*c*d*f + a*e*g) + 2*a*c*(c*d*(2*e*f - d*g) + a*e*(e*g - 2*
d*h)) + b*(c^2*d^2*f + a^2*e^2*h - a*c*(3*e^2*f - 2*d*e*g - d^2*h)) + c*(2*
c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*
```

```
d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*x)/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(a + b*x + c*x^2)) + ((4*c^4*d^4*f - b^3*e^3*(2*b*e*f -
b*d*g - a*e*g + 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f - 2*
d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2*d
*e*g + 2*d^2*h)) - c*e*(6*a^2*b*e^3*g - 4*a^3*e^3*h - b^3*d*(4*e^2*f - 3*d*
e*g - 2*d^2*h) - 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3) - (e*
(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h
))*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^3 + (e*(e^2*(2*b*e*f - b*d*g - a*e
*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + b*x + c*x^2])/(2
*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = -\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b\left(c^2d^2f+a^2e^2h-ac\right.}{\left(b\right.}$$

$$= -\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b\left(c^2d^2f+a^2e^2h-ac\right.}{\left(b\right.}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-}{}$$

Mathematica [A] time = 2.47566, size = 650, normalized size = 0.97

$$\frac{b\left(-a^2e^2h+ac\left(d^2(-h)-2de(g-hx)+e^2(3f+gx)\right)+c^2d(-df+dgx+2efx)\right)+2c\left(a^2(-e)(e(g+hx)-2dh)+ac\left(d^2(g\right.\right.}{\left(b^2-4ac\right)(a+x(b+cx))\left(e(ae-bd)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x))) + (
-(b^3*e^2*f) + b^2*(a*e^2*g - c*(-2*d*e*f + e^2*f*x + d^2*h*x)) + b*(-(a^2*
e^2*h) + c^2*d*(-(d*f) + 2*e*f*x + d*g*x) + a*c*(-(d^2*h) + e^2*(3*f + g*x)
 - 2*d*e*(g - h*x))) + 2*c*(-(c^2*d^2*f*x) + a*c*(e^2*f*x - 2*d*e*(f + g*x)
 + d^2*(g + h*x)) - a^2*e*(-2*d*h + e*(g + h*x))))/((b^2 - 4*a*c)*(c*d^2 +
e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) - ((4*c^4*d^4*f + b^3*e^3*(-2*b*e*f
+ b*d*g + a*e*g - 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f -
2*d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2
*d*e*g + 2*d^2*h)) + c*e*(-6*a^2*b*e^3*g + 4*a^3*e^3*h + b^3*d*(4*e^2*f - 3
*d*e*g - 2*d^2*h) + 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTan[(b + 2*c
*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(-(c*d^2) + e*(b*d - a*e))^3
) + ((e^3*(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g +
 2*d^2*h))*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 - ((e^3*(-2*b*e*f + b
*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + x*(b
 + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Maple [B] time = 0.215, size = 4716, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x)
```

```
[Out] -e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*f+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*
ln(c*x^2+b*x+a)*a*b*e^4*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-
b^2)*x*b^2*d^4*h-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*a*
d*e^3*f-24/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*a*b*c^2*d*e^3*f-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+
b*x+a)*a*b*d*e^3*g+12/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*d^2*e^2*h-6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2
)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*d*e^3*g+4/(a*e^2-b*d*e+
c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c^2*d^3*e*g-6/(a*e^2-b*d*e+c*d^2)^3/
(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c^2*d^2*e^2*f+3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+
b*x+a)*c^2/(4*a*c-b^2)*x*b^2*d^2*e^2*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a
)*c^2/(4*a*c-b^2)*x*b^2*d^3*e*g-3/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(
c*x^2+b*x+a)*b^2*d^2*e^2*g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b
*x+a)*b^2*d^3*e*h-4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*a
```

*d^3*e*h+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*a*d^2*e^2*
g-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*a*c^3*d^3*e*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*a^
2*d*e^3*h+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b^2*c^2*d^3*e*g-24/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*d^2*e^2*h+24/(a*e^2-b*d*e+c*d^2)^3
/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^3*d^2*e^2*f-2/(a
*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b
^3*c*d^3*e*h-3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))*b^3*c*d^2*e^2*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d*e^3*f+2/(a*e^2-b*d*e+c*d^2)^3/(4
*a*c-b^2)*c*ln(c*x^2+b*x+a)*b^2*d*e^3*f-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)
*ln(c*x^2+b*x+a)*a*b^2*d*e^3*h+24/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*d*e^3*g-2/(a*e^2-b*d*e+c*d^2)^3/
(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*d*e^3*h+12/(a*e
^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b
^2*c*e^4*f-6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*a^2*b*c*e^4*g-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*d^3*e*f-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2
+b*x+a)/(4*a*c-b^2)*a*c^3*d^4*g+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*
a*c-b^2)*x*a*b^2*e^4*f-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*
x*a^2*b*e^4*g+4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*a^2*d
*e^3*g-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^3*d*e^3*f-6/
(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*a*b*d^2*e^2*g-4/(a*e^
2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a^2*b*d*e^3*h+3/(a*e^2-b*d*e
+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a*b^2*d^2*e^2*h+1/(a*e^2-b*d*e+c*d^
2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a*b^2*d*e^3*g-2/(a*e^2-b*d*e+c*d^2)^3/(4
*a*c-b^2)*c*ln(c*x^2+b*x+a)*a^2*e^4*g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^4*g+4/(a*e^2-b*d*e+c*d^2)^
3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^3*d^4*h-1/(a*e^
2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b^2*e^4*g+1/(a*e^2-b*d*e+c*d
^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*d*e^3*g+1/(
a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*e^4*f-4/(a*e^2-b*d*e+c
*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*b*d^3*e*f+6/(a*e^2-b*d*e+c*d^2)^3/(
c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*c*d^2*e^2*h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b
*x+a)/(4*a*c-b^2)*a*b^2*c*d^3*e*h-3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*
a*c-b^2)*a*b^2*c*d^2*e^2*g-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*d^4*g+1/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-
b^2)*ln(c*x^2+b*x+a)*a*b^2*e^4*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*c*e^4*h-12/(a*e^2-b*d*e+c*d^2)^3/(4
*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*e^4*f-1/(a*e^2-
b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^
3/(c*x^2+b*x+a)/(4*a*c-b^2)*b*c^3*d^4*f+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+
a)/(4*a*c-b^2)*a^3*c*e^4*g+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^4/(4*a*c
-b^2)*x*d^4*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*b*e^4*h

```
-e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*d^2*h+e^2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*d*
g+e^4/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*g-2*e^4/(a*e^2-b*d*e+c*d^2)^3*ln(e*
x+d)*b*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c^2*d^4*h+4/
(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^3*d^3*e*f+3/(a*e^2-b*d*
e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^3*c*d^2*e^2*f-3/(a*e^2-b*d*e+c*d^2)^
3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c^2*d^3*e*f+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+
b*x+a)*c/(4*a*c-b^2)*x*a^3*e^4*h-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/
(4*a*c-b^2)*x*a^2*e^4*f-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^
2)*x*b*d^4*g-4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*c*d*e^3*
h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b^2*d*e^3*h-3/(a*e^
2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*c*e^4*f-4/(a*e^2-b*d*e+c*d
^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c^2*d^3*e*h+4/(a*e^2-b*d*e+c*d^2)^3/(c*
x^2+b*x+a)/(4*a*c-b^2)*a^2*c^2*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a
)/(4*a*c-b^2)*a*b^3*d*e^3*g-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*
c-b^2)*x*a*d^4*h+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*
d*e^3*f-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^3*d^3*e*h+4
/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*a*d^3*e*g+1/2/(a*e^2
-b*d*e+c*d^2)^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3*d*e^3*g-3*e^2/(a*e^2-b*d*e+
c*d^2)^3*ln(e*x+d)*c*d^2*g+4*e^3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*c*d*f-1/(a
*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3*e^4*f+4/(a*e^2-b*d*e+c*
d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^4*d^4*f-2/(a
*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b
^4*e^4*f-2*e^3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*d*h+e^3/(a*e^2-b*d*e+c*d^2
)^3*ln(e*x+d)*b*d*g+2*e/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*c*d^3*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42511, size = 1940, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(4*c^4*d^4*f*e^2 - 2*b*c^3*d^4*g*e^2 + 4*a*c^3*d^4*h*e^2 - 8*b*c^3*d^3*f*e
^3 + 6*b^2*c^2*d^3*g*e^3 - 8*a*c^3*d^3*g*e^3 - 2*b^3*c*d^3*h*e^3 + 24*a*c^3
*d^2*f*e^4 - 3*b^3*c*d^2*g*e^4 + 12*a*b^2*c*d^2*h*e^4 - 24*a^2*c^2*d^2*h*e^
4 + 4*b^3*c*d*f*e^5 - 24*a*b*c^2*d*f*e^5 + b^4*d*g*e^5 - 6*a*b^2*c*d*g*e^5
+ 24*a^2*c^2*d*g*e^5 - 2*a*b^3*d*h*e^5 - 2*b^4*f*e^6 + 12*a*b^2*c*f*e^6 - 1
2*a^2*c^2*f*e^6 + a*b^3*g*e^6 - 6*a^2*b*c*g*e^6 + 4*a^3*c*h*e^6)*arctan((2*
c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*a*e^2/(x*e + d))*e^(-
1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((b^2*c^3*d^6 - 4*a*c^4*d^6 - 3*b^3*c^2*d^5*e
 + 12*a*b*c^3*d^5*e + 3*b^4*c*d^4*e^2 - 9*a*b^2*c^2*d^4*e^2 - 12*a^2*c^3*d^
4*e^2 - b^5*d^3*e^3 - 2*a*b^3*c*d^3*e^3 + 24*a^2*b*c^2*d^3*e^3 + 3*a*b^4*d^
2*e^4 - 9*a^2*b^2*c*d^2*e^4 - 12*a^3*c^2*d^2*e^4 - 3*a^2*b^3*d*e^5 + 12*a^3
*b*c*d*e^5 + a^3*b^2*e^6 - 4*a^4*c*e^6)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^3*
h*e - 3*c*d^2*g*e^2 + 4*c*d*f*e^3 + b*d*g*e^3 - 2*a*d*h*e^3 - 2*b*f*e^4 + a
*g*e^4)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e
/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^
2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) - (d^2*h*e^5/(x*e + d) - d*g*e^6/(x
```

```
*e + d) + f*e^7/(x*e + d))/(c^2*d^4*e^4 - 2*b*c*d^3*e^5 + b^2*d^2*e^6 + 2*a
*c*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8) - ((2*c^4*d^3*f*e - b*c^3*d^3*g*e + b^2
*c^2*d^3*h*e - 2*a*c^3*d^3*h*e - 3*b*c^3*d^2*f*e^2 + 6*a*c^3*d^2*g*e^2 - 3*
a*b*c^2*d^2*h*e^2 + 3*b^2*c^2*d*f*e^3 - 6*a*c^3*d*f*e^3 - 3*a*b*c^2*d*g*e^3
 + 6*a^2*c^2*d*h*e^3 - b^3*c*f*e^4 + 3*a*b*c^2*f*e^4 + a*b^2*c*g*e^4 - 2*a^
2*c^2*g*e^4 - a^2*b*c*h*e^4)/(c*d^2 - b*d*e + a*e^2) - (2*c^4*d^4*f*e^2 - b
*c^3*d^4*g*e^2 + b^2*c^2*d^4*h*e^2 - 2*a*c^3*d^4*h*e^2 - 4*b*c^3*d^3*f*e^3
+ 8*a*c^3*d^3*g*e^3 - 4*a*b*c^2*d^3*h*e^3 + 6*b^2*c^2*d^2*f*e^4 - 12*a*c^3*
d^2*f*e^4 - 6*a*b*c^2*d^2*g*e^4 + 12*a^2*c^2*d^2*h*e^4 - 4*b^3*c*d*f*e^5 +
12*a*b*c^2*d*f*e^5 + 4*a*b^2*c*d*g*e^5 - 8*a^2*c^2*d*g*e^5 - 4*a^2*b*c*d*h*
e^5 + b^4*f*e^6 - 4*a*b^2*c*f*e^6 + 2*a^2*c^2*f*e^6 - a*b^3*g*e^6 + 3*a^2*b
*c*g*e^6 + a^2*b^2*h*e^6 - 2*a^3*c*h*e^6)*e^(-1)/((c*d^2 - b*d*e + a*e^2)*(
x*e + d)))/((c*d^2 - b*d*e + a*e^2)^2*(b^2 - 4*a*c)*(c - 2*c*d/(x*e + d) +
c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2))
```

3.160 $\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x^2}{2}+\frac{2(2-x)}{3(x^2-x+1)}+2\log(x^2-x+1)+3x+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 3*x + x^2/2 + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqrt[3]]
)/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Rubi [A] time = 0.0743509, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{x^2}{2}+\frac{2(2-x)}{3(x^2-x+1)}+2\log(x^2-x+1)+3x+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] 3*x + x^2/2 + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqrt[3]]
)/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

```
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(1+x+x^2\right)}{\left(1-x+x^2\right)^2}\,dx &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{-2+6x+6x^2+3x^3}{1-x+x^2}\,dx\\ &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(9+3x-\frac{11-12x}{1-x+x^2}\right)dx\\ &= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{1}{3}\int \frac{11-12x}{1-x+x^2}\,dx\\ &= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx+2\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+2\log\left(1-x+x^2\right)+\frac{10}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0332674, size = 60, normalized size = 0.97

$$\frac{x^2}{2}-\frac{2(x-2)}{3\left(x^2-x+1\right)}+2\log\left(x^2-x+1\right)+3x-\frac{10\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] 3*x + x^2/2 - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/Sqrt[3
]])/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Maple [A] time = 0.049, size = 53, normalized size = 0.9

$$\frac{x^2}{2}+3\,x+\frac{1}{x^2-x+1}\left(-\frac{2\,x}{3}+\frac{4}{3}\right)+2\,\ln\left(x^2-x+1\right)-\frac{10\,\sqrt{3}}{9}\arctan\left(\frac{(2\,x-1)\,\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] 1/2*x^2+3*x+(-2/3*x+4/3)/(x^2-x+1)+2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(2
*x-1)*3^(1/2))
```

Maxima [A] time = 1.43281, size = 69, normalized size = 1.11

$$\frac{1}{2}\,x^2-\frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+3\,x-\frac{2\,(x-2)}{3\left(x^2-x+1\right)}+2\,\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

Fricas [A] time = 2.34005, size = 204, normalized size = 3.29

$$\frac{9\,x^4+45\,x^3-20\,\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-45\,x^2+36\left(x^2-x+1\right)\log\left(x^2-x+1\right)+42\,x+24}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(9*x^4 + 45*x^3 - 20*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1
)) - 45*x^2 + 36*(x^2 - x + 1)*log(x^2 - x + 1) + 42*x + 24)/(x^2 - x + 1)
```

Sympy [A] time = 0.143439, size = 60, normalized size = 0.97

$$\frac{x^2}{2}+3x-\frac{2x-4}{3x^2-3x+3}+2\log\left(x^2-x+1\right)-\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x**2/2 + 3*x - (2*x - 4)/(3*x**2 - 3*x + 3) + 2*log(x**2 - x + 1) - 10*sqrt
(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.19867, size = 69, normalized size = 1.11

$$\frac{1}{2}\,x^{2}-\frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+3\,x-\frac{2\,(x-2)}{3\,\left(x^{2}-x+1\right)}+2\,\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

3.161 $\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} dx$

Optimal. Leaf size=55

$$\frac{2(1-2x)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)+x-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x + (2*(1 - 2*x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt
[3]) + (3*Log[1 - x + x^2])/2
```

Rubi [A] time = 0.0662584, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{2(1-2x)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)+x-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] x + (2*(1 - 2*x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt
[3]) + (3*Log[1 - x + x^2])/2
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

```
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(1+x+x^2\right)}{\left(1-x+x^2\right)^2}\,dx &= \frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{2+6x+3x^2}{1-x+x^2}\,dx \\
&= \frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(3-\frac{1-9x}{1-x+x^2}\right)dx \\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}-\frac{1}{3}\int \frac{1-9x}{1-x+x^2}\,dx \\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{7}{6}\int \frac{1}{1-x+x^2}\,dx+\frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{3}{2}\log\left(1-x+x^2\right)-\frac{7}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.026913, size = 55, normalized size = 1.

$$
-\frac{2(2x-1)}{3\left(x^2-x+1\right)}+\frac{3}{2}\log\left(x^2-x+1\right)+x+\frac{7\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] x - (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sq
rt[3]) + (3*Log[1 - x + x^2])/2
```

Maple [A] time = 0.048, size = 46, normalized size = 0.8

$$
x+\frac{1}{x^2-x+1}\left(-\frac{4\,x}{3}+\frac{2}{3}\right)+\frac{3\,\ln\left(x^2-x+1\right)}{2}+\frac{7\,\sqrt{3}}{9}\arctan\left(\frac{(2\,x-1)\,\sqrt{3}}{3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] x+(-4/3*x+2/3)/(x^2-x+1)+3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))
```

Maxima [A] time = 1.52309, size = 62, normalized size = 1.13

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{2(2x-1)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

Fricas [A] time = 2.32918, size = 192, normalized size = 3.49

$$\frac{18x^3+14\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-18x^2+27\left(x^2-x+1\right)\log\left(x^2-x+1\right)-6x+12}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*x^3 + 14*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 18*
x^2 + 27*(x^2 - x + 1)*log(x^2 - x + 1) - 6*x + 12)/(x^2 - x + 1)
```

Sympy [A] time = 0.13327, size = 54, normalized size = 0.98

$$x-\frac{4x-2}{3x^2-3x+3}+\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x - (4*x - 2)/(3*x**2 - 3*x + 3) + 3*log(x**2 - x + 1)/2 + 7*sqrt(3)*atan(2
*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.17382, size = 62, normalized size = 1.13

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{2(2x-1)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

3.162 $\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx$

Optimal. Leaf size=52

$$-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
 + Log[1 - x + x^2]/2
```

Rubi [A] time = 0.0515259, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1660, 634, 618, 204, 628}

$$-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
 + Log[1 - x + x^2]/2
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(1+x+x^2\right)}{\left(1-x+x^2\right)^2}\,dx &= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{4+3x}{1-x+x^2}\,dx\\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{11}{6}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{2}\log\left(1-x+x^2\right)-\frac{11}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0195145, size = 52, normalized size = 1.

$$-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)+\frac{11\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
) + Log[1 - x + x^2]/2
```

Maple [A] time = 0.047, size = 45, normalized size = 0.9

$$\frac{1}{x^2-x+1}\left(-\frac{2x}{3}-\frac{2}{3}\right)+\frac{\ln\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] (-2/3*x-2/3)/(x^2-x+1)+1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

Maxima [A] time = 1.49714, size = 58, normalized size = 1.12

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

Fricas [A] time = 2.34933, size = 167, normalized size = 3.21

$$\frac{22\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+9\left(x^2-x+1\right)\log\left(x^2-x+1\right)-12x-12}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9*(x^2 - x +
 1)*log(x^2 - x + 1) - 12*x - 12)/(x^2 - x + 1)
```

Sympy [A] time = 0.13767, size = 51, normalized size = 0.98

$$-\frac{2x+2}{3x^2-3x+3}+\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] -(2*x + 2)/(3*x**2 - 3*x + 3) + log(x**2 - x + 1)/2 + 11*sqrt(3)*atan(2*sqr
t(3)*x/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.33756, size = 58, normalized size = 1.12

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

3.163 $\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2} dx$

Optimal. Leaf size=41

$$-\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*(2 - x))/(3*(1 - x + x^2)) - (10*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0329209, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1660, 12, 618, 204}

$$-\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(2 - x))/(3*(1 - x + x^2)) - (10*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx &= -\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{5}{1-x+x^2}\,dx \\
&= -\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{10}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.021879, size = 39, normalized size = 0.95

$$
\frac{2(x-2)}{3\left(x^2-x+1\right)}+\frac{10\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

```
[Out] (2*(-2 + x))/(3*(1 - x + x^2)) + (10*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
)
```

Maple [A] time = 0.044, size = 34, normalized size = 0.8

$$\frac{1}{x^2-x+1}\left(\frac{2x}{3}-\frac{4}{3}\right)+\frac{10\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] (2/3*x-4/3)/(x^2-x+1)+10/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.51796, size = 43, normalized size = 1.05

$$\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2(x-2)}{3(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

Fricas [A] time = 1.90323, size = 115, normalized size = 2.8

$$\frac{2\left(5\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3x-6\right)}{9\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 2/9*(5*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 6)/(x^2
- x + 1)
```

Sympy [A] time = 0.125305, size = 41, normalized size = 1.

$$\frac{2x-4}{3x^2-3x+3}+\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] (2*x - 4)/(3*x**2 - 3*x + 3) + 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.28502, size = 43, normalized size = 1.05

$$\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2(x-2)}{3\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

3.164 $\int \frac{1+x+x^2}{x(1-x+x^2)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{2(1-2x)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*(1 - 2*x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3
]) + Log[x] - Log[1 - x + x^2]/2
```

Rubi [A] time = 0.0894687, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1646, 800, 634, 618, 204, 628}

$$-\frac{2(1-2x)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

```
[Out] (-2*(1 - 2*x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3
]) + Log[x] - Log[1 - x + x^2]/2
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+4x}{x\left(1-x+x^2\right)}\,dx\\ &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x}+\frac{7-3x}{1-x+x^2}\right)dx\\ &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\log(x)+\frac{1}{3}\int \frac{7-3x}{1-x+x^2}\,dx\\ &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\log(x)-\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{11}{6}\int \frac{1}{1-x+x^2}\,dx\\ &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\log(x)-\frac{1}{2}\log\left(1-x+x^2\right)-\frac{11}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0249741, size = 56, normalized size = 1.

$$\frac{2(2x-1)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)+\frac{11\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

```
[Out] (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[
3]) + Log[x] - Log[1 - x + x^2]/2
```

Maple [A] time = 0.048, size = 48, normalized size = 0.9

$$\ln(x)-\frac{1}{x^2-x+1}\left(-\frac{4x}{3}+\frac{2}{3}\right)-\frac{\ln\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x/(x^2-x+1)^2,x)
```

```
[Out] ln(x)-(-4/3*x+2/3)/(x^2-x+1)-1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-1
)*3^(1/2))
```

Maxima [A] time = 1.53454, size = 63, normalized size = 1.12

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2(2x-1)}{3(x^2-x+1)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(x)
```

Fricas [A] time = 1.62718, size = 203, normalized size = 3.62

$$\frac{22\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-9\left(x^2-x+1\right)\log\left(x^2-x+1\right)+18\left(x^2-x+1\right)\log(x)+24x-12}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 9*(x^2 - x +
 1)*log(x^2 - x + 1) + 18*(x^2 - x + 1)*log(x) + 24*x - 12)/(x^2 - x + 1)
```

Sympy [A] time = 0.178549, size = 54, normalized size = 0.96

$$\frac{4x-2}{3x^2-3x+3}+\log(x)-\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x/(x**2-x+1)**2,x)
```

```
[Out] (4*x - 2)/(3*x**2 - 3*x + 3) + log(x) - log(x**2 - x + 1)/2 + 11*sqrt(3)*at
an(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.23469, size = 65, normalized size = 1.16

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2(2x-1)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(abs(x))
```

3.165 $\int \frac{1+x+x^2}{x^2(1-x+x^2)^2} \, dx$

Optimal. Leaf size=61

$$\frac{2(x+1)}{3(x^2-x+1)} - \frac{3}{2}\log\left(x^2-x+1\right) - \frac{1}{x} + 3\log(x) - \frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*
Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Rubi [A] time = 0.128098, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{2(x+1)}{3(x^2-x+1)} - \frac{3}{2}\log\left(x^2-x+1\right) - \frac{1}{x} + 3\log(x) - \frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*
Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx &= \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+6x+2x^2}{x^2\left(1-x+x^2\right)}\,dx\\ &= \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x^2}+\frac{9}{x}+\frac{8-9x}{1-x+x^2}\right)dx\\ &= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{1}{3}\int \frac{8-9x}{1-x+x^2}\,dx\\ &= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{7}{6}\int \frac{1}{1-x+x^2}\,dx-\frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)-\frac{7}{3}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0226778, size = 61, normalized size = 1.

$$\frac{2(x+1)}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)-\frac{1}{x}+3\log(x)+\frac{7\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Maple [A] time = 0.049, size = 55, normalized size = 0.9

$$-x^{-1}+3\,\ln(x)-\frac{1}{x^2-x+1}\left(-\frac{2\,x}{3}-\frac{2}{3}\right)-\frac{3\,\ln\left(x^2-x+1\right)}{2}+\frac{7\,\sqrt{3}}{9}\arctan\left(\frac{(2\,x-1)\,\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^2/(x^2-x+1)^2,x)
```

```
[Out] -1/x+3*ln(x)-(-2/3*x-2/3)/(x^2-x+1)-3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*
(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.463, size = 73, normalized size = 1.2

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x^2-5x+3}{3\left(x^3-x^2+x\right)}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(x)
```

Fricas [A] time = 1.85105, size = 225, normalized size = 3.69

$$\frac{14\sqrt{3}\left(x^3-x^2+x\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6x^2-27\left(x^3-x^2+x\right)\log\left(x^2-x+1\right)+54\left(x^3-x^2+x\right)\log\left(x\right)+30x-}{18\left(x^3-x^2+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(14*sqrt(3)*(x^3 - x^2 + x)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^2 - 27
*(x^3 - x^2 + x)*log(x^2 - x + 1) + 54*(x^3 - x^2 + x)*log(x) + 30*x - 18)/
(x^3 - x^2 + x)
```

Sympy [A] time = 0.176784, size = 65, normalized size = 1.07

$$-\frac{x^2-5x+3}{3x^3-3x^2+3x}+3\log\left(x\right)-\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**2/(x**2-x+1)**2,x)
```

```
[Out] -(x**2 - 5*x + 3)/(3*x**3 - 3*x**2 + 3*x) + 3*log(x) - 3*log(x**2 - x + 1)/
2 + 7*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.27312, size = 74, normalized size = 1.21

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x^2-5x+3}{3\left(x^3-x^2+x\right)}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(abs(x))
```

3.166 $\int \frac{1+x+x^2}{x^3(1-x+x^2)^2} dx$

Optimal. Leaf size=68

$$\frac{2(2-x)}{3(x^2-x+1)} - \frac{1}{2x^2} - 2\log(x^2-x+1) - \frac{3}{x} + 4\log(x) + \frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/(2*x^2) - 3/x + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqr
t[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Rubi [A] time = 0.109582, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{2(2-x)}{3(x^2-x+1)} - \frac{1}{2x^2} - 2\log(x^2-x+1) - \frac{3}{x} + 4\log(x) + \frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/(2*x^2) - 3/x + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqr
t[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x^3(1-x+x^2)^2}\,dx &= \frac{2(2-x)}{3(1-x+x^2)}+\frac{1}{3}\int \frac{3+6x+6x^2-2x^3}{x^3(1-x+x^2)}\,dx \\
&= \frac{2(2-x)}{3(1-x+x^2)}+\frac{1}{3}\int \left(\frac{3}{x^3}+\frac{9}{x^2}+\frac{12}{x}+\frac{1-12x}{1-x+x^2}\right)dx \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3(1-x+x^2)}+4\log(x)+\frac{1}{3}\int \frac{1-12x}{1-x+x^2}\,dx \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3(1-x+x^2)}+4\log(x)-\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx-2\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3(1-x+x^2)}+4\log(x)-2\log\left(1-x+x^2\right)+\frac{10}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+\right. \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3(1-x+x^2)}+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+4\log(x)-2\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.029881, size = 66, normalized size = 0.97

$$-\frac{2(x-2)}{3(x^2-x+1)}-\frac{1}{2x^2}-2\log\left(x^2-x+1\right)-\frac{3}{x}+4\log(x)-\frac{10\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/(2*x^2) - 3/x - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Maple [A] time = 0.048, size = 60, normalized size = 0.9

$$-\frac{1}{2x^2}-3x^{-1}+4\ln(x)-\frac{1}{x^2-x+1}\left(\frac{2x}{3}-\frac{4}{3}\right)-2\ln\left(x^2-x+1\right)-\frac{10\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^3/(x^2-x+1)^2,x)
```

```
[Out] -1/2/x^2-3/x+4*ln(x)-(2/3*x-4/3)/(x^2-x+1)-2*ln(x^2-x+1)-10/9*3^(1/2)*arcta
n(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.50708, size = 85, normalized size = 1.25

$$-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{22x^3-23x^2+15x+3}{6\left(x^4-x^3+x^2\right)}-2\log\left(x^2-x+1\right)+4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/(x^4 - x^3 + x^2) - 2*log(x^2 - x + 1) + 4*log(x)
```

Fricas [A] time = 1.74637, size = 250, normalized size = 3.68

$$-\frac{66x^3+20\sqrt{3}\left(x^4-x^3+x^2\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-69x^2+36\left(x^4-x^3+x^2\right)\log\left(x^2-x+1\right)-72\left(x^4-x^3+x^2\right)\log}{18\left(x^4-x^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(66*x^3 + 20*sqrt(3)*(x^4 - x^3 + x^2)*arctan(1/3*sqrt(3)*(2*x - 1))
- 69*x^2 + 36*(x^4 - x^3 + x^2)*log(x^2 - x + 1) - 72*(x^4 - x^3 + x^2)*log
(x) + 45*x + 9)/(x^4 - x^3 + x^2)
```

Sympy [A] time = 0.187243, size = 71, normalized size = 1.04

$$4\log(x)-2\log\left(x^2-x+1\right)-\frac{10\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{22x^3-23x^2+15x+3}{6x^4-6x^3+6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**3/(x**2-x+1)**2,x)
```

```
[Out] 4*log(x) - 2*log(x**2 - x + 1) - 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)
/9 - (22*x**3 - 23*x**2 + 15*x + 3)/(6*x**4 - 6*x**3 + 6*x**2)
```

Giac [A] time = 1.26833, size = 85, normalized size = 1.25

$$-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{22x^3-23x^2+15x+3}{6\left(x^2-x+1\right)x^2}-2\log\left(x^2-x+1\right)+4\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/((x^2 - x + 1)*x^2) - 2*log(x^2 - x + 1) + 4*log(abs(x))
```

3.167 $\int \frac{1-x^2}{\left(1+x+x^2\right)^2}\,dx$

Optimal. Leaf size=10

$$\frac{x}{x^2+x+1}$$

```
[Out] x/(1 + x + x^2)
```

Rubi [A] time = 0.0107474, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1588}

$$\frac{x}{x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x + x^2)^2,x]
```

```
[Out] x/(1 + x + x^2)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2}\,dx=\frac{x}{1+x+x^2}$$

Mathematica [A] time = 0.0052611, size = 10, normalized size = 1.

$$\frac{x}{x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x + x^2)^2,x]
```

```
[Out] x/(1 + x + x^2)
```

Maple [A] time = 0.042, size = 11, normalized size = 1.1

$$\frac{x}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+x+1)^2,x)
```

```
[Out] x/(x^2+x+1)
```

Maxima [A] time = 1.02832, size = 14, normalized size = 1.4

$$\frac{x}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + x + 1)
```

Fricas [A] time = 1.6145, size = 23, normalized size = 2.3

$$\frac{x}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] x/(x^2 + x + 1)
```

Sympy [A] time = 0.086572, size = 7, normalized size = 0.7

$$\frac{x}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+x+1)**2,x)
```

```
[Out] x/(x**2 + x + 1)
```

Giac [A] time = 1.20656, size = 11, normalized size = 1.1

$$\frac{1}{x + \frac{1}{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1/x + 1)
```

3.168 $\int \frac{1+x^2}{1+x+x^2} \, dx$

Optimal. Leaf size=31

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Rubi [A] time = 0.0318015, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1657, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x + x^2),x]
```

```
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{1+x+x^2}\,dx &= \int \left(1-\frac{x}{1+x+x^2}\right) dx\\
&= x-\int \frac{x}{1+x+x^2}\,dx\\
&= x+\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\
&= x-\frac{1}{2}\log\left(1+x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= x+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0076332, size = 31, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x + x^2),x]
```

```
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Maple [A] time = 0.042, size = 28, normalized size = 0.9

$$x-\frac{\ln\left(x^{2}+x+1\right)}{2}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+x+1),x)
```

```
[Out] x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.48653, size = 36, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+x-\frac{1}{2}\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Fricas [A] time = 1.64362, size = 95, normalized size = 3.06

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+x-\frac{1}{2}\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Sympy [A] time = 0.099471, size = 36, normalized size = 1.16

$$x-\frac{\log\left(x^{2}+x+1\right)}{2}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+x+1),x)
```

```
[Out] x - log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Giac [A] time = 1.22783, size = 36, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

3.169 $\int \frac{-1+x^2}{25-6x+x^2}\,dx$

Optimal. Leaf size=23

$$3\log\left(x^2-6x+25\right)+x-2\tan^{-1}\left(\frac{x-3}{4}\right)$$

```
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

Rubi [A] time = 0.0349412, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1657, 634, 618, 204, 628}

$$3\log\left(x^2-6x+25\right)+x-2\tan^{-1}\left(\frac{x-3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(25 - 6*x + x^2),x]
```

```
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^2}{25-6x+x^2}\,dx &= \int \left(1-\frac{2(13-3x)}{25-6x+x^2}\right) dx \\
&= x-2\int \frac{13-3x}{25-6x+x^2}\,dx \\
&= x+3\int \frac{-6+2x}{25-6x+x^2}\,dx-8\int \frac{1}{25-6x+x^2}\,dx \\
&= x+3\log\left(25-6x+x^2\right)+16\,\mathrm{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,-6+2x\right) \\
&= x-2\tan^{-1}\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0050814, size = 23, normalized size = 1.

$$3\log\left(x^2-6x+25\right)+x-2\tan^{-1}\left(\frac{x-3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(25 - 6*x + x^2),x]
```

```
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

Maple [A] time = 0.05, size = 22, normalized size = 1.

$$x-2\,\arctan\left(-3/4+x/4\right)+3\,\ln\left(x^2-6\,x+25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2-6*x+25),x)
```

```
[Out] x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)
```

Maxima [A] time = 1.4844, size = 28, normalized size = 1.22

$$x - 2\arctan\left(\frac{1}{4}x - \frac{3}{4}\right) + 3\log\left(x^2 - 6x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="maxima")
```

```
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

Fricas [A] time = 1.70121, size = 69, normalized size = 3.

$$x - 2\arctan\left(\frac{1}{4}x - \frac{3}{4}\right) + 3\log\left(x^2 - 6x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="fricas")
```

```
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

Sympy [A] time = 0.100963, size = 22, normalized size = 0.96

$$x + 3\log\left(x^2 - 6x + 25\right) - 2\operatorname{atan}\left(\frac{x}{4} - \frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2-6*x+25),x)
```

```
[Out] x + 3*log(x**2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

Giac [A] time = 1.33842, size = 28, normalized size = 1.22

$$x - 2\arctan\left(\frac{1}{4}x - \frac{3}{4}\right) + 3\log\left(x^2 - 6x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="giac")
```

```
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

3.170 $\int \frac{-10+3x^2}{4-4x+x^2}\,dx$

Optimal. Leaf size=21

$$3x+\frac{2}{2-x}+12\log(2-x)$$

```
[Out] 2/(2 - x) + 3*x + 12*Log[2 - x]
```

Rubi [A] time = 0.0132023, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 697}

$$3x+\frac{2}{2-x}+12\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
```

```
[Out] 2/(2 - x) + 3*x + 12*Log[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = \int \frac{-10+3x^2}{(-2+x)^2}\,dx$$
$$= \int \left(3+\frac{2}{(-2+x)^2}+\frac{12}{-2+x}\right)dx$$
$$= \frac{2}{2-x}+3x+12\log(2-x)$$

Mathematica [A] time = 0.009009, size = 19, normalized size = 0.9

$$3(x-2)-\frac{2}{x-2}+12\log(x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
```

```
[Out] -2/(-2 + x) + 3*(-2 + x) + 12*Log[-2 + x]
```

Maple [A] time = 0.049, size = 18, normalized size = 0.9

$$3x-2(-2+x)^{-1}+12\ln(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-10)/(x^2-4*x+4),x)
```

```
[Out] 3*x-2/(-2+x)+12*ln(-2+x)
```

Maxima [A] time = 1.01219, size = 23, normalized size = 1.1

$$3x-\frac{2}{x-2}+12\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="maxima")
```

[Out] 3*x - 2/(x - 2) + 12*log(x - 2)

Fricas [A] time = 1.67132, size = 69, normalized size = 3.29

$$\frac{3\,x^{2}+12\,(x-2)\log(x-2)-6\,x-2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="fricas")

[Out] (3*x^2 + 12*(x - 2)*log(x - 2) - 6*x - 2)/(x - 2)

Sympy [A] time = 0.081477, size = 14, normalized size = 0.67

$$3x+12\log(x-2)-\frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2-10)/(x**2-4*x+4),x)

[Out] 3*x + 12*log(x - 2) - 2/(x - 2)

Giac [A] time = 1.2725, size = 24, normalized size = 1.14

$$3\,x-\frac{2}{x-2}+12\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="giac")

[Out] 3*x - 2/(x - 2) + 12*log(abs(x - 2))

3.171 $\int \frac{8+x^2}{6-5x+x^2} \, dx$

Optimal. Leaf size=18

$$x - 12\log(2 - x) + 17\log(3 - x)$$

```
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

Rubi [A] time = 0.017693, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1657, 632, 31}

$$x - 12\log(2 - x) + 17\log(3 - x)$$

Antiderivative was successfully verified.

```
[In] Int[(8 + x^2)/(6 - 5*x + x^2),x]
```

```
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{8+x^2}{6-5x+x^2}\,dx &= \int \left(1+\frac{2+5x}{6-5x+x^2}\right)dx\\ &= x+\int \frac{2+5x}{6-5x+x^2}\,dx\\ &= x-12\int \frac{1}{-2+x}\,dx+17\int \frac{1}{-3+x}\,dx\\ &= x-12\log(2-x)+17\log(3-x)\end{aligned}$$

Mathematica [A] time = 0.0043541, size = 18, normalized size = 1.

$$x-12\log(2-x)+17\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + x^2)/(6 - 5*x + x^2),x]
```

```
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

Maple [A] time = 0.049, size = 15, normalized size = 0.8

$$x+17\ \ln(-3+x)-12\ \ln(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+8)/(x^2-5*x+6),x)
```

```
[Out] x+17*ln(-3+x)-12*ln(-2+x)
```

Maxima [A] time = 0.991097, size = 19, normalized size = 1.06

$$x-12\ \log(x-2)+17\ \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="maxima")
```

```
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

Fricas [A] time = 1.65817, size = 47, normalized size = 2.61

$$x - 12 \log(x - 2) + 17 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="fricas")
```

```
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

Sympy [A] time = 0.099877, size = 14, normalized size = 0.78

$$x + 17 \log(x - 3) - 12 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+8)/(x**2-5*x+6),x)
```

```
[Out] x + 17*log(x - 3) - 12*log(x - 2)
```

Giac [A] time = 1.27402, size = 22, normalized size = 1.22

$$x - 12 \log(|x - 2|) + 17 \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="giac")
```

```
[Out] x - 12*log(abs(x - 2)) + 17*log(abs(x - 3))
```

3.172 $\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx$

Optimal. Leaf size=14

$$x + 4\log(4 - x) + \log(x + 2)$$

```
[Out] x + 4*Log[4 - x] + Log[2 + x]
```

Rubi [A] time = 0.0165158, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1657, 632, 31}

$$x + 4\log(4 - x) + \log(x + 2)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
```

```
[Out] x + 4*Log[4 - x] + Log[2 + x]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx &= \int \left(1+\frac{4+5x}{-8-2x+x^2}\right)dx\\ &= x+\int \frac{4+5x}{-8-2x+x^2}\,dx\\ &= x+4\int \frac{1}{-4+x}\,dx+\int \frac{1}{2+x}\,dx\\ &= x+4\log(4-x)+\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0051096, size = 14, normalized size = 1.

$$x+4\log(4-x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]

[Out] x + 4*Log[4 - x] + Log[2 + x]
```

Maple [A] time = 0.049, size = 13, normalized size = 0.9

$$x+\ln(2+x)+4\ln(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(x^2-2*x-8),x)

[Out] x+ln(2+x)+4*ln(x-4)
```

Maxima [A] time = 0.990675, size = 16, normalized size = 1.14

$$x+\log(x+2)+4\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="maxima")
```

```
[Out] x + log(x + 2) + 4*log(x - 4)
```

Fricas [A] time = 1.61126, size = 42, normalized size = 3.

$$x + \log(x + 2) + 4 \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="fricas")
```

```
[Out] x + log(x + 2) + 4*log(x - 4)
```

Sympy [A] time = 0.097405, size = 12, normalized size = 0.86

$$x + 4\log(x - 4) + \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(x**2-2*x-8),x)
```

```
[Out] x + 4*log(x - 4) + log(x + 2)
```

Giac [A] time = 1.24607, size = 19, normalized size = 1.36

$$x + \log(|x + 2|) + 4 \log(|x - 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="giac")
```

```
[Out] x + log(abs(x + 2)) + 4*log(abs(x - 4))
```

3.173 $\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rubi [A] time = 0.0276561, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx &= \int \left(1+\frac{2+x}{5+4x+4x^2}\right) dx \\
&= x+\int \frac{2+x}{5+4x+4x^2}\,dx \\
&= x+\frac{1}{8}\int \frac{4+8x}{5+4x+4x^2}\,dx+\frac{3}{2}\int \frac{1}{5+4x+4x^2}\,dx \\
&= x+\frac{1}{8}\log\left(5+4x+4x^2\right)-3\,\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,4+8x\right) \\
&= x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0054358, size = 31, normalized size = 1.15

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

Maple [A] time = 0.047, size = 22, normalized size = 0.8

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{\ln\left(4\,x^2+4\,x+5\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+5*x+7)/(4*x^2+4*x+5),x)
```

```
[Out] x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

Maxima [A] time = 1.52614, size = 28, normalized size = 1.04

$$x + \frac{3}{8}\,\arctan\left(x + \frac{1}{2}\right) + \frac{1}{8}\,\log\left(4\,x^2 + 4\,x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="maxima")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Fricas [A] time = 1.60817, size = 70, normalized size = 2.59

$$x + \frac{3}{8}\,\arctan\left(x + \frac{1}{2}\right) + \frac{1}{8}\,\log\left(4\,x^2 + 4\,x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="fricas")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Sympy [A] time = 0.10131, size = 22, normalized size = 0.81

$$x + \frac{\log\left(x^2 + x + \frac{5}{4}\right)}{8} + \frac{3\,\operatorname{atan}\left(x + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
```

```
[Out] x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

Giac [A] time = 1.24282, size = 28, normalized size = 1.04

$$x + \frac{3}{8}\arctan\left(x + \frac{1}{2}\right) + \frac{1}{8}\log\left(4x^2 + 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="giac")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

3.174 $\int \frac{2-x+x^2}{-5+2x+x^2} \, dx$

Optimal. Leaf size=48

$$x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

```
[Out] x - ((9 - 5*Sqrt[6])*Log[1 - Sqrt[6] + x])/6 - ((9 + 5*Sqrt[6])*Log[1 + Sqr
t[6] + x])/6
```

Rubi [A] time = 0.0446019, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1657, 632, 31}

$$x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

```
[Out] x - ((9 - 5*Sqrt[6])*Log[1 - Sqrt[6] + x])/6 - ((9 + 5*Sqrt[6])*Log[1 + Sqr
t[6] + x])/6
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2-x+x^2}{-5+2x+x^2}\,dx &= \int \left(1+\frac{7-3x}{-5+2x+x^2}\right) dx\\ &= x+\int \frac{7-3x}{-5+2x+x^2}\,dx\\ &= x+\frac{1}{6}\left(-9+5\sqrt{6}\right)\int \frac{1}{1-\sqrt{6}+x}\,dx-\frac{1}{6}\left(9+5\sqrt{6}\right)\int \frac{1}{1+\sqrt{6}+x}\,dx\\ &= x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(1-\sqrt{6}+x\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)\end{aligned}$$

Mathematica [A] time = 0.038973, size = 48, normalized size = 1.

$$x+\frac{1}{6}\left(5\sqrt{6}-9\right)\log\left(-x+\sqrt{6}-1\right)+\frac{1}{6}\left(-9-5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

```
[Out] x + ((-9 + 5*Sqrt[6])*Log[-1 + Sqrt[6] - x])/6 + ((-9 - 5*Sqrt[6])*Log[1 +
Sqrt[6] + x])/6
```

Maple [A] time = 0.05, size = 30, normalized size = 0.6

$$x-\frac{3\,\ln\left(x^2+2\,x-5\right)}{2}-\frac{5\,\sqrt{6}}{3}\mathrm{Artanh}\left(\frac{(2\,x+2)\,\sqrt{6}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+2)/(x^2+2*x-5),x)
```

```
[Out] x-3/2*ln(x^2+2*x-5)-5/3*6^(1/2)*arctanh(1/12*(2*x+2)*6^(1/2))
```

Maxima [A] time = 1.57666, size = 49, normalized size = 1.02

$$\frac{5}{6}\,\sqrt{6}\log\left(\frac{x-\sqrt{6}+1}{x+\sqrt{6}+1}\right)+x-\frac{3}{2}\,\log\left(x^2+2\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(6)*log((x - sqrt(6) + 1)/(x + sqrt(6) + 1)) + x - 3/2*log(x^2 + 2*
x - 5)
```

Fricas [A] time = 1.76941, size = 157, normalized size = 3.27

$$\frac{5}{6}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}(x+1)-x^2-2x-7}{x^2+2x-5}\right)+x-\frac{3}{2}\log\left(x^2+2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="fricas")
```

```
[Out] 5/6*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(x + 1) - x^2 - 2*x - 7)/(x^2 +
 2*x - 5)) + x - 3/2*log(x^2 + 2*x - 5)
```

Sympy [A] time = 0.112266, size = 46, normalized size = 0.96

$$x+\left(-\frac{5\sqrt{6}}{6}-\frac{3}{2}\right)\log\left(x+1+\sqrt{6}\right)+\left(-\frac{3}{2}+\frac{5\sqrt{6}}{6}\right)\log\left(x-\sqrt{6}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+2)/(x**2+2*x-5),x)
```

```
[Out] x + (-5*sqrt(6)/6 - 3/2)*log(x + 1 + sqrt(6)) + (-3/2 + 5*sqrt(6)/6)*log(x
- sqrt(6) + 1)
```

Giac [A] time = 1.16208, size = 61, normalized size = 1.27

$$\frac{5}{6}\sqrt{6}\log\left(\frac{\left|2x-2\sqrt{6}+2\right|}{\left|2x+2\sqrt{6}+2\right|}\right)+x-\frac{3}{2}\log\left(\left|x^2+2x-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="giac")
```

```
[Out] 5/6*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2)/abs(2*x + 2*sqrt(6) + 2)) + x - 3/
2*log(abs(x^2 + 2*x - 5))
```

3.175 $\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2} dx$

Optimal. Leaf size=21

$$-\frac{3x+2}{2\left(2x^2+7x+4\right)}$$

```
[Out] -(2 + 3*x)/(2*(4 + 7*x + 2*x^2))
```

Rubi [A] time = 0.0119356, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1660, 8}

$$-\frac{3x+2}{2\left(2x^2+7x+4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
```

```
[Out] -(2 + 3*x)/(2*(4 + 7*x + 2*x^2))
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2}\,dx = -\frac{2+3x}{2\left(4+7x+2x^2\right)} - \frac{\int 0\,dx}{17}$$
$$= -\frac{2+3x}{2\left(4+7x+2x^2\right)}$$

Mathematica [A] time = 0.0074776, size = 21, normalized size = 1.

$$\frac{-3x-2}{2\left(2x^2+7x+4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
```

```
[Out] (-2 - 3*x)/(2*(4 + 7*x + 2*x^2))
```

Maple [A] time = 0.048, size = 17, normalized size = 0.8

$$\left(-\frac{3x}{4}-\frac{1}{2}\right)\left(x^2+\frac{7x}{2}+2\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x)
```

```
[Out] (-3/4*x-1/2)/(x^2+7/2*x+2)
```

Maxima [A] time = 1.00437, size = 26, normalized size = 1.24

$$-\frac{3x+2}{2\left(2x^2+7x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

Fricas [A] time = 1.6484, size = 46, normalized size = 2.19

$$-\frac{3\,x+2}{2\left(2\,x^2+7\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

Sympy [A] time = 0.112659, size = 15, normalized size = 0.71

$$-\frac{3x+2}{4x^2+14x+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x+1)/(2*x**2+7*x+4)**2,x)
```

```
[Out] -(3*x + 2)/(4*x**2 + 14*x + 8)
```

Giac [A] time = 1.22712, size = 26, normalized size = 1.24

$$-\frac{3\,x+2}{2\left(2\,x^2+7\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

3.176 $\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2}\,dx$

Optimal. Leaf size=39

$$\frac{1-x}{4\left(x^2+2x+3\right)}+\frac{3\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

Rubi [A] time = 0.0233002, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1660, 12, 618, 204}

$$\frac{1-x}{4\left(x^2+2x+3\right)}+\frac{3\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
```

```
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2}\,dx &= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{1}{8}\int \frac{6}{3+2x+x^2}\,dx \\
&= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3}{4}\int \frac{1}{3+2x+x^2}\,dx \\
&= \frac{1-x}{4\left(3+2x+x^2\right)}-\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right) \\
&= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0246723, size = 39, normalized size = 1.

$$\frac{1-x}{4\left(x^2+2x+3\right)}+\frac{3\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
```

```
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

Maple [A] time = 0.048, size = 34, normalized size = 0.9

$$\frac{1}{x^2+2x+3}\left(-\frac{x}{4}+\frac{1}{4}\right)+\frac{3\sqrt{2}}{8}\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2+2*x+3)^2,x)
```

```
[Out] (-1/4*x+1/4)/(x^2+2*x+3)+3/8*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))
```

Maxima [A] time = 1.56463, size = 41, normalized size = 1.05

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\frac{x-1}{4\left(x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

Fricas [A] time = 1.68439, size = 117, normalized size = 3.

$$\frac{3\sqrt{2}\left(x^2+2x+3\right)\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-2x+2}{8\left(x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(x^2 + 2*x + 3)*arctan(1/2*sqrt(2)*(x + 1)) - 2*x + 2)/(x^2
+ 2*x + 3)
```

Sympy [A] time = 0.122881, size = 37, normalized size = 0.95

$$-\frac{x-1}{4x^2+8x+12}+\frac{3\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2+2*x+3)**2,x)
```

```
[Out] -(x - 1)/(4*x**2 + 8*x + 12) + 3*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/8
```

Giac [A] time = 1.24636, size = 41, normalized size = 1.05

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\frac{x-1}{4\left(x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

3.177 $\int \frac{-1+2x+5x^2}{(1+x+x^2)^4} dx$

Optimal. Leaf size=11

$$-\frac{x}{(x^2+x+1)^3}$$

```
[Out] -(x/(1 + x + x^2)^3)
```

Rubi [A] time = 0.0076456, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$-\frac{x}{(x^2+x+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
```

```
[Out] -(x/(1 + x + x^2)^3)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4} dx = -\frac{x}{(1+x+x^2)^3}$$

Mathematica [A] time = 0.0060935, size = 11, normalized size = 1.

$$-\frac{x}{(x^2+x+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
```

```
[Out] -(x/(1 + x + x^2)^3)
```

Maple [A] time = 0.047, size = 12, normalized size = 1.1

$$-\frac{x}{\left(x^{2}+x+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x-1)/(x^2+x+1)^4,x)
```

```
[Out] -x/(x^2+x+1)^3
```

Maxima [B] time = 1.0292, size = 45, normalized size = 4.09

$$-\frac{x}{x^{6}+3\,x^{5}+6\,x^{4}+7\,x^{3}+6\,x^{2}+3\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="maxima")
```

```
[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

Fricas [B] time = 1.62136, size = 70, normalized size = 6.36

$$-\frac{x}{x^{6}+3\,x^{5}+6\,x^{4}+7\,x^{3}+6\,x^{2}+3\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="fricas")
```

```
[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

Sympy [B] time = 0.129674, size = 31, normalized size = 2.82

$$-\frac{x}{x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x-1)/(x**2+x+1)**4,x)
```

```
[Out] -x/(x**6 + 3*x**5 + 6*x**4 + 7*x**3 + 6*x**2 + 3*x + 1)
```

Giac [A] time = 1.26893, size = 15, normalized size = 1.36

$$-\frac{x}{\left(x^2 + x + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="giac")
```

```
[Out] -x/(x^2 + x + 1)^3
```

3.178 $\int \left(a+bx+cx^2\right)^{5/2} \left(A+Cx^2\right) dx$

Optimal. Leaf size=267

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}\left(-4acC+32Ac^2+9b^2C\right)}{384c^3}-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+32Ac^2+9b^2C\right)}{6144c^4}$$

```
[Out] (5*(b^2 - 4*a*c)^2*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x
+ c*x^2])/(16384*c^5) - (5*(b^2 - 4*a*c)*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^4) + ((32*A*c^2 + 9*b^2*C - 4*a*c
*C)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(384*c^3) - (9*b*C*(a + b*x + c*x^
2)^(7/2))/(112*c^2) + (C*x*(a + b*x + c*x^2)^(7/2))/(8*c) - (5*(b^2 - 4*a*c
)^3*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(32768*c^(11/2))
```

Rubi [A] time = 0.23808, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}\left(-4acC+32Ac^2+9b^2C\right)}{384c^3}-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+32Ac^2+9b^2C\right)}{6144c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```


```
[Out] (5*(b^2 - 4*a*c)^2*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x
+ c*x^2])/(16384*c^5) - (5*(b^2 - 4*a*c)*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^4) + ((32*A*c^2 + 9*b^2*C - 4*a*c
*C)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(384*c^3) - (9*b*C*(a + b*x + c*x^
2)^(7/2))/(112*c^2) + (C*x*(a + b*x + c*x^2)^(7/2))/(8*c) - (5*(b^2 - 4*a*c
)^3*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(32768*c^(11/2))
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
```

```
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)dx = \frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c} + \frac{\int \left(8Ac-aC-\frac{9bCx}{2}\right)\left(a+bx+cx^2\right)^{5/2}dx}{8c}$$

$$= -\frac{9bC\left(a+bx+cx^2\right)^{7/2}}{112c^2} + \frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c} + \frac{\left(\frac{9b^2C}{2}+2c(8Ac-aC)\right)\int \left(a+bx\right.}{16c^2}$$

$$= \frac{\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{384c^3} - \frac{9bC\left(a+bx+cx^2\right)^{7/2}}{112c^2} + \frac{Cx\left(\right.}{}$$

$$= -\frac{5\left(b^2-4ac\right)\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{6144c^4} + \frac{\left(32Ac^2+9b^2C\right.}{}$$

$$= \frac{5\left(b^2-4ac\right)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5\left(b^2-4ac\right)\left(32Ac\right.}{}$$

$$= \frac{5\left(b^2-4ac\right)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5\left(b^2-4ac\right)\left(32Ac\right.}{}$$

$$= \frac{5\left(b^2-4ac\right)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5\left(b^2-4ac\right)\left(32Ac\right.}{}$$

Mathematica [A] time = 0.851162, size = 344, normalized size = 1.29

$$\frac{-\frac{1120A\left(b^2-4ac\right)\left(16c^{3/2}(b+2cx)(a+x(b+cx))^{3/2}-3\left(b^2-4ac\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\right)}{c^{5/2}} + 57344A(b+2cx)(a+x(b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (57344*A*(b + 2*c*x)*(a + x*(b + c*x))^(5/2) - (55296*b*C*(a + x*(b + c*x))
^(7/2))/c + 86016*C*x*(a + x*(b + c*x))^(7/2) - (1120*A*(b^2 - 4*a*c)*(16*c
^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (7*(9*b^2 - 4*a*c)*C*(256*c^(5/2)*(
b + 2*c*x)*(a + x*(b + c*x))^(5/2) - 5*(b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x
)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a +
 x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])]))))/c^(9/2))/(688128*c)
```

Maple [B] time = 0.055, size = 997, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x)
```

```
[Out] -15/64*A/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a^2-5/96*A
/c*(c*x^2+b*x+a)^(3/2)*x*b^2+15/256*A/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*b^4*a+5/256*A/c^2*(c*x^2+b*x+a)^(1/2)*x*b^4+5/32*A/c*(c*x^2+
b*x+a)^(1/2)*b*a^2-5/64*A/c^2*(c*x^2+b*x+a)^(1/2)*b^3*a+15/128*C*b^2/c^(5/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-15/1024*C*b^4/c^3*(c*x^2+
b*x+a)^(3/2)*x-5/192*C*a^2/c*(c*x^2+b*x+a)^(3/2)*x+3/64*C*b^2/c^2*(c*x^2+b*
x+a)^(5/2)*x+35/2048*C*b^6/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*a+1/6*A*(c*x^2+b*x+a)^(5/2)*x-1/48*C*a/c*(c*x^2+b*x+a)^(5/2)*x+5/48*A/c
*(c*x^2+b*x+a)^(3/2)*b*a-1/96*C*a/c^2*(c*x^2+b*x+a)^(5/2)*b-75/1024*C*b^4/c
^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-5/256*C*a^3/c^2*(c*x
^2+b*x+a)^(1/2)*b-5/128*C*a^3/c*(c*x^2+b*x+a)^(1/2)*x+3/128*C*b^3/c^3*(c*x^
2+b*x+a)^(5/2)+45/8192*C*b^6/c^4*(c*x^2+b*x+a)^(1/2)*x+55/1024*C*b^3/c^3*(c
*x^2+b*x+a)^(1/2)*a^2-95/4096*C*b^5/c^4*(c*x^2+b*x+a)^(1/2)*a-5/384*C*a^2/c
^2*(c*x^2+b*x+a)^(3/2)*b-5/1024*A/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*b^6-5/32*A/c*(c*x^2+b*x+a)^(1/2)*x*a*b^2+1/12*A/c*(c*x^2+b*x+a)^
(5/2)*b+5/24*A*(c*x^2+b*x+a)^(3/2)*x*a-5/192*A/c^2*(c*x^2+b*x+a)^(3/2)*b^3+
5/16*A*(c*x^2+b*x+a)^(1/2)*x*a^2+5/512*A/c^3*(c*x^2+b*x+a)^(1/2)*b^5+5/16*A
/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-15/2048*C*b^5/c^4*
(c*x^2+b*x+a)^(3/2)-45/32768*C*b^8/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))+45/16384*C*b^7/c^5*(c*x^2+b*x+a)^(1/2)-5/128*C*a^4/c^(3/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-95/2048*C*b^4/c^3*(c*x^2+b*x+a)^(1
/2)*x*a-9/112*b*C*(c*x^2+b*x+a)^(7/2)/c^2+1/8*C*x*(c*x^2+b*x+a)^(7/2)/c+25/
384*C*b^2/c^2*(c*x^2+b*x+a)^(3/2)*x*a+55/512*C*b^2/c^2*(c*x^2+b*x+a)^(1/2)*
x*a^2+25/768*C*b^3/c^3*(c*x^2+b*x+a)^(3/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.88837, size = 2342, normalized size = 8.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/1376256*(105*(9*C*b^8 - 112*C*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*
A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6
)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) + 4*(43008*C*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C
*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*
C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^
3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)
*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 +
 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3 - 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 +
 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 - 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3
- 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 7
0*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/688128*(105*(9*C*b^8 - 112*C
*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^
2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6)*c^2)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(43008*C
*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*
a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a
^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)
*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*
c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 + 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3
- 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 + 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 -
 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3 - 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8
*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 70*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a)
)/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + Cx^{2}\right)\left(a + bx + cx^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x + c*x**2)**(5/2), x)
```

Giac [B] time = 1.25004, size = 651, normalized size = 2.44

$$\frac{1}{344064}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(12\left(14\,Cc^2x+33\,Cbc\right)x+\frac{243\,Cb^2c^7+476\,Cac^8+224\,Ac^9}{c^7}\right)x+\frac{3\,Cb^3c^6+1228\,C}{c}\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/344064*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(14*C*c^2*x + 33*C*b*c)*x
 + (243*C*b^2*c^7 + 476*C*a*c^8 + 224*A*c^9)/c^7)*x + (3*C*b^3*c^6 + 1228*C
*a*b*c^7 + 1120*A*b*c^8)/c^7)*x - (27*C*b^4*c^5 - 216*C*a*b^2*c^6 - 6608*C*
a^2*c^7 - 6048*A*b^2*c^7 - 11648*A*a*c^8)/c^7)*x + (63*C*b^5*c^4 - 568*C*a*
b^3*c^5 + 1392*C*a^2*b*c^6 + 224*A*b^3*c^6 + 34944*A*a*b*c^7)/c^7)*x - (315
*C*b^6*c^3 - 3164*C*a*b^4*c^4 + 9552*C*a^2*b^2*c^5 + 1120*A*b^4*c^5 - 6720*
C*a^3*c^6 - 10752*A*a*b^2*c^6 - 118272*A*a^2*c^7)/c^7)*x + (945*C*b^7*c^2 -
 10500*C*a*b^5*c^3 + 37744*C*a^2*b^3*c^4 + 3360*A*b^5*c^4 - 42432*C*a^3*b*c
^5 - 35840*A*a*b^3*c^5 + 118272*A*a^2*b*c^6)/c^7) + 5/32768*(9*C*b^8 - 112*
C*a*b^6*c + 480*C*a^2*b^4*c^2 + 32*A*b^6*c^2 - 768*C*a^3*b^2*c^3 - 384*A*a*
b^4*c^3 + 256*C*a^4*c^4 + 1536*A*a^2*b^2*c^4 - 2048*A*a^3*c^5)*log(abs(-2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

3.179 $\int \left(a+bx+cx^2\right)^{3/2} \left(A+Cx^2\right) dx$

Optimal. Leaf size=212

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+24Ac^2+7b^2C\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+24Ac^2+7b^2C\right)}{512c^4}+\ldots$$

```
[Out] -((b^2 - 4*a*c)*(24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c
*x^2])/(512*c^4) + ((24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/2))/(192*c^3) - (7*b*C*(a + b*x + c*x^2)^(5/2))/(60*c^2) + (C*x*(a
 + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*c)^2*(24*A*c^2 + 7*b^2*C - 4*a*c
*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rubi [A] time = 0.183374, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+24Ac^2+7b^2C\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+24Ac^2+7b^2C\right)}{512c^4}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c
*x^2])/(512*c^4) + ((24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/2))/(192*c^3) - (7*b*C*(a + b*x + c*x^2)^(5/2))/(60*c^2) + (C*x*(a
 + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*c)^2*(24*A*c^2 + 7*b^2*C - 4*a*c
*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(A+Cx^2\right)dx = \frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\int \left(6Ac-aC-\frac{7bCx}{2}\right)\left(a+bx+cx^2\right)^{3/2}dx}{6c}$$

$$= -\frac{7bC\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\left(\frac{7b^2C}{2}+2c(6Ac-aC)\right)\int \left(a+bx\right.}{12c^2}$$

$$= \frac{\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}-\frac{7bC\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{Cx\left(\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24Ac^2+7b^2C-4a\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24Ac^2+7b^2C-4a\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24Ac^2+7b^2C-4a\right.}{}$$

Mathematica [A] time = 0.550195, size = 267, normalized size = 1.26

$$\frac{\frac{360A\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}}+1920A(b+2cx)(a+x(b+cx))^{3/2}+\frac{C\left(5\left(7b^2-4ac\right)\left(\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\right.}{}\right.\right.}{}}{15360c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

```
[Out] (1920*A*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 2560*C*x*(a + x*(b + c*x))^(5
/2) + (360*A*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] +
(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3
/2) + (C*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*S
qrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])]))/c^(5/2))))/c)/(15360*c)
```

Maple [B] time = 0.057, size = 613, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x)
```

```
[Out] 1/8*C*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a-3/16*A/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*b^2*a+3/16*A/c*(c*x^2+b*x+a)^(1/2)*b*a-1/16*C*a^2/c*(
c*x^2+b*x+a)^(1/2)*x+9/64*C*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a^2-15/256*C*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*a+7/96*C*b^2/c^2*(c*x^2+b*x+a)^(3/2)*x-7/256*C*b^4/c^3*(c*x^2+b*x+a)^(1/
2)*x+1/16*C*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a-1/48*C*a/c^2*(c*x^2+b*x+a)^(3/2)*
b-1/24*C*a/c*(c*x^2+b*x+a)^(3/2)*x-3/32*A/c*(c*x^2+b*x+a)^(1/2)*x*b^2-1/32*
C*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b+1/4*A*(c*x^2+b*x+a)^(3/2)*x+3/8*A*(c*x^2+b*
x+a)^(1/2)*x*a-3/64*A/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*A/c^(1/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*A/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*b^4+7/192*C*b^3/c^3*(c*x^2+b*x+a)^(3/2)-7/512*C*b^5/c
^4*(c*x^2+b*x+a)^(1/2)+1/8*A/c*(c*x^2+b*x+a)^(3/2)*b+7/1024*C*b^6/c^(9/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*C*a^3/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/60*b*C*(c*x^2+b*x+a)^(5/2)/c^2+1/6*C*x*(c*
x^2+b*x+a)^(5/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.30583, size = 1451, normalized size = 6.84

$$\left[\frac{15\left(7\,Cb^6 - 60\,Cab^4c + 384\,Aa^2c^4 - 64\left(Ca^3 + 3\,Aab^2\right)c^3 + 24\left(6\,Ca^2b^2 + Ab^4\right)c^2\right)\sqrt{c}\log\left(-8\,c^2x^2 - 8\,bcx - b^2 - 4\,\sqrt{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*a^3 + 3*A*a*b^
2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b
^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*C*c^6*x
^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*c^4 - 72
*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A*c^6)*x^
3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*c^2 - 21
6*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c*x^2 + b
*x + a))/c^5, -1/15360*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*
a^3 + 3*A*a*b^2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(-c)*arctan(1/2*sq
rt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280
*C*c^6*x^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*
c^4 - 72*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A
*c^6)*x^3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*
c^2 - 216*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c
*x^2 + b*x + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + Cx^2\right)\left(a + bx + cx^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.32111, size = 401, normalized size = 1.89

$$\frac{1}{7680}\sqrt{cx^2 + bx + a}\left(2\left(4\left(2\left(8\,(10\,Ccx + 13\,Cb)x + \frac{3\,Cb^2c^4 + 140\,Cac^5 + 120\,Ac^6}{c^5}\right)x - \frac{7\,Cb^3c^3 - 36\,Cabc^4 - 360\,Abc^5}{c^5}\right)x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*C*c*x + 13*C*b)*x + (3*C*b^2*c
^4 + 140*C*a*c^5 + 120*A*c^6)/c^5)*x - (7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*
b*c^5)/c^5)*x + (35*C*b^4*c^2 - 216*C*a*b^2*c^3 + 240*C*a^2*c^4 + 120*A*b^2
```

```
*c^4 + 2400*A*a*c^5)/c^5)*x - (105*C*b^5*c - 760*C*a*b^3*c^2 + 1296*C*a^2*b
*c^3 + 360*A*b^3*c^3 - 2400*A*a*b*c^4)/c^5) - 1/1024*(7*C*b^6 - 60*C*a*b^4*
c + 144*C*a^2*b^2*c^2 + 24*A*b^4*c^2 - 64*C*a^3*c^3 - 192*A*a*b^2*c^3 + 384
*A*a^2*c^4)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^
(9/2)
```

3.180 $\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)dx$

Optimal. Leaf size=157

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+16Ac^2+5b^2C\right)}{64c^3}-\frac{\left(b^2-4ac\right)\left(-4acC+16Ac^2+5b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}-\frac{5bC\left(\right.}{}$$

```
[Out] ((16*A*c^2 + 5*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^3)
 - (5*b*C*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (C*x*(a + b*x + c*x^2)^(3/2))
/(4*c) - ((b^2 - 4*a*c)*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2))
```

Rubi [A] time = 0.123895, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+16Ac^2+5b^2C\right)}{64c^3}-\frac{\left(b^2-4ac\right)\left(-4acC+16Ac^2+5b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}-\frac{5bC\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```


```
[Out] ((16*A*c^2 + 5*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^3)
 - (5*b*C*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (C*x*(a + b*x + c*x^2)^(3/2))
/(4*c) - ((b^2 - 4*a*c)*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2))
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)dx = \frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\int \left(4Ac-aC-\frac{5bCx}{2}\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= -\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\left(\frac{5b^2C}{2}+2c(4Ac-aC)\right)\int \sqrt{a+bx\cdots}}{8c^2}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{Cx\left(a+b\cdots\right.}{}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{Cx\left(a+b\cdots\right.}{}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{Cx\left(a+b\cdots\right.}{}$$

Mathematica [A] time = 0.20037, size = 144, normalized size = 0.92

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(C\left(b\left(8c^2x^2-52ac\right)+24c^2x\left(a+2cx^2\right)-10b^2cx+15b^3\right)+48Ac^2(b+2cx)\right)-3\left(b^2-4ac\right)\left(-4acC+\right.}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(48*A*c^2*(b + 2*c*x) + C*(15*b^3 - 10*b^2
*c*x + 24*c^2*x*(a + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2))) - 3*(b^2 - 4*a*c)
*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])])/(384*c^(7/2))
```

Maple [B] time = 0.053, size = 327, normalized size = 2.1

$$\frac{Cx}{4c}\left(cx^2+bx+a\right)^{\frac{3}{2}}-\frac{5bC}{24c^2}\left(cx^2+bx+a\right)^{\frac{3}{2}}+\frac{5Cb^2x}{32c^2}\sqrt{cx^2+bx+a}+\frac{5Cb^3}{64c^3}\sqrt{cx^2+bx+a}+\frac{3Cb^2a}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x)
```

```
[Out] 1/4*C*x*(c*x^2+b*x+a)^(3/2)/c-5/24*b*C*(c*x^2+b*x+a)^(3/2)/c^2+5/32*C*b^2/c
^2*(c*x^2+b*x+a)^(1/2)*x+5/64*C*b^3/c^3*(c*x^2+b*x+a)^(1/2)+3/16*C*b^2/c^(5
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*C*b^4/c^(7/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*C*a/c*(c*x^2+b*x+a)^(1/2)*x-1/16
*C*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*C*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+1/2*A*(c*x^2+b*x+a)^(1/2)*x+1/4*A/c*(c*x^2+b*x+a)^(1/2)*b
+1/2*A/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*A/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09802, size = 846, normalized size = 5.39

$$\left[-\frac{3\left(5\,Cb^4-24\,Cab^2c-64\,Aac^3+16\left(Ca^2+Ab^2\right)c^2\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,a\right.}{768\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 + A*b^2)*c^2)*s
qrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(c) - 4*a*c) - 4*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c - 52*C*a*b
*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt(c*x^2 +
 b*x + a))/c^4, 1/384*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 +
 A*b^2)*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)
/(c^2*x^2 + b*c*x + a*c)) + 2*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c -
52*C*a*b*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt
(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A+Cx^2\right)\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.25234, size = 216, normalized size = 1.38

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,Cx+\frac{Cb}{c}\right)x-\frac{5\,Cb^2c-12\,Cac^2-48\,Ac^3}{c^3}\right)x+\frac{15\,Cb^3-52\,Cabc+48\,Abc^2}{c^3}\right)+\frac{\left(5\,Cb^4-24\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*C*x + C*b/c)*x - (5*C*b^2*c - 12*C*a*c
^2 - 48*A*c^3)/c^3)*x + (15*C*b^3 - 52*C*a*b*c + 48*A*b*c^2)/c^3) + 1/128*(
5*C*b^4 - 24*C*a*b^2*c + 16*C*a^2*c^2 + 16*A*b^2*c^2 - 64*A*a*c^3)*log(abs(
-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

3.181 $\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=104

$$\frac{\left(-4acC+8Ac^2+3b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

```
[Out] (-3*b*C*Sqrt[a + b*x + c*x^2])/(4*c^2) + (C*x*Sqrt[a + b*x + c*x^2])/(2*c)
+ ((8*A*c^2 + 3*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*c^(5/2))
```

Rubi [A] time = 0.0795335, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1661, 640, 621, 206}

$$\frac{\left(-4acC+8Ac^2+3b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-3*b*C*Sqrt[a + b*x + c*x^2])/(4*c^2) + (C*x*Sqrt[a + b*x + c*x^2])/(2*c)
+ ((8*A*c^2 + 3*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*c^(5/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx &= \frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\int \frac{2Ac-aC-\frac{3bCx}{2}}{\sqrt{a+bx+cx^2}}\,dx}{2c}\\ &= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{4c^2}\\ &= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{2c^2}\\ &= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\left(8Ac^2+3b^2C-4acC\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.145834, size = 86, normalized size = 0.83

$$\frac{\left(-4acC+8Ac^2+3b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{8c^{5/2}}+\frac{C(2cx-3b)\sqrt{a+x(b+cx)}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (C*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*A*c^2 + 3*b^2*C - 4*
a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(5/2))
```

Maple [A] time = 0.051, size = 136, normalized size = 1.3

$$\frac{Cx}{2c}\sqrt{cx^2+bx+a}-\frac{3bC}{4c^2}\sqrt{cx^2+bx+a}+\frac{3Cb^2}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{5}{2}}-\frac{aC}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*C*x*(c*x^2+b*x+a)^(1/2)/c-3/4*b*C*(c*x^2+b*x+a)^(1/2)/c^2+3/8*C*b^2/c^(
5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*C*a/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+A*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98015, size = 494, normalized size = 4.75

$$\left[\frac{\left(3Cb^2-4Cac+8Ac^2\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)+4\left(2Cc^2x-3Cbc\right)\sqrt{cx^2}}{16c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*C*c^2*x - 3*
```

```
C*b*c)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) - 2*(2*C*c^2*x - 3*C*b*c)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Cx^2}{\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.34466, size = 113, normalized size = 1.09

$$\frac{1}{4}\sqrt{cx^2 + bx + a}\left(\frac{2\,Cx}{c} - \frac{3\,Cb}{c^2}\right) - \frac{\left(3\,Cb^2 - 4\,Cac + 8\,Ac^2\right)\log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*C*x/c - 3*C*b/c^2) - 1/8*(3*C*b^2 - 4*C*a*c +
8*A*c^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/
2)
```

3.182 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}} \, dx$

Optimal. Leaf size=98

$$\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(c*(b^2 - 4*a*c)*S
qrt[a + b*x + c*x^2]) + (C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/c^(3/2)
```

Rubi [A] time = 0.0770029, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1660, 12, 621, 206}

$$\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(c*(b^2 - 4*a*c)*S
qrt[a + b*x + c*x^2]) + (C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/c^(3/2)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx &= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)C}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{C\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2C)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.763087, size = 104, normalized size = 1.06

$$
\frac{\frac{2\sqrt{c}\left(aC(b-2cx)+Ac(b+2cx)+b^2Cx\right)}{\sqrt{a+x(b+cx)}}-C\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}\left(4ac-b^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x)))/Sqrt[a + x*(b +
c*x)] - (b^2 - 4*a*c)*C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])/(c^(3/2)*(-b^2 + 4*a*c))
```

Maple [A] time = 0.052, size = 169, normalized size = 1.7

$$-\frac{Cx}{c}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{Cb}{2c^2}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{Cb^2x}{c\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{Cb^3}{2c^2\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^2+bx+a}}+C\ln\left(\left(\frac{b}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -C*x/c/(c*x^2+b*x+a)^(1/2)+1/2*C*b/c^2/(c*x^2+b*x+a)^(1/2)+C*b^2/c/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*C*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+C/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*A*(2*c*x+b)/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.14125, size = 892, normalized size = 9.1

$$\left[\frac{\left(Cab^2-4\,Ca^2c+\left(Cb^2c-4\,Cac^2\right)x^2+\left(Cb^3-4\,Cabc\right)x\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-\right.}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^2+\left(b^3c^2-4\,abc^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a*c^2)*x^2 + (C*b^3 - 4*C*a*b*c
)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(c) - 4*a*c) - 4*(C*a*b*c + A*b*c^2 + (C*b^2*c - 2*C*a*c^2 + 2*A
*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4
)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a
*c^2)*x^2 + (C*b^3 - 4*C*a*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(C*a*b*c + A*b*c^2 + (C
*b^2*c - 2*C*a*c^2 + 2*A*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Cx^2}{\left(a + bx + cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((A + C*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.33972, size = 149, normalized size = 1.52

$$-\frac{2\left(\frac{\left(Cb^2-2\,Cac+2\,Ac^2\right)x}{b^2c-4\,ac^2}+\frac{Cab+Abc}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{C\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((C*b^2 - 2*C*a*c + 2*A*c^2)*x/(b^2*c - 4*a*c^2) + (C*a*b + A*b*c)/(b^2*
c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - C*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*sqrt(c) - b))/c^(3/2)
```

3.183 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(3*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(3/2)) + (2*(8*A*c + 4*a*C + (b^2*C)/c)*(b + 2*c*x))/(3*
(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.093144, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1660, 12, 613}

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(3*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(3/2)) + (2*(8*A*c + 4*a*C + (b^2*C)/c)*(b + 2*c*x))/(3*
(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2\int \frac{8Ac+4aC+\frac{b^2C}{c}}{2\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{\left(8Ac+4aC+\frac{b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2\left(8Ac+4aC+\frac{b^2C}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.889565, size = 107, normalized size = 0.94

$$\frac{2C\left(8a^2b+4ax\left(3b^2+3bcx+2c^2x^2\right)+b^2x^2(3b+2cx)\right)-2A(b+2cx)\left(-4c\left(3a+2cx^2\right)+b^2-8bcx\right)}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*A*(b + 2*c*x)*(b^2 - 8*b*c*x - 4*c*(3*a + 2*c*x^2)) + 2*C*(8*a^2*b + b^
2*x^2*(3*b + 2*c*x) + 4*a*x*(3*b^2 + 3*b*c*x + 2*c^2*x^2)))/(3*(b^2 - 4*a*c
)^2*(a + x*(b + c*x))^(3/2))
```

Maple [A] time = 0.049, size = 137, normalized size = 1.2

$$\frac{32\,Ax^3c^3+16\,Cac^2x^3+4\,Cb^2cx^3+48\,Ax^2bc^2+24\,Cabcx^2+6\,Cb^3x^2+48\,Aac^2x+12\,Ab^2cx+24\,Cab^2x+24\,Aabc-2}{48\,a^2c^2-24\,acb^2+3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3/(c*x^2+b*x+a)^(3/2)*(16*A*c^3*x^3+8*C*a*c^2*x^3+2*C*b^2*c*x^3+24*A*b*c^
2*x^2+12*C*a*b*c*x^2+3*C*b^3*x^2+24*A*a*c^2*x+6*A*b^2*c*x+12*C*a*b^2*x+12*A
*a*b*c-A*b^3+8*C*a^2*b)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.07228, size = 520, normalized size = 4.56

$$\frac{2\left(8\,Ca^2b - Ab^3 + 12\,Aabc + 2\left(Cb^2c + 4\,Cac^2 + 8\,Ac^3\right)x^3 + 3\left(Cb^3 + 4\,Cabc + 8\,Abc^2\right)x^2 + 6\left(2\,Cab^2 + Ab^2c + \right.\right.}{3\left(a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + \left(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4\right)x^4 + 2\left(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3\right)x^3 + \left(b^6 - 6\,ab^4c + 32\,a^3c^3\right)x^2 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*C*a^2*b - A*b^3 + 12*A*a*b*c + 2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x^3
 + 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)*x^2 + 6*(2*C*a*b^2 + A*b^2*c + 4*A*a*c
^2)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x
^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b
*c^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Cx^2}{\left(a + bx + cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((A + C*x**2)/(a + b*x + c*x**2)**(5/2), x)
```

Giac [B] time = 1.24251, size = 293, normalized size = 2.57

$$\frac{\left(\left(\frac{2\left(Cb^{2}c+4\,Cac^{2}+8\,Ac^{3}\right)x}{b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}}+\frac{3\left(Cb^{3}+4\,Cabc+8\,Abc^{2}\right)}{b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}}\right)x+\frac{6\left(2\,Cab^{2}+Ab^{2}c+4\,Aac^{2}\right)}{b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}}\right)x+\frac{8\,Ca^{2}b-Ab^{3}+12\,Aabc}{b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}}}{3\left(cx^{2}+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4) + 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4))*x + 6*(2*C*a*b^2 + A*b^2*c + 4*A*a*c^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4))*x + (8*C*a^2*b - A*b^3 + 12*A*a*b*c)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4))/(c*x^2 + b*x + a)^(3/2)
```

3.184 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{16(b+2cx)\left(4acC+16Ac^2+3b^2C\right)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2(b+2cx)\left(4aC+16Ac+\frac{3b^2C}{c}\right)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(5*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(5/2)) + (2*(16*A*c + 4*a*C + (3*b^2*C)/c)*(b + 2*c*x))/
(15*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(3/2)) - (16*(16*A*c^2 + 3*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(15*(b^2 - 4*a*c)^3*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.106591, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1660, 12, 614, 613}

$$-\frac{16(b+2cx)\left(4acC+16Ac^2+3b^2C\right)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2(b+2cx)\left(4aC+16Ac+\frac{3b^2C}{c}\right)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(5*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(5/2)) + (2*(16*A*c + 4*a*C + (3*b^2*C)/c)*(b + 2*c*x))/
(15*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(3/2)) - (16*(16*A*c^2 + 3*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(15*(b^2 - 4*a*c)^3*Sqrt[a + b*x + c*x^2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}-\frac{2\int \frac{16Ac+4aC+\frac{3b^2C}{c}}{2\left(a+bx+cx^2\right)^{5/2}}\,dx}{5\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}-\frac{\left(16Ac+4aC+\frac{3b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^{5/2}}\,dx}{5\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}+\frac{\left(8\left(16Ac^2\right.\right.}{}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}-\frac{16\left(16Ac^2\right.}{15\left(b^2\right.}$$

Mathematica [A] time = 1.85438, size = 148, normalized size = 0.89

$$\frac{2\left(\left(b^2-4ac\right)(b+2cx)(a+x(b+cx))\left(4acC+16Ac^2+3b^2C\right)-8c(b+2cx)(a+x(b+cx))^2\left(4acC+16Ac^2+3b^2C\right)-3\left(\right.\right.}{15c\left(b^2-4ac\right)^3(a+x(b+cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (2*((b^2 - 4*a*c)*(16*A*c^2 + 3*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a + x*(b + c*
x)) - 8*c*(16*A*c^2 + 3*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a + x*(b + c*x))^2 -
3*(b^2 - 4*a*c)^2*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))))/(15*c*(b^
2 - 4*a*c)^3*(a + x*(b + c*x))^(5/2))
```

Maple [B] time = 0.051, size = 316, normalized size = 1.9

$$512\,Ac^5x^5 + 128\,Cac^4x^5 + 96\,Cb^2c^3x^5 + 1280\,Abc^4x^4 + 320\,Cabc^3x^4 + 240\,Cb^3c^2x^4 + 1280\,Aac^4x^3 + 960\,Ax^3b^2c^3 + 32$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x)
```

```
[Out] 2/15/(c*x^2+b*x+a)^(5/2)*(256*A*c^5*x^5+64*C*a*c^4*x^5+48*C*b^2*c^3*x^5+640
*A*b*c^4*x^4+160*C*a*b*c^3*x^4+120*C*b^3*c^2*x^4+640*A*a*c^4*x^3+480*A*b^2*
c^3*x^3+160*C*a^2*c^3*x^3+240*C*a*b^2*c^2*x^3+90*C*b^4*c*x^3+960*A*a*b*c^3*
x^2+80*A*b^3*c^2*x^2+240*C*a^2*b*c^2*x^2+200*C*a*b^3*c*x^2+15*C*b^5*x^2+480
*A*a^2*c^3*x+240*A*a*b^2*c^2*x-10*A*b^4*c*x+240*C*a^2*b^2*c*x+20*C*a*b^4*x+
240*A*a^2*b*c^2-40*A*a*b^3*c+3*A*b^5+96*C*a^3*b*c+8*C*a^2*b^3)/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 77.4996, size = 1224, normalized size = 7.33

$$-\frac{2\left(8\,Ca^2b^3+3\,Ab^5+240\,Aa^2bc^2+16\left(3\,Cb^2c^3+4\,Cac^4+16\,Ac^5\right)x^5+40\left(3\,Cb^3c^2+4\,Cabc^3+16\,Abc^4\right)x^4+10\right.}{15\left(a^3b^6-12\,a^4b^4c+48\,a^5b^2c^2-64\,a^6c^3+\left(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6\right)x^6+3\left(b^7c^2-12\,ab^5c^3+48\,a^2b^3c^4-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*C*a^2*b^3 + 3*A*b^5 + 240*A*a^2*b*c^2 + 16*(3*C*b^2*c^3 + 4*C*a*c^
4 + 16*A*c^5)*x^5 + 40*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)*x^4 + 10*(9
*C*b^4*c + 24*C*a*b^2*c^2 + 64*A*a*c^4 + 16*(C*a^2 + 3*A*b^2)*c^3)*x^3 + 5*
(3*C*b^5 + 40*C*a*b^3*c + 192*A*a*b*c^3 + 16*(3*C*a^2*b + A*b^3)*c^2)*x^2 +
 8*(12*C*a^3*b - 5*A*a*b^3)*c + 10*(2*C*a*b^4 + 24*A*a*b^2*c^2 + 48*A*a^2*c
^3 + (24*C*a^2*b^2 - A*b^4)*c)*x)*sqrt(c*x^2 + b*x + a)/(a^3*b^6 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^6 + 3*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^5 + 3*(b^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64
*a^4*c^5)*x^4 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a
^4*b*c^4)*x^3 + 3*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 -
 64*a^5*c^4)*x^2 + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^
3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2866, size = 659, normalized size = 3.95

$$-\frac{\left(\left(2\left(4\left(\frac{2\left(3\,Cb^2c^3+4\,Cac^4+16\,Ac^5\right)x}{b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6}+\frac{5\left(3\,Cb^3c^2+4\,Cabc^3+16\,Abc^4\right)}{b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6}\right)x+\frac{5\left(9\,Cb^4c+24\,Cab^2c^2+16\,Ca^2c^3+48\,Ab^2c^3+64\,Aac^4\right)}{b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6}\right)x+\frac{5\left(3\,Cb^5+\right.}{}\right.\right.}{15\left(cx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(((2*(4*(2*(3*C*b^2*c^3 + 4*C*a*c^4 + 16*A*c^5)*x/(b^6*c^3 - 12*a*b^4
*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6) + 5*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b
*c^4)/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*x + 5*(9*C*b^
4*c + 24*C*a*b^2*c^2 + 16*C*a^2*c^3 + 48*A*b^2*c^3 + 64*A*a*c^4)/(b^6*c^3 -
 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*x + 5*(3*C*b^5 + 40*C*a*b^3*c
 + 48*C*a^2*b*c^2 + 16*A*b^3*c^2 + 192*A*a*b*c^3)/(b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6))*x + 10*(2*C*a*b^4 + 24*C*a^2*b^2*c - A*b^4*c
 + 24*A*a*b^2*c^2 + 48*A*a^2*c^3)/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5
- 64*a^3*c^6))*x + (8*C*a^2*b^3 + 3*A*b^5 + 96*C*a^3*b*c - 40*A*a*b^3*c + 2
40*A*a^2*b*c^2)/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))/(c*
x^2 + b*x + a)^(5/2)
```

3.185 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx$

Optimal. Leaf size=220

$$\frac{256c(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}-\frac{32(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right.\right.}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}$$

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(7*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(7/2)) + (2*(24*A*c + 4*a*C + (5*b^2*C)/c)*(b + 2*c*x))/
(35*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(5/2)) - (32*(24*A*c^2 + 5*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^(3/2)) + (256*c*
(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^4*Sqrt[a + b
*x + c*x^2])
```

Rubi [A] time = 0.141816, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1660, 12, 614, 613}

$$\frac{256c(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}-\frac{32(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right.\right.}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```


```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(7*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(7/2)) + (2*(24*A*c + 4*a*C + (5*b^2*C)/c)*(b + 2*c*x))/
(35*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(5/2)) - (32*(24*A*c^2 + 5*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^(3/2)) + (256*c*
(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^4*Sqrt[a + b
*x + c*x^2])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
```

```
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}} - \frac{2\int \frac{24Ac+4aC+\frac{5b^2C}{c}}{2(a+bx+cx^2)^{7/2}}\,dx}{7\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}} - \frac{\left(24Ac+4aC+\frac{5b^2C}{c}\right)\int \frac{1}{(a+bx+cx^2)^{7/2}}\,dx}{7\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}} + \frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}} + \frac{\left(16\left(24Ac^2\right.\right.}{}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}} + \frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}} - \frac{32\left(24Ac^2\right.}{105\left(b^2-\right.}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}} + \frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}} - \frac{32\left(24Ac^2\right.}{105\left(b^2-\right.}$$

Mathematica [A] time = 1.78204, size = 199, normalized size = 0.9

$$\frac{2\left(3\left(b^2-4ac\right)^2(b+2cx)(a+x(b+cx))\left(4acC+24Ac^2+5b^2C\right)-16c\left(b^2-4ac\right)(b+2cx)(a+x(b+cx))^2\left(4acC+24Ac\right.\right.}{105c\left(b^2-4ac\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

```
[Out] (2*(3*(b^2 - 4*a*c)^2*(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a + x*(b
+ c*x)) - 16*c*(b^2 - 4*a*c)*(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a
+ x*(b + c*x))^2 + 128*c^2*(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a +
x*(b + c*x))^3 - 15*(b^2 - 4*a*c)^3*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2
*c*x))))/(105*c*(b^2 - 4*a*c)^4*(a + x*(b + c*x))^(7/2))
```

Maple [B] time = 0.053, size = 555, normalized size = 2.5

$$\frac{12288\,Ac^7x^7+2048\,Cac^6x^7+2560\,Cb^2c^5x^7+43008\,Abc^6x^6+7168\,Cabc^5x^6+8960\,Cb^3c^4x^6+43008\,Aac^6x^5+53760\,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x)
```

```
[Out] 2/105/(c*x^2+b*x+a)^(7/2)*(6144*A*c^7*x^7+1024*C*a*c^6*x^7+1280*C*b^2*c^5*x
^7+21504*A*b*c^6*x^6+3584*C*a*b*c^5*x^6+4480*C*b^3*c^4*x^6+21504*A*a*c^6*x^
5+26880*A*b^2*c^5*x^5+3584*C*a^2*c^5*x^5+8960*C*a*b^2*c^4*x^5+5600*C*b^4*c^
3*x^5+53760*A*a*b*c^5*x^4+13440*A*b^3*c^4*x^4+8960*C*a^2*b*c^4*x^4+13440*C*
a*b^3*c^3*x^4+2800*C*b^5*c^2*x^4+26880*A*a^2*c^5*x^3+40320*A*a*b^2*c^4*x^3+
1680*A*b^4*c^3*x^3+4480*C*a^3*c^4*x^3+12320*C*a^2*b^2*c^3*x^3+8680*C*a*b^4*
c^2*x^3+350*C*b^6*c*x^3+40320*A*a^2*b*c^4*x^2+6720*A*a*b^3*c^3*x^2-168*A*b^
5*c^2*x^2+6720*C*a^3*b*c^3*x^2+9520*C*a^2*b^3*c^2*x^2+1372*C*a*b^5*c*x^2-35
*C*b^7*x^2+13440*A*a^3*c^4*x+10080*A*a^2*b^2*c^3*x-840*A*a*b^4*c^2*x+42*A*b
^6*c*x+6720*C*a^3*b^2*c^2*x+1120*C*a^2*b^4*c*x-28*C*a*b^6*x+6720*A*a^3*b*c^
3-1680*A*a^2*b^3*c^2+252*A*a*b^5*c-15*A*b^7+1920*C*a^4*b*c^2+320*C*a^3*b^3*
c-8*C*a^2*b^5)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35431, size = 1152, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((2*(8*(2*(4*(2*(5*C*b^2*c^5 + 4*C*a*c^6 + 24*A*c^7)*x/(b^8*c^4 - 16
*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8) + 7*(5*C*b^3*c
^4 + 4*C*a*b*c^5 + 24*A*b*c^6)/(b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 2
56*a^3*b^2*c^7 + 256*a^4*c^8))*x + 7*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 16*C*
a^2*c^5 + 120*A*b^2*c^5 + 96*A*a*c^6)/(b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*
c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8))*x + 35*(5*C*b^5*c^2 + 24*C*a*b^3*c^3
+ 16*C*a^2*b*c^4 + 24*A*b^3*c^4 + 96*A*a*b*c^5)/(b^8*c^4 - 16*a*b^6*c^5 + 9
6*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8))*x + 35*(5*C*b^6*c + 124*C*a
*b^4*c^2 + 176*C*a^2*b^2*c^3 + 24*A*b^4*c^3 + 64*C*a^3*c^4 + 576*A*a*b^2*c^
4 + 384*A*a^2*c^5)/(b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c
^7 + 256*a^4*c^8))*x - 7*(5*C*b^7 - 196*C*a*b^5*c - 1360*C*a^2*b^3*c^2 + 24
*A*b^5*c^2 - 960*C*a^3*b*c^3 - 960*A*a*b^3*c^3 - 5760*A*a^2*b*c^4)/(b^8*c^4
 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8))*x - 14*(
2*C*a*b^6 - 80*C*a^2*b^4*c - 3*A*b^6*c - 480*C*a^3*b^2*c^2 + 60*A*a*b^4*c^2
 - 720*A*a^2*b^2*c^3 - 960*A*a^3*c^4)/(b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*
c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8))*x - (8*C*a^2*b^5 + 15*A*b^7 - 320*C*a
^3*b^3*c - 252*A*a*b^5*c - 1920*C*a^4*b*c^2 + 1680*A*a^2*b^3*c^2 - 6720*A*a
^3*b*c^3)/(b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 256*
a^4*c^8))/(c*x^2 + b*x + a)^(7/2)
```

3.186 $\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=930

$$\frac{f\left(cx^2+bx+a\right)^{3/2}(g+hx)^4}{7ch}-\frac{(6cfg-14ceh+11bfh)\left(cx^2+bx+a\right)^{3/2}(g+hx)^3}{84c^2h}+\frac{\left(-4\left(3fg^2-7h(eg+2dh)\right)c^2-2h\right.}{}$$

```
[Out] ((256*c^5*d*g^3 - 33*b^5*f*h^3 + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h))
 - 8*b*c^2*h*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*
g*h + d*h^2)) - 64*c^4*g*(2*b*g*(e*g + 3*d*h) + a*(f*g^2 + 3*h*(e*g + d*h))
) + 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6
*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(10
24*c^6) + ((33*b^2*f*h^2 - 2*c*h*(8*b*f*g + 21*b*e*h + 16*a*f*h) - 4*c^2*(3
*f*g^2 - 7*h*(e*g + 2*d*h)))*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(280*c^3*
h) - ((6*c*f*g - 14*c*e*h + 11*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/
(84*c^2*h) + (f*(g + h*x)^4*(a + b*x + c*x^2)^(3/2))/(7*c*h) + ((1155*b^4*f
*h^4 - 128*c^4*g^2*(3*f*g^2 - 7*h*(e*g + 12*d*h)) - 42*b^2*c*h^3*(78*a*f*h
+ 35*b*(3*f*g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 343*a*b*h*(3*f*g + e*h)
+ b^2*(537*f*g^2 + 245*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(15*f*g^2 + 7*h
*(3*e*g + d*h)) + b*g*(17*f*g^2 + 21*h*(19*e*g + 25*d*h))) - 6*c*h*(231*b^3
*f*h^3 - 6*b*c*h^2*(59*b*f*g + 49*b*e*h + 74*a*f*h) + 16*c^3*g*(3*f*g^2 - 7
*h*(e*g + 7*d*h)) + 8*c^2*h*(a*h*(41*f*g + 35*e*h) + b*(5*f*g^2 + 7*h*(9*e*
g + 7*d*h))))*x)*(a + b*x + c*x^2)^(3/2))/(13440*c^5*h) - ((b^2 - 4*a*c)*(2
56*c^5*d*g^3 - 33*b^5*f*h^3 + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) -
8*b*c^2*h*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h
 + d*h^2)) - 64*c^4*g*(2*b*g*(e*g + 3*d*h) + a*(f*g^2 + 3*h*(e*g + d*h))) +
 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6*a*
b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2048*c^(13/2))
```

Rubi [A] time = 3.01272, antiderivative size = 927, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{f\left(cx^2+bx+a\right)^{3/2}(g+hx)^4}{7ch}-\frac{(6cfg-14ceh+11bfh)\left(cx^2+bx+a\right)^{3/2}(g+hx)^3}{84c^2h}+\frac{\left(-4\left(3fg^2-7h(eg+2dh)\right)c^2-2h\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((256*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*
b*g*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h
*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)
) + 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6
*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(10
24*c^6) + ((33*b^2*f*h^2 - 2*c*h*(8*b*f*g + 21*b*e*h + 16*a*f*h) - 4*c^2*(3
*f*g^2 - 7*h*(e*g + 2*d*h)))*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(280*c^3*
h) - ((6*c*f*g - 14*c*e*h + 11*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/
(84*c^2*h) + (f*(g + h*x)^4*(a + b*x + c*x^2)^(3/2))/(7*c*h) + ((1155*b^4*f
*h^4 - 128*c^4*(3*f*g^4 - 7*g^2*h*(e*g + 12*d*h)) - 42*b^2*c*h^3*(78*a*f*h
+ 35*b*(3*f*g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 343*a*b*h*(3*f*g + e*h)
+ b^2*(537*f*g^2 + 245*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(15*f*g^2 + 7*h
*(3*e*g + d*h)) + b*g*(17*f*g^2 + 21*h*(19*e*g + 25*d*h))) - 6*c*h*(231*b^3
*f*h^3 - 6*b*c*h^2*(59*b*f*g + 49*b*e*h + 74*a*f*h) + 16*c^3*(3*f*g^3 - 7*g
*h*(e*g + 7*d*h)) + 8*c^2*h*(5*b*f*g^2 + 7*b*h*(9*e*g + 7*d*h) + a*h*(41*f*
g + 35*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(13440*c^5*h) - ((b^2 - 4*a*c)*(2
56*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*b*g
*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h*(1
0*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) +
 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6*a*
b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2048*c^(13/2))
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx=\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}+\frac{\int (g+hx)^3\left(-\frac{1}{2}h(3bfg-14cdh+8afh)-\right.}{}$$

$$=-\frac{(6cfg-14ceh+11bfh)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2h}+\frac{f(g+hx)^4\left(a+bx\right.}{7ch}$$

$$=\frac{\left(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2\left(3fg^2-7h(eg+2dh)\right)\right)(g}{280c^3h}$$

$$=\frac{\left(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2\left(3fg^2-7h(eg+2dh)\right)\right)(g}{280c^3h}$$

$$=\frac{\left(256c^5dg^3-33b^5fh^3-64c^4g\left(afg^2+3ah(eg+dh)+2bg(eg+3dh)\right)+6\right.}{}$$

$$=\frac{\left(256c^5dg^3-33b^5fh^3-64c^4g\left(afg^2+3ah(eg+dh)+2bg(eg+3dh)\right)+6\right.}{}$$

$$=\frac{\left(256c^5dg^3-33b^5fh^3-64c^4g\left(afg^2+3ah(eg+dh)+2bg(eg+3dh)\right)+6\right.}{}$$

Mathematica [A] time = 2.44677, size = 1093, normalized size = 1.18

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(-3465fh^3b^6+210ch^2(63fg+21eh+11fhx)b^5-84ch\left(-260afh^2+35c(6eg+2dh+ehx)h+cf\left(21\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^6*f*h^3 + 210*b^5*c*h^2*(63*f*g +
 21*e*h + 11*f*h*x) - 84*b^4*c*h*(-260*a*f*h^2 + 35*c*h*(6*e*g + 2*d*h + e*
h*x) + c*f*(210*g^2 + 105*g*h*x + 22*h^2*x^2)) - 16*b^2*c^2*(2163*a^2*f*h^3
 - 2*a*c*h*(7*h*(345*e*g + 115*d*h + 56*e*h*x) + 3*f*(805*g^2 + 392*g*h*x +
 81*h^2*x^2)) + 2*c^2*(7*d*h*(180*g^2 + 75*g*h*x + 14*h^2*x^2) + 21*e*(20*g
^3 + 25*g^2*h*x + 14*g*h^2*x^2 + 3*h^3*x^3) + f*x*(175*g^3 + 294*g^2*h*x +
189*g*h^2*x^2 + 44*h^3*x^3))) + 16*b^3*c^2*(-42*a*h^2*(35*e*h + 3*f*(35*g +
 6*h*x)) + c*(f*(525*g^3 + 735*g^2*h*x + 441*g*h^2*x^2 + 99*h^3*x^3) + 7*h*
(5*d*h*(45*g + 7*h*x) + 3*e*(75*g^2 + 35*g*h*x + 7*h^2*x^2)))) + 32*b*c^3*(
a^2*h^2*(2373*f*g + 791*e*h + 397*f*h*x) - 2*a*c*(f*(455*g^3 + 609*g^2*h*x
+ 357*g*h^2*x^2 + 79*h^3*x^3) + 7*h*(d*h*(195*g + 29*h*x) + e*(195*g^2 + 87
*g*h*x + 17*h^2*x^2))) + 4*c^2*(21*d*(10*g^3 + 10*g^2*h*x + 5*g*h^2*x^2 + h
```

```
^3*x^3) + x*(7*e*(10*g^3 + 15*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + f*x*(35*
g^3 + 63*g^2*h*x + 42*g*h^2*x^2 + 10*h^3*x^3)))) + 64*c^3*(128*a^3*f*h^3 -
a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x) + f*(672*g^2 + 315*g*h*x + 64*h^2
*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h*x + 8*h^2*x^2) + 7*e*(40*g^3 + 45
*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*f*x*(35*g^3 + 56*g^2*h*x + 35*g*h^
2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4
*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x + 36*g*h^2*x^2 + 10*h^3*x^3) + 3*f*
x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3))))) + 105*(b^2 - 4*a*c)
*(-256*c^5*d*g^3 + 33*b^5*f*h^3 + 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2
*b*g*(e*g + 3*d*h)) - 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) + 8*b*c^2*
h*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2
)) - 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) +
6*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)])])/(215040*c^(13/2))
```

Maple [B] time = 0.067, size = 3543, normalized size = 3.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 7/16*b^2/c^3*(c*x^2+b*x+a)^(3/2)*f*g^2*h+3/16*b/c^(5/2)*a^2*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^3-1/8*d*g^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*b^2-5/128*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*f*g^3-1/8*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*f*g^3+5/64*b^3/c^3*(c*x^2+b*x+a)^(1/2)*f*g^3+(c*x^2+b*x+a)^(3/2)/c*d*g^
2*h-1/8*b^2/c^2*(c*x^2+b*x+a)^(1/2)*e*g^3+1/16*b^3/c^(5/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^3+1/4*x*(c*x^2+b*x+a)^(3/2)/c*f*g^3-5/24*b/
c^2*(c*x^2+b*x+a)^(3/2)*f*g^3+21/512*b^5/c^5*(c*x^2+b*x+a)^(1/2)*e*h^3-21/1
024*b^6/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^3+1/2*x^3*
(c*x^2+b*x+a)^(3/2)/c*f*g*h^2+9/16*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)*f*g^2*h+9/
16*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)*e*g*h^2-21/32*b^2/c^3*a*x*(c*x^2+b*x+a)^(1
/2)*f*g*h^2-1/8*a/c^2*x*(c*x^2+b*x+a)^(3/2)*e*h^3+1/4*d*g^3/c*(c*x^2+b*x+a)
^(1/2)*b+9/16*b/c^(5/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g
*h^2+3/16*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)*d*h^3-21/64*b^3/c^3*x*(c*x^2+b*x+a)
^(1/2)*e*g*h^2+9/32*b^2/c^3*a*(c*x^2+b*x+a)^(1/2)*f*g^2*h+15/32*b^2/c^2*x*(
c*x^2+b*x+a)^(1/2)*d*g*h^2+15/32*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)*e*g^2*h+9/32
*b^2/c^3*a*(c*x^2+b*x+a)^(1/2)*e*g*h^2-21/64*b^3/c^3*x*(c*x^2+b*x+a)^(1/2)*
f*g^2*h-15/32*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g
*h^2-15/32*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g^2*
```

h-3/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*e*g^2*h-3/8*a/c*x*(c*x^2+b*x+a)^(1/2)*e*
g^2*h+9/16*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g^2*
h-3/8*a/c*x*(c*x^2+b*x+a)^(1/2)*d*g*h^2-11/84*f*h^3*b/c^2*x^3*(c*x^2+b*x+a)
^(3/2)-33/320*f*h^3*b^3/c^4*x*(c*x^2+b*x+a)^(3/2)-5/32*f*h^3*b/c^(7/2)*a^3*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+35/128*f*h^3*b^3/c^(9/2)*a^2*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/5*x^2*(c*x^2+b*x+a)^(3/2)/c*e*g
*h^2+21/256*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h^2
-63/512*f*h^3*b^5/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-4/
35*f*h^3*a/c^2*x^2*(c*x^2+b*x+a)^(3/2)-5/32*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*a*d*h^3+21/160*b^2/c^3*x*(c*x^2+b*x+a)^(3/2)*e*h^3
+35/256*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h^3-7/4
0*b/c^2*x*(c*x^2+b*x+a)^(3/2)*d*h^3+21/256*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*f*g^2*h+7/16*b^2/c^3*(c*x^2+b*x+a)^(3/2)*e*g*h^2-21
/128*b^4/c^4*(c*x^2+b*x+a)^(1/2)*e*g*h^2-21/128*b^4/c^4*(c*x^2+b*x+a)^(1/2)
*f*g^2*h+49/240*b/c^3*a*(c*x^2+b*x+a)^(3/2)*e*h^3+21/256*b^4/c^4*x*(c*x^2+b
*x+a)^(1/2)*e*h^3+63/512*b^5/c^5*(c*x^2+b*x+a)^(1/2)*f*g*h^2+3/32*b^2/c^3*a
*(c*x^2+b*x+a)^(1/2)*d*h^3+3/5*x^2*(c*x^2+b*x+a)^(3/2)/c*f*g^2*h-5/8*b/c^2*
(c*x^2+b*x+a)^(3/2)*e*g^2*h+1/2*d*g^3/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*a+3/4*x*(c*x^2+b*x+a)^(3/2)/c*d*g*h^2+3/4*x*(c*x^2+b*x+a)^(3
/2)/c*e*g^2*h+15/64*b^3/c^3*(c*x^2+b*x+a)^(1/2)*e*g^2*h+5/32*b^2/c^2*x*(c*x
^2+b*x+a)^(1/2)*f*g^3+15/64*b^3/c^3*(c*x^2+b*x+a)^(1/2)*d*g*h^2-3/8*a^2/c^(
3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^2*h+1/16*a^2/c^2*x*(c*
x^2+b*x+a)^(1/2)*e*h^3+1/32*a^2/c^3*(c*x^2+b*x+a)^(1/2)*b*e*h^3-7/64*b^3/c^
3*x*(c*x^2+b*x+a)^(1/2)*d*h^3+9/16*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*a*d*g*h^2-2/5*a/c^2*(c*x^2+b*x+a)^(3/2)*e*g*h^2-2/5*a/c^2*(
c*x^2+b*x+a)^(3/2)*f*g^2*h+33/280*f*h^3*b^2/c^3*x^2*(c*x^2+b*x+a)^(3/2)-33/
512*f*h^3*b^5/c^5*x*(c*x^2+b*x+a)^(1/2)+15/128*f*h^3*b^4/c^5*a*(c*x^2+b*x+a
)^(1/2)-39/160*f*h^3*b^2/c^4*a*(c*x^2+b*x+a)^(3/2)-5/64*f*h^3*b^2/c^4*a^2*(
c*x^2+b*x+a)^(1/2)-33/1024*f*h^3*b^6/c^6*(c*x^2+b*x+a)^(1/2)+33/2048*f*h^3*
b^7/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/105*f*h^3*a^2/c^
3*(c*x^2+b*x+a)^(3/2)+7/256*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*d*h^3-2/15*a/c^2*(c*x^2+b*x+a)^(3/2)*d*h^3+1/3*(c*x^2+b*x+a)^(3/2)
/c*e*g^3+1/2*d*g^3*x*(c*x^2+b*x+a)^(1/2)+11/128*f*h^3*b^4/c^5*(c*x^2+b*x+a)
^(3/2)-1/4*b/c*x*(c*x^2+b*x+a)^(1/2)*e*g^3-3/8*b^2/c^2*(c*x^2+b*x+a)^(1/2)*
d*g^2*h-1/4*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g^3+3
/16*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*g^2*h-15/128*
b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^2*h-5/8*b/c^2*(
c*x^2+b*x+a)^(3/2)*d*g*h^2-3/20*b/c^2*x^2*(c*x^2+b*x+a)^(3/2)*e*h^3-21/64*b
^3/c^4*(c*x^2+b*x+a)^(3/2)*f*g*h^2-63/1024*b^6/c^(11/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*f*g*h^2-7/64*b^3/c^4*a*(c*x^2+b*x+a)^(1/2)*e*h^3-1
5/64*b^2/c^(7/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^3-7/12
8*b^4/c^4*(c*x^2+b*x+a)^(1/2)*d*h^3-7/64*b^3/c^4*(c*x^2+b*x+a)^(3/2)*e*h^3+
1/7*f*h^3*x^4*(c*x^2+b*x+a)^(3/2)/c-3/8*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*d*g*h^2-1/8*a/c*x*(c*x^2+b*x+a)^(1/2)*f*g^3-1/16*a/c^2
*(c*x^2+b*x+a)^(1/2)*b*f*g^3+3/16*a^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2

```
+b*x+a)^(1/2))*f*g*h^2+3/16*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a*f*g^3-15/128*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*d*g*h^2+1/16*a^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e
*h^3+1/6*x^3*(c*x^2+b*x+a)^(3/2)/c*e*h^3+1/5*x^2*(c*x^2+b*x+a)^(3/2)/c*d*h^
3+7/48*b^2/c^3*(c*x^2+b*x+a)^(3/2)*d*h^3-21/40*b/c^2*x*(c*x^2+b*x+a)^(3/2)*
e*g*h^2-3/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*d*g*h^2-3/4*b/c*x*(c*x^2+b*x+a)^(1
/2)*d*g^2*h-3/4*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g
^2*h+63/160*b^2/c^3*x*(c*x^2+b*x+a)^(3/2)*f*g*h^2-45/64*b^2/c^(7/2)*a^2*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h^2+49/80*b/c^3*a*(c*x^2+b*x+a
)^(3/2)*f*g*h^2-9/20*b/c^2*x^2*(c*x^2+b*x+a)^(3/2)*f*g*h^2-3/8*a/c^2*x*(c*x
^2+b*x+a)^(3/2)*f*g*h^2+3/16*a^2/c^2*x*(c*x^2+b*x+a)^(1/2)*f*g*h^2+3/32*a^2
/c^3*(c*x^2+b*x+a)^(1/2)*b*f*g*h^2+63/256*b^4/c^4*x*(c*x^2+b*x+a)^(1/2)*f*g
*h^2+105/256*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g*
h^2-7/32*b^2/c^3*a*x*(c*x^2+b*x+a)^(1/2)*e*h^3-21/40*b/c^2*x*(c*x^2+b*x+a)^
(3/2)*f*g^2*h-21/64*b^3/c^4*a*(c*x^2+b*x+a)^(1/2)*f*g*h^2-5/32*f*h^3*b/c^3*
a^2*x*(c*x^2+b*x+a)^(1/2)+15/64*f*h^3*b^3/c^4*a*x*(c*x^2+b*x+a)^(1/2)+111/5
60*f*h^3*b/c^3*a*x*(c*x^2+b*x+a)^(3/2)+9/16*b/c^(5/2)*a^2*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.1682, size = 6407, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/430080*(105*(16*(16*(b^2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e +
(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^
5)*d - 2*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b^3*
c^3 + 48*a^2*b*c^4)*f)*g^2*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)
*d - 4*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a*b^4*
c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3
 + 48*a^2*b*c^4)*d - 2*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3
*c^4)*e + (33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3)*
sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(c) - 4*a*c) + 4*(15360*c^7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + (14*c^
7*e + b*c^6*f)*h^3)*x^5 + 128*(504*c^7*f*g^2*h + 42*(12*c^7*e + b*c^6*f)*g*
h^2 + (168*c^7*d + 14*b*c^6*e - (11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(
48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*c^6)*e + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 -
 168*(80*(3*b^2*c^5 - 8*a*c^6)*d - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^
4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b
*c^5)*d - 4*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 -
1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^
4 + 256*a^2*c^5)*d - 14*(315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e +
 (3465*b^6*c - 21840*a*b^4*c^2 + 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 +
 16*(1680*c^7*f*g^3 + 504*(10*c^7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b
*c^6*e - (9*b^2*c^5 - 20*a*c^6)*f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 2
0*a*c^6)*e + (99*b^3*c^4 - 316*a*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c
^6*f)*g^3 + 168*(80*c^7*d + 10*b*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h +
42*(40*b*c^6*d - 4*(7*b^2*c^5 - 16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*
g*h^2 - (56*(7*b^2*c^5 - 16*a*c^6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (23
1*b^4*c^3 - 972*a*b^2*c^4 + 512*a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8
*b*c^6*e - (5*b^2*c^5 - 12*a*c^6)*f)*g^3 + 168*(80*b*c^6*d - 10*(5*b^2*c^5
- 12*a*c^6)*e + (35*b^3*c^4 - 116*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 1
2*a*c^6)*d - 4*(35*b^3*c^4 - 116*a*b*c^5)*e + (105*b^4*c^3 - 448*a*b^2*c^4
+ 240*a^2*c^5)*f)*g*h^2 + (56*(35*b^3*c^4 - 116*a*b*c^5)*d - 14*(105*b^4*c^
3 - 448*a*b^2*c^4 + 240*a^2*c^5)*e + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*
a^2*b*c^4)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/c^7, 1/215040*(105*(16*(16*(b^
2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e + (5*b^4*c^3 - 24*a*b^2*c^4
+ 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^5)*d - 2*(5*b^4*c^3 - 24*a
*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*f)*g^2
*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 4*(7*b^5*c^2 - 40*a*b
^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64
*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*d - 2*(21
*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*e + (33*b^7 - 252*a*
b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(15360*c^
7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + (14*c^7*e + b*c^6*f)*h^3)*x^5 + 128*(5
04*c^7*f*g^2*h + 42*(12*c^7*e + b*c^6*f)*g*h^2 + (168*c^7*d + 14*b*c^6*e -
(11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*
c^6)*e + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 - 168*(80*(3*b^2*c^5 - 8*a*c^6)*d
```

```
 - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*
c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b*c^5)*d - 4*(105*b^4*c^3 - 460*a
*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)
*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d - 14*(315*b^5
*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e + (3465*b^6*c - 21840*a*b^4*c^2 +
 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*(10*c^
7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b*c^6*e - (9*b^2*c^5 - 20*a*c^6)*
f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 20*a*c^6)*e + (99*b^3*c^4 - 316*a
*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c^6*f)*g^3 + 168*(80*c^7*d + 10*b
*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h + 42*(40*b*c^6*d - 4*(7*b^2*c^5 -
16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*g*h^2 - (56*(7*b^2*c^5 - 16*a*c^
6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (231*b^4*c^3 - 972*a*b^2*c^4 + 512*
a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8*b*c^6*e - (5*b^2*c^5 - 12*a*c^6
)*f)*g^3 + 168*(80*b*c^6*d - 10*(5*b^2*c^5 - 12*a*c^6)*e + (35*b^3*c^4 - 11
6*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 12*a*c^6)*d - 4*(35*b^3*c^4 - 116
*a*b*c^5)*e + (105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*f)*g*h^2 + (56*(3
5*b^3*c^4 - 116*a*b*c^5)*d - 14*(105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)
*e + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*a^2*b*c^4)*f)*h^3)*x)*sqrt(c*x^2
 + b*x + a))/c^7]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A] time = 1.32431, size = 2298, normalized size = 2.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*f*h^3*x + (42*c^6*f*g*h^
2 + b*c^5*f*h^3 + 14*c^6*h^3*e)/c^6)*x + (504*c^6*f*g^2*h + 42*b*c^5*f*g*h^
```

2 + 168*c^6*d*h^3 - 11*b^2*c^4*f*h^3 + 24*a*c^5*f*h^3 + 504*c^6*g*h^2*e + 1
4*b*c^5*h^3*e)/c^6)*x + (1680*c^6*f*g^3 + 504*b*c^5*f*g^2*h + 5040*c^6*d*g*
h^2 - 378*b^2*c^4*f*g*h^2 + 840*a*c^5*f*g*h^2 + 168*b*c^5*d*h^3 + 99*b^3*c^
3*f*h^3 - 316*a*b*c^4*f*h^3 + 5040*c^6*g^2*h*e + 504*b*c^5*g*h^2*e - 126*b^
2*c^4*h^3*e + 280*a*c^5*h^3*e)/c^6)*x + (560*b*c^5*f*g^3 + 13440*c^6*d*g^2*
h - 1176*b^2*c^4*f*g^2*h + 2688*a*c^5*f*g^2*h + 1680*b*c^5*d*g*h^2 + 882*b^
3*c^3*f*g*h^2 - 2856*a*b*c^4*f*g*h^2 - 392*b^2*c^4*d*h^3 + 896*a*c^5*d*h^3
- 231*b^4*c^2*f*h^3 + 972*a*b^2*c^3*f*h^3 - 512*a^2*c^4*f*h^3 + 4480*c^6*g^
3*e + 1680*b*c^5*g^2*h*e - 1176*b^2*c^4*g*h^2*e + 2688*a*c^5*g*h^2*e + 294*
b^3*c^3*h^3*e - 952*a*b*c^4*h^3*e)/c^6)*x + (26880*c^6*d*g^3 - 2800*b^2*c^4
*f*g^3 + 6720*a*c^5*f*g^3 + 13440*b*c^5*d*g^2*h + 5880*b^3*c^3*f*g^2*h - 19
488*a*b*c^4*f*g^2*h - 8400*b^2*c^4*d*g*h^2 + 20160*a*c^5*d*g*h^2 - 4410*b^4
*c^2*f*g*h^2 + 18816*a*b^2*c^3*f*g*h^2 - 10080*a^2*c^4*f*g*h^2 + 1960*b^3*c
^3*d*h^3 - 6496*a*b*c^4*d*h^3 + 1155*b^5*c*f*h^3 - 6048*a*b^3*c^2*f*h^3 + 6
352*a^2*b*c^3*f*h^3 + 4480*b*c^5*g^3*e - 8400*b^2*c^4*g^2*h*e + 20160*a*c^5
*g^2*h*e + 5880*b^3*c^3*g*h^2*e - 19488*a*b*c^4*g*h^2*e - 1470*b^4*c^2*h^3*
e + 6272*a*b^2*c^3*h^3*e - 3360*a^2*c^4*h^3*e)/c^6)*x + (26880*b*c^5*d*g^3
+ 8400*b^3*c^3*f*g^3 - 29120*a*b*c^4*f*g^3 - 40320*b^2*c^4*d*g^2*h + 107520
*a*c^5*d*g^2*h - 17640*b^4*c^2*f*g^2*h + 77280*a*b^2*c^3*f*g^2*h - 43008*a^
2*c^4*f*g^2*h + 25200*b^3*c^3*d*g*h^2 - 87360*a*b*c^4*d*g*h^2 + 13230*b^5*c
*f*g*h^2 - 70560*a*b^3*c^2*f*g*h^2 + 75936*a^2*b*c^3*f*g*h^2 - 5880*b^4*c^2
*d*h^3 + 25760*a*b^2*c^3*d*h^3 - 14336*a^2*c^4*d*h^3 - 3465*b^6*f*h^3 + 218
40*a*b^4*c*f*h^3 - 34608*a^2*b^2*c^2*f*h^3 + 8192*a^3*c^3*f*h^3 - 13440*b^2
*c^4*g^3*e + 35840*a*c^5*g^3*e + 25200*b^3*c^3*g^2*h*e - 87360*a*b*c^4*g^2*
h*e - 17640*b^4*c^2*g*h^2*e + 77280*a*b^2*c^3*g*h^2*e - 43008*a^2*c^4*g*h^2
*e + 4410*b^5*c*h^3*e - 23520*a*b^3*c^2*h^3*e + 25312*a^2*b*c^3*h^3*e)/c^6)
+ 1/2048*(256*b^2*c^5*d*g^3 - 1024*a*c^6*d*g^3 + 80*b^4*c^3*f*g^3 - 384*a*
b^2*c^4*f*g^3 + 256*a^2*c^5*f*g^3 - 384*b^3*c^4*d*g^2*h + 1536*a*b*c^5*d*g^
2*h - 168*b^5*c^2*f*g^2*h + 960*a*b^3*c^3*f*g^2*h - 1152*a^2*b*c^4*f*g^2*h
+ 240*b^4*c^3*d*g*h^2 - 1152*a*b^2*c^4*d*g*h^2 + 768*a^2*c^5*d*g*h^2 + 126*
b^6*c*f*g*h^2 - 840*a*b^4*c^2*f*g*h^2 + 1440*a^2*b^2*c^3*f*g*h^2 - 384*a^3*
c^4*f*g*h^2 - 56*b^5*c^2*d*h^3 + 320*a*b^3*c^3*d*h^3 - 384*a^2*b*c^4*d*h^3
- 33*b^7*f*h^3 + 252*a*b^5*c*f*h^3 - 560*a^2*b^3*c^2*f*h^3 + 320*a^3*b*c^3*
f*h^3 - 128*b^3*c^4*g^3*e + 512*a*b*c^5*g^3*e + 240*b^4*c^3*g^2*h*e - 1152*
a*b^2*c^4*g^2*h*e + 768*a^2*c^5*g^2*h*e - 168*b^5*c^2*g*h^2*e + 960*a*b^3*c
^3*g*h^2*e - 1152*a^2*b*c^4*g*h^2*e + 42*b^6*c*h^3*e - 280*a*b^4*c^2*h^3*e
+ 480*a^2*b^2*c^3*h^3*e - 128*a^3*c^4*h^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)

3.187 $\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=584

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(dh^2+2egh+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c^3\cdots\right)}{512c^5}$$

```
[Out] ((128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 3
2*c^3*(2*b*g*(e*g + 2*d*h) + a*(f*g^2 + 2*e*g*h + d*h^2)) + 8*c^2*(2*a^2*f*
h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*g^2 + 2*e*g*h + d*h^2)))*(b + 2*c*x)
*Sqrt[a + b*x + c*x^2])/(512*c^5) - ((2*c*f*g - 4*c*e*h + 3*b*f*h)*(g + h*x
)^2*(a + b*x + c*x^2)^(3/2))/(20*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^
(3/2))/(6*c*h) - ((105*b^3*f*h^3 + 64*c^3*g*(f*g^2 - 2*h*(e*g + 5*d*h)) - 2
8*b*c*h^2*(7*a*f*h + 5*b*(2*f*g + e*h)) + 8*c^2*h*(16*a*h*(2*f*g + e*h) + b
*(7*f*g^2 + 25*h*(2*e*g + d*h))) - 6*c*h*(21*b^2*f*h^2 - 4*c*h*(2*b*f*g + 7
*b*e*h + 5*a*f*h) - 8*c^2*(f*g^2 - h*(2*e*g + 5*d*h)))*x)*(a + b*x + c*x^2)
^(3/2))/(960*c^4*h) - ((b^2 - 4*a*c)*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2
*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(2*b*g*(e*g + 2*d*h) + a*(f*g^2 +
 2*e*g*h + d*h^2)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*
g^2 + 2*e*g*h + d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(1024*c^(11/2))
```

Rubi [A] time = 1.44001, antiderivative size = 581, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c\cdots\right)}{512c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 3
2*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*
h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)
*Sqrt[a + b*x + c*x^2])/(512*c^5) - ((2*c*f*g - 4*c*e*h + 3*b*f*h)*(g + h*x
)^2*(a + b*x + c*x^2)^(3/2))/(20*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^
(3/2))/(6*c*h) - ((105*b^3*f*h^3 + 64*c^3*(f*g^3 - 2*g*h*(e*g + 5*d*h)) - 2
8*b*c*h^2*(7*a*f*h + 5*b*(2*f*g + e*h)) + 8*c^2*h*(7*b*f*g^2 + 25*b*h*(2*e*
```

```
g + d*h) + 16*a*h*(2*f*g + e*h)) - 6*c*h*(21*b^2*f*h^2 - 4*c*h*(2*b*f*g + 7
*b*e*h + 5*a*f*h) - 8*c^2*(f*g^2 - h*(2*e*g + 5*d*h)))*x)*(a + b*x + c*x^2)
^(3/2))/(960*c^4*h) - ((b^2 - 4*a*c)*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2
*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*
b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*
g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(1024*c^(11/2))
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
```

```
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+hx)^2\left(-\frac{3}{2}h(bfg-4cdh+2afh)-\right.}{6}$$

$$= -\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h} + \frac{f(g+hx)^3\left(a+bx\right.}{6ch}$$

$$= -\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h} + \frac{f(g+hx)^3\left(a+bx\right.}{6ch}$$

$$= \frac{\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3\left(afg^2+ah(2\right.\right.}{}$$

$$= \frac{\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3\left(afg^2+ah(2\right.\right.}{}$$

$$= \frac{\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3\left(afg^2+ah(2\right.\right.}{}$$

Mathematica [A] time = 0.96637, size = 436, normalized size = 0.75

$$\frac{3h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c^3\left(ah(dh+2eg\right.\right.}{512c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((-3*(2*c*f*g - 4*c*e*h + 3*b*f*h)*(g + h*x)^2*(a + x*(b + c*x))^(3/2))/(10
*c) + f*(g + h*x)^3*(a + x*(b + c*x))^(3/2) - ((a + x*(b + c*x))^(3/2)*(105
*b^3*f*h^3 - 14*b*c*h^2*(14*a*f*h + b*(20*f*g + 10*e*h + 9*f*h*x)) + 8*c^2*
h*(b*f*g*(7*g + 6*h*x) + b*h*(50*e*g + 25*d*h + 21*e*h*x) + a*h*(32*f*g + 1
6*e*h + 15*f*h*x)) + 16*c^3*(f*g^2*(4*g + 3*h*x) - h*(2*e*g*(4*g + 3*h*x) +
 5*d*h*(8*g + 3*h*x)))))/(160*c^3) + (3*h*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 2
8*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h)
 + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^
2*(f*g^2 + h*(2*e*g + d*h))))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]
- (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(5
12*c^(9/2)))/(6*c*h)
```

Maple [B] time = 0.059, size = 2179, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -7/32*f*h^2*b^2/c^3*a*x*(c*x^2+b*x+a)^(1/2)-1/2*b/c*x*(c*x^2+b*x+a)^(1/2)*d
*g*h+1/6*f*h^2*x^3*(c*x^2+b*x+a)^(3/2)/c-7/64*b^3/c^3*x*(c*x^2+b*x+a)^(1/2)
*e*h^2-7/64*b^4/c^4*(c*x^2+b*x+a)^(1/2)*f*g*h-5/32*b^3/c^(7/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h^2+7/128*b^5/c^(9/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h+3/32*b^2/c^3*a*(c*x^2+b*x+a)^(1/2)*e*h^2+7
/24*b^2/c^3*(c*x^2+b*x+a)^(3/2)*f*g*h+3/8*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)*f*g
*h+5/64*b^3/c^3*(c*x^2+b*x+a)^(1/2)*d*h^2+5/64*b^3/c^3*(c*x^2+b*x+a)^(1/2)*
f*g^2-5/128*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-5
/128*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-1/8*a^2/
c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/8*a^2/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2+1/4*x*(c*x^2+b*x+a)^(3/2)/
c*d*h^2+1/4*x*(c*x^2+b*x+a)^(3/2)/c*f*g^2-4/15*a/c^2*(c*x^2+b*x+a)^(3/2)*f*
g*h-1/4*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g^2+1/8*b
^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*g*h-3/20*f*h^2*b/c
^2*x^2*(c*x^2+b*x+a)^(3/2)+21/160*f*h^2*b^2/c^3*x*(c*x^2+b*x+a)^(3/2)+21/25
6*f*h^2*b^4/c^4*x*(c*x^2+b*x+a)^(1/2)+35/256*f*h^2*b^4/c^(9/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+7/48*b^2/c^3*(c*x^2+b*x+a)^(3/2)*e*h^2-7/
128*b^4/c^4*(c*x^2+b*x+a)^(1/2)*e*h^2+7/256*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*e*h^2+1/16*f*h^2*a^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))-21/1024*f*h^2*b^6/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c
```

```
*x^2+b*x+a)^(1/2))+1/2*d*g^2/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))*a-2/15*a/c^2*(c*x^2+b*x+a)^(3/2)*e*h^2+1/5*x^2*(c*x^2+b*x+a)^(3/2)/c*
e*h^2-1/8*d*g^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-5/2
4*b/c^2*(c*x^2+b*x+a)^(3/2)*f*g^2+3/16*b/c^(5/2)*a^2*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*e*h^2-1/8*a/c^2*(c*x^2+b*x+a)^(1/2)*b*e*g*h+5/16*b^2/
c^2*x*(c*x^2+b*x+a)^(1/2)*e*g*h+3/8*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*a*e*g*h-1/4*a/c*x*(c*x^2+b*x+a)^(1/2)*e*g*h+3/16*b^2/c^3*a
*(c*x^2+b*x+a)^(1/2)*f*g*h+3/8*b/c^(5/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))*f*g*h-5/16*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))*a*f*g*h+3/16*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)*e*h^2-7/20*b/c^2*x*(c*x^2+
b*x+a)^(3/2)*f*g*h-7/32*b^3/c^3*x*(c*x^2+b*x+a)^(1/2)*f*g*h-1/2*b/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g*h-5/24*b/c^2*(c*x^2+b*x+a)
^(3/2)*d*h^2+2/3*(c*x^2+b*x+a)^(3/2)/c*d*g*h-1/8*b^2/c^2*(c*x^2+b*x+a)^(1/2
)*e*g^2+1/16*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^2+
1/4*d*g^2/c*(c*x^2+b*x+a)^(1/2)*b+1/3*(c*x^2+b*x+a)^(3/2)/c*e*g^2+1/2*d*g^2
*x*(c*x^2+b*x+a)^(1/2)+21/512*f*h^2*b^5/c^5*(c*x^2+b*x+a)^(1/2)-7/64*f*h^2*
b^3/c^4*(c*x^2+b*x+a)^(3/2)-7/40*b/c^2*x*(c*x^2+b*x+a)^(3/2)*e*h^2-7/64*f*h
^2*b^3/c^4*a*(c*x^2+b*x+a)^(1/2)-15/64*f*h^2*b^2/c^(7/2)*a^2*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))+49/240*f*h^2*b/c^3*a*(c*x^2+b*x+a)^(3/2)-1/16
*a/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g^2-1/4*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*e*g*h+3/16*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*a*d*h^2+3/16*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))*a*f*g^2-5/64*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e*g*h-1/8*a/c*x*(c*x^2+b*x+a)^(1/2)*d*h^2+5/32*b^3/c^3*(c*x^2+b*x+a)^(1/2)*
e*g*h+2/5*x^2*(c*x^2+b*x+a)^(3/2)/c*f*g*h-1/4*b^2/c^2*(c*x^2+b*x+a)^(1/2)*d
*g*h+1/2*x*(c*x^2+b*x+a)^(3/2)/c*e*g*h-1/4*b/c*x*(c*x^2+b*x+a)^(1/2)*e*g^2-
1/8*a/c*x*(c*x^2+b*x+a)^(1/2)*f*g^2-1/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*d*h^2+
5/32*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)*d*h^2+5/32*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)
*f*g^2-1/8*f*h^2*a/c^2*x*(c*x^2+b*x+a)^(3/2)+1/16*f*h^2*a^2/c^2*x*(c*x^2+b*
x+a)^(1/2)+1/32*f*h^2*a^2/c^3*(c*x^2+b*x+a)^(1/2)*b-5/12*b/c^2*(c*x^2+b*x+a
)^(3/2)*e*g*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.63003, size = 4019, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/30720*(15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (5
*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*
d - 2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^2 +
 48*a^2*b*c^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - 4*(7
*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*a^2*b
^2*c^2 - 64*a^3*c^3)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*h^2*x^5 + 1
28*(24*c^6*f*g*h + (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*(
10*c^6*e + b*c^5*f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 20*a*c^5)*
f)*h^2)*x^3 + 40*(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^3*c^3 - 52
*a*b*c^4)*f)*g^2 - 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*
c^4)*e + (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c
^3 - 52*a*b*c^4)*d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315
*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f
)*g^2 + 8*(80*c^6*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^
5*d - 4*(7*b^2*c^4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 +
2*(40*(48*c^6*d + 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d
 - 10*(5*b^2*c^4 - 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5
*b^2*c^4 - 12*a*c^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 44
8*a*b^2*c^3 + 240*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/15360*(
15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (5*b^4*c^2 -
24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*d - 2*(5*b^
4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c
^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - 4*(7*b^5*c - 40
*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64
*a^3*c^3)*f)*h^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqr
t(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(1280*c^6*f*h^2*x^5 + 128*(24*c^6*f*g*h
+ (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*(10*c^6*e + b*c^5*
f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 20*a*c^5)*f)*h^2)*x^3 + 40*
(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^3*c^3 - 52*a*b*c^4)*f)*g^2
- 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*c^4)*e + (105*b^4
*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c^3 - 52*a*b*c^4)*
d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315*b^5*c - 1680*a*b
^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f)*g^2 + 8*(80*c^6
```

```
*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^5*d - 4*(7*b^2*c^
4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 + 2*(40*(48*c^6*d +
 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d - 10*(5*b^2*c^4
- 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5*b^2*c^4 - 12*a*c
^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 448*a*b^2*c^3 + 240
*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g+hx\right)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A] time = 1.27079, size = 1366, normalized size = 2.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f*h^2*x + (24*c^5*f*g*h + b*c^
4*f*h^2 + 12*c^5*h^2*e)/c^5)*x + (120*c^5*f*g^2 + 24*b*c^4*f*g*h + 120*c^5*
d*h^2 - 9*b^2*c^3*f*h^2 + 20*a*c^4*f*h^2 + 240*c^5*g*h*e + 12*b*c^4*h^2*e)/
c^5)*x + (40*b*c^4*f*g^2 + 640*c^5*d*g*h - 56*b^2*c^3*f*g*h + 128*a*c^4*f*g
*h + 40*b*c^4*d*h^2 + 21*b^3*c^2*f*h^2 - 68*a*b*c^3*f*h^2 + 320*c^5*g^2*e +
 80*b*c^4*g*h*e - 28*b^2*c^3*h^2*e + 64*a*c^4*h^2*e)/c^5)*x + (1920*c^5*d*g
^2 - 200*b^2*c^3*f*g^2 + 480*a*c^4*f*g^2 + 640*b*c^4*d*g*h + 280*b^3*c^2*f*
g*h - 928*a*b*c^3*f*g*h - 200*b^2*c^3*d*h^2 + 480*a*c^4*d*h^2 - 105*b^4*c*f
*h^2 + 448*a*b^2*c^2*f*h^2 - 240*a^2*c^3*f*h^2 + 320*b*c^4*g^2*e - 400*b^2*
c^3*g*h*e + 960*a*c^4*g*h*e + 140*b^3*c^2*h^2*e - 464*a*b*c^3*h^2*e)/c^5)*x
 + (1920*b*c^4*d*g^2 + 600*b^3*c^2*f*g^2 - 2080*a*b*c^3*f*g^2 - 1920*b^2*c^
3*d*g*h + 5120*a*c^4*d*g*h - 840*b^4*c*f*g*h + 3680*a*b^2*c^2*f*g*h - 2048*
a^2*c^3*f*g*h + 600*b^3*c^2*d*h^2 - 2080*a*b*c^3*d*h^2 + 315*b^5*f*h^2 - 16
80*a*b^3*c*f*h^2 + 1808*a^2*b*c^2*f*h^2 - 960*b^2*c^3*g^2*e + 2560*a*c^4*g^
```

```
2*e + 1200*b^3*c^2*g*h*e - 4160*a*b*c^3*g*h*e - 420*b^4*c*h^2*e + 1840*a*b^
2*c^2*h^2*e - 1024*a^2*c^3*h^2*e)/c^5) + 1/1024*(128*b^2*c^4*d*g^2 - 512*a*
c^5*d*g^2 + 40*b^4*c^2*f*g^2 - 192*a*b^2*c^3*f*g^2 + 128*a^2*c^4*f*g^2 - 12
8*b^3*c^3*d*g*h + 512*a*b*c^4*d*g*h - 56*b^5*c*f*g*h + 320*a*b^3*c^2*f*g*h
- 384*a^2*b*c^3*f*g*h + 40*b^4*c^2*d*h^2 - 192*a*b^2*c^3*d*h^2 + 128*a^2*c^
4*d*h^2 + 21*b^6*f*h^2 - 140*a*b^4*c*f*h^2 + 240*a^2*b^2*c^2*f*h^2 - 64*a^3
*c^3*f*h^2 - 64*b^3*c^3*g^2*e + 256*a*b*c^4*g^2*e + 80*b^4*c^2*g*h*e - 384*
a*b^2*c^3*g*h*e + 256*a^2*c^4*g*h*e - 28*b^5*c*h^2*e + 160*a*b^3*c^2*h^2*e
- 192*a^2*b*c^3*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(
c) - b))/c^(11/2)
```

3.188 $\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=322

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+25b(eh+fg))+35b^2fh^2-6chx(7bfh-10ceh+6cfg)-16c^2\left(3fg^2-5h(dh+eg)\right)\right)}{240c^3h}$$

```
[Out] ((32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) + 2*b*
c*(6*a*f*h + 5*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^4)
 + (f*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c*h) + ((35*b^2*f*h^2 - 16*c^
2*(3*f*g^2 - 5*h*(e*g + d*h)) - 2*c*h*(16*a*f*h + 25*b*(f*g + e*h)) - 6*c*h
*(6*c*f*g - 10*c*e*h + 7*b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(240*c^3*h) - (
(b^2 - 4*a*c)*(32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h +
a*e*h) + 2*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(256*c^(9/2))
```

Rubi [A] time = 0.503646, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 621, 206}

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+25b(eh+fg))+35b^2fh^2-6chx(7bfh-10ceh+6cfg)+c^2\left(-\left(48fg^2-80h(dh+eg)\right.\right.\right.}{240c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) + 2*b*
c*(6*a*f*h + 5*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^4)
 + (f*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c*h) + ((35*b^2*f*h^2 - c^2*(
48*f*g^2 - 80*h*(e*g + d*h)) - 2*c*h*(16*a*f*h + 25*b*(f*g + e*h)) - 6*c*h*
(6*c*f*g - 10*c*e*h + 7*b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(240*c^3*h) - ((
b^2 - 4*a*c)*(32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a
*e*h) + 2*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(256*c^(9/2))
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{\int (g+hx)\left(-\frac{1}{2}h(3bfg-10cdh+4afh)-\right.}{5}$$

$$= \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{\left(35b^2fh^2-c^2\left(48fg^2-80h(eg+dh)\right)-2\right.}{}$$

$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+\right.}{128c^4}$$

$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+\right.}{128c^4}$$

$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+\right.}{128c^4}$$

Mathematica [A] time = 0.493249, size = 258, normalized size = 0.8

$$\frac{\frac{(a+x(b+cx))^{3/2}\left(-2ch(16afh+b(25eh+25fg+21fhx))+35b^2fh^2+c^2(20h(4dh+4eg+3ehx)-12fg(4g+3hx))\right)}{48c^2} - \frac{5h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.\right.}{}}{5ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^2*(a + x*(b + c*x))^(3/2) + ((a + x*(b + c*x))^(3/2)*(35*b^2*f
*h^2 + c^2*(-12*f*g*(4*g + 3*h*x) + 20*h*(4*e*g + 4*d*h + 3*e*h*x)) - 2*c*h
*(16*a*f*h + b*(25*f*g + 25*e*h + 21*f*h*x))))/(48*c^2) - (5*h*(-32*c^3*d*g
 + 7*b^3*f*h + 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) - 2*b*c*(6*a*f*h +
 5*b*(f*g + e*h)))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*
a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(256*c^(7/2))
)/(5*c*h)
```

Maple [B] time = 0.053, size = 1117, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/4*b/c*x*(c*x^2+b*x+a)^(1/2)*e*g+3/16*h*f*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)+1
/3*(c*x^2+b*x+a)^(3/2)/c*d*h+1/3*(c*x^2+b*x+a)^(3/2)/c*e*g+1/2*d*g*x*(c*x^2
+b*x+a)^(1/2)-1/8*b^2/c^2*(c*x^2+b*x+a)^(1/2)*d*h-1/8*b^2/c^2*(c*x^2+b*x+a)
^(1/2)*e*g+1/4*x*(c*x^2+b*x+a)^(3/2)/c*f*g-5/24*b/c^2*(c*x^2+b*x+a)^(3/2)*e
*h-5/24*b/c^2*(c*x^2+b*x+a)^(3/2)*f*g+1/4*d*g/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d
*g/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d*g/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/4*x*(c*x^2+b*x+a)^(3/2)/c*e
*h-5/128*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h+1/16*b
^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h-1/4*b/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h-1/4*b/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g-1/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g-1
/8*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-5/128*b^4/c^
(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/8*a^2/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h+1/16*b^3/c^(5/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g+1/5*h*f*x^2*(c*x^2+b*x+a)^(3/2)/c+7/48*h*f
*b^2/c^3*(c*x^2+b*x+a)^(3/2)-7/128*h*f*b^4/c^4*(c*x^2+b*x+a)^(1/2)+7/256*h*
f*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/15*h*f*a/c^2*(c
*x^2+b*x+a)^(3/2)+5/64*b^3/c^3*(c*x^2+b*x+a)^(1/2)*e*h+5/64*b^3/c^3*(c*x^2+
b*x+a)^(1/2)*f*g+3/16*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*a*f*g-1/8*a/c*x*(c*x^2+b*x+a)^(1/2)*e*h-1/8*a/c*x*(c*x^2+b*x+a)^(1/2)*f*
g-1/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*e*h-1/4*b/c*x*(c*x^2+b*x+a)^(1/2)*d*h-7/
40*h*f*b/c^2*x*(c*x^2+b*x+a)^(3/2)-7/64*h*f*b^3/c^3*x*(c*x^2+b*x+a)^(1/2)-5
/32*h*f*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/32*h*f*
b^2/c^3*a*(c*x^2+b*x+a)^(1/2)+3/16*h*f*b/c^(5/2)*a^2*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))+5/32*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)*f*g+3/16*b^2/c^(5/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h+5/32*b^2/c^2*x*(c*x^2+
b*x+a)^(1/2)*e*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.69966, size = 2275, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 - 4*a*b*c^3)*e + (5*
b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d - 2*(
5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2
)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h*x^4 + 48*(10*c^5*f*g + (10*c^5*
e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d + 10*b*c^4*e
- (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*c^3 - 8*a*c^
4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^4)*d - 10*(1
5*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*f)*h
+ 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (80*b*c^4*d -
 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)*x)*sqrt(c*x
^2 + b*x + a))/c^5, 1/3840*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 -
4*a*b*c^3)*e + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 -
4*a*b*c^3)*d - 2*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^
3*c + 48*a^2*b*c^2)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(384*c^5*f*h*x^4 + 48*(10*c^5*f*
g + (10*c^5*e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d +
 10*b*c^4*e - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*
c^3 - 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^
4)*d - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^
2*c^3)*f)*h + 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (
80*b*c^4*d - 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)
*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g+hx\right)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A] time = 1.34907, size = 668, normalized size = 2.07

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,fhx+\frac{10\,c^4fg+bc^3fh+10\,c^4he}{c^4}\right)x+\frac{10\,bc^3fg+80\,c^4dh-7\,b^2c^2fh+16\,ac^3fh+80\,c^4ge}{c^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h*x + (10*c^4*f*g + b*c^3*f*h +
10*c^4*h*e)/c^4)*x + (10*b*c^3*f*g + 80*c^4*d*h - 7*b^2*c^2*f*h + 16*a*c^3*
f*h + 80*c^4*g*e + 10*b*c^3*h*e)/c^4)*x + (480*c^4*d*g - 50*b^2*c^2*f*g + 1
20*a*c^3*f*g + 80*b*c^3*d*h + 35*b^3*c*f*h - 116*a*b*c^2*f*h + 80*b*c^3*g*e
 - 50*b^2*c^2*h*e + 120*a*c^3*h*e)/c^4)*x + (480*b*c^3*d*g + 150*b^3*c*f*g
- 520*a*b*c^2*f*g - 240*b^2*c^2*d*h + 640*a*c^3*d*h - 105*b^4*f*h + 460*a*b
^2*c*f*h - 256*a^2*c^2*f*h - 240*b^2*c^2*g*e + 640*a*c^3*g*e + 150*b^3*c*h*
e - 520*a*b*c^2*h*e)/c^4) + 1/256*(32*b^2*c^3*d*g - 128*a*c^4*d*g + 10*b^4*
c*f*g - 48*a*b^2*c^2*f*g + 32*a^2*c^3*f*g - 16*b^3*c^2*d*h + 64*a*b*c^3*d*h
 - 7*b^5*f*h + 40*a*b^3*c*f*h - 48*a^2*b*c^2*f*h - 16*b^3*c^2*g*e + 64*a*b*
c^3*g*e + 10*b^4*c*h*e - 48*a*b^2*c^2*h*e + 32*a^2*c^3*h*e)*log(abs(-2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

3.189 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=175

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-8bce+16c^2d\right)}{64c^3}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16\right.}{128c^{7/2}}$$

```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```

Rubi [A] time = 0.166976, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-8bce+16c^2d\right)}{64c^3}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16\right.}{128c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```


```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\int \left(4cd-af+\frac{1}{2}(8ce-5bf)x\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= \frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\left(16c^2d-8bce+5b^2f-4ac\right.}{16c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\left(a+bx+cx^2\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\left(a+bx+cx^2\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\left(a+bx+cx^2\right.}{24c^2}$$

Mathematica [A] time = 0.286353, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(4bc\left(2c\left(6d+2ex+fx^2\right)-13af\right)+8c^2\left(a(8e+3fx)+2cx\left(6d+4ex+3fx^2\right)\right)-2b^2c(12e+5fx)\right.}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c
*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e +
 a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(384*c^(7/2)
)
```

Maple [B] time = 0.052, size = 453, normalized size = 2.6

$$\frac{fx}{4c}\left(cx^2+bx+a\right)^{\frac{3}{2}}-\frac{5bf}{24c^2}\left(cx^2+bx+a\right)^{\frac{3}{2}}+\frac{5b^2fx}{32c^2}\sqrt{cx^2+bx+a}+\frac{5fb^3}{64c^3}\sqrt{cx^2+bx+a}+\frac{3b^2fa}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-5/24*f*b/c^2*(c*x^2+b*x+a)^(3/2)+5/32*f*b^2/c
^2*x*(c*x^2+b*x+a)^(1/2)+5/64*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)+3/16*f*b^2/c^(5
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*f*b^4/c^(7/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*f*a/c*x*(c*x^2+b*x+a)^(1/2)-1/16
*f*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*f*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+1/3*e*(c*x^2+b*x+a)^(3/2)/c-1/4*e*b/c*x*(c*x^2+b*x+a)^(1/
2)-1/8*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)-1/4*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a+1/16*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))+1/2*d*x*(c*x^2+b*x+a)^(1/2)+1/4*d/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d
/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d/c^(3/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73372, size = 1061, normalized size = 6.06

$$\left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d-8\left(b^3c-4\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 24
*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^3*d
 + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c - 52*a
*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c
^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15
*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A] time = 1.24786, size = 286, normalized size = 1.63

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{bc^2f+8\,c^3e}{c^3}\right)x+\frac{48\,c^3d-5\,b^2cf+12\,ac^2f+8\,bc^2e}{c^3}\right)x+\frac{48\,bc^2d+15\,b^3f-52\,abcf-}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48
*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*
f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a
*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*
log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

3.190 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

Optimal. Leaf size=321

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(bfh-2ceh+2cfg)\right)}{16c^{5/2}h^4}-\frac{\sqrt{a+bx+}}{}$$

```
[Out] -((4*c*h*(b*f*g - 2*c*d*h) - (4*c*g - b*h)*(2*c*f*g - 2*c*e*h + b*f*h) + 2*
c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(8*c^2*h^3) + (f*
(a + b*x + c*x^2)^(3/2))/(3*c*h) + ((4*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h)
- (2*c*f*g - 2*c*e*h + b*f*h)*(8*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - a*h)))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*h^4) + (S
qrt[c*g^2 - b*g*h + a*h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(b*g - 2*a*h + (
2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^4
```

Rubi [A] time = 0.778152, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(bfh-2ceh+2cfg)\right)}{16c^{5/2}h^4}-\frac{\sqrt{a+bx+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] -((4*c*h*(b*f*g - 2*c*d*h) - (4*c*g - b*h)*(2*c*f*g - 2*c*e*h + b*f*h) + 2*
c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(8*c^2*h^3) + (f*
(a + b*x + c*x^2)^(3/2))/(3*c*h) + ((4*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h)
- (2*c*f*g - 2*c*e*h + b*f*h)*(8*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - a*h)))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*h^4) + (S
qrt[c*g^2 - b*g*h + a*h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(b*g - 2*a*h + (
2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^4
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
```

```
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch} + \frac{\int \frac{\left(-\frac{3}{2}h(bfg-2cdh)-\frac{3}{2}h(2cfg-2ceh+bfh)x\right)\sqrt{a+bx+cx^2}}{g+hx}\,dx}{3ch^2}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{}}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{}}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{}}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{}}{8c^2h^3}$$

Mathematica [A] time = 0.782375, size = 331, normalized size = 1.03

$$\frac{2\sqrt{c}\left(h\sqrt{a+x(b+cx)}\left(2ch(4afh+b(3eh-3fg+fhx))-3b^2fh^2+4c^2\left(3h(2dh-2eg+ehx)+f\left(6g^2-3ghx+2h^2x^2\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (-3*(-(b^3*f*h^3) + 2*b*c*h^2*(-(b*f*g) + b*e*h + 2*a*f*h) + 16*c^3*g*(f*g^
2 + h*(-(e*g) + d*h)) - 8*c^2*h*(b*f*g^2 + b*h*(-(e*g) + d*h) + a*h*(-(f*g)
 + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c
]*(h*Sqrt[a + x*(b + c*x)]*(-3*b^2*f*h^2 + 2*c*h*(4*a*f*h + b*(-3*f*g + 3*e
*h + f*h*x)) + 4*c^2*(3*h*(-2*e*g + 2*d*h + e*h*x) + f*(6*g^2 - 3*g*h*x + 2
*h^2*x^2))) - 24*c^2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*(f*g^2 + h*(-(e*g) + d*
h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) +
a*h)]*Sqrt[a + x*(b + c*x)])]))/(48*c^(5/2)*h^4)
```

Maple [B] time = 0.352, size = 2549, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x)
```

```
[Out] -1/8/h*f*b^2/c^2*(c*x^2+b*x+a)^(1/2)+1/16/h*f*b^3/c^(5/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))+1/h^3*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+(
(x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))*c^(1/2)*g
^2*e-1/h^4*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g
)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))*c^(1/2)*g^3*f-1/h/((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+g/h))*a*d+1/4/h*e/c*(c*x^2+b*x+a)^(1/2)*b+1/2/h*e/
c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8/h*e/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-1/2/h^2*f*g*x*(c*x^2+b*x+a)^(1/2
)+1/2/h*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h
*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)*b*d-1/h^2*ln((1/2*(b*h-2*c
*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2))*c^(1/2)*g*d+1/2/h*e*x*(c*x^2+b*x+a)^(1/2)-1/h^2*((x+g/h)^2
*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g+1/h^3*((x+g/h)^
2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+1/h*((x+g/h)
^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*d-1/h^3/((a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+
2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g^2+1/2/h^3*ln((1/2*(b*h-2*c*g)/h+(x
+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2))/c^(1/2)*b*f*g^2+1/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2
-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*(
(x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*
a*e*g-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(
b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*
c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*g^2*e-1/2/h^2*ln(
(1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)*b*e*g+1/h^4/((a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2))/(x+g/h))*b*g^3*f+1/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h
^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)
)*b*g*d+1/3*f*(c*x^2+b*x+a)^(3/2)/c/h-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
```

```
*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2))/(x+g/h))*c*g^2*d+1/h^4/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g
*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g
/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^
3*e-1/h^5/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*
h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*
g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^4*f-1/2/h^2*f*g/c
^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8/h^2*f*g/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-1/4/h*f*b/c*x*(c*x^2+b*x+a)^(
1/2)-1/4/h*f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/4/h^
2*f*g/c*(c*x^2+b*x+a)^(1/2)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}\left(d + ex + fx^2\right)}{g + hx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.191 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2} dx$

Optimal. Leaf size=459

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{8c^{3/2}h^4}-\frac{\sqrt{a+bx+cx^2}\left(2ch^2x\left(-afh+\right.\right.}{}$$

```
[Out] -((b*f*h^2*(b*g - a*h) + 4*c^2*g*(3*f*g^2 - h*(2*e*g - d*h)) + c*h*(4*a*h*(
2*f*g - e*h) - b*(13*f*g^2 - 8*e*g*h + 4*d*h^2)) + 2*c*h^2*(2*c*e*g + b*f*g
 - (3*c*f*g^2)/h - 2*c*d*h - a*f*h)*x)*Sqrt[a + b*x + c*x^2])/(4*c*h^3*(c*g
^2 - b*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h
*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((b^2*f*h^2 + 4*c*h*(2*b*f*g - b*e*h
- a*f*h) - 8*c^2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*h^4) - ((2*c*g*(3*f*g^2 - h*(2*e*g -
d*h)) + h*(2*a*h*(2*f*g - e*h) - b*(5*f*g^2 - 3*e*g*h + d*h^2)))*ArcTanh[(b
*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x +
 c*x^2])])/(2*h^4*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rubi [A] time = 1.10023, antiderivative size = 453, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 814, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{8c^{3/2}h^4}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(-afh+b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((b*f*h*(b*g - a*h) - 4*c^2*g*(2*e*g - (3*f*g^2)/h - d*h) + 4*a*c*h*(2*f*g
 - e*h) - b*c*(13*f*g^2 - 8*e*g*h + 4*d*h^2) + 2*c*h*(2*c*e*g + b*f*g - (3*
c*f*g^2)/h - 2*c*d*h - a*f*h)*x)*Sqrt[a + b*x + c*x^2])/(4*c*h^2*(c*g^2 - b
*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^
2 - b*g*h + a*h^2)*(g + h*x)) - ((b^2*f*h^2 + 4*c*h*(2*b*f*g - b*e*h - a*f*
h) - 8*c^2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2]])/(8*c^(3/2)*h^4) - ((2*c*(3*f*g^3 - g*h*(2*e*g - d*h))
- h*(5*b*f*g^2 - b*h*(3*e*g - d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2
*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2
```

```
])])/(2*h^4*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\left(\frac{1}{2}\left(-2cdg+3beg+2afg-\frac{3bfg^2}{h}-bdh-2aeh\right)+\left(2ceg+b\right.\right.}{g+hx}}{cg^2-bgh+a}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-8egh+4\right.\right.}{4ch^2\left(cg^2-bgh+a\right.}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-8egh+4\right.\right.}{4ch^2\left(cg^2-bgh+a\right.}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-8egh+4\right.\right.}{4ch^2\left(cg^2-bgh+a\right.}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-8egh+4\right.\right.}{4ch^2\left(cg^2-bgh+a\right.}$$

Mathematica [A] time = 1.69481, size = 486, normalized size = 1.06

$$-\frac{\frac{\left(h(ah-bg)+cg^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4ch(afh+beh-2bfg)-b^2fh^2+8c^2\left(h(dh-2eg)+3fg^2\right)\right)}{\sqrt{c}}+2h\sqrt{a+x(b+cx)}\left(ch(2ah(2eh-4fg+fhx)+4bh(dh-2eg)+bfg(13g-2hx))+bfh^2\right.}{4h^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((f*(a + x*(b + c*x))^(3/2))/(g + h*x) - ((3*c*f*g^2 + f*h*(-(b*g) + a*h) +
 2*c*h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/((c*g^2 + h*(-(b*g) + a*h))
*(g + h*x)) - (2*h*Sqrt[a + x*(b + c*x)]*(b*f*h^2*(-(b*g) + a*h) + c*h*(4*b
*h*(-2*e*g + d*h) + b*f*g*(13*g - 2*h*x) + 2*a*h*(-4*f*g + 2*e*h + f*h*x))
+ c^2*(6*f*g^2*(-2*g + h*x) + 4*h*(e*g*(2*g - h*x) + d*h*(-g + h*x)))) + ((
c*g^2 + h*(-(b*g) + a*h))*(-(b^2*f*h^2) + 4*c*h*(-2*b*f*g + b*e*h + a*f*h)
+ 8*c^2*(3*f*g^2 + h*(-2*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)])])/Sqrt[c] + 4*c*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*(2*c*(3*f*g
^3 + g*h*(-2*e*g + d*h)) - h*(5*b*f*g^2 + b*h*(-3*e*g + d*h) + 2*a*h*(-2*f*
g + e*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(
b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(4*h^3*(-(c*g^2) + h*(b*g - a*h))))/(
2*c*h)
```

Maple [B] time = 0.273, size = 6218, normalized size = 13.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.192 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=448

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\right.\right.}{8h^4\left(ah^2-bgh+cg^2\right)^{3/2}}$$

```
[Out] (((4*c*g^2*(3*f*g - e*h))/h + 4*a*h*(3*f*g - e*h) - b*(11*f*g^2 - 3*e*g*h -
 d*h^2) - 2*h*(c*e*g + 2*b*f*g - (3*c*f*g^2)/h - c*d*h - 2*a*f*h)*x)*Sqrt[a
 + b*x + c*x^2])/(4*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2 - h*(e
*g - d*h))*(a + b*x + c*x^2)^(3/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^
2) - ((6*c*f*g - 2*c*e*h - b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2*Sqrt[c]*h^4) + ((8*c^2*g^3*(3*f*g - e*h) - 4*c*h*(b*g^2*(
10*f*g - 3*e*h) - a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 - 4*a
*b*h*(6*f*g - e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTanh[(b*g - 2*a*
h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]
)/(8*h^4*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rubi [A] time = 0.874962, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 812, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\right.\right.}{8h^4\left(ah^2-bgh+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -((11*b*f*g^2 - b*h*(3*e*g + d*h) - (4*c*g^2*(3*f*g - e*h))/h - 4*a*h*(3*f*
g - e*h) + 2*h*(c*e*g + 2*b*f*g - (3*c*f*g^2)/h - c*d*h - 2*a*f*h)*x)*Sqrt[
a + b*x + c*x^2])/(4*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2 - h*(
e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)
^2) - ((6*c*f*g - 2*c*e*h - b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(2*Sqrt[c]*h^4) + ((8*c^2*g^3*(3*f*g - e*h) - 4*c*h*(b*g^2*
(10*f*g - 3*e*h) - a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 - 4*
a*b*h*(6*f*g - e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTanh[(b*g - 2*a
*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])
```

```
])/(8*h^4*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx = -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{2h\,(cg^2-bgh+ah^2)\,(g+hx)^2} - \frac{\int \frac{\left(\frac{1}{2}\left(-4cdg+3beg+4afg-\frac{3bfg^2}{h}+bdh-4aeh\right)+(ceg+\cdots\right.}{(g+hx)^2}}{2\,(cg^2-bgh\cdots}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bfg-\frac{3cfg^2}{h}\cdots\right.\right.}{4h^2\,(cg^2-bgh+ah^2)\,(g+hx)}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bfg-\frac{3cfg^2}{h}\cdots\right.\right.}{4h^2\,(cg^2-bgh+ah^2)\,(g+hx)}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bfg-\frac{3cfg^2}{h}\cdots\right.\right.}{4h^2\,(cg^2-bgh+ah^2)\,(g+hx)}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bfg-\frac{3cfg^2}{h}\cdots\right.\right.}{4h^2\,(cg^2-bgh+ah^2)\,(g+hx)}$$

Mathematica [A] time = 3.95224, size = 645, normalized size = 1.44

$$-\frac{\frac{2c\sqrt{a+x(b+cx)}\left(h^2\left(-4a^2fh^2-4abh(eh-4fg)+b^2\left(dh^2+3egh-11fg^2\right)\right)+ch\left(b\left(h(dh(hx-g)+eg(3hx-7g))+fg^2(23g-7hx)\right)-2ah(h(dh-3eg+2ehx)+fg(9g-4hx))\right)-2c^2\left(gh\left(dh^2x+eg(hx-2g)\right)+3fg^3(2\cdots\right.\right.}{h^2}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((f*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 - ((3*c*f*g^2 + 2*f*h*(-(b*g) + a*
h) + c*h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/(2*(c*g^2 + h*(-(b*g) + a
*h))*(g + h*x)^2) - ((-2*c*(6*c*f*g^3 - 2*c*g*h*(e*g + d*h) - 4*a*h^2*(-2*f
*g + e*h) + b*h*(-7*f*g^2 + h*(3*e*g + d*h)))*(a + x*(b + c*x))^(3/2))/(g +
 h*x) + (2*c*Sqrt[a + x*(b + c*x)]*(h^2*(-4*a^2*f*h^2 - 4*a*b*h*(-4*f*g + e
*h) + b^2*(-11*f*g^2 + 3*e*g*h + d*h^2)) - 2*c^2*(3*f*g^3*(2*g - h*x) + g*h
*(d*h^2*x + e*g*(-2*g + h*x))) + c*h*(-2*a*h*(f*g*(9*g - 4*h*x) + h*(-3*e*g
 + d*h + 2*e*h*x)) + b*(f*g^2*(23*g - 7*h*x) + h*(d*h*(-g + h*x) + e*g*(-7*
g + 3*h*x))))))/h^2 + (4*Sqrt[c]*(6*c*f*g - 2*c*e*h - b*f*h)*(c*g^2 + h*(-(
b*g) + a*h))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c*S
qrt[c*g^2 + h*(-(b*g) + a*h)]*(8*c^2*g^3*(3*f*g - e*h) + 4*c*h*(b*g^2*(-10*
f*g + 3*e*h) + a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*
h*(-6*f*g + e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTanh[(-(b*g) + 2*a
*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x
)])])/h^3)/(8*(c*g^2 + h*(-(b*g) + a*h))^2))/(c*h)
```

Maple [B] time = 0.285, size = 12139, normalized size = 27.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.193 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4} dx$

Optimal. Leaf size=603

$$-\frac{\sqrt{a+bx+cx^2}\left(hx\left(h^2\left(8a^2fh^2-2abh(10fg-eh)+b^2\left(11fg^2-h(dh+eg)\right)\right)+2cgh\left(2ah(6fg-eh)-b\left(12fg^2-h(2dh\right.\right.\right.\right.}{}$$

```
[Out] -((8*c^2*f*g^5 - 2*c*g*h*(7*b*f*g^3 - 6*a*f*g^2*h + b*d*g*h^2 - 2*a*d*h^3)
+ h^2*(4*a^2*e*h^3 + b^2*g*(5*f*g^2 + e*g*h + d*h^2) - 2*a*b*h*(3*f*g^2 + 2
*e*g*h + d*h^2)) + h*(4*c^2*(3*f*g^4 - d*g^2*h^2) + h^2*(8*a^2*f*h^2 - 2*a*
b*h*(10*f*g - e*h) + b^2*(11*f*g^2 - h*(e*g + d*h))) + 2*c*g*h*(2*a*h*(6*f*
g - e*h) - b*(12*f*g^2 - h*(e*g + 2*d*h))))*x)*Sqrt[a + b*x + c*x^2])/(8*h^
3*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*
x + c*x^2)^(3/2))/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^4 - ((16*c^3*f*g^5
- 8*c^2*g*h*(5*b*f*g^3 - 5*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a*
b*h*(6*f*g + e*h) + b^2*(5*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(4*a^2*h^2*(4*
f*g - e*h) - 2*a*b*h*(15*f*g^2 - e*g*h - d*h^2) + b^2*(15*f*g^3 + d*g*h^2))
)*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sq
rt[a + b*x + c*x^2])])/(16*h^4*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rubi [A] time = 1.44877, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(hx\left(8a^2fh^3-2b\left(ah^2(10fg-eh)-cgh(2dh+eg)+12cfg^3\right)+4acgh(6fg-eh)+b^2h\left(11fg^2-h(dh+e\right.\right.\right.}{8h}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -(((8*c^2*f*g^5)/h + 4*a^2*e*h^4 + 4*a*c*g*h*(3*f*g^2 + d*h^2) + b^2*g*h*(5
*f*g^2 + h*(e*g + d*h)) - 2*b*(a*h^2*(3*f*g^2 + 2*e*g*h + d*h^2) + c*(7*f*g
^4 + d*g^2*h^2)) + h*(8*a^2*f*h^3 + 4*a*c*g*h*(6*f*g - e*h) + c^2*((12*f*g^
4)/h - 4*d*g^2*h) + b^2*h*(11*f*g^2 - h*(e*g + d*h)) - 2*b*(12*c*f*g^3 - c*
g*h*(e*g + 2*d*h) + a*h^2*(10*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(8*h^2
*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x
```

```
 + c*x^2)^(3/2))/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^4 - ((16*c^3*f*g^5 -
 8*c^2*g*h*(5*b*f*g^3 - 5*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a*b
*h*(6*f*g + e*h) + b^2*(5*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(4*a^2*h^2*(4*f
*g - e*h) - 2*a*b*h*(15*f*g^2 - e*g*h - d*h^2) + b^2*(15*f*g^3 + d*g*h^2)))
*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2])])/(16*h^4*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
```

```
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-\frac{3}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}-bdh+2aeh\right)+3f\left(bg-\frac{cg^2}{h}\right.\right.}{(g+hx)^3}}{3\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-2b\left(ah^2\left(3f\right.\right.\right.}{}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-2b\left(ah^2\left(3f\right.\right.\right.}{}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-2b\left(ah^2\left(3f\right.\right.\right.}{}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-2b\left(ah^2\left(3f\right.\right.\right.}{}$$

Mathematica [A] time = 2.28668, size = 439, normalized size = 0.73

$$\frac{\frac{\left(\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)}{8\left(h(ah-bg)+cg^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ah+b(g-hx)+2cgx)}{4(g+hx)^2\left(h(ah-bg)+cg^2\right)}\right)\left(2ah^2(eh-2fg)-bh\left(h(dh+eg)-3fg^2\right)+c\left(2dgh^2-2fg^3\right)\right)}{2\left(h(ah-bg)+cg^2\right)}-\frac{h(a+x(b+cx))^{3/2}\left(h(dh-eg\right.}{3(g+hx)^3\left(h(ah-bg)+\right.}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] (-(h*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/(3*(c*g^2 + h*(-(b
*g) + a*h))*(g + h*x)^3) + ((2*a*h^2*(-2*f*g + e*h) + c*(-2*f*g^3 + 2*d*g*h
^2) - b*h*(-3*f*g^2 + h*(e*g + d*h)))*((Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c
*g*x + b*(g - h*x)))/(4*(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((b^2 - 4
*a*c)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g)
+ a*h)]*Sqrt[a + x*(b + c*x)])])/(8*(c*g^2 + h*(-(b*g) + a*h))^(3/2))))/(2*
(c*g^2 + h*(-(b*g) + a*h))) + (f*(-((h*Sqrt[a + x*(b + c*x)])/(g + h*x)) +
Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + ((2*c*g -
b*h)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) +
 a*h)]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)])))/h^2)/h
^2
```

Maple [B] time = 0.3, size = 19321, normalized size = 32.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{\left(g+hx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.194 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal. Leaf size=497

$$\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)\left(16a^2fh^2-4c\left(a\left(dh^2-5egh+fg^2\right)+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\right.}{64(g+hx)^2\left(ah^2-bgh+cg^2\right)^3}$$

```
[Out] ((16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) + b^2*(5*f*g^2 + 3*e*
g*h + 5*d*h^2) - 4*c*(2*b*g*(e*g + 2*d*h) + a*(f*g^2 - 5*e*g*h + d*h^2)))*(
b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(64*(c*g^2 - b*g*h +
a*h^2)^3*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(
4*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + ((2*c*g*(3*f*g^2 + h*(e*g - 5*d*
h)) + h*(8*a*h*(2*f*g - e*h) - b*(11*f*g^2 - 3*e*g*h - 5*d*h^2)))*(a + b*x
+ c*x^2)^(3/2))/(24*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((b^2 - 4*a*
c)*(16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) + b^2*(5*f*g^2 + 3*
e*g*h + 5*d*h^2) - 4*c*(2*b*g*(e*g + 2*d*h) + a*(f*g^2 - 5*e*g*h + d*h^2)))
*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2])])/(128*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rubi [A] time = 0.85545, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1650, 806, 720, 724, 206}

$$\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\right.}{64(g+hx)^2\left(ah^2-bgh+cg^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] ((16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*
(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))*(
b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(64*(c*g^2 - b*g*h +
a*h^2)^3*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(
4*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + ((6*c*f*g^3 + 2*c*g*h*(e*g - 5*d
*h) + 8*a*h^2*(2*f*g - e*h) - b*h*(11*f*g^2 - h*(3*e*g + 5*d*h)))*(a + b*x
+ c*x^2)^(3/2))/(24*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((b^2 - 4*a*
c)*(16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*
```

```
h*(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))
*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2])])/(128*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(\frac{1}{2}\left(-8cdg+3beg+8afg-\frac{3bfg^2}{h}+5bdh-8aeh\right)-\left(ceg\right.\right.}{(g+hx)^4}}{4\left(cg^2-bgh\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} + \frac{\left(6cfg^3+2cgh(eg-5dh)+8ah^2(2fg-\right.}{24h\left(cg^2-\right.}$$

$$= \frac{\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c\left(afg^2-ah(5eg-dh)+2bg(eg+2dh\right.\right.}{64\left(cg^2-bgh+ah^2\right)}$$

$$= \frac{\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c\left(afg^2-ah(5eg-dh)+2bg(eg+2dh\right.\right.}{64\left(cg^2-bgh+ah^2\right)}$$

$$= \frac{\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c\left(afg^2-ah(5eg-dh)+2bg(eg+2dh\right.\right.}{64\left(cg^2-bgh+ah^2\right)}$$

Mathematica [A] time = 4.79482, size = 447, normalized size = 0.9

$$\frac{\frac{\frac{3}{2}ch\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)}{8\left(h(ah-bg)+cg^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ah+b(g-hx)+2cgx)}{4(g+hx)^2\left(h(ah-bg)+cg^2\right)}\right)\left(16a^2fh^2-4c\left(ah(dh-5eg)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\left(h(5dh+3eg)\right.\right.}{24\left(h(ah-bg)+cg^2\right)^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (-((f*(a + x*(b + c*x))^(3/2))/(g + h*x)^4) + ((3*c*f*g^2 + 4*f*h*(-(b*g) +
 a*h) + c*h*(e*g - d*h))*(a + x*(b + c*x))^(3/2))/(4*(c*g^2 + h*(-(b*g) + a
*h))*(g + h*x)^4) + ((c*(6*c*f*g^3 + 2*c*g*h*(e*g - 5*d*h) - 8*a*h^2*(-2*f*
g + e*h) + b*h*(-11*f*g^2 + h*(3*e*g + 5*d*h)))*(a + x*(b + c*x))^(3/2))/(g
```

```
 + h*x)^3 + (3*c*h*(16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4
*c*(a*f*g^2 + a*h*(-5*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*
(3*e*g + 5*d*h)))*((Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)))
/(4*(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g
) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(
b + c*x)])])/(8*(c*g^2 + h*(-(b*g) + a*h))^(3/2))))/2)/(24*(c*g^2 + h*(-(b*
g) + a*h))^2))/(c*h)
```

Maple [B] time = 0.284, size = 29161, normalized size = 58.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x)

[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima"
)

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**5,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.195 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

Optimal. Leaf size=824

$$\frac{(4c^2(3fg^2+h(2eg-27dh))g^2-5h^2((3fg^2+3ehg+7dh^2)b^2-2ah(6fg+5eh)b+16a^2fh^2)-2ch(bg(16fg^2-21ehg}{240h(cg^2-bhg+ah^2)^3(g+hx)^3}$$

```
[Out] ((32*c^3*d*g^3 - 8*c^2*g*(2*b*g*(e*g + 3*d*h) + a*(f*g^2 - 6*e*g*h + 3*d*h^
2)) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + 3*e*g*h
+ 7*d*h^2)) + 2*c*(4*a^2*h^2*(6*f*g - e*h) - 6*a*b*h*(3*f*g^2 + 3*e*g*h - d
*h^2) + b^2*g*(5*f*g^2 + 6*e*g*h + 15*d*h^2)))*(b*g - 2*a*h + (2*c*g - b*h)
*x)*Sqrt[a + b*x + c*x^2])/(128*(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) - ((
f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(5*h*(c*g^2 - b*g*h + a*h^2
)*(g + h*x)^5) + ((2*c*g*(3*f*g^2 + h*(2*e*g - 7*d*h)) + h*(10*a*h*(2*f*g -
 e*h) - b*(13*f*g^2 - 3*e*g*h - 7*d*h^2)))*(a + b*x + c*x^2)^(3/2))/(40*h*(
c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^4) + ((4*c^2*g^2*(3*f*g^2 + h*(2*e*g - 2
7*d*h)) - 5*h^2*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + 3*
e*g*h + 7*d*h^2)) - 2*c*h*(b*g*(16*f*g^2 - 21*e*g*h - 54*d*h^2) - 2*a*h*(18
*f*g^2 - 33*e*g*h + 8*d*h^2)))*(a + b*x + c*x^2)^(3/2))/(240*h*(c*g^2 - b*g
*h + a*h^2)^3*(g + h*x)^3) - ((b^2 - 4*a*c)*(32*c^3*d*g^3 - 8*c^2*g*(2*b*g*
(e*g + 3*d*h) + a*(f*g^2 - 6*e*g*h + 3*d*h^2)) - b*h*(16*a^2*f*h^2 - 2*a*b*
h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + 3*e*g*h + 7*d*h^2)) + 2*c*(4*a^2*h^2*(6*
f*g - e*h) - 6*a*b*h*(3*f*g^2 + 3*e*g*h - d*h^2) + b^2*g*(5*f*g^2 + 6*e*g*h
 + 15*d*h^2)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(256*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rubi [A] time = 2.33474, antiderivative size = 826, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{(4(3fg^4+h(2eg-27dh)g^2)c^2-2h(bg(16fg^2-21ehg-54dh^2)-2ah(18fg^2-33ehg+8dh^2))c-5h^2((3fg^2+3ehg}{240h(cg^2-bhg+ah^2)^3(g+hx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] ((32*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*
h)) + 2*c*(4*a^2*h^2*(6*f*g - e*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) +
b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g
 + 5*e*h) + b^2*(3*f*g^2 + h*(3*e*g + 7*d*h))))*(b*g - 2*a*h + (2*c*g - b*h
)*x)*Sqrt[a + b*x + c*x^2])/(128*(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) - (
(f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(5*h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^5) + ((6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g -
 e*h) - b*h*(13*f*g^2 - h*(3*e*g + 7*d*h)))*(a + b*x + c*x^2)^(3/2))/(40*h*
(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^4) + ((4*c^2*(3*f*g^4 + g^2*h*(2*e*g -
27*d*h)) - 5*h^2*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + 3
*e*g*h + 7*d*h^2)) - 2*c*h*(b*g*(16*f*g^2 - 21*e*g*h - 54*d*h^2) - 2*a*h*(1
8*f*g^2 - 33*e*g*h + 8*d*h^2)))*(a + b*x + c*x^2)^(3/2))/(240*h*(c*g^2 - b*
g*h + a*h^2)^3*(g + h*x)^3) - ((b^2 - 4*a*c)*(32*c^3*d*g^3 - 8*c^2*g*(a*f*g
^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*h)) + 2*c*(4*a^2*h^2*(6*f*g - e
*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) + b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5
*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + h*(3
*e*g + 7*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(256*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
```

```
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(\frac{1}{2}\left(-10cdg+3beg+10afg-\frac{3bfg^2}{h}+7bdh-10aeh\right)-\left(\right.\right.}{(g+hx}}{5\left(cg^2-bg\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} + \frac{\left(6cfg^3+2cgh(2eg-7dh)+10ah^2(2fg\right.}{40h\left(cg^2-\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} + \frac{\left(6cfg^3+2cgh(2eg-7dh)+10ah^2(2fg\right.}{40h\left(cg^2-\right.}$$

$$= \frac{\left(32c^3dg^3-8c^2g\left(afg^2-3ah(2eg-dh)+2bg(eg+3dh)\right)+2c\left(4a^2h^2(6fg-eh)\right.\right.}{}$$

$$= \frac{\left(32c^3dg^3-8c^2g\left(afg^2-3ah(2eg-dh)+2bg(eg+3dh)\right)+2c\left(4a^2h^2(6fg-eh)\right.\right.}{}$$

$$= \frac{\left(32c^3dg^3-8c^2g\left(afg^2-3ah(2eg-dh)+2bg(eg+3dh)\right)+2c\left(4a^2h^2(6fg-eh)\right.\right.}{}$$

Mathematica [A] time = 6.32801, size = 1129, normalized size = 1.37

$$\sqrt{a+x(b+cx)}\left(-\frac{\left(\frac{1}{2}h(3bfg+4cdh-10afh)-\frac{1}{2}g(6cfg+4ceh-7bfh)\right)\left(cx^2+bx+a\right)^{3/2}}{5\left(cg^2-bhg+ah^2\right)(g+hx)^5} - \frac{-\frac{\left(2cg\left(3cfg^2-5fh(bg-ah)+2ch(eg-dh)\right)-ch\left(3bfg^2-bh(3eg+7dh)+10h(cdg-afg+\right.\right.}{4\left(cg^2-bhg+ah^2\right)(g+hx)^4}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(f*(a + b*x + c*x^2)*Sqrt[a + x*(b + c*x)])/(2*c*h*(g + h*x)^5) + (Sqrt[a
+ x*(b + c*x)]*(-(((h*(3*b*f*g + 4*c*d*h - 10*a*f*h))/2 - (g*(6*c*f*g + 4*c
*e*h - 7*b*f*h))/2)*(a + b*x + c*x^2)^(3/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g
+ h*x)^5) - (-((2*c*g*(3*c*f*g^2 - 5*f*h*(b*g - a*h) + 2*c*h*(e*g - d*h)) -
 c*h*(3*b*f*g^2 - b*h*(3*e*g + 7*d*h) + 10*h*(c*d*g - a*f*g + a*e*h)))*(a +
 b*x + c*x^2)^(3/2))/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (-((-(c^2*g*
(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(13*f*g
^2 - h*(3*e*g + 7*d*h)))) + (c*h*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)) + 2
*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 16*h*(
5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2)*(a + b*
x + c*x^2)^(3/2))/(3*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) - ((-2*(a*c^2*h*(
6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(13*f*g^
2 - h*(3*e*g + 7*d*h))) + (c^2*g*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)) + 2
*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 16*h*(
5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2) + b*(c^
2*g*(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(13
*f*g^2 - h*(3*e*g + 7*d*h))) + (c*h*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h))
+ 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 16*
h*(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2))*(((
b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a
*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)
*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[c*g^2 -
 b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2)
))/(4*(c*g^2 - b*g*h + a*h^2)))/(5*(c*g^2 - b*g*h + a*h^2))))/(2*c*h*Sqrt[a
 + b*x + c*x^2])
```

Maple [B] time = 0.292, size = 40336, normalized size = 49.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

3.196 $\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=1169

result too large to display

```
[Out] -((b^2 - 4*a*c)*(1536*c^5*d*g^3 - 143*b^5*f*h^3 + 22*b^3*c*h^2*(20*a*f*h +
9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^
2*(3*f*g^2 + 3*e*g*h + d*h^2)) - 256*c^4*g*(3*b*g*(e*g + 3*d*h) + a*(f*g^2
+ 3*h*(e*g + d*h))) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3
*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(32768*c^7) + ((1536*c^5*d*g^3 - 143*b^5*f*h^3 + 22*b^3*c*
h^2*(20*a*f*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f
*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) - 256*c^4*g*(3*b*g*(e*g + 3*
d*h) + a*(f*g^2 + 3*h*(e*g + d*h))) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*
b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(12288*c^6) + ((143*b^2*f*h^2 - 2*c*h*(2
4*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2 - 3*h*(3*e*g + 8*d*h)))*(g
 + h*x)^2*(a + b*x + c*x^2)^(5/2))/(2016*c^3*h) - ((10*c*f*g - 18*c*e*h + 1
3*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(144*c^2*h) + (f*(g + h*x)^4*
(a + b*x + c*x^2)^(5/2))/(9*c*h) + ((3003*b^4*f*h^4 - 192*c^4*g^2*(5*f*g^2
- 3*h*(3*e*g + 64*d*h)) - 198*b^2*c*h^3*(38*a*f*h + 21*b*(3*f*g + e*h)) + 8
*c^2*h^2*(256*a^2*f*h^2 + 837*a*b*h*(3*f*g + e*h) + b^2*(1553*f*g^2 + 756*h
*(3*e*g + d*h))) - 16*c^3*h*(32*a*h*(17*f*g^2 + 9*h*(3*e*g + d*h)) + b*g*(1
3*f*g^2 + 9*h*(141*e*g + 196*d*h))) - 10*c*h*(429*b^3*f*h^3 - 22*b*c*h^2*(2
9*b*f*g + 27*b*e*h + 34*a*f*h) + 16*c^3*g*(5*f*g^2 - 9*h*(e*g + 12*d*h)) +
8*c^2*h*(a*h*(61*f*g + 63*e*h) + 3*b*(f*g^2 + 6*h*(7*e*g + 6*d*h))))*x)*(a
+ b*x + c*x^2)^(5/2))/(80640*c^5*h) + ((b^2 - 4*a*c)^2*(1536*c^5*d*g^3 - 14
3*b^5*f*h^3 + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a
^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) - 256
*c^4*g*(3*b*g*(e*g + 3*d*h) + a*(f*g^2 + 3*h*(e*g + d*h))) + 32*c^3*(3*a^2*
h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2
+ h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(65536*c^(15/2))
```

Rubi [A] time = 3.69766, antiderivative size = 1166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{f\left(cx^2+bx+a\right)^{5/2}(g+hx)^4}{9ch} - \frac{(10cfg-18ceh+13bfh)\left(cx^2+bx+a\right)^{5/2}(g+hx)^3}{144c^2h} + \frac{\left(-12\left(5fg^2-3h(3eg+8dh)\right)c^2 - \cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*
h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g
+ e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2
+ 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3
*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(32768*c^7) + ((1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g
*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f
*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) +
 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*
b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(12288*c^6) + ((143*b^2*f*h^2 - 2*c*h*(2
4*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2 - 3*h*(3*e*g + 8*d*h)))*(g
 + h*x)^2*(a + b*x + c*x^2)^(5/2))/(2016*c^3*h) - ((10*c*f*g - 18*c*e*h + 1
3*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(144*c^2*h) + (f*(g + h*x)^4*
(a + b*x + c*x^2)^(5/2))/(9*c*h) + ((3003*b^4*f*h^4 - 192*c^4*(5*f*g^4 - 3*
g^2*h*(3*e*g + 64*d*h)) - 198*b^2*c*h^3*(38*a*f*h + 21*b*(3*f*g + e*h)) + 8
*c^2*h^2*(256*a^2*f*h^2 + 837*a*b*h*(3*f*g + e*h) + b^2*(1553*f*g^2 + 756*h
*(3*e*g + d*h))) - 16*c^3*h*(32*a*h*(17*f*g^2 + 9*h*(3*e*g + d*h)) + b*g*(1
3*f*g^2 + 9*h*(141*e*g + 196*d*h))) - 10*c*h*(429*b^3*f*h^3 - 22*b*c*h^2*(2
9*b*f*g + 27*b*e*h + 34*a*f*h) + 16*c^3*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)) +
8*c^2*h*(a*h*(61*f*g + 63*e*h) + 3*b*(f*g^2 + 6*h*(7*e*g + 6*d*h))))*x)*(a
+ b*x + c*x^2)^(5/2))/(80640*c^5*h) + ((b^2 - 4*a*c)^2*(1536*c^5*d*g^3 - 14
3*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h))
 + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 +
9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*
h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2
+ h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(65536*c^(15/2))
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)^3\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch} + \frac{\int (g+hx)^3\left(-\frac{1}{2}h(5bfg-18cdh+8af\right.}{}$$

$$= -\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{144c^2h} + \frac{f(g+hx)^4\left(\right.}{}$$

$$= \frac{\left(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2\left(5fg^2-3h(3eg+\right.\right.}{2016c^3h}$$

$$= \frac{\left(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2\left(5fg^2-3h(3eg+\right.\right.}{2016c^3h}$$

$$= \frac{\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(eg+dh)+3bg(eg+3\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(eg+dh)+\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(eg+dh)+\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(eg+dh)+\right.\right.}{}$$

Mathematica [A] time = 3.00461, size = 721, normalized size = 0.62

$$\frac{3h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\left(-48bc^2h\left(5a^2fh^2+9abh(eh+3fg)+6b^2\left(dh^2+3egh+3fg^2\right)\right)+32c^3\left(3a\right.\right.}{655}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (((143*b^2*f*h^2 - 2*c*h*(24*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2
 - 3*h*(3*e*g + 8*d*h)))*(g + h*x)^2*(a + x*(b + c*x))^(5/2))/(224*c^2) - (
(13*b*f*h + 2*c*(5*f*g - 9*e*h))*(g + h*x)^3*(a + x*(b + c*x))^(5/2))/(16*c
) + f*(g + h*x)^4*(a + x*(b + c*x))^(5/2) + ((a + x*(b + c*x))^(5/2)*(3003*
b^4*f*h^4 - 66*b^2*c*h^3*(114*a*f*h + b*(189*f*g + 63*e*h + 65*f*h*x)) - 32
*c^4*(5*f*g^3*(6*g + 5*h*x) - 9*g*h*(e*g*(6*g + 5*h*x) + 4*d*h*(32*g + 15*h
*x))) + 4*c^2*h^2*(512*a^2*f*h^2 + 2*a*b*h*(2511*f*g + 837*e*h + 935*f*h*x)
 + b^2*(f*g*(3106*g + 1595*h*x) + 27*h*(168*e*g + 56*d*h + 55*e*h*x))) - 16
*c^3*h*(a*h*(f*g*(544*g + 305*h*x) + 9*h*(96*e*g + 32*d*h + 35*e*h*x)) + b*
```

```
(f*g^2*(13*g + 15*h*x) + 9*h*(4*d*h*(49*g + 15*h*x) + e*g*(141*g + 70*h*x))
))))/(8960*c^4) + (3*h*(1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^2
 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b*
(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3
*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*
g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(2*Sqrt[c]*
(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2))
+ 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
)/(65536*c^(13/2)))/(9*c*h)
```

Maple [B] time = 0.078, size = 5881, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 30.47, size = 11167, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [-1/41287680*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d - 12*(b^5*
c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b
^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)
*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*e + 3*(3*b
^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h + 6*(32*(7*
b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*(3*b^7*c^2 -
28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c - 336*a*b^6*c
^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^2 - (96*(3*b
^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(33*b^8*c - 33
6*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*e + (143*b
^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840*a^4*b*c^4)*f
)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) - 4*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9*f*g*h^
2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 54*(16*c^9*e
 + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7 + 320*a*c^8)*f
)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^8*f)*g^2*h + 18*
(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b*c^8*d
+ 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 + 128*(
1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*b*c^8*e + 3*(b^2*c^
7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*c^7 + 64*a*c^8)*e - 3*(
11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d - 18*(11*b^
3*c^6 - 52*a*b*c^7)*e + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f)*h^3)
*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a*b^2*c^
6 + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f)*g^3
+ 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5*c^4 -
 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488
*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*b^3*c^
5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5
 - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^3*c^4
- 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2
*b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2
*b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 1383984*a
^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344*(120*c
^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8*d + 1
4*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 18*(224
*(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^4*c^5
- 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^7)*d -
 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 8536*a*b
^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^2*c^7
+ 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 + 32*a*
c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^6 + 12
```

8*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d - 18*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 22968*a*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(120*(b^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d - 14*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(1155*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 - (288*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130284*a*b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/c^8, -1/20643840*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d - 12*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*e + 3*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h + 6*(32*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^2 - (96*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(33*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*e + (143*b^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840*a^4*b*c^4)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9*f*g*h^2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 54*(16*c^9*e + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7 + 320*a*c^8)*f)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^8*f)*g^2*h + 18*(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b*c^8*d + 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 + 128*(1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*b*c^8*e + 3*(b^2*c^7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*c^7 + 64*a*c^8)*e - 3*(11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d - 18*(11*b^3*c^6 - 52*a*b*c^7)*e + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f)*h^3)*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f)*g^3 + 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^3*c^4 - 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 816

```
48*a^2*b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 138
3984*a^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344*
(120*c^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8
*d + 14*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 1
8*(224*(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^
4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^
7)*d - 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 85
36*a*b^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^
2*c^7 + 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 +
 32*a*c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^
6 + 128*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4
*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2
416*a^2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d
 - 18*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 2
2968*a*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(
120*(b^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 -
216*a*b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d -
 14*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b
^3*c^5 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 2
40*a^2*c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(11
55*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 -
(288*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 -
8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130
284*a*b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^
2 + b*x + a))/c^8]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{\frac{3}{2}} \left(d+ex+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)**3*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

Giac [B] time = 1.35419, size = 4019, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/10321920*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*(16*c*f*h^3*x + (54
*c^9*f*g*h^2 + 19*b*c^8*f*h^3 + 18*c^9*h^3*e)/c^8)*x + (864*c^9*f*g^2*h + 9
18*b*c^8*f*g*h^2 + 288*c^9*d*h^3 + 3*b^2*c^7*f*h^3 + 320*a*c^8*f*h^3 + 864*
c^9*g*h^2*e + 306*b*c^8*h^3*e)/c^8)*x + (1344*c^9*f*g^3 + 4320*b*c^8*f*g^2*
h + 4032*c^9*d*g*h^2 + 54*b^2*c^7*f*g*h^2 + 4536*a*c^8*f*g*h^2 + 1440*b*c^8
*d*h^3 - 13*b^3*c^6*f*h^3 + 60*a*b*c^7*f*h^3 + 4032*c^9*g^2*h*e + 4320*b*c^
8*g*h^2*e + 18*b^2*c^7*h^3*e + 1512*a*c^8*h^3*e)/c^8)*x + (17472*b*c^8*f*g^
3 + 48384*c^9*d*g^2*h + 864*b^2*c^7*f*g^2*h + 55296*a*c^8*f*g^2*h + 52416*b
*c^8*d*g*h^2 - 594*b^3*c^6*f*g*h^2 + 2808*a*b*c^7*f*g*h^2 + 288*b^2*c^7*d*h
^3 + 18432*a*c^8*d*h^3 + 143*b^4*c^5*f*h^3 - 804*a*b^2*c^6*f*h^3 + 768*a^2*
c^7*f*h^3 + 16128*c^9*g^3*e + 52416*b*c^8*g^2*h*e + 864*b^2*c^7*g*h^2*e + 5
5296*a*c^8*g*h^2*e - 198*b^3*c^6*h^3*e + 936*a*b*c^7*h^3*e)/c^8)*x + (16128
0*c^9*d*g^3 + 4032*b^2*c^7*f*g^3 + 188160*a*c^8*f*g^3 + 532224*b*c^8*d*g^2*
h - 7776*b^3*c^6*f*g^2*h + 38016*a*b*c^7*f*g^2*h + 12096*b^2*c^7*d*g*h^2 +
564480*a*c^8*d*g*h^2 + 5346*b^4*c^5*f*g*h^2 - 30672*a*b^2*c^6*f*g*h^2 + 302
40*a^2*c^7*f*g*h^2 - 2592*b^3*c^6*d*h^3 + 12672*a*b*c^7*d*h^3 - 1287*b^5*c^
4*f*h^3 + 8536*a*b^3*c^5*f*h^3 - 12912*a^2*b*c^6*f*h^3 + 177408*b*c^8*g^3*e
 + 12096*b^2*c^7*g^2*h*e + 564480*a*c^8*g^2*h*e - 7776*b^3*c^6*g*h^2*e + 38
016*a*b*c^7*g*h^2*e + 1782*b^4*c^5*h^3*e - 10224*a*b^2*c^6*h^3*e + 10080*a^
2*c^7*h^3*e)/c^8)*x + (483840*b*c^8*d*g^3 - 9408*b^3*c^6*f*g^3 + 48384*a*b*
c^7*f*g^3 + 48384*b^2*c^7*d*g^2*h + 1548288*a*c^8*d*g^2*h + 18144*b^4*c^5*f
*g^2*h - 107136*a*b^2*c^6*f*g^2*h + 110592*a^2*c^7*f*g^2*h - 28224*b^3*c^6*
d*g*h^2 + 145152*a*b*c^7*d*g*h^2 - 12474*b^5*c^4*f*g*h^2 + 84240*a*b^3*c^5*
f*g*h^2 - 130464*a^2*b*c^6*f*g*h^2 + 6048*b^4*c^5*d*h^3 - 35712*a*b^2*c^6*d
*h^3 + 36864*a^2*c^7*d*h^3 + 3003*b^6*c^3*f*h^3 - 22968*a*b^4*c^4*f*h^3 + 4
7280*a^2*b^2*c^5*f*h^3 - 16384*a^3*c^6*f*h^3 + 16128*b^2*c^7*g^3*e + 516096
*a*c^8*g^3*e - 28224*b^3*c^6*g^2*h*e + 145152*a*b*c^7*g^2*h*e + 18144*b^4*c
^5*g*h^2*e - 107136*a*b^2*c^6*g*h^2*e + 110592*a^2*c^7*g*h^2*e - 4158*b^5*c
^4*h^3*e + 28080*a*b^3*c^5*h^3*e - 43488*a^2*b*c^6*h^3*e)/c^8)*x + (161280*
b^2*c^7*d*g^3 + 3225600*a*c^8*d*g^3 + 47040*b^4*c^5*f*g^3 - 290304*a*b^2*c^
6*f*g^3 + 322560*a^2*c^7*f*g^3 - 241920*b^3*c^6*d*g^2*h + 1354752*a*b*c^7*d
*g^2*h - 90720*b^5*c^4*f*g^2*h + 628992*a*b^3*c^5*f*g^2*h - 1009152*a^2*b*c
^6*f*g^2*h + 141120*b^4*c^5*d*g*h^2 - 870912*a*b^2*c^6*d*g*h^2 + 967680*a^2
*c^7*d*g*h^2 + 62370*b^6*c^3*f*g*h^2 - 485352*a*b^4*c^4*f*g*h^2 + 1020384*a
^2*b^2*c^5*f*g*h^2 - 362880*a^3*c^6*f*g*h^2 - 30240*b^5*c^4*d*h^3 + 209664*
a*b^3*c^5*d*h^3 - 336384*a^2*b*c^6*d*h^3 - 15015*b^7*c^2*f*h^3 + 130284*a*b
^5*c^3*f*h^3 - 338832*a^2*b^3*c^4*f*h^3 + 236864*a^3*b*c^5*f*h^3 - 80640*b^
3*c^6*g^3*e + 451584*a*b*c^7*g^3*e + 141120*b^4*c^5*g^2*h*e - 870912*a*b^2*
c^6*g^2*h*e + 967680*a^2*c^7*g^2*h*e - 90720*b^5*c^4*g*h^2*e + 628992*a*b^3
*c^5*g*h^2*e - 1009152*a^2*b*c^6*g*h^2*e + 20790*b^6*c^3*h^3*e - 161784*a*b
```

^4*c^4*h^3*e + 340128*a^2*b^2*c^5*h^3*e - 120960*a^3*c^6*h^3*e)/c^8)*x - (4
83840*b^3*c^6*d*g^3 - 3225600*a*b*c^7*d*g^3 + 141120*b^5*c^4*f*g^3 - 102144
0*a*b^3*c^5*f*g^3 + 1741824*a^2*b*c^6*f*g^3 - 725760*b^4*c^5*d*g^2*h + 4838
400*a*b^2*c^6*d*g^2*h - 6193152*a^2*c^7*d*g^2*h - 272160*b^6*c^3*f*g^2*h +
2177280*a*b^4*c^4*f*g^2*h - 4741632*a^2*b^2*c^5*f*g^2*h + 1769472*a^3*c^6*f
*g^2*h + 423360*b^5*c^4*d*g*h^2 - 3064320*a*b^3*c^5*d*g*h^2 + 5225472*a^2*b
*c^6*d*g*h^2 + 187110*b^7*c^2*f*g*h^2 - 1655640*a*b^5*c^3*f*g*h^2 + 4408992
*a^2*b^3*c^4*f*g*h^2 - 3176064*a^3*b*c^5*f*g*h^2 - 90720*b^6*c^3*d*h^3 + 72
5760*a*b^4*c^4*d*h^3 - 1580544*a^2*b^2*c^5*d*h^3 + 589824*a^3*c^6*d*h^3 - 4
5045*b^8*c*f*h^3 + 438900*a*b^6*c^2*f*h^3 - 1383984*a^2*b^4*c^3*f*h^3 + 146
7072*a^3*b^2*c^4*f*h^3 - 262144*a^4*c^5*f*h^3 - 241920*b^4*c^5*g^3*e + 1612
800*a*b^2*c^6*g^3*e - 2064384*a^2*c^7*g^3*e + 423360*b^5*c^4*g^2*h*e - 3064
320*a*b^3*c^5*g^2*h*e + 5225472*a^2*b*c^6*g^2*h*e - 272160*b^6*c^3*g*h^2*e
+ 2177280*a*b^4*c^4*g*h^2*e - 4741632*a^2*b^2*c^5*g*h^2*e + 1769472*a^3*c^6
*g*h^2*e + 62370*b^7*c^2*h^3*e - 551880*a*b^5*c^3*h^3*e + 1469664*a^2*b^3*c
^4*h^3*e - 1058688*a^3*b*c^5*h^3*e)/c^8) - 1/65536*(1536*b^4*c^5*d*g^3 - 12
288*a*b^2*c^6*d*g^3 + 24576*a^2*c^7*d*g^3 + 448*b^6*c^3*f*g^3 - 3840*a*b^4*
c^4*f*g^3 + 9216*a^2*b^2*c^5*f*g^3 - 4096*a^3*c^6*f*g^3 - 2304*b^5*c^4*d*g^
2*h + 18432*a*b^3*c^5*d*g^2*h - 36864*a^2*b*c^6*d*g^2*h - 864*b^7*c^2*f*g^2
*h + 8064*a*b^5*c^3*f*g^2*h - 23040*a^2*b^3*c^4*f*g^2*h + 18432*a^3*b*c^5*f
*g^2*h + 1344*b^6*c^3*d*g*h^2 - 11520*a*b^4*c^4*d*g*h^2 + 27648*a^2*b^2*c^5
*d*g*h^2 - 12288*a^3*c^6*d*g*h^2 + 594*b^8*c*f*g*h^2 - 6048*a*b^6*c^2*f*g*h
^2 + 20160*a^2*b^4*c^3*f*g*h^2 - 23040*a^3*b^2*c^4*f*g*h^2 + 4608*a^4*c^5*f
*g*h^2 - 288*b^7*c^2*d*h^3 + 2688*a*b^5*c^3*d*h^3 - 7680*a^2*b^3*c^4*d*h^3
+ 6144*a^3*b*c^5*d*h^3 - 143*b^9*f*h^3 + 1584*a*b^7*c*f*h^3 - 6048*a^2*b^5*
c^2*f*h^3 + 8960*a^3*b^3*c^3*f*h^3 - 3840*a^4*b*c^4*f*h^3 - 768*b^5*c^4*g^3
*e + 6144*a*b^3*c^5*g^3*e - 12288*a^2*b*c^6*g^3*e + 1344*b^6*c^3*g^2*h*e -
11520*a*b^4*c^4*g^2*h*e + 27648*a^2*b^2*c^5*g^2*h*e - 12288*a^3*c^6*g^2*h*e
 - 864*b^7*c^2*g*h^2*e + 8064*a*b^5*c^3*g*h^2*e - 23040*a^2*b^3*c^4*g*h^2*e
 + 18432*a^3*b*c^5*g*h^2*e + 198*b^8*c*h^3*e - 2016*a*b^6*c^2*h^3*e + 6720*
a^2*b^4*c^3*h^3*e - 7680*a^3*b^2*c^4*h^3*e + 1536*a^4*c^5*h^3*e)*log(abs(-2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(15/2)

3.197 $\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=753

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(dh^2+2egh+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)\right.}{6144c^5}$$

```
[Out] -((b^2 - 4*a*c)*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e
*h + 3*a*f*h) - 128*c^3*(3*b*g*(e*g + 2*d*h) + a*(f*g^2 + 2*e*g*h + d*h^2))
 + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + 2*e*g*h +
 d*h^2)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16384*c^6) + ((768*c^4*d*g^2
+ 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(3*b*g*
(e*g + 2*d*h) + a*(f*g^2 + 2*e*g*h + d*h^2)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b
*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + 2*e*g*h + d*h^2)))*(b + 2*c*x)*(a + b*x
+ c*x^2)^(3/2))/(6144*c^5) - ((10*c*f*g - 16*c*e*h + 11*b*f*h)*(g + h*x)^2*
(a + b*x + c*x^2)^(5/2))/(112*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^(5/
2))/(8*c*h) - ((693*b^3*f*h^3 + 96*c^3*g*(5*f*g^2 - 8*h*(e*g + 7*d*h)) - 36
*b*c*h^2*(31*a*f*h + 28*b*(2*f*g + e*h)) + 8*c^2*h*(96*a*h*(2*f*g + e*h) +
b*(31*f*g^2 + 196*h*(2*e*g + d*h))) - 10*c*h*(99*b^2*f*h^2 - 8*c^2*(5*f*g^2
 - 4*h*(2*e*g + 7*d*h)) - 12*c*h*(7*a*f*h + 2*b*(f*g + 6*e*h)))*x)*(a + b*x
 + c*x^2)^(5/2))/(13440*c^4*h) + ((b^2 - 4*a*c)^2*(768*c^4*d*g^2 + 99*b^4*f
*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(3*b*g*(e*g + 2*d
*h) + a*(f*g^2 + 2*e*g*h + d*h^2)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g
+ e*h) + 14*b^2*(f*g^2 + 2*e*g*h + d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(32768*c^(13/2))
```

Rubi [A] time = 2.10369, antiderivative size = 749, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)\right.}{6144c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e
*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h))
 + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g
```

```
+ d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16384*c^6) + ((768*c^4*d*g^2
+ 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^
2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b
*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)*(a + b*x
+ c*x^2)^(3/2))/(6144*c^5) - ((10*c*f*g - 16*c*e*h + 11*b*f*h)*(g + h*x)^2*
(a + b*x + c*x^2)^(5/2))/(112*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^(5/
2))/(8*c*h) - ((693*b^3*f*h^3 + 96*c^3*(5*f*g^3 - 8*g*h*(e*g + 7*d*h)) - 36
*b*c*h^2*(31*a*f*h + 28*b*(2*f*g + e*h)) + 8*c^2*h*(31*b*f*g^2 + 196*b*h*(2
*e*g + d*h) + 96*a*h*(2*f*g + e*h)) - 10*c*h*(99*b^2*f*h^2 - 8*c^2*(5*f*g^2
 - 4*h*(2*e*g + 7*d*h)) - 12*c*h*(7*a*f*h + 2*b*(f*g + 6*e*h)))*x)*(a + b*x
 + c*x^2)^(5/2))/(13440*c^4*h) + ((b^2 - 4*a*c)^2*(768*c^4*d*g^2 + 99*b^4*f
*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2
*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g
+ e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(32768*c^(13/2))
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
```

```
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^3 \left(a+bx+cx^2\right)^{5/2}}{8ch} + \frac{\int (g+hx)^2 \left(-\frac{1}{2}h(5bfg-16cdh+6af\right.}{}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2 \left(a+bx+cx^2\right)^{5/2}}{112c^2h} + \frac{f(g+hx)^3 \left(\right.}{8}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2 \left(a+bx+cx^2\right)^{5/2}}{112c^2h} + \frac{f(g+hx)^3 \left(\right.}{8}$$

$$= \frac{\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3 \left(afg^2\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right) \left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-1\right.}{}$$

$$= -\frac{\left(b^2-4ac\right) \left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-1\right.}{}$$

$$= -\frac{\left(b^2-4ac\right) \left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-1\right.}{}$$

Mathematica [A] time = 1.67799, size = 468, normalized size = 0.62

$$\frac{h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3af\right.}{12288c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (-((11*b*f*h + 2*c*(5*f*g - 8*e*h))*(g + h*x)^2*(a + x*(b + c*x))^(5/2))/(1
4*c) + f*(g + h*x)^3*(a + x*(b + c*x))^(5/2) - ((a + x*(b + c*x))^(5/2)*(69
3*b^3*f*h^3 + 8*c^2*h*(b*f*g*(31*g + 30*h*x) + 4*b*h*(98*e*g + 49*d*h + 45*
e*h*x) + 3*a*h*(64*f*g + 32*e*h + 35*f*h*x)) - 18*b*c*h^2*(62*a*f*h + b*(11
2*f*g + 56*e*h + 55*f*h*x)) + 16*c^3*(5*f*g^2*(6*g + 5*h*x) - 4*h*(2*e*g*(6
*g + 5*h*x) + 7*d*h*(12*g + 5*h*x)))))/(1680*c^3) + (h*(768*c^4*d*g^2 + 99*
b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a
*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2
*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*(2*Sqrt[c]*(b + 2*c*x)*Sqr
t[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*
c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(12288*c^(11/
```

```
2)))/(8*c*h)
```

Maple [B] time = 0.065, size = 3769, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/16*b^2/c^3*a*(c*x^2+b*x+a)^(3/2)*f*g*h-3/14*b/c^2*x*(c*x^2+b*x+a)^(5/2)*f
*g*h-3/32*b^3/c^3*(c*x^2+b*x+a)^(1/2)*x*a*e*h^2-15/64*b^3/c^(7/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*f*g*h+3/32*b^2/c^3*a^2*(c*x^2+b*x+a)
^(1/2)*f*g*h+21/256*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a*f*g*h-33/640*f*h^2*b^3/c^4*(c*x^2+b*x+a)^(5/2)-99/16384*f*h^2*b^7/c^6*(c
*x^2+b*x+a)^(1/2)-3/64*d*g^2/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d*g^2/c^(1/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d*g^2/c^(5/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4+1/8*d*g^2/c*(c*x^2+b*x+a)^(3/2)*b+
3/8*d*g^2*(c*x^2+b*x+a)^(1/2)*x*a+3/128*f*h^2*a^4/c^(5/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))+33/2048*f*h^2*b^5/c^5*(c*x^2+b*x+a)^(3/2)+99/327
68*f*h^2*b^8/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/8*f*h^2
*x^3*(c*x^2+b*x+a)^(5/2)/c+1/6*x*(c*x^2+b*x+a)^(5/2)/c*f*g^2-7/60*b/c^2*(c*
x^2+b*x+a)^(5/2)*d*h^2-7/60*b/c^2*(c*x^2+b*x+a)^(5/2)*f*g^2+2/5*(c*x^2+b*x+
a)^(5/2)/c*d*g*h-1/16*b^2/c^2*(c*x^2+b*x+a)^(3/2)*e*g^2+7/192*b^3/c^3*(c*x^
2+b*x+a)^(3/2)*d*h^2+7/192*b^3/c^3*(c*x^2+b*x+a)^(3/2)*f*g^2-7/512*b^5/c^4*
(c*x^2+b*x+a)^(1/2)*d*h^2-7/512*b^5/c^4*(c*x^2+b*x+a)^(1/2)*f*g^2+7/1024*b^
6/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+7/1024*b^6/c^(9
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-1/16*a^3/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+9/1024*b^6/c^5*(c*x^2+b*x+a)^
(1/2)*e*h^2-3/32*b^4/c^4*(c*x^2+b*x+a)^(1/2)*a*f*g*h+3/16*b/c^(5/2)*a^3*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h+1/8*b^3/c^3*(c*x^2+b*x+a)^(1
/2)*a*e*g*h+9/32*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^
2*e*g*h-15/128*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*
g*h-1/12*a/c*x*(c*x^2+b*x+a)^(3/2)*e*g*h+1/8*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*
a*d*h^2+1/8*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a*f*g^2-1/4*b/c*x*(c*x^2+b*x+a)^(
3/2)*d*g*h-3/16*b/c*(c*x^2+b*x+a)^(1/2)*x*a*e*g^2+3/32*b^3/c^2*(c*x^2+b*x+a
)^(1/2)*x*d*g*h-3/16*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a*d*g*h-3/8*b/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*g*h+3/16*b^3/c^(5/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g*h-1/16*a^3/c^(3/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2+1/6*x*(c*x^2+b*x+a)^(5/2)/c*d*h^2-2/35*
a/c^2*(c*x^2+b*x+a)^(5/2)*e*h^2+1/7*x^2*(c*x^2+b*x+a)^(5/2)/c*e*h^2+3/40*b^
2/c^3*(c*x^2+b*x+a)^(5/2)*e*h^2-3/128*b^4/c^4*(c*x^2+b*x+a)^(3/2)*e*h^2-9/2
```

048*b^7/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^2+1/8*b/c^
2*a*x*(c*x^2+b*x+a)^(3/2)*f*g*h+3/16*b/c^2*a^2*(c*x^2+b*x+a)^(1/2)*x*f*g*h-
3/8*b/c*(c*x^2+b*x+a)^(1/2)*x*a*d*g*h+1/4*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a*e
*g*h-3/16*b^3/c^3*(c*x^2+b*x+a)^(1/2)*x*a*f*g*h+3/128*b^4/c^3*(c*x^2+b*x+a)
^(1/2)*e*g^2-3/256*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e*g^2+1/4*d*g^2*x*(c*x^2+b*x+a)^(3/2)+1/5*(c*x^2+b*x+a)^(5/2)/c*e*g^2+1/16*
b/c^2*a*x*(c*x^2+b*x+a)^(3/2)*e*h^2+3/32*b/c^2*a^2*(c*x^2+b*x+a)^(1/2)*x*e*
h^2-57/512*f*h^2*b^2/c^3*a^2*(c*x^2+b*x+a)^(1/2)*x+153/2048*f*h^2*b^4/c^4*(
c*x^2+b*x+a)^(1/2)*x*a-9/128*f*h^2*b^2/c^3*a*x*(c*x^2+b*x+a)^(3/2)-1/24*a/c
^2*(c*x^2+b*x+a)^(3/2)*b*e*g*h-1/8*a^2/c*(c*x^2+b*x+a)^(1/2)*x*e*g*h-1/16*a
^2/c^2*(c*x^2+b*x+a)^(1/2)*b*e*g*h+7/48*b^2/c^2*x*(c*x^2+b*x+a)^(3/2)*e*g*h
-7/128*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x*e*g*h-3/32*b^3/c^3*x*(c*x^2+b*x+a)^(3/
2)*f*g*h+9/256*b^5/c^4*(c*x^2+b*x+a)^(1/2)*x*f*g*h+7/96*b^2/c^2*x*(c*x^2+b*
x+a)^(3/2)*d*h^2-7/30*b/c^2*(c*x^2+b*x+a)^(5/2)*e*g*h+9/64*b^2/c^(5/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*f*g^2-15/256*b^4/c^(7/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h^2-15/256*b^4/c^(7/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g^2+7/512*b^6/c^(9/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h-1/24*a/c*x*(c*x^2+b*x+a)^(3/2)*d*h^2-1/2
4*a/c*x*(c*x^2+b*x+a)^(3/2)*f*g^2-1/48*a/c^2*(c*x^2+b*x+a)^(3/2)*b*d*h^2+7/
96*b^2/c^2*x*(c*x^2+b*x+a)^(3/2)*f*g^2+1/3*x*(c*x^2+b*x+a)^(5/2)/c*e*g*h+1/
16*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a*f*g^2-7/256*b^5/c^4*(c*x^2+b*x+a)^(1/2)*e*
g*h+9/64*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*h^2-
1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*f*g^2-1/32*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*d
*h^2-1/32*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g^2-1/8*a^3/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h+7/96*b^3/c^3*(c*x^2+b*x+a)^(3/2)*e*g*h
-7/256*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x*d*h^2-7/256*b^4/c^3*(c*x^2+b*x+a)^(1/2
)*x*f*g^2+1/16*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a*d*h^2-1/48*a/c^2*(c*x^2+b*x+a)
^(3/2)*b*f*g^2+3/128*f*h^2*a^3/c^2*(c*x^2+b*x+a)^(1/2)*x+33/1024*f*h^2*b^4/
c^4*x*(c*x^2+b*x+a)^(3/2)-11/112*f*h^2*b/c^2*x^2*(c*x^2+b*x+a)^(5/2)-3/28*b
/c^2*x*(c*x^2+b*x+a)^(5/2)*e*h^2+3/20*b^2/c^3*(c*x^2+b*x+a)^(5/2)*f*g*h-3/6
4*b^4/c^4*(c*x^2+b*x+a)^(3/2)*f*g*h-3/64*b^3/c^3*x*(c*x^2+b*x+a)^(3/2)*e*h^
2+3/32*b/c^(5/2)*a^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^2-4/35
a/c^2(c*x^2+b*x+a)^(5/2)*f*g*h+2/7*x^2*(c*x^2+b*x+a)^(5/2)/c*f*g*h-1/8*b/
c*x*(c*x^2+b*x+a)^(3/2)*e*g^2-1/8*b^2/c^2*(c*x^2+b*x+a)^(3/2)*d*g*h+3/64*b^
3/c^2*(c*x^2+b*x+a)^(1/2)*x*e*g^2-3/32*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a*e*g^2+
3/64*b^4/c^3*(c*x^2+b*x+a)^(1/2)*d*g*h-3/16*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))*a^2*e*g^2+3/32*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*a*e*g^2-3/128*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))*d*g*h-3/32*d*g^2/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*d*g^2/c*(c*
x^2+b*x+a)^(1/2)*b*a-3/16*d*g^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*b^2*a-99/8192*f*h^2*b^6/c^5*(c*x^2+b*x+a)^(1/2)*x+153/4096*f*h^2*b
^5/c^5*(c*x^2+b*x+a)^(1/2)*a+33/448*f*h^2*b^2/c^3*x*(c*x^2+b*x+a)^(5/2)+105
/1024*f*h^2*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-57/
1024*f*h^2*b^3/c^4*a^2*(c*x^2+b*x+a)^(1/2)-63/2048*f*h^2*b^6/c^(11/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-15/128*f*h^2*b^2/c^(7/2)*a^3*ln((1

```
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+93/1120*f*h^2*b/c^3*a*(c*x^2+b*x+a)^
(5/2)+3/256*f*h^2*a^3/c^3*(c*x^2+b*x+a)^(1/2)*b+1/64*f*h^2*a^2/c^2*x*(c*x^2
+b*x+a)^(3/2)+1/128*f*h^2*a^2/c^3*(c*x^2+b*x+a)^(3/2)*b-1/16*f*h^2*a/c^2*x*
(c*x^2+b*x+a)^(5/2)-9/256*f*h^2*b^3/c^4*a*(c*x^2+b*x+a)^(3/2)+1/32*b^2/c^3*
a*(c*x^2+b*x+a)^(3/2)*e*h^2-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*d*h^2+3/64*b^2
/c^3*a^2*(c*x^2+b*x+a)^(1/2)*e*h^2+9/512*b^6/c^5*(c*x^2+b*x+a)^(1/2)*f*g*h+
21/512*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h^2-15/1
28*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*h^2+9/512*
b^5/c^4*(c*x^2+b*x+a)^(1/2)*x*e*h^2-3/64*b^4/c^4*(c*x^2+b*x+a)^(1/2)*a*e*h^
2-9/1024*b^7/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 13.6364, size = 7302, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [1/6881280*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 12*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e + (7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2
*c^4 - 64*a^3*c^5)*f)*g^2 - 32*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d
 - 2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e + 3*(3*b^7
*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^6*c^2
- 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 48*(3*b^7*c - 28*a*b^5*c
^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*
b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2
```

15040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + (16*c^8*e + 17*b*c^7*f)*h^2)*x^
6 + 1280*(224*c^8*f*g^2 + 32*(14*c^8*e + 15*b*c^7*f)*g*h + (224*c^8*d + 240
*b*c^7*e + 3*(b^2*c^6 + 84*a*c^7)*f)*h^2)*x^5 + 128*(224*(12*c^8*e + 13*b*c
^7*f)*g^2 + 32*(168*c^8*d + 182*b*c^7*e + 3*(b^2*c^6 + 64*a*c^7)*f)*g*h + (
2912*b*c^7*d + 48*(b^2*c^6 + 64*a*c^7)*e - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h
^2)*x^4 + 16*(224*(120*c^8*d + 132*b*c^7*e + (3*b^2*c^6 + 140*a*c^7)*f)*g^2
 + 32*(1848*b*c^7*d + 14*(3*b^2*c^6 + 140*a*c^7)*e - 3*(9*b^3*c^5 - 44*a*b*
c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d - 48*(9*b^3*c^5 - 44*a*b*c^6)*
e + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*
b^3*c^5 - 20*a*b*c^6)*d - 12*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*e +
 (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 32*(168*(15*b^4*c^
4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d - 14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296
*a^2*b*c^5)*e + 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a
^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 48
*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e + 3*(34
65*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f)*h^2 +
8*(224*(360*b*c^7*d + 12*(b^2*c^6 + 32*a*c^7)*e - (7*b^3*c^5 - 36*a*b*c^6)*
f)*g^2 + 32*(168*(b^2*c^6 + 32*a*c^7)*d - 14*(7*b^3*c^5 - 36*a*b*c^6)*e + 3
*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a
*b*c^6)*d - 48*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e + 3*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 2
0*a*c^7)*d - 12*(5*b^3*c^5 - 28*a*b*c^6)*e + (35*b^4*c^4 - 216*a*b^2*c^5 +
240*a^2*c^6)*f)*g^2 - 32*(168*(5*b^3*c^5 - 28*a*b*c^6)*d - 14*(35*b^4*c^4 -
 216*a*b^2*c^5 + 240*a^2*c^6)*e + 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2
*b*c^5)*f)*g*h + (224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 48*(10
5*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e + 3*(1155*b^6*c^2 - 8988*a*b^
4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))
/c^7, -1/3440640*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 12*(
b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e + (7*b^6*c^2 - 60*a*b^4*c^3 + 144*a
^2*b^2*c^4 - 64*a^3*c^5)*f)*g^2 - 32*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*
c^5)*d - 2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e + 3*
(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^
6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 48*(3*b^7*c - 28*a
*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e + 3*(33*b^8 - 336*a*b^6*c + 112
0*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f)*h^2)*sqrt(-c)*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2
15040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + (16*c^8*e + 17*b*c^7*f)*h^2)*x^
6 + 1280*(224*c^8*f*g^2 + 32*(14*c^8*e + 15*b*c^7*f)*g*h + (224*c^8*d + 240
*b*c^7*e + 3*(b^2*c^6 + 84*a*c^7)*f)*h^2)*x^5 + 128*(224*(12*c^8*e + 13*b*c
^7*f)*g^2 + 32*(168*c^8*d + 182*b*c^7*e + 3*(b^2*c^6 + 64*a*c^7)*f)*g*h + (
2912*b*c^7*d + 48*(b^2*c^6 + 64*a*c^7)*e - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h
^2)*x^4 + 16*(224*(120*c^8*d + 132*b*c^7*e + (3*b^2*c^6 + 140*a*c^7)*f)*g^2
 + 32*(1848*b*c^7*d + 14*(3*b^2*c^6 + 140*a*c^7)*e - 3*(9*b^3*c^5 - 44*a*b*
c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d - 48*(9*b^3*c^5 - 44*a*b*c^6)*
e + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*

```
b^3*c^5 - 20*a*b*c^6)*d - 12*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*e +
 (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 32*(168*(15*b^4*c^
4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d - 14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296
*a^2*b*c^5)*e + 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a
^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 48
*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e + 3*(34
65*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f)*h^2 +
8*(224*(360*b*c^7*d + 12*(b^2*c^6 + 32*a*c^7)*e - (7*b^3*c^5 - 36*a*b*c^6)*
f)*g^2 + 32*(168*(b^2*c^6 + 32*a*c^7)*d - 14*(7*b^3*c^5 - 36*a*b*c^6)*e + 3
*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a
*b*c^6)*d - 48*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e + 3*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 2
0*a*c^7)*d - 12*(5*b^3*c^5 - 28*a*b*c^6)*e + (35*b^4*c^4 - 216*a*b^2*c^5 +
240*a^2*c^6)*f)*g^2 - 32*(168*(5*b^3*c^5 - 28*a*b*c^6)*d - 14*(35*b^4*c^4 -
 216*a*b^2*c^5 + 240*a^2*c^6)*e + 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2
*b*c^5)*f)*g*h + (224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 48*(10
5*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e + 3*(1155*b^6*c^2 - 8988*a*b^
4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))
/c^7]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (g + hx)^2 \left(a + bx + cx^2\right)^{\frac{3}{2}} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)**2*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

Giac [B] time = 1.20415, size = 2500, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f*h^2*x + (32*c^8
*f*g*h + 17*b*c^7*f*h^2 + 16*c^8*h^2*e)/c^7)*x + (224*c^8*f*g^2 + 480*b*c^7
*f*g*h + 224*c^8*d*h^2 + 3*b^2*c^6*f*h^2 + 252*a*c^7*f*h^2 + 448*c^8*g*h*e
+ 240*b*c^7*h^2*e)/c^7)*x + (2912*b*c^7*f*g^2 + 5376*c^8*d*g*h + 96*b^2*c^6
*f*g*h + 6144*a*c^7*f*g*h + 2912*b*c^7*d*h^2 - 33*b^3*c^5*f*h^2 + 156*a*b*c
^6*f*h^2 + 2688*c^8*g^2*e + 5824*b*c^7*g*h*e + 48*b^2*c^6*h^2*e + 3072*a*c^
7*h^2*e)/c^7)*x + (26880*c^8*d*g^2 + 672*b^2*c^6*f*g^2 + 31360*a*c^7*f*g^2
+ 59136*b*c^7*d*g*h - 864*b^3*c^5*f*g*h + 4224*a*b*c^6*f*g*h + 672*b^2*c^6*
d*h^2 + 31360*a*c^7*d*h^2 + 297*b^4*c^4*f*h^2 - 1704*a*b^2*c^5*f*h^2 + 1680
*a^2*c^6*f*h^2 + 29568*b*c^7*g^2*e + 1344*b^2*c^6*g*h*e + 62720*a*c^7*g*h*e
 - 432*b^3*c^5*h^2*e + 2112*a*b*c^6*h^2*e)/c^7)*x + (80640*b*c^7*d*g^2 - 15
68*b^3*c^5*f*g^2 + 8064*a*b*c^6*f*g^2 + 5376*b^2*c^6*d*g*h + 172032*a*c^7*d
*g*h + 2016*b^4*c^4*f*g*h - 11904*a*b^2*c^5*f*g*h + 12288*a^2*c^6*f*g*h - 1
568*b^3*c^5*d*h^2 + 8064*a*b*c^6*d*h^2 - 693*b^5*c^3*f*h^2 + 4680*a*b^3*c^4
*f*h^2 - 7248*a^2*b*c^5*f*h^2 + 2688*b^2*c^6*g^2*e + 86016*a*c^7*g^2*e - 31
36*b^3*c^5*g*h*e + 16128*a*b*c^6*g*h*e + 1008*b^4*c^4*h^2*e - 5952*a*b^2*c^
5*h^2*e + 6144*a^2*c^6*h^2*e)/c^7)*x + (26880*b^2*c^6*d*g^2 + 537600*a*c^7*
d*g^2 + 7840*b^4*c^4*f*g^2 - 48384*a*b^2*c^5*f*g^2 + 53760*a^2*c^6*f*g^2 -
26880*b^3*c^5*d*g*h + 150528*a*b*c^6*d*g*h - 10080*b^5*c^3*f*g*h + 69888*a*
b^3*c^4*f*g*h - 112128*a^2*b*c^5*f*g*h + 7840*b^4*c^4*d*h^2 - 48384*a*b^2*c
^5*d*h^2 + 53760*a^2*c^6*d*h^2 + 3465*b^6*c^2*f*h^2 - 26964*a*b^4*c^3*f*h^2
 + 56688*a^2*b^2*c^4*f*h^2 - 20160*a^3*c^5*f*h^2 - 13440*b^3*c^5*g^2*e + 75
264*a*b*c^6*g^2*e + 15680*b^4*c^4*g*h*e - 96768*a*b^2*c^5*g*h*e + 107520*a^
2*c^6*g*h*e - 5040*b^5*c^3*h^2*e + 34944*a*b^3*c^4*h^2*e - 56064*a^2*b*c^5*
h^2*e)/c^7)*x - (80640*b^3*c^5*d*g^2 - 537600*a*b*c^6*d*g^2 + 23520*b^5*c^3
*f*g^2 - 170240*a*b^3*c^4*f*g^2 + 290304*a^2*b*c^5*f*g^2 - 80640*b^4*c^4*d*
g*h + 537600*a*b^2*c^5*d*g*h - 688128*a^2*c^6*d*g*h - 30240*b^6*c^2*f*g*h +
 241920*a*b^4*c^3*f*g*h - 526848*a^2*b^2*c^4*f*g*h + 196608*a^3*c^5*f*g*h +
 23520*b^5*c^3*d*h^2 - 170240*a*b^3*c^4*d*h^2 + 290304*a^2*b*c^5*d*h^2 + 10
395*b^7*c*f*h^2 - 91980*a*b^5*c^2*f*h^2 + 244944*a^2*b^3*c^3*f*h^2 - 176448
*a^3*b*c^4*f*h^2 - 40320*b^4*c^4*g^2*e + 268800*a*b^2*c^5*g^2*e - 344064*a^
2*c^6*g^2*e + 47040*b^5*c^3*g*h*e - 340480*a*b^3*c^4*g*h*e + 580608*a^2*b*c
^5*g*h*e - 15120*b^6*c^2*h^2*e + 120960*a*b^4*c^3*h^2*e - 263424*a^2*b^2*c^
4*h^2*e + 98304*a^3*c^5*h^2*e)/c^7) - 1/32768*(768*b^4*c^4*d*g^2 - 6144*a*b
^2*c^5*d*g^2 + 12288*a^2*c^6*d*g^2 + 224*b^6*c^2*f*g^2 - 1920*a*b^4*c^3*f*g
^2 + 4608*a^2*b^2*c^4*f*g^2 - 2048*a^3*c^5*f*g^2 - 768*b^5*c^3*d*g*h + 6144
*a*b^3*c^4*d*g*h - 12288*a^2*b*c^5*d*g*h - 288*b^7*c*f*g*h + 2688*a*b^5*c^2
*f*g*h - 7680*a^2*b^3*c^3*f*g*h + 6144*a^3*b*c^4*f*g*h + 224*b^6*c^2*d*h^2
- 1920*a*b^4*c^3*d*h^2 + 4608*a^2*b^2*c^4*d*h^2 - 2048*a^3*c^5*d*h^2 + 99*b
^8*f*h^2 - 1008*a*b^6*c*f*h^2 + 3360*a^2*b^4*c^2*f*h^2 - 3840*a^3*b^2*c^3*f
*h^2 + 768*a^4*c^4*f*h^2 - 384*b^5*c^3*g^2*e + 3072*a*b^3*c^4*g^2*e - 6144*
a^2*b*c^5*g^2*e + 448*b^6*c^2*g*h*e - 3840*a*b^4*c^3*g*h*e + 9216*a^2*b^2*c
^4*g*h*e - 4096*a^3*c^5*g*h*e - 144*b^7*c*h^2*e + 1344*a*b^5*c^2*h^2*e - 38
40*a^2*b^3*c^3*h^2*e + 3072*a^3*b*c^4*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

3.198 $\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=418

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(-2ch(24afh+49b(eh+fg))+63b^2fh^2-10chx(9bfh-14ceh+10cfg)-24c^2\left(5fg^2-7h(dh+eg)\right)\right.}{840c^3h}$$

[Out] -((b^2 - 4*a*c)*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(1024*c^5) + ((48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(384*c^4) + (f*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c*h) + ((63*b^2*f*h^2 - 24*c^2*(5*f*g^2 - 7*h*(e*g + d*h)) - 2*c*h*(24*a*f*h + 49*b*(f*g + e*h)) - 10*c*h*(10*c*f*g - 14*c*e*h + 9*b*f*h)*x)*(a + b*x + c*x^2)^(5/2))/(840*c^3*h) + ((b^2 - 4*a*c)^2*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2))

Rubi [A] time = 0.64543, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 621, 206}

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(-2ch(24afh+49b(eh+fg))+63b^2fh^2-10chx(9bfh-14ceh+10cfg)-24c^2\left(5fg^2-7h(dh+eg)\right)\right.}{840c^3h}$$

Antiderivative was successfully verified.

[In] Int[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]

[Out] -((b^2 - 4*a*c)*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(1024*c^5) + ((48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(384*c^4) + (f*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c*h) + ((63*b^2*f*h^2 - 24*c^2*(5*f*g^2 - 7*h*(e*g + d*h)) - 2*c*h*(24*a*f*h + 49*b*(f*g + e*h)) - 10*c*h*(10*c*f*g - 14*c*e*h + 9*b*f*h)*x)*(a + b*x + c*x^2)^(5/2))/(840*c^3*h) + ((b^2 - 4*a*c)^2*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2))

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch} + \frac{\int (g+hx)\left(-\frac{1}{2}h(5bfg-14cdh+4afh)\cdots\right.}{\cdots}$$

$$= \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch} + \frac{\left(63b^2fh^2-24c^2\left(5fg^2-7h(eg+dh)\right)\cdots\right.}{\cdots}$$

$$= \frac{\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg\cdots\right.}{384c^4}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6\cdots\right.}{1024c^5}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6\cdots\right.}{1024c^5}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6\cdots\right.}{1024c^5}$$

Mathematica [A] time = 0.806003, size = 285, normalized size = 0.68

$$\frac{\frac{(a+x(b+cx))^{5/2}\left(-2ch(24afh+b(49eh+49fg+45fhx))+63b^2fh^2-4c^2(5fg(6g+5hx)-7h(6dh+6eg+5ehx))\right)}{120c^2} - \frac{7h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bc\cdots\right.\right.}{\cdots}}{7ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^2*(a + x*(b + c*x))^(5/2) + ((a + x*(b + c*x))^(5/2)*(63*b^2*f
*h^2 - 4*c^2*(5*f*g*(6*g + 5*h*x) - 7*h*(6*e*g + 6*d*h + 5*e*h*x)) - 2*c*h*
(24*a*f*h + b*(49*f*g + 49*e*h + 45*f*h*x))))/(120*c^2) - (7*h*(-48*c^3*d*g
 + 9*b^3*f*h + 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) - 2*b*c*(6*a*f*h +
 7*b*(f*g + e*h)))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8
*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])]))/(6144*c^(9/2)))/(7*c*h)
```

Maple [B] time = 0.059, size = 2026, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)

[Out] 1/8*d*g/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d*g*(c*x^2+b*x+a)^(1/2)*x*a+1/16*h*f*b/c^2*a*x*(c*x^2+b*x+a)^(3/2)-3/16*b/c*(c*x^2+b*x+a)^(1/2)*x*a*d*h-3/16*b/c*(c*x^2+b*x+a)^(1/2)*x*a*e*g+3/32*h*f*b/c^2*a^2*(c*x^2+b*x+a)^(1/2)*x-3/32*h*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)*x*a+1/4*d*g*x*(c*x^2+b*x+a)^(3/2)+1/5*(c*x^2+b*x+a)^(5/2)/c*d*h+1/5*(c*x^2+b*x+a)^(5/2)/c*e*g-3/64*d*g/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d*g/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d*g/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4+3/128*b^4/c^3*(c*x^2+b*x+a)^(1/2)*e*g+1/8*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a*e*h+1/8*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a*f*g+7/1024*b^6/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/16*a^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h-7/512*b^5/c^4*(c*x^2+b*x+a)^(1/2)*f*g+7/192*b^3/c^3*(c*x^2+b*x+a)^(3/2)*f*g-1/16*a^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-7/512*b^5/c^4*(c*x^2+b*x+a)^(1/2)*e*h+1/6*x*(c*x^2+b*x+a)^(5/2)/c*f*g-1/16*b^2/c^2*(c*x^2+b*x+a)^(3/2)*e*g+3/128*b^4/c^3*(c*x^2+b*x+a)^(1/2)*d*h-3/256*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h-3/256*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g+1/7*h*f*x^2*(c*x^2+b*x+a)^(5/2)/c-3/128*h*f*b^4/c^4*(c*x^2+b*x+a)^(3/2)+9/1024*h*f*b^6/c^5*(c*x^2+b*x+a)^(1/2)+3/40*h*f*b^2/c^3*(c*x^2+b*x+a)^(5/2)-2/35*h*f*a/c^2*(c*x^2+b*x+a)^(5/2)-9/2048*h*f*b^7/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*b^2/c^2*(c*x^2+b*x+a)^(3/2)*d*h+7/192*b^3/c^3*(c*x^2+b*x+a)^(3/2)*e*h+1/6*x*(c*x^2+b*x+a)^(5/2)/c*e*h+7/1024*b^6/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h-7/60*b/c^2*(c*x^2+b*x+a)^(5/2)*e*h-7/60*b/c^2*(c*x^2+b*x+a)^(5/2)*f*g+3/64*h*f*b^2/c^3*a^2*(c*x^2+b*x+a)^(1/2)-3/28*h*f*b/c^2*x*(c*x^2+b*x+a)^(5/2)-3/64*h*f*b^3/c^3*x*(c*x^2+b*x+a)^(3/2)+9/512*h*f*b^5/c^4*(c*x^2+b*x+a)^(1/2)*x-3/64*h*f*b^4/c^4*(c*x^2+b*x+a)^(1/2)*a-15/256*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g-1/24*a/c*x*(c*x^2+b*x+a)^(3/2)*e*h+9/64*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*f*g+1/16*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a*f*g+9/64*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*h-1/32*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*e*h-3/16*d*g/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-3/32*d*g/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*d*g/c*(c*x^2+b*x+a)^(1/2)*b*a-3/32*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a*d*h-3/32*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a*e*g-3/16*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*h-3/16*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*g+3/32*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h+3/32*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g-1/8*b/c*x*(c*x^2+b*x+a)^(3/2)*d*h-1/8*b/c*x*(c*x^2+b*x+a)^(3/2)*e*g+3/64*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x*d*h+3/64*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x*e*g-1/32*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g-1/48*a/c^2*(c*x^2+b*x+a)^(3/2)*b*e*h-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*e*h-15/256*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)

```
)*a*e*h+1/32*h*f*b^2/c^3*a*(c*x^2+b*x+a)^(3/2)+3/32*h*f*b/c^(5/2)*a^3*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/48*a/c^2*(c*x^2+b*x+a)^(3/2)*b*f*g
+21/512*h*f*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-15/12
8*h*f*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+7/96*b^2/
c^2*x*(c*x^2+b*x+a)^(3/2)*e*h+7/96*b^2/c^2*x*(c*x^2+b*x+a)^(3/2)*f*g-1/24*a
/c*x*(c*x^2+b*x+a)^(3/2)*f*g-7/256*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x*f*g-1/16*a
^2/c*(c*x^2+b*x+a)^(1/2)*x*f*g+1/16*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a*e*h-7/256
*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x*e*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.75911, size = 4232, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/430080*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2
- 8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3
 - 64*a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(7*b
^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a*b^5
*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x
- b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(15360*c^7
*f*h*x^6 + 1280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(14*(12*
c^7*e + 13*b*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*a*c^6)*f
)*h)*x^4 + 16*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^6)*f)*g +
 (1848*b*c^6*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*
f)*h)*x^3 + 8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*b^3*c^4 - 3
6*a*b*c^5)*f)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 - 36*a*b*c^5)
*e + 3*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(3*b^
```

```
3*c^4 - 20*a*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*e + (
105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(15*b^4*c^3 - 100
*a*b^2*c^4 + 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*
c^4)*e + 3*(315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f
)*h + 2*(14*(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 - 28*a*b*c^5)*e + (
35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5*b^3*c^4 - 28*a*b*c
^5)*d - 14*(35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*e + 3*(105*b^5*c^2 -
728*a*b^3*c^3 + 1168*a^2*b*c^4)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^6, -1/215
040*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2 - 8*a*
b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*
a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(7*b^6*c -
 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a*b^5*c + 8
0*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(15360*c^7*f*h*x^6 + 1
280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(14*(12*c^7*e + 13*b
*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*a*c^6)*f)*h)*x^4 + 1
6*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^6)*f)*g + (1848*b*c^6
*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*f)*h)*x^3 +
8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*b^3*c^4 - 36*a*b*c^5)*f
)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 - 36*a*b*c^5)*e + 3*(21*b
^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(3*b^3*c^4 - 20*a
*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*e + (105*b^5*c^2
- 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(15*b^4*c^3 - 100*a*b^2*c^4 +
 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*e + 3*(
315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f)*h + 2*(14*
(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 - 28*a*b*c^5)*e + (35*b^4*c^3 -
 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5*b^3*c^4 - 28*a*b*c^5)*d - 14*(
35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*e + 3*(105*b^5*c^2 - 728*a*b^3*c^
3 + 1168*a^2*b*c^4)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (g+hx)\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

Giac [B] time = 1.22256, size = 1289, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*c*f*h*x + (14*c^7*f*g +
15*b*c^6*f*h + 14*c^7*h*e)/c^6)*x + (182*b*c^6*f*g + 168*c^7*d*h + 3*b^2*c^
5*f*h + 192*a*c^6*f*h + 168*c^7*g*e + 182*b*c^6*h*e)/c^6)*x + (1680*c^7*d*g
 + 42*b^2*c^5*f*g + 1960*a*c^6*f*g + 1848*b*c^6*d*h - 27*b^3*c^4*f*h + 132*
a*b*c^5*f*h + 1848*b*c^6*g*e + 42*b^2*c^5*h*e + 1960*a*c^6*h*e)/c^6)*x + (5
040*b*c^6*d*g - 98*b^3*c^4*f*g + 504*a*b*c^5*f*g + 168*b^2*c^5*d*h + 5376*a
*c^6*d*h + 63*b^4*c^3*f*h - 372*a*b^2*c^4*f*h + 384*a^2*c^5*f*h + 168*b^2*c
^5*g*e + 5376*a*c^6*g*e - 98*b^3*c^4*h*e + 504*a*b*c^5*h*e)/c^6)*x + (1680*
b^2*c^5*d*g + 33600*a*c^6*d*g + 490*b^4*c^3*f*g - 3024*a*b^2*c^4*f*g + 3360
*a^2*c^5*f*g - 840*b^3*c^4*d*h + 4704*a*b*c^5*d*h - 315*b^5*c^2*f*h + 2184*
a*b^3*c^3*f*h - 3504*a^2*b*c^4*f*h - 840*b^3*c^4*g*e + 4704*a*b*c^5*g*e + 4
90*b^4*c^3*h*e - 3024*a*b^2*c^4*h*e + 3360*a^2*c^5*h*e)/c^6)*x - (5040*b^3*
c^4*d*g - 33600*a*b*c^5*d*g + 1470*b^5*c^2*f*g - 10640*a*b^3*c^3*f*g + 1814
4*a^2*b*c^4*f*g - 2520*b^4*c^3*d*h + 16800*a*b^2*c^4*d*h - 21504*a^2*c^5*d*
h - 945*b^6*c*f*h + 7560*a*b^4*c^2*f*h - 16464*a^2*b^2*c^3*f*h + 6144*a^3*c
^4*f*h - 2520*b^4*c^3*g*e + 16800*a*b^2*c^4*g*e - 21504*a^2*c^5*g*e + 1470*
b^5*c^2*h*e - 10640*a*b^3*c^3*h*e + 18144*a^2*b*c^4*h*e)/c^6) - 1/2048*(48*
b^4*c^3*d*g - 384*a*b^2*c^4*d*g + 768*a^2*c^5*d*g + 14*b^6*c*f*g - 120*a*b^
4*c^2*f*g + 288*a^2*b^2*c^3*f*g - 128*a^3*c^4*f*g - 24*b^5*c^2*d*h + 192*a*
b^3*c^3*d*h - 384*a^2*b*c^4*d*h - 9*b^7*f*h + 84*a*b^5*c*f*h - 240*a^2*b^3*
c^2*f*h + 192*a^3*b*c^3*f*h - 24*b^5*c^2*g*e + 192*a*b^3*c^3*g*e - 384*a^2*
b*c^4*g*e + 14*b^6*c*h*e - 120*a*b^4*c^2*h*e + 288*a^2*b^2*c^3*h*e - 128*a^
3*c^4*h*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(
11/2)
```

3.199 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=236

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)+7b^2f\right.}{512c^4}$$

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(512*c^4) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rubi [A] time = 0.241952, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)+7b^2f\right.}{512c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```


```
[Out] -((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(512*c^4) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
```

```
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx=\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\int\left(6cd-af+\frac{1}{2}(12ce-7bf)x\right)\left(a+bx+cx^2\right)^{3/2}dx}{6c}$$

$$=\frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\left(2c(6cd-af)-\frac{1}{2}b(12c\right.}{}$$

$$=\frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}+\frac{(12ce-7bf)\left(a+b\right.}{60c^2}$$

$$=-\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24c^2d-\right.}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24c^2d-\right.}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24c^2d-\right.}{}$$

Mathematica [A] time = 0.696889, size = 392, normalized size = 1.66

$$\frac{\frac{360d\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}}-60be\left(\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{5/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (1920*d*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 3072*e*(a + x*(b + c*x))^(5/2
) + 2560*f*x*(a + x*(b + c*x))^(5/2) + (360*(b^2 - 4*a*c)*d*(-2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - 60*b*e*((16*(b + 2*c*x)*(a + x*(b +
c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2)) + (f*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*
(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c)/(15360*c)
```

Maple [B] time = 0.053, size = 862, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/16*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a-3/32*d/c*(c*x^2+b*x+a)^(1/2)*x*b^2+1/8
*f*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a-3/16*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a+7/102
4*f*b^6/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/512*f*b^5/c^4
*(c*x^2+b*x+a)^(1/2)-7/60*f*b/c^2*(c*x^2+b*x+a)^(5/2)+7/192*f*b^3/c^3*(c*x^
2+b*x+a)^(3/2)+3/8*d/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^
2-3/256*e*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*e*b^
2/c^2*(c*x^2+b*x+a)^(3/2)+3/128*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)+3/8*d*(c*x^2+
b*x+a)^(1/2)*x*a-3/64*d/c^2*(c*x^2+b*x+a)^(1/2)*b^3+1/8*d/c*(c*x^2+b*x+a)^(
3/2)*b+3/128*d/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-1/16
*f*a^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/16*d/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-1/32*f*a^2/c^2*(c*x^2+b*x+
a)^(1/2)*b-1/24*f*a/c*x*(c*x^2+b*x+a)^(3/2)-1/48*f*a/c^2*(c*x^2+b*x+a)^(3/2
)*b-1/16*f*a^2/c*(c*x^2+b*x+a)^(1/2)*x+7/96*f*b^2/c^2*x*(c*x^2+b*x+a)^(3/2)
-7/256*f*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x-3/32*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a
-3/16*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/32*e*b^
3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*e*b/c*x*(c*x^2+
b*x+a)^(3/2)+3/64*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x+3/16*d/c*(c*x^2+b*x+a)^(1
/2)*b*a+9/64*f*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-
15/256*f*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/5*e*(c
*x^2+b*x+a)^(5/2)/c+1/4*d*x*(c*x^2+b*x+a)^(3/2)+1/6*f*x*(c*x^2+b*x+a)^(5/2)
/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12967, size = 1947, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*
c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f
)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(3
60*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 -
120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c
^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 +
20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 +
 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e +
(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(12
80*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*
e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^
5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 1
2*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2
 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a
*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx + cx^2\right)^{\frac{3}{2}} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

Giac [A] time = 1.19662, size = 563, normalized size = 2.39

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{13\,bc^5f+12\,c^6e}{c^5}\right)x+\frac{120\,c^6d+3\,b^2c^4f+140\,ac^5f+132\,bc^5e}{c^5}\right)x+\frac{360\,bc^5d-}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e
)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (
360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)
*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a
^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b
*c^4*d + 105*b^5*c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e +
 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2
*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^
3*c^3*f - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

3.200 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$

Optimal. Leaf size=660

$$\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg-3ah)-3b^2h^2+16c^2g^2\right)(bfh-2ceh+2cfg)\right)+6b^2ch^3(-\right.}{}$$

```
[Out] ((3*b^4*f*h^4 + 6*b^2*c*h^3*(b*f*g - b*e*h - 2*a*f*h) + 128*c^4*g^2*(f*g^2
- h*(e*g - d*h)) - 32*c^3*h*(5*b*g - 4*a*h)*(f*g^2 - h*(e*g - d*h)) - 8*b*c
^2*h^2*(3*a*h*(f*g - e*h) - 2*b*(f*g^2 - e*g*h + d*h^2)) + 2*c*h*(8*c*h*(2*
c*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*
b^2*h^2 - 4*c*h*(2*b*g - 3*a*h)))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*h^5) -
 ((8*c*h*(b*f*g - 2*c*d*h) - (8*c*g - 3*b*h)*(2*c*f*g - 2*c*e*h + b*f*h) +
6*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*h^3)
+ (f*(a + b*x + c*x^2)^(5/2))/(5*c*h) - ((4*c*h*(2*c*g - b*h)*(8*c*h*(b*g -
 2*a*h)*(b*f*g - 2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g - 2*c*
e*h + b*f*h)) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(8*c*h*(2*c
*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b
^2*h^2 - 4*c*h*(2*b*g - 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(256*c^(7/2)*h^6) + ((c*g^2 - b*g*h + a*h^2)^(3/2)*(f*g^2 -
h*(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^6
```

Rubi [A] time = 1.82506, antiderivative size = 660, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg-3ah)-3b^2h^2+16c^2g^2\right)(bfh-2ceh+2cfg)\right)+6b^2ch^3(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((3*b^4*f*h^4 + 6*b^2*c*h^3*(b*f*g - b*e*h - 2*a*f*h) - 32*c^3*h*(5*b*g - 4
*a*h)*(f*g^2 - h*(e*g - d*h)) + 128*c^4*(f*g^4 - g^2*h*(e*g - d*h)) - 8*b*c
^2*h^2*(3*a*h*(f*g - e*h) - 2*b*(f*g^2 - e*g*h + d*h^2)) + 2*c*h*(8*c*h*(2*
c*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*
b^2*h^2 - 4*c*h*(2*b*g - 3*a*h)))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*h^5) -
```

```
((8*c*h*(b*f*g - 2*c*d*h) - (8*c*g - 3*b*h)*(2*c*f*g - 2*c*e*h + b*f*h) +
6*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*h^3)
+ (f*(a + b*x + c*x^2)^(5/2))/(5*c*h) - ((4*c*h*(2*c*g - b*h)*(8*c*h*(b*g -
 2*a*h)*(b*f*g - 2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g - 2*c*
e*h + b*f*h)) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(8*c*h*(2*c
*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b
^2*h^2 - 4*c*h*(2*b*g - 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(256*c^(7/2)*h^6) + ((c*g^2 - b*g*h + a*h^2)^(3/2)*(f*g^2 -
h*(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^6
```

<u>Rule 1653</u>

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

<u>Rule 814</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

<u>Rule 843</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch} + \frac{\int \frac{\left(-\frac{5}{2}h(bfg-2cdh)-\frac{5}{2}h(2cfg-2ceh+bfh)x\right)\left(a+bx+cx^2\right)^{3/2}}{g+hx}\,dx}{5ch^2}$$

$$= -\frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ceh+bfh}{48c^2h^3}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-dh)\right)+1\right.}{}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-dh)\right)+1\right.}{}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-dh)\right)+1\right.}{}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-dh)\right)+1\right.}{}$$

Mathematica [A] time = 2.40673, size = 635, normalized size = 0.96

$$\frac{\sqrt{c}h\sqrt{a+x(b+cx)}\left(6b^2ch^3(-2afh-beh+bf(g+hx))-16c^3h\left(ah(h(-8dh+8eg-3ehx)+fg(3hx-8g))+2b(5g-h\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (f*(a + x*(b + c*x))^(5/2))/(5*c*h) - ((a + x*(b + c*x))^(3/2)*(3*b^2*f*h^2
 + 6*b*c*h*(-(e*h) + f*(g + h*x)) - 4*c^2*(f*g*(4*g - 3*h*x) + h*(-4*e*g +
4*d*h + 3*e*h*x))))/(48*c^2*h^3) + (Sqrt[c]*h*Sqrt[a + x*(b + c*x)]*(3*b^4*
f*h^4 + 64*c^4*g*(f*g^2 + h*(-(e*g) + d*h))*(2*g - h*x) + 6*b^2*c*h^3*(-(b*
e*h) - 2*a*f*h + b*f*(g + h*x)) + 4*b*c^2*h^2*(6*a*e*h^2 - 6*a*f*h*(g + h*x
) + b*f*g*(4*g + 3*h*x) + b*h*(-4*e*g + 4*d*h - 3*e*h*x)) - 16*c^3*h*(2*b*(
f*g^2 + h*(-(e*g) + d*h))*(5*g - h*x) + a*h*(f*g*(-8*g + 3*h*x) + h*(8*e*g
- 8*d*h - 3*e*h*x)))) - (2*c*h*(2*c*g - b*h)*(8*c*h*(b*g - 2*a*h)*(b*f*g -
2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g - 2*c*e*h + b*f*h)) + (
-4*c^2*g^2 + (b^2*h^2)/2 + 2*c*h*(b*g - a*h))*(8*c*h*(2*c*g - b*h)*(b*f*g -
 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b^2*h^2 + 4*c*h*(-2
*b*g + 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 1
28*c^(7/2)*(c*g^2 + h*(-(b*g) + a*h))^(3/2)*(f*g^2 + h*(-(e*g) + d*h))*ArcT
anh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sq
rt[a + x*(b + c*x)])])/(128*c^(7/2)*h^6)
```

Maple [B] time = 0.263, size = 6715, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.201 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

Optimal. Leaf size=754

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2h^2\left(a^2fh^2-2abh(2fg-eh)+b^2\left(dh^2-2egh+3fg^2\right)\right)+8b^2ch^3(-3afh-beh+2bfg)+19\cdots\right.}{128c^{5/2}h^6}$$

```
[Out] -((3*b^3*f*h^3 + 4*b*c*h^2*(4*b*f*g - 2*b*e*h - 3*a*f*h) + 64*c^3*g*(5*f*g^
2 - h*(4*e*g - 3*d*h)) + 16*c^2*h*(4*a*h*(2*f*g - e*h) - b*(19*f*g^2 - 14*e
*g*h + 9*d*h^2)) + 2*c*h*(3*b^2*f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a*f*h)
 - 16*c^2*(5*f*g^2 - h*(4*e*g - 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(64*c^2*
h^5) - ((3*b*f*h^2*(b*g - a*h) + 8*c^2*g*(5*f*g^2 - h*(4*e*g - 3*d*h)) + c*
h*(8*a*h*(2*f*g - e*h) - b*(43*f*g^2 - 8*h*(4*e*g - 3*d*h))) + 6*c*h^2*(4*c
*e*g + b*f*g - (5*c*f*g^2)/h - 4*c*d*h - a*f*h)*x)*(a + b*x + c*x^2)^(3/2))
/(24*c*h^3*(c*g^2 - b*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c
*x^2)^(5/2))/(h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) + ((3*b^4*f*h^4 + 8*b^2*
c*h^3*(2*b*f*g - b*e*h - 3*a*f*h) + 128*c^4*g^2*(5*f*g^2 - h*(4*e*g - 3*d*h
)) + 48*c^2*h^2*(a^2*f*h^2 - 2*a*b*h*(2*f*g - e*h) + b^2*(3*f*g^2 - 2*e*g*h
 + d*h^2)) + 192*c^3*h*(a*h*(3*f*g^2 - 2*e*g*h + d*h^2) - b*g*(4*f*g^2 - 3*
e*g*h + 2*d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(128*c^(5/2)*h^6) - (Sqrt[c*g^2 - b*g*h + a*h^2]*(2*c*g*(5*f*g^2 - h*(4*e*g
 - 3*d*h)) + h*(2*a*h*(2*f*g - e*h) - b*(7*f*g^2 - 5*e*g*h + 3*d*h^2)))*Arc
Tanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a
+ b*x + c*x^2])])/(2*h^6)
```

Rubi [A] time = 2.50327, antiderivative size = 750, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2h^2\left(a^2fh^2-2abh(2fg-eh)+b^2\left(dh^2-2egh+3fg^2\right)\right)+8b^2ch^3(-3afh-beh+2bfg)+19\cdots\right.}{128c^{5/2}h^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((3*b^3*f*h^3 + 4*b*c*h^2*(4*b*f*g - 2*b*e*h - 3*a*f*h) + 64*c^3*(5*f*g^3
- g*h*(4*e*g - 3*d*h)) - 16*c^2*h*(19*b*f*g^2 - b*h*(14*e*g - 9*d*h) - 4*a*
```

```
h*(2*f*g - e*h)) + 2*c*h*(3*b^2*f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a*f*h)
 - 16*c^2*(5*f*g^2 - h*(4*e*g - 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(64*c^2*
h^5) - ((3*b*f*h*(b*g - a*h) + (8*c^2*(5*f*g^3 - g*h*(4*e*g - 3*d*h)))/h -
c*(43*b*f*g^2 - 8*b*h*(4*e*g - 3*d*h) - 8*a*h*(2*f*g - e*h)) + 6*c*h*(4*c*e
*g + b*f*g - (5*c*f*g^2)/h - 4*c*d*h - a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(
24*c*h^2*(c*g^2 - b*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x
^2)^(5/2))/(h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) + ((3*b^4*f*h^4 + 8*b^2*c*
h^3*(2*b*f*g - b*e*h - 3*a*f*h) + 128*c^4*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h))
 + 48*c^2*h^2*(a^2*f*h^2 - 2*a*b*h*(2*f*g - e*h) + b^2*(3*f*g^2 - 2*e*g*h +
 d*h^2)) + 192*c^3*h*(a*h*(3*f*g^2 - 2*e*g*h + d*h^2) - b*g*(4*f*g^2 - 3*e*
g*h + 2*d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1
28*c^(5/2)*h^6) - (Sqrt[c*g^2 - b*g*h + a*h^2]*(2*c*(5*f*g^3 - g*h*(4*e*g -
 3*d*h)) - h*(7*b*f*g^2 - b*h*(5*e*g - 3*d*h) - 2*a*h*(2*f*g - e*h)))*ArcTa
nh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a +
b*x + c*x^2])])/(2*h^6)
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
```

```
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\left(\frac{1}{2}\left(-2cdg+5beg+2afg-\frac{5bfg^2}{h}-3bdh-2aeh\right)+\left(4c\right.\right.}{g+hx}}{cg^2-bgh}$$

$$= -\frac{\left(3bfh(bg-ah)+\frac{8c^2\left(5fg^3-gh(4eg-3dh)\right)}{h}-c\left(43bfg^2-8bh(4eg-3dh)-8ah(2fg\right.\right.}{24ch^2\left(cg^2-bgh\right.}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)-16c^2h\right.}{}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)-16c^2h\right.}{}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)-16c^2h\right.}{}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)-16c^2h\right.}{}$$

Mathematica [A] time = 4.60272, size = 756, normalized size = 1.

$$-\frac{-2ch\sqrt{a+x(b+cx)}\left(h(ah-bg)+cg^2\right)\left(-4c^2h(ah(8eh-16fg+3fhx)+2b(h(9dh-14eg+ehx)+fg(19g-2hx)))+bch^2(b(-4eh+8fg+3fhx)-6afh)+\frac{3}{2}b^3fh^3+16c^3(2g-hx)\left(h(3dh-4eg)+5fg^2\right)\right)+\sqrt{c}\left(h(ah-b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -(-((f*(a + x*(b + c*x))^(5/2))/(g + h*x)) + ((5*c*f*g^2 + f*h*(-(b*g) + a*
h) + 4*c*h*(-(e*g) + d*h))*(a + x*(b + c*x))^(5/2))/((c*g^2 + h*(-(b*g) + a
*h))*(g + h*x)) + (((a + x*(b + c*x))^(3/2)*(3*b*f*h^2*(-(b*g) + a*h) + c*h
*(8*b*h*(-4*e*g + 3*d*h) + b*f*g*(43*g - 6*h*x) + 2*a*h*(-8*f*g + 4*e*h + 3
*f*h*x)) + c^2*(10*f*g^2*(-4*g + 3*h*x) + 8*h*(e*g*(4*g - 3*h*x) + 3*d*h*(-
g + h*x)))))/(6*h^2) + (-2*c*h*(c*g^2 + h*(-(b*g) + a*h))*Sqrt[a + x*(b + c
```

```
*x)]*((3*b^3*f*h^3)/2 + 16*c^3*(5*f*g^2 + h*(-4*e*g + 3*d*h))*(2*g - h*x) +
 b*c*h^2*(-6*a*f*h + b*(8*f*g - 4*e*h + 3*f*h*x)) - 4*c^2*h*(a*h*(-16*f*g +
 8*e*h + 3*f*h*x) + 2*b*(f*g*(19*g - 2*h*x) + h*(-14*e*g + 9*d*h + e*h*x)))
) + Sqrt[c]*(c*g^2 + h*(-(b*g) + a*h))*(2*c*h*(2*c*g - b*h)*(3*b^2*f*g*h +
4*a*c*h*(5*f*g - 4*e*h) - 8*b*c*(5*f*g^2 + h*(-4*e*g + 3*d*h))) + ((8*c^2*g
^2 - b^2*h^2 + 4*c*h*(-(b*g) + a*h))*(-3*b^2*f*h^2 + 4*c*h*(-4*b*f*g + 2*b*
e*h + 3*a*f*h) + 16*c^2*(5*f*g^2 + h*(-4*e*g + 3*d*h))))/2)*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 32*c^3*(c*g^2 + h*(-(b*g) + a*h))
^(3/2)*(2*c*(5*f*g^3 + g*h*(-4*e*g + 3*d*h)) + h*(-7*b*f*g^2 + b*h*(5*e*g -
 3*d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)
/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(16*c^2*h^5))/(
-(c*g^2) + h*(b*g - a*h)))/(4*c*h)
```

Maple [B] time = 0.264, size = 14734, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.202 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=824

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}-\frac{\left(4cg\left(-\frac{10fg^2}{h}+6eg-3dh\right)-4ah(7fg-3eh)+b\left(31fg^2-3h(5eg-dh)\right)+2h\left(\right.}{12h^2\left(cg^2-bhg+ah^2\right)(g}$$

```
[Out] -((b^2*f*h^3*(b*g - a*h) - 8*c^3*g^2*(10*f*g^2 - 3*h*(2*e*g - d*h)) - 2*c^2
*h*(2*a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2) - 3*b*g*(22*f*g^2 - 12*e*g*h + 5*d
*h^2)) - c*h^2*(8*a^2*f*h^2 - 18*a*b*h*(3*f*g - e*h) + b^2*(53*f*g^2 - 6*h*
(4*e*g - d*h))) + 2*c*h*(b*f*h^2*(b*g - a*h) + 2*c^2*g*(10*f*g^2 - 3*h*(2*e
*g - d*h)) + c*h*(2*a*h*(7*f*g - 3*e*h) - 3*b*(6*f*g^2 - 3*e*g*h + d*h^2)))
*x)*Sqrt[a + b*x + c*x^2])/(8*c*h^5*(c*g^2 - b*g*h + a*h^2)) - ((4*c*g*(6*e
*g - (10*f*g^2)/h - 3*d*h) - 4*a*h*(7*f*g - 3*e*h) + b*(31*f*g^2 - 3*h*(5*e
*g - d*h)) + 2*h*(3*c*e*g + 2*b*f*g - (5*c*f*g^2)/h - 3*c*d*h - 2*a*f*h)*x)
*(a + b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*
g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*
(g + h*x)^2) - ((b^3*f*h^3 + 6*b*c*h^2*(3*b*f*g - b*e*h - 2*a*f*h) + 16*c^3
*g*(10*f*g^2 - 3*h*(2*e*g - d*h)) + 24*c^2*h*(a*h*(3*f*g - e*h) - b*(6*f*g^
2 - 3*e*g*h + d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(16*c^(3/2)*h^6) + ((8*c^2*g^2*(10*f*g^2 - 3*h*(2*e*g - d*h)) + 4*c*h*(
a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2) - b*g*(28*f*g^2 - 15*e*g*h + 6*d*h^2)) +
 h^2*(8*a^2*f*h^2 - 4*a*b*h*(10*f*g - 3*e*h) + b^2*(35*f*g^2 - 3*h*(5*e*g -
 d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*
h^2]*Sqrt[a + b*x + c*x^2])])/(8*h^6*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rubi [A] time = 2.14107, antiderivative size = 819, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 812, 814, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}-\frac{\left(31bfg^2+4c\left(-\frac{10fg^2}{h}+6eg-3dh\right)g-3bh(5eg-dh)-4ah(7fg-3eh)+2h\left(\right.}{12h^2\left(cg^2-bhg+ah^2\right)(g}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -((b^2*f*h^2*(b*g - a*h) + 8*c^3*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) - 2*c^2
*(2*a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2) - 3*b*g*(22*f*g^2 - 12*e*g*h + 5*d*h
```

```
^2)) - c*h*(8*a^2*f*h^2 - 18*a*b*h*(3*f*g - e*h) + b^2*(53*f*g^2 - 6*h*(4*e
*g - d*h))) + 2*c*(b*f*h^2*(b*g - a*h) + 2*c^2*(10*f*g^3 - 3*g*h*(2*e*g - d
*h)) + c*h*(2*a*h*(7*f*g - 3*e*h) - 3*b*(6*f*g^2 - 3*e*g*h + d*h^2))))*x)*Sq
rt[a + b*x + c*x^2])/(8*c*h^4*(c*g^2 - b*g*h + a*h^2)) - ((31*b*f*g^2 + 4*c
*g*(6*e*g - (10*f*g^2)/h - 3*d*h) - 3*b*h*(5*e*g - d*h) - 4*a*h*(7*f*g - 3*
e*h) + 2*h*(3*c*e*g + 2*b*f*g - (5*c*f*g^2)/h - 3*c*d*h - 2*a*f*h)*x)*(a +
b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2 -
h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g + h
*x)^2) - ((b^3*f*h^3 + 6*b*c*h^2*(3*b*f*g - b*e*h - 2*a*f*h) + 16*c^3*(10*f
*g^3 - 3*g*h*(2*e*g - d*h)) - 24*c^2*h*(6*b*f*g^2 - b*h*(3*e*g - d*h) - a*h
*(3*f*g - e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1
6*c^(3/2)*h^6) + ((8*c^2*(10*f*g^4 - 3*g^2*h*(2*e*g - d*h)) - 4*c*h*(28*b*f
*g^3 - 3*b*g*h*(5*e*g - 2*d*h) - a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2)) + h^2*
(8*a^2*f*h^2 - 4*a*b*h*(10*f*g - 3*e*h) + b^2*(35*f*g^2 - 3*h*(5*e*g - d*h)
)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*
Sqrt[a + b*x + c*x^2])])/(8*h^6*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(\frac{1}{2}\left(-4cdg+5beg+4afg-\frac{5bfg^2}{h}-bdh-4aeh\right)+\left(3ceg \ldots\right.\right.}{(g+hx \ldots}}{2\left(cg^2-bg \ldots\right.}$$

$$= -\frac{\left(31bfg^2+4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-3bh(5eg-dh)-4ah(7fg-3eh)+2h\left(3c \ldots\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+ \ldots}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9egh+3dh^2 \ldots\right.\right.\right.}{}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9egh+3dh^2 \ldots\right.\right.\right.}{}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9egh+3dh^2 \ldots\right.\right.\right.}{}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9egh+3dh^2 \ldots\right.\right.\right.}{}$$

Mathematica [B] time = 6.2934, size = 4162, normalized size = 5.05

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(3*c*h*(g + h*x)^2) - ((a + x
*(b + c*x))^(3/2)*(-(((h*(5*b*f*g - 6*c*d*h - 4*a*f*h))/2 - (g*(10*c*f*g -
6*c*e*h + b*f*h))/2)*(a + b*x + c*x^2)^(5/2))/(2*(c*g^2 - b*g*h + a*h^2)*(g
 + h*x)^2) - (((-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h))
+ (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2)*
(a + b*x + c*x^2)^(5/2))/((-(c*g^2) + b*g*h - a*h^2)*(g + h*x)) + (((4*c*(4
*c*g - (3*b*h)/2)*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h
)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/
2) + 4*c*h*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) -
(3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 +
 (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2
```

+ (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*
(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2) - 12*
c^2*h*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*
(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2)*x)*(a + b
*x + c*x^2)^(3/2))/(12*c*h^2) - (((2*c*h*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (
3*b*h)/2))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3
*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*
c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d
*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h
)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*
h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) -
(3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2
)) - (2*c*g - (b*h)/2)*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*
a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*
(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(
2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h))
- (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2
+ (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/
2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*
h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) +
c*h*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*a*h))*(-3*c*g*(5*c*
f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5
*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(2*c*g - b*h)*(-3*a*
c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f*g
^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*f*
g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(5*
c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*h*
(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))*x)*Sqrt[a + b*x + c*x
^2])/(2*c*h^2) - (((2*c*h*(2*c*g - b*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (3
*b*h)/2))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*
c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c
h(b*g - 2*a*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*
h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)
))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h
)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (
3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)
) + (-4*c^2*g^2 + (b^2*h^2)/2 - c*h*(-2*b*g + 2*a*h))*(-4*c*(-8*c^2*g^2 + (
3*b^2*h^2)/2 - c*h*(-4*b*g + 6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h)
- 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g
- a*f*g + a*e*h)))/2) + 4*c*h*(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b
*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) +
4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) +
4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*
h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*
g - a*f*g + a*e*h)))/2))/2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +

```
 c*x^2])])/(Sqrt[c]*h) - (4*Sqrt[c*g^2 - b*g*h + a*h^2]*(-(g*(2*c*h*(2*c*g
- b*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (3*b*h)/2))*(-3*c*g*(5*c*f*g^2 - 2*
f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h
) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c*f
*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(
5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*
(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 -
2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d
*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) + (-4*c^2*g^2 + (b^2*h^2)/2 - c
*h*(-2*b*g + 2*a*h))*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*a*
h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5
*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(2*
c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) -
(3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 +
 (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2
+ (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*
(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)))) +
h*(2*c*h*(b*g - 2*a*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (3*b*h)/2))*(-3*c*g
*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 -
b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(b*g - 2*a*h)*
(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5
*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(
5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c
*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2
- b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) + (2*a*c*g*h +
b*g*(-2*c*g + (b*h)/2))*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6
*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h
*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*
(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h))
 - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/
2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))
/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c
*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))))
*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*
Sqrt[a + b*x + c*x^2])])/(h*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(4*c*h^2))/(8*c
*h^2))/(-(c*g^2) + b*g*h - a*h^2))/(2*(c*g^2 - b*g*h + a*h^2))))/(3*c*h*(a
+ b*x + c*x^2)^(3/2))
```

Maple [B] time = 0.265, size = 26596, normalized size = 32.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.203 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=833

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}-\frac{\left(2cg\left(-\frac{10fg^2}{h}+4eg-dh\right)-6ah(3fg-eh)+b\left(17fg^2-h(5eg+dh)\right)+2h\left(-\frac{5c}{}\right.\right.}{12h^2\left(cg^2-bhg+ah^2\right)(g+\right.}$$

```
[Out] -((8*c^2*g^2*(10*f*g^2 - h*(4*e*g - d*h)) - 2*c*h*(3*b*g*(18*f*g^2 - 6*e*g*
h + d*h^2) - 2*a*h*(23*f*g^2 - 8*e*g*h + 2*d*h^2)) + h^2*(12*a^2*f*h^2 - 6*
a*b*h*(7*f*g - e*h) + b^2*(29*f*g^2 - h*(5*e*g + d*h))) + 2*h*(3*b*f*h^2*(b
*g - a*h) + 2*c^2*g*(10*f*g^2 - h*(4*e*g - d*h)) + c*h*(6*a*h*(3*f*g - e*h)
 - b*(22*f*g^2 - 7*e*g*h + d*h^2)))*x)*Sqrt[a + b*x + c*x^2])/(8*h^5*(c*g^2
 - b*g*h + a*h^2)*(g + h*x)) - ((2*c*g*(4*e*g - (10*f*g^2)/h - d*h) - 6*a*h
*(3*f*g - e*h) + b*(17*f*g^2 - h*(5*e*g + d*h)) + 2*h*(2*c*e*g + 3*b*f*g -
(5*c*f*g^2)/h - 2*c*d*h - 3*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(12*h^2*(c*g
^2 - b*g*h + a*h^2)*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^
2)^(5/2))/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + ((3*b^2*f*h^2 - 12*c*
h*(4*b*f*g - b*e*h - a*f*h) + 8*c^2*(10*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*h^6) - ((16*c^3*g
^3*(10*f*g^2 - h*(4*e*g - d*h)) - b*h^3*(24*a^2*f*h^2 - 6*a*b*h*(10*f*g - e
*h) + b^2*(35*f*g^2 - 5*e*g*h - d*h^2)) + 6*c*h^2*(4*a^2*h^2*(4*f*g - e*h)
+ b^2*g*(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*g^2 - 7*e*g*h + d*h^2
)) - 24*c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*h*(11*f*g^2 - 4*e*g*h
 + d*h^2)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
 a*h^2]*Sqrt[a + b*x + c*x^2])])/(16*h^6*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rubi [A] time = 2.26438, antiderivative size = 829, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 812, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}-\frac{\left(17bfg^2+2c\left(-\frac{10fg^2}{h}+4eg-dh\right)g-bh(5eg+dh)-6ah(3fg-eh)+2h\left(-\frac{5cf}{h}\right.\right.}{12h^2\left(cg^2-bhg+ah^2\right)(g+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -((12*a^2*f*h^3 - 8*c^2*g^2*(4*e*g - (10*f*g^2)/h - d*h) - 6*a*b*h^2*(7*f*g
 - e*h) + 4*a*c*h*(23*f*g^2 - 2*h*(4*e*g - d*h)) - 6*b*c*g*(18*f*g^2 - h*(6
*e*g - d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + d*h)) + 2*(3*b*f*h^2*(b*g - a*h
) + 2*c^2*(10*f*g^3 - g*h*(4*e*g - d*h)) - c*h*(22*b*f*g^2 - b*h*(7*e*g - d
*h) - 6*a*h*(3*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(8*h^4*(c*g^2 - b*g*h
 + a*h^2)*(g + h*x)) - ((17*b*f*g^2 + 2*c*g*(4*e*g - (10*f*g^2)/h - d*h) -
b*h*(5*e*g + d*h) - 6*a*h*(3*f*g - e*h) + 2*h*(2*c*e*g + 3*b*f*g - (5*c*f*g
^2)/h - 2*c*d*h - 3*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b*g
*h + a*h^2)*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2)
)/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + ((3*b^2*f*h^2 - 12*c*h*(4*b*f
*g - b*e*h - a*f*h) + 8*c^2*(10*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*h^6) - ((16*c^3*(10*f*g^5
 - g^3*h*(4*e*g - d*h)) - b*h^3*(24*a^2*f*h^2 - 6*a*b*h*(10*f*g - e*h) + b^
2*(35*f*g^2 - 5*e*g*h - d*h^2)) + 6*c*h^2*(4*a^2*h^2*(4*f*g - e*h) + b^2*g*
(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*g^2 - 7*e*g*h + d*h^2)) - 24*
c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*h*(11*f*g^2 - 4*e*g*h + d*h^2
)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*
Sqrt[a + b*x + c*x^2])])/(16*h^6*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(\frac{1}{2}\left(-6cdg+5beg+6afg-\frac{5bfg^2}{h}+bdh-6aeh\right)+\left(2ceg\right.\right.}{(g+hx}}{3\left(cg^2-bg\right.}$$

$$= -\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-eh)+2h\left(2ceg\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+h}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2-2h(4\right.\right.}{}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2-2h(4\right.\right.}{}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2-2h(4\right.\right.}{}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2-2h(4\right.\right.}{}$$

Mathematica [B] time = 6.50128, size = 7806, normalized size = 9.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.299, size = 40092, normalized size = 48.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 25.7356, size = 9881, normalized size = 11.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*c*f*x/h^4 - (16*c^2*f*g*h^10 - 5*b*c*f*h^11 -
4*c^2*h^11*e)/(c*h^15)) - 1/8*(160*c^3*f*g^5 - 336*b*c^2*f*g^4*h + 16*c^3*d
*g^3*h^2 + 210*b^2*c*f*g^3*h^2 + 264*a*c^2*f*g^3*h^2 - 24*b*c^2*d*g^2*h^3 -
 35*b^3*f*g^2*h^3 - 300*a*b*c*f*g^2*h^3 + 6*b^2*c*d*g*h^4 + 24*a*c^2*d*g*h^
4 + 60*a*b^2*f*g*h^4 + 96*a^2*c*f*g*h^4 + b^3*d*h^5 - 12*a*b*c*d*h^5 - 24*a
^2*b*f*h^5 - 64*c^3*g^4*h*e + 120*b*c^2*g^3*h^2*e - 60*b^2*c*g^2*h^3*e - 96
*a*c^2*g^2*h^3*e + 5*b^3*g*h^4*e + 84*a*b*c*g*h^4*e - 6*a*b^2*h^5*e - 24*a^
2*c*h^5*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt
(-c*g^2 + b*g*h - a*h^2))/((c*g^2*h^6 - b*g*h^7 + a*h^8)*sqrt(-c*g^2 + b*g*
h - a*h^2)) - 1/24*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*f*g^5
*h^2 - 912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*f*g^4*h^3 + 144*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*d*g^3*h^4 + 522*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*f*g^3*h^4 + 552*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*a*c^(5/2)*f*g^3*h^4 - 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*b*c^(5/2)*d*g^2*h^5 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqr
t(c)*f*g^2*h^5 - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*f*g^
2*h^5 + 78*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*d*g*h^6 + 120*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^(5/2)*d*g*h^6 + 108*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*a*b^2*sqrt(c)*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a^2*c^(3/2)*f*g*h^6 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*b^3*sqrt(c)*d*h^7 - 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*
d*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*sqrt(c)*f*h^7 - 288*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*g^4*h^3*e + 504*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*g^3*h^4*e - 252*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^5*b^2*c^(3/2)*g^2*h^5*e - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*a*c^(5/2)*g^2*h^5*e + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqr
t(c)*g*h^6*e + 228*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*g*h^6*
e - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*sqrt(c)*h^7*e - 24*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^(3/2)*h^7*e + 1680*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*c^4*f*g^6*h - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*b*c^3*f*g^5*h^2 + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*d*g^4*h^
3 + 1362*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*f*g^4*h^3 + 1464*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*f*g^4*h^3 - 504*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*b*c^3*d*g^3*h^4 - 147*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^4*b^3*c*f*g^3*h^4 - 876*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*f*
```

g^3*h^4 + 54*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*d*g^2*h^5 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*d*g^2*h^5 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*f*g^2*h^5 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*f*g^2*h^5 + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*d*g*h^6 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*d*g*h^6 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c*f*g*h^6 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*d*h^7 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*d*h^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c*f*h^7 - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*g^5*h^2*e + 1464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^3*g^4*h^3*e - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*g^3*h^4*e - 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*g^3*h^4*e + 21*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*g^2*h^5*e + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*g^2*h^5*e + 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*g*h^6*e + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*g*h^6*e - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c*h^7*e + 1504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*f*g^7 - 1072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*f*g^6*h + 352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*d*g^5*h^2 - 1308*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*f*g^5*h^2 - 656*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*f*g^5*h^2 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*d*g^4*h^3 + 1042*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*f*g^4*h^3 + 4056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*f*g^4*h^3 - 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*d*g^3*h^4 - 272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*d*g^3*h^4 - 136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*f*g^3*h^4 - 2712*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*f*g^3*h^4 - 2208*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*f*g^3*h^4 + 106*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*d*g^2*h^5 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*d*g^2*h^5 + 328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*f*g^2*h^5 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^(3/2)*f*g^2*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*d*g*h^6 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*d*g*h^6 - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*d*g*h^6 - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*sqrt(c)*f*g*h^6 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^(3/2)*f*g*h^6 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*d*h^7 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*sqrt(c)*f*h^7 - 832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*g^6*h*e + 400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*g^5*h^2*e + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*g^4*h^3*e + 512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*g^4*h^3*e - 478*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*g^3*h^4*e - 2232*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*g^3*h^4*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*g^2*h^5*e + 1092*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*g^2*h^5*e + 1104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*g^2*h^5*e - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*g*h^6*e - 576*(sqrt(c)*x - sqrt(c *x^2 + b*x + a))^3*a^2*b*c^(3/2)*g*h^6*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*sqrt(c)*h^7*e + 2256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^4*f*g^7 - 3420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^3*f*g^6*h - 2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^4*f*g^6*h + 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^4*d*g^5*h^2 + 1218*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^2*f*g^5*h^2 + 5976*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^3*f*g^5*h^2 - 516*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^3*d*g^4*h^3 - 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^4*d*g^4*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c*f*g^4*h^3 - 1944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^2*f*g^4*h^3 - 2208*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^3*f*g^4*h^3 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^2*d*g^3*h^4 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^3*d*g^3*h^4 - 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*f*g^3*h^4 + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^2*f*g^3*h^4 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c*d*g^2*h^5 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^2*d*g^2*h^5 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^3*d*g^2*h^5 + 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c*f*g^2*h^5 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^2*f*g^2*h^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*d*g*h^6 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^2*d*g*h^6 - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^2*d*h^7 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c*f*h^7 - 1248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^4*g^6*h*e + 1656*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^3*g^5*h^2*e + 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^4*g^5*h^2*e - 414*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^2*g^4*h^3*e - 2760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^3*g^4*h^3*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c*g^3*h^4*e + 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^2*g^3*h^4*e + 912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^3*g^3*h^4*e + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*g^2*h^5*e + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^2*g^2*h^5*e - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c*g*h^6*e - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^2*g*h^6*e + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c*h^7*e + 1128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^(7/2)*f*g^7 - 1776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^(5/2)*f*g^6*h - 2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(7/2)*f*g^6*h + 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^(7/2)*d*g^5*h^2 + 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c^(3/2)*f*g^5*h^2 + 5580*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^(5/2)*f*g^5*h^2 + 1776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^(7/2)*f*g^5*h^2 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^(5/2)*d*g^4*h^3 - 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(7/2)*d*g^4*h^3 - 57*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*f*g^4*h^3 - 2514*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*f*g^4*h^3 - 5688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^(5/2)*f*g^4*h^3 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c^(3/2)*d*g^3*h^4 + 852*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^(5/2)*d*g^3*h^4 + 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2

*c^(7/2)*d*g^3*h^4 + 198*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*sqrt(c)*
f*g^3*h^4 + 3078*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c^(3/2)*f*g^3*
h^4 + 1848*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^(5/2)*f*g^3*h^4 + 3*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*d*g^2*h^5 - 90*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*d*g^2*h^5 - 864*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^2*b*c^(5/2)*d*g^2*h^5 - 249*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^2*b^3*sqrt(c)*f*g^2*h^5 - 1476*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^
3*b*c^(3/2)*f*g^2*h^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*sqrt(c)
*d*g*h^6 + 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c^(3/2)*d*g*h^6 +
 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^(5/2)*d*g*h^6 + 132*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*sqrt(c)*f*g*h^6 + 192*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^4*c^(3/2)*f*g*h^6 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^2*b^3*sqrt(c)*d*h^7 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^
(3/2)*d*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*sqrt(c)*f*h^7 -
624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^(7/2)*g^6*h*e + 876*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*b^3*c^(5/2)*g^5*h^2*e + 1536*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b*c^(7/2)*g^5*h^2*e - 282*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b^4*c^(3/2)*g^4*h^3*e - 2664*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^
2*c^(5/2)*g^4*h^3*e - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^(7/2)*g
^4*h^3*e + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*g^3*h^4*e + 8
94*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*g^3*h^4*e + 2640*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^(5/2)*g^3*h^4*e - 48*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b^4*sqrt(c)*g^2*h^5*e - 936*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^2*b^2*c^(3/2)*g^2*h^5*e - 816*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^3*c^(5/2)*g^2*h^5*e + 51*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3
*sqrt(c)*g*h^6*e + 300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^(3/2)*g*
h^6*e - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*sqrt(c)*h^7*e + 24*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c^(3/2)*h^7*e + 188*b^3*c^3*f*g^7 -
272*b^4*c^2*f*g^6*h - 708*a*b^2*c^3*f*g^6*h + 44*b^3*c^3*d*g^5*h^2 + 87*b^5
*c*f*g^5*h^2 + 1214*a*b^3*c^2*f*g^5*h^2 + 888*a^2*b*c^3*f*g^5*h^2 - 44*b^4*
c^2*d*g^4*h^3 - 156*a*b^2*c^3*d*g^4*h^3 - 426*a*b^4*c*f*g^4*h^3 - 2010*a^2*
b^2*c^2*f*g^4*h^3 - 376*a^3*c^3*f*g^4*h^3 + 3*b^5*c*d*g^3*h^4 + 182*a*b^3*c
^2*d*g^3*h^4 + 192*a^2*b*c^3*d*g^3*h^4 + 807*a^2*b^3*c*f*g^3*h^4 + 1468*a^3
*b*c^2*f*g^3*h^4 - 6*a*b^4*c*d*g^2*h^5 - 294*a^2*b^2*c^2*d*g^2*h^5 - 88*a^3
*c^3*d*g^2*h^5 - 732*a^3*b^2*c*f*g^2*h^5 - 400*a^4*c^2*f*g^2*h^5 + 3*a^2*b^
3*c*d*g*h^6 + 220*a^3*b*c^2*d*g*h^6 + 312*a^4*b*c*f*g*h^6 - 64*a^4*c^2*d*h^
7 - 48*a^5*c*f*h^7 - 104*b^3*c^3*g^6*h*e + 134*b^4*c^2*g^5*h^2*e + 384*a*b^
2*c^3*g^5*h^2*e - 33*b^5*c*g^4*h^3*e - 578*a*b^3*c^2*g^4*h^3*e - 480*a^2*b*
c^3*g^4*h^3*e + 144*a*b^4*c*g^3*h^4*e + 936*a^2*b^2*c^2*g^3*h^4*e + 208*a^3
*c^3*g^3*h^4*e - 237*a^2*b^3*c*g^2*h^5*e - 676*a^3*b*c^2*g^2*h^5*e + 174*a^
3*b^2*c*g*h^6*e + 184*a^4*c^2*g*h^6*e - 48*a^4*b*c*h^7*e)/((c^(3/2)*g^2*h^6
 - b*sqrt(c)*g*h^7 + a*sqrt(c)*h^8)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^3) - 1/8*(
80*c^2*f*g^2 - 48*b*c*f*g*h + 8*c^2*d*h^2 + 3*b^2*f*h^2 + 12*a*c*f*h^2 - 32
*c^2*g*h*e + 12*b*c*h^2*e)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sq

```
rt(c) + b))/(sqrt(c)*h^6)
```

3.204 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

Optimal. Leaf size=1097

result too large to display

```
[Out] ((64*c^3*g^4*(5*f*g - e*h) - 16*c^2*g^2*h*(b*g*(41*f*g - 7*e*h) - 8*a*h*(5*
f*g - e*h)) + 4*c*h^2*(2*b^2*g^2*(46*f*g - 5*e*h) + 16*a^2*h^2*(5*f*g - e*h
) - a*b*h*(173*f*g^2 - 25*e*g*h - 3*d*h^2)) - b*h^3*(48*a^2*f*h^2 - 8*a*b*h
*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)) + 2*c*h*(16*c^2*g^3*(
5*f*g - e*h) - 4*c*h*(6*b*g^2*(6*f*g - e*h) - a*h*(35*f*g^2 - h*(7*e*g - 3*
d*h))) + h^2*(48*a^2*f*h^2 - 8*a*b*h*(14*f*g - e*h) + b^2*(61*f*g^2 - h*(5*
e*g + 3*d*h))))*x)*Sqrt[a + b*x + c*x^2])/(64*h^5*(c*g^2 - b*g*h + a*h^2)^2
*(g + h*x)) - ((16*c^2*g^4*(5*f*g - e*h) - h^2*(16*a^2*h^2*(f*g - 2*e*h) -
b^2*g*(35*f*g^2 + 5*e*g*h + 3*d*h^2) + 4*a*b*h*(7*f*g^2 + 7*e*g*h + 3*d*h^2
)) - 4*c*g*h*(b*g*(31*f*g^2 - 5*e*g*h + 3*d*h^2) - a*h*(25*f*g^2 - 5*e*g*h
+ 9*d*h^2)) + 3*h*(8*c^2*g^2*(5*f*g^2 - h*(e*g + d*h)) + h^2*(16*a^2*f*h^2
- 8*a*b*h*(6*f*g - e*h) + b^2*(29*f*g^2 - 5*e*g*h - 3*d*h^2)) - 4*c*h*(2*b*
g*(9*f*g^2 - 2*e*g*h - d*h^2) - a*h*(17*f*g^2 - 5*e*g*h + d*h^2)))*x)*(a +
b*x + c*x^2)^(3/2))/(96*h^3*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((f*g^
2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(4*h*(c*g^2 - b*g*h + a*h^2)*(g
 + h*x)^4) - (Sqrt[c]*(10*c*f*g - 2*c*e*h - 3*b*f*h)*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*h^6) + ((128*c^4*g^5*(5*f*g - e*h) - 6
4*c^3*g^3*h*(b*g*(28*f*g - 5*e*h) - 5*a*h*(5*f*g - e*h)) + 8*c*h^3*(24*a^3*
f*h^3 - 12*a^2*b*h^2*(10*f*g - e*h) - 5*b^3*g^2*(14*f*g - e*h) + 3*a*b^2*h*
(55*f*g^2 - 5*e*g*h - d*h^2)) - 48*c^2*h^2*(10*a*b*g^2*h*(6*f*g - e*h) - 5*
b^2*g^3*(7*f*g - e*h) - a^2*h^2*(25*f*g^2 - 5*e*g*h + d*h^2)) + b^2*h^4*(48
*a^2*f*h^2 - 8*a*b*h*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)))*
ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt
[a + b*x + c*x^2])])/(128*h^6*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rubi [A] time = 3.12273, antiderivative size = 1096, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 810, 812, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}-\frac{\left(\frac{16c^2(5fg-eh)g^4}{h}-4c\left(bg\left(31fg^2-5ehg+3dh^2\right)-ah\left(25fg^2-5ehg+9dh^2\right)\right)g-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (((64*c^3*g^4*(5*f*g - e*h))/h - 16*c^2*g^2*(b*g*(41*f*g - 7*e*h) - 8*a*h*(
5*f*g - e*h)) + 4*c*h*(2*b^2*g^2*(46*f*g - 5*e*h) + 16*a^2*h^2*(5*f*g - e*h
) - a*b*h*(173*f*g^2 - 25*e*g*h - 3*d*h^2)) - b*h^2*(48*a^2*f*h^2 - 8*a*b*h
*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)) + 2*c*(16*c^2*g^3*(5*
f*g - e*h) - 4*c*h*(6*b*g^2*(6*f*g - e*h) - a*h*(35*f*g^2 - h*(7*e*g - 3*d*
h))) + h^2*(48*a^2*f*h^2 - 8*a*b*h*(14*f*g - e*h) + b^2*(61*f*g^2 - h*(5*e*
g + 3*d*h))))*x)*Sqrt[a + b*x + c*x^2])/(64*h^4*(c*g^2 - b*g*h + a*h^2)^2*(
g + h*x)) - (((16*c^2*g^4*(5*f*g - e*h))/h - h*(16*a^2*h^2*(f*g - 2*e*h) -
b^2*g*(35*f*g^2 + 5*e*g*h + 3*d*h^2) + 4*a*b*h*(7*f*g^2 + 7*e*g*h + 3*d*h^2
)) - 4*c*g*(b*g*(31*f*g^2 - 5*e*g*h + 3*d*h^2) - a*h*(25*f*g^2 - 5*e*g*h +
9*d*h^2)) + 3*h*((40*c^2*f*g^4)/h + 16*a^2*f*h^3 - 8*c^2*g^2*(e*g + d*h) -
8*a*b*h^2*(6*f*g - e*h) + 4*a*c*h*(17*f*g^2 - h*(5*e*g - d*h)) - 8*b*c*g*(9
*f*g^2 - h*(2*e*g + d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + 3*d*h)))*x)*(a + b
*x + c*x^2)^(3/2))/(96*h^2*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((f*g^2
 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(4*h*(c*g^2 - b*g*h + a*h^2)*(g
+ h*x)^4) - (Sqrt[c]*(10*c*f*g - 2*c*e*h - 3*b*f*h)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*h^6) + ((128*c^4*g^5*(5*f*g - e*h) - 64
*c^3*g^3*h*(b*g*(28*f*g - 5*e*h) - 5*a*h*(5*f*g - e*h)) + 8*c*h^3*(24*a^3*f
*h^3 - 12*a^2*b*h^2*(10*f*g - e*h) - 5*b^3*g^2*(14*f*g - e*h) + 3*a*b^2*h*(
55*f*g^2 - 5*e*g*h - d*h^2)) - 48*c^2*h^2*(10*a*b*g^2*h*(6*f*g - e*h) - 5*b
^2*g^3*(7*f*g - e*h) - a^2*h^2*(25*f*g^2 - 5*e*g*h + d*h^2)) + b^2*h^4*(48*
a^2*f*h^2 - 8*a*b*h*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)))*A
rcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[
a + b*x + c*x^2])])/(128*h^6*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
```

```
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
```

```
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(\frac{1}{2}\left(-8cdg+5beg+8afg-\frac{5bfg^2}{h}+3bdh-8aeh\right)+\left(ce\right.\right.}{(g+hx)}}{4\left(cg^2-bg\right.}$$

$$= -\frac{\left(\frac{16c^2g^4(5fg-eh)}{h}-h\left(16a^2h^2(fg-2eh)-b^2g\left(35fg^2+5egh+3dh^2\right)+4abh\left(7fg\right.\right.\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h}-16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh))+4ch\left(2b^2g^2(46fg-5\right.\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h}-16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh))+4ch\left(2b^2g^2(46fg-5\right.\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h}-16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh))+4ch\left(2b^2g^2(46fg-5\right.\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h}-16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh))+4ch\left(2b^2g^2(46fg-5\right.\right.}{}$$

Mathematica [B] time = 6.63227, size = 46895, normalized size = 42.75

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.28, size = 57957, normalized size = 52.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.205 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

Optimal. Leaf size=1226

result too large to display

```
[Out] -((128*c^4*f*g^7 - 32*c^3*f*g^5*h*(11*b*g - 10*a*h) + 8*c^2*g*h^2*(38*b^2*f
*g^4 + 2*a^2*h^2*(13*f*g^2 + 3*d*h^2) - a*b*g*h*(65*f*g^2 + 3*d*h^2)) - 2*c
*h^3*(8*a^3*h^3*(2*f*g - 3*e*h) - 2*a*b^2*g^2*h*(34*f*g + 3*e*h) + 4*a^2*b*
h^2*(5*f*g^2 + 6*e*g*h + 3*d*h^2) + b^3*(35*f*g^4 - 3*d*g^2*h^2)) - b*h^4*(
b*g - 2*a*h)*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*h*
(e*g + d*h))) + h*(128*c*f*(c*g^2 - h*(b*g - a*h))^3 + (2*c*g - b*h)*(32*c^
3*f*g^5 - 8*c^2*g*h*(10*b*f*g^3 - 11*a*f*g^2*h + 3*a*d*h^3) + 2*c*h^2*(4*a^
2*h^2*(10*f*g - 3*e*h) - 6*a*b*h*(11*f*g^2 - e*g*h - d*h^2) + b^2*(29*f*g^3
 + 3*d*g*h^2)) - b*h^3*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*
g^2 + 3*h*(e*g + d*h)))))*x)*Sqrt[a + b*x + c*x^2])/(128*h^5*(c*g^2 - b*g*h
 + a*h^2)^3*(g + h*x)^2) - ((16*c^2*f*g^5 - 2*c*g*h*(13*b*f*g^3 - 10*a*f*g^
2*h + 3*b*d*g*h^2 - 6*a*d*h^3) - h^2*(4*a^2*h^2*(2*f*g - 3*e*h) - b^2*g*(7*
f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(f*g^2 + 3*h*(2*e*g + d*h))) + h*(4*c^2*
(7*f*g^4 - 3*d*g^2*h^2) + 2*c*g*h*(2*a*h*(14*f*g - 3*e*h) - b*(28*f*g^2 - 3
*e*g*h - 6*d*h^2)) + h^2*(16*a^2*f*h^2 - 2*a*b*h*(22*f*g - 3*e*h) + b^2*(25
*f*g^2 - 3*h*(e*g + d*h))))*x)*(a + b*x + c*x^2)^(3/2))/(48*h^3*(c*g^2 - b*
g*h + a*h^2)^2*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5
/2))/(5*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^6 - ((256*c^5*f*g^7 - 896*c^4*f*
g^5*h*(b*g - a*h) + 32*c^3*g*h^2*(35*b^2*f*g^4 - 70*a*b*f*g^3*h + a^2*h^2*(
35*f*g^2 - 3*d*h^2)) - 16*c^2*h^3*(35*b^3*f*g^4 - 6*a^3*h^3*(6*f*g - e*h) +
 3*a^2*b*h^2*(35*f*g^2 - e*g*h - d*h^2) - 3*a*b^2*g*h*(35*f*g^2 + d*h^2)) +
 b^3*h^5*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*h*(e*g
 + d*h))) - 2*b*c*h^4*(96*a^3*f*h^3 - 24*a^2*b*h^2*(8*f*g + e*h) - b^3*(35*
f*g^3 - 3*d*g*h^2) + 4*a*b^2*h*(35*f*g^2 + 3*h*(e*g + d*h))))*ArcTanh[(b*g
- 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*
x^2])])/(256*h^6*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rubi [A] time = 3.99728, antiderivative size = 1223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 810, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}-\frac{\left(16c^2fg^5-2ch\left(13bfg^3-10afhg^2+3bdh^2g-6adh^3\right)g-h^2\left(-g\left(7fg^2+3h(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(((128*c^4*f*g^7)/h - 32*c^3*f*g^5*(11*b*g - 10*a*h) + 8*c^2*g*h*(38*b^2*f
*g^4 + 2*a^2*h^2*(13*f*g^2 + 3*d*h^2) - a*b*g*h*(65*f*g^2 + 3*d*h^2)) - b*h
^3*(b*g - 2*a*h)*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 +
3*h*(e*g + d*h))) - 2*c*h^2*(8*a^3*h^3*(2*f*g - 3*e*h) - 2*a*b^2*g^2*h*(34*
f*g + 3*e*h) + b^3*(35*f*g^4 - 3*d*g^2*h^2) + 4*a^2*b*h^2*(5*f*g^2 + 3*h*(2
*e*g + d*h))) + (128*c*f*(c*g^2 - h*(b*g - a*h))^3 + (2*c*g - b*h)*(32*c^3*
f*g^5 - 8*c^2*g*h*(10*b*f*g^3 - 11*a*f*g^2*h + 3*a*d*h^3) + 2*c*h^2*(4*a^2*
h^2*(10*f*g - 3*e*h) - 6*a*b*h*(11*f*g^2 - e*g*h - d*h^2) + b^2*(29*f*g^3 +
 3*d*g*h^2)) - b*h^3*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^
2 + 3*h*(e*g + d*h)))))*x)*Sqrt[a + b*x + c*x^2])/(128*h^4*(c*g^2 - b*g*h +
 a*h^2)^3*(g + h*x)^2) - ((16*c^2*f*g^5 - 2*c*g*h*(13*b*f*g^3 - 10*a*f*g^2*
h + 3*b*d*g*h^2 - 6*a*d*h^3) - h^2*(4*a^2*h^2*(2*f*g - 3*e*h) - b^2*g*(7*f*
g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(f*g^2 + 3*h*(2*e*g + d*h))) + h^2*(16*a^2
*f*h^3 + 4*a*c*g*h*(14*f*g - 3*e*h) + c^2*((28*f*g^4)/h - 12*d*g^2*h) + b^2
*h*(25*f*g^2 - 3*h*(e*g + d*h)) - b*(56*c*f*g^3 - 6*c*g*h*(e*g + 2*d*h) + 2
*a*h^2*(22*f*g - 3*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(48*h^3*(c*g^2 - b*g*
h + a*h^2)^2*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2
))/(5*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^6 - ((256*c^5*f*g^7 - 896*c^4*f*g^
5*h*(b*g - a*h) + 32*c^3*g*h^2*(35*b^2*f*g^4 - 70*a*b*f*g^3*h + a^2*h^2*(35
*f*g^2 - 3*d*h^2)) - 16*c^2*h^3*(35*b^3*f*g^4 - 6*a^3*h^3*(6*f*g - e*h) + 3
*a^2*b*h^2*(35*f*g^2 - e*g*h - d*h^2) - 3*a*b^2*g*h*(35*f*g^2 + d*h^2)) + b
^3*h^5*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*h*(e*g +
 d*h))) - 2*b*c*h^4*(96*a^3*f*h^3 - 24*a^2*b*h^2*(8*f*g + e*h) - b^3*(35*f*
g^3 - 3*d*g*h^2) + 4*a*b^2*h*(35*f*g^2 + 3*h*(e*g + d*h))))*ArcTanh[(b*g -
2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^
2])])/(256*h^6*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-\frac{5}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}-bdh+2aeh\right)+5f\left(bg-\right.\right.}{(g+hx)^5}}{5\left(cg^2-bgh+ah^2\right.}$$

$$= -\frac{\left(16c^2fg^5-2cgh\left(13bfg^3-10afg^2h+3bdgh^2-6adh^3\right)-h^2\left(4a^2h^2(2fg-3eh\right.\right.}{}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg^2+3dh^2\right)-\right.\right.}{}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg^2+3dh^2\right)-\right.\right.}{}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg^2+3dh^2\right)-\right.\right.}{}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg^2+3dh^2\right)-\right.\right.}{}$$

Mathematica [A] time = 6.30515, size = 1111, normalized size = 0.91

$$f(a+x(b+cx))^{3/2}\left(\frac{-\frac{(bh-2cg)\left(cx^2+bx+a\right)^{3/2}}{2\left(cg^2-bhg+ah^2\right)(g+hx)^2}-\frac{\frac{\left(\frac{1}{2}h\left(hb^2+2cgb-8ach\right)-cg(2cg-bh)\right)\left(cx^2+bx+a\right)^{3/2}}{\left(-cg^2+bhg-ah^2\right)(g+hx)}+\frac{\left(h\left(4c^2g^2-b^2h^2-4ch(bg-2ah)\right)xc^2-\left(2cg-\frac{bh}{2}\right)\left(4c^2g^2-b^2h^2-4ch(bg-2ah)\right)c+\frac{1}{2}\right.}{2ch^2}}{}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(((a + x*(b + c*x))^(3/2)*(-((-(g*h*(2*f*g - e*h)) + h*(f*g^2 - d*h^2))*(a
 + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a*h^
2*(2*f*g - e*h) + c*g*(f*g^2 - d*h^2)) + b*(g*h*(2*f*g - e*h) + h*(f*g^2 -
d*h^2)))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g
^2 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g
 - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) +
 ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 -
b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c
*g^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g
*h + a*h^2))))/(h^2*(a + b*x + c*x^2)^(3/2))) + (f*(a + x*(b + c*x))^(3/2)*
(-(a + b*x + c*x^2)^(3/2)/(3*h*(g + h*x)^3) + (-((-2*c*g + b*h)*(a + b*x +
c*x^2)^(3/2))/(2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - (((-(c*g*(2*c*g - b
*h)) + (h*(2*b*c*g + b^2*h - 8*a*c*h))/2)*(a + b*x + c*x^2)^(3/2))/((-(c*g^
2) + b*g*h - a*h^2)*(g + h*x)) + (((-(c*(2*c*g - (b*h)/2)*(4*c^2*g^2 - b^2*
h^2 - 4*c*h*(b*g - 2*a*h))) + (c*h*(-10*b^2*c*g*h + 8*a*c^2*g*h - b^3*h^2 +
 4*b*c*(2*c*g^2 + 3*a*h^2)))/2 + c^2*h*(4*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g -
2*a*h))*x)*Sqrt[a + b*x + c*x^2])/(2*c*h^2) - ((-16*c^(5/2)*(c*g^2 - h*(b*g
 - a*h))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h - (4*S
qrt[c*g^2 - b*g*h + a*h^2]*(-(c*h*(c*g^2 - b*g*h + a*h^2)*(8*b*c^2*g^2 - 6*
b^2*c*g*h - 8*a*c^2*g*h - b^3*h^2 + 12*a*b*c*h^2)) + 16*c^3*g*(c*g^2 - h*(b
*g - a*h))^2)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(h*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(4
*c*h^2))/(-(c*g^2) + b*g*h - a*h^2))/(2*(c*g^2 - b*g*h + a*h^2)))/(2*h)))/(
h^2*(a + b*x + c*x^2)^(3/2))
```

Maple [B] time = 0.288, size = 76693, normalized size = 62.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

3.206 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx$

Optimal. Leaf size=657

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)\left(24a^2fh^2-4c\left(a\left(dh^2-7egh+fg^2\right)+3bg(2dh+eg)\right)-12abh(eh+2fg)+\cdots}{192(g+hx)^4\left(ah^2-bgh+cg^2\right)^3}$$

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d*g^2 + 24*a^2*f*h^2 - 12*a*b*h*(2*f*g + e*h) + b^2
*(7*f*g^2 + 5*e*g*h + 7*d*h^2) - 4*c*(3*b*g*(e*g + 2*d*h) + a*(f*g^2 - 7*e*
g*h + d*h^2)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(512*
(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) + ((24*c^2*d*g^2 + 24*a^2*f*h^2 - 12
*a*b*h*(2*f*g + e*h) + b^2*(7*f*g^2 + 5*e*g*h + 7*d*h^2) - 4*c*(3*b*g*(e*g
+ 2*d*h) + a*(f*g^2 - 7*e*g*h + d*h^2)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*(a
 + b*x + c*x^2)^(3/2))/(192*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^4) - ((f*g^
2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(6*h*(c*g^2 - b*g*h + a*h^2)*(g
 + h*x)^6) + ((2*c*g*(5*f*g^2 + h*(e*g - 7*d*h)) + h*(12*a*h*(2*f*g - e*h)
- b*(17*f*g^2 - 5*e*g*h - 7*d*h^2)))*(a + b*x + c*x^2)^(5/2))/(60*h*(c*g^2
- b*g*h + a*h^2)^2*(g + h*x)^5) + ((b^2 - 4*a*c)^2*(24*c^2*d*g^2 + 24*a^2*f
*h^2 - 12*a*b*h*(2*f*g + e*h) + b^2*(7*f*g^2 + 5*e*g*h + 7*d*h^2) - 4*c*(3*
b*g*(e*g + 2*d*h) + a*(f*g^2 - 7*e*g*h + d*h^2)))*ArcTanh[(b*g - 2*a*h + (2
*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(102
4*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rubi [A] time = 1.21644, antiderivative size = 660, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1650, 806, 720, 724, 206}

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-12abh(eh+2fg)+\cdots}{192(g+hx)^4\left(ah^2-bgh+cg^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d*g^2 + 24*a^2*f*h^2 - 12*a*b*h*(2*f*g + e*h) - 4*c
*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*d*h)) + b^2*(7*f*g^2 + h*(5*
e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(512*
```

```
(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) + ((24*c^2*d*g^2 + 24*a^2*f*h^2 - 12
*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*d*
h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*(a
 + b*x + c*x^2)^(3/2))/(192*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^4) - ((f*g^
2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(6*h*(c*g^2 - b*g*h + a*h^2)*(g
 + h*x)^6) + ((2*c*(5*f*g^3 + g*h*(e*g - 7*d*h)) - h*(17*b*f*g^2 - b*h*(5*e
*g + 7*d*h) - 12*a*h*(2*f*g - e*h)))*(a + b*x + c*x^2)^(5/2))/(60*h*(c*g^2
- b*g*h + a*h^2)^2*(g + h*x)^5) + ((b^2 - 4*a*c)^2*(24*c^2*d*g^2 + 24*a^2*f
*h^2 - 12*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e
*g + 2*d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*ArcTanh[(b*g - 2*a*h + (2
*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(102
4*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{6h\left(cg^2-bgh+ah^2\right)(g+hx)^6} - \frac{\int \frac{\left(\frac{1}{2}\left(-12cdg+5beg+12afg-\frac{5bfg^2}{h}+7bdh-12aeh\right)-\cdots\right.}{(g+h\cdots}}{6\left(cg^2-b\cdots\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{6h\left(cg^2-bgh+ah^2\right)(g+hx)^6} + \frac{\left(2c\left(5fg^3+gh(eg-7dh)\right)-h\left(17bfg^2\cdots\right.\right.}{60h\left(cg^2\cdots\right.}$$

$$= \frac{\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c\left(afg^2-ah(7eg-dh)+3bg(eg+2\cdots\right.\right.}{192\left(cg^2-bgh+a\cdots\right.}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c\left(afg^2-ah(7eg-dh)\cdots\right.\right.}{512\left(cg^2-bg\cdots\right.}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c\left(afg^2-ah(7eg-dh)\cdots\right.\right.}{512\left(cg^2-bg\cdots\right.}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c\left(afg^2-ah(7eg-dh)\cdots\right.\right.}{512\left(cg^2-bg\cdots\right.}$$

Mathematica [A] time = 6.24459, size = 765, normalized size = 1.16

$$(a+x(b+cx))^{3/2}\left(-\frac{-\frac{\left(\frac{(a+bx+cx^2)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4(ah^2-bgh+cg^2)}-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2ah-x(2cg-bh)-bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{2\sqrt{ah^2-bgh+cg^2}(4ah^2-4bgh+4cg^2)}+\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)}{4(g+hx)^2(ah^2-bgh+cg^2)}\right)}{16(ah^2-bgh+cg^2)}\right)\left(b\left(-cg\left(-6fh(\cdots\right.\right.\right.}{2(\cdots}}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -((f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(c*h*(g + h*x)^6)) + ((a +
x*(b + c*x))^(3/2)*(-(((h*(5*b*f*g + 2*c*d*h - 12*a*f*h))/2 - (g*(-7*b*f*h
+ 2*c*(5*f*g + e*h)))/2)*(a + b*x + c*x^2)^(5/2))/(6*(c*g^2 - b*g*h + a*h^2
)*(g + h*x)^6) - (-((c*g*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h))
- (c*h*(5*b*f*g^2 - b*h*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h)))/2)
*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(-
(a*c*h*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h))) - (c^2*g*(5*b*f*g
^2 - b*h*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h)))/2) + b*(-(c*g*(5*
c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h))) - (c*h*(5*b*f*g^2 - b*h*(5*
e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h)))/2))*(((b*g - 2*a*h + (2*c*g -
 b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) -
 (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/
(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) +
2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(16*(
c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2)))/(6*(c*g^2 - b*g*h +
a*h^2))))/(c*h*(a + b*x + c*x^2)^(3/2))
```

Maple [B] time = 0.322, size = 100754, normalized size = 153.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] Timed out
```

3.207 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx$

Optimal. Leaf size=1062

result too large to display

```
[Out] -((b^2 - 4*a*c)*(48*c^3*d*g^3 - 8*c^2*g*(3*b*g*(e*g + 3*d*h) + a*(f*g^2 - 8
*e*g*h + 3*d*h^2)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*
f*g^2 + 5*e*g*h + 9*d*h^2)) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*
g^2 + 13*e*g*h - 3*d*h^2) + b^2*g*(7*f*g^2 + 10*e*g*h + 21*d*h^2)))*(b*g -
2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(1024*(c*g^2 - b*g*h + a*h^
2)^5*(g + h*x)^2) + ((48*c^3*d*g^3 - 8*c^2*g*(3*b*g*(e*g + 3*d*h) + a*(f*g^
2 - 8*e*g*h + 3*d*h^2)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^
2*(5*f*g^2 + 5*e*g*h + 9*d*h^2)) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(
13*f*g^2 + 13*e*g*h - 3*d*h^2) + b^2*g*(7*f*g^2 + 10*e*g*h + 21*d*h^2)))*(b
*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(384*(c*g^2 - b*g*h
+ a*h^2)^4*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))
/(7*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^7) + ((2*c*g*(5*f*g^2 + h*(2*e*g -
9*d*h)) + h*(14*a*h*(2*f*g - e*h) - b*(19*f*g^2 - 5*e*g*h - 9*d*h^2)))*(a +
 b*x + c*x^2)^(5/2))/(84*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^6) + ((4*c^2
*g^2*(5*f*g^2 + h*(2*e*g - 51*d*h)) - 7*h^2*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g
 + 7*e*h) + b^2*(5*f*g^2 + 5*e*g*h + 9*d*h^2)) - 2*c*h*(3*b*g*(8*f*g^2 - 15
*e*g*h - 34*d*h^2) - 2*a*h*(26*f*g^2 - 61*e*g*h + 12*d*h^2)))*(a + b*x + c*
x^2)^(5/2))/(840*h*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^5) + ((b^2 - 4*a*c)^
2*(48*c^3*d*g^3 - 8*c^2*g*(3*b*g*(e*g + 3*d*h) + a*(f*g^2 - 8*e*g*h + 3*d*h
^2)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*f*g^2 + 5*e*g*
h + 9*d*h^2)) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*g^2 + 13*e*g*h
 - 3*d*h^2) + b^2*g*(7*f*g^2 + 10*e*g*h + 21*d*h^2)))*ArcTanh[(b*g - 2*a*h
+ (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/
(2048*(c*g^2 - b*g*h + a*h^2)^(11/2))
```

Rubi [A] time = 3.00357, antiderivative size = 1062, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{\left(4\left(5fg^4+h(2eg-51dh)g^2\right)c^2-2h\left(3bg\left(8fg^2-15ehg-34dh^2\right)-2ah\left(26fg^2-61ehg+12dh^2\right)\right)c-7h^2\left(\left(5fg^2+5ehg\cdots\right.\right.\right.}{840h\left(cg^2-bhg+ah^2\right)^3(g+hx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -((b^2 - 4*a*c)*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h) + 3*
b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*
f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*
g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*(b*g -
2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(1024*(c*g^2 - b*g*h + a*h^
2)^5*(g + h*x)^2) + ((48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h)
 + 3*b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^
2*(5*f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(
13*f*g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*(b
*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(384*(c*g^2 - b*g*h
+ a*h^2)^4*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))
/(7*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^7) + ((2*c*(5*f*g^3 + g*h*(2*e*g -
9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h)))*(a +
 b*x + c*x^2)^(5/2))/(84*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^6) + ((4*c^2
*(5*f*g^4 + g^2*h*(2*e*g - 51*d*h)) - 7*h^2*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g
 + 7*e*h) + b^2*(5*f*g^2 + 5*e*g*h + 9*d*h^2)) - 2*c*h*(3*b*g*(8*f*g^2 - 15
*e*g*h - 34*d*h^2) - 2*a*h*(26*f*g^2 - 61*e*g*h + 12*d*h^2)))*(a + b*x + c*
x^2)^(5/2))/(840*h*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^5) + ((b^2 - 4*a*c)^
2*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h) + 3*b*g*(e*g + 3*d
*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*f*g^2 + h*(5*e
*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*g^2 + h*(13*e*
g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*ArcTanh[(b*g - 2*a*h
+ (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/
(2048*(c*g^2 - b*g*h + a*h^2)^(11/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
```

```
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} - \frac{\int \frac{\left(\frac{1}{2}\left(-14cdg+5beg+14afg-\frac{5bfg^2}{h}+9bdh-14aeh\right.\right.}{(g}}{7\left(cg^2-\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} + \frac{\left(2c\left(5fg^3+gh(2eg-9dh)\right)-h\left(19bf\right.\right.}{84h\left(cg\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} + \frac{\left(2c\left(5fg^3+gh(2eg-9dh)\right)-h\left(19bf\right.\right.}{84h\left(cg\right.}$$

$$= \frac{\left(48c^3dg^3-8c^2g\left(afg^2-ah(8eg-3dh)+3bg(eg+3dh)\right)-bh\left(24a^2fh^2-2ab\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg^3-8c^2g\left(afg^2-ah(8eg-3dh)+3bg(eg+3dh)\right)-bh\left(24\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg^3-8c^2g\left(afg^2-ah(8eg-3dh)+3bg(eg+3dh)\right)-bh\left(24\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg^3-8c^2g\left(afg^2-ah(8eg-3dh)+3bg(eg+3dh)\right)-bh\left(24\right.\right.}{}$$

Mathematica [A] time = 6.41862, size = 1222, normalized size = 1.15

$$\frac{(a+x(b+cx))^{3/2}\left(-\frac{\left(\frac{1}{2}h(5bfg+4cdh-14afh)-\frac{1}{2}g(10cfg+4ceh-9bfh)\right)\left(cx^2+bx+a\right)^{5/2}}{7\left(cg^2-bhg+ah^2\right)(g+hx)^7}-\frac{-\frac{\left(2cg\left(5cfg^2-7fh(bg-ah)+2ch(eg-dh)\right)-ch\left(5bfg^2-bh(5eg+9dh)+14h(cdg-af\right.\right.}{6\left(cg^2-bhg+ah^2\right)(g+hx)^6}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -(f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(2*c*h*(g + h*x)^7) + ((a +
x*(b + c*x))^(3/2)*(-(((h*(5*b*f*g + 4*c*d*h - 14*a*f*h))/2 - (g*(10*c*f*g
+ 4*c*e*h - 9*b*f*h))/2)*(a + b*x + c*x^2)^(5/2))/(7*(c*g^2 - b*g*h + a*h^2
)*(g + h*x)^7) - (-((2*c*g*(5*c*f*g^2 - 7*f*h*(b*g - a*h) + 2*c*h*(e*g - d*
h)) - c*h*(5*b*f*g^2 - b*h*(5*e*g + 9*d*h) + 14*h*(c*d*g - a*f*g + a*e*h)))
*(a + b*x + c*x^2)^(5/2))/(6*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^6) - (-((-(c
^2*g*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*
d*h) - 14*a*h*(2*f*g - e*h)))) + (c*h*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9*d*h)
) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h)) +
24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h)))))/2)*(
a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a*c
^2*h*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*
d*h) - 14*a*h*(2*f*g - e*h))) + (c^2*g*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9*d*h
)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h)) +
 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h)))))/2)
+ b*(c^2*g*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*
g + 9*d*h) - 14*a*h*(2*f*g - e*h))) + (c*h*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9
```

```
*d*h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h
)) + 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h)))))
/2))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 -
 b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b
*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b
^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2
 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h +
 a*h^2)))/(6*(c*g^2 - b*g*h + a*h^2)))/(7*(c*g^2 - b*g*h + a*h^2))))/(2*c*h
*(a + b*x + c*x^2)^(3/2))
```

Maple [B] time = 0.396, size = 126612, normalized size = 119.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] Timed out
```

3.208 $\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=143

$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4+\frac{67}{378}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{17}{105}\left(3x^2-x+2\right)^{3/2}(2x+1)^2-\frac{(26982x+75295)\left(3x^2\right.}{68040}$$

```
[Out] (5393*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/15552 + (17*(1 + 2*x)^2*(2 - x + 3*x^2
)^(3/2))/105 + (67*(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/378 + (2*(1 + 2*x)^4*
(2 - x + 3*x^2)^(3/2))/21 - ((75295 + 26982*x)*(2 - x + 3*x^2)^(3/2))/68040
 + (124039*ArcSinh[(1 - 6*x)/Sqrt[23]])/(31104*Sqrt[3])
```

Rubi [A] time = 0.138362, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4+\frac{67}{378}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{17}{105}\left(3x^2-x+2\right)^{3/2}(2x+1)^2-\frac{(26982x+75295)\left(3x^2\right.}{68040}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (5393*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/15552 + (17*(1 + 2*x)^2*(2 - x + 3*x^2
)^(3/2))/105 + (67*(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/378 + (2*(1 + 2*x)^4*
(2 - x + 3*x^2)^(3/2))/21 - ((75295 + 26982*x)*(2 - x + 3*x^2)^(3/2))/68040
 + (124039*ArcSinh[(1 - 6*x)/Sqrt[23]])/(31104*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx = \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}+\frac{1}{84}\int (1+2x)^3(-32+268x)\sqrt{2-x+3x^2}$$

$$= \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}+\frac{\int (1+2x)}{}$$

$$= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{2}{21}(1+2$$

$$= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{2}{21}(1+2$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x$$

Mathematica [A] time = 0.0474035, size = 70, normalized size = 0.49

$$\frac{6\sqrt{3x^2-x+2}\left(2488320x^6+6462720x^5+7491456x^4+5497776x^3+3280872x^2+1493894x-543069\right)-4341365\sqrt{3}}{3265920}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-543069 + 1493894*x + 3280872*x^2 + 5497776*x^3 + 7
491456*x^4 + 6462720*x^5 + 2488320*x^6) - 4341365*Sqrt[3]*ArcSinh[(-1 + 6*x
)/Sqrt[23]])/3265920
```

Maple [A] time = 0.057, size = 115, normalized size = 0.8

$$\frac{32x^4}{21}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{844x^3}{189}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{1594x^2}{315}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{7849x}{3780}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{-5393+32358x}{15552}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)
```

```
[Out] 32/21*x^4*(3*x^2-x+2)^(3/2)+844/189*x^3*(3*x^2-x+2)^(3/2)+1594/315*x^2*(3*x
^2-x+2)^(3/2)+7849/3780*x*(3*x^2-x+2)^(3/2)-5393/15552*(-1+6*x)*(3*x^2-x+2)
^(1/2)-124039/93312*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-45739/68040*(3*x
^2-x+2)^(3/2)
```

Maxima [A] time = 1.52799, size = 170, normalized size = 1.19

$$\frac{32}{21}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^4+\frac{844}{189}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{1594}{315}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{7849}{3780}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x-\frac{45739}{68040}\left(3\,x^2-x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/21*(3*x^2 - x + 2)^(3/2)*x^4 + 844/189*(3*x^2 - x + 2)^(3/2)*x^3 + 1594/
315*(3*x^2 - x + 2)^(3/2)*x^2 + 7849/3780*(3*x^2 - x + 2)^(3/2)*x - 45739/6
8040*(3*x^2 - x + 2)^(3/2) - 5393/2592*sqrt(3*x^2 - x + 2)*x - 124039/93312
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 5393/15552*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.90773, size = 285, normalized size = 1.99

$$\frac{1}{544320}\left(2488320\,x^6+6462720\,x^5+7491456\,x^4+5497776\,x^3+3280872\,x^2+1493894\,x-543069\right)\sqrt{3\,x^2-x+2}+\frac{12}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/544320*(2488320*x^6 + 6462720*x^5 + 7491456*x^4 + 5497776*x^3 + 3280872*x
^2 + 1493894*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/186624*sqrt(3)*log(4*
sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)^3\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**3*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

Giac [A] time = 1.34908, size = 105, normalized size = 0.73

$$\frac{1}{544320}\left(2\left(12\left(6\left(8\left(30\left(72x+187\right)x+6503\right)x+38179\right)x+136703\right)x+746947\right)x-543069\right)\sqrt{3x^2-x+2}+\frac{124039}{93312}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/544320*(2*(12*(6*(8*(30*(72*x + 187)*x + 6503)*x + 38179)*x + 136703)*x +
 746947)*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/93312*sqrt(3)*log(-2*sqrt
(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.209 $\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=118

$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{1}{5}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{1}{810}(306x+25)\left(3x^2-x+2\right)^{3/2}+\frac{235(1-6x)\sqrt{3x^2-x+2}}{1296}$$

```
[Out] (235*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/1296 + ((1 + 2*x)^2*(2 - x + 3*x^2)^(3/
2))/5 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/9 + ((25 + 306*x)*(2 - x + 3*x^
2)^(3/2))/810 + (5405*ArcSinh[(1 - 6*x)/Sqrt[23]])/(2592*Sqrt[3])
```

Rubi [A] time = 0.111741, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{1}{5}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{1}{810}(306x+25)\left(3x^2-x+2\right)^{3/2}+\frac{235(1-6x)\sqrt{3x^2-x+2}}{1296}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (235*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/1296 + ((1 + 2*x)^2*(2 - x + 3*x^2)^(3/
2))/5 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/9 + ((25 + 306*x)*(2 - x + 3*x^
2)^(3/2))/810 + (5405*ArcSinh[(1 - 6*x)/Sqrt[23]])/(2592*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx = \frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{72}\int (1+2x)^2(-12+216x)\sqrt{2-x+3x^2}\,dx$$
$$= \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{\int (1+2x)(-15$$
$$= \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{810}(25+306x$$
$$= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-\right.$$
$$= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-\right.$$
$$= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-\right.$$

Mathematica [A] time = 0.0387511, size = 65, normalized size = 0.55

$$\frac{6\sqrt{3x^2-x+2}\left(17280x^5+35712x^4+33552x^3+22344x^2+14638x+5607\right)-27025\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{38880}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(5607 + 14638*x + 22344*x^2 + 33552*x^3 + 35712*x^4
+ 17280*x^5) - 27025*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/38880
```

Maple [A] time = 0.055, size = 98, normalized size = 0.8

$$\frac{8x^3}{9}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{32x^2}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{83x}{45}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{277}{810}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{-235+1410x}{1296}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)
```

```
[Out] 8/9*x^3*(3*x^2-x+2)^(3/2)+32/15*x^2*(3*x^2-x+2)^(3/2)+83/45*x*(3*x^2-x+2)^(
3/2)+277/810*(3*x^2-x+2)^(3/2)-235/1296*(-1+6*x)*(3*x^2-x+2)^(1/2)-5405/777
6*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A] time = 1.4895, size = 147, normalized size = 1.25

$$\frac{8}{9}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{32}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{83}{45}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{277}{810}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{235}{216}\sqrt{3x^2-x+2}x-\frac{5405}{7776}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(3*x^2 - x + 2)^(3/2)*x^3 + 32/15*(3*x^2 - x + 2)^(3/2)*x^2 + 83/45*(3*
x^2 - x + 2)^(3/2)*x + 277/810*(3*x^2 - x + 2)^(3/2) - 235/216*sqrt(3*x^2 -
 x + 2)*x - 5405/7776*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 235/1296*s
qrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.55612, size = 243, normalized size = 2.06

$$\frac{1}{6480}\left(17280x^5+35712x^4+33552x^3+22344x^2+14638x+5607\right)\sqrt{3x^2-x+2}+\frac{5405}{15552}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6480*(17280*x^5 + 35712*x^4 + 33552*x^3 + 22344*x^2 + 14638*x + 5607)*sqr
t(3*x^2 - x + 2) + 5405/15552*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)^2\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**2*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

Giac [A] time = 1.16029, size = 99, normalized size = 0.84

$$\frac{1}{6480}\left(2\left(12\left(6\left(8\left(15x+31\right)x+233\right)x+931\right)x+7319\right)x+5607\right)\sqrt{3x^2-x+2}+\frac{5405}{7776}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6480*(2*(12*(6*(8*(15*x + 31)*x + 233)*x + 931)*x + 7319)*x + 5607)*sqrt(
3*x^2 - x + 2) + 5405/7776*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 -
 x + 2)) + 1)
```

3.210 $\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=93

$$\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{(738x+745)\left(3x^2-x+2\right)^{3/2}}{1620}+\frac{19(1-6x)\sqrt{3x^2-x+2}}{2592}+\frac{437\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

```
[Out] (19*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(3
/2))/15 + ((745 + 738*x)*(2 - x + 3*x^2)^(3/2))/1620 + (437*ArcSinh[(1 - 6*
x)/Sqrt[23]])/(5184*Sqrt[3])
```

Rubi [A] time = 0.0657437, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{(738x+745)\left(3x^2-x+2\right)^{3/2}}{1620}+\frac{19(1-6x)\sqrt{3x^2-x+2}}{2592}+\frac{437\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (19*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(3
/2))/15 + ((745 + 738*x)*(2 - x + 3*x^2)^(3/2))/1620 + (437*ArcSinh[(1 - 6*
x)/Sqrt[23]])/(5184*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx = \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{60}\int (1+2x)(8+164x)\sqrt{2-x+3x^2}\,dx$$

$$= \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620}-\frac{19}{216}\int \sqrt{2-$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+738x)\left(2-\right.}{1620}$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+738x)\left(2-\right.}{1620}$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+738x)\left(2-\right.}{1620}$$

Mathematica [A] time = 0.0286907, size = 60, normalized size = 0.65

$$\frac{6\sqrt{3x^2-x+2}\left(20736x^4+31536x^3+24072x^2+17374x+15471\right)-2185\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{77760}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(15471 + 17374*x + 24072*x^2 + 31536*x^3 + 20736*x^4
) - 2185*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/77760
```

Maple [A] time = 0.051, size = 81, normalized size = 0.9

$$\frac{8x^2}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{89x}{90}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{961}{1620}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{-19+114x}{2592}\sqrt{3x^2-x+2}-\frac{437\sqrt{3}}{15552}\mathrm{Arcsinh}\left(\frac{6\sqrt{}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)
```

```
[Out] 8/15*x^2*(3*x^2-x+2)^(3/2)+89/90*x*(3*x^2-x+2)^(3/2)+961/1620*(3*x^2-x+2)^(
3/2)-19/2592*(-1+6*x)*(3*x^2-x+2)^(1/2)-437/15552*3^(1/2)*arcsinh(6/23*23^(
1/2)*(x-1/6))
```

Maxima [A] time = 1.53582, size = 124, normalized size = 1.33

$$\frac{8}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{89}{90}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{961}{1620}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{19}{432}\sqrt{3x^2-x+2}x-\frac{437}{15552}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/15*(3*x^2 - x + 2)^(3/2)*x^2 + 89/90*(3*x^2 - x + 2)^(3/2)*x + 961/1620*(
3*x^2 - x + 2)^(3/2) - 19/432*sqrt(3*x^2 - x + 2)*x - 437/15552*sqrt(3)*arc
sinh(1/23*sqrt(23)*(6*x - 1)) + 19/2592*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.62988, size = 228, normalized size = 2.45

$$\frac{1}{12960}\left(20736\,x^4+31536\,x^3+24072\,x^2+17374\,x+15471\right)\sqrt{3\,x^2-x+2}+\frac{437}{31104}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12960*(20736*x^4 + 31536*x^3 + 24072*x^2 + 17374*x + 15471)*sqrt(3*x^2 -
x + 2) + 437/31104*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72
*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

Giac [A] time = 1.16218, size = 92, normalized size = 0.99

$$\frac{1}{12960}\,(2\,(12\,(18\,(48\,x+73)x+1003)x+8687)x+15471)\sqrt{3\,x^2-x+2}+\frac{437}{15552}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12960*(2*(12*(18*(48*x + 73)*x + 1003)*x + 8687)*x + 15471)*sqrt(3*x^2 -
x + 2) + 437/15552*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
 + 1)
```

3.211 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{1+2x}\,dx$

Optimal. Leaf size=101

$$\frac{2}{9}\left(3x^2-x+2\right)^{3/2}+\frac{1}{72}(30x+13)\sqrt{3x^2-x+2}-\frac{1}{8}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}$$

```
[Out] ((13 + 30*x)*Sqrt[2 - x + 3*x^2])/72 + (2*(2 - x + 3*x^2)^(3/2))/9 - (43*Ar
cSinh[(1 - 6*x)/Sqrt[23]])/(144*Sqrt[3]) - (Sqrt[13]*ArcTanh[(9 - 8*x)/(2*S
qrt[13]*Sqrt[2 - x + 3*x^2])])/8
```

Rubi [A] time = 0.116899, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{9}\left(3x^2-x+2\right)^{3/2}+\frac{1}{72}(30x+13)\sqrt{3x^2-x+2}-\frac{1}{8}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((13 + 30*x)*Sqrt[2 - x + 3*x^2])/72 + (2*(2 - x + 3*x^2)^(3/2))/9 - (43*Ar
cSinh[(1 - 6*x)/Sqrt[23]])/(144*Sqrt[3]) - (Sqrt[13]*ArcTanh[(9 - 8*x)/(2*S
qrt[13]*Sqrt[2 - x + 3*x^2])])/8
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{1+2x}\,dx = \frac{2}{9}\left(2-x+3x^2\right)^{3/2}+\frac{1}{36}\int \frac{(48+60x)\sqrt{2-x+3x^2}}{1+2x}\,dx$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}-\frac{\int \frac{-3324-1032x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{1728}$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}+\frac{43}{144}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx+\frac{13}{8}$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}-\frac{13}{4}\operatorname{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\right.$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}-\frac{1}{8}\sqrt{13}\tanh$$

Mathematica [A] time = 0.055795, size = 86, normalized size = 0.85

$$\frac{1}{432}\left(6\sqrt{3x^2-x+2}\left(48x^2+14x+45\right)-54\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+43\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(45 + 14*x + 48*x^2) + 43*Sqrt[3]*ArcSinh[(-1 + 6*x)
/Sqrt[23]] - 54*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])
])/432
```

Maple [A] time = 0.054, size = 95, normalized size = 0.9

$$\frac{2}{9}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{-5+30\,x}{72}\sqrt{3\,x^2-x+2}+\frac{43\,\sqrt{3}}{432}\operatorname{Arcsinh}\left(\frac{6\,\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right)+\frac{1}{8}\sqrt{12\,\left(x+1/2\right)^2-16\,x+5}-\frac{\sqrt{13}}{8}\mathrm{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x)
```

```
[Out] 2/9*(3*x^2-x+2)^(3/2)+5/72*(-1+6*x)*(3*x^2-x+2)^(1/2)+43/432*3^(1/2)*arcsin
h(6/23*23^(1/2)*(x-1/6))+1/8*(12*(x+1/2)^2-16*x+5)^(1/2)-1/8*13^(1/2)*arcta
nh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A] time = 1.52865, size = 130, normalized size = 1.29

$$\frac{2}{9}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{5}{12}\sqrt{3x^2-x+2}x+\frac{43}{432}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{1}{8}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/9*(3*x^2 - x + 2)^(3/2) + 5/12*sqrt(3*x^2 - x + 2)*x + 43/432*sqrt(3)*arc
sinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/8*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 13/72*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.6215, size = 321, normalized size = 3.18

$$\frac{1}{72}\left(48x^2+14x+45\right)\sqrt{3x^2-x+2}+\frac{43}{864}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+\frac{1}{16}\sqrt{13}\log\left(-\frac{4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/72*(48*x^2 + 14*x + 45)*sqrt(3*x^2 - x + 2) + 43/864*sqrt(3)*log(-4*sqrt(
3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 1/16*sqrt(13)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x),x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Giac [A] time = 1.261, size = 170, normalized size = 1.68

$$\frac{1}{72}\left(2\left(24\,x+7\right)x+45\right)\sqrt{3\,x^2-x+2}-\frac{43}{432}\,\sqrt{3}\log\left(-6\,\sqrt{3}x+\sqrt{3}+6\,\sqrt{3\,x^2-x+2}\right)+\frac{1}{8}\,\sqrt{13}\log\left(-\frac{\left|-4\,\sqrt{3}x-2\,\sqrt{13}-\right.}{2\left(2\,\sqrt{3}x-\sqrt{13}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/72*(2*(24*x + 7)*x + 45)*sqrt(3*x^2 - x + 2) - 43/432*sqrt(3)*log(-6*sqrt
(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/8*sqrt(13)*log(-1/2*abs(-4*sqr
t(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqr
t(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

3.212 $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2} dx$

Optimal. Leaf size=108

$$-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)}-\frac{1}{156}(67-96x)\sqrt{3x^2-x+2}+\frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}}-\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

```
[Out] -((67 - 96*x)*Sqrt[2 - x + 3*x^2])/156 - (2 - x + 3*x^2)^(3/2)/(13*(1 + 2*x
)) - (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) + (17*ArcTanh[(9 - 8*x)/(
2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(8*Sqrt[13])
```

Rubi [A] time = 0.116414, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)}-\frac{1}{156}(67-96x)\sqrt{3x^2-x+2}+\frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}}-\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] -((67 - 96*x)*Sqrt[2 - x + 3*x^2])/156 - (2 - x + 3*x^2)^(3/2)/(13*(1 + 2*x
)) - (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) + (17*ArcTanh[(9 - 8*x)/(
2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(8*Sqrt[13])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = -\frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{15}{2}-32x\right)\sqrt{2-x+3x^2}}{1+2x}\,dx$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} + \frac{1}{624}\int \frac{-182+2288x}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} + \frac{11}{6}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx - \frac{17}{8}\int$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} + \frac{17}{4}\operatorname{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{}{\sqrt{2}}\right.$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} - \frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}} + \frac{17\tanh^{-1}\left(\right.}{8\sqrt{}}$$

Mathematica [A] time = 0.0875472, size = 92, normalized size = 0.85

$$\frac{\sqrt{3x^2-x+2}\left(12x^2-2x-7\right)}{24x+12} + \frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}} + \frac{11\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] (Sqrt[2 - x + 3*x^2]*(-7 - 2*x + 12*x^2))/(12 + 24*x) + (11*ArcSinh[(-1 + 6
*x)/Sqrt[23]])/(6*Sqrt[3]) + (17*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/(8*Sqrt[13])
```

Maple [A] time = 0.055, size = 123, normalized size = 1.1

$$\frac{-1+6x}{12}\sqrt{3x^2-x+2} + \frac{11\sqrt{3}}{18}\operatorname{Arcsinh}\left(\frac{6\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right) - \frac{1}{26}\left(3\,(x+1/2)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}\left(x+\frac{1}{2}\right)^{-1} - \frac{17}{104}\sqrt{12\,(x+1/2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x)
```

```
[Out] 1/12*(-1+6*x)*(3*x^2-x+2)^(1/2)+11/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6)
)-1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^(3/2)-17/104*(12*(x+1/2)^2-16*x+5)^(1/
2)+17/104*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1
/2))+1/52*(-1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(1/2)
```

Maxima [A] time = 1.63886, size = 139, normalized size = 1.29

$$\frac{1}{2}\sqrt{3x^2-x+2}x+\frac{11}{18}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{17}{104}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)-\frac{1}{3}\sqrt{3x^2-x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 - x + 2)*x + 11/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sq
rt(23)) - 17/104*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(
23)/abs(2*x + 1)) - 1/3*sqrt(3*x^2 - x + 2) - 1/4*sqrt(3*x^2 - x + 2)/(2*x
+ 1)
```

Fricas [A] time = 1.62976, size = 363, normalized size = 3.36

$$\frac{572\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+153\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220x^2}{4x^2+4x+1}\right.}{1872(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1872*(572*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 153*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
 x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 156*(12*x^2
 - 2*x - 7)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)}{(2x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**2,x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Giac [B] time = 1.6724, size = 513, normalized size = 4.75

$$\frac{17}{104}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{11}{18}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}}\right.}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 17/104*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 11/18*sqrt(3)*log(1/2*abs(-2*sqrt(
3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqr
t(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sgn(1
/(2*x + 1)) - 1/8*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x + 1))
+ 1/12*(67*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3
*sgn(1/(2*x + 1)) - 57*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) +
sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 129*(sqrt(-8/(2*x + 1) + 13/(2*x +
 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 27*sqrt(13)*sgn(1/(2*x
+ 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
3)^2
```

3.213 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}+\frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)}-\frac{803\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}}+\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

```
[Out] (11*(7 + 10*x)*Sqrt[2 - x + 3*x^2])/(104*(1 + 2*x)) - (2 - x + 3*x^2)^(3/2)
/(26*(1 + 2*x)^2) + (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(8*Sqrt[3]) - (803*Arc
Tanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(208*Sqrt[13])
```

Rubi [A] time = 0.116874, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}+\frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)}-\frac{803\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}}+\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] (11*(7 + 10*x)*Sqrt[2 - x + 3*x^2])/(104*(1 + 2*x)) - (2 - x + 3*x^2)^(3/2)
/(26*(1 + 2*x)^2) + (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(8*Sqrt[3]) - (803*Arc
Tanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(208*Sqrt[13])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = -\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{33}{2}-55x\right)\sqrt{2-x+3x^2}}{(1+2x)^2}\,dx$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{517-572x}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} - \frac{11}{8}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx + \frac{803}{208}\int$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} - \frac{803}{104}\,\mathrm{Subst}\left(\int \frac{1}{52-x^2}\,dx, x,\right.$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} + \frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}} - \frac{803\tanh^{-1}\left(\right.}{208}$$

Mathematica [A] time = 0.0815547, size = 93, normalized size = 0.81

$$\frac{\frac{78\sqrt{3x^2-x+2}\left(208x^2+268x+69\right)}{(2x+1)^2} - 2409\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) - 3718\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{8112}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((78*Sqrt[2 - x + 3*x^2]*(69 + 268*x + 208*x^2))/(1 + 2*x)^2 - 3718*Sqrt[3]
*ArcSinh[(-1 + 6*x)/Sqrt[23]] - 2409*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]
*Sqrt[2 - x + 3*x^2])])/8112
```

Maple [A] time = 0.057, size = 125, normalized size = 1.1

$$\frac{11}{338}\left(3\,(x+1/2)^2-4\,x+\frac{5}{4}\right)^{\frac{3}{2}}\left(x+\frac{1}{2}\right)^{-1}+\frac{803}{2704}\sqrt{12\,(x+1/2)^2-16\,x+5}-\frac{11\sqrt{3}}{24}\mathrm{Arcsinh}\left(\frac{6\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right)-\frac{803\sqrt{1}}{2704}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x)
```

```
[Out] 11/338/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^(3/2)+803/2704*(12*(x+1/2)^2-16*x+5)^(
1/2)-11/24*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-803/2704*13^(1/2)*arctanh
(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-11/676*(-1+6*x)*(3*(x
+1/2)^2-4*x+5/4)^(1/2)-1/104/(x+1/2)^2*(3*(x+1/2)^2-4*x+5/4)^(3/2)
```

Maxima [A] time = 1.49728, size = 154, normalized size = 1.34

$$-\frac{11}{24}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{803}{2704}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{55}{104}\sqrt{3\,x^2-x+2}-\frac{(3\,x^2-x\cdots}{26\,(4\,x^2+4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] -11/24*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 803/2704*sqrt(13)
*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 55/10
4*sqrt(3*x^2 - x + 2) - 1/26*(3*x^2 - x + 2)^(3/2)/(4*x^2 + 4*x + 1) + 11/5
2*sqrt(3*x^2 - x + 2)/(2*x + 1)
```

Fricas [A] time = 1.65216, size = 405, normalized size = 3.52

$$\frac{3718\sqrt{3}\left(4\,x^2+4\,x+1\right)\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)+2409\sqrt{13}\left(4\,x^2+4\,x+1\right)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2\cdots}}{\cdots}\right.\cdots}{16224\left(4\,x^2+4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16224*(3718*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(
6*x - 1) - 72*x^2 + 24*x - 25) + 2409*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sq
rt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x
+ 1)) + 156*(208*x^2 + 268*x + 69)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x^2 - x + 2}\,(4x^2 + 3x + 1)}{(2x + 1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**3,x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.214 $\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=158

$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4+\frac{77}{81}x^3\left(3x^2-x+2\right)^{5/2}+\frac{913}{486}x^2\left(3x^2-x+2\right)^{5/2}-\frac{11(283-5850x)\left(3x^2-x+2\right)^{5/2}}{58320}+\frac{545}{}$$

```
[Out] (1255639*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 + (54593*(1 - 6*x)*(2 - x +
 3*x^2)^(3/2))/559872 - (11*(283 - 5850*x)*(2 - x + 3*x^2)^(5/2))/58320 + (
913*x^2*(2 - x + 3*x^2)^(5/2))/486 + (77*x^3*(2 - x + 3*x^2)^(5/2))/81 + (2
*(1 + 2*x)^4*(2 - x + 3*x^2)^(5/2))/27 + (28879697*ArcSinh[(1 - 6*x)/Sqrt[2
3]])/(8957952*Sqrt[3])
```

Rubi [A] time = 0.200635, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 12, 779, 612, 619, 215}

$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4+\frac{77}{81}x^3\left(3x^2-x+2\right)^{5/2}+\frac{913}{486}x^2\left(3x^2-x+2\right)^{5/2}-\frac{11(283-5850x)\left(3x^2-x+2\right)^{5/2}}{58320}+\frac{545}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```


```
[Out] (1255639*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 + (54593*(1 - 6*x)*(2 - x +
 3*x^2)^(3/2))/559872 - (11*(283 - 5850*x)*(2 - x + 3*x^2)^(5/2))/58320 + (
913*x^2*(2 - x + 3*x^2)^(5/2))/486 + (77*x^3*(2 - x + 3*x^2)^(5/2))/81 + (2
*(1 + 2*x)^4*(2 - x + 3*x^2)^(5/2))/27 + (28879697*ArcSinh[(1 - 6*x)/Sqrt[2
3]])/(8957952*Sqrt[3])
```


Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
```

```
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{1}{108} \int 308x(1+2x)^3 \left(2-x+3x^2\right)^{3/2} dx$$
$$= \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{77}{27} \int x(1+2x)^3 \left(2-x+3x^2\right)^{3/2} dx$$
$$= \frac{77}{81}x^3 \left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{77}{648} \int x \left(2-x\right.$$
$$= \frac{913}{486}x^2 \left(2-x+3x^2\right)^{5/2} + \frac{77}{81}x^3 \left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4 \left(2-x+\right.$$
$$= -\frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{913}{486}x^2 \left(2-x+3x^2\right)^{5/2} + \frac{77}{81}x^3 \left(2\right.$$
$$= \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320} + \ldots$$
$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \ldots$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \ldots$$
$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \ldots$$

Mathematica [A] time = 0.0498844, size = 80, normalized size = 0.51

$$\frac{6\sqrt{3x^2-x+2}\left(238878720x^8+510105600x^7+635765760x^6+711210240x^5+649452672x^4+421626672x^3+2012897\ldots\right.}{134369280}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(12499587 + 84014278*x + 201289704*x^2 + 421626672*x
^3 + 649452672*x^4 + 711210240*x^5 + 635765760*x^6 + 510105600*x^7 + 238878
720*x^8) - 144398485*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/134369280
```

Maple [A] time = 0.055, size = 134, normalized size = 0.9

$$\frac{32 x^4}{27}\left(3 x^2-x+2\right)^{\frac{5}{2}}+\frac{269 x^3}{81}\left(3 x^2-x+2\right)^{\frac{5}{2}}+\frac{1777 x^2}{486}\left(3 x^2-x+2\right)^{\frac{5}{2}}+\frac{1099 x}{648}\left(3 x^2-x+2\right)^{\frac{5}{2}}-\frac{-1255639+7533}{4478976}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

```
[Out] 32/27*x^4*(3*x^2-x+2)^(5/2)+269/81*x^3*(3*x^2-x+2)^(5/2)+1777/486*x^2*(3*x^
2-x+2)^(5/2)+1099/648*x*(3*x^2-x+2)^(5/2)-1255639/4478976*(-1+6*x)*(3*x^2-x
+2)^(1/2)-28879697/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-54593/55
9872*(-1+6*x)*(3*x^2-x+2)^(3/2)+1207/58320*(3*x^2-x+2)^(5/2)
```

Maxima [A] time = 1.54024, size = 209, normalized size = 1.32

$$\frac{32}{27}\left(3 x^2-x+2\right)^{\frac{5}{2}} x^4+\frac{269}{81}\left(3 x^2-x+2\right)^{\frac{5}{2}} x^3+\frac{1777}{486}\left(3 x^2-x+2\right)^{\frac{5}{2}} x^2+\frac{1099}{648}\left(3 x^2-x+2\right)^{\frac{5}{2}} x+\frac{1207}{58320}\left(3 x^2-x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 32/27*(3*x^2 - x + 2)^(5/2)*x^4 + 269/81*(3*x^2 - x + 2)^(5/2)*x^3 + 1777/4
86*(3*x^2 - x + 2)^(5/2)*x^2 + 1099/648*(3*x^2 - x + 2)^(5/2)*x + 1207/5832
0*(3*x^2 - x + 2)^(5/2) - 54593/93312*(3*x^2 - x + 2)^(3/2)*x + 54593/55987
2*(3*x^2 - x + 2)^(3/2) - 1255639/746496*sqrt(3*x^2 - x + 2)*x - 28879697/2
6873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 1255639/4478976*sqrt(3*x
^2 - x + 2)
```

Fricas [A] time = 1.40745, size = 354, normalized size = 2.24

$$\frac{1}{22394880}\left(238878720 x^8+510105600 x^7+635765760 x^6+711210240 x^5+649452672 x^4+421626672 x^3+2012897\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/22394880*(238878720*x^8 + 510105600*x^7 + 635765760*x^6 + 711210240*x^5 +
 649452672*x^4 + 421626672*x^3 + 201289704*x^2 + 84014278*x + 12499587)*sqr
t(3*x^2 - x + 2) + 28879697/53747712*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x +
 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{\frac{3}{2}} \left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A] time = 1.17993, size = 119, normalized size = 0.75

$$\frac{1}{22394880}(2(12(6(8(30(36(2(96x+205)x+511)x+20579)x+563761)x+2927963)x+8387071)x+42007139)x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/22394880*(2*(12*(6*(8*(30*(36*(2*(96*x + 205)*x + 511)*x + 20579)*x + 563
761)*x + 2927963)*x + 8387071)*x + 42007139)*x + 12499587)*sqrt(3*x^2 - x +
 2) + 28879697/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x
+ 2)) + 1)
```

3.215 $\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=141

$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3+\frac{8}{63}\left(3x^2-x+2\right)^{5/2}(2x+1)^2+\frac{13(50x+29)\left(3x^2-x+2\right)^{5/2}}{2520}+\frac{91(1-6x)\left(3x^2-x+2\right)}{3456}$$

```
[Out] (2093*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/27648 + (91*(1 - 6*x)*(2 - x + 3*x^2)^
(3/2))/3456 + (8*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/63 + ((1 + 2*x)^3*(2 -
x + 3*x^2)^(5/2))/12 + (13*(29 + 50*x)*(2 - x + 3*x^2)^(5/2))/2520 + (48139
*ArcSinh[(1 - 6*x)/Sqrt[23]])/(55296*Sqrt[3])
```

Rubi [A] time = 0.12095, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3+\frac{8}{63}\left(3x^2-x+2\right)^{5/2}(2x+1)^2+\frac{13(50x+29)\left(3x^2-x+2\right)^{5/2}}{2520}+\frac{91(1-6x)\left(3x^2-x+2\right)}{3456}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2093*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/27648 + (91*(1 - 6*x)*(2 - x + 3*x^2)^
(3/2))/3456 + (8*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/63 + ((1 + 2*x)^3*(2 -
x + 3*x^2)^(5/2))/12 + (13*(29 + 50*x)*(2 - x + 3*x^2)^(5/2))/2520 + (48139
*ArcSinh[(1 - 6*x)/Sqrt[23]])/(55296*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{1}{12}(1+2x)^3 \left(2-x+3x^2\right)^{5/2} + \frac{1}{96}\int (1+2x)^2(20+256x) \left(2-x+3x^2\right.$$

$$= \frac{8}{63}(1+2x)^2 \left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2x)^3 \left(2-x+3x^2\right)^{5/2} + \frac{\int(1+2}{}$$

$$= \frac{8}{63}(1+2x)^2 \left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2x)^3 \left(2-x+3x^2\right)^{5/2} + \frac{13(29+}{}$$

$$= \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}(1+2x)^2 \left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2$$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}(1+2x)$$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}(1+2x)$$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}(1+2x)$$

Mathematica [A] time = 0.0433892, size = 75, normalized size = 0.53

$$\frac{6\sqrt{3x^2-x+2}\left(5806080x^7+9262080x^6+10656000x^5+12173952x^4+10119792x^3+5694024x^2+2735918x+1517\right.}{5806080}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(1517367 + 2735918*x + 5694024*x^2 + 10119792*x^3 +
12173952*x^4 + 10656000*x^5 + 9262080*x^6 + 5806080*x^7) - 1684865*Sqrt[3]*
ArcSinh[(-1 + 6*x)/Sqrt[23]])/5806080
```

Maple [A] time = 0.055, size = 117, normalized size = 0.8

$$\frac{2x^3}{3}\left(3x^2-x+2\right)^{\frac{5}{2}} + \frac{95x^2}{63}\left(3x^2-x+2\right)^{\frac{5}{2}} + \frac{319x}{252}\left(3x^2-x+2\right)^{\frac{5}{2}} - \frac{-2093+12558x}{27648}\sqrt{3x^2-x+2} - \frac{48139\sqrt{3}}{165888}\mathrm{Arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

```
[Out] 2/3*x^3*(3*x^2-x+2)^(5/2)+95/63*x^2*(3*x^2-x+2)^(5/2)+319/252*x*(3*x^2-x+2)
^(5/2)-2093/27648*(-1+6*x)*(3*x^2-x+2)^(1/2)-48139/165888*3^(1/2)*arcsinh(6
/23*23^(1/2)*(x-1/6))-91/3456*(-1+6*x)*(3*x^2-x+2)^(3/2)+907/2520*(3*x^2-x+
2)^(5/2)
```

Maxima [A] time = 1.50437, size = 186, normalized size = 1.32

$$\frac{2}{3}\left(3x^2-x+2\right)^{\frac{5}{2}}x^3+\frac{95}{63}\left(3x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{319}{252}\left(3x^2-x+2\right)^{\frac{5}{2}}x+\frac{907}{2520}\left(3x^2-x+2\right)^{\frac{5}{2}}-\frac{91}{576}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{91}{345}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 2/3*(3*x^2 - x + 2)^(5/2)*x^3 + 95/63*(3*x^2 - x + 2)^(5/2)*x^2 + 319/252*(
3*x^2 - x + 2)^(5/2)*x + 907/2520*(3*x^2 - x + 2)^(5/2) - 91/576*(3*x^2 - x
 + 2)^(3/2)*x + 91/3456*(3*x^2 - x + 2)^(3/2) - 2093/4608*sqrt(3*x^2 - x +
2)*x - 48139/165888*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2093/27648*s
qrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.62997, size = 308, normalized size = 2.18

$$\frac{1}{967680}\left(5806080x^7+9262080x^6+10656000x^5+12173952x^4+10119792x^3+5694024x^2+2735918x+1517367\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/967680*(5806080*x^7 + 9262080*x^6 + 10656000*x^5 + 12173952*x^4 + 1011979
2*x^3 + 5694024*x^2 + 2735918*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/3317
76*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{\frac{3}{2}} \left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A] time = 1.15427, size = 112, normalized size = 0.79

$$\frac{1}{967680}\,(2\,(12\,(2\,(8\,(30\,(12\,(42\,x+67)x+925)x+31703)x+210829)x+237251)x+1367959)x+1517367)\sqrt{3\,x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/967680*(2*(12*(2*(8*(30*(12*(42*x + 67)*x + 925)*x + 31703)*x + 210829)*x
 + 237251)*x + 1367959)*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/165888*sqr
t(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.216 $\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=116

$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}+\frac{1}{378}(102x+109)\left(3x^2-x+2\right)^{5/2}-\frac{71(1-6x)\left(3x^2-x+2\right)^{3/2}}{2592}-\frac{1633(1-6x)\sqrt{3x^2-x+}}{20736}$$

```
[Out] (-1633*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/20736 - (71*(1 - 6*x)*(2 - x + 3*x^2)
^(3/2))/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/21 + ((109 + 102*x)*(2
 - x + 3*x^2)^(5/2))/378 - (37559*ArcSinh[(1 - 6*x)/Sqrt[23]])/(41472*Sqrt[
3])
```

Rubi [A] time = 0.0822707, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}+\frac{1}{378}(102x+109)\left(3x^2-x+2\right)^{5/2}-\frac{71(1-6x)\left(3x^2-x+2\right)^{3/2}}{2592}-\frac{1633(1-6x)\sqrt{3x^2-x+}}{20736}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1633*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/20736 - (71*(1 - 6*x)*(2 - x + 3*x^2)
^(3/2))/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/21 + ((109 + 102*x)*(2
 - x + 3*x^2)^(5/2))/378 - (37559*ArcSinh[(1 - 6*x)/Sqrt[23]])/(41472*Sqrt[
3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)dx = \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{84}\int (1+2x)(40+204x)\left(2-x+3x^2\right)^{3/2}$$

$$= \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/2}+\frac{71}{108}$$

$$= -\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{378}(109+$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2$$

Mathematica [A] time = 0.035977, size = 70, normalized size = 0.6

$$\frac{6\sqrt{3x^2-x+2}\left(497664x^6+518400x^5+653184x^4+744336x^3+531384x^2+275410x+203337\right)+262913\sqrt{3}\sinh^{-1}\left(\right.}{870912}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(203337 + 275410*x + 531384*x^2 + 744336*x^3 + 65318
4*x^4 + 518400*x^5 + 497664*x^6) + 262913*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[2
3]])/870912
```

Maple [A] time = 0.051, size = 100, normalized size = 0.9

$$\frac{8x^2}{21}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{41x}{63}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{145}{378}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{-71+426x}{2592}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{-1633+9798x}{20736}\sqrt{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

```
[Out] 8/21*x^2*(3*x^2-x+2)^(5/2)+41/63*x*(3*x^2-x+2)^(5/2)+145/378*(3*x^2-x+2)^(5
/2)+71/2592*(-1+6*x)*(3*x^2-x+2)^(3/2)+1633/20736*(-1+6*x)*(3*x^2-x+2)^(1/2
)+37559/124416*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A] time = 1.50264, size = 163, normalized size = 1.41

$$\frac{8}{21}\left(3x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{41}{63}\left(3x^2-x+2\right)^{\frac{5}{2}}x+\frac{145}{378}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{71}{432}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{71}{2592}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{16}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/21*(3*x^2 - x + 2)^(5/2)*x^2 + 41/63*(3*x^2 - x + 2)^(5/2)*x + 145/378*(3
*x^2 - x + 2)^(5/2) + 71/432*(3*x^2 - x + 2)^(3/2)*x - 71/2592*(3*x^2 - x +
 2)^(3/2) + 1633/3456*sqrt(3*x^2 - x + 2)*x + 37559/124416*sqrt(3)*arcsinh(
1/23*sqrt(23)*(6*x - 1)) - 1633/20736*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.56758, size = 277, normalized size = 2.39

$$\frac{1}{145152}\left(497664\,x^6+518400\,x^5+653184\,x^4+744336\,x^3+531384\,x^2+275410\,x+203337\right)\sqrt{3\,x^2-x+2}+\frac{37559}{248832}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/145152*(497664*x^6 + 518400*x^5 + 653184*x^4 + 744336*x^3 + 531384*x^2 +
275410*x + 203337)*sqrt(3*x^2 - x + 2) + 37559/248832*sqrt(3)*log(-4*sqrt(3
)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)\left(3x^2-x+2\right)^{\frac{3}{2}}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A] time = 1.17022, size = 105, normalized size = 0.91

$$\frac{1}{145152}\left(2\left(12\left(18\left(24\left(2\left(24x+25\right)x+63\right)x+1723\right)x+22141\right)x+137705\right)x+203337\right)\sqrt{3x^2-x+2}-\frac{37559}{124416}\sqrt{3}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/145152*(2*(12*(18*(24*(2*(24*x + 25)*x + 63)*x + 1723)*x + 22141)*x + 137
705)*x + 203337)*sqrt(3*x^2 - x + 2) - 37559/124416*sqrt(3)*log(-2*sqrt(3)*
(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.217 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx$

Optimal. Leaf size=124

$$\frac{2}{15}\left(3x^2-x+2\right)^{5/2}+\frac{1}{144}(30x+7)\left(3x^2-x+2\right)^{3/2}+\frac{(402x+869)\sqrt{3x^2-x+2}}{1152}-\frac{13}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right.$$

```
[Out] ((869 + 402*x)*Sqrt[2 - x + 3*x^2])/1152 + ((7 + 30*x)*(2 - x + 3*x^2)^(3/2
))/144 + (2*(2 - x + 3*x^2)^(5/2))/15 + (2203*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(2304*Sqrt[3]) - (13*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*
x^2])])/32
```

Rubi [A] time = 0.144169, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{15}\left(3x^2-x+2\right)^{5/2}+\frac{1}{144}(30x+7)\left(3x^2-x+2\right)^{3/2}+\frac{(402x+869)\sqrt{3x^2-x+2}}{1152}-\frac{13}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```



```
[Out] ((869 + 402*x)*Sqrt[2 - x + 3*x^2])/1152 + ((7 + 30*x)*(2 - x + 3*x^2)^(3/2
))/144 + (2*(2 - x + 3*x^2)^(5/2))/15 + (2203*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(2304*Sqrt[3]) - (13*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*
x^2])])/32
```


Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{1+2x}\,dx = \frac{2}{15}\left(2-x+3x^2\right)^{5/2} + \frac{1}{60}\int \frac{(80+100x)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,dx$$

$$= \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x+3x^2\right)^{5/2} - \frac{\int \frac{(-13380-8040x)\sqrt{2-x+3x^2}}{1+2x}\,d}{5760}$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x+3x^2\right)$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x+3x^2\right)$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x+3x^2\right)$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x+3x^2\right)$$

Mathematica [A] time = 0.0577476, size = 96, normalized size = 0.77

$$\frac{6\sqrt{3x^2-x+2}\left(6912x^4-1008x^3+9624x^2+1058x+7977\right)-14040\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-11015\sqrt{3}\sinh^{-1}\left(\frac{6x}{\sqrt{2}}\right.}{34560}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(7977 + 1058*x + 9624*x^2 - 1008*x^3 + 6912*x^4) - 1
1015*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]] - 14040*Sqrt[13]*ArcTanh[(9 - 8*x
)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/34560
```

Maple [A] time = 0.051, size = 151, normalized size = 1.2

$$\frac{2}{15}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{-5+30x}{144}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{-115+690x}{1152}\sqrt{3x^2-x+2}-\frac{2203\sqrt{3}}{6912}\mathrm{Arcsinh}\left(\frac{6\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right)+\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x)
```

```
[Out] 2/15*(3*x^2-x+2)^(5/2)+5/144*(-1+6*x)*(3*x^2-x+2)^(3/2)+115/1152*(-1+6*x)*(
3*x^2-x+2)^(1/2)-2203/6912*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+1/12*(3*(
x+1/2)^2-4*x+5/4)^(3/2)-1/24*(-1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(1/2)+13/32*(12
*(x+1/2)^2-16*x+5)^(1/2)-13/32*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12
*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A] time = 1.48406, size = 169, normalized size = 1.36

$$\frac{2}{15}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{5}{24}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{7}{144}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{67}{192}\sqrt{3x^2-x+2}x-\frac{2203}{6912}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^2 - x + 2)^(5/2) + 5/24*(3*x^2 - x + 2)^(3/2)*x + 7/144*(3*x^2 -
x + 2)^(3/2) + 67/192*sqrt(3*x^2 - x + 2)*x - 2203/6912*sqrt(3)*arcsinh(6/2
3*sqrt(23)*x - 1/23*sqrt(23)) + 13/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(
2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 869/1152*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.6081, size = 367, normalized size = 2.96

$$\frac{1}{5760}\left(6912x^4-1008x^3+9624x^2+1058x+7977\right)\sqrt{3x^2-x+2}+\frac{2203}{13824}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/5760*(6912*x^4 - 1008*x^3 + 9624*x^2 + 1058*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/13824*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2
+ 24*x - 25) + 13/64*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9
) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2 - x + 2\right)^{\frac{3}{2}}\left(4x^2 + 3x + 1\right)}{2x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Giac [A] time = 1.28475, size = 184, normalized size = 1.48

$$\frac{1}{5760}\left(2\left(12\left(6\left(48x - 7\right)x + 401\right)x + 529\right)x + 7977\right)\sqrt{3x^2 - x + 2} + \frac{2203}{6912}\sqrt{3}\log\left(-6\sqrt{3}x + \sqrt{3} + 6\sqrt{3x^2 - x + 2}\right) + \frac{13}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/5760*(2*(12*(6*(48*x - 7)*x + 401)*x + 529)*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/6912*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) +
13/32*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

3.218 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2} dx$

Optimal. Leaf size=131

$$-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}-\frac{1}{104}(23-38x)(3x^2-x+2)^{3/2}-\frac{1}{192}(349-294x)\sqrt{3x^2-x+2}+\frac{25}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+ \dots}}\right.$$

```
[Out] -((349 - 294*x)*Sqrt[2 - x + 3*x^2])/192 - ((23 - 38*x)*(2 - x + 3*x^2)^(3/
2))/104 - (2 - x + 3*x^2)^(5/2)/(13*(1 + 2*x)) - (2327*ArcSinh[(1 - 6*x)/Sq
rt[23]])/(384*Sqrt[3]) + (25*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2
- x + 3*x^2])])/32
```

Rubi [A] time = 0.140166, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}-\frac{1}{104}(23-38x)(3x^2-x+2)^{3/2}-\frac{1}{192}(349-294x)\sqrt{3x^2-x+2}+\frac{25}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+ \dots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```


```
[Out] -((349 - 294*x)*Sqrt[2 - x + 3*x^2])/192 - ((23 - 38*x)*(2 - x + 3*x^2)^(3/
2))/104 - (2 - x + 3*x^2)^(5/2)/(13*(1 + 2*x)) - (2327*ArcSinh[(1 - 6*x)/Sq
rt[23]])/(384*Sqrt[3]) + (25*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2
- x + 3*x^2])])/32
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = -\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{13}{2}-38x\right)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,dx$$

$$= -\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)} + \frac{\int \frac{(-78+7644x)\sqrt{2-x+3x^2}}{1+2x}\,dx}{1248}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{\left(2-x+3x^2\right.}{13(1+2x}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{\left(2-x+3x^2\right.}{13(1+2x}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{\left(2-x+3x^2\right.}{13(1+2x}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{\left(2-x+3x^2\right.}{13(1+2x}$$

Mathematica [A] time = 0.0899109, size = 103, normalized size = 0.79

$$\frac{\frac{6\sqrt{3x^2-x+2}\left(288x^4-96x^3+564x^2-332x-493\right)}{2x+1} + 900\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + 2327\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{1152}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-493 - 332*x + 564*x^2 - 96*x^3 + 288*x^4))/(1 + 2
*x) + 2327*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]] + 900*Sqrt[13]*ArcTanh[(9 -
 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/1152
```

Maple [A] time = 0.055, size = 179, normalized size = 1.4

$$\frac{-1+6x}{24}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{-23+138x}{192}\sqrt{3x^2-x+2}+\frac{2327\sqrt{3}}{1152}\mathrm{Arcsinh}\left(\frac{6\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right)-\frac{1}{26}\Bigg(3\left(x+1/2\right)^2-4x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x)
```

```
[Out] 1/24*(-1+6*x)*(3*x^2-x+2)^(3/2)+23/192*(-1+6*x)*(3*x^2-x+2)^(1/2)+2327/1152
*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^
(5/2)-25/156*(3*(x+1/2)^2-4*x+5/4)^(3/2)+13/96*(-1+6*x)*(3*(x+1/2)^2-4*x+5/
4)^(1/2)-25/32*(12*(x+1/2)^2-16*x+5)^(1/2)+25/32*13^(1/2)*arctanh(2/13*(9/2
-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))+1/52*(-1+6*x)*(3*(x+1/2)^2-4*x+
5/4)^(3/2)
```

Maxima [A] time = 1.49578, size = 178, normalized size = 1.36

$$\frac{1}{4}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{1}{8}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{49}{32}\sqrt{3x^2-x+2}x+\frac{2327}{1152}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{25}{32}\sqrt{13}\,\mathrm{arsinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*x^2 - x + 2)^(3/2)*x - 1/8*(3*x^2 - x + 2)^(3/2) + 49/32*sqrt(3*x^2
- x + 2)*x + 2327/1152*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 2
5/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
+ 1)) - 349/192*sqrt(3*x^2 - x + 2) - 1/4*(3*x^2 - x + 2)^(3/2)/(2*x + 1)
```

Fricas [A] time = 1.71441, size = 396, normalized size = 3.02

$$\frac{2327\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+900\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220}{4x^2+4x+1}\right.}{2304\,(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2304*(2327*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
 - 72*x^2 + 24*x - 25) + 900*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2
- x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12*(288*x^
4 - 96*x^3 + 564*x^2 - 332*x - 493)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2 - x + 2)^{\frac{3}{2}} (4x^2 + 3x + 1)}{(2x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Giac [B] time = 1.84953, size = 770, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 25/32*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt
(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 2327/1152*sqrt(3)*log(1/2*abs(-2*sq
rt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(
sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sg
n(1/(2*x + 1)) - 13/32*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x +
 1)) + 1/192*(5929*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^7*sgn(1/(2*x + 1)) - 7272*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)
^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*x + 1)) + 25101*(sqrt(-8/(2*x + 1)
 + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) - 48*sqrt(1
3)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(
2*x + 1)) + 112359*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^3*sgn(1/(2*x + 1)) - 69336*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1
```

```
)^2 + 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 71955*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 24624*sqrt
(13)*sgn(1/(2*x + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)
/(2*x + 1))^2 - 3)^4
```

3.219 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{(3x^2-x+2)^{5/2}}{26(2x+1)^2}+\frac{(122x+151)(3x^2-x+2)^{3/2}}{312(2x+1)}+\frac{1}{624}(1858-771x)\sqrt{3x^2-x+2}-\frac{1153\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}}+\frac{15}{}$$

```
[Out] ((1858 - 771*x)*Sqrt[2 - x + 3*x^2])/624 + ((151 + 122*x)*(2 - x + 3*x^2)^(
3/2))/(312*(1 + 2*x)) - (2 - x + 3*x^2)^(5/2)/(26*(1 + 2*x)^2) + (1519*ArcS
inh[(1 - 6*x)/Sqrt[23]])/(192*Sqrt[3]) - (1153*ArcTanh[(9 - 8*x)/(2*Sqrt[13
]*Sqrt[2 - x + 3*x^2])])/(64*Sqrt[13])
```

Rubi [A] time = 0.139158, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{5/2}}{26(2x+1)^2}+\frac{(122x+151)(3x^2-x+2)^{3/2}}{312(2x+1)}+\frac{1}{624}(1858-771x)\sqrt{3x^2-x+2}-\frac{1153\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}}+\frac{15}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((1858 - 771*x)*Sqrt[2 - x + 3*x^2])/624 + ((151 + 122*x)*(2 - x + 3*x^2)^(
3/2))/(312*(1 + 2*x)) - (2 - x + 3*x^2)^(5/2)/(26*(1 + 2*x)^2) + (1519*ArcS
inh[(1 - 6*x)/Sqrt[23]])/(192*Sqrt[3]) - (1153*ArcTanh[(9 - 8*x)/(2*Sqrt[13
]*Sqrt[2 - x + 3*x^2])])/(64*Sqrt[13])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
```

```
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = -\frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2}-\frac{1}{26}\int \frac{\left(-\frac{31}{2}-61x\right)\left(2-x+3x^2\right)^{3/2}}{(1+2x)^2}\,dx$$

$$= \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)}-\frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2}+\frac{1}{208}\int \frac{(639-1028x)\sqrt{2-x}}{1+2x}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2}+\frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)}-\frac{\left(2-x+3x^2\right)}{26(1+2x)^2}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2}+\frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)}-\frac{\left(2-x+3x^2\right)}{26(1+2x)^2}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2}+\frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)}-\frac{\left(2-x+3x^2\right)}{26(1+2x)^2}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2}+\frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)}-\frac{\left(2-x+3x^2\right)}{26(1+2x)^2}$$

Mathematica [A] time = 0.103753, size = 103, normalized size = 0.75

$$\frac{\frac{156\sqrt{3x^2-x+2}\left(96x^4-68x^3+390x^2+627x+182\right)}{(2x+1)^2}-10377\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-19747\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{7488}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((156*Sqrt[2 - x + 3*x^2]*(182 + 627*x + 390*x^2 - 68*x^3 + 96*x^4))/(1 + 2
*x)^2 - 19747*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]] - 10377*Sqrt[13]*ArcTanh
[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/7488
```

Maple [A] time = 0.063, size = 162, normalized size = 1.2

$$\frac{15}{338}\left(3\,(x+1/2)^2-4\,x+\frac{5}{4}\right)^{\frac{5}{2}}\left(x+\frac{1}{2}\right)^{-1}+\frac{1153}{4056}\left(3\,(x+1/2)^2-4\,x+\frac{5}{4}\right)^{\frac{3}{2}}-\frac{-257+1542\,x}{1248}\sqrt{3\,(x+1/2)^2-4\,x+\frac{5}{4}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x)
```

```
[Out] 15/338/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^(5/2)+1153/4056*(3*(x+1/2)^2-4*x+5/4)^
(3/2)-257/1248*(-1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(1/2)-1519/576*3^(1/2)*arcsin
h(6/23*23^(1/2)*(x-1/6))+1153/832*(12*(x+1/2)^2-16*x+5)^(1/2)-1153/832*13^(
1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-15/676*(-
1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(3/2)-1/104/(x+1/2)^2*(3*(x+1/2)^2-4*x+5/4)^(5
/2)
```

Maxima [A] time = 1.49185, size = 193, normalized size = 1.4

$$\frac{61}{312}\left(3\,x^2-x+2\right)^{\frac{3}{2}}-\frac{\left(3\,x^2-x+2\right)^{\frac{5}{2}}}{26\left(4\,x^2+4\,x+1\right)}-\frac{257}{208}\sqrt{3\,x^2-x+2}x-\frac{1519}{576}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{1153}{832}\sqrt{13}\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 61/312*(3*x^2 - x + 2)^(3/2) - 1/26*(3*x^2 - x + 2)^(5/2)/(4*x^2 + 4*x + 1)
 - 257/208*sqrt(3*x^2 - x + 2)*x - 1519/576*sqrt(3)*arcsinh(6/23*sqrt(23)*x
 - 1/23*sqrt(23)) + 1153/832*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1)
- 9/23*sqrt(23)/abs(2*x + 1)) + 929/312*sqrt(3*x^2 - x + 2) + 15/52*(3*x^2
```

```
- x + 2)^(3/2)/(2*x + 1)
```

Fricas [A] time = 1.68986, size = 433, normalized size = 3.14

$$\frac{19747\sqrt{3}\left(4x^2+4x+1\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+10377\sqrt{13}\left(4x^2+4x+1\right)\log\left(-\frac{4\sqrt{13}\sqrt{3}}{}\right.}{14976\left(4x^2+4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/14976*(19747*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25) + 10377*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*
sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*
x + 1)) + 312*(96*x^4 - 68*x^3 + 390*x^2 + 627*x + 182)*sqrt(3*x^2 - x + 2)
)/(4*x^2 + 4*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2-x+2\right)^{\frac{3}{2}}\left(4x^2+3x+1\right)}{(2x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.220 $\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=189

$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4+\frac{29}{330}\left(3x^2-x+2\right)^{7/2}(2x+1)^3+\frac{133\left(3x^2-x+2\right)^{7/2}(2x+1)^2}{1485}-\frac{(26353-21350x)\left(3x^2-\right.}{498960}$$

```
[Out] (2692081*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/11943936 + (117047*(1 - 6*x)*(2 - x
 + 3*x^2)^(3/2))/1492992 + (5089*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/155520 -
((26353 - 21350*x)*(2 - x + 3*x^2)^(7/2))/498960 + (133*(1 + 2*x)^2*(2 - x
+ 3*x^2)^(7/2))/1485 + (29*(1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/330 + (2*(1 +
 2*x)^4*(2 - x + 3*x^2)^(7/2))/33 + (61917863*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(23887872*Sqrt[3])
```

Rubi [A] time = 0.155952, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4+\frac{29}{330}\left(3x^2-x+2\right)^{7/2}(2x+1)^3+\frac{133\left(3x^2-x+2\right)^{7/2}(2x+1)^2}{1485}-\frac{(26353-21350x)\left(3x^2-\right.}{498960}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2692081*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/11943936 + (117047*(1 - 6*x)*(2 - x
 + 3*x^2)^(3/2))/1492992 + (5089*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/155520 -
((26353 - 21350*x)*(2 - x + 3*x^2)^(7/2))/498960 + (133*(1 + 2*x)^2*(2 - x
+ 3*x^2)^(7/2))/1485 + (29*(1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/330 + (2*(1 +
 2*x)^4*(2 - x + 3*x^2)^(7/2))/33 + (61917863*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(23887872*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
```

```
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \frac{2}{33}(1+2x)^4 \left(2-x+3x^2\right)^{7/2} + \frac{1}{132} \int (1+2x)^3(32+348x) \left(2-x+3x^2\right.$$

$$= \frac{29}{330}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + \frac{2}{33}(1+2x)^4 \left(2-x+3x^2\right)^{7/2} + \frac{\int (1+2\ldots}{}$$

$$= \frac{133(1+2x)^2 \left(2-x+3x^2\right)^{7/2}}{1485} + \frac{29}{330}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + \frac{2}{33}(1+\ldots$$

$$= -\frac{(26353-21350x) \left(2-x+3x^2\right)^{7/2}}{498960} + \frac{133(1+2x)^2 \left(2-x+3x^2\right)^{7/2}}{1485} + \ldots$$

$$= \frac{5089(1-6x) \left(2-x+3x^2\right)^{5/2}}{155520} - \frac{(26353-21350x) \left(2-x+3x^2\right)^{7/2}}{498960} + \ldots$$

$$= \frac{117047(1-6x) \left(2-x+3x^2\right)^{3/2}}{1492992} + \frac{5089(1-6x) \left(2-x+3x^2\right)^{5/2}}{155520} - \ldots$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x) \left(2-x+3x^2\right)^{3/2}}{1492992} + \ldots$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x) \left(2-x+3x^2\right)^{3/2}}{1492992} + \ldots$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x) \left(2-x+3x^2\right)^{3/2}}{1492992} + \ldots$$

Mathematica [A] time = 0.0599622, size = 90, normalized size = 0.48

$$\frac{6\sqrt{3x^2-x+2}\left(120394874880x^{10}+207681159168x^9+308846297088x^8+419978151936x^7+415908006912x^6+3472\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(9173509857 + 26646633218*x + 72088585464*x^2 + 1612
69204752*x^3 + 263636134272*x^4 + 347247744768*x^5 + 415908006912*x^6 + 419
978151936*x^7 + 308846297088*x^8 + 207681159168*x^9 + 120394874880*x^10) -
23838377255*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/27590492160
```

Maple [A] time = 0.059, size = 153, normalized size = 0.8

$$\frac{92423}{498960}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{32x^4}{33}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{436x^3}{165}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{4258x^2}{1485}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{10073x}{7128}\left(3x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

```
[Out] 92423/498960*(3*x^2-x+2)^(7/2)+32/33*x^4*(3*x^2-x+2)^(7/2)+436/165*x^3*(3*x
^2-x+2)^(7/2)+4258/1485*x^2*(3*x^2-x+2)^(7/2)+10073/7128*x*(3*x^2-x+2)^(7/2
)-2692081/11943936*(-1+6*x)*(3*x^2-x+2)^(1/2)-61917863/71663616*3^(1/2)*arc
sinh(6/23*23^(1/2)*(x-1/6))-5089/155520*(-1+6*x)*(3*x^2-x+2)^(5/2)-117047/1
492992*(-1+6*x)*(3*x^2-x+2)^(3/2)
```

Maxima [A] time = 1.49075, size = 248, normalized size = 1.31

$$\frac{32}{33}\left(3x^2-x+2\right)^{\frac{7}{2}}x^4+\frac{436}{165}\left(3x^2-x+2\right)^{\frac{7}{2}}x^3+\frac{4258}{1485}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2+\frac{10073}{7128}\left(3x^2-x+2\right)^{\frac{7}{2}}x+\frac{92423}{498960}\left(3x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 32/33*(3*x^2 - x + 2)^(7/2)*x^4 + 436/165*(3*x^2 - x + 2)^(7/2)*x^3 + 4258/
1485*(3*x^2 - x + 2)^(7/2)*x^2 + 10073/7128*(3*x^2 - x + 2)^(7/2)*x + 92423
/498960*(3*x^2 - x + 2)^(7/2) - 5089/25920*(3*x^2 - x + 2)^(5/2)*x + 5089/1
55520*(3*x^2 - x + 2)^(5/2) - 117047/248832*(3*x^2 - x + 2)^(3/2)*x + 11704
7/1492992*(3*x^2 - x + 2)^(3/2) - 2692081/1990656*sqrt(3*x^2 - x + 2)*x - 6
1917863/71663616*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2692081/1194393
6*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.45142, size = 444, normalized size = 2.35

$$\frac{1}{4598415360}\left(120394874880x^{10}+207681159168x^9+308846297088x^8+419978151936x^7+415908006912x^6+347\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/4598415360*(120394874880*x^10 + 207681159168*x^9 + 308846297088*x^8 + 419
978151936*x^7 + 415908006912*x^6 + 347247744768*x^5 + 263636134272*x^4 + 16
1269204752*x^3 + 72088585464*x^2 + 26646633218*x + 9173509857)*sqrt(3*x^2 -
 x + 2) + 61917863/143327232*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{\frac{5}{2}} \left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A] time = 1.27783, size = 132, normalized size = 0.7

$$\frac{1}{4598415360}(2(12(6(8(6(36(14(48(18(40x+69)x+1847)x+120557)x+1671441)x+50238389)x+228850811)x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/4598415360*(2*(12*(6*(8*(6*(36*(14*(48*(18*(40*x + 69)*x + 1847)*x + 1205
57)*x + 1671441)*x + 50238389)*x + 228850811)*x + 1119925033)*x + 300369106
1)*x + 13323316609)*x + 9173509857)*sqrt(3*x^2 - x + 2) + 61917863/71663616
*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.221 $\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=164

$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}+\frac{37}{405}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{(3430x+2731)\left(3x^2-x+2\right)^{7/2}}{17010}-\frac{293(1-6x)\left(3x^2-x\right.}{58320}$$

```
[Out] (-154997*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 - (6739*(1 - 6*x)*(2 - x +
3*x^2)^(3/2))/559872 - (293*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/58320 + (37*(1
 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/405 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/
15 + ((2731 + 3430*x)*(2 - x + 3*x^2)^(7/2))/17010 - (3564931*ArcSinh[(1 -
6*x)/Sqrt[23]])/(8957952*Sqrt[3])
```

Rubi [A] time = 0.132842, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}+\frac{37}{405}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{(3430x+2731)\left(3x^2-x+2\right)^{7/2}}{17010}-\frac{293(1-6x)\left(3x^2-x\right.}{58320}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-154997*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 - (6739*(1 - 6*x)*(2 - x +
3*x^2)^(3/2))/559872 - (293*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/58320 + (37*(1
 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/405 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/
15 + ((2731 + 3430*x)*(2 - x + 3*x^2)^(7/2))/17010 - (3564931*ArcSinh[(1 -
6*x)/Sqrt[23]])/(8957952*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
```

```
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \frac{1}{15}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + \frac{1}{120} \int (1+2x)^2(52+296x) \left(2-x+3x\right.$$

$$= \frac{37}{405}(1+2x)^2 \left(2-x+3x^2\right)^{7/2} + \frac{1}{15}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + \frac{\int (1+}{}$$

$$= \frac{37}{405}(1+2x)^2 \left(2-x+3x^2\right)^{7/2} + \frac{1}{15}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + \frac{(2731}{}$$

$$= -\frac{293(1-6x) \left(2-x+3x^2\right)^{5/2}}{58320} + \frac{37}{405}(1+2x)^2 \left(2-x+3x^2\right)^{7/2} + \frac{1}{15}(1$$

$$= -\frac{6739(1-6x) \left(2-x+3x^2\right)^{3/2}}{559872} - \frac{293(1-6x) \left(2-x+3x^2\right)^{5/2}}{58320} + \frac{37}{405}($$

$$= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x) \left(2-x+3x^2\right)^{3/2}}{559872} - \frac{293(}{}$$

$$= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x) \left(2-x+3x^2\right)^{3/2}}{559872} - \frac{293(}{}$$

$$= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x) \left(2-x+3x^2\right)^{3/2}}{559872} - \frac{293(}{}$$

Mathematica [A] time = 0.0550144, size = 85, normalized size = 0.52

$$\frac{6\sqrt{3x^2-x+2} \left(2257403904x^9 + 2675441664x^8 + 4427716608x^7 + 5671627776x^6 + 4996802304x^5 + 4171579776x^4 +\right.}{940584960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(387182961 + 692659234*x + 1693765752*x^2 + 30961049
76*x^3 + 4171579776*x^4 + 4996802304*x^5 + 5671627776*x^6 + 4427716608*x^7
+ 2675441664*x^8 + 2257403904*x^9) + 124772585*Sqrt[3]*ArcSinh[(-1 + 6*x)/S
qrt[23]])/940584960
```

Maple [A] time = 0.056, size = 136, normalized size = 0.8

$$\frac{5419}{17010} \left(3\,x^2-x+2\right)^{\frac{7}{2}} + \frac{8\,x^3}{15} \left(3\,x^2-x+2\right)^{\frac{7}{2}} + \frac{472\,x^2}{405} \left(3\,x^2-x+2\right)^{\frac{7}{2}} + \frac{235\,x}{243} \left(3\,x^2-x+2\right)^{\frac{7}{2}} + \frac{-154997+929982\,x}{4478976}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

```
[Out] 5419/17010*(3*x^2-x+2)^(7/2)+8/15*x^3*(3*x^2-x+2)^(7/2)+472/405*x^2*(3*x^2-
x+2)^(7/2)+235/243*x*(3*x^2-x+2)^(7/2)+154997/4478976*(-1+6*x)*(3*x^2-x+2)^
(1/2)+3564931/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+293/58320*(-1
+6*x)*(3*x^2-x+2)^(5/2)+6739/559872*(-1+6*x)*(3*x^2-x+2)^(3/2)
```

Maxima [A] time = 1.48004, size = 225, normalized size = 1.37

$$\frac{8}{15}\left(3x^2-x+2\right)^{\frac{7}{2}}x^3+\frac{472}{405}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2+\frac{235}{243}\left(3x^2-x+2\right)^{\frac{7}{2}}x+\frac{5419}{17010}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{293}{9720}\left(3x^2-x+2\right)^{\frac{5}{2}}x -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/15*(3*x^2 - x + 2)^(7/2)*x^3 + 472/405*(3*x^2 - x + 2)^(7/2)*x^2 + 235/24
3*(3*x^2 - x + 2)^(7/2)*x + 5419/17010*(3*x^2 - x + 2)^(7/2) + 293/9720*(3*
x^2 - x + 2)^(5/2)*x - 293/58320*(3*x^2 - x + 2)^(5/2) + 6739/93312*(3*x^2
- x + 2)^(3/2)*x - 6739/559872*(3*x^2 - x + 2)^(3/2) + 154997/746496*sqrt(3
*x^2 - x + 2)*x + 3564931/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 154997/4478976*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.32793, size = 390, normalized size = 2.38

$$\frac{1}{156764160}\left(2257403904\,x^9+2675441664\,x^8+4427716608\,x^7+5671627776\,x^6+4996802304\,x^5+4171579776\,x^4+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/156764160*(2257403904*x^9 + 2675441664*x^8 + 4427716608*x^7 + 5671627776*
x^6 + 4996802304*x^5 + 4171579776*x^4 + 3096104976*x^3 + 1693765752*x^2 + 6
92659234*x + 387182961)*sqrt(3*x^2 - x + 2) + 3564931/53747712*sqrt(3)*log(
-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{\frac{5}{2}} \left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A] time = 1.19182, size = 126, normalized size = 0.77

$$\frac{1}{156764160}\left(2\left(12\left(6\left(8\left(6\left(36\left(14\left(24\left(27x+32\right)x+1271\right)x+22793\right)x+722917\right)x+3621163\right)x+21500729\right)x+705735\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/156764160*(2*(12*(6*(8*(6*(36*(14*(24*(27*x + 32)*x + 1271)*x + 22793)*x
+ 722917)*x + 3621163)*x + 21500729)*x + 70573573)*x + 346329617)*x + 38718
2961)*sqrt(3*x^2 - x + 2) - 3564931/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.222 $\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=139

$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{1}{648}(122x+137)\left(3x^2-x+2\right)^{7/2}-\frac{445(1-6x)\left(3x^2-x+2\right)^{5/2}}{15552}-\frac{51175(1-6x)\left(3x^2-x\right.}{746496}$$

```
[Out] (-1177025*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/5971968 - (51175*(1 - 6*x)*(2 - x
+ 3*x^2)^(3/2))/746496 - (445*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/15552 + (2*(
1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + ((137 + 122*x)*(2 - x + 3*x^2)^(7/2)
)/648 - (27071575*ArcSinh[(1 - 6*x)/Sqrt[23]])/(11943936*Sqrt[3])
```

Rubi [A] time = 0.0917934, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{1}{648}(122x+137)\left(3x^2-x+2\right)^{7/2}-\frac{445(1-6x)\left(3x^2-x+2\right)^{5/2}}{15552}-\frac{51175(1-6x)\left(3x^2-x\right.}{746496}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1177025*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/5971968 - (51175*(1 - 6*x)*(2 - x
+ 3*x^2)^(3/2))/746496 - (445*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/15552 + (2*(
1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + ((137 + 122*x)*(2 - x + 3*x^2)^(7/2)
)/648 - (27071575*ArcSinh[(1 - 6*x)/Sqrt[23]])/(11943936*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)dx = \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{108}\int (1+2x)(72+244x)\left(2-x+3x^2\right)^{5}$$

$$= \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2}+\frac{445}{432}$$

$$= -\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137$$

$$= -\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(}{}$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(}{}$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(}{}$$

Mathematica [A] time = 0.0447259, size = 80, normalized size = 0.58

$$\frac{6\sqrt{3x^2-x+2}\left(47775744x^8+30357504x^7+79377408x^6+80034048x^5+66969216x^4+58946544x^3+41031048x^2+\right.}{35831808}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(10960335 + 19860062*x + 41031048*x^2 + 58946544*x^3
 + 66969216*x^4 + 80034048*x^5 + 79377408*x^6 + 30357504*x^7 + 47775744*x^8
) + 27071575*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/35831808
```

Maple [A] time = 0.055, size = 119, normalized size = 0.9

$$\frac{8x^2}{27}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{157x}{324}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{185}{648}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{-445+2670x}{15552}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{-51175+30705}{746496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

```
[Out] 8/27*x^2*(3*x^2-x+2)^(7/2)+157/324*x*(3*x^2-x+2)^(7/2)+185/648*(3*x^2-x+2)^
(7/2)+445/15552*(-1+6*x)*(3*x^2-x+2)^(5/2)+51175/746496*(-1+6*x)*(3*x^2-x+2
)^(3/2)+1177025/5971968*(-1+6*x)*(3*x^2-x+2)^(1/2)+27071575/35831808*3^(1/2
)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A] time = 1.56519, size = 203, normalized size = 1.46

$$\frac{8}{27}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2+\frac{157}{324}\left(3x^2-x+2\right)^{\frac{7}{2}}x+\frac{185}{648}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{445}{2592}\left(3x^2-x+2\right)^{\frac{5}{2}}x-\frac{445}{15552}\left(3x^2-x+2\right)^{\frac{5}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/27*(3*x^2 - x + 2)^(7/2)*x^2 + 157/324*(3*x^2 - x + 2)^(7/2)*x + 185/648*
(3*x^2 - x + 2)^(7/2) + 445/2592*(3*x^2 - x + 2)^(5/2)*x - 445/15552*(3*x^2
 - x + 2)^(5/2) + 51175/124416*(3*x^2 - x + 2)^(3/2)*x - 51175/746496*(3*x^
2 - x + 2)^(3/2) + 1177025/995328*sqrt(3*x^2 - x + 2)*x + 27071575/35831808
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 1177025/5971968*sqrt(3*x^2 - x
+ 2)
```

Fricas [A] time = 1.35887, size = 344, normalized size = 2.47

$$\frac{1}{5971968}\left(47775744\,x^8+30357504\,x^7+79377408\,x^6+80034048\,x^5+66969216\,x^4+58946544\,x^3+41031048\,x^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/5971968*(47775744*x^8 + 30357504*x^7 + 79377408*x^6 + 80034048*x^5 + 6696
9216*x^4 + 58946544*x^3 + 41031048*x^2 + 19860062*x + 10960335)*sqrt(3*x^2
- x + 2) + 27071575/71663616*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)\left(3x^2-x+2\right)^{\frac{5}{2}}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A] time = 1.2123, size = 119, normalized size = 0.86

$$\frac{1}{5971968}\left(2\left(12\left(6\left(8\left(6\left(36\left(2\left(96\,x+61\right)x+319\right)x+11579\right)x+58133\right)x+409351\right)x+1709627\right)x+9930031\right)x+109603$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/5971968*(2*(12*(6*(8*(6*(36*(2*(96*x + 61)*x + 319)*x + 11579)*x + 58133)
*x + 409351)*x + 1709627)*x + 9930031)*x + 10960335)*sqrt(3*x^2 - x + 2) -
27071575/35831808*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

3.223 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x} dx$

Optimal. Leaf size=147

$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}+\frac{(150x+29)\left(3x^2-x+2\right)^{5/2}}{1080}+\frac{(2154x+2449)\left(3x^2-x+2\right)^{3/2}}{10368}+\frac{(221999-17850x)\sqrt{3x^2-x+}}{82944}$$

```
[Out] ((221999 - 17850*x)*Sqrt[2 - x + 3*x^2])/82944 + ((2449 + 2154*x)*(2 - x +
3*x^2)^(3/2))/10368 + ((29 + 150*x)*(2 - x + 3*x^2)^(5/2))/1080 + (2*(2 - x
 + 3*x^2)^(7/2))/21 + (944521*ArcSinh[(1 - 6*x)/Sqrt[23]])/(165888*Sqrt[3])
 - (169*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/128
```

Rubi [A] time = 0.159525, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}+\frac{(150x+29)\left(3x^2-x+2\right)^{5/2}}{1080}+\frac{(2154x+2449)\left(3x^2-x+2\right)^{3/2}}{10368}+\frac{(221999-17850x)\sqrt{3x^2-x+}}{82944}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```



```
[Out] ((221999 - 17850*x)*Sqrt[2 - x + 3*x^2])/82944 + ((2449 + 2154*x)*(2 - x +
3*x^2)^(3/2))/10368 + ((29 + 150*x)*(2 - x + 3*x^2)^(5/2))/1080 + (2*(2 - x
 + 3*x^2)^(7/2))/21 + (944521*ArcSinh[(1 - 6*x)/Sqrt[23]])/(165888*Sqrt[3])
 - (169*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/128
```


Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{1+2x}\,dx = \frac{2}{21}\left(2-x+3x^2\right)^{7/2}+\frac{1}{84}\int \frac{(112+140x)\left(2-x+3x^2\right)^{5/2}}{1+2x}\,dx$$

$$= \frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080}+\frac{2}{21}\left(2-x+3x^2\right)^{7/2}-\frac{\int \frac{(-29708-20104x)\left(2-x+3x^2\right)^{3/2}}{1+2x}}{12096}$$

$$= \frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080}+\frac{2}{21}\left(2-x+3\right.$$

$$= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(29+15}{}$$

$$= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(29+15}{}$$

$$= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(29+15}{}$$

$$= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(29+15}{}$$

Mathematica [A] time = 0.0735996, size = 106, normalized size = 0.72

$$\frac{6\sqrt{3x^2-x+2}\left(7464960x^6-3836160x^5+15700608x^4-3646512x^3+12466776x^2-2120998x+11665053\right)-229975}{17418240}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(11665053 - 2120998*x + 12466776*x^2 - 3646512*x^3 +
 15700608*x^4 - 3836160*x^5 + 7464960*x^6) - 33058235*Sqrt[3]*ArcSinh[(-1 +
 6*x)/Sqrt[23]] - 22997520*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 -
```

```
x + 3*x^2])])/17418240
```

Maple [A] time = 0.052, size = 207, normalized size = 1.4

$$\frac{2}{21}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{-5+30x}{216}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{-575+3450x}{10368}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{-13225+79350x}{82944}\sqrt{3x^2-x+2}-\frac{94}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x)
```

```
[Out] 2/21*(3*x^2-x+2)^(7/2)+5/216*(-1+6*x)*(3*x^2-x+2)^(5/2)+575/10368*(-1+6*x)*
(3*x^2-x+2)^(3/2)+13225/82944*(-1+6*x)*(3*x^2-x+2)^(1/2)-944521/497664*3^(1
/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+1/20*(3*(x+1/2)^2-4*x+5/4)^(5/2)-1/48*(-
1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(3/2)-25/128*(-1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(1
/2)+13/48*(3*(x+1/2)^2-4*x+5/4)^(3/2)+169/128*(12*(x+1/2)^2-16*x+5)^(1/2)-1
69/128*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2)
)
```

Maxima [A] time = 1.51275, size = 208, normalized size = 1.41

$$\frac{2}{21}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{5}{36}\left(3x^2-x+2\right)^{\frac{5}{2}}x+\frac{29}{1080}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{359}{1728}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{2449}{10368}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{29}{138}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^2 - x + 2)^(7/2) + 5/36*(3*x^2 - x + 2)^(5/2)*x + 29/1080*(3*x^2
- x + 2)^(5/2) + 359/1728*(3*x^2 - x + 2)^(3/2)*x + 2449/10368*(3*x^2 - x +
 2)^(3/2) - 2975/13824*sqrt(3*x^2 - x + 2)*x - 944521/497664*sqrt(3)*arcsin
h(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 169/128*sqrt(13)*arcsinh(8/23*sqrt(23)
*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 221999/82944*sqrt(3*x^2 - x
 + 2)
```

Fricas [A] time = 1.37156, size = 440, normalized size = 2.99

$$\frac{1}{2903040}\left(7464960\,x^6-3836160\,x^5+15700608\,x^4-3646512\,x^3+12466776\,x^2-2120998\,x+11665053\right)\sqrt{3\,x^2-x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/2903040*(7464960*x^6 - 3836160*x^5 + 15700608*x^4 - 3646512*x^3 + 1246677
6*x^2 - 2120998*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/995328*sqrt(3)*l
og(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 169/256*
sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x +
 185)/(4*x^2 + 4*x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2-x+2\right)^{\frac{5}{2}}\left(4x^2+3x+1\right)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Giac [A] time = 1.34667, size = 197, normalized size = 1.34

$$\frac{1}{2903040}\left(2\left(12\left(18\left(8\left(30\left(72\,x-37\right)x+4543\right)x-8441\right)x+519449\right)x-1060499\right)x+11665053\right)\sqrt{3\,x^2-x+2}+\frac{94452}{49766}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/2903040*(2*(12*(18*(8*(30*(72*x - 37)*x + 4543)*x - 8441)*x + 519449)*x -
 1060499)*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/497664*sqrt(3)*log(-6*
```

```
sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 169/128*sqrt(13)*log(-1/2*ab
s(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)
*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

3.224 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2} dx$

Optimal. Leaf size=154

$$-\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}-\frac{11(37-60x)(3x^2-x+2)^{5/2}}{2340}-\frac{11}{864}(67-78x)(3x^2-x+2)^{3/2}-\frac{11(4727-3090x)\sqrt{3x^2-x+2}}{6912}+$$

```
[Out] (-11*(4727 - 3090*x)*Sqrt[2 - x + 3*x^2])/6912 - (11*(67 - 78*x)*(2 - x + 3
*x^2)^(3/2))/864 - (11*(37 - 60*x)*(2 - x + 3*x^2)^(5/2))/2340 - (2 - x + 3
*x^2)^(7/2)/(13*(1 + 2*x)) - (315623*ArcSinh[(1 - 6*x)/Sqrt[23]])/(13824*Sq
rt[3]) + (429*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])
/128
```

Rubi [A] time = 0.163071, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}-\frac{11(37-60x)(3x^2-x+2)^{5/2}}{2340}-\frac{11}{864}(67-78x)(3x^2-x+2)^{3/2}-\frac{11(4727-3090x)\sqrt{3x^2-x+2}}{6912}+$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```


```
[Out] (-11*(4727 - 3090*x)*Sqrt[2 - x + 3*x^2])/6912 - (11*(67 - 78*x)*(2 - x + 3
*x^2)^(3/2))/864 - (11*(37 - 60*x)*(2 - x + 3*x^2)^(5/2))/2340 - (2 - x + 3
*x^2)^(7/2)/(13*(1 + 2*x)) - (315623*ArcSinh[(1 - 6*x)/Sqrt[23]])/(13824*Sq
rt[3]) + (429*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])
/128
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = -\frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{11}{2}-44x\right)\left(2-x+3x^2\right)^{5/2}}{1+2x}\,dx$$

$$= -\frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} + \frac{\int \frac{(-286+14872x)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,d}{1872}$$

$$= -\frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x\right.}{13(1+2}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-6}{}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-6}{}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-6}{}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-6}{}$$

Mathematica [A] time = 0.115087, size = 113, normalized size = 0.73

$$\frac{\frac{6\sqrt{3x^2-x+2}\left(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257\right)}{2x+1} + 694980\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + 1578115\sqrt{3}\,\mathrm{si}}{207360}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-364257 - 322972*x + 310660*x^2 - 115680*x^3 + 251
424*x^4 - 65664*x^5 + 103680*x^6))/(1 + 2*x) + 1578115*Sqrt[3]*ArcSinh[(-1
+ 6*x)/Sqrt[23]] + 694980*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x
```

```
+ 3*x^2])])/207360
```

Maple [A] time = 0.061, size = 235, normalized size = 1.5

$$\frac{-1+6x}{36}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{-115+690x}{1728}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{-2645+15870x}{13824}\sqrt{3x^2-x+2}+\frac{315623\sqrt{3}}{41472}\mathrm{Arcsinh}\left(\frac{6\sqrt{23}}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x)
```

```
[Out] 1/36*(-1+6*x)*(3*x^2-x+2)^(5/2)+115/1728*(-1+6*x)*(3*x^2-x+2)^(3/2)+2645/13
824*(-1+6*x)*(3*x^2-x+2)^(1/2)+315623/41472*3^(1/2)*arcsinh(6/23*23^(1/2)*(
x-1/6))-33/260*(3*(x+1/2)^2-4*x+5/4)^(5/2)+19/192*(-1+6*x)*(3*(x+1/2)^2-4*x
+5/4)^(3/2)+965/1536*(-1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(1/2)-11/16*(3*(x+1/2)^
2-4*x+5/4)^(3/2)-429/128*(12*(x+1/2)^2-16*x+5)^(1/2)+429/128*13^(1/2)*arcta
nh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-1/26/(x+1/2)*(3*(x+
1/2)^2-4*x+5/4)^(7/2)+1/52*(-1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(5/2)
```

Maxima [A] time = 1.56935, size = 217, normalized size = 1.41

$$\frac{1}{6}\left(3x^2-x+2\right)^{\frac{5}{2}}x-\frac{7}{90}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{143}{144}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{737}{864}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{\left(3x^2-x+2\right)^{\frac{5}{2}}}{4(2x+1)}+\frac{5665}{1152}\sqrt{3x^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*x^2 - x + 2)^(5/2)*x - 7/90*(3*x^2 - x + 2)^(5/2) + 143/144*(3*x^2 -
 x + 2)^(3/2)*x - 737/864*(3*x^2 - x + 2)^(3/2) - 1/4*(3*x^2 - x + 2)^(5/2)
/(2*x + 1) + 5665/1152*sqrt(3*x^2 - x + 2)*x + 315623/41472*sqrt(3)*arcsinh
(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 429/128*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 51997/6912*sqrt(3*x^2 - x +
2)
```

Fricas [A] time = 1.56708, size = 462, normalized size = 3.

$$\frac{1578115\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+694980\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x}{4x^2}\right.}{41472}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/414720*(1578115*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25) + 694980*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqr
t(3*x^2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12
*(103680*x^6 - 65664*x^5 + 251424*x^4 - 115680*x^3 + 310660*x^2 - 322972*x
- 364257)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2-x+2\right)^{\frac{5}{2}}\left(4x^2+3x+1\right)}{(2x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Giac [B] time = 2.01138, size = 1026, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 429/128*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sq
rt(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 315623/41472*sqrt(3)*log(1/2*abs(
-2*sqrt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x +
```

```
1))/(sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)
))*sgn(1/(2*x + 1)) - 169/128*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1
/(2*x + 1)) + 1/34560*(5154065*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + s
qrt(13)/(2*x + 1))^11*sgn(1/(2*x + 1)) - 7837020*sqrt(13)*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^10*sgn(1/(2*x + 1)) + 3946881
5*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^9*sgn(1/(2
*x + 1)) - 14445540*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1))^8*sgn(1/(2*x + 1)) + 460893402*(sqrt(-8/(2*x + 1) + 13/(2*
x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^7*sgn(1/(2*x + 1)) - 343084680*sqrt(13)
*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*
x + 1)) + 944150094*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*
x + 1))^5*sgn(1/(2*x + 1)) - 22871160*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x
 + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(2*x + 1)) + 1397032245*(sqrt(-8
/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3*sgn(1/(2*x + 1)) -
 683367516*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*
x + 1))^2*sgn(1/(2*x + 1)) + 392684355*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 197538588*sqrt(13)*sgn(1/(2*x
 + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
 3)^6
```

3.225 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

Optimal. Leaf size=161

$$-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}+\frac{(134x+257)(3x^2-x+2)^{5/2}}{520(2x+1)}+\frac{1}{832}(1227-838x)(3x^2-x+2)^{3/2}+\frac{(21317-10470x)\sqrt{3x^2-x+}}{1536}$$

```
[Out] ((21317 - 10470*x)*Sqrt[2 - x + 3*x^2])/1536 + ((1227 - 838*x)*(2 - x + 3*x
^2)^(3/2))/832 + ((257 + 134*x)*(2 - x + 3*x^2)^(5/2))/(520*(1 + 2*x)) - (2
 - x + 3*x^2)^(7/2)/(26*(1 + 2*x)^2) + (118423*ArcSinh[(1 - 6*x)/Sqrt[23]])
/(3072*Sqrt[3]) - (1631*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/256
```

Rubi [A] time = 0.162784, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}+\frac{(134x+257)(3x^2-x+2)^{5/2}}{520(2x+1)}+\frac{1}{832}(1227-838x)(3x^2-x+2)^{3/2}+\frac{(21317-10470x)\sqrt{3x^2-x+}}{1536}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```


```
[Out] ((21317 - 10470*x)*Sqrt[2 - x + 3*x^2])/1536 + ((1227 - 838*x)*(2 - x + 3*x
^2)^(3/2))/832 + ((257 + 134*x)*(2 - x + 3*x^2)^(5/2))/(520*(1 + 2*x)) - (2
 - x + 3*x^2)^(7/2)/(26*(1 + 2*x)^2) + (118423*ArcSinh[(1 - 6*x)/Sqrt[23]])
/(3072*Sqrt[3]) - (1631*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/256
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = -\frac{\left(2-x+3x^2\right)^{7/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{29}{2}-67x\right)\left(2-x+3x^2\right)^{5/2}}{(1+2x)^2}\,dx$$

$$= \frac{(257+134x)\left(2-x+3x^2\right)^{5/2}}{520(1+2x)} - \frac{\left(2-x+3x^2\right)^{7/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{(793-1676x)\left(2-\right.}{1+2x}$$

$$= \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \frac{(257+134x)\left(2-x+3x^2\right)^{5/2}}{520(1+2x)} - \frac{\left(2-x+\right.}{26(1+}$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \frac{(257+}{}$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \frac{(257+}{}$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \frac{(257+}{}$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \frac{(257+}{}$$

Mathematica [A] time = 0.108018, size = 113, normalized size = 0.7

$$\frac{\frac{6\sqrt{3x^2-x+2}(27648x^6-22464x^5+83616x^4-76200x^3+256564x^2+464446x+142057)}{(2x+1)^2} - 293580\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) - 592115\sqrt{3}\sinh^{-1}\left(\right.}{46080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(142057 + 464446*x + 256564*x^2 - 76200*x^3 + 83616
*x^4 - 22464*x^5 + 27648*x^6))/(1 + 2*x)^2 - 592115*Sqrt[3]*ArcSinh[(-1 + 6
*x)/Sqrt[23]] - 293580*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
3*x^2])])/46080
```

Maple [A] time = 0.059, size = 199, normalized size = 1.2

$$\frac{1631}{6760}\left(3\,(x+1/2)^2-4\,x+\frac{5}{4}\right)^{\frac{5}{2}}-\frac{-19+114\,x}{676}\left(3\,(x+1/2)^2-4\,x+\frac{5}{4}\right)^{\frac{5}{2}}+\frac{19}{338}\left(3\,(x+1/2)^2-4\,x+\frac{5}{4}\right)^{\frac{7}{2}}\left(x+\frac{1}{2}\right)^{-1}-\frac{1}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x)
```

```
[Out] 1631/6760*(3*(x+1/2)^2-4*x+5/4)^(5/2)-19/676*(-1+6*x)*(3*(x+1/2)^2-4*x+5/4)
^(5/2)+19/338/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^(7/2)-1/104/(x+1/2)^2*(3*(x+1/2
)^2-4*x+5/4)^(7/2)-1745/1536*(-1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(1/2)-419/2496*
(-1+6*x)*(3*(x+1/2)^2-4*x+5/4)^(3/2)-1631/256*13^(1/2)*arctanh(2/13*(9/2-4*
x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-118423/9216*3^(1/2)*arcsinh(6/23*2
3^(1/2)*(x-1/6))+1631/256*(12*(x+1/2)^2-16*x+5)^(1/2)+1631/1248*(3*(x+1/2)^
2-4*x+5/4)^(3/2)
```

Maxima [A] time = 1.59254, size = 232, normalized size = 1.44

$$\frac{67}{520}\left(3\,x^2-x+2\right)^{\frac{5}{2}}-\frac{\left(3\,x^2-x+2\right)^{\frac{7}{2}}}{26\left(4\,x^2+4\,x+1\right)}-\frac{419}{416}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{1227}{832}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{19\left(3\,x^2-x+2\right)^{\frac{5}{2}}}{52\,(2\,x+1)}-\frac{1745}{256}\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 67/520*(3*x^2 - x + 2)^(5/2) - 1/26*(3*x^2 - x + 2)^(7/2)/(4*x^2 + 4*x + 1)
 - 419/416*(3*x^2 - x + 2)^(3/2)*x + 1227/832*(3*x^2 - x + 2)^(3/2) + 19/52
*(3*x^2 - x + 2)^(5/2)/(2*x + 1) - 1745/256*sqrt(3*x^2 - x + 2)*x - 118423/
9216*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1631/256*sqrt(13)*a
rcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 21317/1
536*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.49135, size = 487, normalized size = 3.02

$$\frac{592115\sqrt{3}\left(4x^2+4x+1\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+293580\sqrt{13}\left(4x^2+4x+1\right)\log\left(-\frac{4\sqrt{}}{}\right.}{9216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/92160*(592115*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)
*(6*x - 1) - 72*x^2 + 24*x - 25) + 293580*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(
4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 +
4*x + 1)) + 12*(27648*x^6 - 22464*x^5 + 83616*x^4 - 76200*x^3 + 256564*x^2
+ 464446*x + 142057)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2-x+2\right)^{\frac{5}{2}}\left(4x^2+3x+1\right)}{(2x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.226 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=693

$$\frac{\sqrt{a+bx+cx^2}\left(8c^2h^2\left(128a^2fh^2+275abh(eh+3fg)+3b^2\left(50h(dh+3eg)+129fg^2\right)\right)-2chx\left(8c^2h\left(ah(45eh+71fg)\right.\right.\right.}{}$$

[Out] ((63*b^2*f*h^2 - 2*c*h*(24*b*f*g + 35*b*e*h + 32*a*f*h) - c^2*(12*f*g^2 - 2
0*h*(3*e*g + 4*d*h)))*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(240*c^3*h) - ((9*
b*f*h + 2*c*(f*g - 5*e*h))*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(40*c^2*h) +
(f*(g + h*x)^4*Sqrt[a + b*x + c*x^2])/(5*c*h) + ((945*b^4*f*h^4 - 64*c^4*g^
2*(3*f*g^2 - 5*h*(3*e*g + 16*d*h)) - 210*b^2*c*h^3*(14*a*f*h + 5*b*(3*f*g +
e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 275*a*b*h*(3*f*g + e*h) + 3*b^2*(129*f*
g^2 + 50*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(13*f*g^2 + 5*h*(3*e*g + d*h)
) + b*g*(39*f*g^2 + 5*h*(47*e*g + 54*d*h))) - 2*c*h*(315*b^3*f*h^3 - 14*b*c
h^2(39*b*f*g + 25*b*e*h + 46*a*f*h) + 16*c^3*g*(3*f*g^2 - 5*h*(3*e*g + 10
*d*h)) + 8*c^2*h*(a*h*(71*f*g + 45*e*h) + b*(21*f*g^2 + 80*e*g*h + 50*d*h^2
)))*x)*Sqrt[a + b*x + c*x^2])/(1920*c^5*h) + ((256*c^5*d*g^3 - 63*b^5*f*h^3
+ 70*b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a
*b*h*(3*f*g + e*h) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) - 128*c^4*g*(b*g*(e*g
+ 3*d*h) + a*(f*g^2 + 3*h*(e*g + d*h))) + 96*c^3*(a^2*h^2*(3*f*g + e*h) +
b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(11/2))

Rubi [A] time = 2.10221, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}\left(8c^2h^2\left(128a^2fh^2+275abh(eh+3fg)+3b^2\left(50h(dh+3eg)+129fg^2\right)\right)-2chx\left(8c^2h\left(ah(45eh+71fg)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]

[Out] ((63*b^2*f*h^2 - 2*c*h*(24*b*f*g + 35*b*e*h + 32*a*f*h) - c^2*(12*f*g^2 - 2
0*h*(3*e*g + 4*d*h)))*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(240*c^3*h) - ((9*
b*f*h + 2*c*(f*g - 5*e*h))*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(40*c^2*h) +
(f*(g + h*x)^4*Sqrt[a + b*x + c*x^2])/(5*c*h) + ((945*b^4*f*h^4 - 64*c^4*(3

```
*f*g^4 - 5*g^2*h*(3*e*g + 16*d*h)) - 210*b^2*c*h^3*(14*a*f*h + 5*b*(3*f*g +
 e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 275*a*b*h*(3*f*g + e*h) + 3*b^2*(129*f*
g^2 + 50*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(13*f*g^2 + 5*h*(3*e*g + d*h)
) + b*g*(39*f*g^2 + 5*h*(47*e*g + 54*d*h))) - 2*c*h*(315*b^3*f*h^3 - 14*b*c
*h^2*(39*b*f*g + 25*b*e*h + 46*a*f*h) + 16*c^3*(3*f*g^3 - 5*g*h*(3*e*g + 10
*d*h)) + 8*c^2*h*(21*b*f*g^2 + 10*b*h*(8*e*g + 5*d*h) + a*h*(71*f*g + 45*e*
h)))*x)*Sqrt[a + b*x + c*x^2])/(1920*c^5*h) + ((256*c^5*d*g^3 - 63*b^5*f*h^
3 + 70*b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*a*h*(
e*g + d*h) + b*g*(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g
+ e*h) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g + e*h) +
 b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(11/2))
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx=\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}+\frac{\int \frac{(g+hx)^3\left(-\frac{1}{2}h(bfg-10cdh+8afh)-\frac{1}{2}h(2cfg-10ceh+9bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{5ch^2}$$

$$=-\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}+\frac{\int \frac{(g+hx}{}}{}$$

$$=\frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+hx)^2\sqrt{a}}{240c^3h}$$

$$=\frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+hx)^2\sqrt{a}}{240c^3h}$$

$$=\frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+hx)^2\sqrt{a}}{240c^3h}$$

$$=\frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+hx)^2\sqrt{a}}{240c^3h}$$

Mathematica [A] time = 1.28092, size = 588, normalized size = 0.85

$$\sqrt{a+x(b+cx)}\left(4c^2h\left(256a^2fh^2+2abh(275eh+825fg+161fhx)+b^2\left(25h(12dh+36eg+7ehx)+3f\left(300g^2+175g\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(945*b^4*f*h^3 - 210*b^2*c*h^2*(5*b*e*h + 14*a*f*h +
 3*b*f*(5*g + h*x)) + 32*c^4*(10*d*h*(18*g^2 + 9*g*h*x + 2*h^2*x^2) + 15*e*
(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*g^3 + 20*g^2*h*x +
15*g*h^2*x^2 + 4*h^3*x^3)) + 4*c^2*h*(256*a^2*f*h^2 + 2*a*b*h*(825*f*g + 27
5*e*h + 161*f*h*x) + b^2*(25*h*(36*e*g + 12*d*h + 7*e*h*x) + 3*f*(300*g^2 +
 175*g*h*x + 42*h^2*x^2))) - 16*c^3*(a*h*(5*h*(48*e*g + 16*d*h + 9*e*h*x) +
 f*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + b*(3*f*(30*g^3 + 50*g^2*h*x + 35*g
*h^2*x^2 + 9*h^3*x^3) + 5*h*(2*d*h*(27*g + 5*h*x) + e*(54*g^2 + 30*g*h*x +
7*h^2*x^2))))))/(1920*c^5) + ((256*c^5*d*g^3 - 63*b^5*f*h^3 + 70*b^3*c*h^2*
(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + b*g*
(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g + e*h) + b^2*(3*f
*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g + e*h) + b^2*g*(f*g^2 + 3
*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(256*c^(11/2))
```

Maple [B] time = 0.063, size = 1869, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 3/c*(c*x^2+b*x+a)^(1/2)*g^2*h*d-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*g^3*e+1/2*x/c*(c*x^2+b*x+a)^(1/2)*g^3*f-1/2*a/c^(3/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^3*f+1/c*(c*x^2+b*x+a)^(1/2)*g^3*e+g
^3*d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-3/2*a/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*d+9/8*b^2/c^(5/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*d-3/4*b/c^2*(c*x^2+b*x+a)^(1/2)*g^3*
f+3/8*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^3*f+1/5*h^3
*f*x^4/c*(c*x^2+b*x+a)^(1/2)+63/128*h^3*f*b^4/c^5*(c*x^2+b*x+a)^(1/2)-63/25
6*h^3*f*b^5/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/15*h^3*f
*a^2/c^3*(c*x^2+b*x+a)^(1/2)-35/64*b^3/c^4*(c*x^2+b*x+a)^(1/2)*h^3*e+35/128
*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e+3/8*a^2/c^(5
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e-2/3*a/c^2*(c*x^2+b*x+
a)^(1/2)*h^3*d-5/16*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*h^3*d+5/8*b^2/c^3*(c*x^2+b*x+a)^(1/2)*h^3*d+1/3*x^2/c*(c*x^2+b*x+a)^(1/2)*
h^3*d+1/4*x^3/c*(c*x^2+b*x+a)^(1/2)*h^3*e-3/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*d+9/8*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*g^2*h*e-3/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
```

```
)^(1/2))*g^2*h*e+3/4*x^3/c*(c*x^2+b*x+a)^(1/2)*g*h^2*f-7/24*b/c^2*x^2*(c*x^
2+b*x+a)^(1/2)*h^3*e+35/96*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)*h^3*e-105/64*b^3/c
^4*(c*x^2+b*x+a)^(1/2)*g*h^2*f+105/128*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*g*h^2*f-15/16*b^2/c^(7/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*h^3*e+55/48*b/c^3*a*(c*x^2+b*x+a)^(1/2)*h^3*e-3/8*a/c^2*x*
(c*x^2+b*x+a)^(1/2)*h^3*e-9/4*b/c^2*(c*x^2+b*x+a)^(1/2)*g^2*h*e-9/40*h^3*f*
b/c^2*x^3*(c*x^2+b*x+a)^(1/2)-49/32*h^3*f*b^2/c^4*a*(c*x^2+b*x+a)^(1/2)-15/
16*h^3*f*b/c^(7/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/15*h^3
*f*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)+21/80*h^3*f*b^2/c^3*x^2*(c*x^2+b*x+a)^(1/2
)-21/64*h^3*f*b^3/c^4*x*(c*x^2+b*x+a)^(1/2)+35/32*h^3*f*b^3/c^(9/2)*a*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/2*x/c*(c*x^2+b*x+a)^(1/2)*g*h^2*d+
3/2*x/c*(c*x^2+b*x+a)^(1/2)*g^2*h*e-9/4*b/c^2*(c*x^2+b*x+a)^(1/2)*g*h^2*d-2
*a/c^2*(c*x^2+b*x+a)^(1/2)*g^2*h*f+x^2/c*(c*x^2+b*x+a)^(1/2)*g^2*h*f-5/12*b
/c^2*x*(c*x^2+b*x+a)^(1/2)*h^3*d+15/8*b^2/c^3*(c*x^2+b*x+a)^(1/2)*g*h^2*e+1
5/8*b^2/c^3*(c*x^2+b*x+a)^(1/2)*g^2*h*f-15/16*b^3/c^(7/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*e-15/16*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*g^2*h*f+3/4*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*h^3*d-2*a/c^2*(c*x^2+b*x+a)^(1/2)*g*h^2*e+x^2/c*(c*x^2+b*x+a
)^(1/2)*g*h^2*e+9/8*a^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*g*h^2*f-45/16*b^2/c^(7/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*
h^2*f-9/8*a/c^2*x*(c*x^2+b*x+a)^(1/2)*g*h^2*f+9/4*b/c^(5/2)*a*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*e+9/4*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*g^2*h*f+55/16*b/c^3*a*(c*x^2+b*x+a)^(1/2)*g*h^2*f-5
/4*b/c^2*x*(c*x^2+b*x+a)^(1/2)*g*h^2*e-5/4*b/c^2*x*(c*x^2+b*x+a)^(1/2)*g^2*
h*f+161/240*h^3*f*b/c^3*a*x*(c*x^2+b*x+a)^(1/2)-7/8*b/c^2*x^2*(c*x^2+b*x+a)
^(1/2)*g*h^2*f+35/32*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)*g*h^2*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.63271, size = 3267, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/7680*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)*g^3 - 48*(
8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)*f)*g^2*h +
 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e + (35*b^4*c -
 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*c^3)*d - 2*
(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*c + 240*a^2
*b*c^2)*f)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h^3*x^4 + 480*(4*c^5*e -
3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^4)*f)*g^
2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^2 - 44*a*
b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*a*b*c
^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^5*f*g
*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^5*e -
 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f)*h^3
)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^5*d -
 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35*b^2*
c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x
 + a))/c^6, -1/3840*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)
*g^3 - 48*(8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)
*f)*g^2*h + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e +
(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*
c^3)*d - 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*
c + 240*a^2*b*c^2)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*f*h^3*x^4 + 480*(4*c^5
*e - 3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^4)*
f)*g^2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^2 -
44*a*b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*
a*b*c^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^
5*f*g*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^
5*e - 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f
)*h^3)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^
5*d - 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35
*b^2*c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2
+ b*x + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.26692, size = 1110, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h^3*x/c + (30*c^4*f*g*h^2 - 9*b*
c^3*f*h^3 + 10*c^4*h^3*e)/c^5)*x + (240*c^4*f*g^2*h - 210*b*c^3*f*g*h^2 + 8
0*c^4*d*h^3 + 63*b^2*c^2*f*h^3 - 64*a*c^3*f*h^3 + 240*c^4*g*h^2*e - 70*b*c^
3*h^3*e)/c^5)*x + (480*c^4*f*g^3 - 1200*b*c^3*f*g^2*h + 1440*c^4*d*g*h^2 +
1050*b^2*c^2*f*g*h^2 - 1080*a*c^3*f*g*h^2 - 400*b*c^3*d*h^3 - 315*b^3*c*f*h
^3 + 644*a*b*c^2*f*h^3 + 1440*c^4*g^2*h*e - 1200*b*c^3*g*h^2*e + 350*b^2*c^
2*h^3*e - 360*a*c^3*h^3*e)/c^5)*x - (1440*b*c^3*f*g^3 - 5760*c^4*d*g^2*h -
3600*b^2*c^2*f*g^2*h + 3840*a*c^3*f*g^2*h + 4320*b*c^3*d*g*h^2 + 3150*b^3*c
*f*g*h^2 - 6600*a*b*c^2*f*g*h^2 - 1200*b^2*c^2*d*h^3 + 1280*a*c^3*d*h^3 - 9
45*b^4*f*h^3 + 2940*a*b^2*c*f*h^3 - 1024*a^2*c^2*f*h^3 - 1920*c^4*g^3*e + 4
320*b*c^3*g^2*h*e - 3600*b^2*c^2*g*h^2*e + 3840*a*c^3*g*h^2*e + 1050*b^3*c*
h^3*e - 2200*a*b*c^2*h^3*e)/c^5) - 1/256*(256*c^5*d*g^3 + 96*b^2*c^3*f*g^3
- 128*a*c^4*f*g^3 - 384*b*c^4*d*g^2*h - 240*b^3*c^2*f*g^2*h + 576*a*b*c^3*f
*g^2*h + 288*b^2*c^3*d*g*h^2 - 384*a*c^4*d*g*h^2 + 210*b^4*c*f*g*h^2 - 720*
a*b^2*c^2*f*g*h^2 + 288*a^2*c^3*f*g*h^2 - 80*b^3*c^2*d*h^3 + 192*a*b*c^3*d*
h^3 - 63*b^5*f*h^3 + 280*a*b^3*c*f*h^3 - 240*a^2*b*c^2*f*h^3 - 128*b*c^4*g^
3*e + 288*b^2*c^3*g^2*h*e - 384*a*c^4*g^2*h*e - 240*b^3*c^2*g*h^2*e + 576*a
*b*c^3*g*h^2*e + 70*b^4*c*h^3*e - 240*a*b^2*c^2*h^3*e + 96*a^2*c^3*h^3*e)*l
og(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

3.227 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=420

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(dh^2+2egh+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-64c^3\left(a\left(\right.\right.\right.}{128c^{9/2}}$$

```
[Out] -((2*c*f*g - 8*c*e*h + 7*b*f*h)*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(24*c^2*
h) + (f*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c*h) - ((105*b^3*f*h^3 + 32*c
^3*g*(f*g^2 - 4*h*(e*g + 3*d*h)) - 20*b*c*h^2*(11*a*f*h + 6*b*(2*f*g + e*h)
) + 8*c^2*h*(16*a*h*(2*f*g + e*h) + b*(11*f*g^2 + 18*h*(2*e*g + d*h))) - 2*
c*h*(35*b^2*f*h^2 - 4*c*h*(6*b*f*g + 10*b*e*h + 9*a*f*h) - 8*c^2*(f*g^2 - 2
*h*(2*e*g + 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(192*c^4*h) + ((128*c^4*d*g^
2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g + b*e*h + 3*a*f*h) - 64*c^3*(b*g*(e*
g + 2*d*h) + a*(f*g^2 + 2*e*g*h + d*h^2)) + 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*
f*g + e*h) + b^2*(f*g^2 + 2*e*g*h + d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rubi [A] time = 1.01109, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-64c^3\left(a\right.\right.}{128c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] -((2*c*f*g - 8*c*e*h + 7*b*f*h)*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(24*c^2*
h) + (f*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c*h) - ((105*b^3*f*h^3 + 32*c
^3*(f*g^3 - 4*g*h*(e*g + 3*d*h)) - 20*b*c*h^2*(11*a*f*h + 6*b*(2*f*g + e*h)
) + 8*c^2*h*(11*b*f*g^2 + 18*b*h*(2*e*g + d*h) + 16*a*h*(2*f*g + e*h)) - 2*
c*h*(35*b^2*f*h^2 - 4*c*h*(6*b*f*g + 10*b*e*h + 9*a*f*h) - 8*c^2*(f*g^2 - 2
*h*(2*e*g + 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(192*c^4*h) + ((128*c^4*d*g^
2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g + b*e*h + 3*a*f*h) - 64*c^3*(a*f*g^2
 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)) + 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*
f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left(-\frac{1}{2}h(bfg-8cdh+6afh)-\frac{1}{2}h(2cfg-8ceh+7bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{4ch^2}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} + \frac{\int \frac{(g+hx)(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{\left(105b^3fh\cdots\right.}{\cdots}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{\left(105b^3fh\cdots\right.}{\cdots}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{\left(105b^3fh\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.669533, size = 343, normalized size = 0.82

$$\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-64c^3\left(\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f*h^2 + 10*b*c*h*(22*a*f*h + b*(
24*f*g + 12*e*h + 7*f*h*x)) + 16*c^3*(6*d*h*(4*g + h*x) + 4*e*(3*g^2 + 3*g*
h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2)) - 8*c^2*(2*b*h*(18*e*g
+ 9*d*h + 5*e*h*x) + a*h*(32*f*g + 16*e*h + 9*f*h*x) + b*f*(18*g^2 + 20*g*h
*x + 7*h^2*x^2))) + 3*(128*c^4*d*g^2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g +
 b*e*h + 3*a*f*h) - 64*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)
) + 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h
))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(384*c^(9/2))
```

Maple [B] time = 0.063, size = 1069, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h+3/8*h^2*f*a^2/
c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*x/c*(c*x^2+b*x+a)^(
1/2)*d*h^2+1/2*x/c*(c*x^2+b*x+a)^(1/2)*f*g^2-3/4*b/c^2*(c*x^2+b*x+a)^(1/2)*
d*h^2+1/c*(c*x^2+b*x+a)^(1/2)*e*g^2+g^2*d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))/c^(1/2)+35/96*h^2*f*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)-15/16*h^2*f*b^
2/c^(7/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/24*h^2*f*b/c^2*x^
2*(c*x^2+b*x+a)^(1/2)-35/64*h^2*f*b^3/c^4*(c*x^2+b*x+a)^(1/2)+1/4*h^2*f*x^3
/c*(c*x^2+b*x+a)^(1/2)+2/3*x^2/c*(c*x^2+b*x+a)^(1/2)*g*h*f+1/3*x^2/c*(c*x^2
+b*x+a)^(1/2)*h^2*e+5/8*b^2/c^3*(c*x^2+b*x+a)^(1/2)*h^2*e-5/16*b^3/c^(7/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e-2/3*a/c^2*(c*x^2+b*x+a)^(
1/2)*h^2*e+35/128*h^2*f*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))+3/8*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+3/8*
b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-1/2*a/c^(3/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/2*a/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2+2/c*(c*x^2+b*x+a)^(1/2)*g*h*d-1/2*b
/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^2-3/4*b/c^2*(c*x^2
+b*x+a)^(1/2)*f*g^2-5/6*b/c^2*x*(c*x^2+b*x+a)^(1/2)*g*h*f-3/8*h^2*f*a/c^2*x
*(c*x^2+b*x+a)^(1/2)+3/2*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))*g*h*f-3/2*b/c^2*(c*x^2+b*x+a)^(1/2)*e*g*h+3/4*b^2/c^(5/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h+55/48*h^2*f*b/c^3*a*(c*x^2+b*x+a)^(1
/2)+x/c*(c*x^2+b*x+a)^(1/2)*e*g*h-b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*g*h*d-5/12*b/c^2*x*(c*x^2+b*x+a)^(1/2)*h^2*e+5/4*b^2/c^3*(c*x^
2+b*x+a)^(1/2)*g*h*f-5/8*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))*g*h*f+3/4*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2
*e-4/3*a/c^2*(c*x^2+b*x+a)^(1/2)*g*h*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17739, size = 1953, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3*(16*(8*c^4*d - 4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b
*c^3*d - 2*(3*b^2*c^2 - 4*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3
*b^2*c^2 - 4*a*c^3)*d - 8*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*h^2*x^3 + 48*(4*c^4*
e - 3*b*c^3*f)*g^2 + 16*(24*c^4*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)
*g*h - (144*b*c^3*d - 8*(15*b^2*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^
2)*f)*h^2 + 8*(16*c^4*f*g*h + (8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*
g^2 + 16*(6*c^4*e - 5*b*c^3*f)*g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 -
 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(16*(8*c^4*d -
4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b*c^3*d - 2*(3*b^2*c^2 - 4
*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3*b^2*c^2 - 4*a*c^3)*d - 8
*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f)*h^2)*sqr
t(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*
x + a*c)) - 2*(48*c^4*f*h^2*x^3 + 48*(4*c^4*e - 3*b*c^3*f)*g^2 + 16*(24*c^4
*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)*g*h - (144*b*c^3*d - 8*(15*b^2
*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^2)*f)*h^2 + 8*(16*c^4*f*g*h + (
8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*g^2 + 16*(6*c^4*e - 5*b*c^3*f)*
g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 - 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^
2 + b*x + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g + hx\right)^2 \left(d + ex + fx^2\right)}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.24578, size = 617, normalized size = 1.47

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6fh^2x}{c}+\frac{16c^3fgh-7bc^2fh^2+8c^3h^2e}{c^4}\right)x+\frac{48c^3fg^2-80bc^2fgh+48c^3dh^2+35b^2cfh^2-3}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*h^2*x/c + (16*c^3*f*g*h - 7*b*c^2*f*
h^2 + 8*c^3*h^2*e)/c^4)*x + (48*c^3*f*g^2 - 80*b*c^2*f*g*h + 48*c^3*d*h^2 +
 35*b^2*c*f*h^2 - 36*a*c^2*f*h^2 + 96*c^3*g*h*e - 40*b*c^2*h^2*e)/c^4)*x -
(144*b*c^2*f*g^2 - 384*c^3*d*g*h - 240*b^2*c*f*g*h + 256*a*c^2*f*g*h + 144*
b*c^2*d*h^2 + 105*b^3*f*h^2 - 220*a*b*c*f*h^2 - 192*c^3*g^2*e + 288*b*c^2*g
*h*e - 120*b^2*c*h^2*e + 128*a*c^2*h^2*e)/c^4) - 1/128*(128*c^4*d*g^2 + 48*
b^2*c^2*f*g^2 - 64*a*c^3*f*g^2 - 128*b*c^3*d*g*h - 80*b^3*c*f*g*h + 192*a*b
*c^2*f*g*h + 48*b^2*c^2*d*h^2 - 64*a*c^3*d*h^2 + 35*b^4*f*h^2 - 120*a*b^2*c
*f*h^2 + 48*a^2*c^2*f*h^2 - 64*b*c^3*g^2*e + 96*b^2*c^2*g*h*e - 128*a*c^3*g
*h*e - 40*b^3*c*h^2*e + 96*a*b*c^2*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

3.228 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=223

$$\frac{\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+fg))+15b^2fh^2-2chx(5bfh-6ceh+2cfg)-8c^2\left(fg^2-3h(dh+eg)\right)\right)}{24c^3h}+\frac{\tanh^{-1}\left(\right.}{}$$

```
[Out] (f*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*h) + ((15*b^2*f*h^2 - 8*c^2*(f*g
^2 - 3*h*(e*g + d*h)) - 2*c*h*(8*a*f*h + 9*b*(f*g + e*h)) - 2*c*h*(2*c*f*g
- 6*c*e*h + 5*b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(24*c^3*h) + ((16*c^3*d*g -
5*b^3*f*h - 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) + 6*b*c*(b*f*g + b*e*h +
2*a*f*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/
2))
```

Rubi [A] time = 0.303192, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1653, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+fg))+15b^2fh^2-2chx(5bfh-6ceh+2cfg)-8c^2\left(fg^2-3h(dh+eg)\right)\right)}{24c^3h}+\frac{\tanh^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (f*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*h) + ((15*b^2*f*h^2 - 8*c^2*(f*g
^2 - 3*h*(e*g + d*h)) - 2*c*h*(8*a*f*h + 9*b*(f*g + e*h)) - 2*c*h*(2*c*f*g
- 6*c*e*h + 5*b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(24*c^3*h) + ((16*c^3*d*g -
5*b^3*f*h - 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) + 6*b*c*(b*f*g + b*e*h +
2*a*f*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/
2))
```


Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
```

```
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\int \frac{(g+hx)\left(-\frac{1}{2}h(bfg-6cdh+4afh)-\frac{1}{2}h(2cfg-6ceh+5bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{3ch^2}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8afh+9b(fg+\cdots}{24c^3h}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8afh+9b(fg+\cdots}{24c^3h}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8afh+9b(fg+\cdots}{24c^3h}$$

Mathematica [A] time = 0.255891, size = 215, normalized size = 0.96

$$\frac{\frac{\sqrt{a+x(b+cx)}\left(-2ch(8afh+b(9eh+9fg+5fhx))+15b^2fh^2-4c^2(fg(2g+hx)-3h(2dh+2eg+ehx))\right)}{8c^2}-\frac{3h\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(8c^2(aeh+afg+bdh+beg)-6bc(2afh+b\right.}{16c^{5/2}}}{3ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + x*(b + c*x)] + (Sqrt[a + x*(b + c*x)]*(15*b^2*f*h^2
 - 4*c^2*(f*g*(2*g + h*x) - 3*h*(2*e*g + 2*d*h + e*h*x)) - 2*c*h*(8*a*f*h +
 b*(9*f*g + 9*e*h + 5*f*h*x))))/(8*c^2) - (3*h*(-16*c^3*d*g + 5*b^3*f*h + 8
*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) - 6*b*c*(b*f*g + b*e*h + 2*a*f*h))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(16*c^(5/2)))/(3*c*h)
```

Maple [B] time = 0.056, size = 505, normalized size = 2.3

$$\frac{fhx^2}{3c}\sqrt{cx^2+bx+a}-\frac{5bfhx}{12c^2}\sqrt{cx^2+bx+a}+\frac{5hfb^2}{8c^3}\sqrt{cx^2+bx+a}-\frac{5b^3fh}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{7}{2}}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*h*f*x^2/c*(c*x^2+b*x+a)^(1/2)-5/12*h*f*b/c^2*x*(c*x^2+b*x+a)^(1/2)+5/8*
h*f*b^2/c^3*(c*x^2+b*x+a)^(1/2)-5/16*h*f*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))+3/4*h*f*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))-2/3*h*f*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*x/c*(c*x^2+b*x+a)^(1/2)*e*
h+1/2*x/c*(c*x^2+b*x+a)^(1/2)*f*g-3/4*b/c^2*(c*x^2+b*x+a)^(1/2)*e*h-3/4*b/c
^2*(c*x^2+b*x+a)^(1/2)*f*g+3/8*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*e*h+3/8*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*f*g-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h-1/2*a/c^
(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g+1/c*(c*x^2+b*x+a)^(1/
2)*d*h+1/c*(c*x^2+b*x+a)^(1/2)*e*g-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*d*h-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*e*g+d*g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.4843, size = 1060, normalized size = 4.75

$$\left[\frac{3\left(2\left(8c^3d-4bc^2e+\left(3b^2c-4ac^2\right)f\right)g-\left(8bc^2d-2\left(3b^2c-4ac^2\right)e+\left(5b^3-12abc\right)f\right)h\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f)*g - (8*b*c^2*d -
2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(c)*log(-8*c^2*x^2 -
 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(
8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c^2*e + (15*b^
2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)*x)*sqrt(c*x^
2 + b*x + a))/c^4, -1/48*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f
)*g - (8*b*c^2*d - 2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(
-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
+ a*c)) - 2*(8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c
^2*e + (15*b^2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)
*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g+hx\right)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.22228, size = 284, normalized size = 1.27

$$\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,fhx}{c}+\frac{6\,c^2fg-5\,bcfh+6\,c^2he}{c^3}\right)x-\frac{18\,bcfg-24\,c^2dh-15\,b^2fh+16\,acfh-24\,c^2ge+18\,bche}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*f*h*x/c + (6*c^2*f*g - 5*b*c*f*h + 6*c^2*h
*e)/c^3)*x - (18*b*c*f*g - 24*c^2*d*h - 15*b^2*f*h + 16*a*c*f*h - 24*c^2*g*
e + 18*b*c*h*e)/c^3) - 1/16*(16*c^3*d*g + 6*b^2*c*f*g - 8*a*c^2*f*g - 8*b*c
^2*d*h - 5*b^3*f*h + 12*a*b*c*f*h - 8*b*c^2*g*e + 6*b^2*c*h*e - 8*a*c^2*h*e
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

3.229 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rubi [A] time = 0.106052, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd-af+\frac{1}{2}(4ce-3bf)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{4c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,d\right.}{2c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(8c^2d+3b^2f-4c(be+af)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+c}}\right.}{8c^{5/2}}$$

Mathematica [A] time = 0.151866, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*c^2*d + 3*b
^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
```

```
])])/(8*c^(5/2))
```

Maple [A] time = 0.054, size = 185, normalized size = 1.6

$$\frac{fx}{2c}\sqrt{cx^2+bx+a}-\frac{3bf}{4c^2}\sqrt{cx^2+bx+a}+\frac{3b^2f}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{5}{2}}-\frac{af}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(c*x^2+b*x+a)^(1/2)/c-3/4*f*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*f*b^2/c^(
5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*f*a/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e/c*(c*x^2+b*x+a)^(1/2)-1/2*e*b/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42946, size = 549, normalized size = 4.73

$$\left[-\frac{\left(8c^2d-4bce+\left(3b^2-4ac\right)f\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2+4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)-4\left(2c^2fx+4c^2e\cdots\right.}{16c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c
^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)
*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2}{\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.31607, size = 132, normalized size = 1.14

$$\frac{1}{4}\sqrt{cx^2 + bx + a}\left(\frac{2\,fx}{c} - \frac{3\,bf - 4\,ce}{c^2}\right) - \frac{\left(8\,c^2d + 3\,b^2f - 4\,acf - 4\,bce\right)\log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d +
3*b^2*f - 4*a*c*f - 4*b*c*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/c^(5/2)
```

3.230 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=179

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{2c^{3/2}h^2}+\frac{\left(fg^2-h(eg-dh)\right)\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^2\sqrt{ah^2-bgh+cg^2}}+\frac{f\sqrt{a+bx+cx^2}}{ch}$$

```
[Out] (f*Sqrt[a + b*x + c*x^2])/(c*h) - ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*h^2) + ((f*g^2 - h*(
e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
 a*h^2]*Sqrt[a + b*x + c*x^2])])/(h^2*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rubi [A] time = 0.293372, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{2c^{3/2}h^2}+\frac{\left(fg^2-h(eg-dh)\right)\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^2\sqrt{ah^2-bgh+cg^2}}+\frac{f\sqrt{a+bx+cx^2}}{ch}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] (f*Sqrt[a + b*x + c*x^2])/(c*h) - ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*h^2) + ((f*g^2 - h*(
e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
 a*h^2]*Sqrt[a + b*x + c*x^2])])/(h^2*Sqrt[c*g^2 - b*g*h + a*h^2])
```


<u>Rule 1653</u>

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
```

```
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \frac{f\sqrt{a+bx+cx^2}}{ch} + \frac{\int \frac{-\frac{1}{2}h(bfg-2cdh)-\frac{1}{2}h(2cfg-2ceh+bfh)x}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{ch^2}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2ch^2} + \frac{\left(fg^2-egh+dh^2\right)\int \frac{1}{(g+hx)\sqrt{a+}}}{h^2}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{ch^2} - \frac{\left(2\left(fg^2-eg\right.\right.}{}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}h^2} + \frac{\left(fg^2-h(eg-dh)\right)\tan}{h^2\sqrt{cg}}$$

Mathematica [A] time = 0.285503, size = 172, normalized size = 0.96

$$-\frac{\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)(bfh-2ceh+2cfg)}{c^{3/2}} + \frac{2\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)}{\sqrt{h(ah-bg)+cg^2}} - \frac{2fh\sqrt{a+x(b+cx)}}{c}}{2h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((-2*f*h*Sqrt[a + x*(b + c*x)])/c + ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (2*(f*g^2 + h*(-(e
*g) + d*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-
(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*g^2 + h*(-(b*g) + a*h)])/(2*h
^2)
```

Maple [B] time = 0.317, size = 599, normalized size = 3.4

$$\frac{f}{ch}\sqrt{cx^2+bx+a} - \frac{bf}{2h}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)c^{-\frac{3}{2}} + \frac{e}{h}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}} - \frac{fg}{h^2}\ln\left(\left(\frac{b}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] f*(c*x^2+b*x+a)^(1/2)/c/h-1/2/h*f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))+1/h*e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h^
2*f*g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h/((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((
2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(
x+g/h))*e*g-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)
/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(
b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2}{(g + hx)\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.231 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}-\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h\left(2ah(2fg-eh)-b\left(-dh^2-egh+3fg^2\right)\right)+2c\left(\right.\right.}{2h^2\left(ah^2-bgh+cg^2\right)^{3/2}}$$

```
[Out] -(((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*h^2
)*(g + h*x))) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(Sqrt[c]*h^2) - ((2*c*(f*g^3 - d*g*h^2) + h*(2*a*h*(2*f*g - e*h) - b*(3*f*g
^2 - e*g*h - d*h^2)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*h^2*(c*g^2 - b*g*h + a*h^2)^(
3/2))
```

Rubi [A] time = 0.37112, antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1650, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}-\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh(dh+eg\right.\right.}{2h^2\left(ah^2-bgh+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*h^2
)*(g + h*x))) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(Sqrt[c]*h^2) - ((2*c*(f*g^3 - d*g*h^2) - h*(3*b*f*g^2 - b*h*(e*g + d*h) -
2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*h^2*(c*g^2 - b*g*h + a*h^2)^(
3/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
```

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\frac{1}{2}\left(-2cdg+beg+2afg-\frac{bfg^2}{h}+bdh-2aeh\right)+f\left(bg-\frac{cg^2}{h}-ah\right)x}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{cg^2-bgh+ah^2}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{h^2} - \frac{\left(2c\left(fg^3-dgh^2\right)-h\left(3bfg^2-\right.\right.}{2}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{h^2} + \frac{\left(2c\left(fg^3-\right.\right.}{}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}h^2} - \frac{\left(2c\left(fg^3-dgh^2\right)-h\left(\right.\right.}{}$$

Mathematica [A] time = 0.477062, size = 227, normalized size = 0.94

$$\frac{\frac{\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(h\left(-2ah(eh-2fg)+bh(dh+eg)-3bfg^2\right)+2c\left(fg^3-dgh^2\right)\right)}{2\left(h(ah-bg)+cg^2\right)^{3/2}} - \frac{h\sqrt{a+x(b+cx)}\left(h(dh-eg)+fg^2\right)}{(g+hx)\left(h(ah-bg)+cg^2\right)} + \frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g)
 + a*h))*(g + h*x))) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*
x)])])/Sqrt[c] + ((2*c*(f*g^3 - d*g*h^2) + h*(-3*b*f*g^2 + b*h*(e*g + d*h)
- 2*a*h*(-2*f*g + e*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt
[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(2*(c*g^2 + h*(-(b*g) +
 a*h))^(3/2)))/h^2
```

Maple [B] time = 0.279, size = 1671, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] f/h^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h^2/((a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*
((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b
*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln
((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/(x+g/h))*f*g-1/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g
/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*d+1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/
h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^
2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)*f*g^2+1/2/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2))/(x+g/h))*b*d-1/2/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/(x+g/h))*b*e*g+1/2/h^2/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2))/(x+g/h))*b*f*g^2-1/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g
^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2))/(x+g/h))*c*g*d+1/h^2/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2))/(x+g/h))*c*g^2*e-1/h^3/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2
)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2))/(x+g/h))*c*g^3*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.232 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=336

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c\left(a\left(dh^2-3egh+fg^2\right)+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(3dh^2+egh\right.\right.}{8\left(ah^2-bgh+cg^2\right)^{5/2}}$$

```
[Out] -((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^2) + ((2*c*g*(f*g^2 + h*(e*g - 3*d*h)) + h*(4*a*h*(2*f*g - e*h
) - b*(5*f*g^2 - e*g*h - 3*d*h^2)))*Sqrt[a + b*x + c*x^2])/(4*h*(c*g^2 - b*
g*h + a*h^2)^2*(g + h*x)) + ((8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g +
e*h) + b^2*(3*f*g^2 + e*g*h + 3*d*h^2) - 4*c*(b*g*(e*g + 2*d*h) + a*(f*g^2
- 3*e*g*h + d*h^2)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
- b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rubi [A] time = 0.656221, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1650, 806, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c\left(-ah(3eg-dh)+afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(3dh+e\right.\right.}{8\left(ah^2-bgh+cg^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^2) + ((2*c*(f*g^3 + g*h*(e*g - 3*d*h)) - h*(5*b*f*g^2 - b*h*(e
*g + 3*d*h) - 4*a*h*(2*f*g - e*h)))*Sqrt[a + b*x + c*x^2])/(4*h*(c*g^2 - b*
g*h + a*h^2)^2*(g + h*x)) + ((8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g +
e*h) - 4*c*(a*f*g^2 - a*h*(3*e*g - d*h) + b*g*(e*g + 2*d*h)) + b^2*(3*f*g^2
 + h*(e*g + 3*d*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
- b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\frac{1}{2}\left(-4cdg+beg+4afg-\frac{bfg^2}{h}+3bdh-4aeh\right)-\left(ceg-2bfg+\frac{cfg^2}{h}-c\cdots\right.}{(g+hx)^2\sqrt{a+bx+cx^2}}}{2\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\left(2c\left(fg^3+gh(eg-3dh)\right)-h\left(5bfg^2-bh(eg+3\cdots\right.\right.}{4h\left(cg^2-bgh+ah^2\right)\cdots}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\left(2c\left(fg^3+gh(eg-3dh)\right)-h\left(5bfg^2-bh(eg+3\cdots\right.\right.}{4h\left(cg^2-bgh+ah^2\right)\cdots}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\left(2c\left(fg^3+gh(eg-3dh)\right)-h\left(5bfg^2-bh(eg+3\cdots\right.\right.}{4h\left(cg^2-bgh+ah^2\right)\cdots}$$

Mathematica [A] time = 1.34882, size = 367, normalized size = 1.09

$$\frac{-\frac{ch\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(8a^2fh^2-4c\left(ah(dh-3eg)+afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(3dh+eg)+3fg^2\right)+8c^2dg^2\right)}{8\left(h(ah-bg)+cg^2\right)^{5/2}}+\frac{c\sqrt{a+x(b+cx)}\left(h\left(-4ah(\cdots\right.\right.}{\cdots}}{ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((f*Sqrt[a + x*(b + c*x)])/(g + h*x)^2) + ((c*f*g^2 + 2*f*h*(-(b*g) + a*h
) + c*h*(e*g - d*h))*Sqrt[a + x*(b + c*x)])/(2*(c*g^2 + h*(-(b*g) + a*h))*(
g + h*x)^2) + (c*(2*c*(f*g^3 + g*h*(e*g - 3*d*h)) + h*(-5*b*f*g^2 + b*h*(e*
g + 3*d*h) - 4*a*h*(-2*f*g + e*h)))*Sqrt[a + x*(b + c*x)])/(4*(c*g^2 + h*(-
(b*g) + a*h))^2*(g + h*x)) - (c*h*(8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f
*g + e*h) - 4*c*(a*f*g^2 + a*h*(-3*e*g + d*h) + b*g*(e*g + 2*d*h)) + b^2*(3
*f*g^2 + h*(e*g + 3*d*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sq
rt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(8*(c*g^2 + h*(-(b*g)
 + a*h))^(5/2)))/(c*h)
```

Maple [B] time = 0.285, size = 3615, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 3/2/h^2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-
b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((
x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b
*c*g^3*f-3/2/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*
(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+
g/h))*b*c*g^2*e-f/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*
g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2
*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))-1/2/h/(a*
h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*d-1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g
)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e-3/2/h^2/(a*h^2-b*g*h+c*g^2)^2/
(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c
*g^3*f-3/8/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a
*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/
h))*b^2*f*g^2+3/2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(
(2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2
)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/
(x+g/h))*b*c*g*d-3/2/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2
)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2))/(x+g/h))*c^2*g^2*d+3/2/h^2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2
)/h^2)^(1/2))/(x+g/h))*c^2*g^3*e-3/2/h^3/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*(
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*
g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^4*f+3/4/h/(a*h^2-b*g*h+c*g^2)^2/(x+g/
h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*f*g^
2-1/h^2/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*
g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+
g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*f
*g-3/2/h^2/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2
-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*(
(x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*
c*g*e+5/2/h^3/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*
h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h
))*c*g^2*f+3/2/h/(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(
```

```
x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c*g^2*e+2/h^2/(a*h^2-b*g*h+c*g^2)/(x+
g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g+
1/2/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*
h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/
h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*e-1
/2/h^3/(a*h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+3/4*h/(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g
/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*d-3/4/(a*h^2
-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g
^2)/h^2)^(1/2)*b*e*g+1/2/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*
h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-3/2/(a*h^2-b*g*h+c*g^
2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*c*g*d-3/8*h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2
*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x
+g/h))*b^2*d+3/8/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((
2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(
x+g/h))*b^2*e*g+1/2/h*c/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2))/(x+g/h))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2}{\left(g + hx\right)^3 \sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)**3*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] time = 1.36301, size = 3114, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c^2*d*g^2 + 3*b^2*f*g^2 - 4*a*c*f*g^2 - 8*b*c*d*g*h - 8*a*b*f*g*h +
3*b^2*d*h^2 - 4*a*c*d*h^2 + 8*a^2*f*h^2 - 4*b*c*g^2*e + b^2*g*h*e + 12*a*c*
g*h*e - 4*a*b*h^2*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(
c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^2*g^4 - 2*b*c*g^3*h + b^2*g^2*h^2 +
 2*a*c*g^2*h^2 - 2*a*b*g*h^3 + a^2*h^4)*sqrt(-c*g^2 + b*g*h - a*h^2)) + 1/4
*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^4*h - 16*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 +
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 + 8*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b*c*d*g*h^4 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*a*b*f*g*h^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*h^5 + 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*b*c*g^2*h^3*e - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*g*h^4*e
- 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*g*h^4*e + 4*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a*b*h^5*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c
```

^(5/2)*f*g^5 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f*g^3*h^2 + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f*g^3*h^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^2*h^3 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^4 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*g*h^4 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*f*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*g^4*h*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*g^3*h^2*e + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*g^2*h^3*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*g^2*h^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*g*h^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*h^5*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f*g^5 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f*g^4*h - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f*g^3*h^2 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f*g^3*h^2 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*g^2*h^3 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^2*h^3 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*g*h^4 - 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*f*g*h^4 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*g^4*h*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*g^3*h^2*e + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*g^2*h^3*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*g^2*h^3*e + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*g*h^4*e + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*g*h^4*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*h^5*e + 2*b^2*c^(3/2)*f*g^5 - 5*b^3*sqrt(c)*f*g^4*h - 4*a*b*c^(3/2)*f*g^4*h - 6*b^2*c^(3/2)*d*g^3*h^2 + 21*a*b^2*sqrt(c)*f*g^3*h^2 + 4*a^2*c^(3/2)*f*g^3*h^2 + 3*b^3*sqrt(c)*d*g^2*h^3 + 20*a*b*c^(3/2)*d*g^2*h^3 - 32*a^2*b*sqrt(c)*f*g^2*h^3 - 11*a*b^2*sqrt(c)*d*g*h^4 - 12*a^2*c^(3/2)*d*g*h^4 + 16*a^3*sqrt(c)*f*g*h^4 + 8*a^2*b*sqrt(c)*d*h^5 + 2*b^2*c^(3/2)*g^4*h*e + b^3*sqrt(c)*g^3*h^2*e - 8*a*b*c^(3/2)*g^3*h^2*e - 5*a*b^2*sqrt(c)*g^2*h^3*e + 4*a^2*c^(3/2)*g^2*h^3*e + 12*a^2*b*sqrt(c)*g*h^4*e - 8*a^3*sqrt(c)*h^5*e)/((c^2*g^4*h^2 - 2*b*c*g^3*h^3 + b^2*g^2*h^4 + 2*a*c*g^2*h^4 - 2*a*b*g*h^5 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^2)

3.233 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=504

$$\frac{h\sqrt{a+bx+cx^2}\left(8c^2\left(32a^2fh^2+39abh(eh+3fg)+b^2\left(9h(dh+3eg)+20fg^2\right)\right)+2chx\left(-8c^2(9aeh+11afg+3bdh+3b\right.\right.}{}$$

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^3)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((12*c^2*d - 6*b*c*e +
 7*b^2*f - 16*a*c*f)*h*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c^2*(b^2 - 4*a
*c)) + (h*(192*c^4*d*g^2 + 105*b^4*f*h^2 - 10*b^2*c*h*(46*a*f*h + 9*b*(3*f*
g + e*h)) - 16*c^3*(3*b*g*(2*e*g + 3*d*h) + 4*a*(7*f*g^2 + 9*e*g*h + 3*d*h^
2)) + 8*c^2*(32*a^2*f*h^2 + 39*a*b*h*(3*f*g + e*h) + b^2*(20*f*g^2 + 9*h*(3
*e*g + d*h))) + 2*c*h*(48*c^3*d*g - 35*b^3*f*h - 8*c^2*(3*b*e*g + 11*a*f*g
+ 3*b*d*h + 9*a*e*h) + 2*b*c*(17*b*f*g + 15*b*e*h + 58*a*f*h))*x)*Sqrt[a +
b*x + c*x^2])/(24*c^4*(b^2 - 4*a*c)) - ((35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*g
 + b*e*h + 2*a*f*h) - 16*c^3*g*(f*g^2 + 3*h*(e*g + d*h)) + 24*c^2*h*(a*h*(3
*f*g + e*h) + b*(3*f*g^2 + 3*e*g*h + d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(16*c^(9/2))
```

Rubi [A] time = 1.1756, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1644, 832, 779, 621, 206}

$$\frac{h\sqrt{a+bx+cx^2}\left(8c\left(32a^2fh^2+39abh(eh+3fg)+b^2\left(9h(dh+3eg)+20fg^2\right)\right)+2hx\left(-8c^2(9aeh+11afg+3bdh+3beg\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^3)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((12*c^2*d - 6*b*c*e +
 7*b^2*f - 16*a*c*f)*h*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c^2*(b^2 - 4*a
*c)) + (h*(192*c^3*d*g^2 + (105*b^4*f*h^2)/c - 10*b^2*h*(46*a*f*h + 9*b*(3*
f*g + e*h)) - 16*c^2*(3*b*g*(2*e*g + 3*d*h) + 4*a*(7*f*g^2 + 9*e*g*h + 3*d*
h^2)) + 8*c*(32*a^2*f*h^2 + 39*a*b*h*(3*f*g + e*h) + b^2*(20*f*g^2 + 9*h*(3
```

```
*e*g + d*h))) + 2*h*(48*c^3*d*g - 35*b^3*f*h - 8*c^2*(3*b*e*g + 11*a*f*g +
3*b*d*h + 9*a*e*h) + 2*b*c*(17*b*f*g + 15*b*e*h + 58*a*f*h))*x)*Sqrt[a + b*
x + c*x^2])/(24*c^3*(b^2 - 4*a*c)) - ((35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*g +
 b*e*h + 2*a*f*h) - 16*c^3*(f*g^3 + 3*g*h*(e*g + d*h)) + 24*c^2*h*(3*b*f*g^
2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(16*c^(9/2))
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
```

```
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(g+hx)^2\left(-\frac{b^2fg+6b(cd+}{}\right.}{}}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+7b^2f-2c(\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+7b^2f-2c(\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+7b^2f-2c(\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+7b^2f-2c(\right.}{}$$

Mathematica [A] time = 1.64845, size = 715, normalized size = 1.42

$$\frac{3\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(24c^2h\left(ah(eh+3fg)+bh(dh+3eg)+3bfg^2\right)-30bch^2(2afh+beh+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*(105*b^5*f*h^3*x + 5*b^4*h^2*(21*a*f*h + c*x*(-54*f*g - 18*e*h
+ 7*f*h*x)) - 2*b^3*c*h*(5*a*h*(27*f*g + 9*e*h + 53*f*h*x) + c*x*(3*h*(-36*
```

```
e*g - 12*d*h + 5*e*h*x) + f*(-108*g^2 + 45*g*h*x + 7*h^2*x^2))) - 16*c^2*(-
16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2) -
3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h*x
+ 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2) +
3*h*(4*d*h + 3*e*(4*g + h*x)))) - 8*b*c^2*(-6*c^2*g^2*(-(d*g) + e*g*x + 3*d
*h*x) - a^2*h^2*(117*f*g + 39*e*h + 61*f*h*x) + a*c*(f*(6*g^3 + 90*g^2*h*x
- 45*g*h^2*x^2 - 7*h^3*x^3) + 3*h*(2*d*h*(3*g + 5*h*x) + e*(6*g^2 + 30*g*h*
x - 5*h^2*x^2)))) + 4*b^2*c*(-115*a^2*f*h^3 + a*c*h*(3*h*(18*e*g + 6*d*h +
31*e*h*x) + f*(54*g^2 + 279*g*h*x - 43*h^2*x^2)) + c^2*x*(f*(-12*g^3 + 18*g
^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + 3*h*(2*d*h*(-6*g + h*x) + e*(-12*g^2 +
6*g*h*x + h^2*x^2))))) + 3*(b^2 - 4*a*c)*(35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*
g + b*e*h + 2*a*f*h) - 16*c^3*g*(f*g^2 + 3*h*(e*g + d*h)) + 24*c^2*h*(3*b*f
*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*Sqrt[a + x*(b + c*x)]*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(48*c^(9/2)*(-b^2 + 4*a*c
)*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.064, size = 2780, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -39/4*b/c^3*a/(c*x^2+b*x+a)^(1/2)*g*h^2*f-13/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*h^3*e-15/4*b/c^2*x^2/(c*x^2+b*x+a)^(1/2)*g*h^2*f-15/4*h^3*f*b
/c^3*a*x/(c*x^2+b*x+a)^(1/2)-8/3*h^3*f*a^2/c^3*b^2/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)+9/2*b/c^2*x/(c*x^2+b*x+a)^(1/2)*g*h^2*e+9/2*b/c^2*x/(c*x^2+b*x+a)^(
1/2)*g^2*h*f-3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^3*d-9/4*b^4/c^
3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*h*f-9/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*
x+a)^(1/2)*g*h^2*e+2*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*h^3*d+x^2/c/
(c*x^2+b*x+a)^(1/2)*h^3*d-3/4*b^2/c^3/(c*x^2+b*x+a)^(1/2)*h^3*d-3/2*b/c^(5/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*d+1/2*x^3/c/(c*x^2+b*x+a
)^(1/2)*h^3*e+15/16*b^3/c^4/(c*x^2+b*x+a)^(1/2)*h^3*e+15/8*b^2/c^(7/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e-3/2*a/c^(5/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e+3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))*g^2*h*e-3/c/(c*x^2+b*x+a)^(1/2)*g^2*h*d+2*g^3*d*(2*c*x+b)/(4*
a*c-b^2)/(c*x^2+b*x+a)^(1/2)+4*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^3*
d-16/3*h^3*f*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+45/8*b^4/c^3/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*f-13/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+
a)^(1/2)*x*h^3*e-39/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h^2*f+6*a
/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h^2*e+6*a/c^2*b^2/(4*a*c-b^2)/(c
```

```
*x^2+b*x+a)^(1/2)*g^2*h*f-9/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h
^2*e-35/16*h^3*f*b^3/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-8/
3*h^3*f*a^2/c^3/(c*x^2+b*x+a)^(1/2)-x/c/(c*x^2+b*x+a)^(1/2)*g^3*f+1/2*b/c^2
/(c*x^2+b*x+a)^(1/2)*g^3*f+3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))*g*h^2*d+2*a/c^2/(c*x^2+b*x+a)^(1/2)*h^3*d-9/2*b^3/c^2/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)*x*g^2*h*f+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*
d+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*e+115/12*h^3*f*b^3/c^3*a/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*g^3*f-1/c/(c*x^2+b*x+a)^(1/2)*g^3*e-39/2*b^2/c^2*a/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*x*g*h^2*f+12*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g
*h^2*e+12*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*f+1/3*h^3*f*x^4/c/(
c*x^2+b*x+a)^(1/2)-35/32*h^3*f*b^4/c^5/(c*x^2+b*x+a)^(1/2)+115/24*h^3*f*b^4
/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-35/16*h^3*f*b^5/c^4/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)*x+b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^3*f+9/2*a/c^2*x
/(c*x^2+b*x+a)^(1/2)*g*h^2*f+3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*
h^2*d+3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*h*e-45/8*b^2/c^3*x/(c
*x^2+b*x+a)^(1/2)*g*h^2*f+15/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^
3*e+45/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h^2*f+3/2*b/c^2*x/(c*x^
2+b*x+a)^(1/2)*h^3*d-9/4*b^2/c^3/(c*x^2+b*x+a)^(1/2)*g*h^2*e-9/4*b^2/c^3/(c
*x^2+b*x+a)^(1/2)*g^2*h*f-3/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*h^3*d
-9/2*b/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*e-9/2*b/c^
(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*f+6*a/c^2/(c*x^2+b*
x+a)^(1/2)*g*h^2*e+6*a/c^2/(c*x^2+b*x+a)^(1/2)*g^2*h*f+3/2*x^3/c/(c*x^2+b*x
+a)^(1/2)*g*h^2*f+3/2*b/c^2/(c*x^2+b*x+a)^(1/2)*g^2*h*e+1/2*b^3/c^2/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)*g^3*f-2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^3*e-
b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^3*e-4/3*h^3*f*a/c^2*x^2/(c*x^2+b*x+
a)^(1/2)+35/16*h^3*f*b^3/c^4*x/(c*x^2+b*x+a)^(1/2)-35/32*h^3*f*b^6/c^5/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2)+115/24*h^3*f*b^2/c^4*a/(c*x^2+b*x+a)^(1/2)+15/4
*h^3*f*b/c^(7/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/12*h^3*f*b
/c^2*x^3/(c*x^2+b*x+a)^(1/2)+35/24*h^3*f*b^2/c^3*x^2/(c*x^2+b*x+a)^(1/2)-3*
x/c/(c*x^2+b*x+a)^(1/2)*g*h^2*d-3*x/c/(c*x^2+b*x+a)^(1/2)*g^2*h*e+3/2*b/c^2
/(c*x^2+b*x+a)^(1/2)*g*h^2*d-5/4*b/c^2*x^2/(c*x^2+b*x+a)^(1/2)*h^3*e-15/8*b
^2/c^3*x/(c*x^2+b*x+a)^(1/2)*h^3*e+45/16*b^3/c^4/(c*x^2+b*x+a)^(1/2)*g*h^2*
f+15/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*h^3*e+45/8*b^2/c^(7/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*f-13/4*b/c^3*a/(c*x^2+b*x+a)^
(1/2)*h^3*e+3/2*a/c^2*x/(c*x^2+b*x+a)^(1/2)*h^3*e-9/2*a/c^(5/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*f+3*x^2/c/(c*x^2+b*x+a)^(1/2)*g^2*h*
f+3*x^2/c/(c*x^2+b*x+a)^(1/2)*g*h^2*e-6*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x
*g^2*h*d-3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*h*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 132.617, size = 6238, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(3*(16*(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)*
e - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d -
 12*(a*b^3*c^2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c^
3)*f)*g*h^2 - (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b^2*c
^2 + 16*a^3*c^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3 + (16
*(b^2*c^4 - 4*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3 - 4*a
*b*c^4)*f)*g^2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*c^4)*e
+ 3*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b
*c^4)*d - 6*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c - 40*a*b
^3*c^2 + 48*a^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3 + 24*(
2*(b^3*c^3 - 4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h + 6*(8*(b^3*
c^3 - 4*a*b*c^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c - 24*a*b^3*c
^2 + 16*a^2*b*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d - 6*(5*b^5*c -
24*a*b^3*c^2 + 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)
*h^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*(b^2*c^4 - 4*a*c^5)*f*h^3*x^4 - 48*(b*c^
5*d - 2*a*c^5*e + a*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c^4*e + (3*a*b^2*c
^3 - 8*a^2*c^4)*f)*g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c^3 - 8*a^2*c^4)*e
+ (15*a*b^3*c^2 - 52*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*c^3 - 8*a^2*c^4)*d
- 6*(15*a*b^3*c^2 - 52*a^2*b*c^3)*e + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*
a^3*c^3)*f)*h^3 + 2*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 + (6*(b^2*c^4 - 4*a*c^5
)*e - 7*(b^3*c^3 - 4*a*b*c^4)*f)*h^3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h
 + 18*(4*(b^2*c^4 - 4*a*c^5)*e - 5*(b^3*c^3 - 4*a*b*c^4)*f)*g*h^2 + (24*(b^
2*c^4 - 4*a*c^5)*d - 30*(b^3*c^3 - 4*a*b*c^4)*e + (35*b^4*c^2 - 172*a*b^2*c
^3 + 128*a^2*c^4)*f)*h^3)*x^2 - (48*(2*c^6*d - b*c^5*e + (b^2*c^4 - 2*a*c^5
)*f)*g^3 - 72*(2*b*c^5*d - 2*(b^2*c^4 - 2*a*c^5)*e + (3*b^3*c^3 - 10*a*b*c^
```

```
4)*f)*g^2*h + 18*(8*(b^2*c^4 - 2*a*c^5)*d - 4*(3*b^3*c^3 - 10*a*b*c^4)*e +
(15*b^4*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 - 10*a*b
*c^4)*d - 6*(15*b^4*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*e + (105*b^5*c - 530*a
*b^3*c^2 + 488*a^2*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*
a^2*c^6 + (b^2*c^6 - 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x), -1/48*(3*(16*
(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)*e - 3*(a*b^3*
c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d - 12*(a*b^3*c^
2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c^3)*f)*g*h^2 -
 (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c
^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3 + (16*(b^2*c^4 - 4
*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3 - 4*a*b*c^4)*f)*g^
2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*c^4)*e + 3*(5*b^4*c^
2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b*c^4)*d - 6*(
5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c - 40*a*b^3*c^2 + 48*a
^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3 + 24*(2*(b^3*c^3 -
4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h + 6*(8*(b^3*c^3 - 4*a*b*c
^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c - 24*a*b^3*c^2 + 16*a^2*b
*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d - 6*(5*b^5*c - 24*a*b^3*c^2
+ 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)*h^3)*x)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) - 2*(8*(b^2*c^4 - 4*a*c^5)*f*h^3*x^4 - 48*(b*c^5*d - 2*a*c^5*e + a
*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c^4*e + (3*a*b^2*c^3 - 8*a^2*c^4)*f)*
g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c^3 - 8*a^2*c^4)*e + (15*a*b^3*c^2 - 5
2*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*c^3 - 8*a^2*c^4)*d - 6*(15*a*b^3*c^2 -
 52*a^2*b*c^3)*e + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*f)*h^3 + 2
*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 + (6*(b^2*c^4 - 4*a*c^5)*e - 7*(b^3*c^3 -
4*a*b*c^4)*f)*h^3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h + 18*(4*(b^2*c^4 -
 4*a*c^5)*e - 5*(b^3*c^3 - 4*a*b*c^4)*f)*g*h^2 + (24*(b^2*c^4 - 4*a*c^5)*d
- 30*(b^3*c^3 - 4*a*b*c^4)*e + (35*b^4*c^2 - 172*a*b^2*c^3 + 128*a^2*c^4)*f
)*h^3)*x^2 - (48*(2*c^6*d - b*c^5*e + (b^2*c^4 - 2*a*c^5)*f)*g^3 - 72*(2*b*
c^5*d - 2*(b^2*c^4 - 2*a*c^5)*e + (3*b^3*c^3 - 10*a*b*c^4)*f)*g^2*h + 18*(8
*(b^2*c^4 - 2*a*c^5)*d - 4*(3*b^3*c^3 - 10*a*b*c^4)*e + (15*b^4*c^2 - 62*a*
b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 - 10*a*b*c^4)*d - 6*(15*b^4
*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*e + (105*b^5*c - 530*a*b^3*c^2 + 488*a^2*
b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*a^2*c^6 + (b^2*c^6
- 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g+hx\right)^{3}\left(d+ex+fx^{2}\right)}{\left(a+bx+cx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [B] time = 1.24518, size = 1423, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(((2*(4*(b^2*c^3*f*h^3 - 4*a*c^4*f*h^3)*x/(b^2*c^4 - 4*a*c^5) + (18*b^
2*c^3*f*g*h^2 - 72*a*c^4*f*g*h^2 - 7*b^3*c^2*f*h^3 + 28*a*b*c^3*f*h^3 + 6*b
^2*c^3*h^3*e - 24*a*c^4*h^3*e)/(b^2*c^4 - 4*a*c^5))*x + (72*b^2*c^3*f*g^2*h
 - 288*a*c^4*f*g^2*h - 90*b^3*c^2*f*g*h^2 + 360*a*b*c^3*f*g*h^2 + 24*b^2*c^
3*d*h^3 - 96*a*c^4*d*h^3 + 35*b^4*c*f*h^3 - 172*a*b^2*c^2*f*h^3 + 128*a^2*c
^3*f*h^3 + 72*b^2*c^3*g*h^2*e - 288*a*c^4*g*h^2*e - 30*b^3*c^2*h^3*e + 120*
a*b*c^3*h^3*e)/(b^2*c^4 - 4*a*c^5))*x - (96*c^5*d*g^3 + 48*b^2*c^3*f*g^3 -
96*a*c^4*f*g^3 - 144*b*c^4*d*g^2*h - 216*b^3*c^2*f*g^2*h + 720*a*b*c^3*f*g^
2*h + 144*b^2*c^3*d*g*h^2 - 288*a*c^4*d*g*h^2 + 270*b^4*c*f*g*h^2 - 1116*a*
b^2*c^2*f*g*h^2 + 432*a^2*c^3*f*g*h^2 - 72*b^3*c^2*d*h^3 + 240*a*b*c^3*d*h^
3 - 105*b^5*f*h^3 + 530*a*b^3*c*f*h^3 - 488*a^2*b*c^2*f*h^3 - 48*b*c^4*g^3*
e + 144*b^2*c^3*g^2*h*e - 288*a*c^4*g^2*h*e - 216*b^3*c^2*g*h^2*e + 720*a*b
*c^3*g*h^2*e + 90*b^4*c*h^3*e - 372*a*b^2*c^2*h^3*e + 144*a^2*c^3*h^3*e)/(b
^2*c^4 - 4*a*c^5))*x - (48*b*c^4*d*g^3 + 48*a*b*c^3*f*g^3 - 288*a*c^4*d*g^2
*h - 216*a*b^2*c^2*f*g^2*h + 576*a^2*c^3*f*g^2*h + 144*a*b*c^3*d*g*h^2 + 27
0*a*b^3*c*f*g*h^2 - 936*a^2*b*c^2*f*g*h^2 - 72*a*b^2*c^2*d*h^3 + 192*a^2*c^
3*d*h^3 - 105*a*b^4*f*h^3 + 460*a^2*b^2*c*f*h^3 - 256*a^3*c^2*f*h^3 - 96*a*
c^4*g^3*e + 144*a*b*c^3*g^2*h*e - 216*a*b^2*c^2*g*h^2*e + 576*a^2*c^3*g*h^2
*e + 90*a*b^3*c*h^3*e - 312*a^2*b*c^2*h^3*e)/(b^2*c^4 - 4*a*c^5))/sqrt(c*x^
2 + b*x + a) - 1/16*(16*c^3*f*g^3 - 72*b*c^2*f*g^2*h + 48*c^3*d*g*h^2 + 90*
b^2*c*f*g*h^2 - 72*a*c^2*f*g*h^2 - 24*b*c^2*d*h^3 - 35*b^3*f*h^3 + 60*a*b*c
*f*h^3 + 48*c^3*g^2*h*e - 72*b*c^2*g*h^2*e + 30*b^2*c*h^3*e - 24*a*c^2*h^3*
e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

3.234 $\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2\left(h(dh+2eg)+fg^2\right)\right)}{8c^{7/2}}+\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right.\right.\right.}{c\left(b^2-4ac\right.}$$

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^2)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (h*(32*c^3*d*g - 15*b^
3*f*h - 8*c^2*(2*b*e*g + 8*a*f*g + b*d*h + 4*a*e*h) + 4*b*c*(6*b*f*g + 3*b*
e*h + 13*a*f*h) + 2*c*(8*c^2*d - 4*b*c*e + 5*b^2*f - 12*a*c*f)*h*x)*Sqrt[a
+ b*x + c*x^2])/(4*c^3*(b^2 - 4*a*c)) + ((15*b^2*f*h^2 - 12*c*h*(2*b*f*g +
b*e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rubi [A] time = 0.391658, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1644, 779, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2\left(h(dh+2eg)+fg^2\right)\right)}{8c^{7/2}}+\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right.\right.\right.}{c\left(b^2-4ac\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^2)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (h*(32*c^2*d*g - (15*b
^3*f*h)/c - 8*c*(2*b*e*g + 8*a*f*g + b*d*h + 4*a*e*h) + 4*b*(6*b*f*g + 3*b*
e*h + 13*a*f*h) + 2*(8*c^2*d - 4*b*c*e + 5*b^2*f - 12*a*c*f)*h*x)*Sqrt[a +
b*x + c*x^2])/(4*c^2*(b^2 - 4*a*c)) + ((15*b^2*f*h^2 - 12*c*h*(2*b*f*g + b*
e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
```

```
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - 2\int \frac{(g+hx)\left(-\frac{b^2fg+4b(cd+a}{\cdots}\right.}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2dg-\frac{15b^3fh}{c}-\cdots\right.}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2dg-\frac{15b^3fh}{c}-\cdots\right.}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2dg-\frac{15b^3fh}{c}-\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.832222, size = 412, normalized size = 1.43

$$\frac{2\sqrt{c}\left(4bc\left(-13a^2fh^2+ac\left(2h(dh+2eg+5ehx)+f\left(2g^2+20ghx-5h^2x^2\right)\right)+2c^2g(d(g-2hx)-egx)\right)+8c^2\left(a^2h(4eh+8\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*(15*b^4*f*h^2*x + b^3*h*(15*a*f*h + c*x*(-24*f*g - 12*e*h + 5*f*
h*x)) + 4*b*c*(-13*a^2*f*h^2 + 2*c^2*g*(-(e*g*x) + d*(g - 2*h*x)) + a*c*(2*
h*(2*e*g + d*h + 5*e*h*x) + f*(2*g^2 + 20*g*h*x - 5*h^2*x^2))) - 2*b^2*c*(a
*h*(12*f*g + 6*e*h + 31*f*h*x) + c*x*(2*h*(-4*e*g - 2*d*h + e*h*x) + f*(-4*
g^2 + 4*g*h*x + h^2*x^2))) + 8*c^2*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h +
3*f*h*x) + a*c*(-2*d*h*(2*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(-
2*g^2 + 4*g*h*x + h^2*x^2)))) - (b^2 - 4*a*c)*(15*b^2*f*h^2 - 12*c*h*(2*b*f
*g + b*e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*Sqrt[a + x*(b + c*x)
]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(8*c^(7/2)*(-b^2
+ 4*a*c)*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.063, size = 1557, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -3/2*h^2*f*a/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+15/8*h^2*f
*b^2/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-x/c/(c*x^2+b*x+a)^
(1/2)*d*h^2-x/c/(c*x^2+b*x+a)^(1/2)*f*g^2+1/2*b/c^2/(c*x^2+b*x+a)^(1/2)*d*h
^2+1/2*b/c^2/(c*x^2+b*x+a)^(1/2)*f*g^2-4*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
x*g*h*d-2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h*d+8*a/c*b/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)*x*g*h*f+2*g^2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
)+x^2/c/(c*x^2+b*x+a)^(1/2)*h^2*e-3/4*b^2/c^3/(c*x^2+b*x+a)^(1/2)*h^2*e-3/2
*b/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e+2*a/c^2/(c*x^2
+b*x+a)^(1/2)*h^2*e+1/2*h^2*f*x^3/c/(c*x^2+b*x+a)^(1/2)+15/16*h^2*f*b^3/c^4
/(c*x^2+b*x+a)^(1/2)+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
d*h^2+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-1/c/(c*x^
2+b*x+a)^(1/2)*e*g^2+2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e*g*h+b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*h^2+b^2/c/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*x*f*g^2+b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e*g*h+15/8*h
^2*f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-3/2*b^3/c^2/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)*x*h^2*e+4*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h*f+2
*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e*g*h-3*b^3/c^2/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*g*h*f+4*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^2*e-13/2*
h^2*f*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-13/4*h^2*f*b^3/c^3*a/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2)-3/2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h
*f+2*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*h^2*e+3*b/c^2*x/(c*x^2+b*x+a
)^(1/2)*g*h*f-2/c/(c*x^2+b*x+a)^(1/2)*g*h*d-2*x/c/(c*x^2+b*x+a)^(1/2)*e*g*h
+b/c^2/(c*x^2+b*x+a)^(1/2)*e*g*h-3*b/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))*g*h*f+4*a/c^2/(c*x^2+b*x+a)^(1/2)*g*h*f-b^2/c/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)*e*g^2+15/16*h^2*f*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-
3/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*h^2*e+3/2*b/c^2*x/(c*x^2+b*x+a)
^(1/2)*h^2*e-3/2*b^2/c^3/(c*x^2+b*x+a)^(1/2)*g*h*f-13/4*h^2*f*b/c^3*a/(c*x^
2+b*x+a)^(1/2)+3/2*h^2*f*a/c^2*x/(c*x^2+b*x+a)^(1/2)-2*b/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*e*g^2+2*x^2/c/(c*x^2+b*x+a)^(1/2)*g*h*f-15/8*h^2*f*b^2/c^3*
x/(c*x^2+b*x+a)^(1/2)+1/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*h^2+1/2
*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f*g^2-5/4*h^2*f*b/c^2*x^2/(c*x^2+b
*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 90.9787, size = 3730, normalized size = 12.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e -
 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b^2*c^2 - 4*a^2*c^3)*d - 12*(a*b^
3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f)*h^2 + (8*
(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b
*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 12*(b^3*c^2 - 4*a*b*c^3)*e + 3*(5
*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)*x^2 + (8*(b^3*c^2 - 4*a*b*c^3)*
f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b^4*c - 4*a*b^2*c^2)*f)*g*h + (8*
(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b^2*c^2)*e + 3*(5*b^5 - 24*a*b^3*
c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*
f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g^2 + 8*(4*a*c^4*d - 2*a*b*
c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b*c^3*d - 4*(3*a*b^2*c^2 -
8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*
f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4*a*b*c^3)*f)*h^2)*x^2 - (8
*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^4*d - 2*(b^2*c^
3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 2*a*c^4)*d
 - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f)
*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6
)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 +
8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e - 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b
^2*c^2 - 4*a^2*c^3)*d - 12*(a*b^3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*
b^2*c + 16*a^3*c^2)*f)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 -
 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 1
2*(b^3*c^2 - 4*a*b*c^3)*e + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)
*x^2 + (8*(b^3*c^2 - 4*a*b*c^3)*f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b
^4*c - 4*a*b^2*c^2)*f)*g*h + (8*(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b
^2*c^2)*e + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(-c)*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) -
```

```
 2*(2*(b^2*c^3 - 4*a*c^4)*f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g
^2 + 8*(4*a*c^4*d - 2*a*b*c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b
*c^3*d - 4*(3*a*b^2*c^2 - 8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2
 + (8*(b^2*c^3 - 4*a*c^4)*f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4
*a*b*c^3)*f)*h^2)*x^2 - (8*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2
- 8*(2*b*c^4*d - 2*(b^2*c^3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h
+ (8*(b^2*c^3 - 2*a*c^4)*d - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*
a*b^2*c^2 + 24*a^2*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^
2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g+hx\right)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [B] time = 1.33493, size = 783, normalized size = 2.71

$$\left(\left(\left(\frac{2\left(b^2c^2fh^2-4ac^3fh^2\right)x}{b^2c^3-4ac^4}+\frac{8b^2c^2fgh-32ac^3fgh-5b^3cfh^2+20abc^2fh^2+4b^2c^2h^2e-16ac^3h^2e}{b^2c^3-4ac^4}\right)x-\frac{16c^4dg^2+8b^2c^2fg^2-16ac^3fg^2-16bc^3dgh-24b^3cfgh+8}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*f*h^2 - 4*a*c^3*f*h^2)*x/(b^2*c^3 - 4*a*c^4) + (8*b^2*c^2
*f*g*h - 32*a*c^3*f*g*h - 5*b^3*c*f*h^2 + 20*a*b*c^2*f*h^2 + 4*b^2*c^2*h^2*
e - 16*a*c^3*h^2*e)/(b^2*c^3 - 4*a*c^4))*x - (16*c^4*d*g^2 + 8*b^2*c^2*f*g^
2 - 16*a*c^3*f*g^2 - 16*b*c^3*d*g*h - 24*b^3*c*f*g*h + 80*a*b*c^2*f*g*h + 8
*b^2*c^2*d*h^2 - 16*a*c^3*d*h^2 + 15*b^4*f*h^2 - 62*a*b^2*c*f*h^2 + 24*a^2*
c^2*f*h^2 - 8*b*c^3*g^2*e + 16*b^2*c^2*g*h*e - 32*a*c^3*g*h*e - 12*b^3*c*h^
2*e + 40*a*b*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d*g^2 + 8*a*b*c^2
```

```
*f*g^2 - 32*a*c^3*d*g*h - 24*a*b^2*c*f*g*h + 64*a^2*c^2*f*g*h + 8*a*b*c^2*d
*h^2 + 15*a*b^3*f*h^2 - 52*a^2*b*c*f*h^2 - 16*a*c^3*g^2*e + 16*a*b*c^2*g*h*
e - 12*a*b^2*c*h^2*e + 32*a^2*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 +
b*x + a) - 1/8*(8*c^2*f*g^2 - 24*b*c*f*g*h + 8*c^2*d*h^2 + 15*b^2*f*h^2 - 1
2*a*c*f*h^2 + 16*c^2*g*h*e - 12*b*c*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

3.235 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{h\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bce+4c^2d\right)}{c^2\left(b^2-4ac\right)}-\text{ta}\cdots$$

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((4*c^2*d - 2*b*c*e + 3*
b^2*f - 8*a*c*f)*h*Sqrt[a + b*x + c*x^2])/(c^2*(b^2 - 4*a*c)) - ((3*b*f*h -
 2*c*(f*g + e*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*c^(5/2))
```

Rubi [A] time = 0.226817, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1644, 640, 621, 206}

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{h\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bce+4c^2d\right)}{c^2\left(b^2-4ac\right)}-\text{ta}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((4*c^2*d - 2*b*c*e + 3*
b^2*f - 8*a*c*f)*h*Sqrt[a + b*x + c*x^2])/(c^2*(b^2 - 4*a*c)) - ((3*b*f*h -
 2*c*(f*g + e*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*c^(5/2))
```


Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
```

```
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - 2\int \frac{-\frac{b^2fg+2b(cd+af)h-4ac(f}{2c}}{b}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3b^2f-2c(be\right.}{c^2\left(b\right.}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3b^2f-2c(be\right.}{c^2\left(b\right.}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3b^2f-2c(be\right.}{c^2\left(b\right.}$$

Mathematica [A] time = 0.780652, size = 205, normalized size = 1.1

$$\frac{\frac{2\sqrt{c}\left(4c\left(2a^2fh-ac(dh+e(g+hx)+fx(g-hx))+c^2dgx\right)+b^2(cx(2eh+2fg-fhx)-3afh)+2bc(aeh+af(g+5hx)+cd(g-hx)-cegx)-3b^3fhx\right)}{\sqrt{a+x(b+cx)}}+\left(b^2-4ac\right)\log\left(2\sqrt{c}\right.}{2c^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(-3*b^3*f*h*x + 2*b*c*(a*e*h - c*e*g*x + c*d*(g - h*x) + a*f*(g
 + 5*h*x)) + b^2*(-3*a*f*h + c*x*(2*f*g + 2*e*h - f*h*x)) + 4*c*(2*a^2*f*h
+ c^2*d*g*x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x)))))/Sqrt[a + x*(b + c*
x)] + (b^2 - 4*a*c)*(3*b*f*h - 2*c*(f*g + e*h))*Log[b + 2*c*x + 2*Sqrt[c]*S
qrt[a + x*(b + c*x)]])/(2*c^(5/2)*(-b^2 + 4*a*c))
```

Maple [B] time = 0.057, size = 735, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e*h-b^2/c/(4*a*c-b^2)/(c*x^2+b*
x+a)^(1/2)*e*g+b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e*h-x/c/(c*x^2+b*x+a
)^(1/2)*e*h-x/c/(c*x^2+b*x+a)^(1/2)*f*g+4*h*f*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+
a)^(1/2)*x+2*h*f*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+b^2/c/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)*x*f*g-3/2*h*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
*x-3/4*h*f*b^2/c^3/(c*x^2+b*x+a)^(1/2)+h*f*x^2/c/(c*x^2+b*x+a)^(1/2)+1/2*b/
c^2/(c*x^2+b*x+a)^(1/2)*f*g+1/2*b/c^2/(c*x^2+b*x+a)^(1/2)*e*h-3/2*h*f*b/c^(
5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*h*f*a/c^2/(c*x^2+b*x+a)^
(1/2)+2*d*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-2*b/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*x*d*h-2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e*g+1/2*b^3/c^2/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f*g-1/c/(c*x^2+b*x+a)^(1/2)*d*h-1/c/(c*x^2+
b*x+a)^(1/2)*e*g+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h+
1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-3/4*h*f*b^4/c^3/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+3/2*h*f*b/c^2*x/(c*x^2+b*x+a)^(1/2)-b^2/c/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*h
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 53.4317, size = 1939, normalized size = 10.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*
c^2 - 4*a*c^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^
2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c
- 4*a*b*c^2)*e - 3*(b^4 - 4*a*b^2*c)*f)*h)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*((b^2*
c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d
```

```
 - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^
2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d - 2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 1
0*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^
4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), -1/2*((2*(a*b^2*c - 4*a^2*c^2)
*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*c^2 - 4*a*c^3)*e - 3*(b^3*c - 4
*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)
*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c - 4*a*b*c^2)*e - 3*(b^4 - 4*a*b
^2*c)*f)*h)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-
c)/(c^2*x^2 + b*c*x + a*c)) - 2*((b^2*c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d -
 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c
^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d -
2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 10*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x
+ a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c
^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.22159, size = 366, normalized size = 1.97

$$\frac{\left(\frac{\left(b^2cfh-4\,ac^2fh\right)x}{b^2c^2-4\,ac^3}-\frac{4\,c^3dg+2\,b^2cfg-4\,ac^2fg-2\,bc^2dh-3\,b^3fh+10\,abcfh-2\,bc^2ge+2\,b^2che-4\,ac^2he}{b^2c^2-4\,ac^3}\right)x-\frac{2\,bc^2dg+2\,abcfg-4\,ac^2dh-3\,ab^2fh+8\,a^2cfh-4\,ac^2g}{b^2c^2-4\,ac^3}}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] (((b^2*c*f*h - 4*a*c^2*f*h)*x/(b^2*c^2 - 4*a*c^3) - (4*c^3*d*g + 2*b^2*c*f*
g - 4*a*c^2*f*g - 2*b*c^2*d*h - 3*b^3*f*h + 10*a*b*c*f*h - 2*b*c^2*g*e + 2*
```

```
b^2*c*h*e - 4*a*c^2*h*e)/(b^2*c^2 - 4*a*c^3))*x - (2*b*c^2*d*g + 2*a*b*c*f*
g - 4*a*c^2*d*h - 3*a*b^2*f*h + 8*a^2*c*f*h - 4*a*c^2*g*e + 2*a*b*c*h*e)/(b
^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a) - 1/2*(2*c*f*g - 3*b*f*h + 2*c*h*e
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

3.236 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rubi [A] time = 0.0658969, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx &= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}\\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}\\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2f)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c}\\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.319114, size = 113, normalized size = 1.02

$$
\frac{\frac{2\sqrt{c}\left(abf-2ac(e+fx)+b^2fx+bc(d-ex)+2c^2dx\right)}{\sqrt{a+x(b+cx)}}-f\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}\left(4ac-b^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x))
)/Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*f*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)]])/(c^(3/2)*(-b^2 + 4*a*c))
```

Maple [B] time = 0.056, size = 249, normalized size = 2.2

$$-\frac{fx}{c}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{bf}{2c^2}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{b^2fx}{c\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{fb^3}{2c^2\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^2+bx+a}}+f\ln\left(\left(\frac{b}{2}+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -f*x/c/(c*x^2+b*x+a)^(1/2)+1/2*f*b/c^2/(c*x^2+b*x+a)^(1/2)+f*b^2/c/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-e/c/(c*x^2+b*x+a)^(1/2)-
2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.56513, size = 941, normalized size = 8.48

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)fx^2+\left(b^3-4\,abc\right)fx+\left(ab^2-4\,a^2c\right)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right.}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^2+\left(b^3c^2-4\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e
 + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*f*
x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*d
- 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c
*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^
2 - 4*a*b*c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2}{\left(a + bx + cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.21096, size = 165, normalized size = 1.49

$$-\frac{2\left(\frac{\left(2\,c^2d+b^2f-2\,acf-bce\right)x}{b^2c-4\,ac^2}+\frac{bcd+abf-2\,ace}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{f\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*
f - 2*a*c*e)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(-2*(sqrt(
```

```
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

3.237 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=225

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2dh\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}+\frac{(f\ldots}{\ldots}$$

```
[Out] (2*(b^2*d*h - b*(c*d*g + a*f*g + a*e*h) + 2*a*(c*e*g - c*d*h + a*f*h) - (2*
c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h))*x))/((b^
2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) + ((f*g^2 - h*(e*
g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a
*h^2]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - b*g*h + a*h^2)^(3/2)
```

Rubi [A] time = 0.265658, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 12, 724, 206}

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2dh\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}+\frac{(f\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```


```
[Out] (2*(b^2*d*h - b*(c*d*g + a*f*g + a*e*h) + 2*a*(c*e*g - c*d*h + a*f*h) - (2*
c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h))*x))/((b^
2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) + ((f*g^2 - h*(e*
g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a
*h^2]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - b*g*h + a*h^2)^(3/2)
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
```

```
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah)-c(beg\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah)-c(beg\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah)-c(beg\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah)-c(beg\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.584231, size = 271, normalized size = 1.2

$$\frac{\frac{-b^2\left(afh^2+2cdh^2+cfg(g-2hx)\right)-2bch(-aeh+af(g+hx)+c(-dg+dhx+egx))+4c^2(ah(dh-eg+ehx)+afg(g-hx)+cdghx)+b^3fgh}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(h(bg-ah)-cg^2\right)}-\frac{ch\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{2a\cdots}{2\sqrt{a+x(b\cdots}}\right.}{\left(h(ah-bg)+cg^2\right)^{3/\cdots}}}{ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-(f/Sqrt[a + x*(b + c*x)]) + (b^3*f*g*h - b^2*(2*c*d*h^2 + a*f*h^2 + c*f*g
*(g - 2*h*x)) - 2*b*c*h*(-(a*e*h) + a*f*(g + h*x) + c*(-(d*g) + e*g*x + d*h
*x)) + 4*c^2*(c*d*g*h*x + a*f*g*(g - h*x) + a*h*(-(e*g) + d*h + e*h*x)))/((
b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) - (c*h*(f*g^
2 + h*(-(e*g) + d*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*
g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))
^(3/2))/(c*h)
```

Maple [B] time = 0.293, size = 2079, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] h/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2
)/h^2)^(1/2)*d-2/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)
/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*f*g^2+2/(a*h^2-b*g*h+c*g^2)
/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*b*c*g*d+4/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x
+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*g*d-1/h/(a*h^2-b*g*h+c*g^2)/(4*a
*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b
^2*f*g^2-4/h^2*f*g/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*c-2/h/(a*h^2-b*g*h+c*g
^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*b*c*g^2*e-2*h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*
g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*d+2/(a*h^2-b*g*h+c*g^2)/(
4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)*x*b*c*e*g+4/h^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/
h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*g^3*f-4/h/(a*h^2-b*g*h+c*g^2
```

```
)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*x*c^2*g^2*e+2/h^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*
c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c*g^3*f-1/h*f*b^2/c/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)+4/h*e/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*c+1/(a*h^2
-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*b^2*e*g-1/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2))/(x+g/h))*f*g^2-2/h*f*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-h/(a*h^2-
b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*b^2*d-2/h^2*f*g/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+1/(a*h^2-
b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+
(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2
*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*e*g+1/h/(a*h^2-b*g
*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*f*g^2+2/h*e/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-h/(a*h^2-b*g*h+c*g^2)/((a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/
h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*d-1/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+
(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-1/h*f/c/(c*x^2+b*x
+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 131.668, size = 3934, normalized size = 17.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b^2 - 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*c
)*d*h^2 + ((b^2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4*a
*c^2)*d*h^2)*x^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3 -
4*a*b*c)*d*h^2)*x)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 -
 (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sq
rt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h
)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*
x + g^2)) - 4*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2*(b^2*c
- a*c^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d - (a
*b^2 + 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)*d)*h^3 +
((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b^2*c + 2*a*
c^2)*e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2)*d - 2*(a*b
^2 - a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)*sqrt(c*x^2 +
 b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h +
 (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3
+ (a^3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c
^3)*g^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*
b*c^2)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^
4 - 2*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2
 - 2*(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*b*c)*h^4)*x), (((a*b^2
- 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*c)*d*h^2 + ((b^
2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4*a*c^2)*d*h^2)*x
^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3 - 4*a*b*c)*d*h^2
)*x)*sqrt(-c*g^2 + b*g*h - a*h^2)*arctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*
sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h +
a^2*h^2 + (c^2*g^2 - b*c*g*h + a*c*h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)
*x)) - 2*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2*(b^2*c - a*c
^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d - (a*b^2
+ 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)*d)*h^3 + ((2*c
^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b^2*c + 2*a*c^2)*
e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2)*d - 2*(a*b^2 -
a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)*sqrt(c*x^2 + b*x
+ a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h + (a*b
^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3 + (a^
3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c^3)*g
^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*b*c^2
)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^4 - 2
*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2 - 2*
(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*b*c)*h^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex+fx^2}{(g+hx)\,(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [B] time = 1.20519, size = 971, normalized size = 4.32

$$-\frac{2\left(\frac{(2c^3dg^3+b^2cfg^3-2ac^2fg^3-3bc^2dg^2h-b^3fg^2h+abcfg^2h+b^2cdgh^2+2ac^2dgh^2+2ab^2fgh^2-2a^2cfgh^2-abcdh^3-a^2bfh^3-bc^2g^3e+b^2cg^2he+2ac^2g^2he-3abcg}{b^2c^2g^4-4ac^3g^4-2b^3cg^3h+8abc^2g^3h+b^4g^2h^2-2ab^2cg^2h^2-8a^2c^2g^2h^2-2ab^3gh^3+8a^2bcgh^3+a^2b^2h^4-4a^3ch^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d*g^3 + b^2*c*f*g^3 - 2*a*c^2*f*g^3 - 3*b*c^2*d*g^2*h - b^3*f*g^
2*h + a*b*c*f*g^2*h + b^2*c*d*g*h^2 + 2*a*c^2*d*g*h^2 + 2*a*b^2*f*g*h^2 - 2
*a^2*c*f*g*h^2 - a*b*c*d*h^3 - a^2*b*f*h^3 - b*c^2*g^3*e + b^2*c*g^2*h*e +
2*a*c^2*g^2*h*e - 3*a*b*c*g*h^2*e + 2*a^2*c*h^3*e)*x/(b^2*c^2*g^4 - 4*a*c^3
*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2 -
8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3*c
*h^4) + (b*c^2*d*g^3 + a*b*c*f*g^3 - 2*b^2*c*d*g^2*h + 2*a*c^2*d*g^2*h - a*
b^2*f*g^2*h - 2*a^2*c*f*g^2*h + b^3*d*g*h^2 - a*b*c*d*g*h^2 + 3*a^2*b*f*g*h
^2 - a*b^2*d*h^3 + 2*a^2*c*d*h^3 - 2*a^3*f*h^3 - 2*a*c^2*g^3*e + 3*a*b*c*g^
2*h*e - a*b^2*g*h^2*e - 2*a^2*c*g*h^2*e + a^2*b*h^3*e)/(b^2*c^2*g^4 - 4*a*c
^3*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2
- 8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3
*c*h^4))/sqrt(c*x^2 + b*x + a) + 2*(f*g^2 + d*h^2 - g*h*e)*arctan(-((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/(
(c*g^2 - b*g*h + a*h^2)*sqrt(-c*g^2 + b*g*h - a*h^2))
```

3.238 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=421

$$-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2fh^2+ac\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bg\right.}$$

```
[Out] (-2*(b^3*d*h^2 - b^2*h*(2*c*d*g + a*e*h) - 2*a*c*(c*g*(e*g - 2*d*h) + a*h*(
2*f*g - e*h)) + b*(c^2*d*g^2 + a^2*f*h^2 + a*c*(f*g^2 + 2*e*g*h - 3*d*h^2))
 + c*(2*c^2*d*g^2 + 2*a^2*f*h^2 - a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + d*h^2)
 - c*(b*g*(e*g + 2*d*h) + 2*a*(f*g^2 - 2*e*g*h + d*h^2)))*x))/((b^2 - 4*a*c
)*(c*g^2 - b*g*h + a*h^2)^2*Sqrt[a + b*x + c*x^2]) - (h*(f*g^2 - h*(e*g - d
*h))*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*h^2)^2*(g + h*x)) + ((2*c*g
*(f*g^2 - h*(2*e*g - 3*d*h)) - h*(2*a*h*(2*f*g - e*h) - b*(f*g^2 + e*g*h -
3*d*h^2)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rubi [A] time = 0.796687, antiderivative size = 418, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 806, 724, 206}

$$-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2fh^2+ac\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bg\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b^3*d*h^2 - b^2*h*(2*c*d*g + a*e*h) - 2*a*c*(c*g*(e*g - 2*d*h) + a*h*(
2*f*g - e*h)) + b*(c^2*d*g^2 + a^2*f*h^2 + a*c*(f*g^2 + 2*e*g*h - 3*d*h^2))
 + c*(2*c^2*d*g^2 + 2*a^2*f*h^2 - a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + d*h^2)
 - c*(b*g*(e*g + 2*d*h) + 2*a*(f*g^2 - 2*e*g*h + d*h^2)))*x))/((b^2 - 4*a*c
)*(c*g^2 - b*g*h + a*h^2)^2*Sqrt[a + b*x + c*x^2]) - (h*(f*g^2 - h*(e*g - d
*h))*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*h^2)^2*(g + h*x)) + ((2*c*(
f*g^3 - g*h*(2*e*g - 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) - 2*a*h*(2*f*
g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+a^2fh^2\right.\right.}{}$$

$$= -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+a^2fh^2\right.\right.}{}$$

$$= -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+a^2fh^2\right.\right.}{}$$

$$= -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+a^2fh^2\right.\right.}{}$$

Mathematica [A] time = 3.26168, size = 487, normalized size = 1.16

$$\frac{\frac{4c\left(-a^2fh^2+ac(2h(dh-eg+ehx)+fg(g-2hx))+2c^2dghx\right)-b^2\left(afh^2+4cdh^2+cfg(g-4hx)\right)-4bch(ah(fx-e)+c(-dg+dhx+egx))+b^3fgh}{\left(b^2-4ac\right)(g+hx)\sqrt{a+x(b+cx)}\left(h(bg-ah)-cg^2\right)}+\frac{ch\left(\frac{\left(4ac-b^2\right)\tanh^{-1}\left(\frac{2ah-}{2\sqrt{a+x(b+}}\right.}{}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-(f/((g + h*x)*Sqrt[a + x*(b + c*x)])) + (b^3*f*g*h - b^2*(4*c*d*h^2 + a*f
*h^2 + c*f*g*(g - 4*h*x)) - 4*b*c*h*(a*h*(-e + f*x) + c*(-(d*g) + e*g*x + d
*h*x)) + 4*c*(-(a^2*f*h^2) + 2*c^2*d*g*h*x + a*c*(f*g*(g - 2*h*x) + 2*h*(-(
e*g) + d*h + e*h*x))))/((b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*(g + h*x)*
Sqrt[a + x*(b + c*x)]) + (c*h*((-2*h*(4*c^2*d*g^2 + 4*a^2*f*h^2 - 2*a*b*h*(
2*f*g + e*h) - 2*c*(4*a*f*g^2 + 2*a*h*(-3*e*g + 2*d*h) + b*g*(e*g + 2*d*h))
 + b^2*(3*f*g^2 + h*(-(e*g) + 3*d*h)))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(
-(b*g) + a*h))^2*(g + h*x)) + ((-b^2 + 4*a*c)*(2*c*(f*g^3 + g*h*(-2*e*g + 3
*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) + 2*a*h*(-2*f*g + e*h)))*ArcTanh[(-
(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a +
 x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(5/2)))/(b^2 - 4*a*c))/(2*c*h)
```

Maple [B] time = 0.301, size = 4930, normalized size = 11.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -3/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*c*g^2*e+2*f/h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-2
/h/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)*f*g-1/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g
)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*e-1/(a*h^2-b*g*h+c*g^2)/((a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+
g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(
a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*e-1/(a*h^2-b*g*h+c*g^2)/(x+g/h)/((x
+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*d-3/2*h^2/(a
*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*b*d-3/2/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)
+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*f*g^2+1/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+
(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e+6/h^2/(a*h^2-b*g*h+c
*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*b*c^2*g^4*f+3/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*
h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b^2*c*f*g^2+12/(a*h^2-b
*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*x*b*c^2*g^2*e-6*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/
h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*c*g*d-6/h/(
a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)*b^2*c*g^3*f-6/h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/(
(x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c^2*g^3*
e-12/h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)
+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^3*g^3*e+3*h^2/(a*h^2-b*g*h+c*g^2)^2/(4*
a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
x*b^2*c*d+12/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(
x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*g*e+12/h^2/(a*h^2-b*g*h+c*g^2)^
2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*x*c^3*g^4*f+6/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*
g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c*g*e-8/h^2/(a*h^2-b*g*h+c*g^
2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*b*c*g^2*f-16/h^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*
c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*g^2*f+4/h/(a*h^2-b*g*h+
c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2)*x*b*c*f*g-3*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h
```

-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b^2*c*e*g-12*h/(a*h^2-b*
g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*x*b*c^2*g*d-12/h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)
^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c^2*g^3*f-8*c
^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)*x*d+3/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2
)/h^2)^(1/2))/(x+g/h))*c*g^2*e+3/2*h^2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*d+1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)/((x+g/h)^
2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^2-b
*g*h+c*g^2)/(x+g/h)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*f*g^2+3/2*h/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+
g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*e*g-4*c/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^
2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*d+3/
2/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h
^2-b*g*h+c*g^2)/h^2)^(1/2)*b^3*f*g^2+3/2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*(
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*
g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*f*g^2+2/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*
((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b
*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g+3/h/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c
+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c*g^3*f+3/2*h^2/(a*h^
2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)*b^3*d+3*h/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g
)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c*g*d-3/h/(a*h^2-b*g*h+c*g^2)^2/
((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h
(x+g/h)+2((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/
h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3*f-2/(a*h^2-b*g*h+c*g^2)/(
4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)*x*b*c*e+2/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x
+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*f*g+12/(a*h^2-b*g*h+c*g^2)^2/(4*a*
c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*
c^3*g^2*d+6/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x
+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c^2*g^2*d-3/2*h/(a*h^2-b*g*h+c*g^2)^
2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)
/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+
g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*e*g-3/2*h/(a*h^2-b*g*h+c*g^
2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2)*b^3*e*g-3*h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*l
n((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h
^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)

```
)/(x+g/h))*c*g*d+6/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*
g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*c*g^2*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + ex + d}{\left(cx^2 + bx + a\right)^{\frac{3}{2}} \left(hx + g\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e*x + d)/((c*x^2 + b*x + a)^(3/2)*(h*x + g)^2), x)
```

3.239 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=713

$$2\left(-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(-dh^2+egh+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\right)-bh^3\left(a^2f-abe+b^2d\right)-c^2g\left(2a\left(3dh^2-3\right.\right.\right.\right.$$

```
[Out] (2*(b^4*d*h^3 - b^3*h^2*(3*c*d*g + a*e*h) + b^2*h*(3*c^2*d*g^2 + a^2*f*h^2
+ a*c*h*(3*e*g - 4*d*h)) - b*c*(c^2*d*g^3 + 3*a^2*h^2*(f*g - e*h) + a*c*g*(
f*g^2 + 3*e*g*h - 9*d*h^2)) - 2*a*c*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*
c*h*(3*f*g^2 - 3*e*g*h + d*h^2)) - c*(2*c^3*d*g^3 - b*(b^2*d - a*b*e + a^2*
f)*h^3 - c^2*g*(b*g*(e*g + 3*d*h) + 2*a*(f*g^2 - 3*e*g*h + 3*d*h^2)) + c*(2
*a^2*h^2*(3*f*g - e*h) - 3*a*b*h*(f*g^2 + e*g*h - d*h^2) + b^2*(f*g^3 + 3*d
*g*h^2)))*x))/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)^3*Sqrt[a + b*x + c*x^2
]) - (h*(f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*(c*g^2 - b*g*h +
a*h^2)^2*(g + h*x)^2) - (h*(2*c*g*(3*f*g^2 - h*(5*e*g - 7*d*h)) - h*(4*a*h*
(2*f*g - e*h) - b*(f*g^2 + 3*e*g*h - 7*d*h^2)))*Sqrt[a + b*x + c*x^2])/(4*(
c*g^2 - b*g*h + a*h^2)^3*(g + h*x)) + ((8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^
2) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(2*f*g - 3*e*h) - b^2*(f*g^2 + 3*h*(e*g - 5
*d*h))) - 4*c*h*(a*h*(11*f*g^2 - 9*e*g*h + 3*d*h^2) - b*g*(2*f*g^2 + 3*h*(e
*g - 4*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rubi [A] time = 2.66961, antiderivative size = 707, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 1650, 806, 724, 206}

$$2\left(-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\right)-bh^3\left(a^2f-abe+b^2d\right)-c^2g\left(-6ah(eg-dh)\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^4*d*h^3 - b^3*h^2*(3*c*d*g + a*e*h) + b^2*h*(3*c^2*d*g^2 + a^2*f*h^2
+ a*c*h*(3*e*g - 4*d*h)) - b*c*(c^2*d*g^3 + 3*a^2*h^2*(f*g - e*h) + a*c*g*(
f*g^2 + 3*e*g*h - 9*d*h^2)) - 2*a*c*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*
```

```
c*h*(3*f*g^2 - 3*e*g*h + d*h^2)) - c*(2*c^3*d*g^3 - b*(b^2*d - a*b*e + a^2*
f)*h^3 - c^2*g*(2*a*f*g^2 - 6*a*h*(e*g - d*h) + b*g*(e*g + 3*d*h)) + c*(2*a
^2*h^2*(3*f*g - e*h) + b^2*(f*g^3 + 3*d*g*h^2) - 3*a*b*h*(f*g^2 + h*(e*g -
d*h))))*x))/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)^3*Sqrt[a + b*x + c*x^2])
 - (h*(f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*(c*g^2 - b*g*h + a*
h^2)^2*(g + h*x)^2) - (h*(6*c*f*g^3 - 2*c*g*h*(5*e*g - 7*d*h) - 4*a*h^2*(2*
f*g - e*h) + b*h*(f*g^2 + h*(3*e*g - 7*d*h)))*Sqrt[a + b*x + c*x^2])/(4*(c*
g^2 - b*g*h + a*h^2)^3*(g + h*x)) + ((8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2)
 + 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g - 4*d*h) - a*h*(11*f*g^2 - 9*e*g*h + 3*d
*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(2*f*g - 3*e*h) - b^2*(f*g^2 + 3*h*(e*g
 - 5*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h
+ a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c^2dg^3+\right.\right.}{}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c^2dg^3+\right.\right.}{}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c^2dg^3+\right.\right.}{}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c^2dg^3+\right.\right.}{}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c^2dg^3+\right.\right.}{}$$

Mathematica [B] time = 6.21183, size = 2046, normalized size = 2.87

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] -(f*(a + b*x + c*x^2))/(3*c*h*(g + h*x)^2*(a + x*(b + c*x))^(3/2)) + ((a +
b*x + c*x^2)^(3/2)*((-2*(((b*c*g - b^2*h + 2*a*c*h)*(-(b*f*g) + 6*c*d*h - 4
*a*f*h))/2 - (a*(2*c*g - b*h)*(-2*c*f*g + 6*c*e*h - 5*b*f*h))/2 + c*(((2*c*
g - b*h)*(-(b*f*g) + 6*c*d*h - 4*a*f*h))/2 - ((b*g - 2*a*h)*(-2*c*f*g + 6*c
*e*h - 5*b*f*h))/2)*x))/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2*
Sqrt[a + b*x + c*x^2]) - (2*(-((-6*c*g*h^2*(2*c^2*d*g + b*f*(b*g - a*h) - c
*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h)) + (3*c*h^2*(4*b*h*(c*d*g + a*f*g + a*
e*h) - b^2*(f*g^2 - h*(e*g - 5*d*h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*(e*
g - d*h))))/2)*Sqrt[a + b*x + c*x^2])/(2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^
2) - (-(((-(c*g*(-6*c*g*h^2*(2*c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f
*g + b*d*h - 2*a*e*h)) + (3*c*h^2*(4*b*h*(c*d*g + a*f*g + a*e*h) - b^2*(f*g
^2 - h*(e*g - 5*d*h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*(e*g - d*h))))/2))
 + h*((b*(6*c*g*h^2*(2*c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d
*h - 2*a*e*h)) - (3*c*h^2*(4*b*h*(c*d*g + a*f*g + a*e*h) - b^2*(f*g^2 - h*(
e*g - 5*d*h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*(e*g - d*h))))/2))/2 - 2*(
6*a*c*h^3*(2*c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e
*h)) + (3*c^2*g*h*(4*b*h*(c*d*g + a*f*g + a*e*h) - b^2*(f*g^2 - h*(e*g - 5*
d*h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*(e*g - d*h))))/2 - (3*b*c*h^2*(4*b
*h*(c*d*g + a*f*g + a*e*h) - b^2*(f*g^2 - h*(e*g - 5*d*h)) + 4*a*(c*f*g^2 -
 2*a*f*h^2 - 3*c*h*(e*g - d*h))))/2)))*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g
*h + a*h^2)*(g + h*x))) + (2*(-2*(a*c*h*(-6*c*g*h^2*(2*c^2*d*g + b*f*(b*g -
 a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h)) + (3*c*h^2*(4*b*h*(c*d*g + a
*f*g + a*e*h) - b^2*(f*g^2 - h*(e*g - 5*d*h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 -
3*c*h*(e*g - d*h))))/2) + c*g*((b*(6*c*g*h^2*(2*c^2*d*g + b*f*(b*g - a*h) -
 c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h)) - (3*c*h^2*(4*b*h*(c*d*g + a*f*g +
a*e*h) - b^2*(f*g^2 - h*(e*g - 5*d*h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*(
e*g - d*h))))/2))/2 - 2*(6*a*c*h^3*(2*c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g
+ 2*a*f*g + b*d*h - 2*a*e*h)) + (3*c^2*g*h*(4*b*h*(c*d*g + a*f*g + a*e*h) -
 b^2*(f*g^2 - h*(e*g - 5*d*h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*(e*g - d*
h))))/2 - (3*b*c*h^2*(4*b*h*(c*d*g + a*f*g + a*e*h) - b^2*(f*g^2 - h*(e*g -
 5*d*h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*(e*g - d*h))))/2))) + b*(c*g*(-
6*c*g*h^2*(2*c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e
*h)) + (3*c*h^2*(4*b*h*(c*d*g + a*f*g + a*e*h) - b^2*(f*g^2 - h*(e*g - 5*d*
h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*(e*g - d*h))))/2) + h*((b*(6*c*g*h^2
*(2*c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h)) - (3
*c*h^2*(4*b*h*(c*d*g + a*f*g + a*e*h) - b^2*(f*g^2 - h*(e*g - 5*d*h)) + 4*a
*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*(e*g - d*h))))/2))/2 - 2*(6*a*c*h^3*(2*c^2*d*
g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h)) + (3*c^2*g*h*(
4*b*h*(c*d*g + a*f*g + a*e*h) - b^2*(f*g^2 - h*(e*g - 5*d*h)) + 4*a*(c*f*g^
2 - 2*a*f*h^2 - 3*c*h*(e*g - d*h))))/2 - (3*b*c*h^2*(4*b*h*(c*d*g + a*f*g +
 a*e*h) - b^2*(f*g^2 - h*(e*g - 5*d*h)) + 4*a*(c*f*g^2 - 2*a*f*h^2 - 3*c*h*
(e*g - d*h))))/2))))*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g
```

```
^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*g^2 - b*g*h + a*h^2]*(
4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(2*(c*g^2 - b*g*h + a*h^2))))/((b^2 - 4*a*c)
*(c*g^2 - b*g*h + a*h^2))))/(3*c*h*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.327, size = 9126, normalized size = 12.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.78442, size = 7610, normalized size = 10.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^7*d*g^9 + b^2*c^5*f*g^9 - 2*a*c^6*f*g^9 - 9*b*c^6*d*g^8*h - 3*b^3*
c^4*f*g^8*h + 3*a*b*c^5*f*g^8*h + 18*b^2*c^5*d*g^7*h^2 + 3*b^4*c^3*f*g^7*h^
2 + 6*a*b^2*c^4*f*g^7*h^2 - 21*b^3*c^4*d*g^6*h^3 - b^5*c^2*f*g^6*h^3 - 13*a
*b^3*c^3*f*g^6*h^3 - 16*a^2*b*c^4*f*g^6*h^3 + 15*b^4*c^3*d*g^5*h^4 + 6*a*b^
2*c^4*d*g^5*h^4 - 12*a^2*c^5*d*g^5*h^4 + 6*a*b^4*c^2*f*g^5*h^4 + 36*a^2*b^2
*c^3*f*g^5*h^4 + 12*a^3*c^4*f*g^5*h^4 - 6*b^5*c^2*d*g^4*h^5 - 15*a*b^3*c^3*
d*g^4*h^5 + 30*a^2*b*c^4*d*g^4*h^5 - 21*a^2*b^3*c^2*f*g^4*h^5 - 42*a^3*b*c^
3*f*g^4*h^5 + b^6*c*d*g^3*h^6 + 12*a*b^4*c^2*d*g^3*h^6 - 18*a^2*b^2*c^3*d*g
^3*h^6 - 16*a^3*c^4*d*g^3*h^6 + a^2*b^4*c*f*g^3*h^6 + 34*a^3*b^2*c^2*f*g^3*
h^6 + 16*a^4*c^3*f*g^3*h^6 - 3*a*b^5*c*d*g^2*h^7 - 3*a^2*b^3*c^2*d*g^2*h^7
+ 24*a^3*b*c^3*d*g^2*h^7 - 3*a^3*b^3*c*f*g^2*h^7 - 24*a^4*b*c^2*f*g^2*h^7 +
 3*a^2*b^4*c*d*g*h^8 - 6*a^3*b^2*c^2*d*g*h^8 - 6*a^4*c^3*d*g*h^8 + 3*a^4*b^
2*c*f*g*h^8 + 6*a^5*c^2*f*g*h^8 - a^3*b^3*c*d*h^9 + 3*a^4*b*c^2*d*h^9 - a^5
*b*c*f*h^9 - b*c^6*g^9*e + 3*b^2*c^5*g^8*h*e + 6*a*c^6*g^8*h*e - 3*b^3*c^4*
g^7*h^2*e - 24*a*b*c^5*g^7*h^2*e + b^4*c^3*g^6*h^3*e + 34*a*b^2*c^4*g^6*h^3
*e + 16*a^2*c^5*g^6*h^3*e - 21*a*b^3*c^3*g^5*h^4*e - 42*a^2*b*c^4*g^5*h^4*e
 + 6*a*b^4*c^2*g^4*h^5*e + 36*a^2*b^2*c^3*g^4*h^5*e + 12*a^3*c^4*g^4*h^5*e
- a*b^5*c*g^3*h^6*e - 13*a^2*b^3*c^2*g^3*h^6*e - 16*a^3*b*c^3*g^3*h^6*e + 3
*a^2*b^4*c*g^2*h^7*e + 6*a^3*b^2*c^2*g^2*h^7*e - 3*a^3*b^3*c*g*h^8*e + 3*a^
4*b*c^2*g*h^8*e + a^4*b^2*c*h^9*e - 2*a^5*c^2*h^9*e)*x/(b^2*c^6*g^12 - 4*a*
c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^4*g^10*h^2 - 54*
a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h^3 + 50*a*b^3*c^
4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 225*a^2*b^2*c^4*g^
8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2*g^7*h^5 + 180*a
```

^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 + 26*a*b^6*c*g^6*h
^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*a^4*c^4*g^6*h^6
- 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*g^5*h^7 + 240*a^
4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*h^8 - 60*a^5*c^3
*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*a^5*b*c^2*g^3*h^
9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^2*g^2*h^10 - 6*a
^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*h^12) + (b*c^6*d
*g^9 + a*b*c^5*f*g^9 - 6*b^2*c^5*d*g^8*h + 6*a*c^6*d*g^8*h - 3*a*b^2*c^4*f*
g^8*h - 6*a^2*c^5*f*g^8*h + 15*b^3*c^4*d*g^7*h^2 - 24*a*b*c^5*d*g^7*h^2 + 3
*a*b^3*c^3*f*g^7*h^2 + 24*a^2*b*c^4*f*g^7*h^2 - 20*b^4*c^3*d*g^6*h^3 + 34*a
*b^2*c^4*d*g^6*h^3 + 16*a^2*c^5*d*g^6*h^3 - a*b^4*c^2*f*g^6*h^3 - 34*a^2*b^
2*c^3*f*g^6*h^3 - 16*a^3*c^4*f*g^6*h^3 + 15*b^5*c^2*d*g^5*h^4 - 15*a*b^3*c^
3*d*g^5*h^4 - 54*a^2*b*c^4*d*g^5*h^4 + 21*a^2*b^3*c^2*f*g^5*h^4 + 42*a^3*b*
c^3*f*g^5*h^4 - 6*b^6*c*d*g^4*h^5 - 9*a*b^4*c^2*d*g^4*h^5 + 66*a^2*b^2*c^3*
d*g^4*h^5 + 12*a^3*c^4*d*g^4*h^5 - 6*a^2*b^4*c*f*g^4*h^5 - 36*a^3*b^2*c^2*f
*g^4*h^5 - 12*a^4*c^3*f*g^4*h^5 + b^7*d*g^3*h^6 + 11*a*b^5*c*d*g^3*h^6 - 31
*a^2*b^3*c^2*d*g^3*h^6 - 32*a^3*b*c^3*d*g^3*h^6 + a^2*b^5*f*g^3*h^6 + 13*a^
3*b^3*c*f*g^3*h^6 + 16*a^4*b*c^2*f*g^3*h^6 - 3*a*b^6*d*g^2*h^7 + 30*a^3*b^2
*c^2*d*g^2*h^7 - 3*a^3*b^4*f*g^2*h^7 - 6*a^4*b^2*c*f*g^2*h^7 + 3*a^2*b^5*d*
g*h^8 - 9*a^3*b^3*c*d*g*h^8 - 3*a^4*b*c^2*d*g*h^8 + 3*a^4*b^3*f*g*h^8 - 3*a
^5*b*c*f*g*h^8 - a^3*b^4*d*h^9 + 4*a^4*b^2*c*d*h^9 - 2*a^5*c^2*d*h^9 - a^5*
b^2*f*h^9 + 2*a^6*c*f*h^9 - 2*a*c^6*g^9*e + 9*a*b*c^5*g^8*h*e - 18*a*b^2*c^
4*g^7*h^2*e + 21*a*b^3*c^3*g^6*h^3*e - 15*a*b^4*c^2*g^5*h^4*e - 6*a^2*b^2*c
^3*g^5*h^4*e + 12*a^3*c^4*g^5*h^4*e + 6*a*b^5*c*g^4*h^5*e + 15*a^2*b^3*c^2*
g^4*h^5*e - 30*a^3*b*c^3*g^4*h^5*e - a*b^6*g^3*h^6*e - 12*a^2*b^4*c*g^3*h^6
*e + 18*a^3*b^2*c^2*g^3*h^6*e + 16*a^4*c^3*g^3*h^6*e + 3*a^2*b^5*g^2*h^7*e
+ 3*a^3*b^3*c*g^2*h^7*e - 24*a^4*b*c^2*g^2*h^7*e - 3*a^3*b^4*g*h^8*e + 6*a^
4*b^2*c*g*h^8*e + 6*a^5*c^2*g*h^8*e + a^4*b^3*h^9*e - 3*a^5*b*c*h^9*e)/(b^2
*c^6*g^12 - 4*a*c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^
4*g^10*h^2 - 54*a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h
^3 + 50*a*b^3*c^4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 22
5*a^2*b^2*c^4*g^8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2
*g^7*h^5 + 180*a^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 +
26*a*b^6*c*g^6*h^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*
a^4*c^4*g^6*h^6 - 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*
g^5*h^7 + 240*a^4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*
h^8 - 60*a^5*c^3*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*
a^5*b*c^2*g^3*h^9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^
2*g^2*h^10 - 6*a^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*
h^12))/sqrt(c*x^2 + b*x + a) + 1/4*(8*c^2*f*g^4 + 8*b*c*f*g^3*h + 48*c^2*d*
g^2*h^2 - b^2*f*g^2*h^2 - 44*a*c*f*g^2*h^2 - 48*b*c*d*g*h^3 + 8*a*b*f*g*h^3
 + 15*b^2*d*h^4 - 12*a*c*d*h^4 + 8*a^2*f*h^4 - 24*c^2*g^3*h*e + 12*b*c*g^2*
h^2*e - 3*b^2*g*h^3*e + 36*a*c*g*h^3*e - 12*a*b*h^4*e)*arctan(-((sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^3
*g^6 - 3*b*c^2*g^5*h + 3*b^2*c*g^4*h^2 + 3*a*c^2*g^4*h^2 - b^3*g^3*h^3 - 6*

a*b*c*g^3*h^3 + 3*a*b^2*g^2*h^4 + 3*a^2*c*g^2*h^4 - 3*a^2*b*g*h^5 + a^3*h^6
)*sqrt(-c*g^2 + b*g*h - a*h^2)) - 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*c^2*f*g^4*h + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*g^2*h^3 -
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 - 20*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b*c*d*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 + 7*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a*c*d*h^5 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*g^3*h^
2*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*g^2*h^3*e - 3*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*b^2*g*h^4*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*a*c*g*h^4*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*h^5*e + 2
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f*g^5 - 8*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*c^(5/2)*d*g^3*h^2 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt
(c)*f*g^3*h^2 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f*g^3*h^
2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*g^2*h^3 + 13*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^4 - 28*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*c^(3/2)*d*g*h^4 + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*a^2*sqrt(c)*f*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sq
rt(c)*d*h^5 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*g^4*h*e + 28
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*g^3*h^2*e - 9*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*g^2*h^3*e + 36*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a*c^(3/2)*g^2*h^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a*b*sqrt(c)*g*h^4*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c
)*h^5*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f*g^5 - 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a*c^2*f*g^4*h + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*g^3*h^
2 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f*g^3*h^2 - 28*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b*c*f*g^3*h^2 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*b^2*c*d*g^2*h^3 - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3
+ 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^2*h^3 + 44*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b^3*d*g*h^4 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*g*h^4 - 8*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*f*g*h^4 - 9*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a*b^2*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h
^5 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*g^4*h*e + 24*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*b^2*c*g^3*h^2*e + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a*c^2*g^3*h^2*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*g^2*h^3*e
 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*g^2*h^3*e + (sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b^2*g*h^4*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^2*c*g*h^4*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*h^5*e + 6*b^2*
c^(3/2)*f*g^5 + b^3*sqrt(c)*f*g^4*h - 20*a*b*c^(3/2)*f*g^4*h + 14*b^2*c^(3/
2)*d*g^3*h^2 - 9*a*b^2*sqrt(c)*f*g^3*h^2 + 12*a^2*c^(3/2)*f*g^3*h^2 - 7*b^3
*sqrt(c)*d*g^2*h^3 - 44*a*b*c^(3/2)*d*g^2*h^3 + 24*a^2*b*sqrt(c)*f*g^2*h^3
+ 23*a*b^2*sqrt(c)*d*g*h^4 + 28*a^2*c^(3/2)*d*g*h^4 - 16*a^3*sqrt(c)*f*g*h^

```
4 - 16*a^2*b*sqrt(c)*d*h^5 - 10*b^2*c^(3/2)*g^4*h*e + 3*b^3*sqrt(c)*g^3*h^2
*e + 32*a*b*c^(3/2)*g^3*h^2*e - 7*a*b^2*sqrt(c)*g^2*h^3*e - 20*a^2*c^(3/2)*
g^2*h^3*e - 4*a^2*b*sqrt(c)*g*h^4*e + 8*a^3*sqrt(c)*h^5*e)/((c^3*g^6 - 3*b*
c^2*g^5*h + 3*b^2*c*g^4*h^2 + 3*a*c^2*g^4*h^2 - b^3*g^3*h^3 - 6*a*b*c*g^3*h
^3 + 3*a*b^2*g^2*h^4 + 3*a^2*c*g^2*h^4 - 3*a^2*b*g*h^5 + a^3*h^6)*((sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqr
t(c)*g + b*g - a*h)^2)
```

3.240 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4+\frac{19}{60}\sqrt{3x^2-x+2}(2x+1)^3+\frac{44}{135}\sqrt{3x^2-x+2}(2x+1)^2-\frac{(6298x+24897)\sqrt{3x^2-x+2}}{3240}+\frac{92}{}$$

```
[Out] (44*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/135 + (19*(1 + 2*x)^3*Sqrt[2 - x + 3*x
^2])/60 + (2*(1 + 2*x)^4*Sqrt[2 - x + 3*x^2])/15 - ((24897 + 6298*x)*Sqrt[2
 - x + 3*x^2])/3240 + (9211*ArcSinh[(1 - 6*x)/Sqrt[23]])/(1296*Sqrt[3])
```

Rubi [A] time = 0.134877, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 619, 215}

$$\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4+\frac{19}{60}\sqrt{3x^2-x+2}(2x+1)^3+\frac{44}{135}\sqrt{3x^2-x+2}(2x+1)^2-\frac{(6298x+24897)\sqrt{3x^2-x+2}}{3240}+\frac{92}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (44*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/135 + (19*(1 + 2*x)^3*Sqrt[2 - x + 3*x
^2])/60 + (2*(1 + 2*x)^4*Sqrt[2 - x + 3*x^2])/15 - ((24897 + 6298*x)*Sqrt[2
 - x + 3*x^2])/3240 + (9211*ArcSinh[(1 - 6*x)/Sqrt[23]])/(1296*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} + \frac{1}{60}\int \frac{(1+2x)^3(-64+228x)}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} + \frac{1}{720}\int \frac{(1+2x)^2(-3390+2112}{\sqrt{2-x+3x^2}}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} + \int$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} - (2$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} - (2$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} - (2$$

Mathematica [A] time = 0.046816, size = 60, normalized size = 0.5

$$\frac{6\sqrt{3x^2-x+2}\left(6912x^4+22032x^3+26904x^2+7538x-22383\right)-46055\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{19440}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-22383 + 7538*x + 26904*x^2 + 22032*x^3 + 6912*x^4)
 - 46055*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/19440
```

Maple [A] time = 0.056, size = 96, normalized size = 0.8

$$\frac{32\,x^4}{15}\sqrt{3\,x^2-x+2}+\frac{34\,x^3}{5}\sqrt{3\,x^2-x+2}+\frac{1121\,x^2}{135}\sqrt{3\,x^2-x+2}+\frac{3769\,x}{1620}\sqrt{3\,x^2-x+2}-\frac{829}{120}\sqrt{3\,x^2-x+2}-\frac{9211\,\sqrt{3}}{3888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 32/15*x^4*(3*x^2-x+2)^(1/2)+34/5*x^3*(3*x^2-x+2)^(1/2)+1121/135*x^2*(3*x^2-
x+2)^(1/2)+3769/1620*x*(3*x^2-x+2)^(1/2)-829/120*(3*x^2-x+2)^(1/2)-9211/388
8*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A] time = 1.51555, size = 131, normalized size = 1.09

$$\frac{32}{15}\sqrt{3x^2-x+2}x^4+\frac{34}{5}\sqrt{3x^2-x+2}x^3+\frac{1121}{135}\sqrt{3x^2-x+2}x^2+\frac{3769}{1620}\sqrt{3x^2-x+2}x-\frac{9211}{3888}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 - x + 2)*x^4 + 34/5*sqrt(3*x^2 - x + 2)*x^3 + 1121/135*sqr
t(3*x^2 - x + 2)*x^2 + 3769/1620*sqrt(3*x^2 - x + 2)*x - 9211/3888*sqrt(3)*
arcsinh(1/23*sqrt(23)*(6*x - 1)) - 829/120*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.62303, size = 224, normalized size = 1.87

$$\frac{1}{3240}\left(6912x^4+22032x^3+26904x^2+7538x-22383\right)\sqrt{3x^2-x+2}+\frac{9211}{7776}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3240*(6912*x^4 + 22032*x^3 + 26904*x^2 + 7538*x - 22383)*sqrt(3*x^2 - x +
 2) + 9211/7776*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^
2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2x+1)^3\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

Giac [A] time = 1.18289, size = 92, normalized size = 0.77

$$\frac{1}{3240}\left(2\left(12\left(18\left(16x+51\right)x+1121\right)x+3769\right)x-22383\right)\sqrt{3x^2-x+2}+\frac{9211}{3888}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3240*(2*(12*(18*(16*x + 51)*x + 1121)*x + 3769)*x - 22383)*sqrt(3*x^2 - x
 + 2) + 9211/3888*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

3.241 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=95

$$\frac{1}{6}\sqrt{3x^2-x+2}(2x+1)^3+\frac{11}{27}\sqrt{3x^2-x+2}(2x+1)^2-\frac{143}{324}(3-2x)\sqrt{3x^2-x+2}+\frac{4147\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

```
[Out] (-143*(3 - 2*x)*Sqrt[2 - x + 3*x^2])/324 + (11*(1 + 2*x)^2*Sqrt[2 - x + 3*x
^2])/27 + ((1 + 2*x)^3*Sqrt[2 - x + 3*x^2])/6 + (4147*ArcSinh[(1 - 6*x)/Sqr
t[23]])/(648*Sqrt[3])
```

Rubi [A] time = 0.0989095, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 619, 215}

$$\frac{1}{6}\sqrt{3x^2-x+2}(2x+1)^3+\frac{11}{27}\sqrt{3x^2-x+2}(2x+1)^2-\frac{143}{324}(3-2x)\sqrt{3x^2-x+2}+\frac{4147\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (-143*(3 - 2*x)*Sqrt[2 - x + 3*x^2])/324 + (11*(1 + 2*x)^2*Sqrt[2 - x + 3*x
^2])/27 + ((1 + 2*x)^3*Sqrt[2 - x + 3*x^2])/6 + (4147*ArcSinh[(1 - 6*x)/Sqr
t[23]])/(648*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} + \frac{1}{48}\int \frac{(1+2x)^2(-44+176x)}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} + \frac{1}{432}\int \frac{(1+2x)(-1716+1144}{\sqrt{2-x+3x^2}}$$

$$= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2} + \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} -$$

$$= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2} + \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} - -$$

$$= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2} + \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} + -$$

Mathematica [A] time = 0.0327693, size = 55, normalized size = 0.58

$$\frac{6\sqrt{3x^2-x+2}\left(432x^3+1176x^2+1138x-243\right)-4147\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{1944}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-243 + 1138*x + 1176*x^2 + 432*x^3) - 4147*Sqrt[3]*
ArcSinh[(-1 + 6*x)/Sqrt[23]])/1944
```

Maple [A] time = 0.056, size = 79, normalized size = 0.8

$$\frac{4x^3}{3}\sqrt{3x^2-x+2}+\frac{98x^2}{27}\sqrt{3x^2-x+2}+\frac{569x}{162}\sqrt{3x^2-x+2}-\frac{3}{4}\sqrt{3x^2-x+2}-\frac{4147\sqrt{3}}{1944}\mathrm{Arcsinh}\left(\frac{6\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 4/3*x^3*(3*x^2-x+2)^(1/2)+98/27*x^2*(3*x^2-x+2)^(1/2)+569/162*x*(3*x^2-x+2)
^(1/2)-3/4*(3*x^2-x+2)^(1/2)-4147/1944*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6
```

```
))
```

Maxima [A] time = 1.54187, size = 108, normalized size = 1.14

$$\frac{4}{3}\sqrt{3x^2-x+2}x^3+\frac{98}{27}\sqrt{3x^2-x+2}x^2+\frac{569}{162}\sqrt{3x^2-x+2}x-\frac{4147}{1944}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)-\frac{3}{4}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 - x + 2)*x^3 + 98/27*sqrt(3*x^2 - x + 2)*x^2 + 569/162*sqrt(
3*x^2 - x + 2)*x - 4147/1944*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 3/4
*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.70268, size = 201, normalized size = 2.12

$$\frac{1}{324}\left(432x^3+1176x^2+1138x-243\right)\sqrt{3x^2-x+2}+\frac{4147}{3888}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(432*x^3 + 1176*x^2 + 1138*x - 243)*sqrt(3*x^2 - x + 2) + 4147/3888*s
qrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2x+1)^2\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

Giac [A] time = 1.18767, size = 85, normalized size = 0.89

$$\frac{1}{324}\left(2\left(12\left(18x+49\right)x+569\right)x-243\right)\sqrt{3x^2-x+2}+\frac{4147}{1944}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/324*(2*(12*(18*x + 49)*x + 569)*x - 243)*sqrt(3*x^2 - x + 2) + 4147/1944*
sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.242 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=70

$$\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2+\frac{1}{54}(62x+69)\sqrt{3x^2-x+2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + ((69 + 62*x)*Sqrt[2 - x + 3*x^2])/5
4 + (251*ArcSinh[(1 - 6*x)/Sqrt[23]])/(108*Sqrt[3])
```

Rubi [A] time = 0.0580893, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1653, 779, 619, 215}

$$\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2+\frac{1}{54}(62x+69)\sqrt{3x^2-x+2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + ((69 + 62*x)*Sqrt[2 - x + 3*x^2])/5
4 + (251*ArcSinh[(1 - 6*x)/Sqrt[23]])/(108*Sqrt[3])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx &= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{36}\int \frac{(1+2x)(-24+124x)}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}-\frac{251}{108}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}-\frac{251\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1\right.}{108\sqrt{69}} \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0214987, size = 50, normalized size = 0.71

$$\frac{1}{324}\left(6\sqrt{3x^2-x+2}\left(48x^2+110x+81\right)-251\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(81 + 110*x + 48*x^2) - 251*Sqrt[3]*ArcSinh[(-1 + 6*
x)/Sqrt[23]])/324
```

Maple [A] time = 0.057, size = 62, normalized size = 0.9

$$\frac{8x^2}{9}\sqrt{3x^2-x+2}+\frac{55x}{27}\sqrt{3x^2-x+2}+\frac{3}{2}\sqrt{3x^2-x+2}-\frac{251\sqrt{3}}{324}\mathrm{Arcsinh}\left(\frac{6\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 8/9*x^2*(3*x^2-x+2)^(1/2)+55/27*x*(3*x^2-x+2)^(1/2)+3/2*(3*x^2-x+2)^(1/2)-2
51/324*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A] time = 1.53746, size = 85, normalized size = 1.21

$$\frac{8}{9}\sqrt{3x^2-x+2}x^2+\frac{55}{27}\sqrt{3x^2-x+2}x-\frac{251}{324}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)+\frac{3}{2}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 - x + 2)*x^2 + 55/27*sqrt(3*x^2 - x + 2)*x - 251/324*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 3/2*sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.61625, size = 178, normalized size = 2.54

$$\frac{1}{54}\left(48x^2+110x+81\right)\sqrt{3x^2-x+2}+\frac{251}{648}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(48*x^2 + 110*x + 81)*sqrt(3*x^2 - x + 2) + 251/648*sqrt(3)*log(4*sqrt
(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

Giac [A] time = 1.16874, size = 78, normalized size = 1.11

$$\frac{1}{54}\left(2\left(24\,x+55\right)x+81\right)\sqrt{3\,x^2-x+2}+\frac{251}{324}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/54*(2*(24*x + 55)*x + 81)*sqrt(3*x^2 - x + 2) + 251/324*sqrt(3)*log(-2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.243 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 - (5*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) - A
rcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[13])
```

Rubi [A] time = 0.0969312, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 843, 619, 215, 724, 206}

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 - (5*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) - A
rcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[13])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx &= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{12}\int \frac{16+20x}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\ &= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{2}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx+\frac{5}{6}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\ &= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{6\sqrt{69}}-\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3}}\right.\\ &= \frac{2}{3}\sqrt{2-x+3x^2}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2\sqrt{13}}\end{aligned}$$

Mathematica [A] time = 0.0371155, size = 78, normalized size = 1.

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}+\frac{5\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 + (5*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(6*Sqrt[3]) -
ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[13])
```

Maple [A] time = 0.051, size = 60, normalized size = 0.8

$$\frac{2}{3}\sqrt{3\,x^2-x+2}+\frac{5\,\sqrt{3}}{18}\text{Arcsinh}\left(\frac{6\,\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right)-\frac{\sqrt{13}}{26}\text{Artanh}\left(\frac{2\,\sqrt{13}}{13}\left(\frac{9}{2}-4\,x\right)\frac{1}{\sqrt{12\,(x+1/2)^2-16\,x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 2/3*(3*x^2-x+2)^(1/2)+5/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1/26*13^(
1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A] time = 1.51391, size = 90, normalized size = 1.15

$$\frac{5}{18}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{1}{26}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{2}{3}\sqrt{3\,x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/26*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 2/3*sqrt(3*
x^2 - x + 2)
```

Fricas [A] time = 1.65931, size = 289, normalized size = 3.71

$$\frac{5}{36}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)+\frac{1}{52}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-1}{4\,x^2+4\,x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x -
 25) + 1/52*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x
^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 2/3*sqrt(3*x^2 - x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 - x + 2)), x)
```

Giac [A] time = 1.24688, size = 157, normalized size = 2.01

$$-\frac{5}{18}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right)+\frac{1}{26}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2}{3}\sqrt{3x^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -5/18*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/26*sq
rt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 -
x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 2/3*s
qrt(3*x^2 - x + 2)
```

3.244 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}$$

```
[Out] -Sqrt[2 - x + 3*x^2]/(13*(1 + 2*x)) - ArcSinh[(1 - 6*x)/Sqrt[23]]/Sqrt[3] +
 (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(26*Sqrt[13])
```

Rubi [A] time = 0.0950249, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 843, 619, 215, 724, 206}

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -Sqrt[2 - x + 3*x^2]/(13*(1 + 2*x)) - ArcSinh[(1 - 6*x)/Sqrt[23]]/Sqrt[3] +
 (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(26*Sqrt[13])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} - \frac{1}{13}\int \frac{-\frac{17}{2}-26x}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$
$$= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} - \frac{9}{26}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx + \int \frac{1}{\sqrt{2-x+3x^2}}\,dx$$
$$= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} + \frac{9}{13}\,\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9-8x}{\sqrt{2-x+3x^2}}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, \cdots\right.}{\sqrt{69}}$$
$$= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} - \frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}} + \frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{26\sqrt{13}}$$

Mathematica [A] time = 0.0518867, size = 82, normalized size = 0.99

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)} + \frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}} + \frac{\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -Sqrt[2 - x + 3*x^2]/(13*(1 + 2*x)) + ArcSinh[(-1 + 6*x)/Sqrt[23]]/Sqrt[3]
+ (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(26*Sqrt[13])
```

Maple [A] time = 0.056, size = 67, normalized size = 0.8

$$\frac{\sqrt{3}}{3}\text{Arcsinh}\left(\frac{6\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right) + \frac{9\sqrt{13}}{338}\text{Artanh}\left(\frac{2\sqrt{13}}{13}\left(\frac{9}{2}-4x\right)\frac{1}{\sqrt{12\left(x+1/2\right)^2-16x+5}}\right) - \frac{1}{26}\sqrt{3\left(x+1/2\right)^2-4x\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+9/338*13^(1/2)*arctanh(2/13*(9/2
-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5
```

```
/4)^(1/2)
```

Maxima [A] time = 1.54286, size = 100, normalized size = 1.2

$$\frac{1}{3}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{9}{338}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2-x+2}}{13\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 9/338*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 1/13*sqrt(3
*x^2 - x + 2)/(2*x + 1)
```

Fricas [A] time = 1.70077, size = 336, normalized size = 4.05

$$\frac{338\sqrt{3}(2\,x+1)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)+27\sqrt{13}(2\,x+1)\log\left(\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)-220\,x^2+19}{4\,x^2+4\,x+1}\right.}{2028\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2028*(338*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 27*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) - 156*sqrt(3*x
^2 - x + 2))/(2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 - x + 2)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4\,x^2+3\,x+1}{\sqrt{3\,x^2-x+2}(2\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*x + 1)/(sqrt(3*x^2 - x + 2)*(2*x + 1)^2), x)
```

3.245 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=89

$$\frac{7\sqrt{3x^2-x+2}}{169(2x+1)}-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}$$

```
[Out] -Sqrt[2 - x + 3*x^2]/(26*(1 + 2*x)^2) + (7*Sqrt[2 - x + 3*x^2])/(169*(1 + 2
*x)) - (581*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(676*Sqrt[
13])
```

Rubi [A] time = 0.0882716, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1650, 806, 724, 206}

$$\frac{7\sqrt{3x^2-x+2}}{169(2x+1)}-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -Sqrt[2 - x + 3*x^2]/(26*(1 + 2*x)^2) + (7*Sqrt[2 - x + 3*x^2])/(169*(1 + 2
*x)) - (581*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(676*Sqrt[
13])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx &= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}-\frac{1}{26}\int \frac{-\frac{35}{2}-49x}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{581}{676}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581}{338}\operatorname{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{676\sqrt{13}}
\end{aligned}$$

Mathematica [A] time = 0.0447765, size = 69, normalized size = 0.78

$$\frac{\frac{26(28x+1)\sqrt{3x^2-x+2}}{(2x+1)^2}-581\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8788}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] ((26*(1 + 28*x)*Sqrt[2 - x + 3*x^2])/(1 + 2*x)^2 - 581*Sqrt[13]*ArcTanh[(9
- 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/8788
```

Maple [A] time = 0.055, size = 74, normalized size = 0.8

$$-\frac{1}{104}\sqrt{3\,(x+1/2)^2-4\,x+\frac{5}{4}}\left(x+\frac{1}{2}\right)^{-2}+\frac{7}{338}\sqrt{3\,(x+1/2)^2-4\,x+\frac{5}{4}}\left(x+\frac{1}{2}\right)^{-1}-\frac{581\sqrt{13}}{8788}\mathrm{Artanh}\left(\frac{2\sqrt{13}}{13}\left(\frac{9}{2}-4\,x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x)
```

```
[Out] -1/104/(x+1/2)^2*(3*(x+1/2)^2-4*x+5/4)^(1/2)+7/338/(x+1/2)*(3*(x+1/2)^2-4*x
+5/4)^(1/2)-581/8788*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2
-16*x+5)^(1/2))
```

Maxima [A] time = 1.59528, size = 111, normalized size = 1.25

$$\frac{581}{8788}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2-x+2}}{26\left(4\,x^2+4\,x+1\right)}+\frac{7\sqrt{3\,x^2-x+2}}{169\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 581/8788*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) - 1/26*sqrt(3*x^2 - x + 2)/(4*x^2 + 4*x + 1) + 7/169*sqrt(3*x^2 -
 x + 2)/(2*x + 1)
```

Fricas [A] time = 1.28669, size = 252, normalized size = 2.83

$$\frac{581\sqrt{13}\left(4\,x^2+4\,x+1\right)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+52\sqrt{3\,x^2-x+2}(28\,x+1)}{17576\left(4\,x^2+4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/17576*(581*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2
)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 52*sqrt(3*x^2 - x
 + 2)*(28*x + 1))/(4*x^2 + 4*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2 + 3x + 1}{(2x+1)^3 \sqrt{3x^2 - x + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 - x + 2)), x)
```

Giac [B] time = 1.22116, size = 275, normalized size = 3.09

$$\frac{581}{8788}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3x^2 - x + 2}\right|}{2\left(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3x^2 - x + 2}\right)}\right) + \frac{190\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right)^3 - 53\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x}\right.}{338\left(2\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right)^2 + 2\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 581/8788*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sq
rt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2
))) + 1/338*(190*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 53*sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 - x + 2))^2 - 489*sqrt(3)*x + 289*sqrt(3) + 489*sqrt(3*x^2 -
 x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sq
rt(3*x^2 - x + 2)) - 5)^2
```

3.246 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{32}{27}\sqrt{3x^2-x+2}x^2+\frac{412}{81}\sqrt{3x^2-x+2}x+\frac{746}{81}\sqrt{3x^2-x+2}+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}+\frac{353\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

```
[Out] (2*(12839 - 3871*x))/(1863*Sqrt[2 - x + 3*x^2]) + (746*Sqrt[2 - x + 3*x^2])
/81 + (412*x*Sqrt[2 - x + 3*x^2])/81 + (32*x^2*Sqrt[2 - x + 3*x^2])/27 + (3
53*ArcSinh[(1 - 6*x)/Sqrt[23]])/(81*Sqrt[3])
```

Rubi [A] time = 0.12421, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{32}{27}\sqrt{3x^2-x+2}x^2+\frac{412}{81}\sqrt{3x^2-x+2}x+\frac{746}{81}\sqrt{3x^2-x+2}+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}+\frac{353\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(12839 - 3871*x))/(1863*Sqrt[2 - x + 3*x^2]) + (746*Sqrt[2 - x + 3*x^2])
/81 + (412*x*Sqrt[2 - x + 3*x^2])/81 + (32*x^2*Sqrt[2 - x + 3*x^2])/27 + (3
53*ArcSinh[(1 - 6*x)/Sqrt[23]])/(81*Sqrt[3])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{2}{23}\int \frac{\frac{1127}{81}+\frac{7682x}{27}+\frac{2852x^2}{9}+\frac{368x^3}{3}}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{2}{207}\int \frac{\frac{1127}{9}+2070x+\frac{9476x^2}{3}}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{1}{621}\int \frac{-5566+17158\ldots}{\sqrt{2-x+3x^2}}$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} - \frac{35\ldots}{81}$$

$$35\ldots$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} - \ldots$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{35\ldots}{\ldots}$$

Mathematica [A] time = 0.0405456, size = 69, normalized size = 0.67

$$\frac{6\left(3312x^4+13110x^3+23207x^2-2974x+29997\right)-8119\sqrt{9x^2-3x+6}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{5589\sqrt{3x^2-x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (6*(29997 - 2974*x + 23207*x^2 + 13110*x^3 + 3312*x^4) - 8119*Sqrt[6 - 3*x
+ 9*x^2]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(5589*Sqrt[2 - x + 3*x^2])
```

Maple [A] time = 0.053, size = 115, normalized size = 1.1

$$\frac{32\,x^4}{9}\frac{1}{\sqrt{3\,x^2-x+2}}+\frac{380\,x^3}{27}\frac{1}{\sqrt{3\,x^2-x+2}}+\frac{2018\,x^2}{81}\frac{1}{\sqrt{3\,x^2-x+2}}-\frac{353\sqrt{3}}{243}\mathrm{Arcsinh}\left(\frac{6\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right)+\frac{353\,x}{81}\frac{\ldots}{\sqrt{3\,x^2\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 32/9*x^4/(3*x^2-x+2)^(1/2)+380/27*x^3/(3*x^2-x+2)^(1/2)+2018/81*x^2/(3*x^2-
x+2)^(1/2)-353/243*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+353/81*x/(3*x^2-x
+2)^(1/2)-521/414*(-1+6*x)/(3*x^2-x+2)^(1/2)+557/18/(3*x^2-x+2)^(1/2)
```

Maxima [A] time = 1.5473, size = 131, normalized size = 1.27

$$\frac{32\,x^4}{9\,\sqrt{3\,x^2-x+2}}+\frac{380\,x^3}{27\,\sqrt{3\,x^2-x+2}}+\frac{2018\,x^2}{81\,\sqrt{3\,x^2-x+2}}-\frac{353}{243}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(6\,x-1)\right)-\frac{5948\,x}{1863\,\sqrt{3\,x^2-x+2}}+\frac{\cdots}{69\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 32/9*x^4/sqrt(3*x^2 - x + 2) + 380/27*x^3/sqrt(3*x^2 - x + 2) + 2018/81*x^2
/sqrt(3*x^2 - x + 2) - 353/243*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 5
948/1863*x/sqrt(3*x^2 - x + 2) + 2222/69/sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.09804, size = 269, normalized size = 2.61

$$\frac{8119\,\sqrt{3}\left(3\,x^2-x+2\right)\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)+12\left(3312\,x^4+13110\,x^3+23207\,x^2-2974\cdots\right.}{11178\left(3\,x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11178*(8119*sqrt(3)*(3*x^2 - x + 2)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25) + 12*(3312*x^4 + 13110*x^3 + 23207*x^2 - 2974*
x + 29997)*sqrt(3*x^2 - x + 2))/(3*x^2 - x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Giac [A] time = 1.19926, size = 90, normalized size = 0.87

$$\frac{353}{243}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left(\left(23\left(6\left(24x+95\right)x+1009\right)x-2974\right)x+29997\right)}{1863\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 353/243*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1
863*((23*(6*(24*x + 95)*x + 1009)*x - 2974)*x + 29997)/sqrt(3*x^2 - x + 2)
```

3.247 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}+\frac{8}{9}x\sqrt{3x^2-x+2}+\frac{112}{27}\sqrt{3x^2-x+2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] (2*(1249 - 2273*x))/(621*Sqrt[2 - x + 3*x^2]) + (112*Sqrt[2 - x + 3*x^2])/2
7 + (8*x*Sqrt[2 - x + 3*x^2])/9 - (64*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[
3])
```

Rubi [A] time = 0.108118, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}+\frac{8}{9}x\sqrt{3x^2-x+2}+\frac{112}{27}\sqrt{3x^2-x+2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(1249 - 2273*x))/(621*Sqrt[2 - x + 3*x^2]) + (112*Sqrt[2 - x + 3*x^2])/2
7 + (8*x*Sqrt[2 - x + 3*x^2])/9 - (64*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[
3])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx &= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{2116}{27}+\frac{1150x}{9}+\frac{184x^2}{3}}{\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{1}{69}\int \frac{\frac{3128}{9}+\frac{2576x}{3}}{\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{64}{9}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{64\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1\right.}{9\sqrt{69}}\\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0318988, size = 61, normalized size = 0.74

$$\frac{2\left(828x^3+3588x^2+736\sqrt{9x^2-3x+6}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)-3009x+3825\right)}{621\sqrt{3x^2-x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(3825 - 3009*x + 3588*x^2 + 828*x^3 + 736*Sqrt[6 - 3*x + 9*x^2]*ArcSinh[
(-1 + 6*x)/Sqrt[23]]))/(621*Sqrt[2 - x + 3*x^2])
```

Maple [A] time = 0.053, size = 98, normalized size = 1.2

$$\frac{8x^3}{3}\frac{1}{\sqrt{3x^2-x+2}}+\frac{104x^2}{9}\frac{1}{\sqrt{3x^2-x+2}}-\frac{64x}{9}\frac{1}{\sqrt{3x^2-x+2}}+\frac{107}{9}\frac{1}{\sqrt{3x^2-x+2}}-\frac{-89+534x}{207}\frac{1}{\sqrt{3x^2-x+2}}+\frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 8/3*x^3/(3*x^2-x+2)^(1/2)+104/9*x^2/(3*x^2-x+2)^(1/2)-64/9*x/(3*x^2-x+2)^(1
/2)+107/9/(3*x^2-x+2)^(1/2)-89/207*(-1+6*x)/(3*x^2-x+2)^(1/2)+64/27*3^(1/2)
*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A] time = 1.55439, size = 108, normalized size = 1.32

$$\frac{8x^3}{3\sqrt{3x^2-x+2}}+\frac{104x^2}{9\sqrt{3x^2-x+2}}+\frac{64}{27}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)-\frac{2006x}{207\sqrt{3x^2-x+2}}+\frac{850}{69\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^3/sqrt(3*x^2 - x + 2) + 104/9*x^2/sqrt(3*x^2 - x + 2) + 64/27*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2006/207*x/sqrt(3*x^2 - x + 2) + 850/69
/sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.12729, size = 244, normalized size = 2.98

$$\frac{2\left(368\sqrt{3}\left(3x^2-x+2\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+3\left(276x^3+1196x^2-1003x+1275\right)\sqrt{3}\right.}{621\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/621*(368*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 3*(276*x^3 + 1196*x^2 - 1003*x + 1275)*sqrt(3*
x^2 - x + 2))/(3*x^2 - x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Giac [A] time = 1.15107, size = 84, normalized size = 1.02

$$-\frac{64}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left((92(3x+13)x-1003)x+1275\right)}{207\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -64/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/20
7*((92*(3*x + 13)*x - 1003)*x + 1275)/sqrt(3*x^2 - x + 2)
```

3.248 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}+\frac{8}{9}\sqrt{3x^2-x+2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*(73 + 367*x))/(207*Sqrt[2 - x + 3*x^2]) + (8*Sqrt[2 - x + 3*x^2])/9 - (
14*ArcSinh[(1 - 6*x)/Sqrt[23]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0596095, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1660, 640, 619, 215}

$$-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}+\frac{8}{9}\sqrt{3x^2-x+2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```


```
[Out] (-2*(73 + 367*x))/(207*Sqrt[2 - x + 3*x^2]) + (8*Sqrt[2 - x + 3*x^2])/9 - (
14*ArcSinh[(1 - 6*x)/Sqrt[23]])/(3*Sqrt[3])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{437}{9}+\frac{92x}{3}}{\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14}{3}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{3\sqrt{69}}\\
&= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.130231, size = 50, normalized size = 0.79

$$\frac{2\left(92x^2-153x+37\right)}{69\sqrt{3x^2-x+2}}+\frac{14\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(37 - 153*x + 92*x^2))/(69*Sqrt[2 - x + 3*x^2]) + (14*ArcSinh[(-1 + 6*x)
/Sqrt[23]])/(3*Sqrt[3])
```

Maple [A] time = 0.049, size = 81, normalized size = 1.3

$$\frac{8\,x^{2}}{3}\frac{1}{\sqrt{3\,x^{2}-x+2}}-\frac{14\,x}{3}\frac{1}{\sqrt{3\,x^{2}-x+2}}+\frac{10}{9}\frac{1}{\sqrt{3\,x^{2}-x+2}}+\frac{-8+48\,x}{207}\frac{1}{\sqrt{3\,x^{2}-x+2}}+\frac{14\,\sqrt{3}}{9}\mathrm{Arcsinh}\left(\frac{6\,\sqrt{23}}{23}\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 8/3*x^2/(3*x^2-x+2)^(1/2)-14/3*x/(3*x^2-x+2)^(1/2)+10/9/(3*x^2-x+2)^(1/2)+8
/207*(-1+6*x)/(3*x^2-x+2)^(1/2)+14/9*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A] time = 1.5263, size = 85, normalized size = 1.35

$$\frac{8\,x^{2}}{3\,\sqrt{3\,x^{2}-x+2}}+\frac{14}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(6\,x-1)\right)-\frac{102\,x}{23\,\sqrt{3\,x^{2}-x+2}}+\frac{74}{69\,\sqrt{3\,x^{2}-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 - x + 2) + 14/9*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 102/23*x/sqrt(3*x^2 - x + 2) + 74/69/sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 0.940043, size = 224, normalized size = 3.56

$$\frac{161\,\sqrt{3}\left(3\,x^{2}-x+2\right)\log\left(-4\,\sqrt{3}\sqrt{3\,x^{2}-x+2}(6\,x-1)-72\,x^{2}+24\,x-25\right)+6\left(92\,x^{2}-153\,x+37\right)\sqrt{3\,x^{2}-x+2}}{207\left(3\,x^{2}-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/207*(161*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 6*(92*x^2 - 153*x + 37)*sqrt(3*x^2 - x + 2))/(
3*x^2 - x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Giac [A] time = 1.19603, size = 77, normalized size = 1.22

$$-\frac{14}{9}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)+\frac{2\,((92\,x-153)x+37)}{69\,\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -14/9*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/69*
((92*x - 153)*x + 37)/sqrt(3*x^2 - x + 2)
```

3.249 $\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}$$

```
[Out] (-2*(101 - 77*x))/(299*Sqrt[2 - x + 3*x^2]) - (2*ArcTanh[(9 - 8*x)/(2*Sqrt[
13]*Sqrt[2 - x + 3*x^2])])/(13*Sqrt[13])
```

Rubi [A] time = 0.0736133, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 12, 724, 206}

$$-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(299*Sqrt[2 - x + 3*x^2]) - (2*ArcTanh[(9 - 8*x)/(2*Sqrt[
13]*Sqrt[2 - x + 3*x^2])])/(13*Sqrt[13])
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx &= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{23}{13(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}+\frac{2}{13}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{4}{13}\,\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{13\sqrt{13}}
\end{aligned}$$

Mathematica [A] time = 0.0218867, size = 73, normalized size = 1.18

$$-\frac{2\left(23\sqrt{13}\sqrt{3x^2-x+2}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-1001x+1313\right)}{3887\sqrt{3x^2-x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(1313 - 1001*x + 23*Sqrt[13]*Sqrt[2 - x + 3*x^2]*ArcTanh[(9 - 8*x)/(2*S
qrt[13]*Sqrt[2 - x + 3*x^2])]))/(3887*Sqrt[2 - x + 3*x^2])
```

Maple [B] time = 0.061, size = 102, normalized size = 1.7

$$-\frac{2}{3}\frac{1}{\sqrt{3x^2-x+2}}+\frac{-5+30x}{69}\frac{1}{\sqrt{3x^2-x+2}}+\frac{1}{13}\frac{1}{\sqrt{3\left(x+1/2\right)^2-4x+\frac{5}{4}}}+\frac{-4+24x}{299}\frac{1}{\sqrt{3\left(x+1/2\right)^2-4x+\frac{5}{4}}}-\frac{2\sqrt{1}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] -2/3/(3*x^2-x+2)^(1/2)+5/69*(-1+6*x)/(3*x^2-x+2)^(1/2)+1/13/(3*(x+1/2)^2-4*
x+5/4)^(1/2)+4/299*(-1+6*x)/(3*(x+1/2)^2-4*x+5/4)^(1/2)-2/169*13^(1/2)*arct
anh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A] time = 1.93413, size = 86, normalized size = 1.39

$$\frac{2}{169}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)+\frac{154x}{299\sqrt{3x^2-x+2}}-\frac{202}{299\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/169*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
 + 1)) + 154/299*x/sqrt(3*x^2 - x + 2) - 202/299/sqrt(3*x^2 - x + 2)
```

Fricas [A] time = 1.0099, size = 247, normalized size = 3.98

$$\frac{23\sqrt{13}\left(3x^2-x+2\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+26\sqrt{3x^2-x+2}(77x-101)}{3887\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3887*(23*sqrt(13)*(3*x^2 - x + 2)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8
*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 26*sqrt(3*x^2 - x + 2
)*(77*x - 101))/(3*x^2 - x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(3/2)), x)
```

Giac [A] time = 1.21485, size = 123, normalized size = 1.98

$$\frac{2}{169}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2(77x-101)}{299\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/169*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
 + 2/299*(77*x - 101)/sqrt(3*x^2 - x + 2)
```

3.250 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{169(2x+1)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

```
[Out] (-2*(197 - 837*x))/(3887*Sqrt[2 - x + 3*x^2]) - (4*Sqrt[2 - x + 3*x^2])/(16
9*(1 + 2*x)) + (2*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169
*Sqrt[13])
```

Rubi [A] time = 0.092342, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 806, 724, 206}

$$-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{169(2x+1)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```


```
[Out] (-2*(197 - 837*x))/(3887*Sqrt[2 - x + 3*x^2]) - (4*Sqrt[2 - x + 3*x^2])/(16
9*(1 + 2*x)) + (2*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169
*Sqrt[13])
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx &= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{184}{169}-\frac{230x}{169}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{2}{169}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{4}{169}\,\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}
\end{aligned}$$

Mathematica [A] time = 0.0459473, size = 74, normalized size = 0.85

$$\frac{2\left(1536x^2+489x-289\right)}{3887(2x+1)\sqrt{3x^2-x+2}}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(-289 + 489*x + 1536*x^2))/(3887*(1 + 2*x)*Sqrt[2 - x + 3*x^2]) + (2*Arc
Tanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169*Sqrt[13])
```

Maple [A] time = 0.055, size = 109, normalized size = 1.3

$$\frac{-2+12x}{23}\frac{1}{\sqrt{3x^2-x+2}}-\frac{1}{169}\frac{1}{\sqrt{3\,(x+1/2)^2-4x+\frac{5}{4}}}-\frac{-82+492x}{3887}\frac{1}{\sqrt{3\,(x+1/2)^2-4x+\frac{5}{4}}}+\frac{2\sqrt{13}}{2197}\mathrm{Artanh}\left(\frac{2\sqrt{13}}{13}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 2/23*(-1+6*x)/(3*x^2-x+2)^(1/2)-1/169/(3*(x+1/2)^2-4*x+5/4)^(1/2)-82/3887*(
-1+6*x)/(3*(x+1/2)^2-4*x+5/4)^(1/2)+2/2197*13^(1/2)*arctanh(2/13*(9/2-4*x)*
13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-1/26/(x+1/2)/(3*(x+1/2)^2-4*x+5/4)^(1
/2)
```

Maxima [A] time = 1.50075, size = 130, normalized size = 1.49

$$-\frac{2}{2197}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)+\frac{1536x}{3887\sqrt{3x^2-x+2}}-\frac{279}{3887\sqrt{3x^2-x+2}}-\frac{1}{13\left(2\sqrt{3x^2-x+2}x+\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2
*x + 1)) + 1536/3887*x/sqrt(3*x^2 - x + 2) - 279/3887/sqrt(3*x^2 - x + 2) -
 1/13/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

Fricas [A] time = 1.15101, size = 285, normalized size = 3.28

$$\frac{23\,\sqrt{13}\left(6\,x^{3}+x^{2}+3\,x+2\right)\log\left(\frac{4\,\sqrt{13}\sqrt{3\,x^{2}-x+2}(8\,x-9)-220\,x^{2}+196\,x-185}{4\,x^{2}+4\,x+1}\right)+26\left(1536\,x^{2}+489\,x-289\right)\sqrt{3\,x^{2}-x+2}}{50531\left(6\,x^{3}+x^{2}+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50531*(23*sqrt(13)*(6*x^3 + x^2 + 3*x + 2)*log((4*sqrt(13)*sqrt(3*x^2 - x
 + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 26*(1536*x^2
+ 489*x - 289)*sqrt(3*x^2 - x + 2))/(6*x^3 + x^2 + 3*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^{2}+3x+1}{(2x+1)^{2}\left(3x^{2}-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4\,x^{2}+3\,x+1}{\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}(2\,x+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*x + 1)/((3*x^2 - x + 2)^(3/2)*(2*x + 1)^2), x)
```

3.251 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2\sqrt{3x^2-x+2}}{169(2x+1)^2}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

```
[Out] (2*(2363 + 3693*x))/(50531*Sqrt[2 - x + 3*x^2]) - (2*Sqrt[2 - x + 3*x^2])/(
169*(1 + 2*x)^2) - (4*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (487*ArcTanh[
(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rubi [A] time = 0.154798, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2\sqrt{3x^2-x+2}}{169(2x+1)^2}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(50531*Sqrt[2 - x + 3*x^2]) - (2*Sqrt[2 - x + 3*x^2])/(
169*(1 + 2*x)^2) - (4*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (487*ArcTanh[
(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{8349}{2197}+\frac{20838x}{2197}+\frac{23828x^2}{2197}}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{1}{299}\int \frac{-\frac{11615}{169}-\frac{22034x}{169}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}+\frac{487\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{2197}$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{974\,\mathrm{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\right.}{2197}$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

Mathematica [A] time = 0.0584065, size = 79, normalized size = 0.71

$$\frac{2\left(14496x^3+23281x^2+13306x+1673\right)}{50531(2x+1)^2\sqrt{3x^2-x+2}}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(1673 + 13306*x + 23281*x^2 + 14496*x^3))/(50531*(1 + 2*x)^2*Sqrt[2 - x
+ 3*x^2]) - (487*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197
*Sqrt[13])
```

Maple [A] time = 0.057, size = 111, normalized size = 1.

$$-\frac{1}{104}\left(x+\frac{1}{2}\right)^{-2}\frac{1}{\sqrt{3\,(x+1/2)^2-4\,x+\frac{5}{4}}}+\frac{3}{338}\left(x+\frac{1}{2}\right)^{-1}\frac{1}{\sqrt{3\,(x+1/2)^2-4\,x+\frac{5}{4}}}+\frac{487}{4394}\frac{1}{\sqrt{3\,(x+1/2)^2-4\,x+\frac{5}{4}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x)
```

```
[Out] -1/104/(x+1/2)^2/(3*(x+1/2)^2-4*x+5/4)^(1/2)+3/338/(x+1/2)/(3*(x+1/2)^2-4*x
+5/4)^(1/2)+487/4394/(3*(x+1/2)^2-4*x+5/4)^(1/2)+1208/50531*(-1+6*x)/(3*(x+
1/2)^2-4*x+5/4)^(1/2)-487/28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(1
2*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A] time = 1.50668, size = 196, normalized size = 1.75

$$\frac{487}{28561}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{7248\,x}{50531\,\sqrt{3\,x^2-x+2}}+\frac{8785}{101062\,\sqrt{3\,x^2-x+2}}-\frac{}{26\left(4\sqrt{3\,x^2-x+2}x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 487/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) + 7248/50531*x/sqrt(3*x^2 - x + 2) + 8785/101062/sqrt(3*x^2 - x
+ 2) - 1/26/(4*sqrt(3*x^2 - x + 2)*x^2 + 4*sqrt(3*x^2 - x + 2)*x + sqrt(3*x
^2 - x + 2)) + 3/169/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

Fricas [A] time = 1.13226, size = 344, normalized size = 3.07

$$\frac{11201\sqrt{13}\left(12\,x^4+8\,x^3+7\,x^2+7\,x+2\right)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+52\left(14496\,x^3+23281\,x^2+13306\,x\right.}{1313806\left(12\,x^4+8\,x^3+7\,x^2+7\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1313806*(11201*sqrt(13)*(12*x^4 + 8*x^3 + 7*x^2 + 7*x + 2)*log(-(4*sqrt(1
3)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)
) + 52*(14496*x^3 + 23281*x^2 + 13306*x + 1673)*sqrt(3*x^2 - x + 2))/(12*x^
4 + 8*x^3 + 7*x^2 + 7*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)), x)
```

Giac [B] time = 1.24475, size = 301, normalized size = 2.69

$$\frac{487}{28561}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}+\frac{2\left(62\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3 \cdots\right.}{2197\left(2\left(\sqrt{3}x \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 487/28561*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*s
qrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x +
2))) + 2/50531*(3693*x + 2363)/sqrt(3*x^2 - x + 2) + 2/2197*(62*(sqrt(3)*x
- sqrt(3*x^2 - x + 2))^3 - 37*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 +
 263*sqrt(3)*x - 71*sqrt(3) - 263*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt
(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

3.252 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{2(12839-3871x)}{5589\left(3x^2-x+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2-x+2}-\frac{28(42240x+35809)}{128547\sqrt{3x^2-x+2}}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

```
[Out] (2*(12839 - 3871*x))/(5589*(2 - x + 3*x^2)^(3/2)) - (28*(35809 + 42240*x))/
(128547*Sqrt[2 - x + 3*x^2]) + (32*Sqrt[2 - x + 3*x^2])/27 - (296*ArcSinh[(
1 - 6*x)/Sqrt[23]])/(27*Sqrt[3])
```

Rubi [A] time = 0.112569, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 640, 619, 215}

$$\frac{2(12839-3871x)}{5589\left(3x^2-x+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2-x+2}-\frac{28(42240x+35809)}{128547\sqrt{3x^2-x+2}}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```


```
[Out] (2*(12839 - 3871*x))/(5589*(2 - x + 3*x^2)^(3/2)) - (28*(35809 + 42240*x))/
(128547*Sqrt[2 - x + 3*x^2]) + (32*Sqrt[2 - x + 3*x^2])/27 - (296*ArcSinh[(
1 - 6*x)/Sqrt[23]])/(27*Sqrt[3])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{4361}{81}+\frac{7682x}{9}+\frac{2852x^2}{3}+368x^3}{\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{4\int \frac{\frac{37030}{9}+\frac{4232x}{3}}{\sqrt{2-x+3x^2}}\,dx}{1587}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}+\frac{296}{27}\int \frac{1}{\sqrt{2-x+3\ldots}}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}+\frac{296\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\ldots}}\ldots\right)}{27\sqrt{\ldots}}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

Mathematica [A] time = 0.0628087, size = 71, normalized size = 0.83

$$\frac{2\left(228528x^4-743712x^3+25890x^2+78292\sqrt{3}\left(3x^2-x+2\right)^{3/2}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)-358377x-134217\right)}{42849\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-134217 - 358377*x + 25890*x^2 - 743712*x^3 + 228528*x^4 + 78292*Sqrt[3
]*(2 - x + 3*x^2)^(3/2)*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/(42849*(2 - x + 3*x^
2)^(3/2))
```

Maple [B] time = 0.056, size = 163, normalized size = 1.9

$$\frac{32x^4}{3}\left(3x^2-x+2\right)^{-\frac{3}{2}}-\frac{296x^3}{27}\left(3x^2-x+2\right)^{-\frac{3}{2}}+\frac{8x^2}{27}\left(3x^2-x+2\right)^{-\frac{3}{2}}+\frac{296\sqrt{3}}{81}\mathrm{Arcsinh}\left(\frac{6\sqrt{23}}{23}\left(x-\frac{1}{6}\right)\right)-\frac{461x}{81}(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 32/3*x^4/(3*x^2-x+2)^(3/2)-296/27*x^3/(3*x^2-x+2)^(3/2)+8/27*x^2/(3*x^2-x+2
)^(3/2)+296/81*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-461/81*x/(3*x^2-x+2)^
(3/2)-296/27*x/(3*x^2-x+2)^(1/2)+65264/128547*(-1+6*x)/(3*x^2-x+2)^(1/2)+13
763/33534*(-1+6*x)/(3*x^2-x+2)^(3/2)-1727/1458/(3*x^2-x+2)^(3/2)-148/81/(3*
x^2-x+2)^(1/2)
```

Maxima [B] time = 1.52883, size = 273, normalized size = 3.17

$$\frac{32x^4}{3\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{296}{42849}x\left(\frac{426x}{\sqrt{3x^2-x+2}}-\frac{4761x^2}{\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{3x^2-x+2}}+\frac{805x}{\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{2162}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\right)+\frac{2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 - x + 2)^(3/2) + 296/42849*x*(426*x/sqrt(3*x^2 - x + 2) - 4
761*x^2/(3*x^2 - x + 2)^(3/2) - 71/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x +
 2)^(3/2) - 2162/(3*x^2 - x + 2)^(3/2)) + 296/81*sqrt(3)*arcsinh(1/23*sqrt(
23)*(6*x - 1)) - 42032/42849*sqrt(3*x^2 - x + 2) - 47072/42849*x/sqrt(3*x^2
 - x + 2) + 52/9*x^2/(3*x^2 - x + 2)^(3/2) - 23104/14283/sqrt(3*x^2 - x + 2
```

```
) - 7742/1863*x/(3*x^2 - x + 2)^(3/2) + 1666/1863/(3*x^2 - x + 2)^(3/2)
```

Fricas [A] time = 1.14331, size = 325, normalized size = 3.78

$$\frac{2\left(39146\sqrt{3}\left(9x^4-6x^3+13x^2-4x+4\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+3\left(76176x^4-2479\right.\right.}{42849\left(9x^4-6x^3+13x^2-4x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/42849*(39146*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sq
rt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(76176*x^4 - 247904*x
^3 + 8630*x^2 - 119459*x - 44739)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*
x^2 - 4*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Giac [A] time = 1.15634, size = 90, normalized size = 1.05

$$-\frac{296}{81}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left(\left(2\left(8\left(4761x-15494\right)x+4315\right)x-119459\right)x-44739\right)}{14283\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -296/81*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1
4283*((2*(8*(4761*x - 15494)*x + 4315)*x - 119459)*x - 44739)/(3*x^2 - x +
2)^(3/2)
```

3.253 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{3x^2-x+2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] (2*(1249 - 2273*x))/(1863*(2 - x + 3*x^2)^(3/2)) - (8*(23257 - 1473*x))/(42
849*Sqrt[2 - x + 3*x^2]) - (16*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[3])
```

Rubi [A] time = 0.0946948, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 12, 619, 215}

$$\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{3x^2-x+2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```


```
[Out] (2*(1249 - 2273*x))/(1863*(2 - x + 3*x^2)^(3/2)) - (8*(23257 - 1473*x))/(42
849*Sqrt[2 - x + 3*x^2]) - (16*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[3])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx &= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{1802}{27}+\frac{1150x}{3}+184x^2}{\left(2-x+3x^2\right)^{3/2}}\,dx \\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{4\int \frac{2116}{3\sqrt{2-x+3x^2}}\,dx}{1587} \\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{16}{9}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{16\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{9\sqrt{69}} \\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.148035, size = 66, normalized size = 0.97

$$
\frac{2\left(5892x^3-94992x^2+4232\sqrt{3}\left(3x^2-x+2\right)^{3/2}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)+17511x-52443\right)}{14283\left(3x^2-x+2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-52443 + 17511*x - 94992*x^2 + 5892*x^3 + 4232*Sqrt[3]*(2 - x + 3*x^2)^
(3/2)*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/(14283*(2 - x + 3*x^2)^(3/2))
```

Maple [B] time = 0.054, size = 146, normalized size = 2.2

$$-\frac{16\,x^{3}}{9}\left(3\,x^{2}-x+2\right)^{-\frac{3}{2}}-\frac{92\,x^{2}}{9}\left(3\,x^{2}-x+2\right)^{-\frac{3}{2}}-\frac{67\,x}{27}\left(3\,x^{2}-x+2\right)^{-\frac{3}{2}}-\frac{2653}{486}\left(3\,x^{2}-x+2\right)^{-\frac{3}{2}}+\frac{-4585+27510\,x}{11178}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] -16/9*x^3/(3*x^2-x+2)^(3/2)-92/9*x^2/(3*x^2-x+2)^(3/2)-67/27*x/(3*x^2-x+2)^
(3/2)-2653/486/(3*x^2-x+2)^(3/2)+4585/11178*(-1+6*x)/(3*x^2-x+2)^(3/2)+1889
2/42849*(-1+6*x)/(3*x^2-x+2)^(1/2)-16/9*x/(3*x^2-x+2)^(1/2)-8/27/(3*x^2-x+2
)^(1/2)+16/27*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [B] time = 1.4618, size = 250, normalized size = 3.68

$$\frac{16}{14283}\,x\left(\frac{426\,x}{\sqrt{3\,x^{2}-x+2}}-\frac{4761\,x^{2}}{\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{3\,x^{2}-x+2}}+\frac{805\,x}{\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}}-\frac{2162}{\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}}\right)+\frac{16}{27}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/14283*x*(426*x/sqrt(3*x^2 - x + 2) - 4761*x^2/(3*x^2 - x + 2)^(3/2) - 71
/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x + 2)^(3/2) - 2162/(3*x^2 - x + 2)^(
3/2)) + 16/27*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2272/14283*sqrt(3*
x^2 - x + 2) + 28184/14283*x/sqrt(3*x^2 - x + 2) - 28/3*x^2/(3*x^2 - x + 2)
^(3/2) - 2956/4761/sqrt(3*x^2 - x + 2) - 106/621*x/(3*x^2 - x + 2)^(3/2) -
3394/621/(3*x^2 - x + 2)^(3/2)
```

Fricas [B] time = 1.0629, size = 304, normalized size = 4.47

$$\frac{2\left(2116\sqrt{3}\left(9x^4-6x^3+13x^2-4x+4\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+3\left(1964x^3-31664x^2-\right.\right.}{14283\left(9x^4-6x^3+13x^2-4x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/14283*(2116*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sqr
t(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(1964*x^3 - 31664*x^2
+ 5837*x - 17481)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2x+1)^2\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Giac [A] time = 1.15909, size = 84, normalized size = 1.24

$$-\frac{16}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left((4(491x-7916)x+5837)x-17481\right)}{4761\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -16/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/47
61*((4*(491*x - 7916)*x + 5837)*x - 17481)/(3*x^2 - x + 2)^(3/2)
```

3.254 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}}-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

```
[Out] (-2*(73 + 367*x))/(621*(2 - x + 3*x^2)^(3/2)) - (4*(3889 - 4290*x))/(14283*
Sqrt[2 - x + 3*x^2])
```

Rubi [A] time = 0.0476253, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1660, 636}

$$-\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}}-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (-2*(73 + 367*x))/(621*(2 - x + 3*x^2)^(3/2)) - (4*(3889 - 4290*x))/(14283*
Sqrt[2 - x + 3*x^2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
```

```
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(73+367x)}{621\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{577}{9}+92x}{\left(2-x+3x^2\right)^{3/2}}\,dx$$
$$= -\frac{2(73+367x)}{621\left(2-x+3x^2\right)^{3/2}}-\frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}$$

Mathematica [A] time = 0.0602662, size = 33, normalized size = 0.7

$$\frac{2\left(2860x^3-3546x^2+1833x-1915\right)}{1587\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]

[Out] (2*(-1915 + 1833*x - 3546*x^2 + 2860*x^3))/(1587*(2 - x + 3*x^2)^(3/2))
```

Maple [A] time = 0.046, size = 30, normalized size = 0.6

$$\frac{5720\,x^3-7092\,x^2+3666\,x-3830}{1587}\left(3\,x^2-x+2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)

[Out] 2/1587/(3*x^2-x+2)^(3/2)*(2860*x^3-3546*x^2+1833*x-1915)
```

Maxima [A] time = 1.00424, size = 103, normalized size = 2.19

$$\frac{5720\,x}{4761\,\sqrt{3\,x^2-x+2}}-\frac{8\,x^2}{3\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{2860}{14283\,\sqrt{3\,x^2-x+2}}-\frac{182\,x}{621\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{1250}{621\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5720/4761*x/sqrt(3*x^2 - x + 2) - 8/3*x^2/(3*x^2 - x + 2)^(3/2) - 2860/1428
3/sqrt(3*x^2 - x + 2) - 182/621*x/(3*x^2 - x + 2)^(3/2) - 1250/621/(3*x^2 -
 x + 2)^(3/2)
```

Fricas [A] time = 1.00179, size = 136, normalized size = 2.89

$$\frac{2\left(2860\,x^{3}-3546\,x^{2}+1833\,x-1915\right)\sqrt{3\,x^{2}-x+2}}{1587\left(9\,x^{4}-6\,x^{3}+13\,x^{2}-4\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1587*(2860*x^3 - 3546*x^2 + 1833*x - 1915)*sqrt(3*x^2 - x + 2)/(9*x^4 - 6
*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)\left(4x^{2}+3x+1\right)}{\left(3x^{2}-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Giac [A] time = 1.18293, size = 38, normalized size = 0.81

$$\frac{2\,((2\,(1430\,x-1773)x+1833)x-1915)}{1587\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/1587*((2*(1430*x - 1773)*x + 1833)*x - 1915)/(3*x^2 - x + 2)^(3/2)
```

3.255 $\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}}-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

```
[Out] (-2*(101 - 77*x))/(897*(2 - x + 3*x^2)^(3/2)) - (4*(691 - 13668*x))/(268203
*Sqrt[2 - x + 3*x^2]) - (8*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2
])])/(169*Sqrt[13])
```

Rubi [A] time = 0.0944011, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 822, 12, 724, 206}

$$-\frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}}-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(897*(2 - x + 3*x^2)^(3/2)) - (4*(691 - 13668*x))/(268203
*Sqrt[2 - x + 3*x^2]) - (8*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2
])])/(169*Sqrt[13])
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
```

```
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{223}{13}+\frac{308x}{13}}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} + \frac{4\int \frac{3174}{13(1+2x)\sqrt{2-x+3x^2}}\,dx}{20631}$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} + \frac{8}{169}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{16}{169}\operatorname{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9-}{\sqrt{2-x}}\right.$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

Mathematica [A] time = 0.0482146, size = 72, normalized size = 0.85

$$\frac{2\left(82008x^3-31482x^2+79077x-32963\right)}{268203\left(3x^2-x+2\right)^{3/2}} - \frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(-32963 + 79077*x - 31482*x^2 + 82008*x^3))/(268203*(2 - x + 3*x^2)^(3/2
)) - (8*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169*Sqrt[13])
```

Maple [B] time = 0.056, size = 158, normalized size = 1.9

$$-\frac{2}{9}\left(3x^2-x+2\right)^{-\frac{3}{2}} + \frac{-5+30x}{207}\left(3x^2-x+2\right)^{-\frac{3}{2}} + \frac{-40+240x}{1587}\frac{1}{\sqrt{3x^2-x+2}} + \frac{1}{39}\left(3\left(x+1/2\right)^2-4x+\frac{5}{4}\right)^{-\frac{3}{2}} + \frac{-4-}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] -2/9/(3*x^2-x+2)^(3/2)+5/207*(-1+6*x)/(3*x^2-x+2)^(3/2)+40/1587*(-1+6*x)/(3
*x^2-x+2)^(1/2)+1/39/(3*(x+1/2)^2-4*x+5/4)^(3/2)+4/897*(-1+6*x)/(3*(x+1/2)^
2-4*x+5/4)^(3/2)+784/89401*(-1+6*x)/(3*(x+1/2)^2-4*x+5/4)^(1/2)+4/169/(3*(x
+1/2)^2-4*x+5/4)^(1/2)-8/2197*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*
(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A] time = 1.47288, size = 126, normalized size = 1.48

$$\frac{8}{2197}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{18224\,x}{89401\,\sqrt{3\,x^2-x+2}}-\frac{2764}{268203\,\sqrt{3\,x^2-x+2}}+\frac{154\,x}{897\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*
x + 1)) + 18224/89401*x/sqrt(3*x^2 - x + 2) - 2764/268203/sqrt(3*x^2 - x +
2) + 154/897*x/(3*x^2 - x + 2)^(3/2) - 202/897/(3*x^2 - x + 2)^(3/2)
```

Fricas [A] time = 1.10805, size = 344, normalized size = 4.05

$$\frac{2\left(3174\sqrt{13}\left(9\,x^4-6\,x^3+13\,x^2-4\,x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+13\left(82008\,x^3-31482\,x^2+79077\,x\cdots\right.\right.}{3486639\left(9\,x^4-6\,x^3+13\,x^2-4\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3486639*(3174*sqrt(13)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-(4*sqrt(13
)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
 + 13*(82008*x^3 - 31482*x^2 + 79077*x - 32963)*sqrt(3*x^2 - x + 2))/(9*x^4
 - 6*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(5/2)), x)
```

Giac [A] time = 1.20905, size = 136, normalized size = 1.6

$$\frac{8}{2197}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2\left(3\left(6\left(4556x-1749\right)x+26359\right)x-32963\right)}{268203\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/2197*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt
(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))
) + 2/268203*(3*(6*(4556*x - 1749)*x + 26359)*x - 32963)/(3*x^2 - x + 2)^(3
/2)
```

3.256 $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{24(841-6633x)}{1162213\sqrt{3x^2-x+2}}-\frac{16\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}-\frac{56\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

```
[Out] (-2*(197 - 837*x))/(11661*(2 - x + 3*x^2)^(3/2)) - (24*(841 - 6633*x))/(116
2213*Sqrt[2 - x + 3*x^2]) - (16*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (56
*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rubi [A] time = 0.150562, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 806, 724, 206}

$$-\frac{24(841-6633x)}{1162213\sqrt{3x^2-x+2}}-\frac{16\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}-\frac{56\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```



```
[Out] (-2*(197 - 837*x))/(11661*(2 - x + 3*x^2)^(3/2)) - (24*(841 - 6633*x))/(116
2213*Sqrt[2 - x + 3*x^2]) - (16*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (56
*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
```

```
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{2226}{169}+\frac{462x}{13}+\frac{6696x^2}{169}}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} + \frac{4\int \frac{\frac{50784}{2197}+\frac{19044x}{2197}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx}{1587}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} - \frac{16\sqrt{2-x+3x^2}}{2197(1+2x)} + \frac{56\int \frac{1}{(1+2x)\sqrt{2-x+}}}{2197}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} - \frac{16\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{112\,\mathrm{Subst}\left(\int \frac{}{52}\right.}{2}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} - \frac{16\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{56\tanh^{-1}\left(\frac{}{2\sqrt{13}}\right.}{2197\sqrt{1}}$$

Mathematica [A] time = 0.0616853, size = 111, normalized size = 1.01

$$\frac{26\left(1318464x^4+133308x^3+1021566x^2+569989x-170239\right)-88872\sqrt{13}\sqrt{3x^2-x+2}\left(6x^3+x^2+3x+2\right)\tanh^{-1}\left(\frac{}{2}\right.}{45326307(2x+1)\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (26*(-170239 + 569989*x + 1021566*x^2 + 133308*x^3 + 1318464*x^4) - 88872*S
qrt[13]*Sqrt[2 - x + 3*x^2]*(2 + 3*x + x^2 + 6*x^3)*ArcTanh[(9 - 8*x)/(2*Sq
rt[13]*Sqrt[2 - x + 3*x^2])])/(45326307*(1 + 2*x)*(2 - x + 3*x^2)^(3/2))
```

Maple [A] time = 0.057, size = 165, normalized size = 1.5

$$\frac{-2+12\,x}{69}\left(3\,x^2-x+2\right)^{-\frac{3}{2}} + \frac{-16+96\,x}{529}\frac{1}{\sqrt{3\,x^2-x+2}} + \frac{7}{507}\left(3\,(x+1/2)^2-4\,x+\frac{5}{4}\right)^{-\frac{3}{2}} - \frac{-128+768\,x}{11661}\left(3\,(x+1/2)^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 2/69*(-1+6*x)/(3*x^2-x+2)^(3/2)+16/529*(-1+6*x)/(3*x^2-x+2)^(1/2)+7/507/(3*
(x+1/2)^2-4*x+5/4)^(3/2)-128/11661*(-1+6*x)/(3*(x+1/2)^2-4*x+5/4)^(3/2)-107
36/1162213*(-1+6*x)/(3*(x+1/2)^2-4*x+5/4)^(1/2)+28/2197/(3*(x+1/2)^2-4*x+5/
4)^(1/2)-56/28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16
*x+5)^(1/2))-1/26/(x+1/2)/(3*(x+1/2)^2-4*x+5/4)^(3/2)
```

Maxima [A] time = 1.48197, size = 169, normalized size = 1.54

$$\frac{56}{28561}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{146496\,x}{1162213\,\sqrt{3\,x^2-x+2}}-\frac{9604}{1162213\,\sqrt{3\,x^2-x+2}}+\frac{420\,x}{3887\left(3\,x^2-x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 56/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) + 146496/1162213*x/sqrt(3*x^2 - x + 2) - 9604/1162213/sqrt(3*x^2
- x + 2) + 420/3887*x/(3*x^2 - x + 2)^(3/2) - 1/13/(2*(3*x^2 - x + 2)^(3/2)
*x + (3*x^2 - x + 2)^(3/2)) - 49/11661/(3*x^2 - x + 2)^(3/2)
```

Fricas [A] time = 1.08474, size = 397, normalized size = 3.61

$$\frac{2\left(22218\sqrt{13}\left(18\,x^5-3\,x^4+20\,x^3+5\,x^2+4\,x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+13\left(1318464\,x^4+1333\right.}{45326307\left(18\,x^5-3\,x^4+20\,x^3+5\,x^2+4\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45326307*(22218*sqrt(13)*(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1)) + 13*(1318464*x^4 + 133308*x^3 + 1021566*x^2 + 569989*x - 17023
9)*sqrt(3*x^2 - x + 2))/(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2+3x+1}{(2x+1)^2\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4\,x^2+3\,x+1}{\left(3\,x^2-x+2\right)^{\frac{5}{2}}(2\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*x + 1)/((3*x^2 - x + 2)^(5/2)*(2*x + 1)^2), x)
```

3.257 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}-\frac{144\sqrt{3x^2-x+2}}{28561(2x+1)}-\frac{8\sqrt{3x^2-x+2}}{2197(2x+1)^2}+\frac{12(103526x+25771)}{15108769\sqrt{3x^2-x+2}}-\frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

```
[Out] (2*(2363 + 3693*x))/(151593*(2 - x + 3*x^2)^(3/2)) + (12*(25771 + 103526*x)
)/(15108769*Sqrt[2 - x + 3*x^2]) - (8*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)^
2) - (144*Sqrt[2 - x + 3*x^2])/(28561*(1 + 2*x)) - (2084*ArcTanh[(9 - 8*x)/
(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(28561*Sqrt[13])
```

Rubi [A] time = 0.212556, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}-\frac{144\sqrt{3x^2-x+2}}{28561(2x+1)}-\frac{8\sqrt{3x^2-x+2}}{2197(2x+1)^2}+\frac{12(103526x+25771)}{15108769\sqrt{3x^2-x+2}}-\frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(151593*(2 - x + 3*x^2)^(3/2)) + (12*(25771 + 103526*x)
)/(15108769*Sqrt[2 - x + 3*x^2]) - (8*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)^
2) - (144*Sqrt[2 - x + 3*x^2])/(28561*(1 + 2*x)) - (2084*ArcTanh[(9 - 8*x)/
(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(28561*Sqrt[13])
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
```

```
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{32433}{2197}+\frac{106830x}{2197}+\frac{160116x^2}{2197}+\frac{59088x^3}{2197}}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} + \frac{4\int \frac{\frac{1434648}{28561}+\frac{3345396x}{28561}+\frac{3097824x^2}{28561}}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx}{1587}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{2\int \frac{-\frac{2167842}{2197}-\frac{28\ldots}{\ldots}}{(1+2x)^2\sqrt{2-\ldots}}}{20631\ldots}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{144\sqrt{2-x+\ldots}}{28561(1+2\ldots}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{144\sqrt{2-x+\ldots}}{28561(1+2\ldots}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{144\sqrt{2-x+\ldots}}{28561(1+2\ldots}$$

Mathematica [A] time = 0.0764865, size = 89, normalized size = 0.66

$$\frac{2\left(20304864x^5+20074356x^4+19381992x^3+21890266x^2+10777477x+847141\right)}{45326307(2x+1)^2\left(3x^2-x+2\right)^{3/2}} - \frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(847141 + 10777477*x + 21890266*x^2 + 19381992*x^3 + 20074356*x^4 + 2030
4864*x^5))/(45326307*(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)) - (2084*ArcTanh[(9
- 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(28561*Sqrt[13])
```

Maple [A] time = 0.061, size = 148, normalized size = 1.1

$$-\frac{1}{104}\left(x+\frac{1}{2}\right)^{-2}\left(3\,(x+1/2)^2-4\,x+\frac{5}{4}\right)^{-\frac{3}{2}} - \frac{1}{338}\left(x+\frac{1}{2}\right)^{-1}\left(3\,(x+1/2)^2-4\,x+\frac{5}{4}\right)^{-\frac{3}{2}} + \frac{521}{13182}\left(3\,(x+1/2)^2-4\,x+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x)
```

```
[Out] -1/104/(x+1/2)^2/(3*(x+1/2)^2-4*x+5/4)^(3/2)-1/338/(x+1/2)/(3*(x+1/2)^2-4*x
+5/4)^(3/2)+521/13182/(3*(x+1/2)^2-4*x+5/4)^(3/2)+886/151593*(-1+6*x)/(3*(x
+1/2)^2-4*x+5/4)^(3/2)+188008/15108769*(-1+6*x)/(3*(x+1/2)^2-4*x+5/4)^(1/2)
+1042/28561/(3*(x+1/2)^2-4*x+5/4)^(1/2)-2084/371293*13^(1/2)*arctanh(2/13*(
9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A] time = 1.49437, size = 235, normalized size = 1.74

$$\frac{2084}{371293}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{1128048\,x}{15108769\sqrt{3\,x^2-x+2}}+\frac{363210}{15108769\sqrt{3\,x^2-x+2}}+\frac{1772\,x}{50531\,(3\,x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2084/371293*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/a
bs(2*x + 1)) + 1128048/15108769*x/sqrt(3*x^2 - x + 2) + 363210/15108769/sqr
t(3*x^2 - x + 2) + 1772/50531*x/(3*x^2 - x + 2)^(3/2) - 1/26/(4*(3*x^2 - x
+ 2)^(3/2)*x^2 + 4*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) - 1/169
/(2*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) + 10211/303186/(3*x^2
- x + 2)^(3/2)
```

Fricas [A] time = 1.19942, size = 460, normalized size = 3.41

$$\frac{2\left(826827\sqrt{13}\left(36\,x^6+12\,x^5+37\,x^4+30\,x^3+13\,x^2+12\,x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+13\left(2030486\right.}{589241991\left(36\,x^6+12\,x^5+37\,x^4+30\,x^3+13\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/589241991*(826827*sqrt(13)*(36*x^6 + 12*x^5 + 37*x^4 + 30*x^3 + 13*x^2 +
12*x + 4)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x
```

```
+ 185)/(4*x^2 + 4*x + 1)) + 13*(20304864*x^5 + 20074356*x^4 + 19381992*x^3
+ 21890266*x^2 + 10777477*x + 847141)*sqrt(3*x^2 - x + 2))/(36*x^6 + 12*x^5
 + 37*x^4 + 30*x^3 + 13*x^2 + 12*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^2 + 3x + 1}{(2x+1)^3 \left(3x^2 - x + 2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)), x)
```

Giac [B] time = 1.23155, size = 315, normalized size = 2.33

$$\frac{2084}{371293}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3x^2 - x + 2}\right|}{2\left(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3x^2 - x + 2}\right)}\right) + \frac{2\left(3\left(6\left(310578x - 26213\right)x + 1455755\right)x + 1634293\right)}{45326307\left(3x^2 - x + 2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 2084/371293*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4
*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x
+ 2))) + 2/45326307*(3*(6*(310578*x - 26213)*x + 1455755)*x + 1634293)/(3*x
^2 - x + 2)^(3/2) - 8/28561*(66*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 + 21*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1015*sqrt(3)*x + 431*sqrt(3) +
1015*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3
)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

3.258 $\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=208

$$\frac{(b+2cx)\left(-3b^2fh^2+6bceh^2+4c^2\left(fg^2-h(2dh+eg)\right)\right)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}+\frac{2\left(dh^2-egh+fg^2\right)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-\frac{\cdots}{ch^3\sqrt{-g\cdots}}$$

```
[Out] -(f/(c*h^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2])) + ((6*b*c*e*h^2 -
 3*b^2*f*h^2 + 4*c^2*(f*g^2 - h*(e*g + 2*d*h)))*(b + 2*c*x))/(3*c*h^2*(2*c*
g - b*h)^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2]) + (2*(f*g^2 - e*g*
h + d*h^2))/(3*h^3*(2*c*g - b*h)*(g + h*x)*Sqrt[-(g*(c*g - b*h)) + b*h^2*x
+ c*h^2*x^2])
```

Rubi [A] time = 0.424205, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.064, Rules used = {1638, 792, 613}

$$\frac{(b+2cx)\left(-3b^2fh^2+6bceh^2+4c^2\left(fg^2-h(2dh+eg)\right)\right)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}+\frac{2\left(dh^2-egh+fg^2\right)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-\frac{\cdots}{ch^3\sqrt{-g\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*x^2)^(
3/2)),x]
```

```
[Out] -(f/(c*h^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2])) + ((6*b*c*e*h^2 -
 3*b^2*f*h^2 + 4*c^2*(f*g^2 - h*(e*g + 2*d*h)))*(b + 2*c*x))/(3*c*h^2*(2*c*
g - b*h)^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2]) + (2*(f*g^2 - e*g*
h + d*h^2))/(3*h^3*(2*c*g - b*h)*(g + h*x)*Sqrt[-(g*(c*g - b*h)) + b*h^2*x
+ c*h^2*x^2])
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
```

```
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} - \frac{\int \frac{\frac{1}{2}h^3(bfg-2cdh)+\frac{1}{2}h^3(2cfg-2ceh+bfh)x}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}}{ch^4}$$

$$= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{2\left(fg^2-egh+d\right.}{3h^3(2cg-bh)(g+hx)\sqrt{-g(cg-}}$$

$$= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{\left(6bceh^2-3b^2fh^2+4c^2\left(fg^2-h\right.\right.}{3ch^2(2cg-bh)^3\sqrt{-g(cg-b}}$$

Mathematica [A] time = 0.519542, size = 219, normalized size = 1.05

$$\frac{2b^2h^2\left(f\left(8g^2+12ghx+3h^2x^2\right)-h(dh+2eg+3ehx)\right)-4bch\left(h\left(e\left(g^2+2ghx+3h^2x^2\right)-2dh(2g+hx)\right)+2fg^2(4g+5\right.}{3h^3(g+hx)(bh-2cg)^3\sqrt{(g+hx)(bh-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*
x^2)^(3/2)),x]
```

```
[Out] (2*b^2*h^2*(-(h*(2*e*g + d*h + 3*e*h*x)) + f*(8*g^2 + 12*g*h*x + 3*h^2*x^2)
) + 8*c^2*(f*g^2*(2*g^2 + 2*g*h*x - h^2*x^2) + h*(e*g*(g^2 + g*h*x + h^2*x^
2) + d*h*(-g^2 + 2*g*h*x + 2*h^2*x^2))) - 4*b*c*h*(2*f*g^2*(4*g + 5*h*x) +
h*(-2*d*h*(2*g + h*x) + e*(g^2 + 2*g*h*x + 3*h^2*x^2))))/(3*h^3*(-2*c*g + b
*h)^3*(g + h*x)*Sqrt[(g + h*x)*(-(c*g) + b*h + c*h*x)])
```

Maple [A] time = 0.06, size = 324, normalized size = 1.6

$$-\frac{(2chx+2bh-2cg)(-3b^2fh^4x^2+6bceh^4x^2-8c^2dh^4x^2-4c^2egh^3x^2+4c^2fg^2h^2x^2+3b^2eh^4x-12b^2fgh^3x-4bcdh^4\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x)
```

```
[Out] -2/3*(c*h*x+b*h-c*g)*(-3*b^2*f*h^4*x^2+6*b*c*e*h^4*x^2-8*c^2*d*h^4*x^2-4*c^
2*e*g*h^3*x^2+4*c^2*f*g^2*h^2*x^2+3*b^2*e*h^4*x-12*b^2*f*g*h^3*x-4*b*c*d*h^
4*x+4*b*c*e*g*h^3*x+20*b*c*f*g^2*h^2*x-8*c^2*d*g*h^3*x-4*c^2*e*g^2*h^2*x-8*
c^2*f*g^3*h*x+b^2*d*h^4+2*b^2*e*g*h^3-8*b^2*f*g^2*h^2-8*b*c*d*g*h^3+2*b*c*e
*g^2*h^2+16*b*c*f*g^3*h+4*c^2*d*g^2*h^2-4*c^2*e*g^3*h-8*c^2*f*g^4)/(b^3*h^3
-6*b^2*c*g*h^2+12*b*c^2*g^2*h-8*c^3*g^3)/h^3/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2
)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 134.36, size = 941, normalized size = 4.52

$$\frac{2\left(8c^2fg^4-b^2dh^4+4\left(c^2e-4bcf\right)g^3h-2\left(2c^2d+bce-4b^2f\right)g^2h^2+2\left(4bcd-b^2e\right)gh^3-\left(4c^2fg^2h^2-4c^2egh^3-\left(8c^2\cdots\right.\right.\right.}{3\left(8c^4g^6h^3-20bc^3g^5h^4+18b^2c^2g^4h^5-7b^3cg^3h^6+b^4g^2h^7-\left(8c^4g^3h^6-12bc^3g^2h^7+6b^2c^2gh^8-b^3ch^9\right)x^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="fricas")
```

```
[Out] 2/3*(8*c^2*f*g^4 - b^2*d*h^4 + 4*(c^2*e - 4*b*c*f)*g^3*h - 2*(2*c^2*d + b*c
*e - 4*b^2*f)*g^2*h^2 + 2*(4*b*c*d - b^2*e)*g*h^3 - (4*c^2*f*g^2*h^2 - 4*c^
2*e*g*h^3 - (8*c^2*d - 6*b*c*e + 3*b^2*f)*h^4)*x^2 + (8*c^2*f*g^3*h + 4*(c^
2*e - 5*b*c*f)*g^2*h^2 + 4*(2*c^2*d - b*c*e + 3*b^2*f)*g*h^3 + (4*b*c*d - 3
*b^2*e)*h^4)*x)*sqrt(c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)/(8*c^4*g^6*h^3 -
20*b*c^3*g^5*h^4 + 18*b^2*c^2*g^4*h^5 - 7*b^3*c*g^3*h^6 + b^4*g^2*h^7 - (8*
c^4*g^3*h^6 - 12*b*c^3*g^2*h^7 + 6*b^2*c^2*g*h^8 - b^3*c*h^9)*x^3 - (8*c^4*
g^4*h^5 - 4*b*c^3*g^3*h^6 - 6*b^2*c^2*g^2*h^7 + 5*b^3*c*g*h^8 - b^4*h^9)*x^
2 + (8*c^4*g^5*h^4 - 28*b*c^3*g^4*h^5 + 30*b^2*c^2*g^3*h^6 - 13*b^3*c*g^2*h
^7 + 2*b^4*g*h^8)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*h**2*x**2+b*h**2*x+b*g*h-c*g**2)**(3/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + ex + d}{\left(ch^2x^2 + bh^2x - cg^2 + bgh\right)^{\frac{3}{2}}\left(hx + g\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="giac")
```

```
[Out] integrate((f*x^2 + e*x + d)/((c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)^(3/2)*(h
*x + g)), x)
```

3.259 $\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=906

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{21c^2e}+\frac{2\sqrt{d+ex}\left(d\left(8Cd^2-3e(4Bd-7Ae\right.\right.}{}$$

```
[Out] (2*Sqrt[d + e*x]*(8*b^3*C*e^3 - 3*b*c*e^2*(b*C*d + 4*b*B*e - a*C*e) + c^3*d
*(8*C*d^2 - 3*e*(4*B*d - 7*A*e)) + 3*c^2*e*(a*e*(C*d - 5*B*e) - b*(C*d^2 -
2*B*d*e - 7*A*e^2)) + 3*c*e*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a*C*e)
 - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(315*c^3*e^
3) - (2*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)
)/(21*c^2*e) + (2*C*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*e) + (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(2*(4*c^2*d^2 - b^2*e^2 - (3*c*e*(b*d - 2*a*e))/2)*(
8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a*C*e) - c^2*(2*C*d^2 - 3*e*(B*d +
7*A*e))) - 5*c*e*(2*c*d - b*e)*(6*b^2*C*d*e + c*e*(21*A*c*d - 5*a*C*d - 3*a
*B*e) + b*(2*a*C*e^2 - c*d*(C*d + 9*B*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*
d^2 - b*d*e + a*e^2)*(8*b^3*C*e^3 - 3*c^2*e^2*(b*B*d + 2*a*C*d - 7*A*b*e -
10*a*B*e) + 3*b*c*e^2*(b*C*d - 4*b*B*e - 9*a*C*e) - 2*c^3*d*(8*C*d^2 - 3*e*
(4*B*d - 7*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[d + e*x]*Sqrt[a +
 b*x + c*x^2])
```

Rubi [A] time = 2.70038, antiderivative size = 905, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ =

0.206, Rules used = {1653, 832, 814, 843, 718, 424, 419}

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{21c^2e}+\frac{2\sqrt{d+ex}\left(\left(8Cd^3-3de(4Bd-7A\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(8*b^3*C*e^3 - 3*b*c*e^2*(b*C*d + 4*b*B*e - a*C*e) + c^3*(
8*C*d^3 - 3*d*e*(4*B*d - 7*A*e)) - 3*c^2*e*(b*C*d^2 - b*e*(2*B*d + 7*A*e) -
 a*e*(C*d - 5*B*e)) + 3*c*e*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a*C*e)
 - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(315*c^3*e^
3) - (2*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)
)/(21*c^2*e) + (2*C*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*e) - (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(6*b^2*C*d*e + 2*a*b*C*e^2 - b*
c*d*(C*d + 9*B*e) + c*e*(21*A*c*d - 5*a*C*d - 3*a*B*e)) - 2*(4*c^2*d^2 - b^
2*e^2 - (3*c*e*(b*d - 2*a*e))/2)*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a
*C*e) - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*
d^2 - b*d*e + a*e^2)*(8*b^3*C*e^3 - 3*c^2*e^2*(b*B*d + 2*a*C*d - 7*A*b*e -
10*a*B*e) + 3*b*c*e^2*(b*C*d - 4*b*B*e - 9*a*C*e) - 2*c^3*(8*C*d^3 - 3*d*e*
(4*B*d - 7*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[d + e*x]*Sqrt[a +
 b*x + c*x^2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
```

```
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
```

```
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)dx = \frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce} + \frac{2\int \sqrt{d+ex}\left(-\frac{3}{2}e(bCd-3Ace+aCe)\right.}{}$$

$$= -\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{21c^2e} + \frac{2C(d+ex)^{3/2}\left(a+\right.}{9ce}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3de(4Bd-7\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3de(4Bd-7\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3de(4Bd-7\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3de(4Bd-7\right.\right.}{}$$

Mathematica [C] time = 15.2902, size = 15669, normalized size = 17.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.426, size = 19955, normalized size = 22.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d + ex}\left(A + Bx + Cx^2\right)\sqrt{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.260 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=668

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce(-10aCe-7bBe+8bCd)+c^2\left(48Cd^2-14e(4Bd-\right.\right.}{105c^3e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*Sqrt[d + e*x]*(5*c*e*(3*b*C*d - 7*A*c*e + a*C*e) - (4*c*d - b*e)*(6*c*C
*d - 7*B*c*e + 4*b*C*e) + 3*c*e*(6*c*C*d - 7*B*c*e + 4*b*C*e)*x)*Sqrt[a + b
*x + c*x^2])/(105*c^2*e^3) + (2*C*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7
*c*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(3*b*C*d - 7*A*c*e
+ a*C*e) - (6*c*C*d - 7*B*c*e + 4*b*C*e)*(8*c^2*d^2 - 2*b^2*e^2 - 3*c*e*(b*
d - 2*a*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[
2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^
3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(4*b^2*C*e
^2 + c*e*(8*b*C*d - 7*b*B*e - 10*a*C*e) + c^2*(48*C*d^2 - 14*e*(4*B*d - 5*A
*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^3*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2
])
```

Rubi [A] time = 1.18846, antiderivative size = 668, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1653, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce(-10aCe-7bBe+8bCd)+c^2\left(48Cd^2-14e(4Bd-\right.\right.}{105c^3e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

```
[Out] (-2*Sqrt[d + e*x]*(5*c*e*(3*b*C*d - 7*A*c*e + a*C*e) - (4*c*d - b*e)*(6*c*C
*d - 7*B*c*e + 4*b*C*e) + 3*c*e*(6*c*C*d - 7*B*c*e + 4*b*C*e)*x)*Sqrt[a + b
*x + c*x^2])/(105*c^2*e^3) + (2*C*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7
*c*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(3*b*C*d - 7*A*c*e
+ a*C*e) - (6*c*C*d - 7*B*c*e + 4*b*C*e)*(8*c^2*d^2 - 2*b^2*e^2 - 3*c*e*(b*
d - 2*a*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[
2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^
3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(4*b^2*C*e
^2 + c*e*(8*b*C*d - 7*b*B*e - 10*a*C*e) + c^2*(48*C*d^2 - 14*e*(4*B*d - 5*A
*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^3*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2
])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
```

```
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}}\,dx = \frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce} + \frac{2\int \frac{\left(-\frac{1}{2}e(3bCd-7Ace+aCe)-\frac{1}{2}e(6cCd-7Bce+4bCe)x\right)\sqrt{a+bx+c}}{\sqrt{d+ex}}}{7ce^2}$$

$$= -\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6c}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6c}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6c}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6c}{105c^2e^3}$$

Mathematica [C] time = 14.4392, size = 9965, normalized size = 14.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.426, size = 12761, normalized size = 19.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + Bx + Cx^2)\sqrt{a + bx + cx^2}}{\sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/sqrt(d + e*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.261 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=749

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-ce\left(2ae(9Cd-5Be)-b\left(32Cd^2-5e(5Bd-3Ae)\right)\right)+bCe^2(bd-ae)-2\right.}{15c^2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*Sqrt[d + e*x]*(b*C*e^2*(b*d - a*e) + c^2*d*(24*C*d^2 - 5*e*(4*B*d - 3*A
*e)) + c*e*(a*e*(9*C*d - 5*B*e) - 5*b*(5*C*d^2 - 4*B*d*e + 3*A*e^2)) + 3*c*
e^2*(5*B*c*d + b*C*d - (6*c*C*d^2)/e - 5*A*c*e - a*C*e)*x)*Sqrt[a + b*x + c
*x^2])/(15*c*e^3*(c*d^2 - b*d*e + a*e^2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*S
qrt[b^2 - 4*a*c]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B*e - 6*a*C*e) - c^2*(48
*C*d^2 - 10*e*(4*B*d - 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)])/(15*c^2*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*C*e^2*(b*d -
a*e) - 2*c^2*d*(24*C*d^2 - 5*e*(4*B*d - 3*A*e)) - c*e*(2*a*e*(9*C*d - 5*B*e
) - b*(32*C*d^2 - 5*e*(5*B*d - 3*A*e))))*Sqrt[(c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
(-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^2*e^4*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.51845, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1650, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce\left(-2ae(9Cd-5Be)-5be(5Bd-3Ae)+32bCd^2\right)+bCe^2(bd-ae)-2c\right.}{15c^2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(b*C*e^2*(b*d - a*e) + c^2*(24*C*d^3 - 5*d*e*(4*B*d - 3*A
*e)) + c*e*(a*e*(9*C*d - 5*B*e) - 5*b*(5*C*d^2 - 4*B*d*e + 3*A*e^2)) + 3*c*
e^2*(5*B*c*d + b*C*d - (6*c*C*d^2)/e - 5*A*c*e - a*C*e)*x)*Sqrt[a + b*x + c
*x^2])/(15*c*e^3*(c*d^2 - b*d*e + a*e^2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*S
qrt[b^2 - 4*a*c]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B*e - 6*a*C*e) - c^2*(48
*C*d^2 - 10*e*(4*B*d - 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)])/(15*c^2*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*C*e^2*(b*d -
a*e) - 2*c^2*d*(24*C*d^2 - 5*e*(4*B*d - 3*A*e)) + c*e*(32*b*C*d^2 - 5*b*e*(
5*B*d - 3*A*e) - 2*a*e*(9*C*d - 5*B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
(-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^2*e^4*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd-2Ae)+e(Acd-aCd+aBe)}{2e}+\frac{1}{2}\left(5Bc\right.\right.}{\sqrt{d+}}}{cd^2-bde}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9Cd-5Be)-\right.\right.}{15ce^3\left(\right.}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9Cd-5Be)-\right.\right.}{15ce^3\left(\right.}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9Cd-5Be)-\right.\right.}{15ce^3\left(\right.}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9Cd-5Be)-\right.\right.}{15ce^3\left(\right.}$$

Mathematica [C] time = 14.1107, size = 13240, normalized size = 17.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.379, size = 8221, normalized size = 11.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^2*x^2 + 2
*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + Bx + Cx^2)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.262 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=712

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(e(-2aCe-3bBe+8bCd)-2c\left(8Cd^2-e(4Bd-Ae)\right)\right)\text{EllipticF}\left(\sin^{-1}\right.}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(e*(b*d - a*e)*(7*C*d - 3*B*e) - c*d*(8*C*d^2 - e*(4*B*d - A*e)) + e^2*
(B*c*d + b*C*d - (2*c*C*d^2)/e - A*c*e - a*C*e)*x)*Sqrt[a + b*x + c*x^2])/(
3*e^3*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (2*(C*d^2 - e*(B*d - A*e))*(
a + b*x + c*x^2)^(3/2))/(3*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + (Sq
rt[2]*Sqrt[b^2 - 4*a*c]*(2*(4*c*d - (b*e)/2)*(B*c*d + b*C*d - (2*c*C*d^2)/e
 - A*c*e - a*C*e) + 6*c*(b*d*(C*d - B*e) + e*(A*c*d - a*C*d + a*B*e)))*Sqrt
[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^3*(c*d^2 - b*d*e
+ a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b
*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*(8*b*C*d - 3*b*B*e - 2*a*C*e
) - 2*c*(8*C*d^2 - e*(4*B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^4*Sqrt[d
+ e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.27107, antiderivative size = 711, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1650, 812, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(e(-2aCe-3bBe+8bCd)-2c\left(8Cd^2-e(4Bd-Ae)\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\cdots}{\sqrt{b\cdots}}}}{\sqrt{\cdots}}\right.\right.}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

```
[Out] (2*((8*c*C*d^3)/e - c*d*(4*B*d - A*e) - (b*d - a*e)*(7*C*d - 3*B*e) - e*(B*
c*d + b*C*d - (2*c*C*d^2)/e - A*c*e - a*C*e)*x)*Sqrt[a + b*x + c*x^2])/(3*e
^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(3*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + (Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(2*(4*c*d - (b*e)/2)*(B*c*d + b*C*d - (2*c*C*d^2)/e -
A*c*e - a*C*e) + 6*c*(b*d*(C*d - B*e) + e*(A*c*d - a*C*d + a*B*e)))*Sqrt[d
+ e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^3*(c*d^2 - b*d*e + a
*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*(8*b*C*d - 3*b*B*e - 2*a*C*e) -
 2*c*(8*C*d^2 - e*(4*B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^4*Sqrt[d + e
*x]*Sqrt[a + b*x + c*x^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2\int \frac{\left(-\frac{3(bd(Cd-Be)+e(Acd-aCd+aBe))}{2e}+\frac{3}{2}\left(Bcd+bCd\right.\right.}{(d+ex)^{3/2}}}{3\left(cd^2-bde+\right.}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

Mathematica [C] time = 14.4505, size = 8456, normalized size = 11.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.452, size = 21038, normalized size = 29.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^3*x^3 + 3
*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx + Cx^2\right)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.263 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=992

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{2\left(c^2\left(24Cd^2-e(4Bd+Ae)\right)d^3-ce\left(bd\left(41Cd^2-6Bed+Ae^2\right)-ae\left(37Cd^2\right.\right.\right.}{}$$

```
[Out] (-2*(c^2*d^3*(24*C*d^2 - e*(4*B*d + A*e)) + e^2*(15*b^2*C*d^3 + 5*a^2*e^2*(
C*d + B*e) - a*b*e*(22*C*d^2 + 3*B*d*e + 2*A*e^2)) - c*d*e*(b*d*(41*C*d^2 -
 6*B*d*e + A*e^2) - a*e*(37*C*d^2 - 7*B*d*e + 7*A*e^2)) + e*(5*c^2*d^2*(6*C
*d^2 - e*(B*d + A*e)) + e^2*(15*a^2*C*e^2 - 5*a*b*e*(8*C*d - B*e) + b^2*(23
*C*d^2 - 3*B*d*e - 2*A*e^2)) - c*e*(5*b*d*(11*C*d^2 - 2*B*d*e - A*e^2) - a*
e*(53*C*d^2 - 13*B*d*e + 3*A*e^2)))*x)*Sqrt[a + b*x + c*x^2])/(15*e^3*(c*d^
2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x
 + c*x^2)^(3/2))/(5*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2)) + (Sqrt[2]*S
qrt[b^2 - 4*a*c]*(2*c^2*d^2*(24*C*d^2 - e*(4*B*d + A*e)) + e^2*(30*a^2*C*e^
2 - 5*a*b*e*(14*C*d - B*e) + b^2*(38*C*d^2 - 3*B*d*e - 2*A*e^2)) - c*e*(b*d
*(88*C*d^2 - 13*B*d*e - 2*A*e^2) - 2*a*e*(43*C*d^2 - 8*B*d*e + 3*A*e^2)))*S
qrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^4*(c*d^2 - b*d*
e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a
 + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(15*b*C*e^2*(b*d - a*e) + 2
*c^2*d*(24*C*d^2 - e*(4*B*d + A*e)) + c*e*(10*a*e*(5*C*d - B*e) - b*(64*C*d
^2 - 9*B*d*e - A*e^2)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c*e^4*(c*d^2 - b*d*e + a*
e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.91633, antiderivative size = 989, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ =

0.176, Rules used = {1650, 810, 843, 718, 424, 419}

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{2\left(\left(24Cd^5-d^3e(4Bd+Ae)\right)c^2-de\left(bd\left(41Cd^2-6Bed+Ae^2\right)-ae\left(37Cd^2-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c^2*(24*C*d^5 - d^3*e*(4*B*d + A*e)) + e^2*(15*b^2*C*d^3 + 5*a^2*e^2*(
C*d + B*e) - a*b*e*(22*C*d^2 + 3*B*d*e + 2*A*e^2)) - c*d*e*(b*d*(41*C*d^2 -
 6*B*d*e + A*e^2) - a*e*(37*C*d^2 - 7*B*d*e + 7*A*e^2)) + e^2*((30*c^2*C*d^
4)/e + 15*a^2*C*e^3 - 5*c^2*d^2*(B*d + A*e) - 5*a*b*e^2*(8*C*d - B*e) + a*c
*e*(53*C*d^2 - e*(13*B*d - 3*A*e)) - 5*b*c*d*(11*C*d^2 - e*(2*B*d + A*e)) +
 b^2*e*(23*C*d^2 - e*(3*B*d + 2*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(15*e^3*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
b*x + c*x^2)^(3/2))/(5*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2)) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 - 2*d^2*e*(4*B*d + A*e)) + e^2*(30*a^2*C
*e^2 - 5*a*b*e*(14*C*d - B*e) + b^2*(38*C*d^2 - 3*B*d*e - 2*A*e^2)) - c*e*(
b*d*(88*C*d^2 - 13*B*d*e - 2*A*e^2) - 2*a*e*(43*C*d^2 - 8*B*d*e + 3*A*e^2))
)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^4*(c*d^2 - b
*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqr
t[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(15*b*C*e^2*(b*d - a*e)
+ c^2*(48*C*d^3 - 2*d*e*(4*B*d + A*e)) - c*e*(64*b*C*d^2 - b*e*(9*B*d + A*e
) - 10*a*e*(5*C*d - B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c*e^4*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{7/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+2Ae)+5e(Acd-aCd+aBe)}{2e}+\frac{1}{2}\left(Bc\cdots\right.\right.}{(d+ex\cdots}}{5\left(cd^2-bd\cdots\right.}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\left(22Cd^2+\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\left(22Cd^2+\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\left(22Cd^2+\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\left(22Cd^2+\cdots\right.\right.\right.}{\cdots}$$

Mathematica [C] time = 14.864, size = 12997, normalized size = 13.1

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.566, size = 48427, normalized size = 48.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^4x^4 + 4\,de^3x^3 + 6\,d^2e^2x^2 + 4\,d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^4*x^4 + 4
*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.264 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=1363

result too large to display

```
[Out] (2*(2*c^3*d^3*(24*C*d^2 + e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 14*a*b
*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*e*(69
*C*d^2 + e*(15*B*d - 29*A*e)) - b*d*(128*C*d^2 + e*(19*B*d + 9*A*e))) + c*e
^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e)) + b^
2*d*(103*C*d^2 + e*(9*B*d + 19*A*e))))*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d
^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) - (2*(c^2*d^3*(24*C*d^2 + e*(4*B*d + 3
*A*e)) - e^2*(7*a^2*e^2*(C*d - 3*B*e) - b^2*d*(15*C*d^2 + 6*B*d*e + 8*A*e^2
) + a*b*e*(12*C*d^2 + 23*B*d*e + 12*A*e^2)) - c*d*e*(b*d*(43*C*d^2 + 6*B*d*
e + 15*A*e^2) - a*e*(33*C*d^2 + 9*B*d*e + 19*A*e^2)) + e*(7*c^2*d^2*(6*C*d^
2 + e*(B*d - 3*A*e)) + e^2*(35*a^2*C*e^2 - 7*a*b*e*(12*C*d - B*e) + b^2*(45
*C*d^2 - 3*B*d*e - 4*A*e^2)) + c*e*(a*e*(93*C*d^2 - 9*B*d*e - 5*A*e^2) - b*
(91*C*d^3 - 21*A*d*e^2)))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e
 + a*e^2)^2*(d + e*x)^(5/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)
^(3/2))/(7*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2)) - (Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(2*c^3*d^3*(24*C*d^2 + e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 1
4*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*
e*(69*C*d^2 + e*(15*B*d - 29*A*e)) - b*d*(128*C*d^2 + e*(19*B*d + 9*A*e)))
+ c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e))
 + b^2*d*(103*C*d^2 + e*(9*B*d + 19*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(2*c^2*d^2*(24*C*d^2 + e*(4*B*d + 3*A*e)) + c*e*(2*a*e*(
51*C*d^2 + e*(12*B*d - 5*A*e)) - b*d*(104*C*d^2 + 3*e*(5*B*d + 2*A*e))) + e
^2*(70*a^2*C*e^2 - 7*a*b*e*(18*C*d + B*e) + b^2*(60*C*d^2 + e*(3*B*d + 4*A*
e))))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x
]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 4.19999, antiderivative size = 1363, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ =

0.206, Rules used = {1650, 810, 834, 843, 718, 424, 419}

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}-\frac{2\left(\left(24Cd^5+e(4Bd+3Ae)d^3\right)c^2-de\left(bd\left(43Cd^2+6Bed+15Ae^2\right)-ae\left(33Cd\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

```
[Out] (2*(c^3*(48*C*d^5 + 2*d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 14*a*b
*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*e*(69
*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e))) + c*e
^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e)) + b^
2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d
^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) - (2*(c^2*(24*C*d^5 + d^3*e*(4*B*d + 3
*A*e)) - e^2*(7*a^2*e^2*(C*d - 3*B*e) - b^2*d*(15*C*d^2 + 6*B*d*e + 8*A*e^2
) + a*b*e*(12*C*d^2 + 23*B*d*e + 12*A*e^2)) - c*d*e*(b*d*(43*C*d^2 + 6*B*d*
e + 15*A*e^2) - a*e*(33*C*d^2 + 9*B*d*e + 19*A*e^2)) + e*(7*c^2*(6*C*d^4 +
d^2*e*(B*d - 3*A*e)) + e^2*(35*a^2*C*e^2 - 7*a*b*e*(12*C*d - B*e) + b^2*(45
*C*d^2 - 3*B*d*e - 4*A*e^2)) + c*e*(a*e*(93*C*d^2 - 9*B*d*e - 5*A*e^2) - b*
(91*C*d^3 - 21*A*d*e^2)))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e
 + a*e^2)^2*(d + e*x)^(5/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)
^(3/2))/(7*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2)) - (Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(2*c^3*(24*C*d^5 + d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 1
4*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*
e*(69*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e)))
+ c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e))
 + b^2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 + 2*d^2*e*(4*B*d + 3*A*e)) + c*e*(2*a*e*(
51*C*d^2 + e*(12*B*d - 5*A*e)) - b*(104*C*d^3 + 3*d*e*(5*B*d + 2*A*e))) + e
^2*(70*a^2*C*e^2 - 7*a*b*e*(18*C*d + B*e) + b^2*(60*C*d^2 + e*(3*B*d + 4*A*
e))))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x
]*Sqrt[a + b*x + c*x^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{7e\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+4Ae)+7e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(\cdots\right.\right.}{(d+\cdots}}{7\left(cd^2-\cdots\right.}$$

$$= -\frac{2\left(c^2\left(24Cd^5+d^3e(4Bd+3Ae)\right)-e^2\left(7a^2e^2(Cd-3Be)-b^2d\left(15Cd^2+6Bde\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)+b^2\left(15C\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)+b^2\left(15C\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)+b^2\left(15C\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)+b^2\left(15C\cdots\right.\right.\right.}{\cdots}$$

Mathematica [C] time = 16.486, size = 19853, normalized size = 14.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.738, size = 88790, normalized size = 65.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^5x^5 + 5\,de^4x^4 + 10\,d^2e^3x^3 + 10\,d^3e^2x^2 + 5\,d^4ex + d^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^5*x^5 + 5
*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.265 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=1904

result too large to display

```
[Out] (2*(2*c^3*d^3*(8*C*d^2 + e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a*e*(9*C*d^2 + 7
*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*c*e^2*(2*a^2*e^
2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) + b^2*d*(15*C*d^2
 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) + b^
2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 -
b*d*e + a*e^2)^3*(d + e*x)^(3/2)) + (2*(2*c^4*d^4*(8*C*d^2 + e*(4*B*d + 5*A
*e)) + 2*b^2*e^4*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) + b^2*(5*C*d^2 + 4
*B*d*e + 8*A*e^2)) - 6*c^2*e^2*(a*b*d*e*(30*C*d^2 - 5*B*d*e - 34*A*e^2) - a
^2*e^2*(30*C*d^2 - 36*B*d*e + 7*A*e^2) - b^2*d^2*(11*C*d^2 + 3*B*d*e + 11*A
*e^2)) - c*e^3*(126*a^3*C*e^3 - 3*a^2*b*e^2*(12*C*d + 29*B*e) - 6*a*b^2*e*(
5*C*d^2 + 7*B*d*e - 12*A*e^2) + b^3*d*(20*C*d^2 + 25*B*d*e + 56*A*e^2)) + c
^3*d^2*e*(6*a*e*(11*C*d^2 + 8*B*d*e - 34*A*e^2) - b*d*(56*C*d^2 + 5*e*(5*B*
d + 4*A*e))))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 - b*d*e + a*e^2)^4*Sqr
t[d + e*x]) - (2*(c^2*d^3*(8*C*d^2 + e*(4*B*d + 5*A*e)) - e^2*(3*a^2*e^2*(3
*C*d - 5*B*e) - a*b*e*(2*C*d^2 - 17*B*d*e - 10*A*e^2) - b^2*d*(5*C*d^2 + 4*
B*d*e + 8*A*e^2)) - c*d*e*(3*b*d*(5*C*d^2 + 2*B*d*e + 5*A*e^2) - a*e*(7*C*d
^2 + 11*B*d*e + 13*A*e^2)) + e*(3*c^2*d^2*(6*C*d^2 + e*(3*B*d - 5*A*e)) + c
*e*(a*e*(47*C*d^2 + B*d*e - 7*A*e^2) - 3*b*d*(15*C*d^2 + 2*B*d*e - 5*A*e^2)
) + e^2*(21*a^2*C*e^2 - 3*a*b*e*(16*C*d - B*e) + b^2*(25*C*d^2 - e*(B*d + 2
*A*e))))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e + a*e^2)^2*(d +
e*x)^(7/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(9*e*(c*d
^2 - b*d*e + a*e^2)*(d + e*x)^(9/2)) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c^4*d^
4*(8*C*d^2 + e*(4*B*d + 5*A*e)) + 2*b^2*e^4*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d
+ 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2*(a*b*d*e*(30*C*d^
2 - 5*B*d*e - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e + 7*A*e^2) - b^2*d^2
*(11*C*d^2 + 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3 - 3*a^2*b*e^2*(12*
C*d + 29*B*e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2) + b^3*d*(20*C*d^2
+ 25*B*d*e + 56*A*e^2)) + c^3*d^2*e*(6*a*e*(11*C*d^2 + 8*B*d*e - 34*A*e^2)
- b*d*(56*C*d^2 + 5*e*(5*B*d + 4*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)])/(315*e^4*(c*d^2 - b*d*e + a*e^2)^4*Sqrt[(c*(d + e*x))/
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(2*c^3*d^3*(8*C*d^2 + e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a*e
*(9*C*d^2 + 7*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*c*
e^2*(2*a^2*e^2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) + b^
2*d*(15*C*d^2 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d
```

```
+ 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[(c*(d + e*x))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*e
^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 6.24313, antiderivative size = 1904, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {1650, 810, 834, 843, 718, 424, 419}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

```
[Out] (2*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a*e*(9*C*d^2 + 7
*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*c*e^2*(2*a^2*e^
2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) + b^2*d*(15*C*d^2
 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) + b^
2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 -
b*d*e + a*e^2)^3*(d + e*x)^(3/2)) + (2*(2*c^4*(8*C*d^6 + d^4*e*(4*B*d + 5*A
*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d + 4*A*e) - 6*a*e*(11*C*d^2 +
8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) +
b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2*(a*b*d*e*(30*C*d^2 - 5*B*d*e
 - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e + 7*A*e^2) - b^2*d^2*(11*C*d^2
+ 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3 - 3*a^2*b*e^2*(12*C*d + 29*B*
e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2) + b^3*d*(20*C*d^2 + 25*B*d*e
+ 56*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 - b*d*e + a*e^2)^4*Sqr
t[d + e*x]) - (2*(c^2*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) - e^2*(3*a^2*e^2*(3
*C*d - 5*B*e) - a*b*e*(2*C*d^2 - 17*B*d*e - 10*A*e^2) - b^2*d*(5*C*d^2 + 4*
B*d*e + 8*A*e^2)) - c*d*e*(3*b*d*(5*C*d^2 + 2*B*d*e + 5*A*e^2) - a*e*(7*C*d
^2 + 11*B*d*e + 13*A*e^2)) + e^2*((3*c^2*(6*C*d^4 + d^2*e*(3*B*d - 5*A*e)))
/e + c*(a*e*(47*C*d^2 + e*(B*d - 7*A*e)) - 3*b*(15*C*d^3 + d*e*(2*B*d - 5*A
*e))) + e*(21*a^2*C*e^2 - 3*a*b*e*(16*C*d - B*e) + b^2*(25*C*d^2 - e*(B*d +
 2*A*e))))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e + a*e^2)^2*(d
+ e*x)^(7/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(9*e*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(9/2)) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c^4*
(8*C*d^6 + d^4*e*(4*B*d + 5*A*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d
+ 4*A*e) - 6*a*e*(11*C*d^2 + 8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2
 - 6*a*b*e*(3*C*d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2
*(a*b*d*e*(30*C*d^2 - 5*B*d*e - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e +
```

```
7*A*e^2) - b^2*d^2*(11*C*d^2 + 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3
- 3*a^2*b*e^2*(12*C*d + 29*B*e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2)
+ b^3*d*(20*C*d^2 + 25*B*d*e + 56*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(315*e^4*(c*d^2 - b*d*e + a*e^2)^4*Sqrt[(c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*
Sqrt[b^2 - 4*a*c]*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a
*e*(9*C*d^2 + 7*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*
c*e^2*(2*a^2*e^2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) +
b^2*d*(15*C*d^2 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*
d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[(c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sq
rt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315
*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{9e\left(cd^2-bde+ae^2\right)(d+ex)^{9/2}} - \frac{2\int \frac{\left(-\frac{3\left(bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)\right)}{2e}-\frac{3}{2}\left(Bc\right.}{(d+ex}}{9\left(cd^2-bc\right.}$$

$$= -\frac{2\left(c^2\left(8Cd^5+d^3e(4Bd+5Ae)\right)-e^2\left(3a^2e^2(3Cd-5Be)-abe\left(2Cd^2-17Bde-\right.\right.\right.}{}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-bd\left(16Cd\right.\right.\right.}{}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-bd\left(16Cd\right.\right.\right.}{}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-bd\left(16Cd\right.\right.\right.}{}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-bd\left(16Cd\right.\right.\right.}{}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-bd\left(16Cd\right.\right.\right.}{}$$

Mathematica [C] time = 19.2101, size = 29140, normalized size = 15.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 1.017, size = 153623, normalized size = 80.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{e^6x^6+6\,de^5x^5+15\,d^2e^4x^4+20\,d^3e^3x^3+15\,d^4e^2x^2+6\,d^5ex+d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^6*x^6 + 6
*d*e^5*x^5 + 15*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e*x +
```

```
d^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.266 $\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=724

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-ce(25aCe+28bBe+15bCd)+c^2\left(-\left(6Cd^2-7e(5A\right.\right.\right.}{105c^4e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*
e*(3*B*d + 5*A*e)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(105*c^3*e) - (2*(
2*c*C*d - 7*B*c*e + 6*b*C*e)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c^2
*e) + (2*C*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e) - (Sqrt[2]*Sqrt[b
^2 - 4*a*c]*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*
d*(6*C*d^2 - 7*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C
*d^2 + 91*B*d*e + 70*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(105*c^4*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
a*e^2)*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2
- 7*e*(3*B*d + 5*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^4*e^2*Sqrt[d + e*x]*Sq
rt[a + b*x + c*x^2])
```

Rubi [A] time = 1.77854, antiderivative size = 724, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1653, 832, 843, 718, 424, 419}

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)+c^2\left(-\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{105c^3e}-\frac{2\sqrt{2}\sqrt{b^2-4ac}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*
e*(3*B*d + 5*A*e)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(105*c^3*e) - (2*(
2*c*C*d - 7*B*c*e + 6*b*C*e)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c^2
*e) + (2*C*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e) - (Sqrt[2]*Sqrt[b
^2 - 4*a*c]*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*
(6*C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C
*d^2 + 91*B*d*e + 70*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(105*c^4*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
a*e^2)*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2
- 7*e*(3*B*d + 5*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^4*e^2*Sqrt[d + e*x]*Sq
rt[a + b*x + c*x^2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} + \frac{2\int \frac{(d+ex)^{3/2}\left(-\frac{1}{2}e(bCd-7Ace+5aCe)-\frac{1}{2}e(2cCd-7Bce+6bCe)x\right)}{\sqrt{a+bx+cx^2}}dx}{7ce^2}$$

$$= -\frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e} + \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} + \cdots$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d+ex}\sqrt{a\cdots}}{105c^3e}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d+ex}\sqrt{a\cdots}}{105c^3e}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d+ex}\sqrt{a\cdots}}{105c^3e}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d+ex}\sqrt{a\cdots}}{105c^3e}$$

Mathematica [C] time = 14.5728, size = 9972, normalized size = 13.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.438, size = 14084, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)(ex + d)^{\frac{3}{2}}}{\sqrt{cx^2 + bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cex^3 + (Cd + Be)x^2 + Ad + (Bd + Ae)x\right)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*e*x^3 + (C*d + B*e)*x^2 + A*d + (B*d + A*e)*x)*sqrt(e*x + d)/sq
rt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^{\frac{3}{2}} \left(A + Bx + Cx^2\right)}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.267 $\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=557

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(4bCe-5Bce+2cCd)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{15c^3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(2*c*C*d - 5*B*c*e + 4*b*C*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*e) + (2*C*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e) + (Sqrt[2]*Sqr
t[b^2 - 4*a*c]*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) - c^2*(2*C
*d^2 - 5*e*(B*d + 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(15*c^3*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d - 5*B*c*e +
4*b*C*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*e^2*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.888835, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1653, 832, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)+c^2\left(-\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)E\left(\sin^{-1}\left(\right.\right.}{15c^3e^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(2*c*C*d - 5*B*c*e + 4*b*C*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*e) + (2*C*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e) + (Sqrt[2]*Sqr
t[b^2 - 4*a*c]*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) - c^2*(2*C
*d^2 - 5*e*(B*d + 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(15*c^3*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d - 5*B*c*e +
4*b*C*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*e^2*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{2\int \frac{\sqrt{d+ex}\left(-\frac{1}{2}e(bCd-5Ace+3aCe)-\frac{1}{2}e(2cCd-5Bce+4bCe)x\right)}{\sqrt{a+bx+cx^2}}dx}{5ce^2}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{4\int$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{\left((2cC\right.}{}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{\left(\sqrt{2}\right.}{}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{\sqrt{2}\sqrt{}}{}$$

Mathematica [C] time = 11.7188, size = 992, normalized size = 1.78

$$\frac{\left(\frac{2(cCd+5Bce-4bCe)}{15c^2e}+\frac{2Cx}{5c}\right)\sqrt{d+ex}\left(cx^2+bx+a\right)}{\sqrt{a+x(b+cx)}} - \frac{2(d+ex)^{3/2}\sqrt{cx^2+bx+a}\left(\left(\left(2Cd^2-5e(Bd+3Ae)\right)c^2+e(3bCd+10b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (((2*(c*C*d + 5*B*c*e - 4*b*C*e))/(15*c^2*e) + (2*C*x)/(5*c))*Sqrt[d + e*x]
*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(d + e*x)^(3/2)*Sqrt[a + b*x
```

```
 + c*x^2]*((-8*b^2*C*e^2 + c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) + c^2*(2*C*d^
2 - 5*e*(B*d + 3*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) +
 (a*e)/(d + e*x)))/(d + e*x)) + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e
)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2
 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]
*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*
b*B*e + 9*a*C*e) + c^2*(-2*C*d^2 + 5*e*(B*d + 3*A*e)))*EllipticE[I*ArcSinh[
(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b
*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (8*b^3*C*e^3 - b^2*e^2*(11*c*C*d + 10*B*c
*e + 8*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*(c*d*Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d -
 5*B*e) - 15*A*c*e^2*(2*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(14*c*C*d +
10*B*c*e + 9*C*Sqrt[(b^2 - 4*a*c)*e^2])) + b*c*e*(15*A*c*e^2 - 17*a*C*e^2 +
 3*C*d*Sqrt[(b^2 - 4*a*c)*e^2] + 5*B*(3*c*d*e + 2*e*Sqrt[(b^2 - 4*a*c)*e^2]
)))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*
d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d +
e*x])))/(15*c^3*e^3*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d +
 e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] time = 0.361, size = 8161, normalized size = 14.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.268 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=471

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(Ce(bd-ae)+c\left(2Cd^2-3e(Bd-Ae)\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - (Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*c^2*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(C*e*(b*d - a*e
) + c*(2*C*d^2 - 3*e*(B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*e^2*Sqrt[d
+ e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.481855, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1653, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(Ce(bd-ae)-3ce(Bd-Ae)+2cCd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b}}{2cd-(b+}\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - (Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2)
```

```
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*c^2*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 + C*
e*(b*d - a*e) - 3*c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*e^2*Sqrt[d
+ e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
```

```
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} + \frac{2\int \frac{-\frac{1}{2}e(bCd-3Ace+aCe)-\frac{1}{2}e(2cCd-3Bce+2bCe)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3ce^2}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{(2cCd-3Bce+2bCe)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{3ce^2} + \frac{\left(2cCd^2+Ce(bd-\right.}{}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right.}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a}}$$

Mathematica [C] time = 12.5869, size = 1080, normalized size = 2.29

$$\frac{\sqrt{cx^2+bx+a}\left(-4(2cCd-3Bce+2bCe)\sqrt{\frac{cd^2+e(ae-bd)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}\left(c\left(\frac{d}{d+ex}-1\right)^2+\frac{e\left(-\frac{db}{d+ex}+b+\frac{ae}{d+ex}\right)}{d+ex}\right)+\frac{i\sqrt{2}(2cCd-3Bce+2bCe)\left(2cd-be+\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*C*Sqrt[d + e*x]*(a + b*x + c*x^2))/(3*c*e*Sqrt[a + x*(b + c*x)]) + ((d +
 e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[(c
*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-1 +
 d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) +
(I*Sqrt[2]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*
e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(
d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(
d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b
*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] -
(I*Sqrt[2]*(-2*b^2*C*e^2 + b*e*(3*B*c*e + 2*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*
(-6*A*c*e^2 + 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d - 3*B*e)))*Sqrt[(S
qrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b
*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(S
qrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b
*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Ellipti
cF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^
2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2
])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(6*c^2*e^3*Sq
rt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqr
t[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d
)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] time = 0.404, size = 4251, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)

[Out] -1/3/c^2*(-2*C*a*c*d*e^2-2*C*x^2*c^2*d*e^2-2*C*x*a*c*e^3-4*C*2^(1/2)*(-(e*x
+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/
2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(
-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c
*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d^3+3*C*2^(1/2)*(-(e*x+d)*c/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b
*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4
*a*c+b^2)^(1/2)))^(1/2))*a*b*e^3+6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c
+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1
/2)))^(1/2))*c^2*d^2*e-2*C*x^2*b*c*e^3-4*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(
-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b
^2)^(1/2)))^(1/2))*a*b*e^3+4*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*
e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^
(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^
(1/2))*b^2*d*e^2+3*A*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1
/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)
*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4
*a*c+b^2)^(1/2)*c*e^3+3*A*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)
))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)
,(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)
)*b*c*e^3-6*A*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(
```

e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipt
icF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d*e^2-
6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*
x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(
-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2
)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^3-3*C*2^(1/2)*
(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^
(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*
c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e^2+6*B*2^(1/2)*(-(e*x+d)*
c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*
(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b
e+e(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^3-C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(
-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b
^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*e^3-2*C*x*b*c*d*e^2-2*C*x^3*c^2*e^3
-3*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*
c*d*e^2+2*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b
+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Elliptic
F(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)
^(1/2)*c*d^2*e-4*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1
/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2
)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*E
llipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(
-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d
*e^2-6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2
^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*e^2+C*2^(
1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a

```
*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+
b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*
x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2+
6*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*
x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(
-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2
)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e^2+3*B*2^(1/2
)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+
b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2
)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d
)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*e^2)*(e*x+d)^(1/2)*(c*x^
2+b*x+a)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)/e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{cex^3+(cd+be)x^2+ad+(bd+ae)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e*x^3 + (
c*d + b*e)*x^2 + a*d + (b*d + a*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.269 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=508

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{b\cdots}}{\cdots}$$

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(e*(c*d^2 - b*d*e + a*e^
2)*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(C*e*(b*d - a*e) - c*(2*C*d^
2 - e*(B*d - A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(c*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*C*d -
B*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.648922, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1650, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-Ce(bd-ae)-ce(Bd-Ae)+2cCd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(e*(c*d^2 - b*d*e + a*e^
2)*Sqrt[d + e*x]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - C*e*(b*d - a*e)
```

```
 - c*e*(B*d - A*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(c*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*C*d -
B*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{-\frac{bd(Cd-Be)+e(Acd-aCd+aBe)}{2e}+\frac{1}{2}\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-\cdots\right.}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}}{cd^2-bde+ae^2} \cdots$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{e^2} - \frac{\left(Bcd+bC\cdots\right.}{\cdots}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-a\cdots\right.\right.}{ce\left(cd^2-b\cdots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aC\cdots\right.}{ce\left(cd^2-bde\cdots\right.}$$

Mathematica [C] time = 7.38258, size = 772, normalized size = 1.52

$$2\left(-\frac{i(d+ex)^{3/2}\sqrt{1-\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}-be+2cd\right)}}\sqrt{\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}+be-2cd\right)}+1}\left(\left(b\left(Cde\sqrt{e^2\left(b^2-4ac\right)}+aCe^3+Ace^3+Bcde^2\right)-Ace^2\left(\sqrt{e^2\left(b^2-4ac\right)}+2cd\right)+Bcde\sqrt{e^2\left(\cdots\right.}\right.\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*(-(e^2*(C*d^2 + e*(-(B*d) + A*e))*(a + x*(b + c*x))) + (e^2*(2*c*C*d^2 +
 C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*(a + x*(b + c*x)))/c - ((I/2)*(d
+ e*x)^(3/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(
b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*(2*c*C*d^2 + C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*Ellip
ticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-(b^2*C*d*e^2) + 2*a*c*C*
d*e^2 - 2*a*B*c*e^3 - 2*c*C*d^2*Sqrt[(b^2 - 4*a*c)*e^2] + B*c*d*e*Sqrt[(b^2
 - 4*a*c)*e^2] - a*C*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - A*c*e^2*(2*c*d + Sqrt[(b
^2 - 4*a*c)*e^2]) + b*(B*c*d*e^2 + A*c*e^3 + a*C*e^3 + C*d*e*Sqrt[(b^2 - 4*
a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*
c*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
)))/(e^3*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.376, size = 6053, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{ce^2x^4 + \left(2\,cde + be^2\right)x^3 + ad^2 + \left(cd^2 + 2\,bde + ae^2\right)x^2 + \left(bd^2 + 2\,ade\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^2*x^4 +
 (2*c*d*e + b*e^2)*x^3 + a*d^2 + (c*d^2 + 2*b*d*e + a*e^2)*x^2 + (b*d^2 + 2
*a*d*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{\frac{3}{2}}\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.270 $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=684

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(3Ce(bd-ae)-c\left(e(Bd-Ae)+2Cd^2\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(3/2)) + (2*(c*d*(2*C*d^2 + e*(B*d - 4*A*e)) + e*(3*a*e*(2*C
*d - B*e) - b*(4*C*d^2 - B*d*e - 2*A*e^2)))*Sqrt[a + b*x + c*x^2])/(3*e*(c*
d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d*(2*
C*d^2 + e*(B*d - 4*A*e)) + e*(3*a*e*(2*C*d - B*e) - b*(4*C*d^2 - B*d*e - 2*
A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*e^2*(c*d
^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(3*C*e*(b*d - a*e)
 - c*(2*C*d^2 + e*(B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^2*(c*d^2 - b*d
*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.19205, antiderivative size = 680, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1650, 834, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2}{2cd-(\ldots}\right.}{3ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(3/2)) + (2*(2*c*C*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*
d - B*e) - b*e*(4*C*d^2 - e*(B*d + 2*A*e)))*Sqrt[a + b*x + c*x^2])/(3*e*(c*
d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d
^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d +
2*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*e^2*(c*d
^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - 3*C*e
*(b*d - a*e) + c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^2*(c*d^2 - b*d
*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2\int \frac{-\frac{bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(Bcd-3bCd+\frac{2cCd^2}{e}\right.}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}}{3\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-\right.}{3e\left(cd^2-bde+a\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-\right.}{3e\left(cd^2-bde+a\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-\right.}{3e\left(cd^2-bde+a\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-\right.}{3e\left(cd^2-bde+a\right.}$$

Mathematica [C] time = 12.2044, size = 1194, normalized size = 1.75

$$\frac{2\sqrt{cx^2+bx+a}\left(\frac{i\sqrt{1-\frac{2\left(cd^2+e(ae-bd)\right)}{\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}}\sqrt{\frac{2\left(cd^2+e(ae-bd)\right)}{\left(-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}+1}\left(\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)\left(cd\left(2Cd^2+e(Bd-4Ae)\right)+e\left(-4bCd^2+be(Bd+2Ae)-\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(3*e*(c*d^2
- b*d*e + a*e^2)*(d + e*x)^2) - (2*(-2*c*C*d^3 - B*c*d^2*e + 4*b*C*d^2*e -
b*B*d*e^2 + 4*A*c*d*e^2 - 6*a*C*d*e^2 - 2*A*b*e^3 + 3*a*B*e^3))/(3*e*(c*d^2
 - b*d*e + a*e^2)^2*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)
*Sqrt[a + b*x + c*x^2]*(-((2*c*C*d^3 + c*d*e*(B*d - 4*A*e) - 3*a*e^2*(-2*C*
d + B*e) + b*e*(-4*C*d^2 + e*(B*d + 2*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(
b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x))) + ((I/2)*Sqrt[1 - (2*(c
*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x
))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(c*d*(2*C*d
^2 + e*(B*d - 4*A*e)) + e*(-4*b*C*d^2 + b*e*(B*d + 2*A*e) - 3*a*e*(-2*C*d +
 B*e)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d +
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - (2*a*c*C*d^2*
e^2 - 8*a*B*c*d*e^3 - 6*a^2*C*e^4 + 2*c*C*d^3*Sqrt[(b^2 - 4*a*c)*e^2] + B*c
*d^2*e*Sqrt[(b^2 - 4*a*c)*e^2] + 6*a*C*d*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - 3*a*
B*e^3*Sqrt[(b^2 - 4*a*c)*e^2] + 2*A*c*e^2*(-3*c*d^2 + a*e^2 - 2*d*Sqrt[(b^2
 - 4*a*c)*e^2]) - b^2*e^2*(2*C*d^2 + e*(B*d + 2*A*e)) + b*e*(2*A*e^2*(3*c*d
 + Sqrt[(b^2 - 4*a*c)*e^2]) + 2*C*d*(3*a*e^2 - 2*d*Sqrt[(b^2 - 4*a*c)*e^2])
 + B*e*(3*c*d^2 + 3*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d -
 b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e)
)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(3*e^3*(c*d^2
- b*d*e + a*e^2)^2*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d +
e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] time = 0.463, size = 20481, normalized size = 29.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}(ex + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{ce^3x^5+\left(3\,cde^2+be^3\right)x^4+ad^3+\left(3\,cd^2e+3\,bde^2+ae^3\right)x^3+\left(cd^3+3\,bd^2e+3\,ade^2\right)x^2+\left(bd^3+3\,ad^2e\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^3*x^5 +
 (3*c*d*e^2 + b*e^3)*x^4 + a*d^3 + (3*c*d^2*e + 3*b*d*e^2 + a*e^3)*x^3 + (c
*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^2 + (b*d^3 + 3*a*d^2*e)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.271 $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=944

$$-\frac{2\sqrt{cx^2+bx+a}\left(Cd^2-e(Bd-Ae)\right)}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(c^2\left(2Cd^2+e(3Bd-23Ae)\right)d^2-e^2\left(\left(3Cd^2+2Bed+8Ae^2\right)b^2-10\right.\right.}{}$$

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(5*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(5/2)) + (2*(c*d*(2*C*d^2 + e*(3*B*d - 8*A*e)) + e*(5*a*e*(2
*C*d - B*e) - b*(6*C*d^2 - B*d*e - 4*A*e^2)))*Sqrt[a + b*x + c*x^2])/(15*e*
(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) + (2*(c^2*d^2*(2*C*d^2 + e*(3*B*
d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*
B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^2
 - 29*B*d*e + 9*A*e^2)))*Sqrt[a + b*x + c*x^2])/(15*e*(c*d^2 - b*d*e + a*e^
2)^3*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*d^2*(2*C*d^2 + e*(3*B
*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2
*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^
2 - 29*B*d*e + 9*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*d*(2*C*d^2 + e*(3*B*d - 8*A*e)) + e*(5*a*e*(2*C*d - B*e) - b*(6*C*d^2 -
B*d*e - 4*A*e^2)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*
e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^2*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 2.25575, antiderivative size = 942, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ =

0.176, Rules used = {1650, 834, 843, 718, 424, 419}

$$-\frac{2\sqrt{cx^2+bx+a}\left(Cd^2-e(Bd-Ae)\right)}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e\left(bd\left(7Cd^2-7Bed-23Ae^2\right)-a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(5*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(5/2)) + (2*(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*
C*d - B*e) - b*e*(6*C*d^2 - e*(B*d + 4*A*e)))*Sqrt[a + b*x + c*x^2])/(15*e*
(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) + (2*(c^2*(2*C*d^4 + d^2*e*(3*B*
d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*
B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^2
 - 29*B*d*e + 9*A*e^2)))*Sqrt[a + b*x + c*x^2])/(15*e*(c*d^2 - b*d*e + a*e^
2)^3*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*(2*C*d^4 + d^2*e*(3*B
*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2
*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^
2 - 29*B*d*e + 9*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*C*d - B*e) - b*e*(6*C*d^2 -
 e*(B*d + 4*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*
e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^2*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{2\int \frac{-\frac{bCd^2-be(Bd+4Ae)+5e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(3Bcd-5bCd+\frac{2c\cdots}{\cdots}\right.}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}}{5\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2Cd-Be\cdots\right.}{15e\left(cd^2-bde+\cdots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2Cd-Be\cdots\right.}{15e\left(cd^2-bde+\cdots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2Cd-Be\cdots\right.}{15e\left(cd^2-bde+\cdots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2Cd-Be\cdots\right.}{15e\left(cd^2-bde+\cdots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2Cd-Be\cdots\right.}{15e\left(cd^2-bde+\cdots\right.}$$

Mathematica [C] time = 15.2817, size = 12295, normalized size = 13.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.628, size = 46695, normalized size = 49.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}(ex + d)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{ce^4x^6 + \left(4\,cde^3 + be^4\right)x^5 + ad^4 + \left(6\,cd^2e^2 + 4\,bde^3 + ae^4\right)x^4 + 2\left(2\,cd^3e + 3\,bd^2e^2 + 2\,ade^3\right)x^3 + \left(cd^4 + 4\,bd^3e + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^4*x^6 +
 (4*c*d*e^3 + b*e^4)*x^5 + a*d^4 + (6*c*d^2*e^2 + 4*b*d*e^3 + a*e^4)*x^4 +
2*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^3 + (c*d^4 + 4*b*d^3*e + 6*a*d^2*
e^2)*x^2 + (b*d^4 + 4*a*d^3*e)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.272 $\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=510

$$\frac{(g+hx)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\ldots}\right.}{ch^3(m+1)(m+2p+3)}$$

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) + ((f*h
*(b*g - a*h)*(1 + m) + c*(2*f*g^2*(1 + p) - h*(e*g - d*h)*(3 + m + 2*p)))*(
g + h*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*(g
 + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b +
 Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(1 + m)*(3 + m + 2*p)*(1 - (2*c*(g + h*x))/
(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + S
qrt[b^2 - 4*a*c])*h))^p) - ((b*f*h*(2 + m + p) + c*(2*f*g*(1 + p) - e*h*(3
+ m + 2*p)))*(g + h*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p, -p,
3 + m, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))
/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(2 + m)*(3 + m + 2*p)*(1 - (2
*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(
2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p)
```

Rubi [A] time = 0.833057, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1653, 843, 759, 133}

$$\frac{(g+hx)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\ldots}\right.}{ch^3(m+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) + ((f*h
*(b*g - a*h)*(1 + m) + 2*c*f*g^2*(1 + p) - c*h*(e*g - d*h)*(3 + m + 2*p))*(
g + h*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*(g
 + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b +
 Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(1 + m)*(3 + m + 2*p)*(1 - (2*c*(g + h*x))/
(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + S
qrt[b^2 - 4*a*c])*h))^p) - ((2*c*f*g*(1 + p) + b*f*h*(2 + m + p) - c*e*h*(3
```

```
 + m + 2*p))*(g + h*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p, -p,
3 + m, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))
/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(2 + m)*(3 + m + 2*p)*(1 - (2
*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(
2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p)
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\int (g+hx)^m(-h(afh(1+m)+bfg(1}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{(2cfg(1+p)+bfh(2+m+p)-ceh}{ch^2}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{\Big((2cfg(1+p)+bfh(2+m+p)-ce}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(fh(bg-ah)(1+m)+2cfg^2(1+p)\right.}{}$$

Mathematica [F] time = 2.31765, size = 0, normalized size = 0.

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F] time = 1.428, size = 0, normalized size = 0.

$$\int \left(hx+g\right)^m \left(cx^2+bx+a\right)^p \left(fx^2+ex+d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

```
[Out] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

3.273 $\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=496

$$\frac{\sqrt{a+bx+cx^2}(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)\left(fh(m+1)(bg-ah)+c\left(3fg^2-h\right.\right.}{ch^3(m+1)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*h*(4 + m)) + ((f*h*(b*g -
a*h)*(1 + m) + c*(3*f*g^2 - h*(e*g - d*h)*(4 + m)))*(g + h*x)^(1 + m)*Sqrt[
a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(g + h*x))/(2*c*g
- (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*
c])*h)])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt
[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c]
)*h)]) - ((b*f*h*(5 + 2*m) + c*(6*f*g - 2*e*h*(4 + m)))*(g + h*x)^(2 + m)*S
qrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(g + h*x))/(2*
c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])/(2*c*h^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b
- Sqrt[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])
```

Rubi [A] time = 0.672379, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1653, 843, 759, 133}

$$\frac{\sqrt{a+bx+cx^2}(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)\left(fh(m+1)(bg-ah)-ch(m+4)(\right.}{ch^3(m+1)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 +
 f*h*(b*g - a*h)*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[
a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(g + h*x))/(2*c*g
- (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*
```

```
c])*h)])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt
[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c]
)*h)]) - ((6*c*f*g - 2*c*e*h*(4 + m) + b*f*h*(5 + 2*m))*(g + h*x)^(2 + m)*S
qrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(g + h*x))/(2*
c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])/(2*c*h^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b
- Sqrt[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)}+\frac{\int (g+hx)^m\left(-\frac{1}{2}h(3bfg+2afh(1+m$$

$$=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)}+\frac{\left(3cfg^2+fh(bg-ah)(1+m)-ch(eg}{ch}$$

$$=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)}+\frac{\left(\left(3cfg^2+fh(bg-ah)(1+m)-ch(eg\right.\right.}{}$$

$$=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)}+\frac{\left(3cfg^2+fh(bg-ah)(1+m)-ch(eg\right.}{}$$

Mathematica [F] time = 1.46647, size = 0, normalized size = 0.

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2), x]
```

Maple [F] time = 1.338, size = 0, normalized size = 0.

$$\int \left(hx+g\right)^m\left(fx^2+ex+d\right)\sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}\left(fx^2 + ex + d\right)\left(hx + g\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}\left(fx^2 + ex + d\right)\left(hx + g\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}\left(fx^2 + ex + d\right)\left(hx + g\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

3.274 $\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=590

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\ \frac{\cdots}{2cg \cdots}\right.}{2h^3p}$$

```
[Out] -((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(1 + p))/(2*h*(c*g^2 - b*g*h +
a*h^2)*(1 + p)*(g + h*x)^(2*(1 + p))) - (f*(a + b*x + c*x^2)^p*AppellF1[-2*
p, -p, -p, 1 - 2*p, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2
*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(2*h^3*p*(g + h*x)^(2*p
)*(1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g
+ h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p) - ((2*c*(f*g^3 - d*g*h^2) +
 h*(2*a*h*(2*f*g - e*h) - b*(3*f*g^2 - e*g*h - d*h^2)))*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x)*(g + h*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2F1[-1
- 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(g + h*x))/((2*c*g - (b + Sqrt[b^2
 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*h^2*(2*c*g - (b - Sqrt[
b^2 - 4*a*c])*h)*(c*g^2 - b*g*h + a*h^2)*(1 + 2*p)*(((2*c*g - (b - Sqrt[b^2
 - 4*a*c])*h)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*g - (b + Sqrt[b^2 - 4*
a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rubi [A] time = 0.755247, antiderivative size = 588, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1655, 759, 133, 806, 726}

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\ \frac{\cdots}{2cg \cdots}\right.}{2h^3p}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(1 + p))/(2*h*(c*g^2 - b*g*h +
a*h^2)*(1 + p)*(g + h*x)^(2*(1 + p))) - (f*(a + b*x + c*x^2)^p*AppellF1[-2*
p, -p, -p, 1 - 2*p, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2
*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(2*h^3*p*(g + h*x)^(2*p
)*(1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g
```

```
+ h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p) - ((2*c*(f*g^3 - d*g*h^2) -
 h*(3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h)))*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x)*(g + h*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2F1[-1
- 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(g + h*x))/((2*c*g - (b + Sqrt[b^2
 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*h^2*(2*c*g - (b - Sqrt[
b^2 - 4*a*c])*h)*(c*g^2 - b*g*h + a*h^2)*(1 + 2*p)*(((2*c*g - (b - Sqrt[b^2
 - 4*a*c])*h)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*g - (b + Sqrt[b^2 - 4*
a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rule 1655

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d
 + e*x)^(m + q)*(a + b*x + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(
a + b*x + c*x^2)^p*ExpandToSum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x]
, x]] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ[m, 0] && RationalQ[a, b, c
, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)dx=\frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)\left(a+bx+cx^2\right)^p dx}{h^2}+\cdots$$

$$=-\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}-\frac{\left(2c\left(fg^3-\cdots\right.\right.}{\cdots}$$

$$=-\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}-\frac{f(g+hx)^{-2\cdots}}{\cdots}$$

Mathematica [F] time = 3.63639, size = 0, normalized size = 0.

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F] time = 1.356, size = 0, normalized size = 0.

$$\int \left(hx+g\right)^{-3-2p} \left(cx^{2}+bx+a\right)^{p} \left(f\,x^{2}+ex+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

```
[Out] int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^{2}+ex+d\right)\left(cx^{2}+bx+a\right)^{p}\left(hx+g\right)^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxi
ma")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(fx^{2}+ex+d\right)\left(cx^{2}+bx+a\right)^{p}\left(hx+g\right)^{-2\,p-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fric
as")
```

```
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac
")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

3.275 $\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) d$

Optimal. Leaf size=41

$$\frac{bf(2p+3)\left(d+fx^2\right)^{p+1}}{p+1}+2cfx\left(d+fx^2\right)^{p+1}$$

```
[Out] (b*f*(3 + 2*p)*(d + f*x^2)^(1 + p))/(1 + p) + 2*c*f*x*(d + f*x^2)^(1 + p)
```

Rubi [A] time = 0.0515075, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1815, 12, 261}

$$\frac{bf(2p+3)\left(d+fx^2\right)^{p+1}}{p+1}+2cfx\left(d+fx^2\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x
]
```

```
[Out] (b*f*(3 + 2*p)*(d + f*x^2)^(1 + p))/(1 + p) + 2*c*f*x*(d + f*x^2)^(1 + p)
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx = 2cfx\left(d+fx^2\right)^{1+p}+\frac{\int 2bf^3(3+2p)^2x\left(d+fx^2\right)^p\,dx}{f(3+2p)}$$
$$=2cfx\left(d+fx^2\right)^{1+p}+\left(2bf^2(3+2p)\right)\int x\left(d+fx^2\right)^p\,dx$$
$$=\frac{bf(3+2p)\left(d+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+fx^2\right)^{1+p}$$

Mathematica [C] time = 0.0946046, size = 119, normalized size = 2.9

$$\frac{f\left(d+fx^2\right)^p\left(\frac{fx^2}{d}+1\right)^{-p}\left((2p+3)\left(3b\left(d+fx^2\right)\left(\frac{fx^2}{d}+1\right)^p+2cf(p+1)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{fx^2}{d}\right)\right)+6cd(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\right.\right.}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*
x^2),x]
```

```
[Out] (f*(d + f*x^2)^p*(6*c*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((f*x^2)
/d)] + (3 + 2*p)*(3*b*(d + f*x^2)*(1 + (f*x^2)/d)^p + 2*c*f*(1 + p)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((f*x^2)/d)])))/(3*(1 + p)*(1 + (f*x^2)/d)^p)
```

Maple [A] time = 0.044, size = 36, normalized size = 0.9

$$\frac{f\left(fx^2+d\right)^{1+p}\left(2\,cxp+2\,bp+2\,cx+3\,b\right)}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x)
```

```
[Out] f*(f*x^2+d)^(1+p)*(2*c*p*x+2*b*p+2*c*x+3*b)/(1+p)
```

Maxima [A] time = 1.11117, size = 80, normalized size = 1.95

$$\frac{\left(2\,cf^2(p+1)x^3+bf^2(2\,p+3)x^2+2\,cdf(p+1)x+bdf(2\,p+3)\right)\left(fx^2+d\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*f^2*(2*p + 3)*x^2 + 2*c*d*f*(p + 1)*x + b*d*f*(2*p
 + 3))*(f*x^2 + d)^p/(p + 1)
```

Fricas [A] time = 1.39543, size = 166, normalized size = 4.05

$$\frac{\left(2\,bdfp+2\left(cf^2p+cf^2\right)x^3+3\,bdf+\left(2\,bf^2p+3\,bf^2\right)x^2+2\left(cdfp+cdf\right)x\right)\left(fx^2+d\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="fricas")
```

```
[Out] (2*b*d*f*p + 2*(c*f^2*p + c*f^2)*x^3 + 3*b*d*f + (2*b*f^2*p + 3*b*f^2)*x^2
+ 2*(c*d*f*p + c*d*f)*x)*(f*x^2 + d)^p/(p + 1)
```

Sympy [B] time = 9.56548, size = 221, normalized size = 5.39

$$\begin{cases} \frac{2bdfp(d+fx^2)^p}{p+1}+\frac{3bdf(d+fx^2)^p}{p+1}+\frac{2bf^2px^2(d+fx^2)^p}{p+1}+\frac{3bf^2x^2(d+fx^2)^p}{p+1}+\frac{2cdfpx(d+fx^2)^p}{p+1}+\frac{2cdfx(d+fx^2)^p}{p+1}+\frac{2cf^2px^3(d+fx^2)^p}{p+1}+\frac{2cf^2x^3(d+\cdots}{p+1} \\ bf\log\left(-i\sqrt{d}\sqrt{\frac{1}{f}}+x\right)+bf\log\left(i\sqrt{d}\sqrt{\frac{1}{f}}+x\right)+2cfx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+d)**p*(2*c*d*f+2*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),
x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + f*x**2)**p/(p + 1) + 3*b*d*f*(d + f*x**2)**p/(p +
 1) + 2*b*f**2*p*x**2*(d + f*x**2)**p/(p + 1) + 3*b*f**2*x**2*(d + f*x**2)*
*p/(p + 1) + 2*c*d*f*p*x*(d + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + f*x**2)**
p/(p + 1) + 2*c*f**2*p*x**3*(d + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d + f*
x**2)**p/(p + 1), Ne(p, -1)), (b*f*log(-I*sqrt(d)*sqrt(1/f) + x) + b*f*log(
I*sqrt(d)*sqrt(1/f) + x) + 2*c*f*x, True))
```

Giac [B] time = 1.22614, size = 190, normalized size = 4.63

$$\frac{2\left(fx^2+d\right)^p cf^2px^3+2\left(fx^2+d\right)^p bf^2px^2+2\left(fx^2+d\right)^p cf^2x^3+2\left(fx^2+d\right)^p cdfpx+3\left(fx^2+d\right)^p bf^2x^2+2\left(fx^2+d\right)^p b}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="giac")
```

```
[Out] (2*(f*x^2 + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + d)^p*b*f^2*p*x^2 + 2*(f*x^2 + d)^
p*c*f^2*x^3 + 2*(f*x^2 + d)^p*c*d*f*p*x + 3*(f*x^2 + d)^p*b*f^2*x^2 + 2*(f*
x^2 + d)^p*b*d*f*p + 2*(f*x^2 + d)^p*c*d*f*x + 3*(f*x^2 + d)^p*b*d*f)/(p +
1)
```

3.276 $\int (d+ex+fx^2)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)$

Optimal. Leaf size=46

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{ce(p+2)\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

```
[Out] -((c*e*(2 + p)*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*f*x*(d + e*x + f*x
^2)^(1 + p)
```

Rubi [A] time = 0.0721821, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1661, 629}

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{ce(p+2)\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 + 2*p)*x
^2),x]
```

```
[Out] -((c*e*(2 + p)*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*f*x*(d + e*x + f*x
^2)^(1 + p)
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx = 2cfx\left(d+ex+fx^2\right)^{1+p} + \frac{\int \left(-ce^2f(2+p)(3+2p)-2c\right.}{}$$

$$= -\frac{ce(2+p)\left(d+ex+fx^2\right)^{1+p}}{1+p} + 2cfx\left(d+ex+fx^2\right)^{1+p}$$

Mathematica [A] time = 0.124169, size = 34, normalized size = 0.74

$$\frac{c(2f(p+1)x-e(p+2))(d+x(e+fx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 +
2*p)*x^2),x]
```

```
[Out] (c*(-(e*(2 + p)) + 2*f*(1 + p)*x)*(d + x*(e + f*x))^(1 + p))/(1 + p)
```

Maple [A] time = 0.049, size = 39, normalized size = 0.9

$$-\frac{c\left(fx^2+ex+d\right)^{1+p}\left(-2\,fpx+ep-2\,fx+2\,e\right)}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x)
```

```
[Out] -c*(f*x^2+e*x+d)^(1+p)*(-2*f*p*x+e*p-2*f*x+2*e)/(1+p)
```

Maxima [A] time = 1.11649, size = 89, normalized size = 1.93

$$\frac{\left(2\,cf^2\left(p+1\right)x^3+cefpx^2-cde\left(p+2\right)-\left(e^2\left(p+2\right)-2\,df\left(p+1\right)\right)cx\right)\left(fx^2+ex+d\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + c*e*f*p*x^2 - c*d*e*(p + 2) - (e^2*(p + 2) - 2*d*f*(
p + 1))*c*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

Fricas [A] time = 1.37539, size = 182, normalized size = 3.96

$$\frac{\left(cefpx^2 - cdep + 2\left(cf^2p + cf^2\right)x^3 - 2\,cde - \left(2\,ce^2 - 2\,cdf + \left(ce^2 - 2\,cdf\right)p\right)x\right)\left(fx^2 + ex + d\right)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="fricas")
```

```
[Out] (c*e*f*p*x^2 - c*d*e*p + 2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e - (2*c*e^2 - 2*c
*d*f + (c*e^2 - 2*c*d*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

Sympy [A] time = 95.9917, size = 280, normalized size = 6.09

$$\begin{cases} -\frac{cdep\left(d+ex+fx^2\right)^p}{p+1} - \frac{2cde\left(d+ex+fx^2\right)^p}{p+1} + \frac{2cdfpx\left(d+ex+fx^2\right)^p}{p+1} + \frac{2cdfx\left(d+ex+fx^2\right)^p}{p+1} - \frac{ce^2px\left(d+ex+fx^2\right)^p}{p+1} - \frac{2ce^2x\left(d+ex+fx^2\right)^p}{p+1} + \frac{cefpx^2\left(d+ex+f\right.}{p+1} \\ -ce\log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) - ce\log\left(\frac{e}{2f} + x + \frac{\sqrt{-4df+e^2}}{2f}\right) + 2cfx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f-c*e**2*p+2*c*f**2*(3+2*p)*x*
*2),x)
```

```
[Out] Piecewise((-c*d*e*p*(d + e*x + f*x**2)**p/(p + 1) - 2*c*d*e*(d + e*x + f*x*
*2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d +
 e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*(d + e*x + f*x**2)**p/(p + 1) - 2*c*
e**2*x*(d + e*x + f*x**2)**p/(p + 1) + c*e*f*p*x**2*(d + e*x + f*x**2)**p/(
p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d +
 e*x + f*x**2)**p/(p + 1), Ne(p, -1)), (-c*e*log(e/(2*f) + x - sqrt(-4*d*f
+ e**2)/(2*f)) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x
, True))
```

Giac [B] time = 1.17813, size = 258, normalized size = 5.61

$$\frac{2\left(fx^2+xe+d\right)^p cf^2px^3+2\left(fx^2+xe+d\right)^p cf^2x^3+\left(fx^2+xe+d\right)^p cfpx^2e+2\left(fx^2+xe+d\right)^p cdfpx+2\left(fx^2+xe+d\right)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="giac")
```

```
[Out] (2*(f*x^2 + x*e + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + x*e + d)^p*c*f^2*x^3 + (f*x
^2 + x*e + d)^p*c*f*p*x^2*e + 2*(f*x^2 + x*e + d)^p*c*d*f*p*x + 2*(f*x^2 +
x*e + d)^p*c*d*f*x - (f*x^2 + x*e + d)^p*c*p*x*e^2 - (f*x^2 + x*e + d)^p*c*
d*p*e - 2*(f*x^2 + x*e + d)^p*c*x*e^2 - 2*(f*x^2 + x*e + d)^p*c*d*e)/(p + 1
)
```

3.277 $\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+\right.$

Optimal. Leaf size=57

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{(ce(p+2)-bf(2p+3))\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

```
[Out] -(((c*e*(2 + p) - b*f*(3 + 2*p))*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*
f*x*(d + e*x + f*x^2)^(1 + p)
```

Rubi [A] time = 0.121235, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1661, 629}

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{(ce(p+2)-bf(2p+3))\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b*e*f*p
 + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] -(((c*e*(2 + p) - b*f*(3 + 2*p))*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*
f*x*(d + e*x + f*x^2)^(1 + p)
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = 2cfx\left(d+ex+fx^2\right)^{1+}$$

$$= -\frac{(ce(2+p)-bf(3+}{1}$$

Mathematica [A] time = 0.300149, size = 43, normalized size = 0.75

$$\frac{(d+x(e+fx))^{p+1}(bf(2p+3)-ce(p+2)+2cf(p+1)x)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b
*e*f*p + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] ((-(c*e*(2 + p)) + b*f*(3 + 2*p) + 2*c*f*(1 + p)*x)*(d + x*(e + f*x))^(1 +
p))/(1 + p)
```

Maple [A] time = 0.049, size = 51, normalized size = 0.9

$$\frac{\left(fx^2+ex+d\right)^{1+p}\left(2\,cfxp+2\,bfp-cep+2\,cfx+3\,bf-2\,ce\right)}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f^2*(3+
2*p)*x+2*c*f^2*(3+2*p)*x^2),x)
```

```
[Out] (f*x^2+e*x+d)^(1+p)*(2*c*f*p*x+2*b*f*p-c*e*p+2*c*f*x+3*b*f-2*c*e)/(1+p)
```

Maxima [A] time = 1.14622, size = 132, normalized size = 2.32

$$\frac{\left(2\,cf^2\left(p+1\right)x^3+bdf\left(2\,p+3\right)-cde\left(p+2\right)+\left(bf^2\left(2\,p+3\right)+cefp\right)x^2+\left(bef\left(2\,p+3\right)-\left(e^2\left(p+2\right)-2\,df\left(p+1\right)\right)c\right)x\right)(}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*d*f*(2*p + 3) - c*d*e*(p + 2) + (b*f^2*(2*p + 3) +
 c*e*f*p)*x^2 + (b*e*f*(2*p + 3) - (e^2*(p + 2) - 2*d*f*(p + 1))*c)*x)*(f*x
^2 + e*x + d)^p/(p + 1)
```

Fricas [B] time = 1.37041, size = 269, normalized size = 4.72

$$\frac{\left(2\left(cf^2p + cf^2\right)x^3 - 2\,cde + 3\,bdf + \left(3\,bf^2 + \left(cef + 2\,bf^2\right)p\right)x^2 - \left(cde - 2\,bdf\right)p - \left(2\,ce^2 - (2\,cd + 3\,be)f + \left(ce^2 - 2\,(cd\right.\right.\right.}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="fricas")
```

```
[Out] (2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e + 3*b*d*f + (3*b*f^2 + (c*e*f + 2*b*f^2)
*p)*x^2 - (c*d*e - 2*b*d*f)*p - (2*c*e^2 - (2*c*d + 3*b*e)*f + (c*e^2 - 2*(
c*d + b*e)*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

Sympy [B] time = 113.673, size = 483, normalized size = 8.47

$$\begin{cases} \frac{2bdfp\left(d+ex+fx^2\right)^p}{p+1} + \frac{3bdf\left(d+ex+fx^2\right)^p}{p+1} + \frac{2befpx\left(d+ex+fx^2\right)^p}{p+1} + \frac{3befx\left(d+ex+fx^2\right)^p}{p+1} + \frac{2bf^2px^2\left(d+ex+fx^2\right)^p}{p+1} + \frac{3bf^2x^2\left(d+ex+fx^2\right)^p}{p+1} - \frac{cdep\left(d+ex\right.}{p+1} \\ bf\log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) + bf\log\left(\frac{e}{2f} + x + \frac{\sqrt{-4df+e^2}}{2f}\right) - ce\log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) - ce\log\left(\frac{e}{2f} + x + \frac{\sqrt{-4df+e^2}}{2f}\right) + 2c \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f+3*b*e*f-c*e**2*p+2*b*e*f*p+2
*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + e*x + f*x**2)**p/(p + 1) + 3*b*d*f*(d + e*x + f*x
**2)**p/(p + 1) + 2*b*e*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 3*b*e*f*x*(d
+ e*x + f*x**2)**p/(p + 1) + 2*b*f**2*p*x**2*(d + e*x + f*x**2)**p/(p + 1)
+ 3*b*f**2*x**2*(d + e*x + f*x**2)**p/(p + 1) - c*d*e*p*(d + e*x + f*x**2)*
```

```
*p/(p + 1) - 2*c*d*e*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x +
 f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*
(d + e*x + f*x**2)**p/(p + 1) - 2*c*e**2*x*(d + e*x + f*x**2)**p/(p + 1) +
c*e*f*p*x**2*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x
**2)**p/(p + 1) + 2*c*f**2*x**3*(d + e*x + f*x**2)**p/(p + 1), Ne(p, -1)),
(b*f*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)) + b*f*log(e/(2*f) + x + s
qrt(-4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)
) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x, True))
```

Giac [B] time = 1.22389, size = 424, normalized size = 7.44

$$\frac{2\left(fx^2+xe+d\right)^p cf^2px^3+2\left(fx^2+xe+d\right)^p bf^2px^2+2\left(fx^2+xe+d\right)^p cf^2x^3+\left(fx^2+xe+d\right)^p cfpx^2e+2\left(fx^2+xe+d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="giac")
```

```
[Out] (2*(f*x^2 + x*e + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + x*e + d)^p*b*f^2*p*x^2 + 2*
(f*x^2 + x*e + d)^p*c*f^2*x^3 + (f*x^2 + x*e + d)^p*c*f*p*x^2*e + 2*(f*x^2
+ x*e + d)^p*c*d*f*p*x + 3*(f*x^2 + x*e + d)^p*b*f^2*x^2 + 2*(f*x^2 + x*e +
 d)^p*b*f*p*x*e + 2*(f*x^2 + x*e + d)^p*b*d*f*p + 2*(f*x^2 + x*e + d)^p*c*d
*f*x - (f*x^2 + x*e + d)^p*c*p*x*e^2 - (f*x^2 + x*e + d)^p*c*d*p*e + 3*(f*x
^2 + x*e + d)^p*b*f*x*e + 3*(f*x^2 + x*e + d)^p*b*d*f - 2*(f*x^2 + x*e + d)
^p*c*x*e^2 - 2*(f*x^2 + x*e + d)^p*c*d*e)/(p + 1)
```

3.278 $\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde\right.\right.$

Optimal. Leaf size=20

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

```
[Out] (d + e*x)^5*(a + b*x + c*x^2)^6
```

Rubi [A] time = 0.420043, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 75, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1624, 1590}

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 + 17*b*d
*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] (d + e*x)^5*(a + b*x + c*x^2)^6
```

Rule 1624

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[PolynomialRemainder[Pq, d + e*x, x], 0]
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x+e(29cd+11be)x^2+17ce^2x^3\right) dx = \int (d+ex)^4 \left(a\right.$$
$$= (d+ex)^5 \left(a+\right.$$

Mathematica [B] time = 0.439293, size = 167, normalized size = 8.35

$$x\left(20a^3x^2(b+cx)^3(d+ex)^5+15a^2x^3(b+cx)^4(d+ex)^5+6a^5(b+cx)(d+ex)^5+15a^4x(b+cx)^2(d+ex)^5+a^6e\left(10d^2e^2x^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 +
17*b*d*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] x*(6*a^5*(b + c*x)*(d + e*x)^5 + 15*a^4*x*(b + c*x)^2*(d + e*x)^5 + 20*a^3*
x^2*(b + c*x)^3*(d + e*x)^5 + 15*a^2*x^3*(b + c*x)^4*(d + e*x)^5 + 6*a*x^4*
(b + c*x)^5*(d + e*x)^5 + x^5*(b + c*x)^6*(d + e*x)^5 + a^6*e*(5*d^4 + 10*d
^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4))
```

Maple [B] time = 0.049, size = 8419, normalized size = 421.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)*
x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.05573, size = 2402, normalized size = 120.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="maxima")
```

```
[Out] c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c^
5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5*d
^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*x^1
4 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 + 10*
(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^5)*x^
13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 100*(2*
b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d*e^4 +
 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 + 15*(5*b
^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 150*(b^4*c^
2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^
3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)*x^11 + (3*
(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e + 150*(b^4*c^2
 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3
)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d*e^4 + 6*(a
*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*c^3 + 3*a*b*c^4
)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^5*c + 10*a*b^3*c
^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*
c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*e^4 + 3*(a^2*b^4 +
 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d
^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e + 2*(b^6 + 30*a*b^4*c +
90*a^2*b^2*c^2 + 20*a^3*c^3)*d^3*e^2 + 12*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*
c^2)*d^2*e^3 + 15*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d*e^4 + 2*(2*a^3*b^3 +
3*a^4*b*c)*e^5)*x^8 + (6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^5 + 5*(b^6
 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^4*e + 60*(a*b^5 + 10*a^2*b^3
*c + 10*a^3*b*c^2)*d^3*e^2 + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^3
+ 50*(2*a^3*b^3 + 3*a^4*b*c)*d*e^4 + 3*(5*a^4*b^2 + 2*a^5*c)*e^5)*x^7 + (6*
a^5*b*e^5 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^5 + 30*(a*b^
5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d^4*e + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c
^2)*d^3*e^2 + 100*(2*a^3*b^3 + 3*a^4*b*c)*d^2*e^3 + 15*(5*a^4*b^2 + 2*a^5*c
)*d*e^4)*x^6 + (30*a^5*b*d*e^4 + a^6*e^5 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3
*b*c^2)*d^5 + 75*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^4*e + 100*(2*a^3*b^3 +
 3*a^4*b*c)*d^3*e^2 + 30*(5*a^4*b^2 + 2*a^5*c)*d^2*e^3)*x^5 + 5*(12*a^5*b*d
^2*e^3 + a^6*d*e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^5 + 10*(2*a^3*b^
3 + 3*a^4*b*c)*d^4*e + 6*(5*a^4*b^2 + 2*a^5*c)*d^3*e^2)*x^4 + 5*(12*a^5*b*d
^3*e^2 + 2*a^6*d^2*e^3 + 2*(2*a^3*b^3 + 3*a^4*b*c)*d^5 + 3*(5*a^4*b^2 + 2*a
^5*c)*d^4*e)*x^3 + (30*a^5*b*d^4*e + 10*a^6*d^3*e^2 + 3*(5*a^4*b^2 + 2*a^5*
c)*d^5)*x^2 + (6*a^5*b*d^5 + 5*a^6*d^4*e)*x
```

Fricas [B] time = 1.12122, size = 5253, normalized size = 262.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="fricas")

[Out] x^17*e^5*c^6 + 5*x^16*e^4*d*c^6 + 6*x^16*e^5*c^5*b + 10*x^15*e^3*d^2*c^6 + 30*x^15*e^4*d*c^5*b + 15*x^15*e^5*c^4*b^2 + 6*x^15*e^5*c^5*a + 10*x^14*e^2*d^3*c^6 + 60*x^14*e^3*d^2*c^5*b + 75*x^14*e^4*d*c^4*b^2 + 20*x^14*e^5*c^3*b^3 + 30*x^14*e^4*d*c^5*a + 30*x^14*e^5*c^4*b*a + 5*x^13*e*d^4*c^6 + 60*x^13*e^2*d^3*c^5*b + 150*x^13*e^3*d^2*c^4*b^2 + 100*x^13*e^4*d*c^3*b^3 + 15*x^13*e^5*c^2*b^4 + 60*x^13*e^3*d^2*c^5*a + 150*x^13*e^4*d*c^4*b*a + 60*x^13*e^5*c^3*b^2*a + 15*x^13*e^5*c^4*a^2 + x^12*d^5*c^6 + 30*x^12*e*d^4*c^5*b + 150*x^12*e^2*d^3*c^4*b^2 + 200*x^12*e^3*d^2*c^3*b^3 + 75*x^12*e^4*d*c^2*b^4 + 6*x^12*e^5*c*b^5 + 60*x^12*e^2*d^3*c^5*a + 300*x^12*e^3*d^2*c^4*b*a + 300*x^12*e^4*d*c^3*b^2*a + 60*x^12*e^5*c^2*b^3*a + 75*x^12*e^4*d*c^4*a^2 + 60*x^12*e^5*c^3*b*a^2 + 6*x^11*d^5*c^5*b + 75*x^11*e*d^4*c^4*b^2 + 200*x^11*e^2*d^3*c^3*b^3 + 150*x^11*e^3*d^2*c^2*b^4 + 30*x^11*e^4*d*c*b^5 + x^11*e^5*b^6 + 30*x^11*e*d^4*c^5*a + 300*x^11*e^2*d^3*c^4*b*a + 600*x^11*e^3*d^2*c^3*b^2*a + 300*x^11*e^4*d*c^2*b^3*a + 30*x^11*e^5*c*b^4*a + 150*x^11*e^3*d^2*c^4*a^2 + 300*x^11*e^4*d*c^3*b*a^2 + 90*x^11*e^5*c^2*b^2*a^2 + 20*x^11*e^5*c^3*a^3 + 15*x^10*d^5*c^4*b^2 + 100*x^10*e*d^4*c^3*b^3 + 150*x^10*e^2*d^3*c^2*b^4 + 60*x^10*e^3*d^2*c*b^5 + 5*x^10*e^4*d*b^6 + 6*x^10*d^5*c^5*a + 150*x^10*e*d^4*c^4*b*a + 600*x^10*e^2*d^3*c^3*b^2*a + 600*x^10*e^3*d^2*c^2*b^3*a + 150*x^10*e^4*d*c*b^4*a + 6*x^10*e^5*b^5*a + 150*x^10*e^2*d^3*c^4*a^2 + 600*x^10*e^3*d^2*c^3*b*a^2 + 450*x^10*e^4*d*c^2*b^2*a^2 + 60*x^10*e^5*c*b^3*a^2 + 100*x^10*e^4*d*c^3*a^3 + 60*x^10*e^5*c^2*b*a^3 + 20*x^9*d^5*c^3*b^3 + 75*x^9*e*d^4*c^2*b^4 + 60*x^9*e^2*d^3*c*b^5 + 10*x^9*e^3*d^2*b^6 + 30*x^9*d^5*c^4*b*a + 300*x^9*e*d^4*c^3*b^2*a + 600*x^9*e^2*d^3*c^2*b^3*a + 300*x^9*e^3*d^2*c*b^4*a + 30*x^9*e^4*d*b^5*a + 75*x^9*e*d^4*c^4*a^2 + 600*x^9*e^2*d^3*c^3*b*a^2 + 900*x^9*e^3*d^2*c^2*b^2*a^2 + 300*x^9*e^4*d*c*b^3*a^2 + 15*x^9*e^5*b^4*a^2 + 200*x^9*e^3*d^2*c^3*a^3 + 300*x^9*e^4*d*c^2*b*a^3 + 60*x^9*e^5*c*b^2*a^3 + 15*x^9*e^5*c^2*a^4 + 15*x^8*d^5*c^2*b^4 + 30*x^8*e*d^4*c*b^5 + 10*x^8*e^2*d^3*b^6 + 60*x^8*d^5*c^3*b^2*a + 300*x^8*e*d^4*c^2*b^3*a + 300*x^8*e^2*d^3*c*b^4*a + 60*x^8*e^3*d^2*b^5*a + 15*x^8*d^5*c^4*a^2 + 300*x^8*e*d^4*c^3*b*a^2 + 900*x^8*e^2*d^3*c^2*b^2*a^2 + 600*x^8*e^3*d^2*c*b^3*a^2 + 75*x^8*e^4*d*b^4*a^2 + 200*x^8*e^2*d^3*c^3*a^3 + 600*x^8*e^3*d^2*c^2*b*a^3 + 300*x^8*e^4*d*c*b^2*a^3 + 20*x^8*e^5*b^3*a^3 + 75*x^8*e^4*d*c^2*a^4 + 30*x^8*e^5*c*b*a^4 + 6*x^7*d^5*c*b^5 + 5*x^7*e*d^4*b^6 + 60*x^7*d^5*c^2*b^3*a + 150*x^7*e*d^4*c*b^4*a + 60*x^7*e^2*d^3*b^5*a + 60*x^7*d^5*c^3*b*a^2 + 450*x^7*e*d^4*c^2*b^2*a^2 + 600*x^7*e^2*d^3*c*b^3*a^2 + 150*x^7*e^3*d^2*b^4*a^2 + 100*x^7*e*d^4*c^3*a^3 + 600*x^7*e^2*d^3*c^2*b*a^3 + 600*x^7*e^3*d^2*c*b^2*a^3 + 100*x^7*e^4*d*b^3*a^3 + 150*x^7*e^3*d^2*c^2*a^4 + 150*x^7*e^4*d*c*b*a^4 + 15*x^7*e^5*b^2*a^4 + 6*x^7*e^5*c*a^5 + x^6*d^5*b^6 + 30*x^6*d^5*c*b^4*a + 30*x^6*e*d^4*b^5*a + 90*x^6*d^5*c^2*b^2*a^2 + 300*x^6*e*d^4*c*b^3*a^2 + 150*x^6*e^2*d^3*b^4*a^2 + 20*x^6*d^5*c^3*a^3 + 300*x^6*e*d^4*c^2*b*a^3 + 600*x^6*e^2*d^3*c*b^2*a^3 + 200*x^6*e^3*d^2*b^3*a^3 + 150*x^6*e^2*d^3

```
*c^2*a^4 + 300*x^6*e^3*d^2*c*b*a^4 + 75*x^6*e^4*d*b^2*a^4 + 30*x^6*e^4*d*c*
a^5 + 6*x^6*e^5*b*a^5 + 6*x^5*d^5*b^5*a + 60*x^5*d^5*c*b^3*a^2 + 75*x^5*e*d
^4*b^4*a^2 + 60*x^5*d^5*c^2*b*a^3 + 300*x^5*e*d^4*c*b^2*a^3 + 200*x^5*e^2*d
^3*b^3*a^3 + 75*x^5*e*d^4*c^2*a^4 + 300*x^5*e^2*d^3*c*b*a^4 + 150*x^5*e^3*d
^2*b^2*a^4 + 60*x^5*e^3*d^2*c*a^5 + 30*x^5*e^4*d*b*a^5 + x^5*e^5*a^6 + 15*x
^4*d^5*b^4*a^2 + 60*x^4*d^5*c*b^2*a^3 + 100*x^4*e*d^4*b^3*a^3 + 15*x^4*d^5*
c^2*a^4 + 150*x^4*e*d^4*c*b*a^4 + 150*x^4*e^2*d^3*b^2*a^4 + 60*x^4*e^2*d^3*
c*a^5 + 60*x^4*e^3*d^2*b*a^5 + 5*x^4*e^4*d*a^6 + 20*x^3*d^5*b^3*a^3 + 30*x^
3*d^5*c*b*a^4 + 75*x^3*e*d^4*b^2*a^4 + 30*x^3*e*d^4*c*a^5 + 60*x^3*e^2*d^3*
b*a^5 + 10*x^3*e^3*d^2*a^6 + 15*x^2*d^5*b^2*a^4 + 6*x^2*d^5*c*a^5 + 30*x^2*
e*d^4*b*a^5 + 10*x^2*e^2*d^3*a^6 + 6*x*d^5*b*a^5 + 5*x*e*d^4*a^6
```

Sympy [B] time = 0.393559, size = 2281, normalized size = 114.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**5*(d*(5*a*e+6*b*d)+(5*a*e**2+17*b*d*e+
12*c*d**2)*x+e*(11*b*e+29*c*d)*x**2+17*c*e**2*x**3),x)
```

```
[Out] c**6*e**5*x**17 + x**16*(6*b*c**5*e**5 + 5*c**6*d*e**4) + x**15*(6*a*c**5*e
**5 + 15*b**2*c**4*e**5 + 30*b*c**5*d*e**4 + 10*c**6*d**2*e**3) + x**14*(30
*a*b*c**4*e**5 + 30*a*c**5*d*e**4 + 20*b**3*c**3*e**5 + 75*b**2*c**4*d*e**4
 + 60*b*c**5*d**2*e**3 + 10*c**6*d**3*e**2) + x**13*(15*a**2*c**4*e**5 + 60
*a*b**2*c**3*e**5 + 150*a*b*c**4*d*e**4 + 60*a*c**5*d**2*e**3 + 15*b**4*c**
2*e**5 + 100*b**3*c**3*d*e**4 + 150*b**2*c**4*d**2*e**3 + 60*b*c**5*d**3*e*
*2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d*e**4 + 60
*a*b**3*c**2*e**5 + 300*a*b**2*c**3*d*e**4 + 300*a*b*c**4*d**2*e**3 + 60*a*
c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d*e**4 + 200*b**3*c**3*d**2*e
**3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x**11*(20*a
**3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d*e**4 + 150*a**2*
c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d*e**4 + 600*a*b**2*c**
3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6*e**5 + 30*b*
*5*c*d*e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e**2 + 75*b**2*c
**4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 100*a**3*c**3*d*
e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d*e**4 + 600*a**2*b*c**3*d*
*2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*b**4*c*d*e**4 + 6
00*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 150*a*b*c**4*d**4*e
+ 6*a*c**5*d**5 + 5*b**6*d*e**4 + 60*b**5*c*d**2*e**3 + 150*b**4*c**2*d**3*
e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x**9*(15*a**4*c**2*e**5
+ 60*a**3*b**2*c*e**5 + 300*a**3*b*c**2*d*e**4 + 200*a**3*c**3*d**2*e**3 +
```

```
15*a**2*b**4*e**5 + 300*a**2*b**3*c*d*e**4 + 900*a**2*b**2*c**2*d**2*e**3 +
 600*a**2*b*c**3*d**3*e**2 + 75*a**2*c**4*d**4*e + 30*a*b**5*d*e**4 + 300*a
*b**4*c*d**2*e**3 + 600*a*b**3*c**2*d**3*e**2 + 300*a*b**2*c**3*d**4*e + 30
*a*b*c**4*d**5 + 10*b**6*d**2*e**3 + 60*b**5*c*d**3*e**2 + 75*b**4*c**2*d**
4*e + 20*b**3*c**3*d**5) + x**8*(30*a**4*b*c*e**5 + 75*a**4*c**2*d*e**4 + 2
0*a**3*b**3*e**5 + 300*a**3*b**2*c*d*e**4 + 600*a**3*b*c**2*d**2*e**3 + 200
*a**3*c**3*d**3*e**2 + 75*a**2*b**4*d*e**4 + 600*a**2*b**3*c*d**2*e**3 + 90
0*a**2*b**2*c**2*d**3*e**2 + 300*a**2*b*c**3*d**4*e + 15*a**2*c**4*d**5 + 6
0*a*b**5*d**2*e**3 + 300*a*b**4*c*d**3*e**2 + 300*a*b**3*c**2*d**4*e + 60*a
*b**2*c**3*d**5 + 10*b**6*d**3*e**2 + 30*b**5*c*d**4*e + 15*b**4*c**2*d**5)
 + x**7*(6*a**5*c*e**5 + 15*a**4*b**2*e**5 + 150*a**4*b*c*d*e**4 + 150*a**4
*c**2*d**2*e**3 + 100*a**3*b**3*d*e**4 + 600*a**3*b**2*c*d**2*e**3 + 600*a*
*3*b*c**2*d**3*e**2 + 100*a**3*c**3*d**4*e + 150*a**2*b**4*d**2*e**3 + 600*
a**2*b**3*c*d**3*e**2 + 450*a**2*b**2*c**2*d**4*e + 60*a**2*b*c**3*d**5 + 6
0*a*b**5*d**3*e**2 + 150*a*b**4*c*d**4*e + 60*a*b**3*c**2*d**5 + 5*b**6*d**
4*e + 6*b**5*c*d**5) + x**6*(6*a**5*b*e**5 + 30*a**5*c*d*e**4 + 75*a**4*b**
2*d*e**4 + 300*a**4*b*c*d**2*e**3 + 150*a**4*c**2*d**3*e**2 + 200*a**3*b**3
*d**2*e**3 + 600*a**3*b**2*c*d**3*e**2 + 300*a**3*b*c**2*d**4*e + 20*a**3*c
**3*d**5 + 150*a**2*b**4*d**3*e**2 + 300*a**2*b**3*c*d**4*e + 90*a**2*b**2*
c**2*d**5 + 30*a*b**5*d**4*e + 30*a*b**4*c*d**5 + b**6*d**5) + x**5*(a**6*e
**5 + 30*a**5*b*d*e**4 + 60*a**5*c*d**2*e**3 + 150*a**4*b**2*d**2*e**3 + 30
0*a**4*b*c*d**3*e**2 + 75*a**4*c**2*d**4*e + 200*a**3*b**3*d**3*e**2 + 300*
a**3*b**2*c*d**4*e + 60*a**3*b*c**2*d**5 + 75*a**2*b**4*d**4*e + 60*a**2*b*
*3*c*d**5 + 6*a*b**5*d**5) + x**4*(5*a**6*d*e**4 + 60*a**5*b*d**2*e**3 + 60
*a**5*c*d**3*e**2 + 150*a**4*b**2*d**3*e**2 + 150*a**4*b*c*d**4*e + 15*a**4
*c**2*d**5 + 100*a**3*b**3*d**4*e + 60*a**3*b**2*c*d**5 + 15*a**2*b**4*d**5
) + x**3*(10*a**6*d**2*e**3 + 60*a**5*b*d**3*e**2 + 30*a**5*c*d**4*e + 75*a
**4*b**2*d**4*e + 30*a**4*b*c*d**5 + 20*a**3*b**3*d**5) + x**2*(10*a**6*d**
3*e**2 + 30*a**5*b*d**4*e + 6*a**5*c*d**5 + 15*a**4*b**2*d**5) + x*(5*a**6*
d**4*e + 6*a**5*b*d**5)
```

Giac [B] time = 1.20794, size = 3217, normalized size = 160.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="giac")
```

```
[Out] c^6*x^17*e^5 + 5*c^6*d*x^16*e^4 + 10*c^6*d^2*x^15*e^3 + 10*c^6*d^3*x^14*e^2
 + 5*c^6*d^4*x^13*e + c^6*d^5*x^12 + 6*b*c^5*x^16*e^5 + 30*b*c^5*d*x^15*e^4
```

+ 60*b*c^5*d^2*x^14*e^3 + 60*b*c^5*d^3*x^13*e^2 + 30*b*c^5*d^4*x^12*e + 6*b*c^5*d^5*x^11 + 15*b^2*c^4*x^15*e^5 + 6*a*c^5*x^15*e^5 + 75*b^2*c^4*d*x^14*e^4 + 30*a*c^5*d*x^14*e^4 + 150*b^2*c^4*d^2*x^13*e^3 + 60*a*c^5*d^2*x^13*e^3 + 150*b^2*c^4*d^3*x^12*e^2 + 60*a*c^5*d^3*x^12*e^2 + 75*b^2*c^4*d^4*x^11*e + 30*a*c^5*d^4*x^11*e + 15*b^2*c^4*d^5*x^10 + 6*a*c^5*d^5*x^10 + 20*b^3*c^3*x^14*e^5 + 30*a*b*c^4*x^14*e^5 + 100*b^3*c^3*d*x^13*e^4 + 150*a*b*c^4*d*x^13*e^4 + 200*b^3*c^3*d^2*x^12*e^3 + 300*a*b*c^4*d^2*x^12*e^3 + 200*b^3*c^3*d^3*x^11*e^2 + 300*a*b*c^4*d^3*x^11*e^2 + 100*b^3*c^3*d^4*x^10*e + 150*a*b*c^4*d^4*x^10*e + 20*b^3*c^3*d^5*x^9 + 30*a*b*c^4*d^5*x^9 + 15*b^4*c^2*x^13*e^5 + 60*a*b^2*c^3*x^13*e^5 + 15*a^2*c^4*x^13*e^5 + 75*b^4*c^2*d*x^12*e^4 + 300*a*b^2*c^3*d*x^12*e^4 + 75*a^2*c^4*d*x^12*e^4 + 150*b^4*c^2*d^2*x^11*e^3 + 600*a*b^2*c^3*d^2*x^11*e^3 + 150*a^2*c^4*d^2*x^11*e^3 + 150*b^4*c^2*d^3*x^10*e^2 + 600*a*b^2*c^3*d^3*x^10*e^2 + 150*a^2*c^4*d^3*x^10*e^2 + 75*b^4*c^2*d^4*x^9*e + 300*a*b^2*c^3*d^4*x^9*e + 75*a^2*c^4*d^4*x^9*e + 15*b^4*c^2*d^5*x^8 + 60*a*b^2*c^3*d^5*x^8 + 15*a^2*c^4*d^5*x^8 + 6*b^5*c*x^12*e^5 + 60*a*b^3*c^2*x^12*e^5 + 60*a^2*b*c^3*x^12*e^5 + 30*b^5*c*d*x^11*e^4 + 300*a*b^3*c^2*d*x^11*e^4 + 300*a^2*b*c^3*d*x^11*e^4 + 60*b^5*c*d^2*x^10*e^3 + 600*a*b^3*c^2*d^2*x^10*e^3 + 600*a^2*b*c^3*d^2*x^10*e^3 + 60*b^5*c*d^3*x^9*e^2 + 600*a*b^3*c^2*d^3*x^9*e^2 + 600*a^2*b*c^3*d^3*x^9*e^2 + 30*b^5*c*d^4*x^8*e + 300*a*b^3*c^2*d^4*x^8*e + 300*a^2*b*c^3*d^4*x^8*e + 6*b^5*c*d^5*x^7 + 60*a*b^3*c^2*d^5*x^7 + 60*a^2*b*c^3*d^5*x^7 + b^6*x^11*e^5 + 30*a*b^4*c*x^11*e^5 + 90*a^2*b^2*c^2*x^11*e^5 + 20*a^3*c^3*x^11*e^5 + 5*b^6*d*x^10*e^4 + 150*a*b^4*c*d*x^10*e^4 + 450*a^2*b^2*c^2*d*x^10*e^4 + 100*a^3*c^3*d*x^10*e^4 + 10*b^6*d^2*x^9*e^3 + 300*a*b^4*c*d^2*x^9*e^3 + 900*a^2*b^2*c^2*d^2*x^9*e^3 + 200*a^3*c^3*d^2*x^9*e^3 + 10*b^6*d^3*x^8*e^2 + 300*a*b^4*c*d^3*x^8*e^2 + 900*a^2*b^2*c^2*d^3*x^8*e^2 + 200*a^3*c^3*d^3*x^8*e^2 + 5*b^6*d^4*x^7*e + 150*a*b^4*c*d^4*x^7*e + 450*a^2*b^2*c^2*d^4*x^7*e + 100*a^3*c^3*d^4*x^7*e + b^6*d^5*x^6 + 30*a*b^4*c*d^5*x^6 + 90*a^2*b^2*c^2*d^5*x^6 + 20*a^3*c^3*d^5*x^6 + 6*a*b^5*x^10*e^5 + 60*a^2*b^3*c*x^10*e^5 + 60*a^3*b*c^2*x^10*e^5 + 30*a*b^5*d*x^9*e^4 + 300*a^2*b^3*c*d*x^9*e^4 + 300*a^3*b*c^2*d*x^9*e^4 + 60*a*b^5*d^2*x^8*e^3 + 600*a^2*b^3*c*d^2*x^8*e^3 + 600*a^3*b*c^2*d^2*x^8*e^3 + 60*a*b^5*d^3*x^7*e^2 + 600*a^2*b^3*c*d^3*x^7*e^2 + 600*a^3*b*c^2*d^3*x^7*e^2 + 30*a*b^5*d^4*x^6*e + 300*a^2*b^3*c*d^4*x^6*e + 300*a^3*b*c^2*d^4*x^6*e + 6*a*b^5*d^5*x^5 + 60*a^2*b^3*c*d^5*x^5 + 60*a^3*b*c^2*d^5*x^5 + 15*a^2*b^4*x^9*e^5 + 60*a^3*b^2*c*x^9*e^5 + 15*a^4*c^2*x^9*e^5 + 75*a^2*b^4*d*x^8*e^4 + 300*a^3*b^2*c*d*x^8*e^4 + 75*a^4*c^2*d*x^8*e^4 + 150*a^2*b^4*d^2*x^7*e^3 + 600*a^3*b^2*c*d^2*x^7*e^3 + 150*a^4*c^2*d^2*x^7*e^3 + 150*a^2*b^4*d^3*x^6*e^2 + 600*a^3*b^2*c*d^3*x^6*e^2 + 150*a^4*c^2*d^3*x^6*e^2 + 75*a^2*b^4*d^4*x^5*e + 300*a^3*b^2*c*d^4*x^5*e + 75*a^4*c^2*d^4*x^5*e + 15*a^2*b^4*d^5*x^4 + 60*a^3*b^2*c*d^5*x^4 + 15*a^4*c^2*d^5*x^4 + 20*a^3*b^3*x^8*e^5 + 30*a^4*b*c*x^8*e^5 + 100*a^3*b^3*d*x^7*e^4 + 150*a^4*b*c*d*x^7*e^4 + 200*a^3*b^3*d^2*x^6*e^3 + 300*a^4*b*c*d^2*x^6*e^3 + 200*a^3*b^3*d^3*x^5*e^2 + 300*a^4*b*c*d^3*x^5*e^2 + 100*a^3*b^3*d^4*x^4*e + 150*a^4*b*c*d^4*x^4*e + 20*a^3*b^3*d^5*x^3 + 30*a^4*b*c*d^5*x^3 + 15*a^4*b^2*x^7*e^5 + 6*a^5*c*x^7*e^5 + 75*a^4*b^2*d*x^6*e^4 + 30*a^5*c*d*x^6*e^4 + 150*a^4*b^2*d^2*x^5*e^3 +

60*a^5*c*d^2*x^5*e^3 + 150*a^4*b^2*d^3*x^4*e^2 + 60*a^5*c*d^3*x^4*e^2 + 75*a^4*b^2*d^4*x^3*e + 30*a^5*c*d^4*x^3*e + 15*a^4*b^2*d^5*x^2 + 6*a^5*c*d^5*x^2 + 6*a^5*b*x^6*e^5 + 30*a^5*b*d*x^5*e^4 + 60*a^5*b*d^2*x^4*e^3 + 60*a^5*b*d^3*x^3*e^2 + 30*a^5*b*d^4*x^2*e + 6*a^5*b*d^5*x + a^6*x^5*e^5 + 5*a^6*d*x^4*e^4 + 10*a^6*d^2*x^3*e^3 + 10*a^6*d^3*x^2*e^2 + 5*a^6*d^4*x*e

3.279 $\int \frac{x^2+x^3}{-2+x+x^2} dx$

Optimal. Leaf size=26

$$\frac{x^2}{2} + \frac{2}{3}\log(1-x) + \frac{4}{3}\log(x+2)$$

```
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Rubi [A] time = 0.0245635, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1593, 800, 632, 31}

$$\frac{x^2}{2} + \frac{2}{3}\log(1-x) + \frac{4}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^3)/(-2 + x + x^2),x]

[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2+x^3}{-2+x+x^2}\,dx &= \int \frac{x^2(1+x)}{-2+x+x^2}\,dx \\
&= \int \left(x+\frac{2x}{-2+x+x^2}\right) dx \\
&= \frac{x^2}{2}+2\int \frac{x}{-2+x+x^2}\,dx \\
&= \frac{x^2}{2}+\frac{2}{3}\int \frac{1}{-1+x}\,dx+\frac{4}{3}\int \frac{1}{2+x}\,dx \\
&= \frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(2+x)
\end{aligned}
$$

Mathematica [A] time = 0.0046166, size = 26, normalized size = 1.

$$\frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^3)/(-2 + x + x^2),x]
```

```
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Maple [A] time = 0.047, size = 19, normalized size = 0.7

$$\frac{x^2}{2}+\frac{4\ln(2+x)}{3}+\frac{2\ln(-1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)/(x^2+x-2),x)
```

```
[Out] 1/2*x^2+4/3*ln(2+x)+2/3*ln(-1+x)
```

Maxima [A] time = 0.998585, size = 24, normalized size = 0.92

$$\frac{1}{2}x^2+\frac{4}{3}\log\left(x+2\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

Fricas [A] time = 1.36524, size = 58, normalized size = 2.23

$$\frac{1}{2}x^2+\frac{4}{3}\log\left(x+2\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

Sympy [A] time = 0.102618, size = 20, normalized size = 0.77

$$\frac{x^2}{2}+\frac{2\log\left(x-1\right)}{3}+\frac{4\log\left(x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)/(x**2+x-2),x)
```

```
[Out] x**2/2 + 2*log(x - 1)/3 + 4*log(x + 2)/3
```

Giac [A] time = 1.19172, size = 27, normalized size = 1.04

$$\frac{1}{2}x^2 + \frac{4}{3}\log(|x+2|) + \frac{2}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 4/3*log(abs(x + 2)) + 2/3*log(abs(x - 1))
```

3.280 $\int \frac{x^2\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=346

$$-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+40bc^2(36c\right.}{1920c^5}$$

```
[Out] ((80*c^2*e - 70*b*c*f + 63*b^2*g - 64*a*c*g)*x^2*Sqrt[a + b*x + c*x^2])/(24
0*c^3) + ((10*c*f - 9*b*g)*x^3*Sqrt[a + b*x + c*x^2])/(40*c^2) + (g*x^4*Sqr
t[a + b*x + c*x^2])/(5*c) - ((1050*b^3*c*f + 40*b*c^2*(36*c*d - 55*a*f) - 9
45*b^4*g - 60*b^2*c*(20*c*e - 49*a*g) + 256*a*c^2*(5*c*e - 4*a*g) - 2*c*(48
0*c^3*d - 40*c^2*(10*b*e + 9*a*f) - 315*b^3*g + 14*b*c*(25*b*f + 46*a*g))*x
)*Sqrt[a + b*x + c*x^2])/(1920*c^5) + ((70*b^4*c*f + 48*b^2*c^2*(2*c*d - 5*
a*f) - 32*a*c^3*(4*c*d - 3*a*f) - 63*b^5*g - 40*b^3*c*(2*c*e - 7*a*g) + 48*
a*b*c^2*(4*c*e - 5*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(256*c^(11/2))
```

Rubi [A] time = 0.811677, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1653, 832, 779, 621, 206}

$$-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+40bc^2(36c\right.}{1920c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((80*c^2*e - 70*b*c*f + 63*b^2*g - 64*a*c*g)*x^2*Sqrt[a + b*x + c*x^2])/(24
0*c^3) + ((10*c*f - 9*b*g)*x^3*Sqrt[a + b*x + c*x^2])/(40*c^2) + (g*x^4*Sqr
t[a + b*x + c*x^2])/(5*c) - ((1050*b^3*c*f + 40*b*c^2*(36*c*d - 55*a*f) - 9
45*b^4*g - 60*b^2*c*(20*c*e - 49*a*g) + 256*a*c^2*(5*c*e - 4*a*g) - 2*c*(48
0*c^3*d - 40*c^2*(10*b*e + 9*a*f) - 315*b^3*g + 14*b*c*(25*b*f + 46*a*g))*x
)*Sqrt[a + b*x + c*x^2])/(1920*c^5) + ((70*b^4*c*f + 48*b^2*c^2*(2*c*d - 5*
a*f) - 32*a*c^3*(4*c*d - 3*a*f) - 63*b^5*g - 40*b^3*c*(2*c*e - 7*a*g) + 48*
a*b*c^2*(4*c*e - 5*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(256*c^(11/2))
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^2\left(5cd+(5ce-4ag)x+\frac{1}{2}(10cf-9bg)x^2\right)}{\sqrt{a+bx+cx^2}}\,dx}{5c}$$

$$= \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{gx^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^2\left(\frac{1}{2}\left(40c^2d-30acf+27abg\right)+\frac{1}{4}\left(80c^2e-7\right.\right.}{\sqrt{a+bx+cx^2}}}{20c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} +$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} +$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} +$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} +$$

Mathematica [A] time = 0.72651, size = 282, normalized size = 0.82

$$\frac{\sqrt{a+x(b+cx)}\left(16c^2\left(64a^2g-ac(80e+x(45f+32gx))+2c^2x(30d+x(20e+3x(5f+4gx)))\right)+4b^2c(-735ag+300ce\right.}{1920c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(945*b^4*g - 210*b^3*c*(5*f + 3*g*x) + 4*b^2*c*(300*
c*e - 735*a*g + 7*c*x*(25*f + 18*g*x)) - 8*b*c^2*(-(a*(275*f + 161*g*x)) +
2*c*(90*d + x*(50*e + 35*f*x + 27*g*x^2))) + 16*c^2*(64*a^2*g - a*c*(80*e +
 x*(45*f + 32*g*x)) + 2*c^2*x*(30*d + x*(20*e + 3*x*(5*f + 4*g*x))))))/(192
0*c^5) - ((-70*b^4*c*f - 48*b^2*c^2*(2*c*d - 5*a*f) + 32*a*c^3*(4*c*d - 3*a
*f) + 63*b^5*g + 40*b^3*c*(2*c*e - 7*a*g) + 48*a*b*c^2*(-4*c*e + 5*a*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(256*c^(11/2))
```

Maple [B] time = 0.058, size = 783, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 35/96*f*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)+35/32*g*b^3/c^(9/2)*a*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))-15/16*f*b^2/c^(7/2)*a*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))+55/48*f*b/c^3*a*(c*x^2+b*x+a)^(1/2)-3/8*f*a/c^2*x*(c*x^
2+b*x+a)^(1/2)-7/24*f*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)-9/40*g*b/c^2*x^3*(c*x^2
+b*x+a)^(1/2)+21/80*g*b^2/c^3*x^2*(c*x^2+b*x+a)^(1/2)-21/64*g*b^3/c^4*x*(c*
x^2+b*x+a)^(1/2)-5/12*e*b/c^2*x*(c*x^2+b*x+a)^(1/2)+3/4*e*b/c^(5/2)*a*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-49/32*g*b^2/c^4*a*(c*x^2+b*x+a)^(1/2
)-15/16*g*b/c^(7/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/15*g*
a/c^2*x^2*(c*x^2+b*x+a)^(1/2)+3/8*f*a^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+1/2*d*x/c*(c*x^2+b*x+a)^(1/2)-3/4*d*b/c^2*(c*x^2+b*x+a)^(1
/2)+3/8*d*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*d*a/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*e*x^2/c*(c*x^2+b*x+a
)^(1/2)+5/8*e*b^2/c^3*(c*x^2+b*x+a)^(1/2)-5/16*e*b^3/c^(7/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/3*e*a/c^2*(c*x^2+b*x+a)^(1/2)+8/15*g*a^2/c^
3*(c*x^2+b*x+a)^(1/2)+63/128*g*b^4/c^5*(c*x^2+b*x+a)^(1/2)-63/256*g*b^5/c^(
11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/4*f*x^3/c*(c*x^2+b*x+a)
^(1/2)-35/64*f*b^3/c^4*(c*x^2+b*x+a)^(1/2)+35/128*f*b^4/c^(9/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+161/240*g*b/c^3*a*x*(c*x^2+b*x+a)^(1/2)+1/
5*g*x^4*(c*x^2+b*x+a)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15884, size = 1666, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/7680*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2 - 12*a*b*c^3)*e +
2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 280*a*b^3*c + 240*a
^2*b*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*g*x^4 - 1440*b*c^4*d + 48*(10*
c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3 - 64*a*c^4)
*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*b*c^3)*f + (
945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 400*b*c^4*e +
 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*x)*sqrt(c*x^
2 + b*x + a))/c^6, -1/3840*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2
- 12*a*b*c^3)*e + 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 2
80*a*b^3*c + 240*a^2*b*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*g*x^4 - 1440*b*c^4
*d + 48*(10*c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3
 - 64*a*c^4)*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*
b*c^3)*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 4
00*b*c^4*e + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*
x)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(d + ex + fx^2 + gx^3\right)}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(x**2*(d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.23068, size = 446, normalized size = 1.29

$$\frac{1}{1920}\sqrt{cx^2 + bx + a}\left(2\left(4\left(6\left(\frac{8\,gx}{c} + \frac{10\,c^4 f - 9\,bc^3 g}{c^5}\right)x - \frac{70\,bc^3 f - 63\,b^2c^2 g + 64\,ac^3 g - 80\,c^4 e}{c^5}\right)x + \frac{480\,c^4 d + 350\,b^2c^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*g*x/c + (10*c^4*f - 9*b*c^3*g)/c^5
)*x - (70*b*c^3*f - 63*b^2*c^2*g + 64*a*c^3*g - 80*c^4*e)/c^5)*x + (480*c^4
*d + 350*b^2*c^2*f - 360*a*c^3*f - 315*b^3*c*g + 644*a*b*c^2*g - 400*b*c^3*
e)/c^5)*x - (1440*b*c^3*d + 1050*b^3*c*f - 2200*a*b*c^2*f - 945*b^4*g + 294
0*a*b^2*c*g - 1024*a^2*c^2*g - 1200*b^2*c^2*e + 1280*a*c^3*e)/c^5) - 1/256*
(96*b^2*c^3*d - 128*a*c^4*d + 70*b^4*c*f - 240*a*b^2*c^2*f + 96*a^2*c^3*f -
 63*b^5*g + 280*a*b^3*c*g - 240*a^2*b*c^2*g - 80*b^3*c^2*e + 192*a*b*c^3*e)
*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

3.281 $\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=245

$$\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c^3d\right)}{192c^4}-\ldots$$

```
[Out] ((8*c*f - 7*b*g)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (g*x^3*Sqrt[a + b*x
+ c*x^2])/(4*c) + ((192*c^3*d - 16*c^2*(9*b*e + 8*a*f) - 105*b^3*g + 20*b*c
*(6*b*f + 11*a*g) + 2*c*(48*c^2*e - 40*b*c*f + 35*b^2*g - 36*a*c*g)*x)*Sqrt
[a + b*x + c*x^2])/(192*c^4) - ((40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a*f) - 35
*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rubi [A] time = 0.437547, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1653, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c^3d\right)}{192c^4}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] ((8*c*f - 7*b*g)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (g*x^3*Sqrt[a + b*x
+ c*x^2])/(4*c) + ((192*c^3*d - 16*c^2*(9*b*e + 8*a*f) - 105*b^3*g + 20*b*c
*(6*b*f + 11*a*g) + 2*c*(48*c^2*e - 40*b*c*f + 35*b^2*g - 36*a*c*g)*x)*Sqrt
[a + b*x + c*x^2])/(192*c^4) - ((40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a*f) - 35
*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```


Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
```

```
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}dx = \frac{gx^3\sqrt{a+bx+cx^2}}{4c}+\frac{\int \frac{x\left(4cd+(4ce-3ag)x+\frac{1}{2}(8cf-7bg)x^2\right)}{\sqrt{a+bx+cx^2}}dx}{4c}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}+\frac{\int \frac{x\left(12c^2d-8acf+7abg+\frac{1}{4}\left(48c^2e-40bcf+35b\right.\right.}{\sqrt{a+bx+cx^2}}}{12c^2}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}+\frac{\left(192c^3d-16c^2(9be+8af)-105b^3\right.}{}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}+\frac{\left(192c^3d-16c^2(9be+8af)-105b^3\right.}{}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}+\frac{\left(192c^3d-16c^2(9be+8af)-105b^3\right.}{}$$

Mathematica [A] time = 0.45855, size = 199, normalized size = 0.81

$$\frac{\sqrt{a+x(b+cx)}\left(-8c^2\left(16af+9agx+18be+10bfx+7bgx^2\right)+10bc(22ag+12bf+7bgx)-105b^3g+16c^3\left(12d+x\left(6\right.\right.\right.}{192c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-105*b^3*g + 10*b*c*(12*b*f + 22*a*g + 7*b*g*x) - 8
*c^2*(18*b*e + 16*a*f + 10*b*f*x + 9*a*g*x + 7*b*g*x^2) + 16*c^3*(12*d + x*
(6*e + 4*f*x + 3*g*x^2))))/(192*c^4) + ((-40*b^3*c*f + 32*b*c^2*(-2*c*d + 3
*a*f) + 35*b^4*g + 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(-4*c*e + 3*a*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(9/2))
```

Maple [B] time = 0.055, size = 532, normalized size = 2.2

$$\frac{gx^3}{4c}\sqrt{cx^2+bx+a}-\frac{7bgx^2}{24c^2}\sqrt{cx^2+bx+a}+\frac{35b^2gx}{96c^3}\sqrt{cx^2+bx+a}-\frac{35b^3g}{64c^4}\sqrt{cx^2+bx+a}+\frac{35b^4g}{128}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/4*g*x^3*(c*x^2+b*x+a)^(1/2)/c-7/24*g*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)+35/96*
g*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)-35/64*g*b^3/c^4*(c*x^2+b*x+a)^(1/2)+35/128*
g*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-15/16*g*b^2/c^(7/
2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+55/48*g*b/c^3*a*(c*x^2+b*x
+a)^(1/2)-3/8*g*a/c^2*x*(c*x^2+b*x+a)^(1/2)+3/8*g*a^2/c^(5/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*f*x^2/c*(c*x^2+b*x+a)^(1/2)-5/12*f*b/c^2
*x*(c*x^2+b*x+a)^(1/2)+5/8*f*b^2/c^3*(c*x^2+b*x+a)^(1/2)-5/16*f*b^3/c^(7/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/4*f*b/c^(5/2)*a*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/3*f*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*e*x/c*(
c*x^2+b*x+a)^(1/2)-3/4*e*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*e*b^2/c^(5/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*e*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))+d/c*(c*x^2+b*x+a)^(1/2)-1/2*d*b/c^(3/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62188, size = 1173, normalized size = 4.79

$$\left[-\frac{3\left(64\,bc^3d-16\left(3\,b^2c^2-4\,ac^3\right)e+8\left(5\,b^3c-12\,abc^2\right)f-\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)g\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c
^2)*f - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(48*c
^4*g*x^3 + 192*c^4*d - 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^
2*c^2 - 16*a*c^3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*
```

```
f + (35*b^2*c^2 - 36*a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/384*(3*(64*
b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c^2)*f - (35*b^4
 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*g*x^3 + 192*c^4*d
- 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^2*c^2 - 16*a*c^3)*f -
 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*f + (35*b^2*c^2 - 36*
a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(x*(d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.19921, size = 308, normalized size = 1.26

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,gx}{c}+\frac{8\,c^3f-7\,bc^2g}{c^4}\right)x-\frac{40\,bc^2f-35\,b^2cg+36\,ac^2g-48\,c^3e}{c^4}\right)x+\frac{192\,c^3d+120\,b^2cf-128}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*g*x/c + (8*c^3*f - 7*b*c^2*g)/c^4)*x -
 (40*b*c^2*f - 35*b^2*c*g + 36*a*c^2*g - 48*c^3*e)/c^4)*x + (192*c^3*d + 12
0*b^2*c*f - 128*a*c^2*f - 105*b^3*g + 220*a*b*c*g - 144*b*c^2*e)/c^4) + 1/1
28*(64*b*c^3*d + 40*b^3*c*f - 96*a*b*c^2*f - 35*b^4*g + 120*a*b^2*c*g - 48*
a^2*c^2*g - 48*b^2*c^2*e + 64*a*c^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*sqrt(c) - b))/c^(9/2)
```

3.282 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=177

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{16c^{7/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c\right.}{24c^3}$$

```
[Out] ((24*c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)*Sqrt[a + b*x + c*x^2])/(24*c^3
) + ((6*c*f - 5*b*g)*x*Sqrt[a + b*x + c*x^2])/(12*c^2) + (g*x^2*Sqrt[a + b*
x + c*x^2])/(3*c) + ((16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*g + 6*b*c*(b*f +
 2*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2
))
```

Rubi [A] time = 0.235167, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{16c^{7/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c\right.}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] ((24*c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)*Sqrt[a + b*x + c*x^2])/(24*c^3
) + ((6*c*f - 5*b*g)*x*Sqrt[a + b*x + c*x^2])/(12*c^2) + (g*x^2*Sqrt[a + b*
x + c*x^2])/(3*c) + ((16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*g + 6*b*c*(b*f +
 2*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2
))
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{3cd+(3ce-2ag)x+\frac{1}{2}(6cf-5bg)x^2}{\sqrt{a+bx+cx^2}}\,dx}{3c}$$

$$= \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{\frac{1}{2}\left(12c^2d-6acf+5abg\right)+\frac{1}{4}\left(24c^2e-18bcf+15b^2g-1\right.}{\sqrt{a+bx+cx^2}}}{6c^2}$$

$$= \frac{\left(24c^2e-18bcf+15b^2g-16acg\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+}}{3}$$

$$= \frac{\left(24c^2e-18bcf+15b^2g-16acg\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+}}{3}$$

$$= \frac{\left(24c^2e-18bcf+15b^2g-16acg\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+}}{3}$$

Mathematica [A] time = 0.271913, size = 141, normalized size = 0.8

$$\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)+2\sqrt{c}\sqrt{a+x(b+cx)}\left(-2c(8ag+9bf+5bgx\right.}{48c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*g - 2*c*(9*b*f + 8*a*g + 5*b*g*x)
+ 4*c^2*(6*e + x*(3*f + 2*g*x))) + 3*(16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*
g + 6*b*c*(b*f + 2*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])/(48*c^(7/2))
```

Maple [B] time = 0.086, size = 333, normalized size = 1.9

$$\frac{gx^2}{3c}\sqrt{cx^2+bx+a}-\frac{5\,bgx}{12\,c^2}\sqrt{cx^2+bx+a}+\frac{5\,b^2g}{8\,c^3}\sqrt{cx^2+bx+a}-\frac{5\,b^3g}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{7}{2}}+\frac{3\,bga}{4}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c-5/12*g*b/c^2*x*(c*x^2+b*x+a)^(1/2)+5/8*g*b^
2/c^3*(c*x^2+b*x+a)^(1/2)-5/16*g*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))+3/4*g*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
-2/3*g*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*f*x*(c*x^2+b*x+a)^(1/2)/c-3/4*f*b/c^2*
(c*x^2+b*x+a)^(1/2)+3/8*f*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))-1/2*f*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e/c*(c*x
^2+b*x+a)^(1/2)-1/2*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
+d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53496, size = 803, normalized size = 4.54

$$\left[\frac{3\left(16\,c^3d-8\,bc^2e+2\left(3\,b^2c-4\,ac^2\right)f-\left(5\,b^3-12\,abc\right)g\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-\right.}{96\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b*
c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c
*x + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2
*c - 16*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4, -
1/48*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b*c
)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^
2 + b*c*x + a*c)) - 2*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2*c - 16
*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.18691, size = 201, normalized size = 1.14

$$\frac{1}{24}\,\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,gx}{c}+\frac{6\,c^2f-5\,bcg}{c^3}\right)x-\frac{18\,bcf-15\,b^2g+16\,acg-24\,c^2e}{c^3}\right)-\frac{\left(16\,c^3d+6\,b^2cf-8\,ac^2f-5\,b^3g+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*g*x/c + (6*c^2*f - 5*b*c*g)/c^3)*x - (18*b
*c*f - 15*b^2*g + 16*a*c*g - 24*c^2*e)/c^3) - 1/16*(16*c^3*d + 6*b^2*c*f -
8*a*c^2*f - 5*b^3*g + 12*a*b*c*g - 8*b*c^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

3.283 $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=155

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{4c^2}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+\frac{gx\sqrt{a}\cdots}{}$$

```
[Out] ((4*c*f - 3*b*g)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (g*x*Sqrt[a + b*x + c*x^2
])/(2*c) - (d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[
a] + ((8*c^2*e + 3*b^2*g - 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rubi [A] time = 0.256308, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1653, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{4c^2}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+\frac{gx\sqrt{a}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*c*f - 3*b*g)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (g*x*Sqrt[a + b*x + c*x^2
])/(2*c) - (d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[
a] + ((8*c^2*e + 3*b^2*g - 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \frac{gx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd+(2ce-ag)x+\frac{1}{2}(4cf-3bg)x^2}{x\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2c^2d+\frac{1}{4}\left(8c^2e+3b^2g-4c(bf+ag)\right)x}{x\sqrt{a+bx+cx^2}}\,dx}{2c^2}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} + d\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx + \frac{\left(8c^2e+3b^2g-\right.}{}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} - (2d)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+c}}\right.$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} - \frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} + \frac{\left(8c^2e+3b^2g-\right.}{}$$

Mathematica [A] time = 0.384614, size = 134, normalized size = 0.86

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}(-3bg+4cf+2cgx)}{4c^2} - \frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*c*f - 3*b*g + 2*c*g*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) - (d*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + ((8*c^2*e + 3*b^2*g -
 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(
8*c^(5/2))
```

Maple [A] time = 0.052, size = 220, normalized size = 1.4

$$\frac{gx}{2c}\sqrt{cx^2+bx+a} - \frac{3bg}{4c^2}\sqrt{cx^2+bx+a} + \frac{3b^2g}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{5}{2}} - \frac{ag}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*g*x*(c*x^2+b*x+a)^(1/2)/c-3/4*g*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*g*b^2/c^(
5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*g*a/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+f/c*(c*x^2+b*x+a)^(1/2)-1/2*f*b/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)-d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 19.8931, size = 1759, normalized size = 11.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b
*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b
^2 - 4*a^2*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*
g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/8*(4*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^
2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2)
- (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2
*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/16*(16*sqrt
(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 +
 a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(c)*lo
g(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(
a*c^3), 1/8*(8*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*
sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a
```

```
^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^
2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2
+ b*x + a))/(a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2 + gx^3}{x\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.284 $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=139

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{g\sqrt{a+bx+cx^2}}{c}$$

```
[Out] (g*Sqrt[a + b*x + c*x^2])/c - (d*Sqrt[a + b*x + c*x^2])/(a*x) + ((b*d - 2*a
*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)) + (
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^
(3/2))
```

Rubi [A] time = 0.235271, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1650, 1653, 843, 621, 206, 724}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{g\sqrt{a+bx+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*Sqrt[a + b*x + c*x^2])/c - (d*Sqrt[a + b*x + c*x^2])/(a*x) + ((b*d - 2*a
*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)) + (
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^
(3/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}(bd-2ae)-afx-agx^2}{x\sqrt{a+bx+cx^2}}\,dx}{a}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}c(bd-2ae)-\frac{1}{2}a(2cf-bg)x}{x\sqrt{a+bx+cx^2}}\,dx}{ac}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{(bd-2ae)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2a} + \frac{(2cf-bg)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{a} + \frac{(2cf-bg}{}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}} + \frac{(2cf-bg)\tanh^{-1}}{2c^{3/2}}$$

Mathematica [A] time = 0.403244, size = 127, normalized size = 0.91

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2a^{3/2}} + \frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2c^{3/2}} + \frac{\sqrt{a+x(b+cx)}(agx-cd)}{acx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-(c*d) + a*g*x)*Sqrt[a + x*(b + c*x)])/(a*c*x) + ((b*d - 2*a*e)*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(2*a^(3/2)) + ((2*c*f - b*g)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*c^(3/2))
```

Maple [A] time = 0.054, size = 173, normalized size = 1.2

$$\frac{g}{c}\sqrt{cx^2+bx+a} - \frac{bg}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{3}{2}} + f\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}} - e\ln\left(\frac{1}{x}\left(2a+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g*(c*x^2+b*x+a)^(1/2)/c-1/2*g*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))+f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-e/a^(1/2)*ln
((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-d*(c*x^2+b*x+a)^(1/2)/a/x+1/2*d
*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.8985, size = 1658, normalized size = 11.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*((2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + (b*c^2*d - 2*a*c^2*e)*s
qrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*x + a)
)/(a^2*c^2*x), -1/4*(2*(2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + (b*c^2*d - 2*
a*c^2*e)*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2
+ b*x + a))/(a^2*c^2*x), -1/4*(2*(b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (2*
a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^
2 + b*x + a))/(a^2*c^2*x), -1/2*((b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (2*
a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*
```

```
sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*
x + a))/(a^2*c^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**2*sqrt(a + b*x + c*x**2)), x)
```

Giac [A] time = 1.25539, size = 231, normalized size = 1.66

$$\frac{\sqrt{cx^2+bx+a}\,g}{c} - \frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{\left(2\,cf-bg\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}} + \frac{\left(\sqrt{c}x - \cdots\right.}{\left(\left(\sqrt{c}x \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + b*x + a)*g/c - (b*d - 2*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))/sqrt(-a))/(sqrt(-a)*a) - 1/2*(2*c*f - b*g)*log(abs(2*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2) + ((sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b*d + 2*a*sqrt(c)*d)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a
)*a)
```

3.285 $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=159

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2f-4abe-4acd+3b^2d\right)}{8a^{5/2}}+\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{4a^2x}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(2*a*x^2) + ((3*b*d - 4*a*e)*Sqrt[a + b*x + c*x^
2])/(4*a^2*x) - ((3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*ArcTanh[(2*a + b*x
)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)) + (g*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]
```

Rubi [A] time = 0.244307, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1650, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2f-4abe-4acd+3b^2d\right)}{8a^{5/2}}+\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{4a^2x}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(2*a*x^2) + ((3*b*d - 4*a*e)*Sqrt[a + b*x + c*x^
2])/(4*a^2*x) - ((3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*ArcTanh[(2*a + b*x
)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)) + (g*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} - \frac{\int \frac{\frac{1}{2}(3bd-4ae)+(cd-2af)x-2agx^2}{x^2\sqrt{a+bx+cx^2}}\,dx}{2a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} + \frac{\int \frac{\frac{1}{4}\left(3b^2d-4abe-4a(cd-2af)\right)+2a^2gx}{x\sqrt{a+bx+cx^2}}\,dx}{2a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} + \frac{\left(3b^2d-4acd-4abe+8a^2f\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}}{8a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} - \frac{\left(3b^2d-4acd-4abe+8a^2f\right)\text{Subst}\left(\int \frac{}{4a}\right.}{4a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} - \frac{\left(3b^2d-4acd-4abe+8a^2f\right)\tanh^{-1}\left(\frac{}{2\sqrt{a}}\right.}{8a^{5/2}}$$

Mathematica [A] time = 0.371441, size = 137, normalized size = 0.86

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\left(4abe+4a(cd-2af)-3b^2d\right)}{8a^{5/2}} + \frac{\sqrt{a+x(b+cx)}(3bdx-2a(d+2ex))}{4a^2x^2} + \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(3*b*d*x - 2*a*(d + 2*e*x)))/(4*a^2*x^2) + ((-3*b^2*
d + 4*a*b*e + 4*a*(c*d - 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*
(b + c*x)])])/(8*a^(5/2)) + (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)])])/Sqrt[c]
```

Maple [A] time = 0.056, size = 241, normalized size = 1.5

$$g\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}} - f\ln\left(\frac{1}{x}\left(2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{a}} - \frac{d}{2ax^2}\sqrt{cx^2+bx+a} + \frac{3bd}{4a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-f/a^(1/2)*ln((2*a+b*x
+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-1/2*d*(c*x^2+b*x+a)^(1/2)/a/x^2+3/4*d*b/
a^2/x*(c*x^2+b*x+a)^(1/2)-3/8*d*b^2/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)+1/2*d*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
)-e/a/x*(c*x^2+b*x+a)^(1/2)+1/2*e*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 18.345, size = 1837, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(8*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c -
 4*a*c^2)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 +
 b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(2*a^2*c*d - (3*a*b*c*d - 4
*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/16*(16*a^3*sqrt(-c)*g*x
^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
 a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(a)*x^2*log(-(
8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) +
 8*a^2)/x^2) + 4*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x +
 a))/(a^3*c*x^2), 1/8*(4*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*
c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*a^2*c*d - (3*a*b*c*d -
```

```
 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/8*(8*a^3*sqrt(-c)*g*x
^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
 a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) +
 2*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^
2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] time = 1.29618, size = 475, normalized size = 2.99

$$-\frac{g\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)c-b\sqrt{c}\right|\right)}{\sqrt{c}}+\frac{\left(3b^2d-4acd+8a^2f-4abe\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a^2}-\frac{3\left(\sqrt{c}x-\sqrt{c\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -g*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*c - b*sqrt(c)))/sqrt(c) +
 1/4*(3*b^2*d - 4*a*c*d + 8*a^2*f - 4*a*b*e)*arctan(-(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b^2*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^2*sqrt(c)*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d - 4*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^2*b*e - 8*a^2*b*sqrt(c)*d + 8*a^3*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2 - a)^2*a^2)
```

3.286 $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=186

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{16a^{7/2}}-\frac{\sqrt{a+bx+cx^2}\left(24a^2f-18abe-16acd+15b^2\right.}{24a^3x}$$

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(3*a*x^3) + ((5*b*d - 6*a*e)*Sqrt[a + b*x + c*x^
2])/(12*a^2*x^2) - ((15*b^2*d - 16*a*c*d - 18*a*b*e + 24*a^2*f)*Sqrt[a + b*
x + c*x^2])/(24*a^3*x) + ((5*b^3*d - 6*a*b^2*e - 4*a*b*(3*c*d - 2*a*f) + 8*
a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
(16*a^(7/2))
```

Rubi [A] time = 0.319922, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1650, 806, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{16a^{7/2}}-\frac{\sqrt{a+bx+cx^2}\left(24a^2f-18abe-16acd+15b^2\right.}{24a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```



```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(3*a*x^3) + ((5*b*d - 6*a*e)*Sqrt[a + b*x + c*x^
2])/(12*a^2*x^2) - ((15*b^2*d - 16*a*c*d - 18*a*b*e + 24*a^2*f)*Sqrt[a + b*
x + c*x^2])/(24*a^3*x) + ((5*b^3*d - 6*a*b^2*e - 4*a*b*(3*c*d - 2*a*f) + 8*
a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
(16*a^(7/2))
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} - \frac{\int \frac{\frac{1}{2}(5bd-6ae)+(2cd-3af)x-3agx^2}{x^3\sqrt{a+bx+cx^2}}\,dx}{3a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} + \frac{\int \frac{\frac{1}{4}\left(15b^2d-16acd-18abe+24a^2f\right)+\frac{1}{2}\left(5bcd-6ace+12a^2\right.}{x^2\sqrt{a+bx+cx^2}}}{6a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{\left(15b^2d-16acd-18abe+24a^2f\right)\sqrt{a+bx}}{24a^3x}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{\left(15b^2d-16acd-18abe+24a^2f\right)\sqrt{a+bx}}{24a^3x}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{\left(15b^2d-16acd-18abe+24a^2f\right)\sqrt{a+bx}}{24a^3x}$$

Mathematica [A] time = 0.313395, size = 150, normalized size = 0.81

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\left(8a^2(ce-2ag)-6ab^2e+4ab(2af-3cd)+5b^3d\right)}{16a^{7/2}}-\frac{\sqrt{a+x(b+cx)}\left(4a^2(2d+3x(e+2fx))-2ax\right.}{24a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(Sqrt[a + x*(b + c*x)]*(15*b^2*d*x^2 - 2*a*x*(5*b*d + 8*c*d*x + 9*b*e*x) +
 4*a^2*(2*d + 3*x*(e + 2*f*x))))/(24*a^3*x^3) + ((5*b^3*d - 6*a*b^2*e + 4*a
*b*(-3*c*d + 2*a*f) + 8*a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])])/(16*a^(7/2))
```

Maple [B] time = 0.057, size = 375, normalized size = 2.

$$-g\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{d}{3\,ax^3}\sqrt{cx^2+bx+a}+\frac{5\,bd}{12\,a^2x^2}\sqrt{cx^2+bx+a}-\frac{5\,b^2d}{8\,xa^3}\sqrt{cx^2+bx+a}+\frac{5}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -g/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-1/3*d*(c*x^2+b*x+a
)^(1/2)/a/x^3+5/12*d*b/a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*d*b^2/a^3/x*(c*x^2+b
*x+a)^(1/2)+5/16*d*b^3/a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
)-3/4*d*b/a^(5/2)*c*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+2/3*d*c/a
^2/x*(c*x^2+b*x+a)^(1/2)-1/2*e/a/x^2*(c*x^2+b*x+a)^(1/2)+3/4*e*b/a^2/x*(c*x
^2+b*x+a)^(1/2)-3/8*e*b^2/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2)
)/x)+1/2*e*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-f/a/x*(c
*x^2+b*x+a)^(1/2)+1/2*f*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2)
)/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 21.0011, size = 848, normalized size = 4.56

$$\left[-\frac{3\left(8a^2bf-16a^3g+\left(5b^3-12abc\right)d-2\left(3ab^2-4a^2c\right)e\right)\sqrt{a}x^3\log\left(-\frac{8abx+\left(b^2+4ac\right)x^2-4\sqrt{cx^2+bx+a}(bx+2a)\sqrt{a}+8a^2}{x^2}\right)+4\left(8a\right.}{96a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/96*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d - 2*(3*a*b^2 - 4*a^2
*c)*e)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(8*a^3*d - (18*a^2*b*e - 24*a^3*f
 - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^2 + b
*x + a))/(a^4*x^3), -1/48*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d -
 2*(3*a*b^2 - 4*a^2*c)*e)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*
x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(8*a^3*d - (18*a^2*b*e - 24*
a^3*f - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^
2 + b*x + a))/(a^4*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**4*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] time = 1.18562, size = 930, normalized size = 5.

$$-\frac{\left(5\,b^3d-12\,abcd+8\,a^2bf-16\,a^3g-6\,ab^2e+8\,a^2ce\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^3}+\frac{15\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5b^3d-36\left(\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(5*b^3*d - 12*a*b*c*d + 8*a^2*b*f - 16*a^3*g - 6*a*b^2*e + 8*a^2*c*e)*
arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/24
*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d - 36*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a*b*c*d + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*f
 - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*e + 24*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a^2*c*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*
sqrt(c)*f - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d + 96*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^3*b*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*e + 96*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*d - 96*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^4*sqrt(c)*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a^3*b*sqrt(c)*e + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*d + 36*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d + 24*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^4*b*f - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*e - 24*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c*e + 48*a^3*b^2*sqrt(c)*d - 32*a^4*c
^(3/2)*d + 48*a^5*sqrt(c)*f - 48*a^4*b*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2 - a)^3*a^3)
```

3.287 $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=270

$$\frac{\sqrt{a+bx+cx^2}\left(64a^2(2ce-3ag)-120ab^2e-4ab(55cd-36af)+105b^3d\right)}{192a^4x}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(32a^2b(3ce-2ag)+\right.}{}$$

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(4*a*x^4) + ((7*b*d - 8*a*e)*Sqrt[a + b*x + c*x^
2])/(24*a^2*x^3) - ((35*b^2*d - 36*a*c*d - 40*a*b*e + 48*a^2*f)*Sqrt[a + b*
x + c*x^2])/(96*a^3*x^2) + ((105*b^3*d - 120*a*b^2*e - 4*a*b*(55*c*d - 36*a
*f) + 64*a^2*(2*c*e - 3*a*g))*Sqrt[a + b*x + c*x^2])/(192*a^4*x) - ((35*b^4
*d - 40*a*b^3*e + 16*a^2*c*(3*c*d - 4*a*f) - 24*a*b^2*(5*c*d - 2*a*f) + 32*
a^2*b*(3*c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/(128*a^(9/2))
```

Rubi [A] time = 0.487795, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1650, 834, 806, 724, 206}

$$\frac{\sqrt{a+bx+cx^2}\left(64a^2(2ce-3ag)-120ab^2e-4ab(55cd-36af)+105b^3d\right)}{192a^4x}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(32a^2b(3ce-2ag)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(4*a*x^4) + ((7*b*d - 8*a*e)*Sqrt[a + b*x + c*x^
2])/(24*a^2*x^3) - ((35*b^2*d - 36*a*c*d - 40*a*b*e + 48*a^2*f)*Sqrt[a + b*
x + c*x^2])/(96*a^3*x^2) + ((105*b^3*d - 120*a*b^2*e - 4*a*b*(55*c*d - 36*a
*f) + 64*a^2*(2*c*e - 3*a*g))*Sqrt[a + b*x + c*x^2])/(192*a^4*x) - ((35*b^4
*d - 40*a*b^3*e + 16*a^2*c*(3*c*d - 4*a*f) - 24*a*b^2*(5*c*d - 2*a*f) + 32*
a^2*b*(3*c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/(128*a^(9/2))
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
```

```
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} - \frac{\int \frac{\frac{1}{2}(7bd-8ae)+(3cd-4af)x-4agx^2}{x^4\sqrt{a+bx+cx^2}}\,dx}{4a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} + \frac{\int \frac{\frac{1}{4}\left(35b^2d-40abe-12a(3cd-4af)\right)+\left(7bcd-8ace+12a^2g\right.}{x^3\sqrt{a+bx+cx^2}}}{12a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)\sqrt{a+b}}{96a^3x^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)\sqrt{a+b}}{96a^3x^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)\sqrt{a+b}}{96a^3x^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)\sqrt{a+b}}{96a^3x^2}$$

Mathematica [A] time = 0.555585, size = 212, normalized size = 0.79

$$\frac{\sqrt{a+x(b+cx)}\left(8a^2x(7bd+2bx(5e+9fx)+cx(9d+16ex))-16a^3\left(3d+4ex+6x^2(f+2gx)\right)-10abx^2(7bd+12bex+\right.}{192a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(105*b^3*d*x^3 - 10*a*b*x^2*(7*b*d + 22*c*d*x + 12*b
*e*x) + 8*a^2*x*(7*b*d + c*x*(9*d + 16*e*x) + 2*b*x*(5*e + 9*f*x)) - 16*a^3
*(3*d + 4*e*x + 6*x^2*(f + 2*g*x))))/(192*a^4*x^4) - ((35*b^4*d - 40*a*b^3*
e + 16*a^2*c*(3*c*d - 4*a*f) + 24*a*b^2*(-5*c*d + 2*a*f) + 32*a^2*b*(3*c*e
- 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(128*a^(9
/2))
```

Maple [B] time = 0.061, size = 591, normalized size = 2.2

$$-\frac{e}{3\,ax^3}\sqrt{cx^2+bx+a} + \frac{5\,be}{12\,a^2x^2}\sqrt{cx^2+bx+a} - \frac{5\,b^2e}{8\,xa^3}\sqrt{cx^2+bx+a} + \frac{5\,eb^3}{16}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)a^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3*e/a/x^3*(c*x^2+b*x+a)^(1/2)+5/12*e*b/a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*e
*b^2/a^3/x*(c*x^2+b*x+a)^(1/2)+5/16*e*b^3/a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*
x^2+b*x+a)^(1/2))/x)-3/4*e*b/a^(5/2)*c*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^
(1/2))/x)+2/3*e*c/a^2/x*(c*x^2+b*x+a)^(1/2)-1/4*d*(c*x^2+b*x+a)^(1/2)/a/x^4
+7/24*d*b/a^2/x^3*(c*x^2+b*x+a)^(1/2)-35/96*d*b^2/a^3/x^2*(c*x^2+b*x+a)^(1/
2)+35/64*d*b^3/a^4/x*(c*x^2+b*x+a)^(1/2)-35/128*d*b^4/a^(9/2)*ln((2*a+b*x+2
*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+15/16*d*b^2/a^(7/2)*c*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x)-55/48*d*b/a^3*c/x*(c*x^2+b*x+a)^(1/2)+3/8*d*c/a^2
/x^2*(c*x^2+b*x+a)^(1/2)-3/8*d*c^2/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x
+a)^(1/2))/x)-1/2*f/a/x^2*(c*x^2+b*x+a)^(1/2)+3/4*f*b/a^2/x*(c*x^2+b*x+a)^(
1/2)-3/8*f*b^2/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/2*f*
c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-g/a/x*(c*x^2+b*x+a)
^(1/2)+1/2*g*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 42.7422, size = 1216, normalized size = 4.5

$$\left[\frac{3\left(64\,a^{3}bg-\left(35\,b^{4}-120\,ab^{2}c+48\,a^{2}c^{2}\right)d+8\left(5\,ab^{3}-12\,a^{2}bc\right)e-16\left(3\,a^{2}b^{2}-4\,a^{3}c\right)f\right)\sqrt{a}x^{4}\log\left(-\frac{8\,abx+\left(b^{2}+4\,ac\right)x^{2}+4\,\sqrt{cx^{2}}}{x^{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*a*b^3
- 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(a)*x^4*log(-(8*a*b*x + (
b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2
) - 4*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d -
8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 -
 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*
x^4), -1/384*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*
a*b^3 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(-a)*x^4*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(4
8*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d - 8*(15*a^
2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 - 36*a^3*
c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2 + gx^3}{x^5\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**5/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**5*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] time = 1.21294, size = 1955, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(35*b^4*d - 120*a*b^2*c*d + 48*a^2*c^2*d + 48*a^2*b^2*f - 64*a^3*c*f -
 64*a^3*b*g - 40*a*b^3*e + 96*a^2*b*c*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/192*(105*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c*d + 14
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^2*d + 144*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^2*b^2*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^
3*c*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*g - 120*(sqrt(c)*x
```

```
- sqrt(c*x^2 + b*x + a))^7*a*b^3*e + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^7*a^2*b*c*e - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*sqrt(c)*g - 3
85*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*d + 1320*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*a^2*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a^3*c^2*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*f + 192*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c*f + 576*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a^4*b*g + 440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*e - 1
056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c*e - 384*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*a^4*b*sqrt(c)*f + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a^5*sqrt(c)*g - 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(3/2
)*e + 511*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*d - 1752*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a^4*c^2*d + 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*f +
192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c*f - 576*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a^5*b*g - 584*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*
b^3*e + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c*e - 2048*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(3/2)*d + 768*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a^5*b*sqrt(c)*f - 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a^6*sqrt(c)*g - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*sqrt(c)*
e + 1024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(3/2)*e - 279*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^4*b^2*c*d + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^2*d - 240*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*f - 192*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^6*c*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*g + 26
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*e + 288*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^5*b*c*e - 384*a^4*b^3*sqrt(c)*d + 512*a^5*b*c^(3/2)*d - 38
4*a^6*b*sqrt(c)*f + 384*a^7*sqrt(c)*g + 384*a^5*b^2*sqrt(c)*e - 256*a^6*c^(
3/2)*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^4*a^4)
```

3.288 $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=371

$$-\frac{\sqrt{a+bx+cx^2}\left(40a^2b(55ce-36ag)+256a^2c(4cd-5af)-60ab^2(49cd-20af)-1050ab^3e+945b^4d\right)}{1920a^5x}+\frac{\sqrt{a+bx+c\ldots}}{}$$

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(5*a*x^5) + ((9*b*d - 10*a*e)*Sqrt[a + b*x + c*x
^2])/(40*a^2*x^4) - ((63*b^2*d - 64*a*c*d - 70*a*b*e + 80*a^2*f)*Sqrt[a + b
*x + c*x^2])/(240*a^3*x^3) + ((315*b^3*d - 350*a*b^2*e - 4*a*b*(161*c*d - 1
00*a*f) + 120*a^2*(3*c*e - 4*a*g))*Sqrt[a + b*x + c*x^2])/(960*a^4*x^2) - (
(945*b^4*d - 1050*a*b^3*e - 60*a*b^2*(49*c*d - 20*a*f) + 256*a^2*c*(4*c*d -
 5*a*f) + 40*a^2*b*(55*c*e - 36*a*g))*Sqrt[a + b*x + c*x^2])/(1920*a^5*x) +
 ((63*b^5*d - 70*a*b^4*e + 48*a^2*b*c*(5*c*d - 4*a*f) - 40*a*b^3*(7*c*d - 2
*a*f) - 32*a^3*c*(3*c*e - 4*a*g) + 48*a^2*b^2*(5*c*e - 2*a*g))*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(256*a^(11/2))
```

Rubi [A] time = 0.81717, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1650, 834, 806, 724, 206}

$$-\frac{\sqrt{a+bx+cx^2}\left(40a^2b(55ce-36ag)+256a^2c(4cd-5af)-60ab^2(49cd-20af)-1050ab^3e+945b^4d\right)}{1920a^5x}+\frac{\sqrt{a+bx+c\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(5*a*x^5) + ((9*b*d - 10*a*e)*Sqrt[a + b*x + c*x
^2])/(40*a^2*x^4) - ((63*b^2*d - 64*a*c*d - 70*a*b*e + 80*a^2*f)*Sqrt[a + b
*x + c*x^2])/(240*a^3*x^3) + ((315*b^3*d - 350*a*b^2*e - 4*a*b*(161*c*d - 1
00*a*f) + 120*a^2*(3*c*e - 4*a*g))*Sqrt[a + b*x + c*x^2])/(960*a^4*x^2) - (
(945*b^4*d - 1050*a*b^3*e - 60*a*b^2*(49*c*d - 20*a*f) + 256*a^2*c*(4*c*d -
 5*a*f) + 40*a^2*b*(55*c*e - 36*a*g))*Sqrt[a + b*x + c*x^2])/(1920*a^5*x) +
 ((63*b^5*d - 70*a*b^4*e + 48*a^2*b*c*(5*c*d - 4*a*f) - 40*a*b^3*(7*c*d - 2
*a*f) - 32*a^3*c*(3*c*e - 4*a*g) + 48*a^2*b^2*(5*c*e - 2*a*g))*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(256*a^(11/2))
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} - \frac{\int \frac{\frac{1}{2}(9bd-10ae)+(4cd-5af)x-5agx^2}{x^5\sqrt{a+bx+cx^2}}\,dx}{5a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} + \frac{\int \frac{\frac{1}{4}\left(63b^2d-64acd-70abe+80a^2f\right)+\frac{1}{2}\left(27bcd-30ace+\right.}{x^4\sqrt{a+bx+cx^2}}}{20a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)\sqrt{a+b}}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)\sqrt{a+b}}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)\sqrt{a+b}}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)\sqrt{a+b}}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)\sqrt{a+b}}{240a^3x^3}$$

Mathematica [A] time = 0.78017, size = 299, normalized size = 0.81

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\left(-48a^2b^2(2ag-5ce)-48a^2bc(4af-5cd)+32a^3c(4ag-3ce)+40ab^3(2af-7cd)-70ab^4e+63\right.}{256a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(Sqrt[a + x*(b + c*x)]*(945*b^4*d*x^4 - 210*a*b^2*x^3*(3*b*d + 14*c*d*x +
5*b*e*x) + 32*a^4*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)) + 4*a^2*x^2*(256*c^2
*d*x^2 + 2*b*c*x*(161*d + 275*e*x) + b^2*(126*d + 25*x*(7*e + 12*f*x))) - 1
6*a^3*x*(c*x*(32*d + 5*x*(9*e + 16*f*x)) + b*(27*d + 5*x*(7*e + 2*x*(5*f +
9*g*x))))))/(1920*a^5*x^5) + ((63*b^5*d - 70*a*b^4*e + 40*a*b^3*(-7*c*d + 2
*a*f) - 48*a^2*b*c*(-5*c*d + 4*a*f) - 48*a^2*b^2*(-5*c*e + 2*a*g) + 32*a^3*
c*(-3*c*e + 4*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)]])
/(256*a^(11/2))
```

Maple [B] time = 0.062, size = 859, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 7/24*e*b/a^2/x^3*(c*x^2+b*x+a)^(1/2)+3/4*g*b/a^2/x*(c*x^2+b*x+a)^(1/2)+5/12
*f*b/a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*f*b^2/a^3/x*(c*x^2+b*x+a)^(1/2)-3/4*f*
b/a^(5/2)*c*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+2/3*f*c/a^2/x*(c*
x^2+b*x+a)^(1/2)+21/64*d*b^3/a^4/x^2*(c*x^2+b*x+a)^(1/2)+9/40*d*b/a^2/x^4*(
c*x^2+b*x+a)^(1/2)-21/80*d*b^2/a^3/x^3*(c*x^2+b*x+a)^(1/2)+15/16*e*b^2/a^(7
/2)*c*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-35/96*e*b^2/a^3/x^2*(c*
x^2+b*x+a)^(1/2)+63/256*d*b^5/a^(11/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^
(1/2))/x)-1/4*e/a/x^4*(c*x^2+b*x+a)^(1/2)-35/128*e*b^4/a^(9/2)*ln((2*a+b*x+
2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-3/8*e*c^2/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(
c*x^2+b*x+a)^(1/2))/x)-1/2*g/a/x^2*(c*x^2+b*x+a)^(1/2)-3/8*g*b^2/a^(5/2)*ln
((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/2*g*c/a^(3/2)*ln((2*a+b*x+2*a
^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-1/3*f/a/x^3*(c*x^2+b*x+a)^(1/2)+5/16*f*b^3/a
^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+3/8*e*c/a^2/x^2*(c*x^2
+b*x+a)^(1/2)+35/64*e*b^3/a^4/x*(c*x^2+b*x+a)^(1/2)-63/128*d*b^4/a^5/x*(c*x
^2+b*x+a)^(1/2)-35/32*d*b^3/a^(9/2)*c*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(
1/2))/x)+15/16*d*b/a^(7/2)*c^2*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
)+4/15*d*c/a^2/x^3*(c*x^2+b*x+a)^(1/2)-8/15*d*c^2/a^3/x*(c*x^2+b*x+a)^(1/2)
-1/5*d*(c*x^2+b*x+a)^(1/2)/a/x^5-55/48*e*b/a^3*c/x*(c*x^2+b*x+a)^(1/2)+49/3
2*d*b^2/a^4*c/x*(c*x^2+b*x+a)^(1/2)-161/240*d*b/a^3*c/x^2*(c*x^2+b*x+a)^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 115.592, size = 1708, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/7680*(15*((63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a
^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 -
4*a^4*c)*g)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 +
b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(384*a^5*d - (1440*a^4*b*g -
 (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50*(21*a^2*b^3 - 44*a^3*b*
c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a^4*b*f - 480*a^5*g + 7*(
45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^4*c)*e)*x^3 - 8*(70*a^4*
b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 48*(9*a^4*b*d - 10*a^5*e)
*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5), -1/3840*(15*((63*b^5 - 280*a*b^3*c +
240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*
b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 - 4*a^4*c)*g)*sqrt(-a)*x^5*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(384
*a^5*d - (1440*a^4*b*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50
*(21*a^2*b^3 - 44*a^3*b*c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a
^4*b*f - 480*a^5*g + 7*(45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^
4*c)*e)*x^3 - 8*(70*a^4*b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 4
8*(9*a^4*b*d - 10*a^5*e)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2 + gx^3}{x^6\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**6/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**6*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] time = 1.26123, size = 2939, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/128*(63*b^5*d - 280*a*b^3*c*d + 240*a^2*b*c^2*d + 80*a^2*b^3*f - 192*a^3
*b*c*f - 96*a^3*b^2*g + 128*a^4*c*g - 70*a*b^4*e + 240*a^2*b^2*c*e - 96*a^3
*c^2*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^5
) + 1/1920*(945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d - 4200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d + 3600*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a^2*b*c^2*d + 1200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*
f - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c*f - 1440*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a^3*b^2*g + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^9*a^4*c*g - 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*e + 3600*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*e - 1440*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*a^3*c^2*e - 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a*b^5*d + 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*d - 16800*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*d - 5600*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^3*b^3*f + 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a^4*b*c*f + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^2*g - 3840*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*c*g + 4900*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a^2*b^4*e - 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^
2*c*e + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^2*e + 7680*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(3/2)*f + 3840*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^6*a^5*b*sqrt(c)*g + 8064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^2*b^5*d - 35840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c*d + 3072
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^2*d + 10240*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*a^4*b^3*f - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^5*a^5*b*c*f - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*g - 896
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*e + 30720*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^4*b^2*c*e + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*a^5*c^(5/2)*d + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*sqrt
(c)*f - 17920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(3/2)*f - 11520*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*sqrt(c)*g + 20480*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^5*b*c^(3/2)*e - 7110*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^3*b^5*d + 31600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c
*d + 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^2*d - 8480*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*f + 1920*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^6*b*c*f + 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^2*g
 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*c*g + 7900*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a^4*b^4*e - 13920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^5*b^2*c*e - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^2*e + 3
8400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)*d - 10240*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(5/2)*d - 7680*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^6*b^2*sqrt(c)*f + 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
```

))^2*a^7*c^(3/2)*f + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*sqrt(c)*g + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*sqrt(c)*e - 25600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(3/2)*e + 2895*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5*d + 4200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c*d - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*d + 2640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*f + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b*c*f - 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^2*g - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c*g - 2790*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^4*e - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^2*c*e + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c^2*e + 3840*a^5*b^4*sqrt(c)*d - 7680*a^6*b^2*c^(3/2)*d + 2048*a^7*c^(5/2)*d + 3840*a^7*b^2*sqrt(c)*f - 2560*a^8*c^(3/2)*f - 3840*a^8*b*sqrt(c)*g - 3840*a^6*b^3*sqrt(c)*e + 5120*a^7*b*c^(3/2)*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^5*a^5)

3.289 $\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=258

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^8}{8e^7}-\frac{2\left(85d^2e+200d^3+34de^2+2e^3\right)(d+ex)^7}{7e^7}+\frac{\left(102d^2e^2+170d^3e+300d^4+12de^3+21\right.}{6e^7}$$

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^4)/(4*e^7) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^
4 - 7*e^5)*(d + e*x)^5)/(5*e^7) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*
d*e^3 + 21*e^4)*(d + e*x)^6)/(6*e^7) - (2*(200*d^3 + 85*d^2*e + 34*d*e^2 +
2*e^3)*(d + e*x)^7)/(7*e^7) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^8)/(8*
e^7) - ((120*d + 17*e)*(d + e*x)^9)/(9*e^7) + (2*(d + e*x)^10)/e^7
```

Rubi [A] time = 0.256623, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^8}{8e^7}-\frac{2\left(85d^2e+200d^3+34de^2+2e^3\right)(d+ex)^7}{7e^7}+\frac{\left(102d^2e^2+170d^3e+300d^4+12de^3+21\right.}{6e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```


```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^4)/(4*e^7) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^
4 - 7*e^5)*(d + e*x)^5)/(5*e^7) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*
d*e^3 + 21*e^4)*(d + e*x)^6)/(6*e^7) - (2*(200*d^3 + 85*d^2*e + 34*d*e^2 +
2*e^3)*(d + e*x)^7)/(7*e^7) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^8)/(8*
e^7) - ((120*d + 17*e)*(d + e*x)^9)/(9*e^7) + (2*(d + e*x)^10)/e^7
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \int \left(\frac{\left(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6\right)}{e^6} \right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^7}$$

Mathematica [A] time = 0.0406716, size = 212, normalized size = 0.82

$$\frac{1}{8}ex^8\left(60d^2-51de+17e^2\right)+\frac{1}{7}x^7\left(-51d^2e+20d^3+51de^2-4e^3\right)+\frac{1}{6}x^6\left(51d^2e-17d^3-12de^2+21e^3\right)+\frac{1}{5}x^5\left(-12d^2e+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^3*x + (d^2*(7*d + 18*e)*x^2)/2 + d*(7*d^2 + 7*d*e + 6*e^2)*x^3 + ((-4*d
^3 + 63*d^2*e + 21*d*e^2 + 6*e^3)*x^4)/4 + ((17*d^3 - 12*d^2*e + 63*d*e^2 +
 7*e^3)*x^5)/5 + ((-17*d^3 + 51*d^2*e - 12*d*e^2 + 21*e^3)*x^6)/6 + ((20*d^
3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7)/7 + (e*(60*d^2 - 51*d*e + 17*e^2)*x^8
)/8 + ((60*d - 17*e)*e^2*x^9)/9 + 2*e^3*x^10
```

Maple [A] time = 0.043, size = 208, normalized size = 0.8

$$2e^3x^{10}+\frac{\left(60de^2-17e^3\right)x^9}{9}+\frac{\left(60d^2e-51de^2+17e^3\right)x^8}{8}+\frac{\left(20d^3-51d^2e+51de^2-4e^3\right)x^7}{7}+\frac{\left(-17d^3+51d^2e-12\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 2*e^3*x^10+1/9*(60*d*e^2-17*e^3)*x^9+1/8*(60*d^2*e-51*d*e^2+17*e^3)*x^8+1/7
*(20*d^3-51*d^2*e+51*d*e^2-4*e^3)*x^7+1/6*(-17*d^3+51*d^2*e-12*d*e^2+21*e^3
)*x^6+1/5*(17*d^3-12*d^2*e+63*d*e^2+7*e^3)*x^5+1/4*(-4*d^3+63*d^2*e+21*d*e^
2+6*e^3)*x^4+1/3*(21*d^3+21*d^2*e+18*d*e^2)*x^3+1/2*(7*d^3+18*d^2*e)*x^2+6*
d^3*x
```

Maxima [A] time = 0.984523, size = 278, normalized size = 1.08

$$2\,e^3x^{10} + \frac{1}{9}\left(60\,de^2 - 17\,e^3\right)x^9 + \frac{1}{8}\left(60\,d^2e - 51\,de^2 + 17\,e^3\right)x^8 + \frac{1}{7}\left(20\,d^3 - 51\,d^2e + 51\,de^2 - 4\,e^3\right)x^7 - \frac{1}{6}\left(17\,d^3 - 51\,d^2e +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 2*e^3*x^10 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e^
3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 51*
d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e^3)
*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3 + 7
*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

Fricas [A] time = 0.831358, size = 560, normalized size = 2.17

$$2x^{10}e^3 - \frac{17}{9}x^9e^3 + \frac{20}{3}x^9e^2d + \frac{17}{8}x^8e^3 - \frac{51}{8}x^8e^2d + \frac{15}{2}x^8ed^2 - \frac{4}{7}x^7e^3 + \frac{51}{7}x^7e^2d - \frac{51}{7}x^7ed^2 + \frac{20}{7}x^7d^3 + \frac{7}{2}x^6e^3 - 2x^6e^2d +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 2*x^10*e^3 - 17/9*x^9*e^3 + 20/3*x^9*e^2*d + 17/8*x^8*e^3 - 51/8*x^8*e^2*d
+ 15/2*x^8*e*d^2 - 4/7*x^7*e^3 + 51/7*x^7*e^2*d - 51/7*x^7*e*d^2 + 20/7*x^7
*d^3 + 7/2*x^6*e^3 - 2*x^6*e^2*d + 17/2*x^6*e*d^2 - 17/6*x^6*d^3 + 7/5*x^5*
e^3 + 63/5*x^5*e^2*d - 12/5*x^5*e*d^2 + 17/5*x^5*d^3 + 3/2*x^4*e^3 + 21/4*x
^4*e^2*d + 63/4*x^4*e*d^2 - x^4*d^3 + 6*x^3*e^2*d + 7*x^3*e*d^2 + 7*x^3*d^3
 + 9*x^2*e*d^2 + 7/2*x^2*d^3 + 6*x*d^3
```

Sympy [A] time = 0.104844, size = 230, normalized size = 0.89

$$6d^3x + 2e^3x^{10} + x^9\left(\frac{20de^2}{3} - \frac{17e^3}{9}\right) + x^8\left(\frac{15d^2e}{2} - \frac{51de^2}{8} + \frac{17e^3}{8}\right) + x^7\left(\frac{20d^3}{7} - \frac{51d^2e}{7} + \frac{51de^2}{7} - \frac{4e^3}{7}\right) + x^6\left(-\frac{17d^3}{6} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**3*x + 2*e**3*x**10 + x**9*(20*d*e**2/3 - 17*e**3/9) + x**8*(15*d**2*e/
2 - 51*d*e**2/8 + 17*e**3/8) + x**7*(20*d**3/7 - 51*d**2*e/7 + 51*d*e**2/7
- 4*e**3/7) + x**6*(-17*d**3/6 + 17*d**2*e/2 - 2*d*e**2 + 7*e**3/2) + x**5*
(17*d**3/5 - 12*d**2*e/5 + 63*d*e**2/5 + 7*e**3/5) + x**4*(-d**3 + 63*d**2*
e/4 + 21*d*e**2/4 + 3*e**3/2) + x**3*(7*d**3 + 7*d**2*e + 6*d*e**2) + x**2*
(7*d**3/2 + 9*d**2*e)
```

Giac [A] time = 1.12525, size = 311, normalized size = 1.21

$$2\,x^{10}e^{3}+\frac{20}{3}\,dx^{9}e^{2}+\frac{15}{2}\,d^{2}x^{8}e+\frac{20}{7}\,d^{3}x^{7}-\frac{17}{9}\,x^{9}e^{3}-\frac{51}{8}\,dx^{8}e^{2}-\frac{51}{7}\,d^{2}x^{7}e-\frac{17}{6}\,d^{3}x^{6}+\frac{17}{8}\,x^{8}e^{3}+\frac{51}{7}\,dx^{7}e^{2}+\frac{17}{2}\,d^{2}x^{6}e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 2*x^10*e^3 + 20/3*d*x^9*e^2 + 15/2*d^2*x^8*e + 20/7*d^3*x^7 - 17/9*x^9*e^3
- 51/8*d*x^8*e^2 - 51/7*d^2*x^7*e - 17/6*d^3*x^6 + 17/8*x^8*e^3 + 51/7*d*x^
7*e^2 + 17/2*d^2*x^6*e + 17/5*d^3*x^5 - 4/7*x^7*e^3 - 2*d*x^6*e^2 - 12/5*d^
2*x^5*e - d^3*x^4 + 7/2*x^6*e^3 + 63/5*d*x^5*e^2 + 63/4*d^2*x^4*e + 7*d^3*x
^3 + 7/5*x^5*e^3 + 21/4*d*x^4*e^2 + 7*d^2*x^3*e + 7/2*d^3*x^2 + 3/2*x^4*e^3
 + 6*d*x^3*e^2 + 9*d^2*x^2*e + 6*d^3*x
```

3.290 $\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=157

$$\frac{1}{7}x^7 \left(20d^2-34de+17e^2\right)-\frac{1}{6}x^6 \left(17d^2-34de+4e^2\right)+\frac{1}{5}x^5 \left(17d^2-8de+21e^2\right)-\frac{1}{4}x^4 \left(4d^2-42de-7e^2\right)+\frac{1}{3}x^3 \left(21d^2+\right.$$

```
[Out] 6*d^2*x + (d*(7*d + 12*e)*x^2)/2 + ((21*d^2 + 14*d*e + 6*e^2)*x^3)/3 - ((4*
d^2 - 42*d*e - 7*e^2)*x^4)/4 + ((17*d^2 - 8*d*e + 21*e^2)*x^5)/5 - ((17*d^2
 - 34*d*e + 4*e^2)*x^6)/6 + ((20*d^2 - 34*d*e + 17*e^2)*x^7)/7 + ((40*d - 1
7*e)*e*x^8)/8 + (20*e^2*x^9)/9
```

Rubi [A] time = 0.166052, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{1}{7}x^7 \left(20d^2-34de+17e^2\right)-\frac{1}{6}x^6 \left(17d^2-34de+4e^2\right)+\frac{1}{5}x^5 \left(17d^2-8de+21e^2\right)-\frac{1}{4}x^4 \left(4d^2-42de-7e^2\right)+\frac{1}{3}x^3 \left(21d^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^2*x + (d*(7*d + 12*e)*x^2)/2 + ((21*d^2 + 14*d*e + 6*e^2)*x^3)/3 - ((4*
d^2 - 42*d*e - 7*e^2)*x^4)/4 + ((17*d^2 - 8*d*e + 21*e^2)*x^5)/5 - ((17*d^2
 - 34*d*e + 4*e^2)*x^6)/6 + ((20*d^2 - 34*d*e + 17*e^2)*x^7)/7 + ((40*d - 1
7*e)*e*x^8)/8 + (20*e^2*x^9)/9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(6d^2+d(7d+12e)x+\left(21d^2+14de+6e^2\right)x^2-\left(4d^2-42d\right.\right.$$

$$= 6d^2x+\frac{1}{2}d(7d+12e)x^2+\frac{1}{3}\left(21d^2+14de+6e^2\right)x^3-\frac{1}{4}\left(4d^2-\right.$$

Mathematica [A] time = 0.0332222, size = 136, normalized size = 0.87

$$d^2\left(\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x\right)+de\left(5x^8-\frac{34x^7}{7}+\frac{17x^6}{3}-\frac{8x^5}{5}+\frac{21x^4}{2}+\frac{14x^3}{3}+6x^2\right)+\frac{e^2\left(5600x^6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] (e^2*x^3*(5040 + 4410*x + 10584*x^2 - 1680*x^3 + 6120*x^4 - 5355*x^5 + 5600
*x^6))/2520 + d^2*(6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6
+ (20*x^7)/7) + d*e*(6*x^2 + (14*x^3)/3 + (21*x^4)/2 - (8*x^5)/5 + (17*x^6)
/3 - (34*x^7)/7 + 5*x^8)
```

Maple [A] time = 0.044, size = 146, normalized size = 0.9

$$\frac{20\,e^2x^9}{9}+\frac{\left(40\,de-17\,e^2\right)x^8}{8}+\frac{\left(20\,d^2-34\,de+17\,e^2\right)x^7}{7}+\frac{\left(-17\,d^2+34\,de-4\,e^2\right)x^6}{6}+\frac{\left(17\,d^2-8\,de+21\,e^2\right)x^5}{5}+\frac{(-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 20/9*e^2*x^9+1/8*(40*d*e-17*e^2)*x^8+1/7*(20*d^2-34*d*e+17*e^2)*x^7+1/6*(-1
7*d^2+34*d*e-4*e^2)*x^6+1/5*(17*d^2-8*d*e+21*e^2)*x^5+1/4*(-4*d^2+42*d*e+7*
e^2)*x^4+1/3*(21*d^2+14*d*e+6*e^2)*x^3+1/2*(7*d^2+12*d*e)*x^2+6*d^2*x
```

Maxima [A] time = 0.95401, size = 196, normalized size = 1.25

$$\frac{20}{9}\,e^2x^9+\frac{1}{8}\left(40\,de-17\,e^2\right)x^8+\frac{1}{7}\left(20\,d^2-34\,de+17\,e^2\right)x^7-\frac{1}{6}\left(17\,d^2-34\,de+4\,e^2\right)x^6+\frac{1}{5}\left(17\,d^2-8\,de+21\,e^2\right)x^5-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*x
^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^5
```

```
- 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 + 6*
d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

Fricas [A] time = 0.840011, size = 389, normalized size = 2.48

$$\frac{20}{9}x^9e^2 - \frac{17}{8}x^8e^2 + 5x^8ed + \frac{17}{7}x^7e^2 - \frac{34}{7}x^7ed + \frac{20}{7}x^7d^2 - \frac{2}{3}x^6e^2 + \frac{17}{3}x^6ed - \frac{17}{6}x^6d^2 + \frac{21}{5}x^5e^2 - \frac{8}{5}x^5ed + \frac{17}{5}x^5d^2 + \frac{7}{4}x^4e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 20/9*x^9*e^2 - 17/8*x^8*e^2 + 5*x^8*e*d + 17/7*x^7*e^2 - 34/7*x^7*e*d + 20/
7*x^7*d^2 - 2/3*x^6*e^2 + 17/3*x^6*e*d - 17/6*x^6*d^2 + 21/5*x^5*e^2 - 8/5*
x^5*e*d + 17/5*x^5*d^2 + 7/4*x^4*e^2 + 21/2*x^4*e*d - x^4*d^2 + 2*x^3*e^2 +
 14/3*x^3*e*d + 7*x^3*d^2 + 6*x^2*e*d + 7/2*x^2*d^2 + 6*x*d^2
```

Sympy [A] time = 0.094552, size = 158, normalized size = 1.01

$$6d^2x + \frac{20e^2x^9}{9} + x^8\left(5de - \frac{17e^2}{8}\right) + x^7\left(\frac{20d^2}{7} - \frac{34de}{7} + \frac{17e^2}{7}\right) + x^6\left(-\frac{17d^2}{6} + \frac{17de}{3} - \frac{2e^2}{3}\right) + x^5\left(\frac{17d^2}{5} - \frac{8de}{5} + \frac{21e^2}{5}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**2*x + 20*e**2*x**9/9 + x**8*(5*d*e - 17*e**2/8) + x**7*(20*d**2/7 - 34
*d*e/7 + 17*e**2/7) + x**6*(-17*d**2/6 + 17*d*e/3 - 2*e**2/3) + x**5*(17*d*
*2/5 - 8*d*e/5 + 21*e**2/5) + x**4*(-d**2 + 21*d*e/2 + 7*e**2/4) + x**3*(7*
d**2 + 14*d*e/3 + 2*e**2) + x**2*(7*d**2/2 + 6*d*e)
```

Giac [A] time = 1.12306, size = 216, normalized size = 1.38

$$\frac{20}{9}x^9e^2 + 5dx^8e + \frac{20}{7}d^2x^7 - \frac{17}{8}x^8e^2 - \frac{34}{7}dx^7e - \frac{17}{6}d^2x^6 + \frac{17}{7}x^7e^2 + \frac{17}{3}dx^6e + \frac{17}{5}d^2x^5 - \frac{2}{3}x^6e^2 - \frac{8}{5}dx^5e - d^2x^4 + \frac{21}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 20/9*x^9*e^2 + 5*d*x^8*e + 20/7*d^2*x^7 - 17/8*x^8*e^2 - 34/7*d*x^7*e - 17/
6*d^2*x^6 + 17/7*x^7*e^2 + 17/3*d*x^6*e + 17/5*d^2*x^5 - 2/3*x^6*e^2 - 8/5*
d*x^5*e - d^2*x^4 + 21/5*x^5*e^2 + 21/2*d*x^4*e + 7*d^2*x^3 + 7/4*x^4*e^2 +
 14/3*d*x^3*e + 7/2*d^2*x^2 + 2*x^3*e^2 + 6*d*x^2*e + 6*d^2*x
```

3.291 $\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$

Optimal. Leaf size=93

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)-\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 - ((4*d - 21*e)*x^4)/4 +
((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x^
8)/2
```

Rubi [A] time = 0.107599, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1628}

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)-\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 - ((4*d - 21*e)*x^4)/4 +
((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x^
8)/2
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx=\int \left(6d+(7d+6e)x+7(3d+e)x^2-(4d-21e)x^3+(17d-4e)x\right.$$

$$=6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3-\frac{1}{4}(4d-21e)x^4+\frac{1}{5}(17d-$$

Mathematica [A] time = 0.0144416, size = 93, normalized size = 1.

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)+\frac{1}{4}x^4(21e-4d)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 + ((-4*d + 21*e)*x^4)/4 +
 ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x
^8)/2
```

Maple [A] time = 0.043, size = 84, normalized size = 0.9

$$\frac{5ex^8}{2}+\frac{(20d-17e)x^7}{7}+\frac{(-17d+17e)x^6}{6}+\frac{(17d-4e)x^5}{5}+\frac{(-4d+21e)x^4}{4}+\frac{(21d+7e)x^3}{3}+\frac{(7d+6e)x^2}{2}+6dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 5/2*e*x^8+1/7*(20*d-17*e)*x^7+1/6*(-17*d+17*e)*x^6+1/5*(17*d-4*e)*x^5+1/4*(
-4*d+21*e)*x^4+1/3*(21*d+7*e)*x^3+1/2*(7*d+6*e)*x^2+6*d*x
```

Maxima [A] time = 0.991202, size = 107, normalized size = 1.15

$$\frac{5}{2}ex^8+\frac{1}{7}(20d-17e)x^7-\frac{17}{6}(d-e)x^6+\frac{1}{5}(17d-4e)x^5-\frac{1}{4}(4d-21e)x^4+\frac{7}{3}(3d+e)x^3+\frac{1}{2}(7d+6e)x^2+6dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxim
a")
```

```
[Out] 5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^5
 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x
```

Fricas [A] time = 0.839921, size = 217, normalized size = 2.33

$$\frac{5}{2}x^8e - \frac{17}{7}x^7e + \frac{20}{7}x^7d + \frac{17}{6}x^6e - \frac{17}{6}x^6d - \frac{4}{5}x^5e + \frac{17}{5}x^5d + \frac{21}{4}x^4e - x^4d + \frac{7}{3}x^3e + 7x^3d + 3x^2e + \frac{7}{2}x^2d + 6xd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="frica
s")
```

```
[Out] 5/2*x^8*e - 17/7*x^7*e + 20/7*x^7*d + 17/6*x^6*e - 17/6*x^6*d - 4/5*x^5*e +
 17/5*x^5*d + 21/4*x^4*e - x^4*d + 7/3*x^3*e + 7*x^3*d + 3*x^2*e + 7/2*x^2*
d + 6*x*d
```

Sympy [A] time = 0.083282, size = 87, normalized size = 0.94

$$6dx + \frac{5ex^8}{2} + x^7\left(\frac{20d}{7} - \frac{17e}{7}\right) + x^6\left(-\frac{17d}{6} + \frac{17e}{6}\right) + x^5\left(\frac{17d}{5} - \frac{4e}{5}\right) + x^4\left(-d + \frac{21e}{4}\right) + x^3\left(7d + \frac{7e}{3}\right) + x^2\left(\frac{7d}{2} + 3e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d*x + 5*e*x**8/2 + x**7*(20*d/7 - 17*e/7) + x**6*(-17*d/6 + 17*e/6) + x**
5*(17*d/5 - 4*e/5) + x**4*(-d + 21*e/4) + x**3*(7*d + 7*e/3) + x**2*(7*d/2
+ 3*e)
```

Giac [A] time = 1.14264, size = 122, normalized size = 1.31

$$\frac{5}{2}x^8e + \frac{20}{7}dx^7 - \frac{17}{7}x^7e - \frac{17}{6}dx^6 + \frac{17}{6}x^6e + \frac{17}{5}dx^5 - \frac{4}{5}x^5e - dx^4 + \frac{21}{4}x^4e + 7dx^3 + \frac{7}{3}x^3e + \frac{7}{2}dx^2 + 3x^2e + 6dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac"
)
```

```
[Out] 5/2*x^8*e + 20/7*d*x^7 - 17/7*x^7*e - 17/6*d*x^6 + 17/6*x^6*e + 17/5*d*x^5
- 4/5*x^5*e - d*x^4 + 21/4*x^4*e + 7*d*x^3 + 7/3*x^3*e + 7/2*d*x^2 + 3*x^2*
e + 6*d*x
```

3.292 $\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$

Optimal. Leaf size=42

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

```
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Rubi [A] time = 0.0248029, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1657}

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \int \left(6+7x+21x^2-4x^3+17x^4-17x^5+20x^6\right)\,dx\\ &= 6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0014784, size = 42, normalized size = 1.

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Maple [A] time = 0.043, size = 35, normalized size = 0.8

$$6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

Maxima [A] time = 0.969959, size = 46, normalized size = 1.1

$$\frac{20}{7}x^7-\frac{17}{6}x^6+\frac{17}{5}x^5-x^4+7x^3+\frac{7}{2}x^2+6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Fricas [A] time = 0.868683, size = 84, normalized size = 2.

$$\frac{20}{7}x^7-\frac{17}{6}x^6+\frac{17}{5}x^5-x^4+7x^3+\frac{7}{2}x^2+6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Sympy [A] time = 0.063344, size = 37, normalized size = 0.88

$$\frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**2/2 + 6*x
```

Giac [A] time = 1.14635, size = 46, normalized size = 1.1

$$\frac{20}{7}\,x^7 - \frac{17}{6}\,x^6 + \frac{17}{5}\,x^5 - x^4 + 7\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

3.293 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

Optimal. Leaf size=228

$$\frac{x^4(20d^2+17de+17e^2)}{4e^3}-\frac{x^3(17d^2e+20d^3+17de^2+4e^3)}{3e^4}+\frac{x^2(17d^2e^2+17d^3e+20d^4+4de^3+21e^4)}{2e^5}-\frac{x(17d^3e^2+4$$

```
[Out] -(((20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6)
+ ((20*d^4 + 17*d^3*e + 17*d^2*e^2 + 4*d*e^3 + 21*e^4)*x^2)/(2*e^5) - ((20*
d^3 + 17*d^2*e + 17*d*e^2 + 4*e^3)*x^3)/(3*e^4) + ((20*d^2 + 17*d*e + 17*e^
2)*x^4)/(4*e^3) - ((20*d + 17*e)*x^5)/(5*e^2) + (10*x^6)/(3*e) + ((5*d^2 -
2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/
e^7
```

Rubi [A] time = 0.193499, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{x^4(20d^2+17de+17e^2)}{4e^3}-\frac{x^3(17d^2e+20d^3+17de^2+4e^3)}{3e^4}+\frac{x^2(17d^2e^2+17d^3e+20d^4+4de^3+21e^4)}{2e^5}-\frac{x(17d^3e^2+4$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```



```
[Out] -(((20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6)
+ ((20*d^4 + 17*d^3*e + 17*d^2*e^2 + 4*d*e^3 + 21*e^4)*x^2)/(2*e^5) - ((20*
d^3 + 17*d^2*e + 17*d*e^2 + 4*e^3)*x^3)/(3*e^4) + ((20*d^2 + 17*d*e + 17*e^
2)*x^4)/(4*e^3) - ((20*d + 17*e)*x^5)/(5*e^2) + (10*x^6)/(3*e) + ((5*d^2 -
2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/
e^7
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = \int\left(\frac{-20d^5-17d^4e-17d^3e^2-4d^2e^3-21de^4+7e^5}{e^6}+\frac{(20d^4+17d^3e\right.$$
$$= -\frac{(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)\,x}{e^6}+\frac{(20d^4+17d^3e$$

Mathematica [A] time = 0.0585377, size = 179, normalized size = 0.79

$$\frac{ex\left(-10d^3e^2(40x^2-51x+102)+10d^2e^3(30x^3-34x^2+51x-24)+60d^4e(10x-17)-1200d^5-5de^4(48x^4-51x^3+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] (e*x*(-1200*d^5 + 60*d^4*e*(-17 + 10*x) - 10*d^3*e^2*(102 - 51*x + 40*x^2)
+ 10*d^2*e^3*(-24 + 51*x - 34*x^2 + 30*x^3) - 5*d*e^4*(252 - 24*x + 68*x^2
- 51*x^3 + 48*x^4) + e^5*(420 + 630*x - 80*x^2 + 255*x^3 - 204*x^4 + 200*x^
5)) + 60*(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5
 + 6*e^6)*Log[d + e*x])/(60*e^7)
```

Maple [A] time = 0.05, size = 286, normalized size = 1.3

$$7\,\frac{x}{e}+6\,\frac{\ln(ex+d)}{e}-\frac{4\,x^3}{3\,e}+\frac{17\,x^4}{4\,e}-\frac{17\,x^5}{5\,e}-\frac{17\,dx^3}{3\,e^2}-\frac{17\,x^3d^2}{3\,e^3}-4\,\frac{x^5d}{e^2}+5\,\frac{x^4d^2}{e^3}+\frac{17\,dx^4}{4\,e^2}+\frac{17\,x^2d^3}{2\,e^4}-4\,\frac{d^2x}{e^3}+10\,\frac{x^2d^4}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x)
```

```
[Out] 7/e*x+6/e*ln(e*x+d)-4/3/e*x^3+17/4*x^4/e-17/5/e*x^5-17/3/e^2*x^3*d-17/3/e^3
*x^3*d^2-4/e^2*x^5*d+5/e^3*x^4*d^2+17/4/e^2*x^4*d+17/2/e^4*x^2*d^3-4/e^3*x*
d^2+10/e^5*x^2*d^4-17/e^5*x*d^4-17/e^4*x*d^3+2/e^2*x^2*d-20/e^6*d^5*x-21/e^
2*x*d+17/2/e^3*x^2*d^2+17/e^6*ln(e*x+d)*d^5+17/e^5*ln(e*x+d)*d^4+20/e^7*ln(
e*x+d)*d^6+4/e^4*ln(e*x+d)*d^3+21/e^3*ln(e*x+d)*d^2-7/e^2*ln(e*x+d)*d-20/3/
e^4*x^3*d^3+21/2*x^2/e+10/3*x^6/e
```

Maxima [A] time = 1.00043, size = 308, normalized size = 1.35

$$\frac{200\,e^5x^6-12\left(20\,de^4+17\,e^5\right)x^5+15\left(20\,d^2e^3+17\,de^4+17\,e^5\right)x^4-20\left(20\,d^3e^2+17\,d^2e^3+17\,de^4+4\,e^5\right)x^3+30\left(20\,d^4e\right.}{60\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="maxim
a")
```

```
[Out] 1/60*(200*e^5*x^6 - 12*(20*d*e^4 + 17*e^5)*x^5 + 15*(20*d^2*e^3 + 17*d*e^4
+ 17*e^5)*x^4 - 20*(20*d^3*e^2 + 17*d^2*e^3 + 17*d*e^4 + 4*e^5)*x^3 + 30*(2
0*d^4*e + 17*d^3*e^2 + 17*d^2*e^3 + 4*d*e^4 + 21*e^5)*x^2 - 60*(20*d^5 + 17
*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6 + (20*d^6 + 17*d
^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x + d)/
e^7
```

Fricas [A] time = 0.996529, size = 520, normalized size = 2.28

$$\frac{200\,e^6x^6-12\left(20\,de^5+17\,e^6\right)x^5+15\left(20\,d^2e^4+17\,de^5+17\,e^6\right)x^4-20\left(20\,d^3e^3+17\,d^2e^4+17\,de^5+4\,e^6\right)x^3+30\left(20\,d^4e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="frica
s")
```

```
[Out] 1/60*(200*e^6*x^6 - 12*(20*d*e^5 + 17*e^6)*x^5 + 15*(20*d^2*e^4 + 17*d*e^5
+ 17*e^6)*x^4 - 20*(20*d^3*e^3 + 17*d^2*e^4 + 17*d*e^5 + 4*e^6)*x^3 + 30*(2
0*d^4*e^2 + 17*d^3*e^3 + 17*d^2*e^4 + 4*d*e^5 + 21*e^6)*x^2 - 60*(20*d^5*e
+ 17*d^4*e^2 + 17*d^3*e^3 + 4*d^2*e^4 + 21*d*e^5 - 7*e^6)*x + 60*(20*d^6 +
17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x +
 d))/e^7
```

Sympy [A] time = 0.573867, size = 221, normalized size = 0.97

$$\frac{10x^6}{3e}-\frac{x^5\,(20d+17e)}{5e^2}+\frac{x^4\left(20d^2+17de+17e^2\right)}{4e^3}-\frac{x^3\left(20d^3+17d^2e+17de^2+4e^3\right)}{3e^4}+\frac{x^2\left(20d^4+17d^3e+17d^2e^2+4de^3\right.}{2e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] 10*x**6/(3*e) - x**5*(20*d + 17*e)/(5*e**2) + x**4*(20*d**2 + 17*d*e + 17*e
**2)/(4*e**3) - x**3*(20*d**3 + 17*d**2*e + 17*d*e**2 + 4*e**3)/(3*e**4) +
x**2*(20*d**4 + 17*d**3*e + 17*d**2*e**2 + 4*d*e**3 + 21*e**4)/(2*e**5) - x
*(20*d**5 + 17*d**4*e + 17*d**3*e**2 + 4*d**2*e**3 + 21*d*e**4 - 7*e**5)/e*
*6 + (5*d**2 - 2*d*e + 3*e**2)*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 +
2*e**4)*log(d + e*x)/e**7
```

Giac [A] time = 1.16144, size = 308, normalized size = 1.35

$$\left(20\,d^{6} + 17\,d^{5}e + 17\,d^{4}e^{2} + 4\,d^{3}e^{3} + 21\,d^{2}e^{4} - 7\,de^{5} + 6\,e^{6}\right)e^{(-7)}\log\left(|xe + d|\right) + \frac{1}{60}\left(200\,x^{6}e^{5} - 240\,dx^{5}e^{4} + 300\,d^{2}x^{4}e^{3} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="giac"
)
```

```
[Out] (20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)
*e^(-7)*log(abs(x*e + d)) + 1/60*(200*x^6*e^5 - 240*d*x^5*e^4 + 300*d^2*x^4
*e^3 - 400*d^3*x^3*e^2 + 600*d^4*x^2*e - 1200*d^5*x - 204*x^5*e^5 + 255*d*x
^4*e^4 - 340*d^2*x^3*e^3 + 510*d^3*x^2*e^2 - 1020*d^4*x*e + 255*x^4*e^5 - 3
40*d*x^3*e^4 + 510*d^2*x^2*e^3 - 1020*d^3*x*e^2 - 80*x^3*e^5 + 120*d*x^2*e^
4 - 240*d^2*x*e^3 + 630*x^2*e^5 - 1260*d*x*e^4 + 420*x*e^5)*e^(-6)
```

3.294 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx$

Optimal. Leaf size=228

$$\frac{x^3(60d^2+34de+17e^2)}{3e^4}-\frac{x^2(51d^2e+80d^3+34de^2+4e^3)}{2e^5}+\frac{x(51d^2e^2+68d^3e+100d^4+8de^3+21e^4)}{e^6}-\frac{(5d^2-2de+$$

```
[Out] ((100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - ((80*d^3 + 5
1*d^2*e + 34*d*e^2 + 4*e^3)*x^2)/(2*e^5) + ((60*d^2 + 34*d*e + 17*e^2)*x^3)
/(3*e^4) - ((40*d + 17*e)*x^4)/(4*e^3) + (4*x^5)/e^2 - ((5*d^2 - 2*d*e + 3*
e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^7*(d + e*x)) - ((120
*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])
/e^7
```

Rubi [A] time = 0.191275, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{x^3(60d^2+34de+17e^2)}{3e^4}-\frac{x^2(51d^2e+80d^3+34de^2+4e^3)}{2e^5}+\frac{x(51d^2e^2+68d^3e+100d^4+8de^3+21e^4)}{e^6}-\frac{(5d^2-2de+$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```


```
[Out] ((100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - ((80*d^3 + 5
1*d^2*e + 34*d*e^2 + 4*e^3)*x^2)/(2*e^5) + ((60*d^2 + 34*d*e + 17*e^2)*x^3)
/(3*e^4) - ((40*d + 17*e)*x^4)/(4*e^3) + (4*x^5)/e^2 - ((5*d^2 - 2*d*e + 3*
e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^7*(d + e*x)) - ((120
*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])
/e^7
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx = \int \left(\frac{100d^4+68d^3e+51d^2e^2+8de^3+21e^4}{e^6} - \frac{(80d^3+51d^2e+34de^2}{e^5}\right.$$
$$= \frac{(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x}{e^6} - \frac{(80d^3+51d^2e+34de^2}{2e^5}$$

Mathematica [A] time = 0.0886713, size = 223, normalized size = 0.98

$$\frac{4e^3x^3(60d^2+34de+17e^2)-6e^2x^2(51d^2e+80d^3+34de^2+4e^3)+12ex(51d^2e^2+68d^3e+100d^4+8de^3+21e^4)-\frac{12(}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x
]
```

```
[Out] (12*e*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x - 6*e^2*(80*d^
3 + 51*d^2*e + 34*d*e^2 + 4*e^3)*x^2 + 4*e^3*(60*d^2 + 34*d*e + 17*e^2)*x^3
 - 3*e^4*(40*d + 17*e)*x^4 + 48*e^5*x^5 - (12*(20*d^6 + 17*d^5*e + 17*d^4*e
^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6))/(d + e*x) - 12*(120*d^5 + 8
5*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])/(12*e^7
)
```

Maple [A] time = 0.057, size = 313, normalized size = 1.4

$$-120\,\frac{\ln(ex+d)\,d^5}{e^7}-\frac{17\,x^4}{4\,e^2}+\frac{17\,x^3}{3\,e^2}-2\,\frac{x^2}{e^2}+7\,\frac{\ln(ex+d)}{e^2}-6\,\frac{1}{e\,(ex+d)}-85\,\frac{\ln(ex+d)\,d^4}{e^6}-68\,\frac{\ln(ex+d)\,d^3}{e^5}-12\,\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x)
```

```
[Out] -120/e^7*ln(e*x+d)*d^5-17/4/e^2*x^4+17/3/e^2*x^3-2/e^2*x^2+7/e^2*ln(e*x+d)-
6/e/(e*x+d)-85/e^6*ln(e*x+d)*d^4-68/e^5*ln(e*x+d)*d^3-12/e^4*ln(e*x+d)*d^2-
42/e^3*ln(e*x+d)*d+100/e^6*d^4*x+68/e^5*x*d^3+51/e^4*x*d^2+8/e^3*x*d-10/e^3
*x^4*d+20/e^4*x^3*d^2+34/3/e^3*x^3*d-40/e^5*x^2*d^3-51/2/e^4*x^2*d^2-17/e^3
*x^2*d-20/e^7/(e*x+d)*d^6-17/e^6/(e*x+d)*d^5-17/e^5/(e*x+d)*d^4-4/e^4/(e*x+
d)*d^3-21/e^3/(e*x+d)*d^2+7/e^2/(e*x+d)*d+21*x/e^2+4*x^5/e^2
```

Maxima [A] time = 0.994265, size = 316, normalized size = 1.39

$$-\frac{20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6}{e^8x+de^7}+\frac{48\,e^4x^5-3\left(40\,de^3+17\,e^4\right)x^4+4\left(60\,d^2e^2+34\,de^3+17\,e^4\right)x^3 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="max
ima")
```

```
[Out] -(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6
)/(e^8*x + d*e^7) + 1/12*(48*e^4*x^5 - 3*(40*d*e^3 + 17*e^4)*x^4 + 4*(60*d^
2*e^2 + 34*d*e^3 + 17*e^4)*x^3 - 6*(80*d^3*e + 51*d^2*e^2 + 34*d*e^3 + 4*e^
4)*x^2 + 12*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - (
120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*log(e*x +
d)/e^7
```

Fricas [A] time = 0.951872, size = 728, normalized size = 3.19

$$\frac{48\,e^6x^6-240\,d^6-204\,d^5e-204\,d^4e^2-48\,d^3e^3-252\,d^2e^4+84\,de^5-72\,e^6-3\left(24\,de^5+17\,e^6\right)x^5+\left(120\,d^2e^4+85\,de^5+\cdots\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="fri
cas")
```

```
[Out] 1/12*(48*e^6*x^6 - 240*d^6 - 204*d^5*e - 204*d^4*e^2 - 48*d^3*e^3 - 252*d^2
*e^4 + 84*d*e^5 - 72*e^6 - 3*(24*d*e^5 + 17*e^6)*x^5 + (120*d^2*e^4 + 85*d*
e^5 + 68*e^6)*x^4 - 2*(120*d^3*e^3 + 85*d^2*e^4 + 68*d*e^5 + 12*e^6)*x^3 +
6*(120*d^4*e^2 + 85*d^3*e^3 + 68*d^2*e^4 + 12*d*e^5 + 42*e^6)*x^2 + 12*(100
*d^5*e + 68*d^4*e^2 + 51*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5)*x - 12*(120*d^6 +
85*d^5*e + 68*d^4*e^2 + 12*d^3*e^3 + 42*d^2*e^4 - 7*d*e^5 + (120*d^5*e + 85
*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 - 7*e^6)*x)*log(e*x + d))/(e^
8*x + d*e^7)
```

Sympy [A] time = 1.04, size = 226, normalized size = 0.99

$$-\frac{20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6}{de^7+e^8x}+\frac{4x^5}{e^2}-\frac{x^4\left(40d+17e\right)}{4e^3}+\frac{x^3\left(60d^2+34de+17e^2\right)}{3e^4}-\frac{x^2\left(80d^3+\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] -(20*d**6 + 17*d**5*e + 17*d**4*e**2 + 4*d**3*e**3 + 21*d**2*e**4 - 7*d*e**
5 + 6*e**6)/(d*e**7 + e**8*x) + 4*x**5/e**2 - x**4*(40*d + 17*e)/(4*e**3) +
 x**3*(60*d**2 + 34*d*e + 17*e**2)/(3*e**4) - x**2*(80*d**3 + 51*d**2*e + 3
4*d*e**2 + 4*e**3)/(2*e**5) + x*(100*d**4 + 68*d**3*e + 51*d**2*e**2 + 8*d*
e**3 + 21*e**4)/e**6 - (120*d**5 + 85*d**4*e + 68*d**3*e**2 + 12*d**2*e**3
+ 42*d*e**4 - 7*e**5)*log(d + e*x)/e**7
```

Giac [A] time = 1.16966, size = 416, normalized size = 1.82

$$-\frac{1}{12}(xe+d)^5\left(\frac{3\left(120\,de+17\,e^2\right)e^{(-1)}}{xe+d}-\frac{4\left(300\,d^2e^2+85\,de^3+17\,e^4\right)e^{(-2)}}{(xe+d)^2}+\frac{12\left(200\,d^3e^3+85\,d^2e^4+34\,de^5+2\,e^6\right)e^{(-3)}}{(xe+d)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="gia
c")
```

```
[Out] -1/12*(x*e + d)^5*(3*(120*d*e + 17*e^2)*e^(-1)/(x*e + d) - 4*(300*d^2*e^2 +
 85*d*e^3 + 17*e^4)*e^(-2)/(x*e + d)^2 + 12*(200*d^3*e^3 + 85*d^2*e^4 + 34*
d*e^5 + 2*e^6)*e^(-3)/(x*e + d)^3 - 12*(300*d^4*e^4 + 170*d^3*e^5 + 102*d^2
*e^6 + 12*d*e^7 + 21*e^8)*e^(-4)/(x*e + d)^4 - 48)*e^(-7) + (120*d^5 + 85*d
^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*e^(-7)*log(abs(x*e + d)*
e^(-1)/(x*e + d)^2) - (20*d^6*e^5/(x*e + d) + 17*d^5*e^6/(x*e + d) + 17*d^4
*e^7/(x*e + d) + 4*d^3*e^8/(x*e + d) + 21*d^2*e^9/(x*e + d) - 7*d*e^10/(x*e
 + d) + 6*e^11/(x*e + d))*e^(-12)
```

3.295 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

Optimal. Leaf size=231

$$\frac{x^2\left(120d^2+51de+17e^2\right)}{2e^5}-\frac{x\left(102d^2e+200d^3+51de^2+4e^3\right)}{e^6}+\frac{68d^3e^2+12d^2e^3+85d^4e+120d^5+42de^4-7e^5}{e^7(d+ex)}-\frac{(5d^2}{}$$

```
[Out] -((((200*d^3 + 102*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6) + ((120*d^2 + 51*d*e +
17*e^2)*x^2)/(2*e^5) - ((60*d + 17*e)*x^3)/(3*e^4) + (5*x^4)/e^3 - ((5*d^2
- 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(2*e^7*(d +
 e*x)^2) + (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5
)/(e^7*(d + e*x)) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4
)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.203744, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{x^2\left(120d^2+51de+17e^2\right)}{2e^5}-\frac{x\left(102d^2e+200d^3+51de^2+4e^3\right)}{e^6}+\frac{68d^3e^2+12d^2e^3+85d^4e+120d^5+42de^4-7e^5}{e^7(d+ex)}-\frac{(5d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -((((200*d^3 + 102*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6) + ((120*d^2 + 51*d*e +
17*e^2)*x^2)/(2*e^5) - ((60*d + 17*e)*x^3)/(3*e^4) + (5*x^4)/e^3 - ((5*d^2
- 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(2*e^7*(d +
 e*x)^2) + (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5
)/(e^7*(d + e*x)) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4
)*Log[d + e*x])/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx = \int \left(\frac{-200d^3-102d^2e-51de^2-4e^3}{e^6} + \frac{(120d^2+51de+17e^2)\,x}{e^5} - \frac{(60\right.$$
$$= -\frac{(200d^3+102d^2e+51de^2+4e^3)\,x}{e^6} + \frac{(120d^2+51de+17e^2)\,x^2}{2e^5} - \frac{(6}{}$$

Mathematica [A] time = 0.0682934, size = 204, normalized size = 0.88

$$\frac{-51d^4e^2(40x^2+2x-7)-3d^3e^3(200x^3+357x^2-34x-20)+d^2e^4(150x^4-340x^3-561x^2+48x+189)+6(102d^2e^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x
]
```

```
[Out] (660*d^6 + d^5*e*(459 - 480*x) - 51*d^4*e^2*(-7 + 2*x + 40*x^2) - 3*d^3*e^3
*(-20 - 34*x + 357*x^2 + 200*x^3) + d^2*e^4*(189 + 48*x - 561*x^2 - 340*x^3
 + 150*x^4) - d*e^5*(21 - 252*x + 48*x^2 + 204*x^3 - 85*x^4 + 60*x^5) + e^6
*(-18 - 42*x - 24*x^3 + 51*x^4 - 34*x^5 + 30*x^6) + 6*(300*d^4 + 170*d^3*e
+ 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^2*Log[d + e*x])/(6*e^7*(d + e*
x)^2)
```

Maple [A] time = 0.051, size = 336, normalized size = 1.5

$$21\,\frac{\ln(ex+d)}{e^3} - 7\,\frac{1}{e^2(ex+d)} - 3\,\frac{1}{e(ex+d)^2} + \frac{17\,x^2}{2\,e^3} - 4\,\frac{x}{e^3} - \frac{17\,x^3}{3\,e^3} + 120\,\frac{d^5}{e^7(ex+d)} + 85\,\frac{d^4}{e^6(ex+d)} + 300\,\frac{\ln(ex+d)}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x)
```

```
[Out] 21/e^3*ln(e*x+d)-7/e^2/(e*x+d)-3/e/(e*x+d)^2+17/2/e^3*x^2-4/e^3*x-17/3/e^3*
x^3+120/e^7/(e*x+d)*d^5+85/e^6/(e*x+d)*d^4+300/e^7*ln(e*x+d)*d^4+170/e^6*ln
(e*x+d)*d^3+102/e^5*ln(e*x+d)*d^2+60/e^5*x^2*d^2+51/2/e^4*x^2*d+68/e^5/(e*x
+d)*d^3+12/e^4/(e*x+d)*d^2+42/e^3/(e*x+d)*d-10/e^7/(e*x+d)^2*d^6-17/2/e^6/(
e*x+d)^2*d^5-17/2/e^5/(e*x+d)^2*d^4-2/e^4/(e*x+d)^2*d^3-21/2/e^3/(e*x+d)^2*
d^2+7/2/e^2/(e*x+d)^2*d+12/e^4*ln(e*x+d)*d-200/e^6*d^3*x-102/e^5*x*d^2-51/e
```

```
^4*x*d-20/e^4*x^3*d+5*x^4/e^3
```

Maxima [A] time = 0.99386, size = 324, normalized size = 1.4

$$\frac{220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4-7\,de^5-6\,e^6+2\left(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2e^4+42\,de^5-7\,e^6\right)x}{2\left(e^9x^2+2\,de^8x+d^2e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="max
ima")
```

```
[Out] 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 - 7*d*e^5
- 6*e^6 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 -
7*e^6)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7) + 1/6*(30*e^3*x^4 - 2*(60*d*e^2 +
 17*e^3)*x^3 + 3*(120*d^2*e + 51*d*e^2 + 17*e^3)*x^2 - 6*(200*d^3 + 102*d^2
*e + 51*d*e^2 + 4*e^3)*x)/e^6 + (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e
^3 + 21*e^4)*log(e*x + d)/e^7
```

Fricas [A] time = 0.923654, size = 819, normalized size = 3.55

$$\frac{30\,e^6x^6+660\,d^6+459\,d^5e+357\,d^4e^2+60\,d^3e^3+189\,d^2e^4-21\,de^5-18\,e^6-2\left(30\,de^5+17\,e^6\right)x^5+\left(150\,d^2e^4+85\,de^5+\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="fri
cas")
```

```
[Out] 1/6*(30*e^6*x^6 + 660*d^6 + 459*d^5*e + 357*d^4*e^2 + 60*d^3*e^3 + 189*d^2*
e^4 - 21*d*e^5 - 18*e^6 - 2*(30*d*e^5 + 17*e^6)*x^5 + (150*d^2*e^4 + 85*d*e
^5 + 51*e^6)*x^4 - 4*(150*d^3*e^3 + 85*d^2*e^4 + 51*d*e^5 + 6*e^6)*x^3 - 3*
(680*d^4*e^2 + 357*d^3*e^3 + 187*d^2*e^4 + 16*d*e^5)*x^2 - 6*(80*d^5*e + 17
*d^4*e^2 - 17*d^3*e^3 - 8*d^2*e^4 - 42*d*e^5 + 7*e^6)*x + 6*(300*d^6 + 170*
d^5*e + 102*d^4*e^2 + 12*d^3*e^3 + 21*d^2*e^4 + (300*d^4*e^2 + 170*d^3*e^3
+ 102*d^2*e^4 + 12*d*e^5 + 21*e^6)*x^2 + 2*(300*d^5*e + 170*d^4*e^2 + 102*d
^3*e^3 + 12*d^2*e^4 + 21*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2
*e^7)
```

Sympy [A] time = 1.82195, size = 238, normalized size = 1.03

$$\frac{220d^6+153d^5e+119d^4e^2+20d^3e^3+63d^2e^4-7de^5-6e^6+x\left(240d^5e+170d^4e^2+136d^3e^3+24d^2e^4+84de^5-14e^6\right)}{2d^2e^7+4de^8x+2e^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

```
[Out] (220*d**6 + 153*d**5*e + 119*d**4*e**2 + 20*d**3*e**3 + 63*d**2*e**4 - 7*d*
e**5 - 6*e**6 + x*(240*d**5*e + 170*d**4*e**2 + 136*d**3*e**3 + 24*d**2*e**
4 + 84*d*e**5 - 14*e**6))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + 5*x**4
/e**3 - x**3*(60*d + 17*e)/(3*e**4) + x**2*(120*d**2 + 51*d*e + 17*e**2)/(2
*e**5) - x*(200*d**3 + 102*d**2*e + 51*d*e**2 + 4*e**3)/e**6 + (300*d**4 +
170*d**3*e + 102*d**2*e**2 + 12*d*e**3 + 21*e**4)*log(d + e*x)/e**7
```

Giac [A] time = 1.16683, size = 292, normalized size = 1.26

$$\left(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{6}\left(30\,x^4e^9-120\,dx^3e^8+360\,d^2x^2e^7-1200\,d^3xe^6-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="gia
c")
```

```
[Out] (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*e^(-7)*log(abs(x*e
+ d)) + 1/6*(30*x^4*e^9 - 120*d*x^3*e^8 + 360*d^2*x^2*e^7 - 1200*d^3*x*e^6
- 34*x^3*e^9 + 153*d*x^2*e^8 - 612*d^2*x*e^7 + 51*x^2*e^9 - 306*d*x*e^8 - 2
4*x*e^9)*e^(-12) + 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63
*d^2*e^4 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 -
 7*e^6)*x - 7*d*e^5 - 6*e^6)*e^(-7)/(x*e + d)^2
```

3.296 $\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=391

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{\left(945d^2e+5600d^3+666de^2+37e^3\right)(d+ex)^9}{9e^9}+\frac{\left(1665d^2e^2+1575d^3e+7000d^4+\right.}{8e^9}$$

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^4)/(4*e^9) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3
*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^5)/(5*e^9) + ((2800*d^6 + 9
45*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*
(d + e*x)^6)/(6*e^9) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3
 + 592*d*e^4 - 65*e^5)*(d + e*x)^7)/(7*e^9) + ((7000*d^4 + 1575*d^3*e + 166
5*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^8)/(8*e^9) - ((5600*d^3 + 945*d^
2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^9)/(9*e^9) + ((2800*d^2 + 315*d*e + 111
*e^2)*(d + e*x)^10)/(10*e^9) - (5*(160*d + 9*e)*(d + e*x)^11)/(11*e^9) + (2
5*(d + e*x)^12)/(3*e^9)
```

Rubi [A] time = 0.385555, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{\left(945d^2e+5600d^3+666de^2+37e^3\right)(d+ex)^9}{9e^9}+\frac{\left(1665d^2e^2+1575d^3e+7000d^4+\right.}{8e^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```


```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^4)/(4*e^9) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3
*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^5)/(5*e^9) + ((2800*d^6 + 9
45*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*
(d + e*x)^6)/(6*e^9) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3
 + 592*d*e^4 - 65*e^5)*(d + e*x)^7)/(7*e^9) + ((7000*d^4 + 1575*d^3*e + 166
5*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^8)/(8*e^9) - ((5600*d^3 + 945*d^
2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^9)/(9*e^9) + ((2800*d^2 + 315*d*e + 111
*e^2)*(d + e*x)^10)/(10*e^9) - (5*(160*d + 9*e)*(d + e*x)^11)/(11*e^9) + (2
5*(d + e*x)^12)/(3*e^9)
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d}{e^8}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex}{4e^9}$$

Mathematica [A] time = 0.0430987, size = 277, normalized size = 0.71

$$\frac{3}{10}ex^{10}\left(100d^2-45de+37e^2\right)+\frac{1}{9}x^9\left(-135d^2e+100d^3+333de^2-37e^3\right)+\frac{1}{8}x^8\left(333d^2e-45d^3-111de^2+148e^3\right)+\frac{1}{7}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^3*x + (3*d^2*(11*d + 18*e)*x^2)/2 + (d*(107*d^2 + 99*d*e + 54*e^2)*x^3
)/3 + ((65*d^3 + 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4)/4 + ((148*d^3 + 195*d^
2*e + 321*d*e^2 + 33*e^3)*x^5)/5 + ((-37*d^3 + 444*d^2*e + 195*d*e^2 + 107*
e^3)*x^6)/6 + ((111*d^3 - 111*d^2*e + 444*d*e^2 + 65*e^3)*x^7)/7 + ((-45*d^
3 + 333*d^2*e - 111*d*e^2 + 148*e^3)*x^8)/8 + ((100*d^3 - 135*d^2*e + 333*d
*e^2 - 37*e^3)*x^9)/9 + (3*e*(100*d^2 - 45*d*e + 37*e^2)*x^10)/10 + (15*(20
*d - 3*e)*e^2*x^11)/11 + (25*e^3*x^12)/3
```

Maple [A] time = 0.058, size = 264, normalized size = 0.7

$$\frac{25\,e^3x^{12}}{3}+\frac{\left(300\,de^2-45\,e^3\right)x^{11}}{11}+\frac{\left(300\,d^2e-135\,de^2+111\,e^3\right)x^{10}}{10}+\frac{\left(100\,d^3-135\,d^2e+333\,de^2-37\,e^3\right)x^9}{9}+\frac{\left(-45\,d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 25/3*e^3*x^12+1/11*(300*d*e^2-45*e^3)*x^11+1/10*(300*d^2*e-135*d*e^2+111*e^
3)*x^10+1/9*(100*d^3-135*d^2*e+333*d*e^2-37*e^3)*x^9+1/8*(-45*d^3+333*d^2*e
-111*d*e^2+148*e^3)*x^8+1/7*(111*d^3-111*d^2*e+444*d*e^2+65*e^3)*x^7+1/6*(-
37*d^3+444*d^2*e+195*d*e^2+107*e^3)*x^6+1/5*(148*d^3+195*d^2*e+321*d*e^2+33
*e^3)*x^5+1/4*(65*d^3+321*d^2*e+99*d*e^2+18*e^3)*x^4+1/3*(107*d^3+99*d^2*e+
54*d*e^2)*x^3+1/2*(33*d^3+54*d^2*e)*x^2+18*d^3*x
```

Maxima [A] time = 0.962501, size = 355, normalized size = 0.91

$$\frac{25}{3}\,e^{3}x^{12}+\frac{15}{11}\left(20\,de^{2}-3\,e^{3}\right)x^{11}+\frac{3}{10}\left(100\,d^{2}e-45\,de^{2}+37\,e^{3}\right)x^{10}+\frac{1}{9}\left(100\,d^{3}-135\,d^{2}e+333\,de^{2}-37\,e^{3}\right)x^{9}-\frac{1}{8}\left(45\,d^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 25/3*e^3*x^12 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
+ 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8*(
45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e +
444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3)*x^
6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 + 321*
d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e + 54*d*
e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

Fricas [A] time = 0.92602, size = 778, normalized size = 1.99

$$\frac{25}{3}x^{12}e^{3}-\frac{45}{11}x^{11}e^{3}+\frac{300}{11}x^{11}e^{2}d+\frac{111}{10}x^{10}e^{3}-\frac{27}{2}x^{10}e^{2}d+30x^{10}ed^{2}-\frac{37}{9}x^{9}e^{3}+37x^{9}e^{2}d-15x^{9}ed^{2}+\frac{100}{9}x^{9}d^{3}+\frac{37}{2}x^{8}e^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 25/3*x^12*e^3 - 45/11*x^11*e^3 + 300/11*x^11*e^2*d + 111/10*x^10*e^3 - 27/2
*x^10*e^2*d + 30*x^10*e*d^2 - 37/9*x^9*e^3 + 37*x^9*e^2*d - 15*x^9*e*d^2 +
100/9*x^9*d^3 + 37/2*x^8*e^3 - 111/8*x^8*e^2*d + 333/8*x^8*e*d^2 - 45/8*x^8
*d^3 + 65/7*x^7*e^3 + 444/7*x^7*e^2*d - 111/7*x^7*e*d^2 + 111/7*x^7*d^3 + 1
07/6*x^6*e^3 + 65/2*x^6*e^2*d + 74*x^6*e*d^2 - 37/6*x^6*d^3 + 33/5*x^5*e^3
```

```
+ 321/5*x^5*e^2*d + 39*x^5*e*d^2 + 148/5*x^5*d^3 + 9/2*x^4*e^3 + 99/4*x^4*e
^2*d + 321/4*x^4*e*d^2 + 65/4*x^4*d^3 + 18*x^3*e^2*d + 33*x^3*e*d^2 + 107/3
*x^3*d^3 + 27*x^2*e*d^2 + 33/2*x^2*d^3 + 18*x*d^3
```

Sympy [A] time = 0.118886, size = 298, normalized size = 0.76

$$18d^3x + \frac{25e^3x^{12}}{3} + x^{11}\left(\frac{300de^2}{11} - \frac{45e^3}{11}\right) + x^{10}\left(30d^2e - \frac{27de^2}{2} + \frac{111e^3}{10}\right) + x^9\left(\frac{100d^3}{9} - 15d^2e + 37de^2 - \frac{37e^3}{9}\right) + x^8\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**3*x + 25*e**3*x**12/3 + x**11*(300*d*e**2/11 - 45*e**3/11) + x**10*(3
0*d**2*e - 27*d*e**2/2 + 111*e**3/10) + x**9*(100*d**3/9 - 15*d**2*e + 37*d
*e**2 - 37*e**3/9) + x**8*(-45*d**3/8 + 333*d**2*e/8 - 111*d*e**2/8 + 37*e*
*3/2) + x**7*(111*d**3/7 - 111*d**2*e/7 + 444*d*e**2/7 + 65*e**3/7) + x**6*
(-37*d**3/6 + 74*d**2*e + 65*d*e**2/2 + 107*e**3/6) + x**5*(148*d**3/5 + 39
*d**2*e + 321*d*e**2/5 + 33*e**3/5) + x**4*(65*d**3/4 + 321*d**2*e/4 + 99*d
*e**2/4 + 9*e**3/2) + x**3*(107*d**3/3 + 33*d**2*e + 18*d*e**2) + x**2*(33*
d**3/2 + 27*d**2*e)
```

Giac [A] time = 1.14521, size = 400, normalized size = 1.02

$$\frac{25}{3}x^{12}e^3 + \frac{300}{11}dx^{11}e^2 + 30\,d^2x^{10}e + \frac{100}{9}d^3x^9 - \frac{45}{11}x^{11}e^3 - \frac{27}{2}dx^{10}e^2 - 15\,d^2x^9e - \frac{45}{8}d^3x^8 + \frac{111}{10}x^{10}e^3 + 37\,dx^9e^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 25/3*x^12*e^3 + 300/11*d*x^11*e^2 + 30*d^2*x^10*e + 100/9*d^3*x^9 - 45/11*x
^11*e^3 - 27/2*d*x^10*e^2 - 15*d^2*x^9*e - 45/8*d^3*x^8 + 111/10*x^10*e^3 +
 37*d*x^9*e^2 + 333/8*d^2*x^8*e + 111/7*d^3*x^7 - 37/9*x^9*e^3 - 111/8*d*x^
8*e^2 - 111/7*d^2*x^7*e - 37/6*d^3*x^6 + 37/2*x^8*e^3 + 444/7*d*x^7*e^2 + 7
4*d^2*x^6*e + 148/5*d^3*x^5 + 65/7*x^7*e^3 + 65/2*d*x^6*e^2 + 39*d^2*x^5*e
+ 65/4*d^3*x^4 + 107/6*x^6*e^3 + 321/5*d*x^5*e^2 + 321/4*d^2*x^4*e + 107/3*
d^3*x^3 + 33/5*x^5*e^3 + 99/4*d*x^4*e^2 + 33*d^2*x^3*e + 33/2*d^3*x^2 + 9/2
*x^4*e^3 + 18*d*x^3*e^2 + 27*d^2*x^2*e + 18*d^3*x
```

3.297 $\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=201

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}x^5\left(\right.$$

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 - ((37*d^2 - 296*d*e - 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7)
/7 - ((45*d^2 - 222*d*e + 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*x^
9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Rubi [A] time = 0.240234, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}x^5\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 - ((37*d^2 - 296*d*e - 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7)
/7 - ((45*d^2 - 222*d*e + 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*x^
9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(18d^2+3d(11d+12e)x+\left(107d^2+66de+18e^2\right)x^2+\left(6\right.\right.$$
$$= 18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+18e^2\right)x^3+\frac{1}{4}$$

Mathematica [A] time = 0.0267461, size = 201, normalized size = 1.

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)+\frac{1}{8}x^8\left(-45d^2+222de-37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)+\frac{1}{6}x^6\left(-37d^2+296de+65e^2\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 + ((-37*d^2 + 296*d*e + 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7
)/7 + ((-45*d^2 + 222*d*e - 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*
x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Maple [A] time = 0.041, size = 186, normalized size = 0.9

$$\frac{100\,e^2x^{11}}{11}+\frac{\left(200\,de-45\,e^2\right)x^{10}}{10}+\frac{\left(100\,d^2-90\,de+111\,e^2\right)x^9}{9}+\frac{\left(-45\,d^2+222\,de-37\,e^2\right)x^8}{8}+\frac{\left(111\,d^2-74\,de+148\,e\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 100/11*e^2*x^11+1/10*(200*d*e-45*e^2)*x^10+1/9*(100*d^2-90*d*e+111*e^2)*x^9
+1/8*(-45*d^2+222*d*e-37*e^2)*x^8+1/7*(111*d^2-74*d*e+148*e^2)*x^7+1/6*(-37
*d^2+296*d*e+65*e^2)*x^6+1/5*(148*d^2+130*d*e+107*e^2)*x^5+1/4*(65*d^2+214*
d*e+33*e^2)*x^4+1/3*(107*d^2+66*d*e+18*e^2)*x^3+1/2*(33*d^2+36*d*e)*x^2+18*
d^2*x
```

Maxima [A] time = 0.995225, size = 250, normalized size = 1.24

$$\frac{100}{11}e^2x^{11}+\frac{1}{2}\left(40\,de-9\,e^2\right)x^{10}+\frac{1}{9}\left(100\,d^2-90\,de+111\,e^2\right)x^9-\frac{1}{8}\left(45\,d^2-222\,de+37\,e^2\right)x^8+\frac{37}{7}\left(3\,d^2-2\,de+4\,e^2\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 100/11*e^2*x^11 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*e
^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e^2
)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 107*
e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e + 18
*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

Fricas [A] time = 0.825465, size = 540, normalized size = 2.69

$$\frac{100}{11}x^{11}e^2-\frac{9}{2}x^{10}e^2+20x^{10}ed+\frac{37}{3}x^9e^2-10x^9ed+\frac{100}{9}x^9d^2-\frac{37}{8}x^8e^2+\frac{111}{4}x^8ed-\frac{45}{8}x^8d^2+\frac{148}{7}x^7e^2-\frac{74}{7}x^7ed+\frac{111}{7}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 100/11*x^11*e^2 - 9/2*x^10*e^2 + 20*x^10*e*d + 37/3*x^9*e^2 - 10*x^9*e*d +
100/9*x^9*d^2 - 37/8*x^8*e^2 + 111/4*x^8*e*d - 45/8*x^8*d^2 + 148/7*x^7*e^2
 - 74/7*x^7*e*d + 111/7*x^7*d^2 + 65/6*x^6*e^2 + 148/3*x^6*e*d - 37/6*x^6*d
^2 + 107/5*x^5*e^2 + 26*x^5*e*d + 148/5*x^5*d^2 + 33/4*x^4*e^2 + 107/2*x^4*
e*d + 65/4*x^4*d^2 + 6*x^3*e^2 + 22*x^3*e*d + 107/3*x^3*d^2 + 18*x^2*e*d +
33/2*x^2*d^2 + 18*x*d^2
```

Sympy [A] time = 0.104831, size = 206, normalized size = 1.02

$$18d^2x+\frac{100e^2x^{11}}{11}+x^{10}\left(20de-\frac{9e^2}{2}\right)+x^9\left(\frac{100d^2}{9}-10de+\frac{37e^2}{3}\right)+x^8\left(-\frac{45d^2}{8}+\frac{111de}{4}-\frac{37e^2}{8}\right)+x^7\left(\frac{111d^2}{7}-\frac{74de}{7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**2*x + 100*e**2*x**11/11 + x**10*(20*d*e - 9*e**2/2) + x**9*(100*d**2/
9 - 10*d*e + 37*e**2/3) + x**8*(-45*d**2/8 + 111*d*e/4 - 37*e**2/8) + x**7*
(111*d**2/7 - 74*d*e/7 + 148*e**2/7) + x**6*(-37*d**2/6 + 148*d*e/3 + 65*e*
*2/6) + x**5*(148*d**2/5 + 26*d*e + 107*e**2/5) + x**4*(65*d**2/4 + 107*d*e
/2 + 33*e**2/4) + x**3*(107*d**2/3 + 22*d*e + 6*e**2) + x**2*(33*d**2/2 + 1
8*d*e)
```

Giac [A] time = 1.153, size = 278, normalized size = 1.38

$$\frac{100}{11}\,x^{11}e^2+20\,dx^{10}e+\frac{100}{9}\,d^2x^9-\frac{9}{2}\,x^{10}e^2-10\,dx^9e-\frac{45}{8}\,d^2x^8+\frac{37}{3}\,x^9e^2+\frac{111}{4}\,dx^8e+\frac{111}{7}\,d^2x^7-\frac{37}{8}\,x^8e^2-\frac{74}{7}\,dx^7e-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 100/11*x^11*e^2 + 20*d*x^10*e + 100/9*d^2*x^9 - 9/2*x^10*e^2 - 10*d*x^9*e -
 45/8*d^2*x^8 + 37/3*x^9*e^2 + 111/4*d*x^8*e + 111/7*d^2*x^7 - 37/8*x^8*e^2
 - 74/7*d*x^7*e - 37/6*d^2*x^6 + 148/7*x^7*e^2 + 148/3*d*x^6*e + 148/5*d^2*
x^5 + 65/6*x^6*e^2 + 26*d*x^5*e + 65/4*d^2*x^4 + 107/5*x^5*e^2 + 107/2*d*x^
4*e + 107/3*d^2*x^3 + 33/4*x^4*e^2 + 22*d*x^3*e + 33/2*d^2*x^2 + 6*x^3*e^2
+ 18*d*x^2*e + 18*d^2*x
```

3.298 $\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$

Optimal. Leaf size=121

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e$$

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Rubi [A] time = 0.160346, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx=\int \left(18d+3(11d+6e)x+(107d+33e)x^2+(65d+107e)x^3+\right.$$

$$=18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+107e)x^4$$

Mathematica [A] time = 0.0175363, size = 121, normalized size = 1.

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+3$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Maple [A] time = 0.043, size = 108, normalized size = 0.9

$$10\,e{x}^{10}+{\frac{(100\,d-45\,e){x}^{9}}{9}}+{\frac{(-45\,d+111\,e){x}^{8}}{8}}+{\frac{(111\,d-37\,e){x}^{7}}{7}}+{\frac{(-37\,d+148\,e){x}^{6}}{6}}+{\frac{(148\,d+65\,e){x}^{5}}{5}}+{\frac{(65\,d+10}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 10*e*x^10+1/9*(100*d-45*e)*x^9+1/8*(-45*d+111*e)*x^8+1/7*(111*d-37*e)*x^7+1
/6*(-37*d+148*e)*x^6+1/5*(148*d+65*e)*x^5+1/4*(65*d+107*e)*x^4+1/3*(107*d+3
3*e)*x^3+1/2*(33*d+18*e)*x^2+18*d*x
```

Maxima [A] time = 0.964713, size = 142, normalized size = 1.17

$$10\,ex^{10}+\frac{5}{9}\,(20\,d-9\,e)x^9-\frac{3}{8}\,(15\,d-37\,e)x^8+\frac{37}{7}\,(3\,d-e)x^7-\frac{37}{6}\,(d-4\,e)x^6+\frac{1}{5}\,(148\,d+65\,e)x^5+\frac{1}{4}\,(65\,d+107\,e)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 10*e*x^10 + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*x
^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4 +
 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

Fricas [A] time = 0.871077, size = 302, normalized size = 2.5

$$10x^{10}e-5x^9e+\frac{100}{9}x^9d+\frac{111}{8}x^8e-\frac{45}{8}x^8d-\frac{37}{7}x^7e+\frac{111}{7}x^7d+\frac{74}{3}x^6e-\frac{37}{6}x^6d+13x^5e+\frac{148}{5}x^5d+\frac{107}{4}x^4e+\frac{65}{4}x^4d+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 10*x^10*e - 5*x^9*e + 100/9*x^9*d + 111/8*x^8*e - 45/8*x^8*d - 37/7*x^7*e +
 111/7*x^7*d + 74/3*x^6*e - 37/6*x^6*d + 13*x^5*e + 148/5*x^5*d + 107/4*x^4
*e + 65/4*x^4*d + 11*x^3*e + 107/3*x^3*d + 9*x^2*e + 33/2*x^2*d + 18*x*d
```

Sympy [A] time = 0.089445, size = 112, normalized size = 0.93

$$18dx+10ex^{10}+x^9\left(\frac{100d}{9}-5e\right)+x^8\left(-\frac{45d}{8}+\frac{111e}{8}\right)+x^7\left(\frac{111d}{7}-\frac{37e}{7}\right)+x^6\left(-\frac{37d}{6}+\frac{74e}{3}\right)+x^5\left(\frac{148d}{5}+13e\right)+x^4\left(\frac{65}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d*x + 10*e*x**10 + x**9*(100*d/9 - 5*e) + x**8*(-45*d/8 + 111*e/8) + x**
7*(111*d/7 - 37*e/7) + x**6*(-37*d/6 + 74*e/3) + x**5*(148*d/5 + 13*e) + x*
*4*(65*d/4 + 107*e/4) + x**3*(107*d/3 + 11*e) + x**2*(33*d/2 + 9*e)
```

Giac [A] time = 1.13902, size = 157, normalized size = 1.3

$$10\,x^{10}e+\frac{100}{9}\,dx^9-5\,x^9e-\frac{45}{8}\,dx^8+\frac{111}{8}\,x^8e+\frac{111}{7}\,dx^7-\frac{37}{7}\,x^7e-\frac{37}{6}\,dx^6+\frac{74}{3}\,x^6e+\frac{148}{5}\,dx^5+13\,x^5e+\frac{65}{4}\,dx^4+\frac{107}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 10*x^10*e + 100/9*d*x^9 - 5*x^9*e - 45/8*d*x^8 + 111/8*x^8*e + 111/7*d*x^7
- 37/7*x^7*e - 37/6*d*x^6 + 74/3*x^6*e + 148/5*d*x^5 + 13*x^5*e + 65/4*d*x^
4 + 107/4*x^4*e + 107/3*d*x^3 + 11*x^3*e + 33/2*d*x^2 + 9*x^2*e + 18*d*x
```

3.299 $\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=60

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Rubi [A] time = 0.0362729, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1657}

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(18+33x+107x^2+65x^3+148x^4-37x^5+111x^6-45x^7+100x$$
$$= 18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{10}{}$$

Mathematica [A] time = 0.0011501, size = 60, normalized size = 1.

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Maple [A] time = 0.043, size = 45, normalized size = 0.8

$$18\,x+\frac{33\,x^{2}}{2}+\frac{107\,x^{3}}{3}+\frac{65\,x^{4}}{4}+\frac{148\,x^{5}}{5}-\frac{37\,x^{6}}{6}+\frac{111\,x^{7}}{7}-\frac{45\,x^{8}}{8}+\frac{100\,x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/
9*x^9
```

Maxima [A] time = 0.994845, size = 59, normalized size = 0.98

$$\frac{100}{9}\,x^{9}-\frac{45}{8}\,x^{8}+\frac{111}{7}\,x^{7}-\frac{37}{6}\,x^{6}+\frac{148}{5}\,x^{5}+\frac{65}{4}\,x^{4}+\frac{107}{3}\,x^{3}+\frac{33}{2}\,x^{2}+18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

Fricas [A] time = 0.915561, size = 132, normalized size = 2.2

$$\frac{100}{9}x^{9}-\frac{45}{8}x^{8}+\frac{111}{7}x^{7}-\frac{37}{6}x^{6}+\frac{148}{5}x^{5}+\frac{65}{4}x^{4}+\frac{107}{3}x^{3}+\frac{33}{2}x^{2}+18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

Sympy [A] time = 0.06944, size = 56, normalized size = 0.93

$$\frac{100x^9}{9} - \frac{45x^8}{8} + \frac{111x^7}{7} - \frac{37x^6}{6} + \frac{148x^5}{5} + \frac{65x^4}{4} + \frac{107x^3}{3} + \frac{33x^2}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**5/5 + 65*x**4/4 +
107*x**3/3 + 33*x**2/2 + 18*x
```

Giac [A] time = 1.15295, size = 59, normalized size = 0.98

$$\frac{100}{9}\,x^9 - \frac{45}{8}\,x^8 + \frac{111}{7}\,x^7 - \frac{37}{6}\,x^6 + \frac{148}{5}\,x^5 + \frac{65}{4}\,x^4 + \frac{107}{3}\,x^3 + \frac{33}{2}\,x^2 + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

3.300 $\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{d+ex} dx$

Optimal. Leaf size=352

$$\frac{x^6(100d^2+45de+111e^2)}{6e^3} - \frac{x^5(45d^2e+100d^3+111de^2+37e^3)}{5e^4} + \frac{x^4(111d^2e^2+45d^3e+100d^4+37de^3+148e^4)}{4e^5} - x^3($$

```
[Out] -(((100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e^
5 + 107*d*e^6 - 33*e^7)*x)/e^8) + ((100*d^6 + 45*d^5*e + 111*d^4*e^2 + 37*d
^3*e^3 + 148*d^2*e^4 - 65*d*e^5 + 107*e^6)*x^2)/(2*e^7) - ((100*d^5 + 45*d^
4*e + 111*d^3*e^2 + 37*d^2*e^3 + 148*d*e^4 - 65*e^5)*x^3)/(3*e^6) + ((100*d
^4 + 45*d^3*e + 111*d^2*e^2 + 37*d*e^3 + 148*e^4)*x^4)/(4*e^5) - ((100*d^3
+ 45*d^2*e + 111*d*e^2 + 37*e^3)*x^5)/(5*e^4) + ((100*d^2 + 45*d*e + 111*e^
2)*x^6)/(6*e^3) - (5*(20*d + 9*e)*x^7)/(7*e^2) + (25*x^8)/(2*e) + ((5*d^2 -
 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x
])/e^9
```

Rubi [A] time = 0.316476, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{x^6(100d^2+45de+111e^2)}{6e^3} - \frac{x^5(45d^2e+100d^3+111de^2+37e^3)}{5e^4} + \frac{x^4(111d^2e^2+45d^3e+100d^4+37de^3+148e^4)}{4e^5} - x^3($$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```


```
[Out] -(((100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e^
5 + 107*d*e^6 - 33*e^7)*x)/e^8) + ((100*d^6 + 45*d^5*e + 111*d^4*e^2 + 37*d
^3*e^3 + 148*d^2*e^4 - 65*d*e^5 + 107*e^6)*x^2)/(2*e^7) - ((100*d^5 + 45*d^
4*e + 111*d^3*e^2 + 37*d^2*e^3 + 148*d*e^4 - 65*e^5)*x^3)/(3*e^6) + ((100*d
^4 + 45*d^3*e + 111*d^2*e^2 + 37*d*e^3 + 148*e^4)*x^4)/(4*e^5) - ((100*d^3
+ 45*d^2*e + 111*d*e^2 + 37*e^3)*x^5)/(5*e^4) + ((100*d^2 + 45*d*e + 111*e^
2)*x^6)/(6*e^3) - (5*(20*d + 9*e)*x^7)/(7*e^2) + (25*x^8)/(2*e) + ((5*d^2 -
 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x
])/e^9
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{d+ex}\,dx = \int \left(\frac{-100d^7-45d^6e-111d^5e^2-37d^4e^3-148d^3e^4+65d^2e^5-107de}{e^8}\right.$$

$$= -\frac{\left(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6\right.}{e^8}$$

Mathematica [A] time = 0.1217, size = 262, normalized size = 0.74

$$\frac{x\left(-70d^5e^2\left(200x^2-135x+666\right)+210d^4e^3\left(50x^3-30x^2+111x-74\right)-105d^3e^4\left(80x^4-45x^3+148x^2-74x+592\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x
]
```

```
[Out] (x*(-42000*d^7 + 2100*d^6*e*(-9 + 10*x) - 70*d^5*e^2*(666 - 135*x + 200*x^2
) + 210*d^4*e^3*(-74 + 111*x - 30*x^2 + 50*x^3) - 105*d^3*e^4*(592 - 74*x +
 148*x^2 - 45*x^3 + 80*x^4) + 35*d^2*e^5*(780 + 888*x - 148*x^2 + 333*x^3 -
 108*x^4 + 200*x^5) - d*e^6*(44940 + 13650*x + 20720*x^2 - 3885*x^3 + 9324*
x^4 - 3150*x^5 + 6000*x^6) + 2*e^7*(6930 + 11235*x + 4550*x^2 + 7770*x^3 -
1554*x^4 + 3885*x^5 - 1350*x^6 + 2625*x^7)))/(420*e^8) + ((5*d^2 - 2*d*e +
3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/e^9
```

Maple [A] time = 0.052, size = 465, normalized size = 1.3

$$-\frac{45\,x^7}{7\,e}+33\,\frac{x}{e}+18\,\frac{\ln\left(ex+d\right)}{e}+\frac{65\,x^3}{3\,e}+37\,\frac{x^4}{e}-\frac{37\,x^5}{5\,e}-\frac{148\,dx^3}{3\,e^2}-\frac{37\,x^3d^2}{3\,e^3}-\frac{111\,x^5d}{5\,e^2}+\frac{111\,x^4d^2}{4\,e^3}+\frac{37\,dx^4}{4\,e^2}+\frac{37\,x^2d^3}{2\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x)
```

```
[Out] -45/7/e*x^7+33/e*x+18/e*ln(e*x+d)+65/3/e*x^3+37*x^4/e-37/5/e*x^5-148/3/e^2*
x^3*d-37/3/e^3*x^3*d^2-111/5/e^2*x^5*d+111/4/e^3*x^4*d^2+37/4/e^2*x^4*d+37/
2/e^4*x^2*d^3+65/e^3*x*d^2+111/2/e^5*x^2*d^4-37/e^5*x*d^4-148/e^4*x*d^3-65/
2/e^2*x^2*d-111/e^6*d^5*x+100/e^9*ln(e*x+d)*d^8-107/e^2*x*d+74/e^3*x^2*d^2+
37/e^6*ln(e*x+d)*d^5+148/e^5*ln(e*x+d)*d^4+111/e^7*ln(e*x+d)*d^6-65/e^4*ln(
e*x+d)*d^3+107/e^3*ln(e*x+d)*d^2-33/e^2*ln(e*x+d)*d+25/e^5*x^4*d^4-15/e^5*x
^3*d^4-100/e^8*d^7*x+15/2/e^2*x^6*d+45/e^8*ln(e*x+d)*d^7+50/3/e^3*x^6*d^2-4
5/e^7*x*d^6+50/e^7*x^2*d^6-100/3/e^6*x^3*d^5-20/e^4*x^5*d^3-37/e^4*x^3*d^3+
45/2/e^6*x^2*d^5-9/e^3*x^5*d^2-100/7/e^2*x^7*d+45/4/e^4*x^4*d^3+107/2*x^2/e
+25/2*x^8/e+37/2*x^6/e
```

Maxima [A] time = 1.01593, size = 494, normalized size = 1.4

$$\frac{5250\,e^{7}x^{8} - 300\left(20\,de^{6} + 9\,e^{7}\right)x^{7} + 70\left(100\,d^{2}e^{5} + 45\,de^{6} + 111\,e^{7}\right)x^{6} - 84\left(100\,d^{3}e^{4} + 45\,d^{2}e^{5} + 111\,de^{6} + 37\,e^{7}\right)x^{5} + 105}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] 1/420*(5250*e^7*x^8 - 300*(20*d*e^6 + 9*e^7)*x^7 + 70*(100*d^2*e^5 + 45*d*e
^6 + 111*e^7)*x^6 - 84*(100*d^3*e^4 + 45*d^2*e^5 + 111*d*e^6 + 37*e^7)*x^5
+ 105*(100*d^4*e^3 + 45*d^3*e^4 + 111*d^2*e^5 + 37*d*e^6 + 148*e^7)*x^4 - 1
40*(100*d^5*e^2 + 45*d^4*e^3 + 111*d^3*e^4 + 37*d^2*e^5 + 148*d*e^6 - 65*e^
7)*x^3 + 210*(100*d^6*e + 45*d^5*e^2 + 111*d^4*e^3 + 37*d^3*e^4 + 148*d^2*e
^5 - 65*d*e^6 + 107*e^7)*x^2 - 420*(100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d
^4*e^3 + 148*d^3*e^4 - 65*d^2*e^5 + 107*d*e^6 - 33*e^7)*x)/e^8 + (100*d^8 +
 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*d^2*e
^6 - 33*d*e^7 + 18*e^8)*log(e*x + d)/e^9
```

Fricas [A] time = 1.05444, size = 875, normalized size = 2.49

$$\frac{5250\,e^{8}x^{8} - 300\left(20\,de^{7} + 9\,e^{8}\right)x^{7} + 70\left(100\,d^{2}e^{6} + 45\,de^{7} + 111\,e^{8}\right)x^{6} - 84\left(100\,d^{3}e^{5} + 45\,d^{2}e^{6} + 111\,de^{7} + 37\,e^{8}\right)x^{5} + 105}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] 1/420*(5250*e^8*x^8 - 300*(20*d*e^7 + 9*e^8)*x^7 + 70*(100*d^2*e^6 + 45*d*e
^7 + 111*e^8)*x^6 - 84*(100*d^3*e^5 + 45*d^2*e^6 + 111*d*e^7 + 37*e^8)*x^5
+ 105*(100*d^4*e^4 + 45*d^3*e^5 + 111*d^2*e^6 + 37*d*e^7 + 148*e^8)*x^4 - 1
40*(100*d^5*e^3 + 45*d^4*e^4 + 111*d^3*e^5 + 37*d^2*e^6 + 148*d*e^7 - 65*e^
8)*x^3 + 210*(100*d^6*e^2 + 45*d^5*e^3 + 111*d^4*e^4 + 37*d^3*e^5 + 148*d^2
*e^6 - 65*d*e^7 + 107*e^8)*x^2 - 420*(100*d^7*e + 45*d^6*e^2 + 111*d^5*e^3
+ 37*d^4*e^4 + 148*d^3*e^5 - 65*d^2*e^6 + 107*d*e^7 - 33*e^8)*x + 420*(100*
d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*
d^2*e^6 - 33*d*e^7 + 18*e^8)*log(e*x + d))/e^9
```

Sympy [A] time = 0.717569, size = 347, normalized size = 0.99

$$\frac{25x^8}{2e} - \frac{x^7(100d+45e)}{7e^2} + \frac{x^6\left(100d^2+45de+111e^2\right)}{6e^3} - \frac{x^5\left(100d^3+45d^2e+111de^2+37e^3\right)}{5e^4} + \frac{x^4\left(100d^4+45d^3e+111\right.}{4e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] 25*x**8/(2*e) - x**7*(100*d + 45*e)/(7*e**2) + x**6*(100*d**2 + 45*d*e + 11
1*e**2)/(6*e**3) - x**5*(100*d**3 + 45*d**2*e + 111*d*e**2 + 37*e**3)/(5*e*
*4) + x**4*(100*d**4 + 45*d**3*e + 111*d**2*e**2 + 37*d*e**3 + 148*e**4)/(4
*e**5) - x**3*(100*d**5 + 45*d**4*e + 111*d**3*e**2 + 37*d**2*e**3 + 148*d*
e**4 - 65*e**5)/(3*e**6) + x**2*(100*d**6 + 45*d**5*e + 111*d**4*e**2 + 37*
d**3*e**3 + 148*d**2*e**4 - 65*d*e**5 + 107*e**6)/(2*e**7) - x*(100*d**7 +
45*d**6*e + 111*d**5*e**2 + 37*d**4*e**3 + 148*d**3*e**4 - 65*d**2*e**5 + 1
07*d*e**6 - 33*e**7)/e**8 + (5*d**2 - 2*d*e + 3*e**2)**2*(4*d**4 + 5*d**3*e
 + 3*d**2*e**2 - d*e**3 + 2*e**4)*log(d + e*x)/e**9
```

Giac [A] time = 1.14073, size = 510, normalized size = 1.45

$$\left(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8\right)e^{(-9)}\log\left(|xe+d|\right)+\frac{1}{420}\left(5250\,x^8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] (100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 +
 107*d^2*e^6 - 33*d*e^7 + 18*e^8)*e^(-9)*log(abs(x*e + d)) + 1/420*(5250*x^
8*e^7 - 6000*d*x^7*e^6 + 7000*d^2*x^6*e^5 - 8400*d^3*x^5*e^4 + 10500*d^4*x^
4*e^3 - 14000*d^5*x^3*e^2 + 21000*d^6*x^2*e - 42000*d^7*x - 2700*x^7*e^7 +
3150*d*x^6*e^6 - 3780*d^2*x^5*e^5 + 4725*d^3*x^4*e^4 - 6300*d^4*x^3*e^3 + 9
450*d^5*x^2*e^2 - 18900*d^6*x*e + 7770*x^6*e^7 - 9324*d*x^5*e^6 + 11655*d^2
*x^4*e^5 - 15540*d^3*x^3*e^4 + 23310*d^4*x^2*e^3 - 46620*d^5*x*e^2 - 3108*x
^5*e^7 + 3885*d*x^4*e^6 - 5180*d^2*x^3*e^5 + 7770*d^3*x^2*e^4 - 15540*d^4*x
*e^3 + 15540*x^4*e^7 - 20720*d*x^3*e^6 + 31080*d^2*x^2*e^5 - 62160*d^3*x*e^
4 + 9100*x^3*e^7 - 13650*d*x^2*e^6 + 27300*d^2*x*e^5 + 22470*x^2*e^7 - 4494
0*d*x*e^6 + 13860*x*e^7)*e^(-8)
```

3.301 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx$

Optimal. Leaf size=353

$$\frac{3x^5(100d^2+30de+37e^2)}{5e^4}-\frac{x^4(135d^2e+400d^3+222de^2+37e^3)}{4e^5}+\frac{x^3(333d^2e^2+180d^3e+500d^4+74de^3+148e^4)}{3e^6}$$

```
[Out] ((700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5
 + 107*e^6)*x)/e^8 - ((600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*e^3 + 29
6*d*e^4 - 65*e^5)*x^2)/(2*e^7) + ((500*d^4 + 180*d^3*e + 333*d^2*e^2 + 74*d
*e^3 + 148*e^4)*x^3)/(3*e^6) - ((400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e^3)*
x^4)/(4*e^5) + (3*(100*d^2 + 30*d*e + 37*e^2)*x^5)/(5*e^4) - (5*(40*d + 9*e
)*x^6)/(6*e^3) + (100*x^7)/(7*e^2) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*
d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^9*(d + e*x)) - ((5*d^2 - 2*d*e + 3*e
^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*Log[
d + e*x])/e^9
```

Rubi [A] time = 0.32805, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{3x^5(100d^2+30de+37e^2)}{5e^4}-\frac{x^4(135d^2e+400d^3+222de^2+37e^3)}{4e^5}+\frac{x^3(333d^2e^2+180d^3e+500d^4+74de^3+148e^4)}{3e^6}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```


```
[Out] ((700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5
 + 107*e^6)*x)/e^8 - ((600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*e^3 + 29
6*d*e^4 - 65*e^5)*x^2)/(2*e^7) + ((500*d^4 + 180*d^3*e + 333*d^2*e^2 + 74*d
*e^3 + 148*e^4)*x^3)/(3*e^6) - ((400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e^3)*
x^4)/(4*e^5) + (3*(100*d^2 + 30*d*e + 37*e^2)*x^5)/(5*e^4) - (5*(40*d + 9*e
)*x^6)/(6*e^3) + (100*x^7)/(7*e^2) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*
d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^9*(d + e*x)) - ((5*d^2 - 2*d*e + 3*e
^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*Log[
d + e*x])/e^9
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^2}\,dx = \int\left(\frac{700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6}{e^8}\right.$$

$$= \frac{\left(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6\right)x}{e^8}$$

Mathematica [A] time = 0.136448, size = 342, normalized size = 0.97

$$\frac{252e^5x^5\left(100d^2+30de+37e^2\right)-105e^4x^4\left(135d^2e+400d^3+222de^2+37e^3\right)+140e^3x^3\left(333d^2e^2+180d^3e+500d^4+74d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2
,x]
```

```
[Out] (420*e*(700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130
*d*e^5 + 107*e^6)*x - 210*e^2*(600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*
e^3 + 296*d*e^4 - 65*e^5)*x^2 + 140*e^3*(500*d^4 + 180*d^3*e + 333*d^2*e^2
+ 74*d*e^3 + 148*e^4)*x^3 - 105*e^4*(400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e
^3)*x^4 + 252*e^5*(100*d^2 + 30*d*e + 37*e^2)*x^5 - 350*e^6*(40*d + 9*e)*x^
6 + 6000*e^7*x^7 - (420*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*
e^2 - d*e^3 + 2*e^4))/(d + e*x) - 420*(800*d^7 + 315*d^6*e + 666*d^5*e^2 +
185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*Log[d + e*x])
/(420*e^9)
```

Maple [A] time = 0.058, size = 500, normalized size = 1.4

$$-666\,\frac{\ln(ex+d)\,d^5}{e^7}-\frac{15\,x^6}{2\,e^2}-\frac{37\,x^4}{4\,e^2}+\frac{148\,x^3}{3\,e^2}+\frac{65\,x^2}{2\,e^2}+33\,\frac{\ln(ex+d)}{e^2}-18\,\frac{1}{e\,(ex+d)}-185\,\frac{\ln(ex+d)\,d^4}{e^6}-592\,\frac{\ln(ex+}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x)
```

```
[Out] -666/e^7*ln(e*x+d)*d^5-15/2/e^2*x^6-37/4/e^2*x^4+148/3/e^2*x^3+65/2/e^2*x^2
+33/e^2*ln(e*x+d)-18/e/(e*x+d)-185/e^6*ln(e*x+d)*d^4-592/e^5*ln(e*x+d)*d^3+
195/e^4*ln(e*x+d)*d^2-214/e^3*ln(e*x+d)*d+555/e^6*d^4*x+148/e^5*x*d^3+444/e
^4*x*d^2-130/e^3*x*d-111/2/e^3*x^4*d+111/e^4*x^3*d^2+74/3/e^3*x^3*d-222/e^5
*x^2*d^3-111/2/e^4*x^2*d^2-148/e^3*x^2*d-111/e^7/(e*x+d)*d^6-37/e^6/(e*x+d)
*d^5+270/e^7*x*d^5+18/e^3*x^5*d-135/4/e^4*x^4*d^2+60/e^4*x^5*d^2-100/3/e^3*
x^6*d+60/e^5*x^3*d^3-100/e^5*x^4*d^3-225/2/e^6*x^2*d^4-300/e^7*x^2*d^5-100/
e^9/(e*x+d)*d^8-45/e^8/(e*x+d)*d^7-800/e^9*ln(e*x+d)*d^7-315/e^8*ln(e*x+d)*
d^6+500/3/e^6*x^3*d^4+700/e^8*d^6*x-148/e^5/(e*x+d)*d^4+65/e^4/(e*x+d)*d^3-
107/e^3/(e*x+d)*d^2+33/e^2/(e*x+d)*d+107*x/e^2+100/7*x^7/e^2+111/5*x^5/e^2
```

Maxima [A] time = 0.996031, size = 502, normalized size = 1.42

$$-\frac{100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8}{e^{10}x+de^9}+\frac{6000\,e^6x^7-350\left(40\,de^5+9\,e^6\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="m
axima")
```

```
[Out] -(100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5
+ 107*d^2*e^6 - 33*d*e^7 + 18*e^8)/(e^10*x + d*e^9) + 1/420*(6000*e^6*x^7 -
 350*(40*d*e^5 + 9*e^6)*x^6 + 252*(100*d^2*e^4 + 30*d*e^5 + 37*e^6)*x^5 - 1
05*(400*d^3*e^3 + 135*d^2*e^4 + 222*d*e^5 + 37*e^6)*x^4 + 140*(500*d^4*e^2
+ 180*d^3*e^3 + 333*d^2*e^4 + 74*d*e^5 + 148*e^6)*x^3 - 210*(600*d^5*e + 22
5*d^4*e^2 + 444*d^3*e^3 + 111*d^2*e^4 + 296*d*e^5 - 65*e^6)*x^2 + 420*(700*
d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5 + 107
*e^6)*x)/e^8 - (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e
^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*log(e*x + d)/e^9
```

Fricas [A] time = 0.984773, size = 1211, normalized size = 3.43

$$\frac{6000\,e^8x^8-42000\,d^8-18900\,d^7e-46620\,d^6e^2-15540\,d^5e^3-62160\,d^4e^4+27300\,d^3e^5-44940\,d^2e^6+13860\,de^7-756}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="f
ricas")
```

```
[Out] 1/420*(6000*e^8*x^8 - 42000*d^8 - 18900*d^7*e - 46620*d^6*e^2 - 15540*d^5*e
^3 - 62160*d^4*e^4 + 27300*d^3*e^5 - 44940*d^2*e^6 + 13860*d*e^7 - 7560*e^8
 - 50*(160*d*e^7 + 63*e^8)*x^7 + 14*(800*d^2*e^6 + 315*d*e^7 + 666*e^8)*x^6
 - 21*(800*d^3*e^5 + 315*d^2*e^6 + 666*d*e^7 + 185*e^8)*x^5 + 35*(800*d^4*e
^4 + 315*d^3*e^5 + 666*d^2*e^6 + 185*d*e^7 + 592*e^8)*x^4 - 70*(800*d^5*e^3
 + 315*d^4*e^4 + 666*d^3*e^5 + 185*d^2*e^6 + 592*d*e^7 - 195*e^8)*x^3 + 210
*(800*d^6*e^2 + 315*d^5*e^3 + 666*d^4*e^4 + 185*d^3*e^5 + 592*d^2*e^6 - 195
*d*e^7 + 214*e^8)*x^2 + 420*(700*d^7*e + 270*d^6*e^2 + 555*d^5*e^3 + 148*d^
4*e^4 + 444*d^3*e^5 - 130*d^2*e^6 + 107*d*e^7)*x - 420*(800*d^8 + 315*d^7*e
 + 666*d^6*e^2 + 185*d^5*e^3 + 592*d^4*e^4 - 195*d^3*e^5 + 214*d^2*e^6 - 33
*d*e^7 + (800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5
 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

Sympy [A] time = 1.45614, size = 367, normalized size = 1.04

$$-\frac{100d^8+45d^7e+111d^6e^2+37d^5e^3+148d^4e^4-65d^3e^5+107d^2e^6-33de^7+18e^8}{de^9+e^{10}x}+\frac{100x^7}{7e^2}-\frac{x^6\left(200d+45e\right)}{6e^3}+\frac{x^5\left(300d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] -(100*d**8 + 45*d**7*e + 111*d**6*e**2 + 37*d**5*e**3 + 148*d**4*e**4 - 65*
d**3*e**5 + 107*d**2*e**6 - 33*d*e**7 + 18*e**8)/(d*e**9 + e**10*x) + 100*x
**7/(7*e**2) - x**6*(200*d + 45*e)/(6*e**3) + x**5*(300*d**2 + 90*d*e + 111
*e**2)/(5*e**4) - x**4*(400*d**3 + 135*d**2*e + 222*d*e**2 + 37*e**3)/(4*e*
*5) + x**3*(500*d**4 + 180*d**3*e + 333*d**2*e**2 + 74*d*e**3 + 148*e**4)/(
3*e**6) - x**2*(600*d**5 + 225*d**4*e + 444*d**3*e**2 + 111*d**2*e**3 + 296
*d*e**4 - 65*e**5)/(2*e**7) + x*(700*d**6 + 270*d**5*e + 555*d**4*e**2 + 14
8*d**3*e**3 + 444*d**2*e**4 - 130*d*e**5 + 107*e**6)/e**8 - (5*d**2 - 2*d*e
 + 3*e**2)*(160*d**5 + 127*d**4*e + 88*d**3*e**2 - 4*d**2*e**3 + 64*d*e**4
- 11*e**5)*log(d + e*x)/e**9
```

Giac [A] time = 1.16612, size = 620, normalized size = 1.76

$$-\frac{1}{420}\left(xe+d\right)^7\left(\frac{350\left(160\,de+9\,e^2\right)e^{(-1)}}{xe+d}-\frac{84\left(2800\,d^2e^2+315\,de^3+111\,e^4\right)e^{(-2)}}{\left(xe+d\right)^2}+\frac{105\left(5600\,d^3e^3+945\,d^2e^4+666\,de^5+\right.}{\left(xe+d\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="g
iac")
```

```
[Out] -1/420*(x*e + d)^7*(350*(160*d*e + 9*e^2)*e^(-1)/(x*e + d) - 84*(2800*d^2*e
^2 + 315*d*e^3 + 111*e^4)*e^(-2)/(x*e + d)^2 + 105*(5600*d^3*e^3 + 945*d^2*
e^4 + 666*d*e^5 + 37*e^6)*e^(-3)/(x*e + d)^3 - 140*(7000*d^4*e^4 + 1575*d^3
*e^5 + 1665*d^2*e^6 + 185*d*e^7 + 148*e^8)*e^(-4)/(x*e + d)^4 + 210*(5600*d
^5*e^5 + 1575*d^4*e^6 + 2220*d^3*e^7 + 370*d^2*e^8 + 592*d*e^9 - 65*e^10)*e
^(-5)/(x*e + d)^5 - 420*(2800*d^6*e^6 + 945*d^5*e^7 + 1665*d^4*e^8 + 370*d^
3*e^9 + 888*d^2*e^10 - 195*d*e^11 + 107*e^12)*e^(-6)/(x*e + d)^6 - 6000)*e^
(-9) + (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195
*d^2*e^5 + 214*d*e^6 - 33*e^7)*e^(-9)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)
- (100*d^8*e^7/(x*e + d) + 45*d^7*e^8/(x*e + d) + 111*d^6*e^9/(x*e + d) + 3
7*d^5*e^10/(x*e + d) + 148*d^4*e^11/(x*e + d) - 65*d^3*e^12/(x*e + d) + 107
*d^2*e^13/(x*e + d) - 33*d*e^14/(x*e + d) + 18*e^15/(x*e + d))*e^(-16)
```

3.302 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

Optimal. Leaf size=354

$$\frac{3x^4\left(200d^2+45de+37e^2\right)}{4e^5}-\frac{x^3\left(270d^2e+1000d^3+333de^2+37e^3\right)}{3e^6}+\frac{x^2\left(666d^2e^2+450d^3e+1500d^4+111de^3+148e^4\right)}{2e^7}$$

```
[Out] -(((2100*d^5 + 675*d^4*e + 1110*d^3*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5)
*x)/e^8) + ((1500*d^4 + 450*d^3*e + 666*d^2*e^2 + 111*d*e^3 + 148*e^4)*x^2)
/(2*e^7) - ((1000*d^3 + 270*d^2*e + 333*d*e^2 + 37*e^3)*x^3)/(3*e^6) + (3*(
200*d^2 + 45*d*e + 37*e^2)*x^4)/(4*e^5) - (3*(20*d + 3*e)*x^5)/e^4 + (50*x^
6)/(3*e^3) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^
3 + 2*e^4))/(2*e^9*(d + e*x)^2) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d
^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(e^9*(d + e*x)) + ((280
0*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 +
107*e^6)*Log[d + e*x])/e^9
```

Rubi [A] time = 0.343337, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{3x^4\left(200d^2+45de+37e^2\right)}{4e^5}-\frac{x^3\left(270d^2e+1000d^3+333de^2+37e^3\right)}{3e^6}+\frac{x^2\left(666d^2e^2+450d^3e+1500d^4+111de^3+148e^4\right)}{2e^7}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -(((2100*d^5 + 675*d^4*e + 1110*d^3*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5)
*x)/e^8) + ((1500*d^4 + 450*d^3*e + 666*d^2*e^2 + 111*d*e^3 + 148*e^4)*x^2)
/(2*e^7) - ((1000*d^3 + 270*d^2*e + 333*d*e^2 + 37*e^3)*x^3)/(3*e^6) + (3*(
200*d^2 + 45*d*e + 37*e^2)*x^4)/(4*e^5) - (3*(20*d + 3*e)*x^5)/e^4 + (50*x^
6)/(3*e^3) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^
3 + 2*e^4))/(2*e^9*(d + e*x)^2) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d
^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(e^9*(d + e*x)) + ((280
0*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 +
107*e^6)*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^3}\,dx = \int\left(\frac{-2100d^5-675d^4e-1110d^3e^2-222d^2e^3-444de^4+65e^5}{e^8}+\frac{(15\cdots}{\cdots}\right.$$

$$= -\frac{\left(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5\right)x}{e^8}+\frac{(15\cdots}{\cdots}$$

Mathematica [A] time = 0.104478, size = 311, normalized size = 0.88

$$\frac{-18d^6e^2\left(2300x^2+240x-407\right)-2d^5e^3\left(5600x^3+6750x^2+2664x-999\right)+4d^4e^4\left(700x^4-945x^3-5661x^2-111x+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3
,x]
```

```
[Out] (9000*d^8 - 390*d^7*e*(-9 + 40*x) - 18*d^6*e^2*(-407 + 240*x + 2300*x^2) -
2*d^5*e^3*(-999 + 2664*x + 6750*x^2 + 5600*x^3) + 4*d^4*e^4*(1554 - 111*x -
 5661*x^2 - 945*x^3 + 700*x^4) - d^3*e^5*(1950 - 1776*x + 4662*x^2 + 6660*x
^3 - 945*x^4 + 1120*x^5) + d^2*e^6*(1926 - 1560*x - 9768*x^2 - 1480*x^3 + 1
665*x^4 - 378*x^5 + 560*x^6) + d*e^7*(-198 + 2568*x + 1560*x^2 - 3552*x^3 +
 370*x^4 - 666*x^5 + 189*x^6 - 320*x^7) + e^8*(-108 - 396*x + 780*x^3 + 888
*x^4 - 148*x^5 + 333*x^6 - 108*x^7 + 200*x^8) + 12*(2800*d^6 + 945*d^5*e +
1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*(d + e*x)^2
*Log[d + e*x])/(12*e^9*(d + e*x)^2)
```

Maple [A] time = 0.056, size = 531, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x)
```

```
[Out] 107/e^3*ln(e*x+d)-33/e^2/(e*x+d)-9/e/(e*x+d)^2+74/e^3*x^2+65/e^3*x-37/3/e^3
*x^3+666/e^7/(e*x+d)*d^5+185/e^6/(e*x+d)*d^4+1665/e^7*ln(e*x+d)*d^4+370/e^6
*ln(e*x+d)*d^3+888/e^5*ln(e*x+d)*d^2+333/e^5*x^2*d^2+111/2/e^4*x^2*d-9/e^3*
x^5-60/e^4*x^5*d+150/e^5*x^4*d^2+135/4/e^4*x^4*d-1000/3/e^6*x^3*d^3-90/e^5*
x^3*d^2+750/e^7*x^2*d^4+2800/e^9*ln(e*x+d)*d^6+945/e^8*ln(e*x+d)*d^5+800/e^
9/(e*x+d)*d^7+315/e^8/(e*x+d)*d^6-50/e^9/(e*x+d)^2*d^8-45/2/e^8/(e*x+d)^2*d
^7+225/e^6*x^2*d^3-2100/e^8*d^5*x-675/e^7*x*d^4+592/e^5/(e*x+d)*d^3-195/e^4
/(e*x+d)*d^2+214/e^3/(e*x+d)*d-111/2/e^7/(e*x+d)^2*d^6-37/2/e^6/(e*x+d)^2*d
^5-74/e^5/(e*x+d)^2*d^4+65/2/e^4/(e*x+d)^2*d^3-107/2/e^3/(e*x+d)^2*d^2+33/2
/e^2/(e*x+d)^2*d-195/e^4*ln(e*x+d)*d-1110/e^6*d^3*x-222/e^5*x*d^2-444/e^4*x
*d-111/e^4*x^3*d+50/3*x^6/e^3+111/4*x^4/e^3
```

Maxima [A] time = 0.986461, size = 510, normalized size = 1.44

$$\frac{1500\,d^8+585\,d^7e+1221\,d^6e^2+333\,d^5e^3+1036\,d^4e^4-325\,d^3e^5+321\,d^2e^6-33\,de^7-18\,e^8+2\left(800\,d^7e+315\,d^6e^2+66\right.}{2\left(e^{11}x^2+2\,de^{10}x+d^2e^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="m
axima")
```

```
[Out] 1/2*(1500*d^8 + 585*d^7*e + 1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325
*d^3*e^5 + 321*d^2*e^6 - 33*d*e^7 - 18*e^8 + 2*(800*d^7*e + 315*d^6*e^2 + 6
66*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*
x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/12*(200*e^5*x^6 - 36*(20*d*e^4 + 3
*e^5)*x^5 + 9*(200*d^2*e^3 + 45*d*e^4 + 37*e^5)*x^4 - 4*(1000*d^3*e^2 + 270
*d^2*e^3 + 333*d*e^4 + 37*e^5)*x^3 + 6*(1500*d^4*e + 450*d^3*e^2 + 666*d^2*
e^3 + 111*d*e^4 + 148*e^5)*x^2 - 12*(2100*d^5 + 675*d^4*e + 1110*d^3*e^2 +
222*d^2*e^3 + 444*d*e^4 - 65*e^5)*x)/e^8 + (2800*d^6 + 945*d^5*e + 1665*d^4
*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*log(e*x + d)/e^9
```

Fricas [A] time = 1.00041, size = 1323, normalized size = 3.74

$$\frac{200\,e^8x^8+9000\,d^8+3510\,d^7e+7326\,d^6e^2+1998\,d^5e^3+6216\,d^4e^4-1950\,d^3e^5+1926\,d^2e^6-198\,de^7-108\,e^8-4\left(80\,de\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="f
ricas")
```

```
[Out] 1/12*(200*e^8*x^8 + 9000*d^8 + 3510*d^7*e + 7326*d^6*e^2 + 1998*d^5*e^3 + 6
216*d^4*e^4 - 1950*d^3*e^5 + 1926*d^2*e^6 - 198*d*e^7 - 108*e^8 - 4*(80*d*e
^7 + 27*e^8)*x^7 + (560*d^2*e^6 + 189*d*e^7 + 333*e^8)*x^6 - 2*(560*d^3*e^5
 + 189*d^2*e^6 + 333*d*e^7 + 74*e^8)*x^5 + (2800*d^4*e^4 + 945*d^3*e^5 + 16
65*d^2*e^6 + 370*d*e^7 + 888*e^8)*x^4 - 4*(2800*d^5*e^3 + 945*d^4*e^4 + 166
5*d^3*e^5 + 370*d^2*e^6 + 888*d*e^7 - 195*e^8)*x^3 - 6*(6900*d^6*e^2 + 2250
*d^5*e^3 + 3774*d^4*e^4 + 777*d^3*e^5 + 1628*d^2*e^6 - 260*d*e^7)*x^2 - 12*
(1300*d^7*e + 360*d^6*e^2 + 444*d^5*e^3 + 37*d^4*e^4 - 148*d^3*e^5 + 130*d^
2*e^6 - 214*d*e^7 + 33*e^8)*x + 12*(2800*d^8 + 945*d^7*e + 1665*d^6*e^2 + 3
70*d^5*e^3 + 888*d^4*e^4 - 195*d^3*e^5 + 107*d^2*e^6 + (2800*d^6*e^2 + 945*
d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x
^2 + 2*(2800*d^7*e + 945*d^6*e^2 + 1665*d^5*e^3 + 370*d^4*e^4 + 888*d^3*e^5
 - 195*d^2*e^6 + 107*d*e^7)*x)*log(e*x + d))/(e^11*x^2 + 2*d*e^10*x + d^2*e
^9)
```

Sympy [A] time = 2.82858, size = 379, normalized size = 1.07

$$\frac{1500d^8 + 585d^7e + 1221d^6e^2 + 333d^5e^3 + 1036d^4e^4 - 325d^3e^5 + 321d^2e^6 - 33de^7 - 18e^8 + x\left(1600d^7e + 630d^6e^2 + 1332\right.}{2d^2e^9 + 4de^{10}x + 2e^{11}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

```
[Out] (1500*d**8 + 585*d**7*e + 1221*d**6*e**2 + 333*d**5*e**3 + 1036*d**4*e**4 -
 325*d**3*e**5 + 321*d**2*e**6 - 33*d*e**7 - 18*e**8 + x*(1600*d**7*e + 630
*d**6*e**2 + 1332*d**5*e**3 + 370*d**4*e**4 + 1184*d**3*e**5 - 390*d**2*e**
6 + 428*d*e**7 - 66*e**8))/(2*d**2*e**9 + 4*d*e**10*x + 2*e**11*x**2) + 50*
x**6/(3*e**3) - x**5*(60*d + 9*e)/e**4 + x**4*(600*d**2 + 135*d*e + 111*e**
2)/(4*e**5) - x**3*(1000*d**3 + 270*d**2*e + 333*d*e**2 + 37*e**3)/(3*e**6)
 + x**2*(1500*d**4 + 450*d**3*e + 666*d**2*e**2 + 111*d*e**3 + 148*e**4)/(2
*e**7) - x*(2100*d**5 + 675*d**4*e + 1110*d**3*e**2 + 222*d**2*e**3 + 444*d
*e**4 - 65*e**5)/e**8 + (2800*d**6 + 945*d**5*e + 1665*d**4*e**2 + 370*d**3
*e**3 + 888*d**2*e**4 - 195*d*e**5 + 107*e**6)*log(d + e*x)/e**9
```

Giac [A] time = 1.1506, size = 478, normalized size = 1.35

$$\left(2800\, d^{6} + 945\, d^{5} e + 1665\, d^{4} e^{2} + 370\, d^{3} e^{3} + 888\, d^{2} e^{4} - 195\, d e^{5} + 107\, e^{6}\right) e^{(-9)} \log\left(|xe + d|\right) + \frac{1}{12}\left(200\, x^{6} e^{15} - 720\, d x^{5} e^{14}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="g
iac")
```

```
[Out] (2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^
5 + 107*e^6)*e^(-9)*log(abs(x*e + d)) + 1/12*(200*x^6*e^15 - 720*d*x^5*e^14
 + 1800*d^2*x^4*e^13 - 4000*d^3*x^3*e^12 + 9000*d^4*x^2*e^11 - 25200*d^5*x*
e^10 - 108*x^5*e^15 + 405*d*x^4*e^14 - 1080*d^2*x^3*e^13 + 2700*d^3*x^2*e^1
2 - 8100*d^4*x*e^11 + 333*x^4*e^15 - 1332*d*x^3*e^14 + 3996*d^2*x^2*e^13 -
13320*d^3*x*e^12 - 148*x^3*e^15 + 666*d*x^2*e^14 - 2664*d^2*x*e^13 + 888*x^
2*e^15 - 5328*d*x*e^14 + 780*x*e^15)*e^(-18) + 1/2*(1500*d^8 + 585*d^7*e +
1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325*d^3*e^5 + 321*d^2*e^6 + 2*(
800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2
*e^6 + 214*d*e^7 - 33*e^8)*x - 33*d*e^7 - 18*e^8)*e^(-9)/(x*e + d)^2
```

3.303 $\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{(d+ex)^4} dx$

Optimal. Leaf size=360

$$\frac{x^3 (1000d^2 + 180de + 111e^2)}{3e^6} - \frac{x^2 (450d^2e + 2000d^3 + 444de^2 + 37e^3)}{2e^7} + \frac{2x (555d^2e^2 + 450d^3e + 1750d^4 + 74de^3 + 74e^4)}{e^8}$$

```
[Out] (2*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x)/e^8 - ((2000
*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2)/(2*e^7) + ((1000*d^2 + 180*d*e
+ 111*e^2)*x^3)/(3*e^6) - (5*(80*d + 9*e)*x^4)/(4*e^5) + (20*x^5)/e^4 - ((5
*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(3*e
^9*(d + e*x)^3) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^
2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(2*e^9*(d + e*x)^2) - (2800*d^6 + 945*d
^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)/(e^9
*(d + e*x)) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*
e^4 - 65*e^5)*Log[d + e*x])/e^9
```

Rubi [A] time = 0.358292, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{x^3 (1000d^2 + 180de + 111e^2)}{3e^6} - \frac{x^2 (450d^2e + 2000d^3 + 444de^2 + 37e^3)}{2e^7} + \frac{2x (555d^2e^2 + 450d^3e + 1750d^4 + 74de^3 + 74e^4)}{e^8}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4,x]
```



```
[Out] (2*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x)/e^8 - ((2000
*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2)/(2*e^7) + ((1000*d^2 + 180*d*e
+ 111*e^2)*x^3)/(3*e^6) - (5*(80*d + 9*e)*x^4)/(4*e^5) + (20*x^5)/e^4 - ((5
*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(3*e
^9*(d + e*x)^3) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^
2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(2*e^9*(d + e*x)^2) - (2800*d^6 + 945*d
^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)/(e^9
*(d + e*x)) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*
e^4 - 65*e^5)*Log[d + e*x])/e^9
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^4}\,dx = \int\left(\frac{2\left(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4\right)}{e^8} - \frac{\left(2000d^3+450\right.}{}\right.$$

$$= \frac{2\left(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4\right)x}{e^8} - \frac{\left(2000d^3+450d^2\right.}{}$$

Mathematica [A] time = 0.121456, size = 344, normalized size = 0.96

$$\frac{4e^3x^3\left(1000d^2+180de+111e^2\right)-6e^2x^2\left(450d^2e+2000d^3+444de^2+37e^3\right)+24ex\left(555d^2e^2+450d^3e+1750d^4+74de^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4
,x]
```

```
[Out] (24*e*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x - 6*e^2*(2
000*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2 + 4*e^3*(1000*d^2 + 180*d*e +
 111*e^2)*x^3 - 15*e^4*(80*d + 9*e)*x^4 + 240*e^5*x^5 - (4*(5*d^2 - 2*d*e +
 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(d + e*x)^3 + (6*(
800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5
 + 214*d*e^6 - 33*e^7))/(d + e*x)^2 - (12*(2800*d^6 + 945*d^5*e + 1665*d^4*
e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6))/(d + e*x) - 12*(560
0*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*Log[d
 + e*x])/(12*e^9)
```

Maple [A] time = 0.056, size = 558, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x)
```

```
[Out] 65/e^4*ln(e*x+d)-33/2/e^2/(e*x+d)^2-107/e^3/(e*x+d)-6/e/(e*x+d)^3+37/e^4*x^
3-37/2/e^4*x^2+148/e^4*x-45/4/e^4*x^4-148/3/e^5/(e*x+d)^3*d^4+65/3/e^4/(e*x
+d)^3*d^3-107/3/e^3/(e*x+d)^3*d^2+185/2/e^6/(e*x+d)^2*d^4+296/e^5/(e*x+d)^2
*d^3-195/2/e^4/(e*x+d)^2*d^2+107/e^3/(e*x+d)^2*d+11/e^2/(e*x+d)^3*d-5600/e^
9*ln(e*x+d)*d^5-1575/e^8*ln(e*x+d)*d^4-2220/e^7*ln(e*x+d)*d^3-370/e^6*ln(e*
x+d)*d^2-592/e^5*ln(e*x+d)*d-2800/e^9/(e*x+d)*d^6-945/e^8/(e*x+d)*d^5-1665/
e^7/(e*x+d)*d^4-100/e^5*x^4*d+1000/3/e^6*x^3*d^2+60/e^5*x^3*d-1000/e^7*x^2*
d^3-225/e^6*x^2*d^2-222/e^5*x^2*d+3500/e^8*d^4*x+900/e^7*x*d^3+1110/e^6*x*d
^2+148/e^5*x*d-100/3/e^9/(e*x+d)^3*d^8-15/e^8/(e*x+d)^3*d^7-37/e^7/(e*x+d)^
3*d^6-37/3/e^6/(e*x+d)^3*d^5-370/e^6/(e*x+d)*d^3-888/e^5/(e*x+d)*d^2+195/e^
4/(e*x+d)*d+400/e^9/(e*x+d)^2*d^7+315/2/e^8/(e*x+d)^2*d^6+333/e^7/(e*x+d)^2
*d^5+20*x^5/e^4
```

Maxima [A] time = 1.0135, size = 527, normalized size = 1.46

$$-\frac{14600\,d^8+4815\,d^7e+8214\,d^6e^2+1739\,d^5e^3+3848\,d^4e^4-715\,d^3e^5+214\,d^2e^6+33\,de^7+36\,e^8+6\left(2800\,d^6e^2+945\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="m
axima")
```

```
[Out] -1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4 -
 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 + 6*(2800*d^6*e^2 + 945*d^5*
e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 +
 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5
 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^
10*x + d^3*e^9) + 1/12*(240*e^4*x^5 - 15*(80*d*e^3 + 9*e^4)*x^4 + 4*(1000*d
^2*e^2 + 180*d*e^3 + 111*e^4)*x^3 - 6*(2000*d^3*e + 450*d^2*e^2 + 444*d*e^3
 + 37*e^4)*x^2 + 24*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4
)*x)/e^8 - (5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4
- 65*e^5)*log(e*x + d)/e^9
```

Fricas [A] time = 1.02185, size = 1434, normalized size = 3.98

$$\frac{240\,e^8x^8-29200\,d^8-9630\,d^7e-16428\,d^6e^2-3478\,d^5e^3-7696\,d^4e^4+1430\,d^3e^5-428\,d^2e^6-66\,de^7-72\,e^8-15\left(32\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="f
ricas")
```

```
[Out] 1/12*(240*e^8*x^8 - 29200*d^8 - 9630*d^7*e - 16428*d^6*e^2 - 3478*d^5*e^3 -
 7696*d^4*e^4 + 1430*d^3*e^5 - 428*d^2*e^6 - 66*d*e^7 - 72*e^8 - 15*(32*d*e
^7 + 9*e^8)*x^7 + (1120*d^2*e^6 + 315*d*e^7 + 444*e^8)*x^6 - 3*(1120*d^3*e^
5 + 315*d^2*e^6 + 444*d*e^7 + 74*e^8)*x^5 + 3*(5600*d^4*e^4 + 1575*d^3*e^5
+ 2220*d^2*e^6 + 370*d*e^7 + 592*e^8)*x^4 + 2*(47000*d^5*e^3 + 12510*d^4*e^
4 + 16206*d^3*e^5 + 2331*d^2*e^6 + 2664*d*e^7)*x^3 + 6*(13400*d^6*e^2 + 306
0*d^5*e^3 + 2886*d^4*e^4 + 111*d^3*e^5 - 888*d^2*e^6 + 390*d*e^7 - 214*e^8)
*x^2 - 6*(3400*d^7*e + 1665*d^6*e^2 + 3774*d^5*e^3 + 999*d^4*e^4 + 2664*d^3
*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x - 12*(5600*d^8 + 1575*d^7*e + 22
20*d^6*e^2 + 370*d^5*e^3 + 592*d^4*e^4 - 65*d^3*e^5 + (5600*d^5*e^3 + 1575*
d^4*e^4 + 2220*d^3*e^5 + 370*d^2*e^6 + 592*d*e^7 - 65*e^8)*x^3 + 3*(5600*d^
6*e^2 + 1575*d^5*e^3 + 2220*d^4*e^4 + 370*d^3*e^5 + 592*d^2*e^6 - 65*d*e^7)
*x^2 + 3*(5600*d^7*e + 1575*d^6*e^2 + 2220*d^5*e^3 + 370*d^4*e^4 + 592*d^3*
e^5 - 65*d^2*e^6)*x)*log(e*x + d))/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^10*x
+ d^3*e^9)
```

Sympy [A] time = 5.23281, size = 391, normalized size = 1.09

$$-\frac{14600d^8 + 4815d^7e + 8214d^6e^2 + 1739d^5e^3 + 3848d^4e^4 - 715d^3e^5 + 214d^2e^6 + 33de^7 + 36e^8 + x^2\left(16800d^6e^2 + 5670d^5e\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**4,x)
```

```
[Out] -(14600*d**8 + 4815*d**7*e + 8214*d**6*e**2 + 1739*d**5*e**3 + 3848*d**4*e*
*4 - 715*d**3*e**5 + 214*d**2*e**6 + 33*d*e**7 + 36*e**8 + x**2*(16800*d**6
*e**2 + 5670*d**5*e**3 + 9990*d**4*e**4 + 2220*d**3*e**5 + 5328*d**2*e**6 -
 1170*d*e**7 + 642*e**8) + x*(31200*d**7*e + 10395*d**6*e**2 + 17982*d**5*e
**3 + 3885*d**4*e**4 + 8880*d**3*e**5 - 1755*d**2*e**6 + 642*d*e**7 + 99*e*
*8))/(6*d**3*e**9 + 18*d**2*e**10*x + 18*d*e**11*x**2 + 6*e**12*x**3) + 20*
x**5/e**4 - x**4*(400*d + 45*e)/(4*e**5) + x**3*(1000*d**2 + 180*d*e + 111*
e**2)/(3*e**6) - x**2*(2000*d**3 + 450*d**2*e + 444*d*e**2 + 37*e**3)/(2*e*
*7) + x*(3500*d**4 + 900*d**3*e + 1110*d**2*e**2 + 148*d*e**3 + 148*e**4)/e
**8 - (5600*d**5 + 1575*d**4*e + 2220*d**3*e**2 + 370*d**2*e**3 + 592*d*e**
4 - 65*e**5)*log(d + e*x)/e**9
```

Giac [A] time = 1.1417, size = 466, normalized size = 1.29

$$-\left(5600\,d^5+1575\,d^4e+2220\,d^3e^2+370\,d^2e^3+592\,de^4-65\,e^5\right)e^{(-9)}\log\left(|xe+d|\right)+\frac{1}{12}\left(240\,x^5e^{16}-1200\,dx^4e^{15}+4000$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="g
iac")
```

```
[Out] -(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*
e^(-9)*log(abs(x*e + d)) + 1/12*(240*x^5*e^16 - 1200*d*x^4*e^15 + 4000*d^2*
x^3*e^14 - 12000*d^3*x^2*e^13 + 42000*d^4*x*e^12 - 135*x^4*e^16 + 720*d*x^3
*e^15 - 2700*d^2*x^2*e^14 + 10800*d^3*x*e^13 + 444*x^3*e^16 - 2664*d*x^2*e^
15 + 13320*d^2*x*e^14 - 222*x^2*e^16 + 1776*d*x*e^15 + 1776*x*e^16)*e^(-20)
 - 1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4
 - 715*d^3*e^5 + 6*(2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5
 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 + 214*d^2*e^6 + 3*(10400*d^7*e +
3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5 - 585*d^2*e^6 + 2
14*d*e^7 + 33*e^8)*x + 33*d*e^7 + 36*e^8)*e^(-9)/(x*e + d)^3
```

3.304 $\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$

Optimal. Leaf size=221

$$\frac{3}{500}ex^4\left(100d^2-165de+27e^2\right)+\frac{x^3\left(-2475d^2e+500d^3+1215de^2+458e^3\right)}{1875}-\frac{x^2\left(-6075d^2e+4125d^3-6870de^2+881e\right.}{6250}$$

```
[Out] ((10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3)*x)/15625 - ((4125*d^3 -
 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2)/6250 + ((500*d^3 - 2475*d^2*e + 12
15*d*e^2 + 458*e^3)*x^3)/1875 + (3*e*(100*d^2 - 165*d*e + 27*e^2)*x^4)/500
+ (3*(20*d - 11*e)*e^2*x^5)/125 + (2*e^3*x^6)/15 - ((52875*d^3 + 449175*d^2
*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(78125*Sqrt[14])
 + ((57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[3 + 2*x + 5*x^2
])/156250
```

Rubi [A] time = 0.190129, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{3}{500}ex^4\left(100d^2-165de+27e^2\right)+\frac{x^3\left(-2475d^2e+500d^3+1215de^2+458e^3\right)}{1875}-\frac{x^2\left(-6075d^2e+4125d^3-6870de^2+881e\right.}{6250}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```


```
[Out] ((10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3)*x)/15625 - ((4125*d^3 -
 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2)/6250 + ((500*d^3 - 2475*d^2*e + 12
15*d*e^2 + 458*e^3)*x^3)/1875 + (3*e*(100*d^2 - 165*d*e + 27*e^2)*x^4)/500
+ (3*(20*d - 11*e)*e^2*x^5)/125 + (2*e^3*x^6)/15 - ((52875*d^3 + 449175*d^2
*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(78125*Sqrt[14])
 + ((57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[3 + 2*x + 5*x^2
])/156250
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int\left(\frac{10125d^3+34350d^2e-13215de^2-5108e^3}{15625} - \frac{\left(4125d^3-6075d^2e-687\right.}{3125}\right.$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{\left(4125d^3-6075d^2e-687\right.}{6250}$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{\left(4125d^3-6075d^2e-687\right.}{6250}$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{\left(4125d^3-6075d^2e-687\right.}{6250}$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{\left(4125d^3-6075d^2e-687\right.}{6250}$$

Mathematica [A] time = 0.128532, size = 178, normalized size = 0.81

$$\frac{35x\left(450d^2e\left(250x^3-550x^2+405x+916\right)+250d^3\left(200x^2-495x+486\right)+45de^2\left(2000x^4-4125x^3+2700x^2+4580x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] (35*x*(250*d^3*(486 - 495*x + 200*x^2) + 450*d^2*e*(916 + 405*x - 550*x^2 +
 250*x^3) + 45*d*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4) + e^
3*(-61296 - 26430*x + 45800*x^2 + 30375*x^3 - 49500*x^4 + 25000*x^5)) - 6*S
qrt[14]*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5
*x)/Sqrt[14]] + 42*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[
3 + 2*x + 5*x^2])/6562500
```

Maple [A] time = 0.051, size = 291, normalized size = 1.3

$$-\frac{17967\sqrt{14}d^2e}{43750}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)+\frac{54969\sqrt{14}de^2}{218750}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)+\frac{81d^3x}{125}+\frac{23431\ln\left(5x^2+2x+3\right)}{156250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] -17967/43750*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2*e+54969/218750*14^
(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^2+81/125*d^3*x+23431/156250*ln(5*x
^2+2*x+3)*e^3+229/625*ln(5*x^2+2*x+3)*d^3+458/1875*x^3*e^3-33/50*x^2*d^3-33
/125*x^5*e^3+4/15*x^3*d^3-99/100*d*e^2*x^4+687/625*x^2*e^2*d-7662/15625*ln(
5*x^2+2*x+3)*d*e^2-2643/6250*ln(5*x^2+2*x+3)*d^2*e+53189/1093750*14^(1/2)*a
rctan(1/28*(10*x+2)*14^(1/2))*e^3-423/8750*14^(1/2)*arctan(1/28*(10*x+2)*14
^(1/2))*d^3-33/25*x^3*d^2*e+12/25*x^5*d*e^2+81/125*x^3*e^2*d-2643/3125*x*d*
e^2+3/5*x^4*d^2*e+1374/625*x*d^2*e+243/250*x^2*d^2*e-881/6250*e^3*x^2+81/50
0*e^3*x^4+2/15*e^3*x^6-5108/15625*e^3*x
```

Maxima [A] time = 1.49614, size = 278, normalized size = 1.26

$$\frac{2}{15}e^3x^6+\frac{3}{125}\left(20de^2-11e^3\right)x^5+\frac{3}{500}\left(100d^2e-165de^2+27e^3\right)x^4+\frac{1}{1875}\left(500d^3-2475d^2e+1215de^2+458e^3\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2
 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 -
 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*sqrt
(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt
(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3
)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^
2 + 2*x + 3)
```

Fricas [A] time = 1.00415, size = 626, normalized size = 2.83

$$\frac{2}{15}e^3x^6 + \frac{3}{125}\left(20\,de^2 - 11\,e^3\right)x^5 + \frac{3}{500}\left(100\,d^2e - 165\,de^2 + 27\,e^3\right)x^4 + \frac{1}{1875}\left(500\,d^3 - 2475\,d^2e + 1215\,de^2 + 458\,e^3\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2
 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 -
 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*sqrt
(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt
(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3
)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^
2 + 2*x + 3)
```

Sympy [C] time = 1.24721, size = 450, normalized size = 2.04

$$\frac{2e^3x^6}{15} + x^5\left(\frac{12de^2}{25} - \frac{33e^3}{125}\right) + x^4\left(\frac{3d^2e}{5} - \frac{99de^2}{100} + \frac{81e^3}{500}\right) + x^3\left(\frac{4d^3}{15} - \frac{33d^2e}{25} + \frac{81de^2}{125} + \frac{458e^3}{1875}\right) + x^2\left(-\frac{33d^3}{50} + \frac{243d^2e}{250}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 2*e**3*x**6/15 + x**5*(12*d*e**2/25 - 33*e**3/125) + x**4*(3*d**2*e/5 - 99*
d*e**2/100 + 81*e**3/500) + x**3*(4*d**3/15 - 33*d**2*e/25 + 81*d*e**2/125
+ 458*e**3/1875) + x**2*(-33*d**3/50 + 243*d**2*e/250 + 687*d*e**2/625 - 88
1*e**3/6250) + x*(81*d**3/125 + 1374*d**2*e/625 - 2643*d*e**2/3125 - 5108*e
**3/15625) + (229*d**3/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e
**3/156250 - sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189
*e**3)/2187500)*log(x + (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e
**3/5 + sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3
)/5)/(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)) + (229*d**3
/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e**3/156250 + sqrt(14)*
I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/2187500)*log(x
+ (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e**3/5 - sqrt(14)*I*(52
875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/5)/(52875*d**3 + 449
175*d**2*e - 274845*d*e**2 - 53189*e**3))
```

Giac [A] time = 1.1473, size = 286, normalized size = 1.29

$$\frac{2}{15}\,x^6e^3+\frac{12}{25}\,dx^5e^2+\frac{3}{5}\,d^2x^4e+\frac{4}{15}\,d^3x^3-\frac{33}{125}\,x^5e^3-\frac{99}{100}\,dx^4e^2-\frac{33}{25}\,d^2x^3e-\frac{33}{50}\,d^3x^2+\frac{81}{500}\,x^4e^3+\frac{81}{125}\,dx^3e^2+\frac{243}{250}\,d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 2/15*x^6*e^3 + 12/25*d*x^5*e^2 + 3/5*d^2*x^4*e + 4/15*d^3*x^3 - 33/125*x^5*
e^3 - 99/100*d*x^4*e^2 - 33/25*d^2*x^3*e - 33/50*d^3*x^2 + 81/500*x^4*e^3 +
 81/125*d*x^3*e^2 + 243/250*d^2*x^2*e + 81/125*d^3*x + 458/1875*x^3*e^3 + 6
87/625*d*x^2*e^2 + 1374/625*d^2*x*e - 881/6250*x^2*e^3 - 2643/3125*d*x*e^2
- 1/1093750*sqrt(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*
arctan(1/14*sqrt(14)*(5*x + 1)) - 5108/15625*x*e^3 + 1/156250*(57250*d^3 -
66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^2 + 2*x + 3)
```

3.305 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal. Leaf size=156

$$\frac{1}{375}x^3\left(100d^2-330de+81e^2\right)-\frac{x^2\left(825d^2-810de-458e^2\right)}{1250}+\frac{\left(5725d^2-4405de-2554e^2\right)\log\left(5x^2+2x+3\right)}{15625}+\frac{x\left(20\right.}{}$$

```
[Out] ((2025*d^2 + 4580*d*e - 881*e^2)*x)/3125 - ((825*d^2 - 810*d*e - 458*e^2)*x
^2)/1250 + ((100*d^2 - 330*d*e + 81*e^2)*x^3)/375 + ((40*d - 33*e)*e*x^4)/1
00 + (4*e^2*x^5)/25 - ((10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)
/Sqrt[14]])/(15625*Sqrt[14]) + ((5725*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*
x + 5*x^2])/15625
```

Rubi [A] time = 0.162131, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{1}{375}x^3\left(100d^2-330de+81e^2\right)-\frac{x^2\left(825d^2-810de-458e^2\right)}{1250}+\frac{\left(5725d^2-4405de-2554e^2\right)\log\left(5x^2+2x+3\right)}{15625}+\frac{x\left(20\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```


```
[Out] ((2025*d^2 + 4580*d*e - 881*e^2)*x)/3125 - ((825*d^2 - 810*d*e - 458*e^2)*x
^2)/1250 + ((100*d^2 - 330*d*e + 81*e^2)*x^3)/375 + ((40*d - 33*e)*e*x^4)/1
00 + (4*e^2*x^5)/25 - ((10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)
/Sqrt[14]])/(15625*Sqrt[14]) + ((5725*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*
x + 5*x^2])/15625
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \left(\frac{2025d^2+4580de-881e^2}{3125}-\frac{1}{625}\left(825d^2-810de-458e^2\right)x+\frac{1}{125}\left(100d\right.\right.$$

$$= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(100d^2-\right.$$

$$= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(100d^2-\right.$$

$$= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(100d^2-\right.$$

$$= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(100d^2-\right.$$

Mathematica [A] time = 0.0839269, size = 130, normalized size = 0.83

$$\frac{35x\left(50d^2\left(200x^2-495x+486\right)+60de\left(250x^3-550x^2+405x+916\right)+3e^2\left(2000x^4-4125x^3+2700x^2+4580x-352\right.\right.}{1312500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] (35*x*(50*d^2*(486 - 495*x + 200*x^2) + 60*d*e*(916 + 405*x - 550*x^2 + 250
*x^3) + 3*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4)) - 6*Sqrt[1
4]*(10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 84*(572
5*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*x + 5*x^2])/1312500
```

Maple [A] time = 0.052, size = 191, normalized size = 1.2

$$\frac{4\,e^2x^5}{25}+\frac{2\,x^4de}{5}-\frac{33\,x^4e^2}{100}+\frac{4\,x^3d^2}{15}-\frac{22\,x^3de}{25}+\frac{27\,e^2x^3}{125}-\frac{33\,x^2d^2}{50}+\frac{81\,x^2de}{125}+\frac{229\,x^2e^2}{625}+\frac{81\,d^2x}{125}+\frac{916\,xde}{625}-\frac{881\,e^2x}{3125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] 4/25*e^2*x^5+2/5*x^4*d*e-33/100*x^4*e^2+4/15*x^3*d^2-22/25*x^3*d*e+27/125*e
^2*x^3-33/50*x^2*d^2+81/125*x^2*d*e+229/625*x^2*e^2+81/125*d^2*x+916/625*x*
d*e-881/3125*e^2*x+229/625*ln(5*x^2+2*x+3)*d^2-881/3125*ln(5*x^2+2*x+3)*d*e
-2554/15625*ln(5*x^2+2*x+3)*e^2-423/8750*14^(1/2)*arctan(1/28*(10*x+2)*14^(
1/2))*d^2-5989/21875*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e+18323/2187
50*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^2
```

Maxima [A] time = 1.53978, size = 190, normalized size = 1.22

$$\frac{4}{25}\,e^2x^5+\frac{1}{100}\left(40\,de-33\,e^2\right)x^4+\frac{1}{375}\left(100\,d^2-330\,de+81\,e^2\right)x^3-\frac{1}{1250}\left(825\,d^2-810\,de-458\,e^2\right)x^2-\frac{1}{218750}\sqrt{14}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81*
e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(10
575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(
```

```
2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)
*log(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.00717, size = 436, normalized size = 2.79

$$\frac{4}{25}e^2x^5 + \frac{1}{100}\left(40\,de - 33\,e^2\right)x^4 + \frac{1}{375}\left(100\,d^2 - 330\,de + 81\,e^2\right)x^3 - \frac{1}{1250}\left(825\,d^2 - 810\,de - 458\,e^2\right)x^2 - \frac{1}{218750}\sqrt{14}(10$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81*
e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(10
575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(
2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)
*log(5*x^2 + 2*x + 3)
```

Sympy [C] time = 0.998055, size = 303, normalized size = 1.94

$$\frac{4e^2x^5}{25} + x^4\left(\frac{2de}{5} - \frac{33e^2}{100}\right) + x^3\left(\frac{4d^2}{15} - \frac{22de}{25} + \frac{27e^2}{125}\right) + x^2\left(-\frac{33d^2}{50} + \frac{81de}{125} + \frac{229e^2}{625}\right) + x\left(\frac{81d^2}{125} + \frac{916de}{625} - \frac{881e^2}{3125}\right) + \left(\frac{229}{62}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*e**2*x**5/25 + x**4*(2*d*e/5 - 33*e**2/100) + x**3*(4*d**2/15 - 22*d*e/25
 + 27*e**2/125) + x**2*(-33*d**2/50 + 81*d*e/125 + 229*e**2/625) + x*(81*d*
*2/125 + 916*d*e/625 - 881*e**2/3125) + (229*d**2/625 - 881*d*e/3125 - 2554
*e**2/15625 - sqrt(14)*I*(10575*d**2 + 59890*d*e - 18323*e**2)/437500)*log(
x + (2115*d**2 + 11978*d*e - 18323*e**2/5 + sqrt(14)*I*(10575*d**2 + 59890*
d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*e**2)) + (229*d**2/625
 - 881*d*e/3125 - 2554*e**2/15625 + sqrt(14)*I*(10575*d**2 + 59890*d*e - 18
323*e**2)/437500)*log(x + (2115*d**2 + 11978*d*e - 18323*e**2/5 - sqrt(14)*
I*(10575*d**2 + 59890*d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*
e**2))
```

Giac [A] time = 1.13646, size = 196, normalized size = 1.26

$$\frac{4}{25}\,x^5e^2+\frac{2}{5}\,dx^4e+\frac{4}{15}\,d^2x^3-\frac{33}{100}\,x^4e^2-\frac{22}{25}\,dx^3e-\frac{33}{50}\,d^2x^2+\frac{27}{125}\,x^3e^2+\frac{81}{125}\,dx^2e+\frac{81}{125}\,d^2x+\frac{229}{625}\,x^2e^2+\frac{916}{625}\,dxe-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 4/25*x^5*e^2 + 2/5*d*x^4*e + 4/15*d^2*x^3 - 33/100*x^4*e^2 - 22/25*d*x^3*e
- 33/50*d^2*x^2 + 27/125*x^3*e^2 + 81/125*d*x^2*e + 81/125*d^2*x + 229/625*
x^2*e^2 + 916/625*d*x*e - 1/218750*sqrt(14)*(10575*d^2 + 59890*d*e - 18323*
e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) - 881/3125*x*e^2 + 1/15625*(5725*d^2 -
 4405*d*e - 2554*e^2)*log(5*x^2 + 2*x + 3)
```

3.306 $\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$

Optimal. Leaf size=99

$$\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)+\frac{(2290d-881e)\log\left(5x^2+2x+3\right)}{6250}+\frac{1}{625}x(405d+458e)-\frac{(2115d+5989e)\tan^{-1}}{3125\sqrt{14}}$$

```
[Out] ((405*d + 458*e)*x)/625 - (3*(55*d - 27*e)*x^2)/250 + ((20*d - 33*e)*x^3)/7
5 + (e*x^4)/5 - ((2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(3125*Sqrt[1
4]) + ((2290*d - 881*e)*Log[3 + 2*x + 5*x^2])/6250
```

Rubi [A] time = 0.108266, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1628, 634, 618, 204, 628}

$$\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)+\frac{(2290d-881e)\log\left(5x^2+2x+3\right)}{6250}+\frac{1}{625}x(405d+458e)-\frac{(2115d+5989e)\tan^{-1}}{3125\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```


```
[Out] ((405*d + 458*e)*x)/625 - (3*(55*d - 27*e)*x^2)/250 + ((20*d - 33*e)*x^3)/7
5 + (e*x^4)/5 - ((2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(3125*Sqrt[1
4]) + ((2290*d - 881*e)*Log[3 + 2*x + 5*x^2])/6250
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \left(\frac{1}{625}(405d+458e)-\frac{3}{125}(55d-27e)x+\frac{1}{25}(20d-33e)x^2+\frac{4ex^3}{5}+\frac{35d}{\ldots}\right.$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}+\frac{1}{625}\int\ldots$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}+\frac{(-2115\ldots}{\ldots}$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}+\frac{(2290d\ldots}{\ldots}$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}-\frac{(2115d\ldots}{\ldots}$$

Mathematica [A] time = 0.0503886, size = 86, normalized size = 0.87

$$\frac{35x\left(5d\left(200x^2-495x+486\right)+3e\left(250x^3-550x^2+405x+916\right)\right)+21(2290d-881e)\log\left(5x^2+2x+3\right)-3\sqrt{14}(211\ldots}{131250}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] (35*x*(5*d*(486 - 495*x + 200*x^2) + 3*e*(916 + 405*x - 550*x^2 + 250*x^3))
 - 3*Sqrt[14]*(2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]] + 21*(2290*d - 8
81*e)*Log[3 + 2*x + 5*x^2])/131250
```

Maple [A] time = 0.046, size = 102, normalized size = 1.

$$\frac{ex^4}{5}+\frac{4\,dx^3}{15}-\frac{11\,x^3e}{25}-\frac{33\,dx^2}{50}+\frac{81\,ex^2}{250}+\frac{81\,dx}{125}+\frac{458\,ex}{625}+\frac{229\,\ln\left(5\,x^2+2\,x+3\right)d}{625}-\frac{881\,e\ln\left(5\,x^2+2\,x+3\right)}{6250}-\frac{423\,\sqrt{}}{875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] 1/5*e*x^4+4/15*d*x^3-11/25*x^3*e-33/50*d*x^2+81/250*e*x^2+81/125*d*x+458/62
5*e*x+229/625*ln(5*x^2+2*x+3)*d-881/6250*e*ln(5*x^2+2*x+3)-423/8750*14^(1/2
)*arctan(1/28*(10*x+2)*14^(1/2))*d-5989/43750*14^(1/2)*arctan(1/28*(10*x+2)
*14^(1/2))*e
```

Maxima [A] time = 1.56544, size = 113, normalized size = 1.14

$$\frac{1}{5}\,ex^4+\frac{1}{75}\,(20\,d-33\,e)x^3-\frac{3}{250}\,(55\,d-27\,e)x^2-\frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{1}{625}\,(405\,d+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] 1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqrt
(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 458
*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.31163, size = 275, normalized size = 2.78

$$\frac{1}{5}\,ex^4+\frac{1}{75}\,(20\,d-33\,e)x^3-\frac{3}{250}\,(55\,d-27\,e)x^2-\frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{1}{625}\,(405\,d+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] 1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqrt
(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 458
*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Sympy [C] time = 0.686389, size = 163, normalized size = 1.65

$$\frac{ex^4}{5} + x^3\left(\frac{4d}{15} - \frac{11e}{25}\right) + x^2\left(-\frac{33d}{50} + \frac{81e}{250}\right) + x\left(\frac{81d}{125} + \frac{458e}{625}\right) + \left(\frac{229d}{625} - \frac{881e}{6250} - \frac{\sqrt{14}i\,(2115d + 5989e)}{87500}\right)\log\left(x + \frac{423d -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] e*x**4/5 + x**3*(4*d/15 - 11*e/25) + x**2*(-33*d/50 + 81*e/250) + x*(81*d/1
25 + 458*e/625) + (229*d/625 - 881*e/6250 - sqrt(14)*I*(2115*d + 5989*e)/87
500)*log(x + (423*d + 5989*e/5 + sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d +
5989*e)) + (229*d/625 - 881*e/6250 + sqrt(14)*I*(2115*d + 5989*e)/87500)*lo
g(x + (423*d + 5989*e/5 - sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d + 5989*e)
)
```

Giac [A] time = 1.1224, size = 119, normalized size = 1.2

$$\frac{1}{5}x^4e + \frac{4}{15}dx^3 - \frac{11}{25}x^3e - \frac{33}{50}dx^2 + \frac{81}{250}x^2e - \frac{1}{43750}\sqrt{14}(2115\,d + 5989\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x + 1)\right) + \frac{81}{125}dx + \frac{458}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] 1/5*x^4*e + 4/15*d*x^3 - 11/25*x^3*e - 33/50*d*x^2 + 81/250*x^2*e - 1/43750
*sqrt(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 81/125*d*x +
458/625*x*e + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

3.307 $\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} \, dx$

Optimal. Leaf size=56

$$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{229}{625}\log\left(5x^2+2x+3\right)+\frac{81x}{125}-\frac{423\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}$$

```
[Out] (81*x)/125 - (33*x^2)/50 + (4*x^3)/15 - (423*ArcTan[(1 + 5*x)/Sqrt[14]])/(6
25*Sqrt[14]) + (229*Log[3 + 2*x + 5*x^2])/625
```

Rubi [A] time = 0.0486828, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{229}{625}\log\left(5x^2+2x+3\right)+\frac{81x}{125}-\frac{423\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (81*x)/125 - (33*x^2)/50 + (4*x^3)/15 - (423*ArcTan[(1 + 5*x)/Sqrt[14]])/(6
25*Sqrt[14]) + (229*Log[3 + 2*x + 5*x^2])/625
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx &= \int \left(\frac{81}{125}-\frac{33x}{25}+\frac{4x^2}{5}+\frac{7+458x}{125\left(3+2x+5x^2\right)}\right)dx \\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{1}{125}\int \frac{7+458x}{3+2x+5x^2}\,dx \\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\int \frac{2+10x}{3+2x+5x^2}\,dx-\frac{423}{625}\int \frac{1}{3+2x+5x^2}\,dx \\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\log\left(3+2x+5x^2\right)+\frac{846}{625}\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx,x,2+10x
\\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}-\frac{423\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{229}{625}\log\left(3+2x+5x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0180964, size = 50, normalized size = 0.89

$$\frac{35x\left(200x^2-495x+486\right)+9618\log\left(5x^2+2x+3\right)-1269\sqrt{14}\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{26250}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (35*x*(486 - 495*x + 200*x^2) - 1269*Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]] +
9618*Log[3 + 2*x + 5*x^2])/26250
```

Maple [A] time = 0.048, size = 44, normalized size = 0.8

$$\frac{4 x^{3}}{15}-\frac{33 x^{2}}{50}+\frac{81 x}{125}+\frac{229 \ln \left(5 x^{2}+2 x+3\right)}{625}-\frac{423 \sqrt{14}}{8750} \arctan \left(\frac{(10 x+2) \sqrt{14}}{28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] 4/15*x^3-33/50*x^2+81/125*x+229/625*ln(5*x^2+2*x+3)-423/8750*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2))
```

Maxima [A] time = 1.54292, size = 58, normalized size = 1.04

$$\frac{4}{15} x^{3}-\frac{33}{50} x^{2}-\frac{423}{8750} \sqrt{14} \arctan \left(\frac{1}{14} \sqrt{14}(5 x+1)\right)+\frac{81}{125} x+\frac{229}{625} \log \left(5 x^{2}+2 x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.23313, size = 157, normalized size = 2.8

$$\frac{4}{15} x^{3}-\frac{33}{50} x^{2}-\frac{423}{8750} \sqrt{14} \arctan \left(\frac{1}{14} \sqrt{14}(5 x+1)\right)+\frac{81}{125} x+\frac{229}{625} \log \left(5 x^{2}+2 x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Sympy [A] time = 0.123366, size = 61, normalized size = 1.09

$$\frac{4x^3}{15} - \frac{33x^2}{50} + \frac{81x}{125} + \frac{229\log\left(x^2 + \frac{2x}{5} + \frac{3}{5}\right)}{625} - \frac{423\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{8750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*x**3/15 - 33*x**2/50 + 81*x/125 + 229*log(x**2 + 2*x/5 + 3/5)/625 - 423*s
qrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/8750
```

Giac [A] time = 1.17232, size = 58, normalized size = 1.04

$$\frac{4}{15}\,x^3 - \frac{33}{50}\,x^2 - \frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x + 1)\right) + \frac{81}{125}\,x + \frac{229}{625}\,\log\left(5\,x^2 + 2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

3.308 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx$

Optimal. Leaf size=168

$$\frac{(458d-7e)\log\left(5x^2+2x+3\right)}{250\left(5d^2-2de+3e^2\right)}+\frac{\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-\frac{(423d-1367e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}-\frac{x(20d+}{25e}$$

```
[Out] -((20*d + 33*e)*x)/(25*e^2) + (2*x^2)/(5*e) - ((423*d - 1367*e)*ArcTan[(1 +
 5*x)/Sqrt[14]])/(125*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + ((4*d^4 + 5*d^3*e
 + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)) +
 ((458*d - 7*e)*Log[3 + 2*x + 5*x^2])/(250*(5*d^2 - 2*d*e + 3*e^2))
```

Rubi [A] time = 0.193678, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{(458d-7e)\log\left(5x^2+2x+3\right)}{250\left(5d^2-2de+3e^2\right)}+\frac{\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-\frac{(423d-1367e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}-\frac{x(20d+}{25e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -((20*d + 33*e)*x)/(25*e^2) + (2*x^2)/(5*e) - ((423*d - 1367*e)*ArcTan[(1 +
 5*x)/Sqrt[14]])/(125*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + ((4*d^4 + 5*d^3*e
 + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)) +
 ((458*d - 7*e)*Log[3 + 2*x + 5*x^2])/(250*(5*d^2 - 2*d*e + 3*e^2))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx = \int \left(\frac{-20d-33e}{25e^2}+\frac{4x}{5e}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)}+\frac{7d+272e+(458d-7\cdots}{25\left(5d^2-2de+3e^2\right)\left(3+2\cdots\right.}\right.$$

$$= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}+\frac{\int \frac{7d+272e+(458\cdots}{3+2x+5x\cdots}}{25\left(5d^2-2d\cdots\right.}$$

$$= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-\frac{(423d-1367\cdots}{125\left(5d^2-\cdots\right.}$$

$$= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}+\frac{(458d-7e)\log\cdots}{250\left(5d^2\cdots\right.}$$

$$= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}-\frac{(423d-1367e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2\cdots\right.}{e^3\left(5d^2-2de+3\cdots\right.}$$

Mathematica [A] time = 0.10388, size = 146, normalized size = 0.87

$$\frac{70ex\left(5d^2-2de+3e^2\right)(e(10x-33)-20d)+1750\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)\log(d+ex)+7e^3(458d-7e)\log\left(5\cdots\right.}{1750e^3\left(5d^2-2de+3e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (70*e*(5*d^2 - 2*d*e + 3*e^2)*x*(-20*d + e*(-33 + 10*x)) - Sqrt[14]*(423*d
- 1367*e)*e^3*ArcTan[(1 + 5*x)/Sqrt[14]] + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*Log[d + e*x] + 7*(458*d - 7*e)*e^3*Log[3 + 2*x + 5*x^2])
/(1750*e^3*(5*d^2 - 2*d*e + 3*e^2))
```

Maple [A] time = 0.058, size = 298, normalized size = 1.8

$$\frac{2x^2}{5e}-\frac{4dx}{5e^2}-\frac{33x}{25e}+\frac{229\ln\left(5x^2+2x+3\right)d}{625d^2-250de+375e^2}-\frac{7e\ln\left(5x^2+2x+3\right)}{1250d^2-500de+750e^2}-\frac{423\sqrt{14}d}{8750d^2-3500de+5250e^2}\arctan\left(\frac{(10x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x)
```

```
[Out] 2/5*x^2/e-4/5/e^2*x*d-33/25/e*x+229/5/(125*d^2-50*d*e+75*e^2)*ln(5*x^2+2*x+
3)*d-7/10/(125*d^2-50*d*e+75*e^2)*ln(5*x^2+2*x+3)*e-423/70/(125*d^2-50*d*e+
75*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d+1367/70/(125*d^2-50*d*e+7
5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e+4/e^3/(5*d^2-2*d*e+3*e^2)*
ln(e*x+d)*d^4+5/e^2/(5*d^2-2*d*e+3*e^2)*ln(e*x+d)*d^3+3/e/(5*d^2-2*d*e+3*e^
2)*ln(e*x+d)*d^2-1/(5*d^2-2*d*e+3*e^2)*ln(e*x+d)*d+2*e/(5*d^2-2*d*e+3*e^2)*
ln(e*x+d)
```

Maxima [A] time = 1.4724, size = 216, normalized size = 1.29

$$-\frac{\sqrt{14}(423d-1367e)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)}{1750\left(5d^2-2de+3e^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\left(ex+d\right)}{5d^2e^3-2de^4+3e^5}+\frac{(458d-7e)\log\left(5x\right.}{250\left(5d^2-2de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] -1/1750*sqrt(14)*(423*d - 1367*e)*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 -
2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)
```

```
/(5*d^2*e^3 - 2*d*e^4 + 3*e^5) + 1/250*(458*d - 7*e)*log(5*x^2 + 2*x + 3)/(
5*d^2 - 2*d*e + 3*e^2) + 1/25*(10*e*x^2 - (20*d + 33*e)*x)/e^2
```

Fricas [A] time = 1.52551, size = 414, normalized size = 2.46

$$\frac{700\left(5\,d^2e^2-2\,de^3+3\,e^4\right)x^2-\sqrt{14}\left(423\,de^3-1367\,e^4\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)-70\left(100\,d^3e+125\,d^2e^2-6\,de^3+99\,e\right.}{1750\left(5\,d^2e^3-2\,de^4+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] 1/1750*(700*(5*d^2*e^2 - 2*d*e^3 + 3*e^4)*x^2 - sqrt(14)*(423*d*e^3 - 1367*
e^4)*arctan(1/14*sqrt(14)*(5*x + 1)) - 70*(100*d^3*e + 125*d^2*e^2 - 6*d*e^
3 + 99*e^4)*x + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x
+ d) + 7*(458*d*e^3 - 7*e^4)*log(5*x^2 + 2*x + 3))/(5*d^2*e^3 - 2*d*e^4 + 3
*e^5)
```

Sympy [C] time = 11.239, size = 4106, normalized size = 24.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3),x)
```

```
[Out] (-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7
*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))*log(x + (-392000000*d**10*(-sqrt(14)*I
*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*
d**2 - 2*d*e + 3*e**2))) - 823200000*d**9*e*(-sqrt(14)*I*(423*d - 1367*e)/(
3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e*
*2))) + 153104000*d**9 + 490000000*d**8*e**3*(-sqrt(14)*I*(423*d - 1367*e)/
(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e
**2)))**2 - 1043700000*d**8*e**2*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**
2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 349
944000*d**8*e + 220500000*d**7*e**4*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*
d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2
 - 646800000*d**7*e**3*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e
```

+ 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 386841000*d**
7*e**2 + 617925000*d**6*e**5*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 -
2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 8722
00000*d**6*e**4*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**
2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 39565736*d**6*e**3 -
356370000*d**5*e**6*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e +
3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 573645520*d*
*5*e**5*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (4
58*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 14633332*d**5*e**4 + 1259909
000*d**4*e**7*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)
) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 115902052*d**4*e**6
*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d -
7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 107677543*d**4*e**5 - 1045744000*d*
*3*e**8*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (4
58*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 126665168*d**3*e**7*(-sqr
t(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(
250*(5*d**2 - 2*d*e + 3*e**2))) + 129989935*d**3*e**6 + 850339000*d**2*e**9
*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d -
7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 218333192*d**2*e**8*(-sqrt(14)*I
*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*
d**2 - 2*d*e + 3*e**2))) - 50221473*d**2*e**7 - 358554000*d*e**10*(-sqrt(14
)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*
(5*d**2 - 2*d*e + 3*e**2)))**2 + 113884512*d*e**9*(-sqrt(14)*I*(423*d - 136
7*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e
+ 3*e**2))) + 17826327*d*e**8 + 106659000*e**11*(-sqrt(14)*I*(423*d - 1367*
e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e +
3*e**2)))**2 - 89860932*e**10*(-sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 -
2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 125032
88*e**9)/(47376000*d**9 - 34664000*d**8*e - 237671000*d**7*e**2 - 447135416
*d**6*e**3 - 79441992*d**5*e**4 + 39361392*d**4*e**5 + 28919955*d**3*e**6 -
233063217*d**2*e**7 + 141064083*d*e**8 - 59791213*e**9)) + (sqrt(14)*I*(42
3*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2
- 2*d*e + 3*e**2)))*log(x + (-392000000*d**10*(sqrt(14)*I*(423*d - 1367*e)
/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*
e**2))) - 823200000*d**9*e*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d
*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 153104000*
d**9 + 490000000*d**8*e**3*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d
*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 1043700
000*d**8*e**2*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2))
+ (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 349944000*d**8*e + 2205
00000*d**7*e**4*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2
)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 646800000*d**7*e**
3*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d -
7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 386841000*d**7*e**2 + 617925000*d**
6*e**5*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458

```
*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 872200000*d**6*e**4*(sqrt(1
4)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250
*(5*d**2 - 2*d*e + 3*e**2))) + 39565736*d**6*e**3 - 356370000*d**5*e**6*(sq
rt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/
(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 573645520*d**5*e**5*(sqrt(14)*I*(423*
d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 -
 2*d*e + 3*e**2))) + 14633332*d**5*e**4 + 1259909000*d**4*e**7*(sqrt(14)*I*
(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d
**2 - 2*d*e + 3*e**2)))**2 - 115902052*d**4*e**6*(sqrt(14)*I*(423*d - 1367*
e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e +
3*e**2))) + 107677543*d**4*e**5 - 1045744000*d**3*e**8*(sqrt(14)*I*(423*d -
 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*
d*e + 3*e**2)))**2 - 126665168*d**3*e**7*(sqrt(14)*I*(423*d - 1367*e)/(3500
*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))
) + 129989935*d**3*e**6 + 850339000*d**2*e**9*(sqrt(14)*I*(423*d - 1367*e)/
(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e
**2)))**2 - 218333192*d**2*e**8*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2
- 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) - 50221
473*d**2*e**7 - 358554000*d*e**10*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**
2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 +
113884512*d*e**9*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**
2)) + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2))) + 17826327*d*e**8 + 10
6659000*e**11*(sqrt(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2))
 + (458*d - 7*e)/(250*(5*d**2 - 2*d*e + 3*e**2)))**2 - 89860932*e**10*(sqrt
(14)*I*(423*d - 1367*e)/(3500*(5*d**2 - 2*d*e + 3*e**2)) + (458*d - 7*e)/(2
50*(5*d**2 - 2*d*e + 3*e**2))) + 12503288*e**9)/(47376000*d**9 - 34664000*d
**8*e - 237671000*d**7*e**2 - 447135416*d**6*e**3 - 79441992*d**5*e**4 + 39
361392*d**4*e**5 + 28919955*d**3*e**6 - 233063217*d**2*e**7 + 141064083*d*e
**8 - 59791213*e**9)) + 2*x**2/(5*e) - x*(20*d + 33*e)/(25*e**2) + (4*d**4
+ 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)*log(x + (-392000000*d**10*(4*d*
*4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(e**3*(5*d**2 - 2*d*e + 3*e*
*2)) + 153104000*d**9 - 823200000*d**9*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d
*e**3 + 2*e**4)/(e**2*(5*d**2 - 2*d*e + 3*e**2)) + 349944000*d**8*e - 10437
00000*d**8*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(e*(5*d**2 -
 2*d*e + 3*e**2)) + 490000000*d**8*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**
3 + 2*e**4)**2/(e**3*(5*d**2 - 2*d*e + 3*e**2)**2) + 386841000*d**7*e**2 -
646800000*d**7*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2
- 2*d*e + 3*e**2) + 220500000*d**7*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**
3 + 2*e**4)**2/(e**2*(5*d**2 - 2*d*e + 3*e**2)**2) + 39565736*d**6*e**3 - 8
72200000*d**6*e*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2
 - 2*d*e + 3*e**2) + 617925000*d**6*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e*
*3 + 2*e**4)**2/(e*(5*d**2 - 2*d*e + 3*e**2)**2) + 14633332*d**5*e**4 - 573
645520*d**5*e**2*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**
2 - 2*d*e + 3*e**2) - 356370000*d**5*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e
**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**2 + 107677543*d**4*e**5 - 11590
```

```
2052*d**4*e**3*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2
- 2*d*e + 3*e**2) + 1259909000*d**4*e*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*
e**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**2 + 129989935*d**3*e**6 - 1266
65168*d**3*e**4*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2
 - 2*d*e + 3*e**2) - 1045744000*d**3*e**2*(4*d**4 + 5*d**3*e + 3*d**2*e**2
- d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**2 - 50221473*d**2*e**7 - 2
18333192*d**2*e**5*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d
**2 - 2*d*e + 3*e**2) + 850339000*d**2*e**3*(4*d**4 + 5*d**3*e + 3*d**2*e**
2 - d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**2 + 17826327*d*e**8 + 11
3884512*d*e**6*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2
- 2*d*e + 3*e**2) - 358554000*d*e**4*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e
**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**2 + 12503288*e**9 - 89860932*e*
*7*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3*
e**2) + 106659000*e**5*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)*
*2/(5*d**2 - 2*d*e + 3*e**2)**2)/(47376000*d**9 - 34664000*d**8*e - 2376710
00*d**7*e**2 - 447135416*d**6*e**3 - 79441992*d**5*e**4 + 39361392*d**4*e**
5 + 28919955*d**3*e**6 - 233063217*d**2*e**7 + 141064083*d*e**8 - 59791213*
e**9))/(e**3*(5*d**2 - 2*d*e + 3*e**2))
```

Giac [A] time = 1.12489, size = 213, normalized size = 1.27

$$\frac{1}{25}\left(10\,x^2e-20\,dx-33\,xe\right)e^{(-2)}-\frac{\sqrt{14}(423\,d-1367\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{1750\,\left(5\,d^2-2\,de+3\,e^2\right)}+\frac{(458\,d-7\,e)\log\left(5\,x^2+2\,x+3\right)}{250\,\left(5\,d^2-2\,de+3\,e^2\right)}+\frac{(4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] 1/25*(10*x^2*e - 20*d*x - 33*x*e)*e^(-2) - 1/1750*sqrt(14)*(423*d - 1367*e)
*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 - 2*d*e + 3*e^2) + 1/250*(458*d - 7
*e)*log(5*x^2 + 2*x + 3)/(5*d^2 - 2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2
*e^2 - d*e^3 + 2*e^4)*log(abs(x*e + d))/(5*d^2*e^3 - 2*d*e^4 + 3*e^5)
```

3.309 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$

Optimal. Leaf size=233

$$\frac{(229d^2-7de-136e^2)\log(5x^2+2x+3)}{25(5d^2-2de+3e^2)^2}-\frac{3d^2e^2+5d^3e+4d^4-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(28d^3e^2+44d^2e^3+d^4e+40d^5-2de^4+}{e^3(5d^2-2de+3e^2)^2}$$

```
[Out] (4*x)/(5*e^2) - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 -
 2*d*e + 3*e^2)*(d + e*x)) - ((423*d^2 - 2734*d*e + 293*e^2)*ArcTan[(1 + 5*
x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) - ((40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e
+ 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*d^2
 - 2*d*e + 3*e^2)^2)
```

Rubi [A] time = 0.251182, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{(229d^2-7de-136e^2)\log(5x^2+2x+3)}{25(5d^2-2de+3e^2)^2}-\frac{3d^2e^2+5d^3e+4d^4-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(28d^3e^2+44d^2e^3+d^4e+40d^5-2de^4+}{e^3(5d^2-2de+3e^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (4*x)/(5*e^2) - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 -
 2*d*e + 3*e^2)*(d + e*x)) - ((423*d^2 - 2734*d*e + 293*e^2)*ArcTan[(1 + 5*
x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) - ((40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e
+ 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*d^2
 - 2*d*e + 3*e^2)^2)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = \int \left(\frac{4}{5e^2} + \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2(5d^2-2de+3e^2)(d+ex)^2} + \frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-}{e^2(5d^2-2de+3e^2)^2(d+ex)} \right.$$

$$= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\,\mathrm{lo}}{e^3(5d^2-2de+3e^2)^2}$$

$$= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\,\mathrm{lo}}{e^3(5d^2-2de+3e^2)^2}$$

$$= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\,\mathrm{lo}}{e^3(5d^2-2de+3e^2)^2}$$

$$= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(423d^2-2734de+293e^2)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2-2de+3e^2)^2} - (4$$

Mathematica [A] time = 0.149533, size = 233, normalized size = 1.

$$\frac{(229d^2-7de-136e^2)\log(5x^2+2x+3)}{25(5d^2-2de+3e^2)^2} + \frac{-3d^2e^2-5d^3e-4d^4+de^3-2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} + \frac{(-28d^3e^2-44d^2e^3-d^4e-40d^5+2de^4}{e^3(5d^2-2de+3e^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x
]
```

```
[Out] (4*x)/(5*e^2) + (-4*d^4 - 5*d^3*e - 3*d^2*e^2 + d*e^3 - 2*e^4)/(e^3*(5*d^2
- 2*d*e + 3*e^2)*(d + e*x)) + ((-423*d^2 + 2734*d*e - 293*e^2)*ArcTan[(1 +
5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((-40*d^5 - d^4*e
 - 28*d^3*e^2 - 44*d^2*e^3 + 2*d*e^4 - e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d
*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*
d^2 - 2*d*e + 3*e^2)^2)
```

Maple [B] time = 0.063, size = 538, normalized size = 2.3

$$\frac{4x}{5e^2} + \frac{229\ln(5x^2+2x+3)d^2}{25(5d^2-2de+3e^2)^2} - \frac{7\ln(5x^2+2x+3)de}{25(5d^2-2de+3e^2)^2} - \frac{136\ln(5x^2+2x+3)e^2}{25(5d^2-2de+3e^2)^2} - \frac{423\sqrt{14}d^2}{350(5d^2-2de+3e^2)^2}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x)
```

```
[Out] 4/5*x/e^2+229/25/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)*d^2-7/25/(5*d^2-2*d*
e+3*e^2)^2*ln(5*x^2+2*x+3)*d*e-136/25/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)
*e^2-423/350/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*
d^2+1367/175/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*
d*e-293/350/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e
^2-4/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)*d^4-5/e^2/(5*d^2-2*d*e+3*e^2)/(e*x+d)*
d^3-3/e/(5*d^2-2*d*e+3*e^2)/(e*x+d)*d^2+1/(5*d^2-2*d*e+3*e^2)/(e*x+d)*d-2*e
/(5*d^2-2*d*e+3*e^2)/(e*x+d)-40/e^3/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^5-1/e
^2/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^4-28/e/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)
*d^3-44/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^2+2*e/(5*d^2-2*d*e+3*e^2)^2*ln(e*
x+d)*d-e^2/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)
```

<u>Maxima</u> [A] time = 1.47393, size = 397, normalized size = 1.7

$$-\frac{\sqrt{14}\left(423\,d^2-2734\,de+293\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{350\left(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4\right)}-\frac{\left(40\,d^5+d^4e+28\,d^3e^2+44\,d^2e^3-2\,de^4+e^5\right)\log\left(ex+d\right)}{25\,d^4e^3-20\,d^3e^4+34\,d^2e^5-12\,de^6+9\,e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] -1/350*sqrt(14)*(423*d^2 - 2734*d*e + 293*e^2)*arctan(1/14*sqrt(14)*(5*x +
1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (40*d^5 + d^4*e +
 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*log(e*x + d)/(25*d^4*e^3 - 20*d^3
*e^4 + 34*d^2*e^5 - 12*d*e^6 + 9*e^7) + 1/25*(229*d^2 - 7*d*e - 136*e^2)*lo
g(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4
*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(5*d^3*e^3 - 2*d^2*e^4 + 3*d*e^
5 + (5*d^2*e^4 - 2*d*e^5 + 3*e^6)*x) + 4/5*x/e^2
```

<u>Fricas</u> [A] time = 1.82684, size = 995, normalized size = 4.27

$$-\frac{7000\,d^6+5950\,d^5e+5950\,d^4e^2+1400\,d^3e^3+7350\,d^2e^4-2450\,de^5+2100\,e^6-280\left(25\,d^4e^2-20\,d^3e^3+34\,d^2e^4-12\,de\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] -1/350*(7000*d^6 + 5950*d^5*e + 5950*d^4*e^2 + 1400*d^3*e^3 + 7350*d^2*e^4
- 2450*d*e^5 + 2100*e^6 - 280*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*d*
e^5 + 9*e^6)*x^2 + sqrt(14)*(423*d^3*e^3 - 2734*d^2*e^4 + 293*d*e^5 + (423*
d^2*e^4 - 2734*d*e^5 + 293*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) - 280*(2
5*d^5*e - 20*d^4*e^2 + 34*d^3*e^3 - 12*d^2*e^4 + 9*d*e^5)*x + 350*(40*d^6 +
 d^5*e + 28*d^4*e^2 + 44*d^3*e^3 - 2*d^2*e^4 + d*e^5 + (40*d^5*e + d^4*e^2
+ 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)*log(e*x + d) - 14*(229*d^3*e^
3 - 7*d^2*e^4 - 136*d*e^5 + (229*d^2*e^4 - 7*d*e^5 - 136*e^6)*x)*log(5*x^2
+ 2*x + 3))/(25*d^5*e^3 - 20*d^4*e^4 + 34*d^3*e^5 - 12*d^2*e^6 + 9*d*e^7 +
(25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8)*x)
```

Sympy [C] time = 20.7065, size = 8391, normalized size = 36.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3),x)
```

```
[Out] (-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 3
4*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d
**2 - 2*d*e + 3*e**2)**2))*log(x + (-7840000000*d**14*(-sqrt(14)*I*(423*d**
2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e*
*3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)
**2)) - 4900000000*d**13*e**3*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)
/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**
2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 5880000000*d*
*13*e*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3
*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(2
5*(5*d**2 - 2*d*e + 3*e**2)**2)) + 7717500000*d**12*e**4*(-sqrt(14)*I*(423*
d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d
*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e*
*2)**2))**2 - 21329700000*d**12*e**2*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 29
3*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (
229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) + 306208000
0*d**12 - 19327875000*d**11*e**5*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e*
*2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*
```

d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 5507600000
*d**11*e**3*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 2
0*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e*
2)/(25(5*d**2 - 2*d*e + 3*e**2)**2)) - 1159536000*d**11*e + 10872225000*d
10*e6*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*
d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2
)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 7039144000*d**10*e**4*(-sqrt(14)*
I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2
 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e
 + 3*e**2)**2)) + 2648473800*d**10*e**2 - 10871735000*d**9*e**7*(-sqrt(14)*
I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2
 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e
 + 3*e**2)**2))**2 - 28626939600*d**9*e**5*(-sqrt(14)*I*(423*d**2 - 2734*d*
e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4
)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) + 563
1029040*d**9*e**3 - 12890563000*d**8*e**8*(-sqrt(14)*I*(423*d**2 - 2734*d*e
 + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)
) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 3
140906580*d**8*e**6*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*
d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e
- 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) - 3844841924*d**8*e**4 + 148
66261200*d**7*e**9*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d
4 - 20*d3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e -
 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 16078247136*d**7*e**7*(-
sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*
d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**
2 - 2*d*e + 3*e**2)**2)) - 1183700793*d**7*e**5 - 24188575600*d**6*e**10*(-
sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*
d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**
2 - 2*d*e + 3*e**2)**2))**2 - 7728337232*d**6*e**8*(-sqrt(14)*I*(423*d**2 -
 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3
+ 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2
)) + 1694057982*d**6*e**6 + 14439653200*d**5*e**11*(-sqrt(14)*I*(423*d**2 -
 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3
+ 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2
))**2 + 2286078144*d**5*e**9*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/
(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2
 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) - 5520804349*d**5*e
7 - 10082618000*d4*e**12*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/
(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2
 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 7135930760*d**
4*e**10*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d*
*3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/
(25*(5*d**2 - 2*d*e + 3*e**2)**2)) - 4247714700*d**4*e**8 + 3006129000*d**3
*e**13*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**

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3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(
25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 2323015520*d**3*e**11*(-sqrt(14)*I*(
423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 -
12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e +
3*e**2)**2)) + 1298698281*d**3*e**9 - 918199800*d**2*e**14*(-sqrt(14)*I*(42
3*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12
*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*
e**2)**2))**2 - 1227448656*d**2*e**12*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 2
93*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) +
(229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) - 12857701
8*d**2*e**10 - 38820600*d*e**15*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**
2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d
**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 157117968*d
*e**13*(-sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**
3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(
25*(5*d**2 - 2*d*e + 3*e**2)**2)) + 25259757*d*e**11 + 63844200*e**16*(-sqr
t(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**
2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 -
 2*d*e + 3*e**2)**2))**2 + 38078964*e**14*(-sqrt(14)*I*(423*d**2 - 2734*d*e
 + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)
) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) + 3442
796*e**12)/(947520000*d**12 - 6076784000*d**11*e + 1677232200*d**10*e**2 -
5993164240*d**9*e**3 - 15153874456*d**8*e**4 + 607741008*d**7*e**5 - 813150
0617*d**6*e**6 - 9569972586*d**5*e**7 + 3091977675*d**4*e**8 + 698760764*d*
*3*e**9 + 9842433*d**2*e**10 - 95316042*d*e**11 + 9092669*e**12)) + (sqrt(1
4)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e
**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*
d*e + 3*e**2)**2))*log(x + (-7840000000*d**14*(sqrt(14)*I*(423*d**2 - 2734*
d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e*
*4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) - 4
900000000*d**13*e**3*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*
d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e
- 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 5880000000*d**13*e*(sqr
t(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**
2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 -
 2*d*e + 3*e**2)**2)) + 7717500000*d**12*e**4*(sqrt(14)*I*(423*d**2 - 2734*
d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e*
*4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2
- 21329700000*d**12*e**2*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*
(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*
d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) + 3062080000*d**12 - 193
27875000*d**11*e**5*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d
**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e -
 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 5507600000*d**11*e**3*(s
qrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d
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**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2
 - 2*d*e + 3*e**2)**2)) - 1159536000*d**11*e + 10872225000*d**10*e**6*(sqrt
(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2
*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 -
2*d*e + 3*e**2)**2))**2 - 7039144000*d**10*e**4*(sqrt(14)*I*(423*d**2 - 273
4*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*
e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) +
 2648473800*d**10*e**2 - 10871735000*d**9*e**7*(sqrt(14)*I*(423*d**2 - 2734
*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e
**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2
 - 28626939600*d**9*e**5*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*
(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*
d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) + 5631029040*d**9*e**3 -
 12890563000*d**8*e**8*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(2
5*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*
e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 3140906580*d**8*e**6*
(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34
*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d*
*2 - 2*d*e + 3*e**2)**2)) - 3844841924*d**8*e**4 + 14866261200*d**7*e**9*(s
qrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d
**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2
 - 2*d*e + 3*e**2)**2))**2 - 16078247136*d**7*e**7*(sqrt(14)*I*(423*d**2 -
2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 +
 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)
) - 1183700793*d**7*e**5 - 24188575600*d**6*e**10*(sqrt(14)*I*(423*d**2 - 2
734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 +
9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))
**2 - 7728337232*d**6*e**8*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(70
0*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 -
7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) + 1694057982*d**6*e**6
 + 14439653200*d**5*e**11*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700
*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7
*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 2286078144*d**5*e*
*9*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e +
 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5
*d**2 - 2*d*e + 3*e**2)**2)) - 5520804349*d**5*e**7 - 10082618000*d**4*e**1
2*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e +
34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*
d**2 - 2*d*e + 3*e**2)**2))**2 - 7135930760*d**4*e**10*(sqrt(14)*I*(423*d**
2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e*
*3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)
**2)) - 4247714700*d**4*e**8 + 3006129000*d**3*e**13*(sqrt(14)*I*(423*d**2
- 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3
 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**
2))**2 + 2323015520*d**3*e**11*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)
```

/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**
2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) + 1298698281*d**3*
e**9 - 918199800*d**2*e**14*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(7
00*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 -
 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 1227448656*d**2*
e**12*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*
e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25
*(5*d**2 - 2*d*e + 3*e**2)**2)) - 128577018*d**2*e**10 - 38820600*d*e**15*(
sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*
d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**
2 - 2*d*e + 3*e**2)**2))**2 + 157117968*d*e**13*(sqrt(14)*I*(423*d**2 - 273
4*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*
e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2)) +
 25259757*d*e**11 + 63844200*e**16*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e
2)/(700*(25*d4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229
*d**2 - 7*d*e - 136*e**2)/(25*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 38078964*
e**14*(sqrt(14)*I*(423*d**2 - 2734*d*e + 293*e**2)/(700*(25*d**4 - 20*d**3*
e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) + (229*d**2 - 7*d*e - 136*e**2)/(25
*(5*d**2 - 2*d*e + 3*e**2)**2)) + 3442796*e**12)/(947520000*d**12 - 6076784
000*d**11*e + 1677232200*d**10*e**2 - 5993164240*d**9*e**3 - 15153874456*d*
*8*e**4 + 607741008*d**7*e**5 - 8131500617*d**6*e**6 - 9569972586*d**5*e**7
 + 3091977675*d**4*e**8 + 698760764*d**3*e**9 + 9842433*d**2*e**10 - 953160
42*d*e**11 + 9092669*e**12)) - (4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 +
2*e**4)/(5*d**3*e**3 - 2*d**2*e**4 + 3*d*e**5 + x*(5*d**2*e**4 - 2*d*e**5 +
 3*e**6)) + 4*x/(5*e**2) - (40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3
- 2*d*e**4 + e**5)*log(x + (7840000000*d**14*(40*d**5 + d**4*e + 28*d**3*e*
*2 + 44*d**2*e**3 - 2*d*e**4 + e**5)/(e**3*(5*d**2 - 2*d*e + 3*e**2)**2) -
5880000000*d**13*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4
 + e**5)/(e**2*(5*d**2 - 2*d*e + 3*e**2)**2) - 4900000000*d**13*(40*d**5 +
d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)**2/(e**3*(5*d**2 -
2*d*e + 3*e**2)**4) + 3062080000*d**12 + 21329700000*d**12*(40*d**5 + d**4*
e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)/(e*(5*d**2 - 2*d*e + 3*e
2)2) + 7717500000*d**12*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3
 - 2*d*e**4 + e**5)**2/(e**2*(5*d**2 - 2*d*e + 3*e**2)**4) - 1159536000*d**
11*e + 5507600000*d**11*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2
*d*e**4 + e**5)/(5*d**2 - 2*d*e + 3*e**2)**2 - 19327875000*d**11*(40*d**5 +
 d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)**2/(e*(5*d**2 - 2*
d*e + 3*e**2)**4) + 2648473800*d**10*e**2 + 7039144000*d**10*e*(40*d**5 + d
4*e + 28*d3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)/(5*d**2 - 2*d*e + 3*
e**2)**2 + 10872225000*d**10*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**
3 - 2*d*e**4 + e**5)**2/(5*d**2 - 2*d*e + 3*e**2)**4 + 5631029040*d**9*e**3
 + 28626939600*d**9*e**2*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 -
2*d*e**4 + e**5)/(5*d**2 - 2*d*e + 3*e**2)**2 - 10871735000*d**9*e*(40*d**5
 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)**2/(5*d**2 - 2*d
*e + 3*e**2)**4 - 3844841924*d**8*e**4 - 3140906580*d**8*e**3*(40*d**5 + d*

```
*4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)/(5*d**2 - 2*d*e + 3*e
**2)**2 - 12890563000*d**8*e**2*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*
e**3 - 2*d*e**4 + e**5)**2/(5*d**2 - 2*d*e + 3*e**2)**4 - 1183700793*d**7*e
**5 + 16078247136*d**7*e**4*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3
 - 2*d*e**4 + e**5)/(5*d**2 - 2*d*e + 3*e**2)**2 + 14866261200*d**7*e**3*(4
0*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)**2/(5*d**2
 - 2*d*e + 3*e**2)**4 + 1694057982*d**6*e**6 + 7728337232*d**6*e**5*(40*d**
5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)/(5*d**2 - 2*d*e
 + 3*e**2)**2 - 24188575600*d**6*e**4*(40*d**5 + d**4*e + 28*d**3*e**2 + 44
*d**2*e**3 - 2*d*e**4 + e**5)**2/(5*d**2 - 2*d*e + 3*e**2)**4 - 5520804349*
d**5*e**7 - 2286078144*d**5*e**6*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2
*e**3 - 2*d*e**4 + e**5)/(5*d**2 - 2*d*e + 3*e**2)**2 + 14439653200*d**5*e*
*5*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)**2/(5
*d**2 - 2*d*e + 3*e**2)**4 - 4247714700*d**4*e**8 + 7135930760*d**4*e**7*(4
0*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)/(5*d**2 -
2*d*e + 3*e**2)**2 - 10082618000*d**4*e**6*(40*d**5 + d**4*e + 28*d**3*e**2
 + 44*d**2*e**3 - 2*d*e**4 + e**5)**2/(5*d**2 - 2*d*e + 3*e**2)**4 + 129869
8281*d**3*e**9 - 2323015520*d**3*e**8*(40*d**5 + d**4*e + 28*d**3*e**2 + 44
*d**2*e**3 - 2*d*e**4 + e**5)/(5*d**2 - 2*d*e + 3*e**2)**2 + 3006129000*d**
3*e**7*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)**
2/(5*d**2 - 2*d*e + 3*e**2)**4 - 128577018*d**2*e**10 + 1227448656*d**2*e**
9*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)/(5*d**
2 - 2*d*e + 3*e**2)**2 - 918199800*d**2*e**8*(40*d**5 + d**4*e + 28*d**3*e*
*2 + 44*d**2*e**3 - 2*d*e**4 + e**5)**2/(5*d**2 - 2*d*e + 3*e**2)**4 + 2525
9757*d*e**11 - 157117968*d*e**10*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2
*e**3 - 2*d*e**4 + e**5)/(5*d**2 - 2*d*e + 3*e**2)**2 - 38820600*d*e**9*(40
*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)**2/(5*d**2
- 2*d*e + 3*e**2)**4 + 3442796*e**12 - 38078964*e**11*(40*d**5 + d**4*e + 2
8*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4 + e**5)/(5*d**2 - 2*d*e + 3*e**2)**2
+ 63844200*e**10*(40*d**5 + d**4*e + 28*d**3*e**2 + 44*d**2*e**3 - 2*d*e**4
 + e**5)**2/(5*d**2 - 2*d*e + 3*e**2)**4)/(947520000*d**12 - 6076784000*d**
11*e + 1677232200*d**10*e**2 - 5993164240*d**9*e**3 - 15153874456*d**8*e**4
 + 607741008*d**7*e**5 - 8131500617*d**6*e**6 - 9569972586*d**5*e**7 + 3091
977675*d**4*e**8 + 698760764*d**3*e**9 + 9842433*d**2*e**10 - 95316042*d*e*
*11 + 9092669*e**12))/(e**3*(5*d**2 - 2*d*e + 3*e**2)**2)
```

Giac [A] time = 1.17144, size = 479, normalized size = 2.06

$$\frac{1}{25}\,(40\,d+33\,e)e^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)-\frac{\sqrt{14}\left(423\,d^2e^2-2734\,de^3+293\,e^4\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\left(5\,d-\frac{5\,d^2}{xe+d}+\frac{2\,de}{xe+d}-\frac{3\,e^2}{xe+d}-e\right.\right.}{350\left(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 1/25*(40*d + 33*e)*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - 1/350*sqrt
(14)*(423*d^2*e^2 - 2734*d*e^3 + 293*e^4)*arctan(1/14*sqrt(14)*(5*d - 5*d^2
/(x*e + d) + 2*d*e/(x*e + d) - 3*e^2/(x*e + d) - e)*e^(-1))*e^(-2)/(25*d^4
- 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + 4/5*(x*e + d)*e^(-3) + 1/25*(
229*d^2 - 7*d*e - 136*e^2)*log(-10*d/(x*e + d) + 5*d^2/(x*e + d)^2 + 2*e/(x
*e + d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(25*d^4 - 20*d^3*e + 3
4*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4*d^4*e^3/(x*e + d) + 5*d^3*e^4/(x*e + d)
+ 3*d^2*e^5/(x*e + d) - d*e^6/(x*e + d) + 2*e^7/(x*e + d))/(5*d^2*e^6 - 2*d
*e^7 + 3*e^8)
```

3.310 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx$

Optimal. Leaf size=317

$$\frac{(-21d^2e+458d^3-816de^2+113e^3)\log(5x^2+2x+3)}{10(5d^2-2de+3e^2)^3}-\frac{3d^2e^2+5d^3e+4d^4-de^3+2e^4}{2e^3(5d^2-2de+3e^2)(d+ex)^2}+\frac{28d^3e^2+44d^2e^3+d^4e+40d}{e^3(5d^2-2de+3e^2)^2\,(}$$

```
[Out] -(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e^3*(5*d^2 - 2*d*e + 3*e^
2)*(d + e*x)^2) + (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5
)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) - ((423*d^3 - 4101*d^2*e + 879*
d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d*e + 3
*e^2)^3) + ((100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^3 + 141*d^2*e^4
+ 120*d*e^5 - e^6)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3) + ((458*d^
3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*x + 5*x^2])/(10*(5*d^2 - 2*d*
e + 3*e^2)^3)
```

Rubi [A] time = 0.290028, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{(-21d^2e+458d^3-816de^2+113e^3)\log(5x^2+2x+3)}{10(5d^2-2de+3e^2)^3}-\frac{3d^2e^2+5d^3e+4d^4-de^3+2e^4}{2e^3(5d^2-2de+3e^2)(d+ex)^2}+\frac{28d^3e^2+44d^2e^3+d^4e+40d}{e^3(5d^2-2de+3e^2)^2\,(}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e^3*(5*d^2 - 2*d*e + 3*e^
2)*(d + e*x)^2) + (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5
)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) - ((423*d^3 - 4101*d^2*e + 879*
d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d*e + 3
*e^2)^3) + ((100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^3 + 141*d^2*e^4
+ 120*d*e^5 - e^6)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3) + ((458*d^
3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*x + 5*x^2])/(10*(5*d^2 - 2*d*
e + 3*e^2)^3)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)}\,dx = \int \left(\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)^3} + \frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-e^5}{e^2\left(5d^2-2de+3e^2\right)^2(d+ex)^2} + \frac{100\ldots}{\ldots}\right.$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)} + \frac{\left(100d^6\ldots\right.}{\ldots}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)} + \frac{\left(100d^6\ldots\right.}{\ldots}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)} + \frac{\left(100d^6\ldots\right.}{\ldots}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)} - \frac{\left(423d^3\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.432427, size = 278, normalized size = 0.88

$$-\frac{-7\left(-21d^2e+458d^3-816de^2+113e^3\right)\log\left(5x^2+2x+3\right) + \frac{35\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)\left(5d^2-2de+3e^2\right)^2}{e^3(d+ex)^2} - \frac{70\left(28d^3e^2+44d^2e^3+d^4e+40\ldots\right.}{e^3(\ldots}}{7\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x
]
```

```
[Out] -((35*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^
4))/(e^3*(d + e*x)^2) - (70*(5*d^2 - 2*d*e + 3*e^2)*(40*d^5 + d^4*e + 28*d^
3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5))/(e^3*(d + e*x)) + Sqrt[14]*(423*d^3 -
4101*d^2*e + 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + (70*(-100*d^
6 + 120*d^5*e - 228*d^4*e^2 + 242*d^3*e^3 - 141*d^2*e^4 - 120*d*e^5 + e^6)*
Log[d + e*x])/e^3 - 7*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*
x + 5*x^2])/(70*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Maple [B] time = 0.062, size = 819, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x)
```

```
[Out] -2*e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d+40/e^3/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d
^5+141/(5*d^2-2*d*e+3*e^2)^3*e*ln(e*x+d)*d^2+120/(5*d^2-2*d*e+3*e^2)^3*e^2*
ln(e*x+d)*d-703/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(
1/2))*e^3+1/e^2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^4-423/70/(5*d^2-2*d*e+3*e^2
)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3-3/2/e/(5*d^2-2*d*e+3*e^2)/(
e*x+d)^2*d^2-879/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^
(1/2))*d*e^2+4101/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14
^(1/2))*d^2*e+229/5/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^3+113/10/(5*d^2
-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*e^3-e/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2+1/2/(5*
d^2-2*d*e+3*e^2)/(e*x+d)^2*d+44/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^2+e^2/(5*d^
2-2*d*e+3*e^2)^2/(e*x+d)-242/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^3-1/(5*d^2-2
*d*e+3*e^2)^3*e^3*ln(e*x+d)-2/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d^4-5/2/e^2
/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d^3+28/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^3-2
1/10/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^2*e-408/5/(5*d^2-2*d*e+3*e^2)^
3*ln(5*x^2+2*x+3)*d*e^2+100/(5*d^2-2*d*e+3*e^2)^3/e^3*ln(e*x+d)*d^6-120/(5*
d^2-2*d*e+3*e^2)^3/e^2*ln(e*x+d)*d^5+228/(5*d^2-2*d*e+3*e^2)^3/e*ln(e*x+d)*
d^4
```

Maxima [A] time = 1.50774, size = 672, normalized size = 2.12

$$-\frac{\sqrt{14}\left(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{70\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{\left(100\,d^6-120\,d^5e+228\,d^4e^2-242\,d^3e^3+14\right.}{125\,d^6e^3-150\,d^5e^4+285\,d^4e^5-188\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^
3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(e*x + d)/(125*d^6*e^3 - 150*d^5*e^4
+ 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) + 1/10*(458*
d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d
^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + 1/2*(
60*d^6 - 15*d^5*e + 39*d^4*e^2 + 84*d^3*e^3 - 25*d^2*e^4 + 9*d*e^5 - 6*e^6
+ 2*(40*d^5*e + d^4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)/(25*d
```

```
^6*e^3 - 20*d^5*e^4 + 34*d^4*e^5 - 12*d^3*e^6 + 9*d^2*e^7 + (25*d^4*e^5 - 2
0*d^3*e^6 + 34*d^2*e^7 - 12*d*e^8 + 9*e^9)*x^2 + 2*(25*d^5*e^4 - 20*d^4*e^5
 + 34*d^3*e^6 - 12*d^2*e^7 + 9*d*e^8)*x)
```

Fricas [B] time = 2.30544, size = 1674, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 1/70*(10500*d^8 - 6825*d^7*e + 14175*d^6*e^2 + 10395*d^5*e^3 - 6160*d^4*e^4
 + 12145*d^3*e^5 - 4305*d^2*e^6 + 1365*d*e^7 - 630*e^8 - sqrt(14)*(423*d^5*
e^3 - 4101*d^4*e^4 + 879*d^3*e^5 + 703*d^2*e^6 + (423*d^3*e^5 - 4101*d^2*e^
6 + 879*d*e^7 + 703*e^8)*x^2 + 2*(423*d^4*e^4 - 4101*d^3*e^5 + 879*d^2*e^6
+ 703*d*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 70*(200*d^7*e - 75*d^6*e^
2 + 258*d^5*e^3 + 167*d^4*e^4 - 14*d^3*e^5 + 141*d^2*e^6 - 8*d*e^7 + 3*e^8)
*x + 70*(100*d^8 - 120*d^7*e + 228*d^6*e^2 - 242*d^5*e^3 + 141*d^4*e^4 + 12
0*d^3*e^5 - d^2*e^6 + (100*d^6*e^2 - 120*d^5*e^3 + 228*d^4*e^4 - 242*d^3*e^
5 + 141*d^2*e^6 + 120*d*e^7 - e^8)*x^2 + 2*(100*d^7*e - 120*d^6*e^2 + 228*d
^5*e^3 - 242*d^4*e^4 + 141*d^3*e^5 + 120*d^2*e^6 - d*e^7)*x)*log(e*x + d) +
 7*(458*d^5*e^3 - 21*d^4*e^4 - 816*d^3*e^5 + 113*d^2*e^6 + (458*d^3*e^5 - 2
1*d^2*e^6 - 816*d*e^7 + 113*e^8)*x^2 + 2*(458*d^4*e^4 - 21*d^3*e^5 - 816*d^
2*e^6 + 113*d*e^7)*x)*log(5*x^2 + 2*x + 3))/(125*d^8*e^3 - 150*d^7*e^4 + 28
5*d^6*e^5 - 188*d^5*e^6 + 171*d^4*e^7 - 54*d^3*e^8 + 27*d^2*e^9 + (125*d^6*
e^5 - 150*d^5*e^6 + 285*d^4*e^7 - 188*d^3*e^8 + 171*d^2*e^9 - 54*d*e^10 + 2
7*e^11)*x^2 + 2*(125*d^7*e^4 - 150*d^6*e^5 + 285*d^5*e^6 - 188*d^4*e^7 + 17
1*d^3*e^8 - 54*d^2*e^9 + 27*d*e^10)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16568, size = 548, normalized size = 1.73

$$-\frac{\sqrt{14}\left(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{70\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{\left(458\,d^3-21\,d^2e-816\,de^2+113\,e^3\right)}{10\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + 1/10*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)
*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 17
1*d^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d
^3*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(abs(x*e + d))/(125*d^6*e^3 - 15
0*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) +
1/2*(2*(200*d^7 - 75*d^6*e + 258*d^5*e^2 + 167*d^4*e^3 - 14*d^3*e^4 + 141*d
^2*e^5 - 8*d*e^6 + 3*e^7)*x + (300*d^8 - 195*d^7*e + 405*d^6*e^2 + 297*d^5*
e^3 - 176*d^4*e^4 + 347*d^3*e^5 - 123*d^2*e^6 + 39*d*e^7 - 18*e^8)*e^(-1))*
e^(-2)/((5*d^2 - 2*d*e + 3*e^2)^3*(x*e + d)^2)
```

3.311 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=189

$$\frac{ex^2(840d^2-1722de+373e^2)}{3500}-\frac{(-1545d^2e+1025d^3-2601de^2+832e^3)\log(5x^2+2x+3)}{6250}+\frac{x(-17220d^2e+2800d^3 -}{1750}$$

```
[Out] ((2800*d^3 - 17220*d^2*e + 9921*d*e^2 + 6053*e^3)*x)/17500 + (e*(840*d^2 -
1722*d*e + 373*e^2)*x^2)/3500 + ((60*d - 41*e)*e^2*x^3)/375 + (e^3*x^4)/25
- ((1367 + 423*x)*(d + e*x)^3)/(3500*(3 + 2*x + 5*x^2)) + ((32825*d^3 + 317
565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sq
rt[14]) - ((1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*Log[3 + 2*x + 5*x
^2])/6250
```

Rubi [A] time = 0.255334, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1628, 634, 618, 204, 628}

$$\frac{ex^2(840d^2-1722de+373e^2)}{3500}-\frac{(-1545d^2e+1025d^3-2601de^2+832e^3)\log(5x^2+2x+3)}{6250}+\frac{x(-17220d^2e+2800d^3 -}{1750}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((2800*d^3 - 17220*d^2*e + 9921*d*e^2 + 6053*e^3)*x)/17500 + (e*(840*d^2 -
1722*d*e + 373*e^2)*x^2)/3500 + ((60*d - 41*e)*e^2*x^3)/375 + (e^3*x^4)/25
- ((1367 + 423*x)*(d + e*x)^3)/(3500*(3 + 2*x + 5*x^2)) + ((32825*d^3 + 317
565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sq
rt[14]) - ((1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*Log[3 + 2*x + 5*x
^2])/6250
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
```

```
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}dx = -\frac{(1367+423x)(d+ex)^3}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{(d+ex)^2\left(\frac{6}{125}(615d+1367e)-\frac{12}{125}(770d\right.}{3+2x+}$$

$$= -\frac{(1367+423x)(d+ex)^3}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int\left(\frac{2}{625}\left(2800d^3-17220d^2e+9921de^2+60\right.\right.$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500}+\frac{e\left(840d^2-1722de+373e^2\right)x}{3500}$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500}+\frac{e\left(840d^2-1722de+373e^2\right)x}{3500}$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500}+\frac{e\left(840d^2-1722de+373e^2\right)x}{3500}$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500}+\frac{e\left(840d^2-1722de+373e^2\right)x}{3500}$$

Mathematica [A] time = 0.155091, size = 209, normalized size = 1.11

$$\frac{14700ex^2\left(300d^2-615de+103e^2\right)-\frac{42\left(75d^2e(5989x-1269)+125d^3(423x+1367)-15de^2(18323x+17967)+e^3(54969-53189x)\right)}{5x^2+2x+3}+2940\left(1545d^2e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(500*d^3 - 3075*d^2*e + 1545*d*e^2 + 867*e^3)*x + 14700*e*(300*d^2 -
615*d*e + 103*e^2)*x^2 + 49000*(60*d - 41*e)*e^2*x^3 + 735000*e^3*x^4 - (42
*(e^3*(54969 - 53189*x) + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x
) - 15*d*e^2*(17967 + 18323*x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(32825*d^3
 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 29
40*(-1025*d^3 + 1545*d^2*e + 2601*d*e^2 - 832*e^3)*Log[3 + 2*x + 5*x^2])/18
375000
```

Maple [A] time = 0.055, size = 283, normalized size = 1.5

$$\frac{e^3x^4}{25}+\frac{4\,x^3e^2d}{25}-\frac{41\,x^3e^3}{375}+\frac{6\,x^2d^2e}{25}-\frac{123\,x^2e^2d}{250}+\frac{103\,e^3x^2}{1250}+\frac{4\,d^3x}{25}-\frac{123\,xd^2e}{125}+\frac{309\,xde^2}{625}+\frac{867\,e^3x}{3125}-\frac{1}{3125}\left(\left(\frac{2115\,d}{28}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 1/25*e^3*x^4+4/25*x^3*e^2*d-41/375*x^3*e^3+6/25*x^2*d^2*e-123/250*x^2*e^2*d
+103/1250*e^3*x^2+4/25*d^3*x-123/125*x*d^2*e+309/625*x*d*e^2+867/3125*e^3*x
-1/3125*((2115/28*d^3+17967/28*d^2*e-54969/140*d*e^2-53189/700*e^3)*x+6835/
28*d^3-3807/28*d^2*e-53901/140*d*e^2+54969/700*e^3)/(x^2+2/5*x+3/5)-41/250*
ln(5*x^2+2*x+3)*d^3+309/1250*ln(5*x^2+2*x+3)*d^2*e+2601/6250*ln(5*x^2+2*x+3
)*d*e^2-416/3125*ln(5*x^2+2*x+3)*e^3+1313/49000*14^(1/2)*arctan(1/28*(10*x+
2)*14^(1/2))*d^3+63513/245000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2*e
-221643/1225000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^2-67499/1225000
*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^3
```

Maxima [A] time = 1.6389, size = 286, normalized size = 1.51

$$\frac{1}{25}\,e^3x^4+\frac{1}{375}\left(60\,de^2-41\,e^3\right)x^3+\frac{1}{1250}\left(300\,d^2e-615\,de^2+103\,e^3\right)x^2+\frac{1}{1225000}\,\sqrt{14}\left(32825\,d^3+317565\,d^2e-2216\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/25*e^3*x^4 + 1/375*(60*d*e^2 - 41*e^3)*x^3 + 1/1250*(300*d^2*e - 615*d*e^
2 + 103*e^3)*x^2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*d*
e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(500*d^3 - 3075*d
^2*e + 1545*d*e^2 + 867*e^3)*x - 1/6250*(1025*d^3 - 1545*d^2*e - 2601*d*e^2
 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3 - 95175*d^2*e - 269
505*d*e^2 + 54969*e^3 + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^
3)*x)/(5*x^2 + 2*x + 3)
```

Fricas [B] time = 1.28878, size = 1046, normalized size = 5.53

$$\frac{3675000\,e^{3}x^{6} + 1225000\left(12\,de^{2} - 7\,e^{3}\right)x^{5} + 122500\left(180\,d^{2}e - 321\,de^{2} + 47\,e^{3}\right)x^{4} + 147000\left(100\,d^{3} - 555\,d^{2}e + 246\,de^{2} + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/18375000*(3675000*e^3*x^6 + 1225000*(12*d*e^2 - 7*e^3)*x^5 + 122500*(180*
d^2*e - 321*d*e^2 + 47*e^3)*x^4 + 147000*(100*d^3 - 555*d^2*e + 246*d*e^2 +
 153*e^3)*x^3 - 7176750*d^3 + 3997350*d^2*e + 11319210*d*e^2 - 2308698*e^3
+ 2940*(2000*d^3 - 7800*d^2*e - 3045*d*e^2 + 5013*e^3)*x^2 + 15*sqrt(14)*(9
8475*d^3 + 952695*d^2*e - 664929*d*e^2 - 202497*e^3 + 5*(32825*d^3 + 317565
*d^2*e - 221643*d*e^2 - 67499*e^3)*x^2 + 2*(32825*d^3 + 317565*d^2*e - 2216
43*d*e^2 - 67499*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(157125*d^3 -
 1740675*d^2*e + 923745*d*e^2 + 417329*e^3)*x - 2940*(3075*d^3 - 4635*d^2*e
 - 7803*d*e^2 + 2496*e^3 + 5*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)
*x^2 + 2*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*x)*log(5*x^2 + 2*x
+ 3))/(5*x^2 + 2*x + 3)
```

Sympy [C] time = 2.55615, size = 444, normalized size = 2.35

$$\frac{e^{3}x^{4}}{25} + x^{3}\left(\frac{4de^{2}}{25} - \frac{41e^{3}}{375}\right) + x^{2}\left(\frac{6d^{2}e}{25} - \frac{123de^{2}}{250} + \frac{103e^{3}}{1250}\right) + x\left(\frac{4d^{3}}{25} - \frac{123d^{2}e}{125} + \frac{309de^{2}}{625} + \frac{867e^{3}}{3125}\right) + \left(-\frac{41d^{3}}{250} + \frac{309d^{2}e}{1250} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] e**3*x**4/25 + x**3*(4*d*e**2/25 - 41*e**3/375) + x**2*(6*d**2*e/25 - 123*d
*e**2/250 + 103*e**3/1250) + x*(4*d**3/25 - 123*d**2*e/125 + 309*d*e**2/625
 + 867*e**3/3125) + (-41*d**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 41
6*e**3/3125 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 6749
9*e**3)/2450000)*log(x + (6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 6749
9*e**3/5 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e
**3)/5)/(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-41*d
**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 416*e**3/3125 + sqrt(14)*I*(
```

```
32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/2450000)*log(x + (
6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 67499*e**3/5 + sqrt(14)*I*(328
25*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/5)/(32825*d**3 + 3175
65*d**2*e - 221643*d*e**2 - 67499*e**3)) - (170875*d**3 - 95175*d**2*e - 26
9505*d*e**2 + 54969*e**3 + x*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 -
53189*e**3))/(2187500*x**2 + 875000*x + 1312500)
```

Giac [A] time = 1.13359, size = 278, normalized size = 1.47

$$\frac{1}{25}x^4e^3 + \frac{4}{25}dx^3e^2 + \frac{6}{25}d^2x^2e + \frac{4}{25}d^3x - \frac{41}{375}x^3e^3 - \frac{123}{250}dx^2e^2 - \frac{123}{125}d^2xe + \frac{103}{1250}x^2e^3 + \frac{309}{625}dxe^2 + \frac{1}{1225000}\sqrt{14}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/25*x^4*e^3 + 4/25*d*x^3*e^2 + 6/25*d^2*x^2*e + 4/25*d^3*x - 41/375*x^3*e^
3 - 123/250*d*x^2*e^2 - 123/125*d^2*x*e + 103/1250*x^2*e^3 + 309/625*d*x*e^
2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3
)*arctan(1/14*sqrt(14)*(5*x + 1)) + 867/3125*x*e^3 - 1/6250*(1025*d^3 - 154
5*d^2*e - 2601*d*e^2 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3
 - 95175*d^2*e + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*x -
269505*d*e^2 + 54969*e^3)/(5*x^2 + 2*x + 3)
```

3.312 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=140

$$-\frac{(1025d^2-1030de-867e^2)\log(5x^2+2x+3)}{6250}+\frac{x(2800d^2-11480de+3307e^2)}{17500}+\frac{(32825d^2+211710de-73881e^2)\,\text{t}}{87500\sqrt{14}}$$

```
[Out] ((2800*d^2 - 11480*d*e + 3307*e^2)*x)/17500 + ((40*d - 41*e)*e*x^2)/250 + (
4*e^2*x^3)/75 - ((1367 + 423*x)*(d + e*x)^2)/(3500*(3 + 2*x + 5*x^2)) + ((3
2825*d^2 + 211710*d*e - 73881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sqrt[
14]) - ((1025*d^2 - 1030*d*e - 867*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rubi [A] time = 0.208256, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1628, 634, 618, 204, 628}

$$-\frac{(1025d^2-1030de-867e^2)\log(5x^2+2x+3)}{6250}+\frac{x(2800d^2-11480de+3307e^2)}{17500}+\frac{(32825d^2+211710de-73881e^2)\,\text{t}}{87500\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```


```
[Out] ((2800*d^2 - 11480*d*e + 3307*e^2)*x)/17500 + ((40*d - 41*e)*e*x^2)/250 + (
4*e^2*x^3)/75 - ((1367 + 423*x)*(d + e*x)^2)/(3500*(3 + 2*x + 5*x^2)) + ((3
2825*d^2 + 211710*d*e - 73881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sqrt[
14]) - ((1025*d^2 - 1030*d*e - 867*e^2)*Log[3 + 2*x + 5*x^2])/6250
```


Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
```

```
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367+423x)(d+ex)^2}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int \frac{(d+ex)\left(\frac{2}{125}(1845d+2734e)-\frac{6}{125}(1540d\right.}{3+2x+}$$

$$= -\frac{(1367+423x)(d+ex)^2}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int\left(\frac{2}{625}\left(2800d^2-11480de+3307e^2\right)+\frac{56}{125}\right.$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500} + \frac{1}{250}(40d-41e)ex^2 + \frac{4e^2x^3}{75} - \frac{(1367+42}{3500\,(3+}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500} + \frac{1}{250}(40d-41e)ex^2 + \frac{4e^2x^3}{75} - \frac{(1367+42}{3500\,(3+}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500} + \frac{1}{250}(40d-41e)ex^2 + \frac{4e^2x^3}{75} - \frac{(1367+42}{3500\,(3+}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500} + \frac{1}{250}(40d-41e)ex^2 + \frac{4e^2x^3}{75} - \frac{(1367+42}{3500\,(3+}$$

Mathematica [A] time = 0.109274, size = 150, normalized size = 1.07

$$\frac{-\frac{42\left(25d^2(423x+1367)+10de(5989x-1269)-e^2(18323x+17967)\right)}{5x^2+2x+3}+588\left(-1025d^2+1030de+867e^2\right)\log\left(5x^2+2x+3\right)+5880x\left(100d^2\right.}{3675000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(100*d^2 - 410*d*e + 103*e^2)*x + 14700*(40*d - 41*e)*e*x^2 + 196000*
e^2*x^3 - (42*(25*d^2*(1367 + 423*x) + 10*d*e*(-1269 + 5989*x) - e^2*(17967
 + 18323*x)))/(3 + 2*x + 5*x^2) + 3*Sqrt[14]*(32825*d^2 + 211710*d*e - 7388
1*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 588*(-1025*d^2 + 1030*d*e + 867*e^2)*Lo
g[3 + 2*x + 5*x^2])/3675000
```

Maple [A] time = 0.056, size = 189, normalized size = 1.4

$$\frac{4e^2x^3}{75}+\frac{4x^2de}{25}-\frac{41x^2e^2}{250}+\frac{4d^2x}{25}-\frac{82xde}{125}+\frac{103e^2x}{625}-\frac{1}{625}\left(\left(\left(\frac{423d^2}{28}+\frac{5989de}{70}-\frac{18323e^2}{700}\right)x+\frac{1367d^2}{28}-\frac{1269de}{70}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 4/75*e^2*x^3+4/25*x^2*d*e-41/250*x^2*e^2+4/25*d^2*x-82/125*x*d*e+103/625*e^
2*x-1/625*((423/28*d^2+5989/70*d*e-18323/700*e^2)*x+1367/28*d^2-1269/70*d*e
-17967/700*e^2)/(x^2+2/5*x+3/5)-41/250*ln(5*x^2+2*x+3)*d^2+103/625*ln(5*x^2
+2*x+3)*d*e+867/6250*ln(5*x^2+2*x+3)*e^2+1313/49000*14^(1/2)*arctan(1/28*(1
0*x+2)*14^(1/2))*d^2+21171/122500*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d
*e-73881/1225000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^2
```

Maxima [A] time = 1.5442, size = 198, normalized size = 1.41

$$\frac{4}{75}e^2x^3 + \frac{1}{250}\left(40\,de - 41\,e^2\right)x^2 + \frac{1}{1225000}\sqrt{14}\left(32825\,d^2 + 211710\,de - 73881\,e^2\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right) + \frac{1}{625}\left(10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 4/75*e^2*x^3 + 1/250*(40*d*e - 41*e^2)*x^2 + 1/1225000*sqrt(14)*(32825*d^2
+ 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(100*d^2
- 410*d*e + 103*e^2)*x - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5*x^2 +
 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d^2 + 59890
*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.21679, size = 747, normalized size = 5.34

$$\frac{980000\,e^2x^5 + 24500\left(120\,de - 107\,e^2\right)x^4 + 58800\left(50\,d^2 - 185\,de + 41\,e^2\right)x^3 + 2940\left(400\,d^2 - 1040\,de - 203\,e^2\right)x^2 + 3\,\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3675000*(980000*e^2*x^5 + 24500*(120*d*e - 107*e^2)*x^4 + 58800*(50*d^2 -
 185*d*e + 41*e^2)*x^3 + 2940*(400*d^2 - 1040*d*e - 203*e^2)*x^2 + 3*sqrt(1
4)*(5*(32825*d^2 + 211710*d*e - 73881*e^2)*x^2 + 98475*d^2 + 635130*d*e - 2
21643*e^2 + 2*(32825*d^2 + 211710*d*e - 73881*e^2)*x)*arctan(1/14*sqrt(14)*
(5*x + 1)) - 1435350*d^2 + 532980*d*e + 754614*e^2 + 42*(31425*d^2 - 232090
*d*e + 61583*e^2)*x - 588*(5*(1025*d^2 - 1030*d*e - 867*e^2)*x^2 + 3075*d^2
 - 3090*d*e - 2601*e^2 + 2*(1025*d^2 - 1030*d*e - 867*e^2)*x)*log(5*x^2 + 2
*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [C] time = 1.74327, size = 298, normalized size = 2.13

$$\frac{4e^2x^3}{75} + x^2\left(\frac{4de}{25} - \frac{41e^2}{250}\right) + x\left(\frac{4d^2}{25} - \frac{82de}{125} + \frac{103e^2}{625}\right) + \left(-\frac{41d^2}{250} + \frac{103de}{625} + \frac{867e^2}{6250} - \frac{\sqrt{14}i\left(32825d^2 + 211710de - 73881e^2\right.}{2450000}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*e**2*x**3/75 + x**2*(4*d*e/25 - 41*e**2/250) + x*(4*d**2/25 - 82*d*e/125
+ 103*e**2/625) + (-41*d**2/250 + 103*d*e/625 + 867*e**2/6250 - sqrt(14)*I*
(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(x + (6565*d**2 + 42342*
d*e - 73881*e**2/5 - sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/5)/(
32825*d**2 + 211710*d*e - 73881*e**2)) + (-41*d**2/250 + 103*d*e/625 + 867*
e**2/6250 + sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(
x + (6565*d**2 + 42342*d*e - 73881*e**2/5 + sqrt(14)*I*(32825*d**2 + 211710
*d*e - 73881*e**2)/5)/(32825*d**2 + 211710*d*e - 73881*e**2)) - (34175*d**2
 - 12690*d*e - 17967*e**2 + x*(10575*d**2 + 59890*d*e - 18323*e**2))/(43750
0*x**2 + 175000*x + 262500)
```

Giac [A] time = 1.16566, size = 196, normalized size = 1.4

$$\frac{4}{75}x^3e^2 + \frac{4}{25}dx^2e + \frac{4}{25}d^2x - \frac{41}{250}x^2e^2 - \frac{82}{125}dxe + \frac{1}{1225000}\sqrt{14}\left(32825\,d^2 + 211710\,de - 73881\,e^2\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 4/75*x^3*e^2 + 4/25*d*x^2*e + 4/25*d^2*x - 41/250*x^2*e^2 - 82/125*d*x*e +
1/1225000*sqrt(14)*(32825*d^2 + 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14
)*(5*x + 1)) + 103/625*x*e^2 - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5
*x^2 + 2*x + 3) - 1/87500*(34175*d^2 + (10575*d^2 + 59890*d*e - 18323*e^2)*
x - 12690*d*e - 17967*e^2)/(5*x^2 + 2*x + 3)
```

3.313 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}-\frac{(205d-103e)\log\left(5x^2+2x+3\right)}{1250}+\frac{1}{125}x(20d-41e)+\frac{(6565d+21171e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{17500\sqrt{14}}+\frac{2ex^2}{25}$$

```
[Out] ((20*d - 41*e)*x)/125 + (2*e*x^2)/25 - ((1367 + 423*x)*(d + e*x))/(3500*(3
+ 2*x + 5*x^2)) + ((6565*d + 21171*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(17500*Sq
rt[14]) - ((205*d - 103*e)*Log[3 + 2*x + 5*x^2])/1250
```

Rubi [A] time = 0.193461, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1644, 1657, 634, 618, 204, 628}

$$-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}-\frac{(205d-103e)\log\left(5x^2+2x+3\right)}{1250}+\frac{1}{125}x(20d-41e)+\frac{(6565d+21171e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{17500\sqrt{14}}+\frac{2ex^2}{25}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((20*d - 41*e)*x)/125 + (2*e*x^2)/25 - ((1367 + 423*x)*(d + e*x))/(3500*(3
+ 2*x + 5*x^2)) + ((6565*d + 21171*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(17500*Sq
rt[14]) - ((205*d - 103*e)*Log[3 + 2*x + 5*x^2])/1250
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
```

```
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int \frac{\frac{2}{125}(1845d+1367e)-\frac{168}{125}(55d-27e)x+\frac{56}{25}(}{3+2x+5x^2}$$

$$= -\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int\left(\frac{56}{125}(20d-41e)+\frac{224ex}{25}+\frac{2(165d+4811e}{125\left(3\right.}\right.$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{\int\frac{165d+4811e-28(205d-103e)x}{3+2x+5x^2}}{3500}$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{(-205d+103e)\int\frac{2+10x}{3+2x+5}}{1250}$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}-\frac{(205d-103e)\log\left(3+2\right.}{1250}$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{(6565d+21171e)\tan^{-1}}{17500\sqrt{14}}$$

Mathematica [A] time = 0.0662719, size = 96, normalized size = 0.99

$$\frac{-\frac{14(5d(423x+1367)+e(5989x-1269))}{5x^2+2x+3}+196(103e-205d)\log\left(5x^2+2x+3\right)+1960x(20d-41e)+\sqrt{14}(6565d+21171e)\tan^{-1}\left(\right.}{245000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x
]
```

```
[Out] (1960*(20*d - 41*e)*x + 19600*e*x^2 - (14*(5*d*(1367 + 423*x) + e*(-1269 +
5989*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d + 21171*e)*ArcTan[(1 + 5*x)/
Sqrt[14]] + 196*(-205*d + 103*e)*Log[3 + 2*x + 5*x^2])/245000
```

Maple [A] time = 0.055, size = 106, normalized size = 1.1

$$\frac{2\,ex^2}{25}+\frac{4\,dx}{25}-\frac{41\,ex}{125}-\frac{1}{125}\left(\left(\frac{423\,d}{140}+\frac{5989\,e}{700}\right)x+\frac{1367\,d}{140}-\frac{1269\,e}{700}\right)\left(x^2+\frac{2\,x}{5}+\frac{3}{5}\right)^{-1}-\frac{41\,\ln\left(5\,x^2+2\,x+3\right)d}{250}+\frac{103\,e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 2/25*e*x^2+4/25*d*x-41/125*e*x-1/125*((423/140*d+5989/700*e)*x+1367/140*d-1
269/700*e)/(x^2+2/5*x+3/5)-41/250*ln(5*x^2+2*x+3)*d+103/1250*e*ln(5*x^2+2*x
+3)+1313/49000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d+21171/245000*14^(1
/2)*arctan(1/28*(10*x+2)*14^(1/2))*e
```

Maxima [A] time = 1.49595, size = 122, normalized size = 1.26

$$\frac{2}{25}ex^2+\frac{1}{245000}\sqrt{14}(6565d+21171e)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{1}{125}(20d-41e)x-\frac{1}{1250}(205d-103e)\log\left(5x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 2/25*e*x^2 + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 1/125*(20*d - 41*e)*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.20687, size = 462, normalized size = 4.76

$$\frac{98000ex^4+9800(20d-37e)x^3+7840(10d-13e)x^2+\sqrt{14}\left(5(6565d+21171e)x^2+2(6565d+21171e)x+19695d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] 1/245000*(98000*e*x^4 + 9800*(20*d - 37*e)*x^3 + 7840*(10*d - 13*e)*x^2 + s
qrt(14)*(5*(6565*d + 21171*e)*x^2 + 2*(6565*d + 21171*e)*x + 19695*d + 6351
3*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(6285*d - 23209*e)*x - 196*(5*(20
5*d - 103*e)*x^2 + 2*(205*d - 103*e)*x + 615*d - 309*e)*log(5*x^2 + 2*x + 3
) - 95690*d + 17766*e)/(5*x^2 + 2*x + 3)
```

Sympy [C] time = 1.11261, size = 163, normalized size = 1.68

$$\frac{2ex^2}{25} + x\left(\frac{4d}{25} - \frac{41e}{125}\right) + \left(-\frac{41d}{250} + \frac{103e}{1250} - \frac{\sqrt{14}i\,(6565d + 21171e)}{490000}\right)\log\left(x + \frac{1313d + \frac{21171e}{5} - \frac{\sqrt{14}i(6565d+21171e)}{5}}{6565d + 21171e}\right) + \left(-\frac{41d}{250}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 2*e*x**2/25 + x*(4*d/25 - 41*e/125) + (-41*d/250 + 103*e/1250 - sqrt(14)*I*
(6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 - sqrt(14)*I*(6565*d
 + 21171*e)/5)/(6565*d + 21171*e)) + (-41*d/250 + 103*e/1250 + sqrt(14)*I*(
6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 + sqrt(14)*I*(6565*d
+ 21171*e)/5)/(6565*d + 21171*e)) - (6835*d - 1269*e + x*(2115*d + 5989*e))
/(87500*x**2 + 35000*x + 52500)
```

Giac [A] time = 1.14407, size = 127, normalized size = 1.31

$$\frac{2}{25}\,x^2e + \frac{1}{245000}\,\sqrt{14}(6565\,d + 21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x + 1)\right) + \frac{4}{25}\,dx - \frac{41}{125}\,xe - \frac{1}{1250}\,(205\,d - 103\,e)\log\left(5\,x^2 + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 2/25*x^2*e + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 4/25*d*x - 41/125*x*e - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

3.314 $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2} dx$

Optimal. Leaf size=63

$$-\frac{423x+1367}{3500(5x^2+2x+3)}-\frac{41}{250}\log(5x^2+2x+3)+\frac{4x}{25}+\frac{1313\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}$$

```
[Out] (4*x)/25 - (1367 + 423*x)/(3500*(3 + 2*x + 5*x^2)) + (1313*ArcTan[(1 + 5*x)
/Sqrt[14]])/(3500*Sqrt[14]) - (41*Log[3 + 2*x + 5*x^2])/250
```

Rubi [A] time = 0.0773302, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1660, 1657, 634, 618, 204, 628}

$$-\frac{423x+1367}{3500(5x^2+2x+3)}-\frac{41}{250}\log(5x^2+2x+3)+\frac{4x}{25}+\frac{1313\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (4*x)/25 - (1367 + 423*x)/(3500*(3 + 2*x + 5*x^2)) + (1313*ArcTan[(1 + 5*x)
/Sqrt[14]])/(3500*Sqrt[14]) - (41*Log[3 + 2*x + 5*x^2])/250
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

```
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367+423x}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int \frac{\frac{738}{25}-\frac{1848x}{25}+\frac{224x^2}{5}}{3+2x+5x^2}\,dx$$

$$= -\frac{1367+423x}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int \left(\frac{224}{25}+\frac{2(33-1148x)}{25\left(3+2x+5x^2\right)}\right)dx$$

$$= \frac{4x}{25} - \frac{1367+423x}{3500\left(3+2x+5x^2\right)} + \frac{1}{700}\int \frac{33-1148x}{3+2x+5x^2}\,dx$$

$$= \frac{4x}{25} - \frac{1367+423x}{3500\left(3+2x+5x^2\right)} - \frac{41}{250}\int \frac{2+10x}{3+2x+5x^2}\,dx + \frac{1313\int \frac{1}{3+2x+5x^2}\,dx}{3500}$$

$$= \frac{4x}{25} - \frac{1367+423x}{3500\left(3+2x+5x^2\right)} - \frac{41}{250}\log\left(3+2x+5x^2\right) - \frac{1313\,\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx, x, 2+\right.}{1750}$$

$$= \frac{4x}{25} - \frac{1367+423x}{3500\left(3+2x+5x^2\right)} + \frac{1313\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}} - \frac{41}{250}\log\left(3+2x+5x^2\right)$$

Mathematica [A] time = 0.0367986, size = 59, normalized size = 0.94

$$\frac{-\frac{14(423x+1367)}{5x^2+2x+3} - 8036\log\left(5x^2+2x+3\right) + 7840x + 1313\sqrt{14}\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{49000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (7840*x - (14*(1367 + 423*x))/(3 + 2*x + 5*x^2) + 1313*Sqrt[14]*ArcTan[(1 +
 5*x)/Sqrt[14]] - 8036*Log[3 + 2*x + 5*x^2])/49000
```

Maple [A] time = 0.048, size = 51, normalized size = 0.8

$$\frac{4\,x}{25} - \frac{1}{25}\left(\frac{423\,x}{700}+\frac{1367}{700}\right)\left(x^2+\frac{2\,x}{5}+\frac{3}{5}\right)^{-1} - \frac{41\,\ln\left(5\,x^2+2\,x+3\right)}{250} + \frac{1313\,\sqrt{14}}{49000}\arctan\left(\frac{(10\,x+2)\,\sqrt{14}}{28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 4/25*x-1/25*(423/700*x+1367/700)/(x^2+2/5*x+3/5)-41/250*ln(5*x^2+2*x+3)+131
3/49000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Maxima [A] time = 1.50011, size = 70, normalized size = 1.11

$$\frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{4}{25}x-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}-\frac{41}{250}\log\left(5x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.23975, size = 244, normalized size = 3.87

$$\frac{39200x^3+1313\sqrt{14}\left(5x^2+2x+3\right)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+15680x^2-8036\left(5x^2+2x+3\right)\log\left(5x^2+2x+3\right)+1}{49000\left(5x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/49000*(39200*x^3 + 1313*sqrt(14)*(5*x^2 + 2*x + 3)*arctan(1/14*sqrt(14)*(
5*x + 1)) + 15680*x^2 - 8036*(5*x^2 + 2*x + 3)*log(5*x^2 + 2*x + 3) + 17598
*x - 19138)/(5*x^2 + 2*x + 3)
```

Sympy [A] time = 0.171049, size = 63, normalized size = 1.

$$\frac{4x}{25}-\frac{423x+1367}{17500x^2+7000x+10500}-\frac{41\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{250}+\frac{1313\sqrt{14}\operatorname{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*x/25 - (423*x + 1367)/(17500*x**2 + 7000*x + 10500) - 41*log(x**2 + 2*x/5
 + 3/5)/250 + 1313*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/49000
```

Giac [A] time = 1.16164, size = 70, normalized size = 1.11

$$\frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{4}{25}x-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}-\frac{41}{250}\log\left(5x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="giac")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

3.315 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2} dx$

Optimal. Leaf size=224

$$-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}-\frac{\left(-61d^2e+205d^3+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)}{50\left(5d^2-2de+3e^2\right)^2}+\frac{\left(3d^2e^2+5d^3e+4d^4-\right.}{e\left(5d^2-2d\right.}$$

```
[Out] -(1367*d - 293*e + (423*d - 1367*e)*x)/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*
x + 5*x^2)) + ((6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(700*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((4*d^4 + 5*d^
3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e*(5*d^2 - 2*d*e + 3*e^2)^2
) - ((205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3 + 2*x + 5*x^2])/(50*(5*
d^2 - 2*d*e + 3*e^2)^2)
```

Rubi [A] time = 0.340275, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}-\frac{\left(-61d^2e+205d^3+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)}{50\left(5d^2-2de+3e^2\right)^2}+\frac{\left(3d^2e^2+5d^3e+4d^4-\right.}{e\left(5d^2-2d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x]
```


```
[Out] -(1367*d - 293*e + (423*d - 1367*e)*x)/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*
x + 5*x^2)) + ((6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(700*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((4*d^4 + 5*d^
3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e*(5*d^2 - 2*d*e + 3*e^2)^2
) - ((205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3 + 2*x + 5*x^2])/(50*(5*
d^2 - 2*d*e + 3*e^2)^2)
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
```

```
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{\frac{2\left(369d^2-421de+280e^2\right)}{5\left(5d^2-2de+3e^2\right)}-\frac{2\left(924d^2-285de+281e^2\right)}{5\left(5d^2-2de+3e^2\right)}}{(d+ex)\left(3+2x+5x^2\right)}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{1}{56}\int\left(\frac{56\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^2(d+ex)}+\right.$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)\text{ta}}{700\sqrt{14}\left(5d^2-2de+3e^2\right)^2}$$

Mathematica [A] time = 0.159759, size = 186, normalized size = 0.83

$$\frac{\frac{14\left(5d^2-2de+3e^2\right)(e(1367x+293)-d(423x+1367))}{5x^2+2x+3}-196\left(-61d^2e+205d^3+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)+\frac{9800\left(3d^2e^2+5d^3e+4d^4-de^3+\right.}{e}}{9800\left(5d^2-2de+3e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x
]
```

```
[Out] ((14*(5*d^2 - 2*d*e + 3*e^2)*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(3 +
 2*x + 5*x^2) + Sqrt[14]*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*
ArcTan[(1 + 5*x)/Sqrt[14]] + (9800*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2
*e^4)*Log[d + e*x])/e - 196*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3
+ 2*x + 5*x^2])/(9800*(5*d^2 - 2*d*e + 3*e^2)^2)
```

Maple [B] time = 0.1, size = 691, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x)
```

```
[Out] -423/700/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d^3*x+7681/3500/(5*d^2-2*d*e
+3*e^2)^2/(x^2+2/5*x+3/5)*x*d^2*e-4003/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*
x+3/5)*x*d*e^2+4101/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*x*e^3-1367/7
00/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d^3+4199/3500/(5*d^2-2*d*e+3*e^2)^
2/(x^2+2/5*x+3/5)*d^2*e-4687/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d*e
^2+879/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*e^3-41/10/(5*d^2-2*d*e+3*
e^2)^2*ln(5*x^2+2*x+3)*d^3+61/50/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)*d^2*
e-23/50/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)*d*e^2-7/25/(5*d^2-2*d*e+3*e^2
)^2*ln(5*x^2+2*x+3)*e^3+1313/1960/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/2
8*(10*x+2)*14^(1/2))*d^3-26423/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1
/28*(10*x+2)*14^(1/2))*d^2*e+11089/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arct
an(1/28*(10*x+2)*14^(1/2))*d*e^2-6623/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*a
rctan(1/28*(10*x+2)*14^(1/2))*e^3+4/(5*d^2-2*d*e+3*e^2)^2/e*ln(e*x+d)*d^4+5
/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^3+3/(5*d^2-2*d*e+3*e^2)^2*e*ln(e*x+d)*d^
2-1/(5*d^2-2*d*e+3*e^2)^2*e^2*ln(e*x+d)*d+2/(5*d^2-2*d*e+3*e^2)^2*e^3*ln(e*
x+d)
```

Maxima [A] time = 1.60398, size = 390, normalized size = 1.74

$$\frac{\sqrt{14}\left(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{9800\left(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4\right)}+\frac{\left(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4\right)\log\left(ex\right.}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,de^4+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
+ (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)/(25*d^4*e - 20
*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5) - 1/50*(205*d^3 - 61*d^2*e + 23*d
*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*
e^3 + 9*e^4) - 1/700*((423*d - 1367*e)*x + 1367*d - 293*e)/(5*(5*d^2 - 2*d*
```

```
e + 3*e^2)*x^2 + 15*d^2 - 6*d*e + 9*e^2 + 2*(5*d^2 - 2*d*e + 3*e^2)*x)
```

Fricas [B] time = 1.67982, size = 1191, normalized size = 5.32

$$-\frac{95690\,d^3e - 58786\,d^2e^2 + 65618\,de^3 - 12306\,e^4 - \sqrt{14}\left(19695\,d^3e - 79269\,d^2e^2 + 33267\,de^3 - 19869\,e^4 + 5\left(6565\,d^3e - 2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] -1/9800*(95690*d^3*e - 58786*d^2*e^2 + 65618*d*e^3 - 12306*e^4 - sqrt(14)*(
19695*d^3*e - 79269*d^2*e^2 + 33267*d*e^3 - 19869*e^4 + 5*(6565*d^3*e - 264
23*d^2*e^2 + 11089*d*e^3 - 6623*e^4)*x^2 + 2*(6565*d^3*e - 26423*d^2*e^2 +
11089*d*e^3 - 6623*e^4)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(2115*d^3*e
 - 7681*d^2*e^2 + 4003*d*e^3 - 4101*e^4)*x - 9800*(12*d^4 + 15*d^3*e + 9*d^
2*e^2 - 3*d*e^3 + 6*e^4 + 5*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x
^2 + 2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x)*log(e*x + d) + 196*
(615*d^3*e - 183*d^2*e^2 + 69*d*e^3 + 42*e^4 + 5*(205*d^3*e - 61*d^2*e^2 +
23*d*e^3 + 14*e^4)*x^2 + 2*(205*d^3*e - 61*d^2*e^2 + 23*d*e^3 + 14*e^4)*x)*
log(5*x^2 + 2*x + 3))/(75*d^4*e - 60*d^3*e^2 + 102*d^2*e^3 - 36*d*e^4 + 27*
e^5 + 5*(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x^2 + 2*(25
*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x)
```

Sympy [C] time = 18.3775, size = 8322, normalized size = 37.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**2,x)
```

```
[Out] (-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(
25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*
d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))*log(x + (-
6252890000000*d**12*e*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2
 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e*
*4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3
```

*e**2)**2))**2 + 1721036800000*d**12*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e
 + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 1
2*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d
2 - 2*d*e + 3*e2)**2)) - 3349326400000*d**11*e**2*(-sqrt(14)*I*(6565*d
3 - 26423*d2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e
 + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 +
 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 4402940080000*d**11*e*(-s
qrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*
d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**
2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) + 30530841600
0*d**11 + 5503256640000*d**10*e**3*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e
 + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12
*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d*
*2 - 2*d*e + 3*e**2)**2))**2 - 533211796600*d**10*e**2*(-sqrt(14)*I*(6565*
d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e
 + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2
 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) + 1028468958725*d**10*e - 141
469554240000*d**9*e**4*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**
2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e
4)) - (205*d3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e +
3*e**2)**2))**2 + 17262989570400*d**9*e**3*(-sqrt(14)*I*(6565*d**3 - 26423*
d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e*
*2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(5
0*(5*d**2 - 2*d*e + 3*e**2)**2)) + 95412070955*d**9*e**2 + 139354879664000*
d**8*e**5*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3
)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*
d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))*
*2 - 11862414903920*d**8*e**4*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 1108
9*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**
3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2
*d*e + 3*e**2)**2)) + 927554565402*d**8*e**3 - 160769212620800*d**7*e**6*(-
sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25
*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d*
*2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 1322030
0596608*d**7*e**5*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6
623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4))
 - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**
2)**2)) - 1587450017342*d**7*e**4 + 92712805606400*d**6*e**7*(-sqrt(14)*I*(
6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d
3*e + 34*d2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*
e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 376982672864*d**6*e
6*(-sqrt(14)*I*(6565*d3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(196
00*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 -
 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) + 1705
352927600*d**6*e**5 - 61599603788800*d**5*e**8*(-sqrt(14)*I*(6565*d**3 - 26

423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**
2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3
)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 1766518292672*d**5*e**7*(-sqrt(14
)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 -
 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e +
23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) - 927094311444*d**5
*e**6 + 13267673552000*d**4*e**9*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 1
1089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*
e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2
- 2*d*e + 3*e**2)**2))**2 + 6357651035680*d**4*e**8*(-sqrt(14)*I*(6565*d**3
 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 3
4*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14
*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) + 5551412790*d**4*e**7 - 36177335
04000*d**3*e**10*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 66
23*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4))
- (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2
)**2))**2 - 3730299722240*d**3*e**9*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e
+ 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12
*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d*
*2 - 2*d*e + 3*e**2)**2)) + 227625566062*d**3*e**8 - 3887664076800*d**2*e**
11*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(1960
0*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 -
61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 25
47991828368*d**2*e**10*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**
2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e
4)) - (205*d3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e +
3*e**2)**2)) - 201172444677*d**2*e**9 + 1207100966400*d*e**12*(-sqrt(14)*I*
(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*
d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d
*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 703802088864*d*e**
11*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(1960
0*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 -
61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) + 62145
783705*d*e**10 - 676838332800*e**13*(-sqrt(14)*I*(6565*d**3 - 26423*d**2*e
+ 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12
*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d*
*2 - 2*d*e + 3*e**2)**2))**2 + 221086021968*e**12*(-sqrt(14)*I*(6565*d**3 -
 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*
d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e
3)/(50*(5*d2 - 2*d*e + 3*e**2)**2)) - 15706111904*e**11)/(115291904000*
d**11 + 60548006400*d**10*e - 1205961319355*d**9*e**2 - 1979222576837*d**8*
e**3 + 528572641642*d**7*e**4 - 1648297602686*d**6*e**5 + 151381570368*d**5
*e**6 - 924616717780*d**4*e**7 + 478372778758*d**3*e**8 - 478669057938*d**2
*e**9 + 139540516779*d*e**10 - 49409758967*e**11)) + (sqrt(14)*I*(6565*d**3
 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 3

4*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14
*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))*log(x + (-6252890000000*d**12*e*(
sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25
*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d*
*2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 1721036
800000*d**12*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e*
3)/(19600(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (20
5*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)
) - 33493264000000*d**11*e**2*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089
*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3
 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*
d*e + 3*e**2)**2))**2 + 4402940080000*d**11*e*(sqrt(14)*I*(6565*d**3 - 2642
3*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*
e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/
(50*(5*d**2 - 2*d*e + 3*e**2)**2)) + 305308416000*d**11 + 55032566400000*d*
*10*e**3*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/
(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d*
*3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2
 - 5332117966000*d**10*e**2*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d
*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 +
 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*
e + 3*e**2)**2)) + 1028468958725*d**10*e - 141469554240000*d**9*e**4*(sqrt(
14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4
 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e
+ 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 172629895704
00*d**9*e**3*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e*
3)/(19600(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (20
5*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)
) + 95412070955*d**9*e**2 + 139354879664000*d**8*e**5*(sqrt(14)*I*(6565*d**
3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e +
34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 1
4*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 - 11862414903920*d**8*e**4*(s
qrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*
d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**
2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) + 92755456540
2*d**8*e**3 - 16076921262080 0*d**7*e**6*(sqrt(14)*I*(6565*d**3 - 26423*d**2
*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 -
 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5
*d**2 - 2*d*e + 3*e**2)**2))**2 + 13220300596608*d**7*e**5*(sqrt(14)*I*(656
5*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3
*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**
2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) - 1587450017342*d**7*e**4 +
 92712805606400*d**6*e**7*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e
2 - 6623*e3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9
*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e

+ 3*e**2)**2))**2 - 376982672864*d**6*e**6*(sqrt(14)*I*(6565*d**3 - 26423*d
2*e + 11089*d*e2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**
2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50
*(5*d**2 - 2*d*e + 3*e**2)**2)) + 1705352927600*d**6*e**5 - 61599603788800*
d**5*e**8*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)
/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d
3 - 61*d2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**
2 - 1766518292672*d**5*e**7*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d
*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 +
 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*
e + 3*e**2)**2)) - 927094311444*d**5*e**6 + 13267673552000*d**4*e**9*(sqrt(
14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4
 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e
+ 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 635765103568
0*d**4*e**8*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**
3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205
*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))
 + 5551412790*d**4*e**7 - 3617733504000*d**3*e**10*(sqrt(14)*I*(6565*d**3 -
 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*
d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e
3)/(50*(5*d2 - 2*d*e + 3*e**2)**2))**2 - 3730299722240*d**3*e**9*(sqrt(
14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4
 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e
+ 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) + 227625566062*d*
*3*e**8 - 3887664076800*d**2*e**11*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e +
11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d
*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2
 - 2*d*e + 3*e**2)**2))**2 + 2547991828368*d**2*e**10*(sqrt(14)*I*(6565*d**
3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e +
34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 1
4*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) - 201172444677*d**2*e**9 + 12071
00966400*d*e**12*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 662
3*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) -
 (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)
2))2 - 703802088864*d*e**11*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 110
89*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e*
*3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 -
2*d*e + 3*e**2)**2)) + 62145783705*d*e**10 - 676838332800*e**13*(sqrt(14)*I
*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*(25*d**4 - 20
*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61*d**2*e + 23*
d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2))**2 + 221086021968*e**1
2*(sqrt(14)*I*(6565*d**3 - 26423*d**2*e + 11089*d*e**2 - 6623*e**3)/(19600*
(25*d**4 - 20*d**3*e + 34*d**2*e**2 - 12*d*e**3 + 9*e**4)) - (205*d**3 - 61
*d**2*e + 23*d*e**2 + 14*e**3)/(50*(5*d**2 - 2*d*e + 3*e**2)**2)) - 1570611
1904*e**11)/(115291904000*d**11 + 60548006400*d**10*e - 1205961319355*d**9*

e**2 - 1979222576837*d**8*e**3 + 528572641642*d**7*e**4 - 1648297602686*d**
6*e**5 + 151381570368*d**5*e**6 - 924616717780*d**4*e**7 + 478372778758*d**
3*e**8 - 478669057938*d**2*e**9 + 139540516779*d*e**10 - 49409758967*e**11)
) - (1367*d - 293*e + x*(423*d - 1367*e))/(10500*d**2 - 4200*d*e + 6300*e**
2 + x**2*(17500*d**2 - 7000*d*e + 10500*e**2) + x*(7000*d**2 - 2800*d*e + 4
200*e**2)) + (4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)*log(x + (1
721036800000*d**12*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(e*(
5*d**2 - 2*d*e + 3*e**2)**2) - 6252890000000*d**12*(4*d**4 + 5*d**3*e + 3*d
2*e2 - d*e**3 + 2*e**4)**2/(e*(5*d**2 - 2*d*e + 3*e**2)**4) + 305308416
000*d**11 + 4402940080000*d**11*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 +
 2*e**4)/(5*d**2 - 2*d*e + 3*e**2)**2 - 33493264000000*d**11*(4*d**4 + 5*d*
*3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**4 + 102
8468958725*d**10*e - 5332117966000*d**10*e*(4*d**4 + 5*d**3*e + 3*d**2*e**2
 - d*e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3*e**2)**2 + 55032566400000*d**10*e*(
4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e
2)4 + 95412070955*d**9*e**2 + 17262989570400*d**9*e**2*(4*d**4 + 5*d**3
*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3*e**2)**2 - 14146955
4240000*d**9*e**2*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)**2/(5
*d**2 - 2*d*e + 3*e**2)**4 + 927554565402*d**8*e**3 - 11862414903920*d**8*e
3*(4*d4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3
*e**2)**2 + 139354879664000*d**8*e**3*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*
e**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**4 - 1587450017342*d**7*e**4 +
13220300596608*d**7*e**4*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4
)/(5*d**2 - 2*d*e + 3*e**2)**2 - 160769212620800*d**7*e**4*(4*d**4 + 5*d**3
*e + 3*d**2*e**2 - d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**4 + 17053
52927600*d**6*e**5 - 376982672864*d**6*e**5*(4*d**4 + 5*d**3*e + 3*d**2*e**
2 - d*e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3*e**2)**2 + 92712805606400*d**6*e**
5*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*e +
3*e**2)**4 - 927094311444*d**5*e**6 - 1766518292672*d**5*e**6*(4*d**4 + 5*d
3*e + 3*d2*e**2 - d*e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3*e**2)**2 - 61599
603788800*d**5*e**6*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)**2/
(5*d**2 - 2*d*e + 3*e**2)**4 + 5551412790*d**4*e**7 + 6357651035680*d**4*e*
7(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3*
e**2)**2 + 13267673552000*d**4*e**7*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e*
*3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**4 + 227625566062*d**3*e**8 - 373
0299722240*d**3*e**8*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)/(5
*d**2 - 2*d*e + 3*e**2)**2 - 3617733504000*d**3*e**8*(4*d**4 + 5*d**3*e + 3
*d**2*e**2 - d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**4 - 20117244467
7*d**2*e**9 + 2547991828368*d**2*e**9*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*
e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3*e**2)**2 - 3887664076800*d**2*e**9*(4*d*
*4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)
4 + 62145783705*d*e10 - 703802088864*d*e**10*(4*d**4 + 5*d**3*e + 3*d**
2*e**2 - d*e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3*e**2)**2 + 1207100966400*d*e*
10(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*e
+ 3*e**2)**4 - 15706111904*e**11 + 221086021968*e**11*(4*d**4 + 5*d**3*e +

```
3*d**2*e**2 - d*e**3 + 2*e**4)/(5*d**2 - 2*d*e + 3*e**2)**2 - 676838332800*
e**11*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)**2/(5*d**2 - 2*d*
e + 3*e**2)**4)/(115291904000*d**11 + 60548006400*d**10*e - 1205961319355*d
**9*e**2 - 1979222576837*d**8*e**3 + 528572641642*d**7*e**4 - 1648297602686
*d**6*e**5 + 151381570368*d**5*e**6 - 924616717780*d**4*e**7 + 478372778758
*d**3*e**8 - 478669057938*d**2*e**9 + 139540516779*d*e**10 - 49409758967*e*
*11))/(e*(5*d**2 - 2*d*e + 3*e**2)**2)
```

Giac [A] time = 1.17548, size = 383, normalized size = 1.71

$$\frac{\sqrt{14}\left(6565\,d^3 - 26423\,d^2e + 11089\,de^2 - 6623\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{9800\left(25\,d^4 - 20\,d^3e + 34\,d^2e^2 - 12\,de^3 + 9\,e^4\right)} - \frac{\left(205\,d^3 - 61\,d^2e + 23\,de^2 + 14\,e^3\right)\log\left(5\,x^2\right.}{50\left(25\,d^4 - 20\,d^3e + 34\,d^2e^2 - 12\,de^3 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
- 1/50*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^
4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + (4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*log(abs(x*e + d))/(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 -
12*d*e^4 + 9*e^5) - 1/700*(6835*d^3 - 4199*d^2*e + (2115*d^3 - 7681*d^2*e +
 4003*d*e^2 - 4101*e^3)*x + 4687*d*e^2 - 879*e^3)/((5*d^2 - 2*d*e + 3*e^2)^
2*(5*x^2 + 2*x + 3))
```

3.316 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=313

$$-\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{140\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}-\frac{\left(-60d^2e^2-8d^3e+41d^4+24de^3-5e^4\right)\log\left(5x^2+2x+3\right)}{2\left(5d^2-2de+3e^2\right)^3}-$$

```
[Out] -((4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e*(5*d^2 - 2*d*e + 3*e^2)^
2*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e
^2)*x)/(140*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)) + ((1313*d^4 - 100
44*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(28*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + ((41*d^4 - 8*d^3*e - 60*d^2*e^2 +
 24*d*e^3 - 5*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - ((41*d^4 - 8*d
^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d
*e + 3*e^2)^3)
```

Rubi [A] time = 0.497972, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{140\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}-\frac{\left(-60d^2e^2-8d^3e+41d^4+24de^3-5e^4\right)\log\left(5x^2+2x+3\right)}{2\left(5d^2-2de+3e^2\right)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2),x]
```


```
[Out] -((4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e*(5*d^2 - 2*d*e + 3*e^2)^
2*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e
^2)*x)/(140*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)) + ((1313*d^4 - 100
44*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(28*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + ((41*d^4 - 8*d^3*e - 60*d^2*e^2 +
 24*d*e^3 - 5*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - ((41*d^4 - 8*d
^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d
*e + 3*e^2)^3)
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{\frac{2\left(369d^4-842d^3e+787d^2\cdots\right.}{\left(5d^2-2de+\cdots\right.}}{\cdots}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}+\frac{1}{56}\int \left(\frac{56\left(4d^4+5d^3e+\cdots\right.}{\left(5d^2-2de+\cdots\right.}\right.$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+29\cdots\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+29\cdots\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+29\cdots\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+29\cdots\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}$$

Mathematica [A] time = 0.254812, size = 270, normalized size = 0.86

$$\frac{-\frac{14\left(5d^2-2de+3e^2\right)\left(d^2(423x+1367)-2de(1367x+293)+e^2(293x-703)\right)}{5x^2+2x+3}+980\left(60d^2e^2+8d^3e-41d^4-24de^3+5e^4\right)\log\left(5x^2+2x+3\right)-\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-1960*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^
4))/(e*(d + e*x)) - (14*(5*d^2 - 2*d*e + 3*e^2)*(e^2*(-703 + 293*x) + d^2*(
1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(1313
*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/S
qrt[14]] + 1960*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[d +
e*x] + 980*(-41*d^4 + 8*d^3*e + 60*d^2*e^2 - 24*d*e^3 + 5*e^4)*Log[3 + 2*x
+ 5*x^2])/(1960*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Maple [B] time = 0.072, size = 986, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x)
```

```
[Out] -8/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^3*e-4/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)*
d^4-3/(5*d^2-2*d*e+3*e^2)^2*e/(e*x+d)*d^2-879/350/(5*d^2-2*d*e+3*e^2)^3/(x^
2+2/5*x+3/5)*d^2*e^2-423/140/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d^4*x-87
9/700/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*x*e^4-12/(5*d^2-2*d*e+3*e^2)^3*
ln(5*x^2+2*x+3)*d*e^3+1313/392/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(
10*x+2)*14^(1/2))*d^4-5/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^3-5/(5*d^2-2*d*e+3*
e^2)^3*ln(e*x+d)*e^4-1367/140/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d^4+210
9/700/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*e^4-41/2/(5*d^2-2*d*e+3*e^2)^3*
ln(5*x^2+2*x+3)*d^4+5/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*e^4-2/(5*d^2-
2*d*e+3*e^2)^2*e^3/(e*x+d)+41/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^4-4101/350/
(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*x*d^2*e^2+2197/175/(5*d^2-2*d*e+3*e^2
)^3/(x^2+2/5*x+3/5)*x*d*e^3+3629/175/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*
x*d^3*e-2511/98/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2
))*d^3*e+2145/196/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1
/2))*d^2*e^2+39/98/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(
1/2))*d*e^3-60/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^2*e^2+24/(5*d^2-2*d*e+3*e^
2)^3*ln(e*x+d)*d*e^3-271/392/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10
*x+2)*14^(1/2))*e^4+1/(5*d^2-2*d*e+3*e^2)^2*e^2/(e*x+d)*d+4/(5*d^2-2*d*e+3*
e^2)^3*ln(5*x^2+2*x+3)*d^3*e+30/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^2*e
^2+88/175/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d*e^3+1416/175/(5*d^2-2*d*e
+3*e^2)^3/(x^2+2/5*x+3/5)*d^3*e
```

Maxima [A] time = 1.5756, size = 740, normalized size = 2.36

$$\frac{\sqrt{14}\left(1313\,d^4-10044\,d^3e+4290\,d^2e^2+156\,de^3-271\,e^4\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{392\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{\left(41\,d^4-8\,d^3e-60\,d^2e^2+2\cdots\right.}{125\,d^6-150\,d^5e+285\,d^4e^2-188\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(1313*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4
)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*
d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (41*d^4 - 8*d^3*e - 60*d^2*e^2
 + 24*d*e^3 - 5*e^4)*log(e*x + d)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*
d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 60*d^2
*e^2 + 24*d*e^3 - 5*e^4)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^
4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/140*(1680*d^4 +
3467*d^3*e + 674*d^2*e^2 - 1123*d*e^3 + 840*e^4 + (2800*d^4 + 3500*d^3*e +
2523*d^2*e^2 - 3434*d*e^3 + 1693*e^4)*x^2 + (1120*d^4 + 1823*d^3*e - 527*d^
2*e^2 - 573*d*e^3 - 143*e^4)*x)/(75*d^5*e - 60*d^4*e^2 + 102*d^3*e^3 - 36*d
^2*e^4 + 27*d*e^5 + 5*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*d*e^5 + 9*
e^6)*x^3 + (125*d^5*e - 50*d^4*e^2 + 130*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5 + 1
8*e^6)*x^2 + (50*d^5*e + 35*d^4*e^2 + 8*d^3*e^3 + 78*d^2*e^4 - 18*d*e^5 + 2
7*e^6)*x)
```

Fricas [B] time = 1.91916, size = 2264, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/1960*(117600*d^6 + 195650*d^5*e + 20664*d^4*e^2 + 48132*d^3*e^3 + 118552
*d^2*e^4 - 70686*d*e^5 + 35280*e^6 + 14*(14000*d^6 + 11900*d^5*e + 14015*d^
4*e^2 - 11716*d^3*e^3 + 22902*d^2*e^4 - 13688*d*e^5 + 5079*e^6)*x^2 - 5*sqr
t(14)*(3939*d^5*e - 30132*d^4*e^2 + 12870*d^3*e^3 + 468*d^2*e^4 - 813*d*e^5
 + 5*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 271*e^6)*x^
3 + (6565*d^5*e - 47594*d^4*e^2 + 1362*d^3*e^3 + 9360*d^2*e^4 - 1043*d*e^5
- 542*e^6)*x^2 + (2626*d^5*e - 16149*d^4*e^2 - 21552*d^3*e^3 + 13182*d^2*e^
4 - 74*d*e^5 - 813*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(5600*d^6 +
 6875*d^5*e - 2921*d^4*e^2 + 3658*d^3*e^3 - 1150*d^2*e^4 - 1433*d*e^5 - 429
*e^6)*x - 1960*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^
5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d
^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*
d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(e*x + d) +
980*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^5 + 5*(41*d
^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d^5*e + 42*d
^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*d^4*e^2 - 1
44*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(5*x^2 + 2*x + 3))/(375
*d^7*e - 450*d^6*e^2 + 855*d^5*e^3 - 564*d^4*e^4 + 513*d^3*e^5 - 162*d^2*e^
```

```
6 + 81*d*e^7 + 5*(125*d^6*e^2 - 150*d^5*e^3 + 285*d^4*e^4 - 188*d^3*e^5 + 1
71*d^2*e^6 - 54*d*e^7 + 27*e^8)*x^3 + (625*d^7*e - 500*d^6*e^2 + 1125*d^5*e
^3 - 370*d^4*e^4 + 479*d^3*e^5 + 72*d^2*e^6 + 27*d*e^7 + 54*e^8)*x^2 + (250
*d^7*e + 75*d^6*e^2 + 120*d^5*e^3 + 479*d^4*e^4 - 222*d^3*e^5 + 405*d^2*e^6
 - 108*d*e^7 + 81*e^8)*x)
```

Sympy [C] time = 27.3404, size = 13362, normalized size = 42.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**2,x)
```

```
[Out] (-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*
e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**
2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*
e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))*log(x + (4503590000000*d**
17*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 2
71*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*
d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24
*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 79236430000000*d**
16*e*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 -
 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 17
1*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 +
24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 219307065600000*
d**15*e**2*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*
e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**
3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e
**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 1477177520
000*d**15*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e
**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3
 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e*
*2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) - 47010208000000
0*d**14*e**3*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*
d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e
**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2
*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 80627382
22000*d**14*e*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156
*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*
e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**
2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 6698206076
```

80000*d**13*e**4*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d*
*3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*
d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 1340
1619991200*d**13*e**2*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e*
*2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 1
88*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e
- 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 13
2446413125*d**13 - 748279970905600*d**12*e**5*(-sqrt(14)*I*(1313*d**4 - 100
44*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d*
*5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)
) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2
*d*e + 3*e**2)**3))**2 - 8564369003120*d**12*e**3*(-sqrt(14)*I*(1313*d**4 -
10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 15
0*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e
6)) - (41*d4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2
- 2*d*e + 3*e**2)**3)) + 684029295980*d**12*e + 599319595212800*d**11*e**6
*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271
*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d*
*2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d
*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 12510243478208*d**11
*e**4*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3
- 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 1
71*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 +
24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 764206623630*d**11
*e**2 - 291411662710784*d**10*e**7*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e +
4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*
d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**
4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e*
*2)**3))**2 + 49159980986704*d**10*e**5*(-sqrt(14)*I*(1313*d**4 - 10044*d**
3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e +
285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (4
1*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e +
3*e**2)**3)) - 14657220189100*d**10*e**3 - 27190445185792*d**9*e**8*(-sqrt
(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/
(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4
- 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 -
5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 77659175364512*d**9*e**6*(-
sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e*
4)/(784(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*
e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e*
*3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 16942805253691*d**9*e**4 +
253830846834432*d**8*e**9*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d*
*2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**
2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d*

*3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))
2 + 91313688339216*d8*e**7*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 429
0*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4
*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 -
8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)*
*3)) + 6404919470120*d**8*e**5 - 308064129587200*d**7*e**10*(-sqrt(14)*I*(1
313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125
*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e
5 + 27*e6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/
(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 78573287795968*d**7*e**8*(-sqrt(14)*
I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*
(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54
*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e*
4)/(2(5*d**2 - 2*d*e + 3*e**2)**3)) - 16998879119292*d**7*e**6 + 26246800
5502976*d**6*e**11*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2
+ 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*
d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 6
0*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 55
676827575152*d**6*e**9*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e
2 + 156*d*e3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 -
188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e
 - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 5
633839731848*d**6*e**7 - 162086347196928*d**5*e**12*(-sqrt(14)*I*(1313*d**4
 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 -
150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27
*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d*
*2 - 2*d*e + 3*e**2)**3))**2 - 30431528150688*d**5*e**10*(-sqrt(14)*I*(1313
*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d*
*6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5
 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*
(5*d**2 - 2*d*e + 3*e**2)**3)) + 3033254622763*d**5*e**8 + 82236632099328*d
4*e13*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e
3 - 271*e4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3
 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e*
*2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 13587008752
688*d**4*e**11*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 15
6*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3
*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d*
*2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) - 350682737
9684*d**4*e**9 - 30865482805248*d**3*e**14*(-sqrt(14)*I*(1313*d**4 - 10044*
d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*
e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) -
 (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*
e + 3*e**2)**3))**2 - 4535008734144*d**3*e**12*(-sqrt(14)*I*(1313*d**4 - 10
044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d

5*e + 285*d4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6
)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 -
2*d*e + 3*e**2)**3)) + 1484229456462*d**3*e**10 + 9233948989440*d**2*e**15*
(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*
e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**
2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*
e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 1144385029872*d**2*e*
13(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 -
271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171
*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 2
4*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) - 361088969436*d**2*e*
*11 - 1739174903424*d*e**16*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d
2*e2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e*
*2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d
3*e - 60*d2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)
)**2 - 187156660320*d*e**14*(-sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d
2*e2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e*
*2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d
3*e - 60*d2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)
) + 50336842869*d*e**12 + 196869004416*e**17*(-sqrt(14)*I*(1313*d**4 - 1004
4*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**
5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6))
 - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*
d*e + 3*e**2)**3))**2 + 17373868848*e**15*(-sqrt(14)*I*(1313*d**4 - 10044*d
3*e + 4290*d2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e
 + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) -
(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e
 + 3*e**2)**3)) - 3533954480*e**13)/(1101474866245*d**12*e - 9024487794180*
d**11*e**2 + 5764879624590*d**10*e**3 + 17969136971220*d**9*e**4 - 16485388
615365*d**8*e**5 - 12221510721480*d**7*e**6 + 21212253502020*d**6*e**7 - 11
710335235320*d**5*e**8 + 3048287389995*d**4*e**9 - 183650820660*d**3*e**10
- 118302770610*d**2*e**11 + 34222696740*d*e**12 - 3445820555*e**13)) + (sqr
t(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)
/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**
4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3
- 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))*log(x + (4503590000000*d**17*(s
qrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**
4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e
4 - 54*d*e5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**
3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 79236430000000*d**16*e*(
sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e*
4)/(784(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*
e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e*
*3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 219307065600000*d**15*e
2*(sqrt(14)*I*(1313*d4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 2

71*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*
d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24
*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 1477177520000*d**1
5*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271
*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d*
*2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d
*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) - 470102080000000*d**14*e
3*(sqrt(14)*I*(1313*d4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 2
71*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*
d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24
*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 8062738222000*d**1
4*e*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 2
71*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*
d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24
*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 669820607680000*d**13
*e**4*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 -
 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 17
1*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 +
24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 13401619991200*d
13*e2*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e*
*3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3
+ 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**
2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 132446413125*d*
*13 - 748279970905600*d**12*e**5*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 42
90*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**
4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 -
 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)
3))2 - 8564369003120*d**12*e**3*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e +
 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*
d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**
4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e*
*2)**3)) + 684029295980*d**12*e + 599319595212800*d**11*e**6*(sqrt(14)*I*(1
313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125
*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e
5 + 27*e6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/
(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 12510243478208*d**11*e**4*(sqrt(14)*
I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*
(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54
*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e*
4)/(2(5*d**2 - 2*d*e + 3*e**2)**3)) + 764206623630*d**11*e**2 - 291411662
710784*d**10*e**7*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 +
156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d*
*3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*
d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 4915
9980986704*d**10*e**5*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**

2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 18
8*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e -
60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) - 146
57220189100*d**10*e**3 - 27190445185792*d**9*e**8*(sqrt(14)*I*(1313*d**4 -
10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150
*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e*
*6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2
- 2*d*e + 3*e**2)**3))**2 - 77659175364512*d**9*e**6*(sqrt(14)*I*(1313*d**4
- 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 -
150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27
*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d*
*2 - 2*d*e + 3*e**2)**3)) + 16942805253691*d**9*e**4 + 253830846834432*d**8
*e**9*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 -
271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 17
1*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 +
24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 91313688339216*d
8*e7*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**
3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 +
171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2
+ 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 6404919470120*d*
*8*e**5 - 308064129587200*d**7*e**10*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e
+ 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285
*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d*
*4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e
2)3))**2 - 78573287795968*d**7*e**8*(sqrt(14)*I*(1313*d**4 - 10044*d**3
*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e +
285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41
*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e +
3*e**2)**3)) - 16998879119292*d**7*e**6 + 262468005502976*d**6*e**11*(sqrt(
14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(
784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4
- 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 -
5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 55676827575152*d**6*e**9*(sq
rt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4
)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e*
*4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3
- 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 5633839731848*d**6*e**7 - 16
2086347196928*d**5*e**12*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*
e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 -
188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*
e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2
- 30431528150688*d**5*e**10*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d
2*e2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e*
*2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d
3*e - 60*d2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)

) + 3033254622763*d**5*e**8 + 82236632099328*d**4*e**13*(sqrt(14)*I*(1313*d
4 - 10044*d3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6
 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 +
 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5
*d**2 - 2*d*e + 3*e**2)**3))**2 + 13587008752688*d**4*e**11*(sqrt(14)*I*(13
13*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*
d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e*
*5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(
2*(5*d**2 - 2*d*e + 3*e**2)**3)) - 3506827379684*d**4*e**9 - 30865482805248
*d**3*e**14*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*
e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**
3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e
2 + 24*d*e3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 4535008734
144*d**3*e**12*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156
*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*
e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**
2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 1484229456
462*d**3*e**10 + 9233948989440*d**2*e**15*(sqrt(14)*I*(1313*d**4 - 10044*d*
*3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5*e
+ 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (
41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e
+ 3*e**2)**3))**2 + 1144385029872*d**2*e**13*(sqrt(14)*I*(1313*d**4 - 10044
*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(784*(125*d**6 - 150*d**5
*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 - 54*d*e**5 + 27*e**6))
- (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d
*e + 3*e**2)**3)) - 361088969436*d**2*e**11 - 1739174903424*d*e**16*(sqrt(1
4)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(7
84*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 -
 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5
*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 - 187156660320*d*e**14*(sqrt(14
)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 271*e**4)/(78
4*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*d**2*e**4 -
54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*
e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) + 50336842869*d*e**12 + 19686900441
6*e**17*(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3
 - 271*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 +
171*d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2
+ 24*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3))**2 + 17373868848*e*
15(sqrt(14)*I*(1313*d**4 - 10044*d**3*e + 4290*d**2*e**2 + 156*d*e**3 - 2
71*e**4)/(784*(125*d**6 - 150*d**5*e + 285*d**4*e**2 - 188*d**3*e**3 + 171*
d**2*e**4 - 54*d*e**5 + 27*e**6)) - (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24
*d*e**3 - 5*e**4)/(2*(5*d**2 - 2*d*e + 3*e**2)**3)) - 3533954480*e**13)/(11
01474866245*d**12*e - 9024487794180*d**11*e**2 + 5764879624590*d**10*e**3 +
 17969136971220*d**9*e**4 - 16485388615365*d**8*e**5 - 12221510721480*d**7*
e**6 + 21212253502020*d**6*e**7 - 11710335235320*d**5*e**8 + 3048287389995*

d**4*e**9 - 183650820660*d**3*e**10 - 118302770610*d**2*e**11 + 34222696740
*d*e**12 - 3445820555*e**13)) - (1680*d**4 + 3467*d**3*e + 674*d**2*e**2 -
1123*d*e**3 + 840*e**4 + x**2*(2800*d**4 + 3500*d**3*e + 2523*d**2*e**2 - 3
434*d*e**3 + 1693*e**4) + x*(1120*d**4 + 1823*d**3*e - 527*d**2*e**2 - 573*
d*e**3 - 143*e**4))/(10500*d**5*e - 8400*d**4*e**2 + 14280*d**3*e**3 - 5040
*d**2*e**4 + 3780*d*e**5 + x**3*(17500*d**4*e**2 - 14000*d**3*e**3 + 23800*
d**2*e**4 - 8400*d*e**5 + 6300*e**6) + x**2*(17500*d**5*e - 7000*d**4*e**2
+ 18200*d**3*e**3 + 1120*d**2*e**4 + 2940*d*e**5 + 2520*e**6) + x*(7000*d**
5*e + 4900*d**4*e**2 + 1120*d**3*e**3 + 10920*d**2*e**4 - 2520*d*e**5 + 378
0*e**6)) + (41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)*log(x +
(4503590000000*d**17*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e*
*4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 - 79236430000000*d**16*e*(41*d**4 - 8*d
3*e - 60*d2*e**2 + 24*d*e**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6
+ 219307065600000*d**15*e**2*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3
- 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 + 1477177520000*d**15*(41*d**4 -
8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e + 3*e**2)**3
- 470102080000000*d**14*e**3*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**
3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 - 8062738222000*d**14*e*(41*d**
4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e + 3*e**2)
3 + 669820607680000*d13*e**4*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*
e**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 + 13401619991200*d**13*e**2*
(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e +
3*e**2)**3 + 132446413125*d**13 - 748279970905600*d**12*e**5*(41*d**4 - 8*d
3*e - 60*d2*e**2 + 24*d*e**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6
- 8564369003120*d**12*e**3*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 -
5*e**4)/(5*d**2 - 2*d*e + 3*e**2)**3 + 684029295980*d**12*e + 599319595212
800*d**11*e**6*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)**2/
(5*d**2 - 2*d*e + 3*e**2)**6 - 12510243478208*d**11*e**4*(41*d**4 - 8*d**3*
e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e + 3*e**2)**3 + 76420
6623630*d**11*e**2 - 291411662710784*d**10*e**7*(41*d**4 - 8*d**3*e - 60*d*
*2*e**2 + 24*d*e**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 + 49159980986
704*d**10*e**5*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*
d**2 - 2*d*e + 3*e**2)**3 - 14657220189100*d**10*e**3 - 27190445185792*d**9
*e**8*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)**2/(5*d**2 -
2*d*e + 3*e**2)**6 - 77659175364512*d**9*e**6*(41*d**4 - 8*d**3*e - 60*d**
2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e + 3*e**2)**3 + 16942805253691*
d**9*e**4 + 253830846834432*d**8*e**9*(41*d**4 - 8*d**3*e - 60*d**2*e**2 +
24*d*e**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 + 91313688339216*d**8*e
7*(41*d4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*
e + 3*e**2)**3 + 6404919470120*d**8*e**5 - 308064129587200*d**7*e**10*(41*d
4 - 8*d3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*
e**2)**6 - 78573287795968*d**7*e**8*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24
*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e + 3*e**2)**3 - 16998879119292*d**7*e**6 +
262468005502976*d**6*e**11*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3
- 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 + 55676827575152*d**6*e**9*(41*d*

```
*4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e + 3*e**2
)**3 + 5633839731848*d**6*e**7 - 162086347196928*d**5*e**12*(41*d**4 - 8*d*
*3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 -
 30431528150688*d**5*e**10*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 -
 5*e**4)/(5*d**2 - 2*d*e + 3*e**2)**3 + 3033254622763*d**5*e**8 + 822366320
99328*d**4*e**13*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)**
2/(5*d**2 - 2*d*e + 3*e**2)**6 + 13587008752688*d**4*e**11*(41*d**4 - 8*d**
3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e + 3*e**2)**3 - 350
6827379684*d**4*e**9 - 30865482805248*d**3*e**14*(41*d**4 - 8*d**3*e - 60*d
**2*e**2 + 24*d*e**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 - 4535008734
144*d**3*e**12*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*
d**2 - 2*d*e + 3*e**2)**3 + 1484229456462*d**3*e**10 + 9233948989440*d**2*e
**15*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)**2/(5*d**2 -
2*d*e + 3*e**2)**6 + 1144385029872*d**2*e**13*(41*d**4 - 8*d**3*e - 60*d**2
*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e + 3*e**2)**3 - 361088969436*d**
2*e**11 - 1739174903424*d*e**16*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e
**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 - 187156660320*d*e**14*(41*d*
*4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2*d*e + 3*e**2
)**3 + 50336842869*d*e**12 + 196869004416*e**17*(41*d**4 - 8*d**3*e - 60*d*
*2*e**2 + 24*d*e**3 - 5*e**4)**2/(5*d**2 - 2*d*e + 3*e**2)**6 + 17373868848
*e**15*(41*d**4 - 8*d**3*e - 60*d**2*e**2 + 24*d*e**3 - 5*e**4)/(5*d**2 - 2
*d*e + 3*e**2)**3 - 3533954480*e**13)/(1101474866245*d**12*e - 902448779418
0*d**11*e**2 + 5764879624590*d**10*e**3 + 17969136971220*d**9*e**4 - 164853
88615365*d**8*e**5 - 12221510721480*d**7*e**6 + 21212253502020*d**6*e**7 -
11710335235320*d**5*e**8 + 3048287389995*d**4*e**9 - 183650820660*d**3*e**1
0 - 118302770610*d**2*e**11 + 34222696740*d*e**12 - 3445820555*e**13))/(5*d
**2 - 2*d*e + 3*e**2)**3
```

Giac [A] time = 1.22489, size = 771, normalized size = 2.46

$$\frac{\sqrt{14}\left(1313\,d^4e^2-10044\,d^3e^3+4290\,d^2e^4+156\,de^5-271\,e^6\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\left(5\,d-\frac{5\,d^2}{xe+d}+\frac{2\,de}{xe+d}-\frac{3\,e^2}{xe+d}-e\right)e^{(-1)}\right)e^{(-2)}}{392\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}-\frac{(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 2
71*e^6)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(x*e + d) + 2*d*e/(x*e + d) - 3*e
^2/(x*e + d) - e)*e^(-1))*e^(-2)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d
```

```
^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 60*d^2*
e^2 + 24*d*e^3 - 5*e^4)*log(-10*d/(x*e + d) + 5*d^2/(x*e + d)^2 + 2*e/(x*e
+ d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(125*d^6 - 150*d^5*e + 28
5*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - (4*d^4*e^3/(x*
e + d) + 5*d^3*e^4/(x*e + d) + 3*d^2*e^5/(x*e + d) - d*e^6/(x*e + d) + 2*e^
7/(x*e + d))/(25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8) + 1/
28*((423*d^3*e - 4101*d^2*e^2 + 879*d*e^3 + 703*e^4)/(5*d^2 - 2*d*e + 3*e^2
) - (423*d^4*e^2 - 5468*d^3*e^3 + 1758*d^2*e^4 + 2812*d*e^5 - 457*e^6)*e^(-
1)/((5*d^2 - 2*d*e + 3*e^2)*(x*e + d)))/((5*d^2 - 2*d*e + 3*e^2)^2*(10*d/(x
*e + d) - 5*d^2/(x*e + d)^2 - 2*e/(x*e + d) + 2*d*e/(x*e + d)^2 - 3*e^2/(x*
e + d)^2 - 5))
```

3.317 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=412

$$-\frac{x\left(-4101d^2e+423d^3+879de^2+703e^3\right)-879d^2e+1367d^3-2109de^2+457e^3}{28\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}-\frac{\left(-846d^3e^2+396d^2e^3-19d^4e+205d\right.}{2\left(5d^2-2a\right.}$$

```
[Out] -(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e*(5*d^2 - 2*d*e + 3*e^2)
^2*(d + e*x)^2) - (41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)/((5*d^
2 - 2*d*e + 3*e^2)^3*(d + e*x)) - (1367*d^3 - 879*d^2*e - 2109*d*e^2 + 457*
e^3 + (423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*x)/(28*(5*d^2 - 2*d*e +
3*e^2)^3*(3 + 2*x + 5*x^2)) + ((6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42
858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(28*Sqrt[1
4]*(5*d^2 - 2*d*e + 3*e^2)^4) + ((205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^
2*e^3 + 57*d*e^4 - 21*e^5)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^4 - ((205*
d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[3 + 2*x
 + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rubi [A] time = 0.714754, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{x\left(-4101d^2e+423d^3+879de^2+703e^3\right)-879d^2e+1367d^3-2109de^2+457e^3}{28\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}-\frac{\left(-846d^3e^2+396d^2e^3-19d^4e+205d\right.}{2\left(5d^2-2a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2),x]
```



```
[Out] -(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e*(5*d^2 - 2*d*e + 3*e^2)
^2*(d + e*x)^2) - (41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)/((5*d^
2 - 2*d*e + 3*e^2)^3*(d + e*x)) - (1367*d^3 - 879*d^2*e - 2109*d*e^2 + 457*
e^3 + (423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*x)/(28*(5*d^2 - 2*d*e +
3*e^2)^3*(3 + 2*x + 5*x^2)) + ((6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42
858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(28*Sqrt[1
4]*(5*d^2 - 2*d*e + 3*e^2)^4) + ((205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^
2*e^3 + 57*d*e^4 - 21*e^5)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^4 - ((205*
d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[3 + 2*x
```

```
+ 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}dx = -\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}+\frac{1}{56}$$

$$= -\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}+\frac{1}{56}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^3-87}{}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^3-87}{}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^3-87}{}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^3-87}{}$$

Mathematica [A] time = 0.370097, size = 363, normalized size = 0.88

$$-\frac{\frac{14\left(5d^2-2de+3e^2\right)\left(-3d^2e(1367x+293)+d^3(423x+1367)+3de^2(293x-703)+e^3(703x+457)\right)}{5x^2+2x+3}+196\left(846d^3e^2-396d^2e^3+19d^4e-205d^5-57de^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-196*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e
^4))/(e*(d + e*x)^2) + (392*(5*d^2 - 2*d*e + 3*e^2)*(-41*d^4 + 8*d^3*e + 60
*d^2*e^2 - 24*d*e^3 + 5*e^4))/(d + e*x) - (14*(5*d^2 - 2*d*e + 3*e^2)*(3*d*
e^2*(-703 + 293*x) + d^3*(1367 + 423*x) + e^3*(457 + 703*x) - 3*d^2*e*(293
+ 1367*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d^5 - 74017*d^4*e + 35022*d^
```

```
3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] +
 392*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*L
og[d + e*x] + 196*(-205*d^5 + 19*d^4*e + 846*d^3*e^2 - 396*d^2*e^3 - 57*d*e
^4 + 21*e^5)*Log[3 + 2*x + 5*x^2])/(392*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Maple [B] time = 0.073, size = 1314, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x)
```

```
[Out] 57/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d*e^4-19/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)
*d^4*e-846/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^3*e^2+205/(5*d^2-2*d*e+3*e^2)^
4*ln(e*x+d)*d^5-21/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*e^5-1367/28/(5*d^2-2*d*e
+3*e^2)^4/(x^2+2/5*x+3/5)*d^5-1371/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5
)*e^5-205/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^5+21/2/(5*d^2-2*d*e+3*e
^2)^4*ln(5*x^2+2*x+3)*e^5-1/(5*d^2-2*d*e+3*e^2)^2*e^3/(e*x+d)^2-41/(5*d^2-2
*d*e+3*e^2)^3/(e*x+d)*d^4+5/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*e^4-5/2/(5*d^2-2*
d*e+3*e^2)^2/(e*x+d)^2*d^3+21351/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*
x*d^4*e-6933/70/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^3*e^2+5273/70/(5*
d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^2*e^3+21429/196/(5*d^2-2*d*e+3*e^2)^
4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2*e^3+1/2/(5*d^2-2*d*e+3*e^2)^2
*e^2/(e*x+d)^2*d+8/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^3*e+60/(5*d^2-2*d*e+3*e^
2)^3/(e*x+d)*d^2*e^2-24/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d*e^3-423/28/(5*d^2-2
*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^5*x-2109/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5
*x+3/5)*x*e^5+7129/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^4*e+2343/70/
(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^3*e^2-1933/70/(5*d^2-2*d*e+3*e^2)^4
/(x^2+2/5*x+3/5)*d^2*e^3+7241/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d*e
^4+19/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^4*e+423/(5*d^2-2*d*e+3*e^2)
^4*ln(5*x^2+2*x+3)*d^3*e^2+396/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^2*e^3-1724
7/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^4-1
231/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d*e^4-74017/392/(5*d^2-2*d*
e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^4*e+17511/196/(5*d^2-2
*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3*e^2-198/(5*d^2-2*
d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^2*e^3-57/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*
x+3)*d*e^4+6565/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^
(1/2))*d^5+579/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(
1/2))*e^5-2/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)^2*d^4-3/2/(5*d^2-2*d*e+3*e^2)^2
*e/(e*x+d)^2*d^2
```

Maxima [B] time = 1.66047, size = 1149, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) + (205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57
*d*e^4 - 21*e^5)*log(e*x + d)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d
^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) -
1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*lo
g(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 20
86*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/28*(840*d
^6 + 5525*d^5*e - 837*d^4*e^2 - 6981*d^3*e^3 + 3355*d^2*e^4 - 714*d*e^5 + 2
52*e^6 + (5740*d^4*e^2 - 697*d^3*e^3 - 12501*d^2*e^4 + 4239*d*e^5 + 3*e^6)*
x^3 + (1400*d^6 + 6930*d^5*e + 3212*d^4*e^2 - 15403*d^3*e^3 + 2349*d^2*e^4
- 549*d*e^5 + 597*e^6)*x^2 + (560*d^6 + 3195*d^5*e + 2105*d^4*e^2 - 4799*d^
3*e^3 - 6623*d^2*e^4 + 2454*d*e^5 - 252*e^6)*x)/(375*d^8*e - 450*d^7*e^2 +
855*d^6*e^3 - 564*d^5*e^4 + 513*d^4*e^5 - 162*d^3*e^6 + 81*d^2*e^7 + 5*(125
*d^6*e^3 - 150*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8
 + 27*e^9)*x^4 + 2*(625*d^7*e^2 - 625*d^6*e^3 + 1275*d^5*e^4 - 655*d^4*e^5
+ 667*d^3*e^6 - 99*d^2*e^7 + 81*d*e^8 + 27*e^9)*x^3 + (625*d^8*e - 250*d^7*
e^2 + 1200*d^6*e^3 - 250*d^5*e^4 + 958*d^4*e^5 - 150*d^3*e^6 + 432*d^2*e^7
- 54*d*e^8 + 81*e^9)*x^2 + 2*(125*d^8*e + 225*d^7*e^2 - 165*d^6*e^3 + 667*d
^5*e^4 - 393*d^4*e^5 + 459*d^3*e^6 - 135*d^2*e^7 + 81*d*e^8)*x)
```

Fricas [B] time = 2.90437, size = 3853, normalized size = 9.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/392*(58800*d^8 + 363230*d^7*e - 178010*d^6*e^2 - 233184*d^5*e^3 + 395164
*d^4*e^4 - 437122*d^3*e^5 + 178542*d^2*e^6 - 37044*d*e^7 + 10584*e^8 + 14*(
28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 45966*d^2*e
^6 + 12711*d*e^7 + 9*e^8)*x^3 + 14*(7000*d^8 + 31850*d^7*e + 6400*d^6*e^2 -
 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*d^2*e^6 - 2841*d*e^7
 + 1791*e^8)*x^2 - sqrt(14)*(19695*d^7*e - 222051*d^6*e^2 + 105066*d^5*e^3
+ 128574*d^4*e^4 - 51741*d^3*e^5 + 1737*d^2*e^6 + 5*(6565*d^5*e^3 - 74017*d
^4*e^4 + 35022*d^3*e^5 + 42858*d^2*e^6 - 17247*d*e^7 + 579*e^8)*x^4 + 2*(32
825*d^6*e^2 - 363520*d^5*e^3 + 101093*d^4*e^4 + 249312*d^3*e^5 - 43377*d^2*
e^6 - 14352*d*e^7 + 579*e^8)*x^3 + (32825*d^7*e - 343825*d^6*e^2 - 101263*d
^5*e^3 + 132327*d^4*e^4 + 190263*d^3*e^5 + 62481*d^2*e^6 - 49425*d*e^7 + 17
37*e^8)*x^2 + 2*(6565*d^7*e - 54322*d^6*e^2 - 187029*d^5*e^3 + 147924*d^4*e
^4 + 111327*d^3*e^5 - 51162*d^2*e^6 + 1737*d*e^7)*x)*arctan(1/14*sqrt(14)*(
5*x + 1)) + 14*(2800*d^8 + 14855*d^7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 172
02*d^4*e^4 + 11119*d^3*e^5 - 26037*d^2*e^6 + 7866*d*e^7 - 756*e^8)*x - 392*
(615*d^7*e - 57*d^6*e^2 - 2538*d^5*e^3 + 1188*d^4*e^4 + 171*d^3*e^5 - 63*d^
2*e^6 + 5*(205*d^5*e^3 - 19*d^4*e^4 - 846*d^3*e^5 + 396*d^2*e^6 + 57*d*e^7
- 21*e^8)*x^4 + 2*(1025*d^6*e^2 + 110*d^5*e^3 - 4249*d^4*e^4 + 1134*d^3*e^5
 + 681*d^2*e^6 - 48*d*e^7 - 21*e^8)*x^3 + (1025*d^7*e + 725*d^6*e^2 - 3691*
d^5*e^3 - 1461*d^4*e^4 - 669*d^3*e^5 + 1311*d^2*e^6 + 87*d*e^7 - 63*e^8)*x^
2 + 2*(205*d^7*e + 596*d^6*e^2 - 903*d^5*e^3 - 2142*d^4*e^4 + 1245*d^3*e^5
+ 150*d^2*e^6 - 63*d*e^7)*x)*log(e*x + d) + 196*(615*d^7*e - 57*d^6*e^2 - 2
538*d^5*e^3 + 1188*d^4*e^4 + 171*d^3*e^5 - 63*d^2*e^6 + 5*(205*d^5*e^3 - 19
*d^4*e^4 - 846*d^3*e^5 + 396*d^2*e^6 + 57*d*e^7 - 21*e^8)*x^4 + 2*(1025*d^6
*e^2 + 110*d^5*e^3 - 4249*d^4*e^4 + 1134*d^3*e^5 + 681*d^2*e^6 - 48*d*e^7 -
 21*e^8)*x^3 + (1025*d^7*e + 725*d^6*e^2 - 3691*d^5*e^3 - 1461*d^4*e^4 - 66
9*d^3*e^5 + 1311*d^2*e^6 + 87*d*e^7 - 63*e^8)*x^2 + 2*(205*d^7*e + 596*d^6*
e^2 - 903*d^5*e^3 - 2142*d^4*e^4 + 1245*d^3*e^5 + 150*d^2*e^6 - 63*d*e^7)*x
)*log(5*x^2 + 2*x + 3))/(1875*d^10*e - 3000*d^9*e^2 + 6300*d^8*e^3 - 5880*d
^7*e^4 + 6258*d^6*e^5 - 3528*d^5*e^6 + 2268*d^4*e^7 - 648*d^3*e^8 + 243*d^2
*e^9 + 5*(625*d^8*e^3 - 1000*d^7*e^4 + 2100*d^6*e^5 - 1960*d^5*e^6 + 2086*d
^4*e^7 - 1176*d^3*e^8 + 756*d^2*e^9 - 216*d*e^10 + 81*e^11)*x^4 + 2*(3125*d
^9*e^2 - 4375*d^8*e^3 + 9500*d^7*e^4 - 7700*d^6*e^5 + 8470*d^5*e^6 - 3794*d
^4*e^7 + 2604*d^3*e^8 - 324*d^2*e^9 + 189*d*e^10 + 81*e^11)*x^3 + (3125*d^1
0*e - 2500*d^9*e^2 + 8375*d^8*e^3 - 4400*d^7*e^4 + 8890*d^6*e^5 - 3416*d^5*
e^6 + 5334*d^4*e^7 - 1584*d^3*e^8 + 1809*d^2*e^9 - 324*d*e^10 + 243*e^11)*x
^2 + 2*(625*d^10*e + 875*d^9*e^2 - 900*d^8*e^3 + 4340*d^7*e^4 - 3794*d^6*e^
5 + 5082*d^5*e^6 - 2772*d^4*e^7 + 2052*d^3*e^8 - 567*d^2*e^9 + 243*d*e^10)*
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25734, size = 803, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3
+ 57*d*e^4 - 21*e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*
e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
+ 81*e^8) + (205*d^5*e - 19*d^4*e^2 - 846*d^3*e^3 + 396*d^2*e^4 + 57*d*e^5
- 21*e^6)*log(abs(x*e + d))/(625*d^8*e - 1000*d^7*e^2 + 2100*d^6*e^3 - 1960
*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*d^2*e^7 - 216*d*e^8 + 81*e^9)
- 1/28*(4200*d^8 + 25945*d^7*e - 12715*d^6*e^2 - 16656*d^5*e^3 + 28226*d^4*
e^4 + (28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 4596
6*d^2*e^6 + 12711*d*e^7 + 9*e^8)*x^3 - 31223*d^3*e^5 + (7000*d^8 + 31850*d^
7*e + 6400*d^6*e^2 - 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*
d^2*e^6 - 2841*d*e^7 + 1791*e^8)*x^2 + 12753*d^2*e^6 + (2800*d^8 + 14855*d^
7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 17202*d^4*e^4 + 11119*d^3*e^5 - 26037*
d^2*e^6 + 7866*d*e^7 - 756*e^8)*x - 2646*d*e^7 + 756*e^8)*e^(-1)/((5*d^2 -
2*d*e + 3*e^2)^4*(5*x^2 + 2*x + 3)*(x*e + d)^2)
```

3.318 $\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$

Optimal. Leaf size=171

$$\frac{3e\left(100d^2-245de+47e^2\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{3\left(-855175d^2e+353125d^3+74085de^2+556349e^3\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{4900000\sqrt{14}}+\frac{e^{}}{}$$

```
[Out] ((83065*d - 126009*e)*e^2*x)/980000 + (2*e^3*x^2)/125 - ((1367 + 423*x)*(d
+ e*x)^3)/(7000*(3 + 2*x + 5*x^2)^2) + ((d + e*x)^2*(3*(11449*d - 2105*e) +
 (11015*d + 49177*e)*x))/(196000*(3 + 2*x + 5*x^2)) + (3*(353125*d^3 - 8551
75*d^2*e + 74085*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(4900000*S
qrt[14]) + (3*e*(100*d^2 - 245*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rubi [A] time = 0.336016, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1628, 634, 618, 204, 628}

$$\frac{3e\left(100d^2-245de+47e^2\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{3\left(-855175d^2e+353125d^3+74085de^2+556349e^3\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{4900000\sqrt{14}}+\frac{e^{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((83065*d - 126009*e)*e^2*x)/980000 + (2*e^3*x^2)/125 - ((1367 + 423*x)*(d
+ e*x)^3)/(7000*(3 + 2*x + 5*x^2)^2) + ((d + e*x)^2*(3*(11449*d - 2105*e) +
 (11015*d + 49177*e)*x))/(196000*(3 + 2*x + 5*x^2)) + (3*(353125*d^3 - 8551
75*d^2*e + 74085*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(4900000*S
qrt[14]) + (3*e*(100*d^2 - 245*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = -\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2}+\frac{1}{112}\int \frac{(d+ex)^2\left(\frac{6}{125}(1089d+1367e)-\frac{336}{125}(55\right.}{(3+2x}$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2}+\frac{(d+ex)^2(3(11449d-2105e)+(11015d+4917}{196000(3+2x+5x^2)}$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2}+\frac{(d+ex)^2(3(11449d-2105e)+(11015d+4917}{196000(3+2x+5x^2)}$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2}+\frac{(d+ex)^2(3(1}{}$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2}+\frac{(d+ex)^2(3(1}{}$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2}+\frac{(d+ex)^2(3(1}{}$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2}+\frac{(d+ex)^2(3(1}{}$$

Mathematica [A] time = 0.20124, size = 209, normalized size = 1.22

$$-\frac{392\left(75d^2e(5989x-1269)+125d^3(423x+1367)-15de^2(18323x+17967)+e^3(54969-53189x)\right)}{(5x^2+2x+3)^2}+\frac{14\left(75d^2e(181765x-44399)+125d^3(11015x+34347)-15de^2(647\right.}{5x^2+2x+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]
```

```
[Out] (548800*(60*d - 49*e)*e^2*x + 5488000*e^3*x^2 - (392*(e^3*(54969 - 53189*x)
 + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x) - 15*d*e^2*(17967 + 1
8323*x)))/(3 + 2*x + 5*x^2)^2 + (14*(e^3*(2639639 - 3109005*x) + 125*d^3*(3
4347 + 11015*x) + 75*d^2*e*(-44399 + 181765*x) - 15*d*e^2*(809167 + 647195*
x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(353125*d^3 - 855175*d^2*e + 74085*d*e
```

```
^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 164640*e*(100*d^2 - 245*d*e +
 47*e^2)*Log[3 + 2*x + 5*x^2])/343000000
```

Maple [A] time = 0.059, size = 267, normalized size = 1.6

$$\frac{2\,e^3x^2}{125}+\frac{12\,xde^2}{125}-\frac{49\,e^3x}{625}+\frac{1}{25\,\left(5\,x^2+2\,x+3\right)^2}\left(\left(\left(\frac{11015\,d^3}{1568}+\frac{109059\,d^2e}{1568}-\frac{388317\,de^2}{7840}-\frac{621801\,e^3}{39200}\right)x^3+\left(\frac{38753\,d^3}{1568}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 2/125*e^3*x^2+12/125*x*d*e^2-49/625*e^3*x+1/25*((11015/1568*d^3+109059/1568
*d^2*e-388317/7840*d*e^2-621801/39200*e^3)*x^3+(38753/1568*d^3+84921/7840*d
^2*e-640827/7840*d*e^2+1396037/196000*e^3)*x^2+(17979/1568*d^3+173283/7840*
d^2*e-73125/1568*d*e^2-511689/196000*e^3)*x+12953/1568*d^3-58599/7840*d^2*e
-230931/7840*d*e^2+1275957/196000*e^3)/(5*x^2+2*x+3)^2+6/125*ln(5*x^2+2*x+3
)*d^2*e-147/1250*ln(5*x^2+2*x+3)*d*e^2+141/6250*ln(5*x^2+2*x+3)*e^3+339/219
52*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3-102621/2744000*14^(1/2)*arct
an(1/28*(10*x+2)*14^(1/2))*d^2*e+44451/13720000*14^(1/2)*arctan(1/28*(10*x+
2)*14^(1/2))*d*e^2+1669047/68600000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
*e^3
```

Maxima [A] time = 1.55006, size = 300, normalized size = 1.75

$$\frac{2}{125}\,e^3x^2+\frac{3}{68600000}\,\sqrt{14}\left(353125\,d^3-855175\,d^2e+74085\,de^2+556349\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{1}{625}\,\left(60\,de^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 2/125*e^3*x^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d^2*e + 74085*d*e^
2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(60*d*e^2 - 49*e^3)
*x + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/49000
00*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 16191
25*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3 + 21230
```

```
25*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 1444025*d^2*
e - 3046875*d*e^2 - 170563*e^3)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Fricas [B] time = 1.25751, size = 1339, normalized size = 7.83

$$\frac{27440000\,e^3x^6 + 2744000\left(60\,de^2 - 41\,e^3\right)x^5 + 8780800\left(15\,de^2 - 8\,e^3\right)x^4 + 70\left(275375\,d^3 + 2726475\,d^2e + 1257135\,de^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/68600000*(27440000*e^3*x^6 + 2744000*(60*d*e^2 - 41*e^3)*x^5 + 8780800*(1
5*d*e^2 - 8*e^3)*x^4 + 70*(275375*d^3 + 2726475*d^2*e + 1257135*d*e^2 - 304
5929*e^3)*x^3 + 22667750*d^3 - 20509650*d^2*e - 80825850*d*e^2 + 17863398*e
^3 + 14*(4844125*d^3 + 2123025*d^2*e - 10375875*d*e^2 - 2508283*e^3)*x^2 +
3*sqrt(14)*(25*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^4 +
 20*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^3 + 3178125*d^
3 - 7696575*d^2*e + 666765*d*e^2 + 5007141*e^3 + 34*(353125*d^3 - 855175*d^
2*e + 74085*d*e^2 + 556349*e^3)*x^2 + 12*(353125*d^3 - 855175*d^2*e + 74085
*d*e^2 + 556349*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(749125*d^3 +
1444025*d^2*e - 1635675*d*e^2 - 1323043*e^3)*x + 32928*(25*(100*d^2*e - 245
*d*e^2 + 47*e^3)*x^4 + 20*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^3 + 900*d^2*e
- 2205*d*e^2 + 423*e^3 + 34*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^2 + 12*(100*
d^2*e - 245*d*e^2 + 47*e^3)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9)
```

Sympy [C] time = 5.28771, size = 469, normalized size = 2.74

$$\frac{2e^3x^2}{125} + x\left(\frac{12de^2}{125} - \frac{49e^3}{625}\right) + \left(\frac{3e\left(100d^2 - 245de + 47e^2\right)}{6250} - \frac{3\sqrt{14}i\left(353125d^3 - 855175d^2e + 74085de^2 + 556349e^3\right)}{137200000}\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 2*e**3*x**2/125 + x*(12*d*e**2/125 - 49*e**3/625) + (3*e*(100*d**2 - 245*d*
e + 47*e**2)/6250 - 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**
2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3271395
*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 - 3*sqrt(1
4)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1059375
*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (3*e*(100*d**2 -
245*d*e + 47*e**2)/6250 + 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085
*d*e**2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3
271395*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 + 3*
sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1
059375*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (1619125*d*
*3 - 1464975*d**2*e - 5773275*d*e**2 + 1275957*e**3 + x**3*(1376875*d**3 +
13632375*d**2*e - 9707925*d*e**2 - 3109005*e**3) + x**2*(4844125*d**3 + 212
3025*d**2*e - 16020675*d*e**2 + 1396037*e**3) + x*(2247375*d**3 + 4332075*d
**2*e - 9140625*d*e**2 - 511689*e**3))/(122500000*x**4 + 98000000*x**3 + 16
6600000*x**2 + 58800000*x + 44100000)
```

Giac [A] time = 1.14852, size = 271, normalized size = 1.58

$$\frac{2}{125}\,x^2e^3+\frac{12}{125}\,dxe^2+\frac{3}{68600000}\,\sqrt{14}\left(353125\,d^3-855175\,d^2e+74085\,de^2+556349\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)-\frac{49}{62}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 2/125*x^2*e^3 + 12/125*d*x*e^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d
^2*e + 74085*d*e^2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) - 49/625*x
*e^3 + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/490
0000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 161
9125*d^3 + (4844125*d^3 + 2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2
 - 1464975*d^2*e + 3*(749125*d^3 + 1444025*d^2*e - 3046875*d*e^2 - 170563*e
^3)*x - 5773275*d*e^2 + 1275957*e^3)/(5*x^2 + 2*x + 3)^2
```

3.319 $\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$

Optimal. Leaf size=134

$$\frac{\left(211875d^2-342070de+14817e^2\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{980000\sqrt{14}}+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{196000\left(5x^2+2x+3\right)}-\frac{(423x+1367)(d}{7000\left(5x^2+2x\right.}$$

```
[Out] (4*e^2*x)/125 - ((1367 + 423*x)*(d + e*x)^2)/(7000*(3 + 2*x + 5*x^2)^2) + (
(d + e*x)*(34347*d - 6413*e + 5*(2203*d + 8553*e)*x))/(196000*(3 + 2*x + 5*
x^2)) + ((211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(980000*Sqrt[14]) + ((40*d - 49*e)*e*Log[3 + 2*x + 5*x^2])/1250
```

Rubi [A] time = 0.238742, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1657, 634, 618, 204, 628}

$$\frac{\left(211875d^2-342070de+14817e^2\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{980000\sqrt{14}}+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{196000\left(5x^2+2x+3\right)}-\frac{(423x+1367)(d}{7000\left(5x^2+2x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] (4*e^2*x)/125 - ((1367 + 423*x)*(d + e*x)^2)/(7000*(3 + 2*x + 5*x^2)^2) + (
(d + e*x)*(34347*d - 6413*e + 5*(2203*d + 8553*e)*x))/(196000*(3 + 2*x + 5*
x^2)) + ((211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(980000*Sqrt[14]) + ((40*d - 49*e)*e*Log[3 + 2*x + 5*x^2])/1250
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
```

```
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = -\frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{1}{112}\int \frac{(d+ex)\left(\frac{2}{125}(3267d+2734e)-\frac{6}{125}(308\right.}{(3+2}$$

$$= -\frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+8553e)x}{196000(3+2x+5x^2)}$$

$$= -\frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+8553e)x}{196000(3+2x+5x^2)}$$

$$= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+85}{196000(3+2x+5x^2)}$$

$$= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+85}{196000(3+2x+5x^2)}$$

$$= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+85}{196000(3+2x+5x^2)}$$

$$= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+85}{196000(3+2x+5x^2)}$$

Mathematica [A] time = 0.176357, size = 146, normalized size = 1.09

$$\frac{70\left(\frac{5\left(5d^2\left(11015x^3+38753x^2+17979x+12953\right)+2de\left(181765x^3+28307x^2+57761x-19533\right)+e^2\left(156800x^5+125440x^4+83809x^3-138345x^2-65427x-76977\right)\right)}{\left(5x^2+2x+3\right)^2}+\right.}{68600000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]
```

```
[Out] (5*Sqrt[14]*(211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]
] + 70*((5*(5*d^2*(12953 + 17979*x + 38753*x^2 + 11015*x^3) + 2*d*e*(-19533
 + 57761*x + 28307*x^2 + 181765*x^3) + e^2*(-76977 - 65427*x - 138345*x^2 +
 83809*x^3 + 125440*x^4 + 156800*x^5)))/(3 + 2*x + 5*x^2)^2 + 784*(40*d - 4
9*e)*e*Log[3 + 2*x + 5*x^2]))/68600000
```

Maple [A] time = 0.053, size = 179, normalized size = 1.3

$$\frac{4\,e^{2}x}{125}+\frac{1}{5\left(5\,x^{2}+2\,x+3\right)^{2}}\left(\left(\frac{2203\,d^{2}}{1568}+\frac{36353\,de}{3920}-\frac{129439\,e^{2}}{39200}\right)x^{3}+\left(\frac{38753\,d^{2}}{7840}+\frac{28307\,de}{19600}-\frac{213609\,e^{2}}{39200}\right)x^{2}+\left(\frac{17979\,d}{7840}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 4/125*e^2*x+1/5*((2203/1568*d^2+36353/3920*d*e-129439/39200*e^2)*x^3+(38753
/7840*d^2+28307/19600*d*e-213609/39200*e^2)*x^2+(17979/7840*d^2+57761/19600
*d*e-4875/1568*e^2)*x+12953/7840*d^2-19533/19600*d*e-76977/39200*e^2)/(5*x^
2+2*x+3)^2+4/125*ln(5*x^2+2*x+3)*d*e-49/1250*ln(5*x^2+2*x+3)*e^2+339/21952*
14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2-34207/1372000*14^(1/2)*arctan(1
/28*(10*x+2)*14^(1/2))*d*e+14817/13720000*14^(1/2)*arctan(1/28*(10*x+2)*14^
(1/2))*e^2
```

Maxima [A] time = 1.54316, size = 209, normalized size = 1.56

$$\frac{4}{125}\,e^{2}x+\frac{1}{13720000}\,\sqrt{14}\left(211875\,d^{2}-342070\,de+14817\,e^{2}\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{1}{1250}\left(40\,de-49\,e^{2}\right)\log\left(5\,x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 4/125*e^2*x + 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arc
tan(1/14*sqrt(14)*(5*x + 1)) + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 - 39066*d*e - 76977*e^2 + (89895*d^2 +
 115522*d*e - 121875*e^2)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Fricas [B] time = 1.25355, size = 933, normalized size = 6.96

$$10976000\,e^{2}x^{5}+8780800\,e^{2}x^{4}+70\left(55075\,d^{2}+363530\,de+83809\,e^{2}\right)x^{3}+70\left(193765\,d^{2}+56614\,de-138345\,e^{2}\right)x^{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/13720000*(10976000*e^2*x^5 + 8780800*e^2*x^4 + 70*(55075*d^2 + 363530*d*e
 + 83809*e^2)*x^3 + 70*(193765*d^2 + 56614*d*e - 138345*e^2)*x^2 + sqrt(14)
*(25*(211875*d^2 - 342070*d*e + 14817*e^2)*x^4 + 20*(211875*d^2 - 342070*d*
e + 14817*e^2)*x^3 + 34*(211875*d^2 - 342070*d*e + 14817*e^2)*x^2 + 1906875
*d^2 - 3078630*d*e + 133353*e^2 + 12*(211875*d^2 - 342070*d*e + 14817*e^2)*
x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4533550*d^2 - 2734620*d*e - 5388390*e^
2 + 70*(89895*d^2 + 115522*d*e - 65427*e^2)*x + 10976*(25*(40*d*e - 49*e^2)
*x^4 + 20*(40*d*e - 49*e^2)*x^3 + 34*(40*d*e - 49*e^2)*x^2 + 360*d*e - 441*
e^2 + 12*(40*d*e - 49*e^2)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 + 34*x
^2 + 12*x + 9)
```

Sympy [C] time = 3.38472, size = 304, normalized size = 2.27

$$\frac{4e^2x}{125} + \left(\frac{e(40d-49e)}{1250} - \frac{\sqrt{14}i\left(211875d^2-342070de+14817e^2\right)}{27440000}\right)\log\left(x + \frac{42375d^2 - 244030de + 218093e^2 + \frac{21952e(40}{5}}{211875d^2 - 342070}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 4*e**2*x/125 + (e*(40*d - 49*e)/1250 - sqrt(14)*I*(211875*d**2 - 342070*d*e
 + 14817*e**2)/27440000)*log(x + (42375*d**2 - 244030*d*e + 218093*e**2 + 2
1952*e*(40*d - 49*e)/5 - sqrt(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)
/5)/(211875*d**2 - 342070*d*e + 14817*e**2)) + (e*(40*d - 49*e)/1250 + sqrt
(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)/27440000)*log(x + (42375*d**
2 - 244030*d*e + 218093*e**2 + 21952*e*(40*d - 49*e)/5 + sqrt(14)*I*(211875
*d**2 - 342070*d*e + 14817*e**2)/5)/(211875*d**2 - 342070*d*e + 14817*e**2)
) + (64765*d**2 - 39066*d*e - 76977*e**2 + x**3*(55075*d**2 + 363530*d*e -
129439*e**2) + x**2*(193765*d**2 + 56614*d*e - 213609*e**2) + x*(89895*d**2
 + 115522*d*e - 121875*e**2))/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 +
 2352000*x + 1764000)
```

<u>Giac</u> [A] time = 1.14012, size = 194, normalized size = 1.45

$$\frac{1}{13720000}\sqrt{14}\left(211875\,d^2-342070\,de+14817\,e^2\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)+\frac{4}{125}\,xe^2+\frac{1}{1250}\left(40\,de-49\,e^2\right)\log\left(5\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arctan(1/14*sqrt(
14)*(5*x + 1)) + 4/125*x*e^2 + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 + (89895*d^2 + 115522*d*e - 121875*e^2
)*x - 39066*d*e - 76977*e^2)/(5*x^2 + 2*x + 3)^2
```

3.320 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx$

Optimal. Leaf size=103

$$-\frac{(423x+1367)(d+ex)}{7000(5x^2+2x+3)^2}+\frac{x(11015d+36353e)+34347d-6511e}{196000(5x^2+2x+3)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}e\log(5x^2$$

```
[Out] -((1367 + 423*x)*(d + e*x))/(7000*(3 + 2*x + 5*x^2)^2) + (34347*d - 6511*e
+ (11015*d + 36353*e)*x)/(196000*(3 + 2*x + 5*x^2)) + ((42375*d - 34207*e)*
ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3 + 2*x + 5*x^2])/
125
```

Rubi [A] time = 0.145351, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1644, 1660, 634, 618, 204, 628}

$$-\frac{(423x+1367)(d+ex)}{7000(5x^2+2x+3)^2}+\frac{x(11015d+36353e)+34347d-6511e}{196000(5x^2+2x+3)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}e\log(5x^2$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```


```
[Out] -((1367 + 423*x)*(d + e*x))/(7000*(3 + 2*x + 5*x^2)^2) + (34347*d - 6511*e
+ (11015*d + 36353*e)*x)/(196000*(3 + 2*x + 5*x^2)) + ((42375*d - 34207*e)*
ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3 + 2*x + 5*x^2])/
125
```


Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
```

```
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}dx = -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{2}{125}(3267d+1367e)-\frac{12}{25}(308d-123e)x}{\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}+\frac{\int \frac{4}{25}(84}{}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}+\frac{(42375}{}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}+\frac{2}{125}e\,\mathrm{lo}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}+\frac{(42375}{}$$

Mathematica [A] time = 0.0804348, size = 107, normalized size = 1.04

$$\frac{-2115dx-6835d-5989ex+1269e}{35000\left(5x^2+2x+3\right)^2}+\frac{55075dx+171735d+181765ex-44399e}{980000\left(5x^2+2x+3\right)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}}+$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x
]
```

```
[Out] (-6835*d + 1269*e - 2115*d*x - 5989*e*x)/(35000*(3 + 2*x + 5*x^2)^2) + (171
735*d - 44399*e + 55075*d*x + 181765*e*x)/(980000*(3 + 2*x + 5*x^2)) + ((42
375*d - 34207*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3
 + 2*x + 5*x^2])/125
```

Maple [A] time = 0.053, size = 102, normalized size = 1.

$$25\frac{1}{\left(5x^2+2x+3\right)^2}\left(\left(\frac{36353e}{980000}+\frac{2203d}{196000}\right)x^3+\left(\frac{28307e}{4900000}+\frac{38753d}{980000}\right)x^2+\left(\frac{57761e}{4900000}+\frac{17979d}{980000}\right)x+\frac{12953d}{980000}-\frac{1953}{4900}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 25*((36353/980000*e+2203/196000*d)*x^3+(28307/4900000*e+38753/980000*d)*x^2
+(57761/4900000*e+17979/980000*d)*x+12953/980000*d-19533/4900000*e)/(5*x^2+
2*x+3)^2+2/125*e*ln(5*x^2+2*x+3)+339/21952*14^(1/2)*arctan(1/28*(10*x+2)*14
^(1/2))*d-34207/2744000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e
```

Maxima [A] time = 1.48458, size = 136, normalized size = 1.32

$$\frac{1}{2744000}\sqrt{14}(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)+\frac{5\,(11015\,d+36353\,e)x^3+(19\cdots}{196\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(25*x^4 + 20*
x^3 + 34*x^2 + 12*x + 9)
```

Fricas [A] time = 1.26263, size = 558, normalized size = 5.42

$$\frac{70\,(11015\,d+36353\,e)x^3+14\,(193765\,d+28307\,e)x^2+\sqrt{14}\Big(25\,(42375\,d-34207\,e)x^4+20\,(42375\,d-34207\,e)x^3+34\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2744000*(70*(11015*d + 36353*e)*x^3 + 14*(193765*d + 28307*e)*x^2 + sqrt(
14)*(25*(42375*d - 34207*e)*x^4 + 20*(42375*d - 34207*e)*x^3 + 34*(42375*d
- 34207*e)*x^2 + 12*(42375*d - 34207*e)*x + 381375*d - 307863*e)*arctan(1/1
4*sqrt(14)*(5*x + 1)) + 14*(89895*d + 57761*e)*x + 43904*(25*e*x^4 + 20*e*x
```

```
^3 + 34*e*x^2 + 12*e*x + 9*e)*log(5*x^2 + 2*x + 3) + 906710*d - 273462*e)/(
25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Sympy [C] time = 1.65198, size = 163, normalized size = 1.58

$$\left(\frac{2e}{125}-\frac{\sqrt{14}i\,(42375d-34207e)}{5488000}\right)\log\left(x+\frac{8475d-\frac{34207e}{5}-\frac{\sqrt{14}i(42375d-34207e)}{5}}{42375d-34207e}\right)+\left(\frac{2e}{125}+\frac{\sqrt{14}i\,(42375d-34207e)}{5488000}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (2*e/125 - sqrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*
e/5 - sqrt(14)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (2*e/125 + s
qrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*e/5 + sqrt(1
4)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (64765*d - 19533*e + x**
3*(55075*d + 181765*e) + x**2*(193765*d + 28307*e) + x*(89895*d + 57761*e))
/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 + 2352000*x + 1764000)
```

Giac [A] time = 1.16485, size = 131, normalized size = 1.27

$$\frac{1}{2744000}\sqrt{14}(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)+\frac{5\,(11015\,d+36353\,e)x^3+(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(5*x^2 + 2*x
+ 3)^2
```

3.321 $\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{423x+1367}{7000\left(5x^2+2x+3\right)^2}+\frac{11015x+34347}{196000\left(5x^2+2x+3\right)}+\frac{339\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

```
[Out] -(1367 + 423*x)/(7000*(3 + 2*x + 5*x^2)^2) + (34347 + 11015*x)/(196000*(3 +
 2*x + 5*x^2)) + (339*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt[14])
```

Rubi [A] time = 0.0497837, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1660, 12, 618, 204}

$$-\frac{423x+1367}{7000\left(5x^2+2x+3\right)^2}+\frac{11015x+34347}{196000\left(5x^2+2x+3\right)}+\frac{339\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```


```
[Out] -(1367 + 423*x)/(7000*(3 + 2*x + 5*x^2)^2) + (34347 + 11015*x)/(196000*(3 +
 2*x + 5*x^2)) + (339*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt[14])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx &= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{6534}{125}-\frac{3696x}{25}+\frac{448x^2}{5}}{\left(3+2x+5x^2\right)^2}\,dx \\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}+\frac{\int \frac{1356}{3+2x+5x^2}\,dx}{6272} \\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}+\frac{339\int \frac{1}{3+2x+5x^2}\,dx}{1568} \\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}-\frac{339}{784}\,\mathrm{Subst}\left(\int \frac{1}{-56-x^2}\,dx,x,2+ \\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}+\frac{339\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}}
\end{aligned}
$$

Mathematica [A] time = 0.0382764, size = 53, normalized size = 0.83

$$
\frac{\frac{14\left(11015x^3+38753x^2+17979x+12953\right)}{\left(5x^2+2x+3\right)^2}+8475\sqrt{14}\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{548800}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((14*(12953 + 17979*x + 38753*x^2 + 11015*x^3))/(3 + 2*x + 5*x^2)^2 + 8475*
Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]])/548800
```

Maple [A] time = 0.047, size = 47, normalized size = 0.7

$$25\,\frac{1}{\left(5\,x^2+2\,x+3\right)^2}\left(\frac{2203\,x^3}{196000}+\frac{38753\,x^2}{980000}+\frac{17979\,x}{980000}+\frac{12953}{980000}\right)+\frac{339\,\sqrt{14}}{21952}\arctan\left(\frac{(10\,x+2)\,\sqrt{14}}{28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 25*(2203/196000*x^3+38753/980000*x^2+17979/980000*x+12953/980000)/(5*x^2+2*
x+3)^2+339/21952*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Maxima [A] time = 1.50018, size = 76, normalized size = 1.19

$$\frac{339}{21952}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{11015\,x^3+38753\,x^2+17979\,x+12953}{39200\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="maxima")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Fricas [A] time = 1.29036, size = 243, normalized size = 3.8

$$\frac{154210\,x^3+8475\,\sqrt{14}\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+542542\,x^2+251706\,x+181342}{548800\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/548800*(154210*x^3 + 8475*sqrt(14)*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*
arctan(1/14*sqrt(14)*(5*x + 1)) + 542542*x^2 + 251706*x + 181342)/(25*x^4 +
 20*x^3 + 34*x^2 + 12*x + 9)
```

Sympy [A] time = 0.184545, size = 61, normalized size = 0.95

$$\frac{11015x^3 + 38753x^2 + 17979x + 12953}{980000x^4 + 784000x^3 + 1332800x^2 + 470400x + 352800} + \frac{339\sqrt{14}\operatorname{atan}\left(\frac{5\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{21952}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (11015*x**3 + 38753*x**2 + 17979*x + 12953)/(980000*x**4 + 784000*x**3 + 13
32800*x**2 + 470400*x + 352800) + 339*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(
14)/14)/21952
```

Giac [A] time = 1.14812, size = 62, normalized size = 0.97

$$\frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{11015x^3 + 38753x^2 + 17979x + 12953}{39200\left(5x^2 + 2x + 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="giac")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(5*x^2 + 2*x + 3)^2
```

3.322 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$

Optimal. Leaf size=329

$$-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}+\frac{25x\left(-9033d^2e+2203d^3+3635de^2-1829e^3\right)-92989d^2e+171735d^3+362}{39200\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}$$

```
[Out] -(1367*d - 293*e + (423*d - 1367*e)*x)/(1400*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2
*x + 5*x^2)^2) + (171735*d^3 - 92989*d^2*e + 36207*d*e^2 + 1831*e^3 + 25*(2
203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x)/(39200*(5*d^2 - 2*d*e + 3*
e^2)^2*(3 + 2*x + 5*x^2)) + ((42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 560
58*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt
[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + (e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - (e*(4*d^4 + 5*d^3*e + 3*d
^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Rubi [A] time = 0.496008, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 800, 634, 618, 204, 628}

$$-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}+\frac{25x\left(-9033d^2e+2203d^3+3635de^2-1829e^3\right)-92989d^2e+171735d^3+362}{39200\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x]
```



```
[Out] -(1367*d - 293*e + (423*d - 1367*e)*x)/(1400*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2
*x + 5*x^2)^2) + (171735*d^3 - 92989*d^2*e + 36207*d*e^2 + 1831*e^3 + 25*(2
203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x)/(39200*(5*d^2 - 2*d*e + 3*
e^2)^2*(3 + 2*x + 5*x^2)) + ((42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 560
58*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt
[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + (e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - (e*(4*d^4 + 5*d^3*e + 3*d
^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^3)
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx = -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{2\left(3267d^2-2843de+2800e^2\right)}{25\left(5d^2-2de+3e^2\right)}-\frac{6\left(3080d^2-809d\right.}{25\left(5d^2-2de\right.}}{(d+ex)\left(3+2x+5x^2\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+1831e^3}{39200\left(5d^2-2de\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+1831e^3}{39200\left(5d^2-2de\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+1831e^3}{39200\left(5d^2-2de\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+1831e^3}{39200\left(5d^2-2de\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+1831e^3}{39200\left(5d^2-2de\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+1831e^3}{39200\left(5d^2-2de\right.}$$

Mathematica [A] time = 0.29743, size = 282, normalized size = 0.86

$$\frac{\frac{392\left(5d^2-2de+3e^2\right)^2(e(1367x+293)-d(423x+1367))}{\left(5x^2+2x+3\right)^2}+\frac{14\left(5d^2-2de+3e^2\right)\left(-d^2e(225825x+92989)+5d^3(11015x+34347)+de^2(90875x+36207)+e^3(1831-45725x)\right)}{5x^2+2x+3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x
]
```

```
[Out] ((392*(5*d^2 - 2*d*e + 3*e^2)^2*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(
3 + 2*x + 5*x^2)^2 + (14*(5*d^2 - 2*d*e + 3*e^2)*(e^3*(1831 - 45725*x) + 5*
d^3*(34347 + 11015*x) + d*e^2*(36207 + 90875*x) - d^2*e*(92989 + 225825*x))
```

```
)/(3 + 2*x + 5*x^2) + 25*Sqrt[14]*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2
- 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] + 5488
00*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x] - 274400*e*
(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(548800
*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Maple [B] time = 0.071, size = 1437, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 4*e/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^4-27435/1568/(5*d^2-2*d*e+3*e^2)^3/(5
*x^2+2*x+3)^2*x^3*e^5+193765/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^2
*d^5-49377/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^2*e^5-8623/21952/(5
*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^5+3*e^3/(5*d^
2-2*d*e+3*e^2)^3*ln(e*x+d)*d^2-e^4/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d+5*e^2/
(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^3+64765/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2
+2*x+3)^2*d^5+18063/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*e^5-1/(5*d^2
-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*e^5+2*e^5/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)-1
1211/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*e^5-25611/7840/(5*d^2-2*d
*e+3*e^2)^3/(5*x^2+2*x+3)^2*d*e^4-3/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)
*d^2*e^3-5/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^3*e^2-58185/1568/(5*d^
2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*d^4*e+118119/3920/(5*d^2-2*d*e+3*e^2)^3/(5
*x^2+2*x+3)^2*d^3*e^2-28843/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*d^2*
e^3+42375/21952/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2
))*d^5-250449/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d^2*e^3+147247/7
840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d*e^4+1/2/(5*d^2-2*d*e+3*e^2)^3
*ln(5*x^2+2*x+3)*d*e^4-2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^4*e+55075/
1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*d^5+89895/1568/(5*d^2-2*d*e+
3*e^2)^3/(5*x^2+2*x+3)^2*d^5*x+31811/21952/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*a
rctan(1/28*(10*x+2)*14^(1/2))*d*e^4+29265/10976/(5*d^2-2*d*e+3*e^2)^3*14^(1
/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3*e^2-247855/1568/(5*d^2-2*d*e+3*e^2)^
3/(5*x^2+2*x+3)^2*x^3*d^4*e+107125/784/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^
2*x^3*d^3*e^2-108785/784/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*d^2*e^3+
72815/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*d*e^4-388683/3920/(5*d
^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^2*d^2*e^3+250589/7840/(5*d^2-2*d*e+3*e^
2)^3/(5*x^2+2*x+3)^2*x^2*d*e^4-16643/21952/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*a
rctan(1/28*(10*x+2)*14^(1/2))*d^4*e-28029/10976/(5*d^2-2*d*e+3*e^2)^3*14^(1
/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2*e^3-260825/1568/(5*d^2-2*d*e+3*e^2)^
```

```
3/(5*x^2+2*x+3)^2*x^2*d^4*e+655359/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)
^2*x^2*d^3*e^2+380997/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d^3*e^2-
165635/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d^4*e
```

Maxima [A] time = 1.57757, size = 771, normalized size = 2.34

$$\frac{\sqrt{14}\left(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{21952\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{\left(4\,d^4e\right.}{125\,d^6-150\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (4*d^4*
e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(e*x + d)/(125*d^6 - 150*d^5*
e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*d
^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^6
 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)
 + 1/7840*(25*(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x^3 + 64765*d
^3 - 32279*d^2*e - 4523*d*e^2 + 6021*e^3 + (193765*d^3 - 183319*d^2*e + 725
57*d*e^2 - 16459*e^3)*x^2 + (89895*d^3 - 129677*d^2*e + 46591*d*e^2 - 3737*
e^3)*x)/(25*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^4 + 225*d
^4 - 180*d^3*e + 306*d^2*e^2 - 108*d*e^3 + 81*e^4 + 20*(25*d^4 - 20*d^3*e +
 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^3 + 34*(25*d^4 - 20*d^3*e + 34*d^2*e^2 -
12*d*e^3 + 9*e^4)*x^2 + 12*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e
^4)*x)
```

Fricas [B] time = 2.32411, size = 2678, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/109760*(4533550*d^5 - 4072950*d^4*e + 3307332*d^3*e^2 - 807604*d^2*e^3 -
358554*d*e^4 + 252882*e^5 + 350*(11015*d^5 - 49571*d^4*e + 42850*d^3*e^2 -
43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 14*(968825*d^5 - 1304125*d^4*
e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 250589*d*e^4 - 49377*e^5)*x^2 + 5*sq
rt(14)*(381375*d^5 - 149787*d^4*e + 526770*d^3*e^2 - 504522*d^2*e^3 + 28629
9*d*e^4 - 77607*e^5 + 25*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d
^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^4 + 20*(42375*d^5 - 16643*d^4*e + 58530*
d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^3 + 34*(42375*d^5 - 166
43*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^2 + 12
*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8
623*e^5)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(449475*d^5 - 828175*d^4*e
 + 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*x + 109760*(
36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*d^4*e + 5*d^3
*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3
 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5
)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x)*log(e*x + d
) - 54880*(36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*d^
4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e^2
+ 3*d^2*e^3 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*
e^4 + 2*e^5)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x)*
log(5*x^2 + 2*x + 3))/(1125*d^6 - 1350*d^5*e + 2565*d^4*e^2 - 1692*d^3*e^3
+ 1539*d^2*e^4 - 486*d*e^5 + 243*e^6 + 25*(125*d^6 - 150*d^5*e + 285*d^4*e^
2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*x^4 + 20*(125*d^6 - 150*
d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*x^3 +
34*(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^
5 + 27*e^6)*x^2 + 12*(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171
*d^2*e^4 - 54*d*e^5 + 27*e^6)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15806, size = 621, normalized size = 1.89

$$\frac{\sqrt{14}\left(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{21952\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}-\frac{\left(4\,d^4e+\right.}{2\left(125\,d^6-150\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^
6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6
) + (4*d^4*e^2 + 5*d^3*e^3 + 3*d^2*e^4 - d*e^5 + 2*e^6)*log(abs(x*e + d))/(
125*d^6*e - 150*d^5*e^2 + 285*d^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^
6 + 27*e^7) + 1/7840*(323825*d^5 - 290925*d^4*e + 25*(11015*d^5 - 49571*d^4
*e + 42850*d^3*e^2 - 43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 236238*d
^3*e^2 + (968825*d^5 - 1304125*d^4*e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 2
50589*d*e^4 - 49377*e^5)*x^2 - 57686*d^2*e^3 + (449475*d^5 - 828175*d^4*e +
 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*x - 25611*d*e^
4 + 18063*e^5)/((5*d^2 - 2*d*e + 3*e^2)^3*(5*x^2 + 2*x + 3)^2)
```

3.323 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3} dx$

Optimal. Leaf size=443

$$\frac{5x\left(34698d^2e^2-85924d^3e+11015d^4+10348de^3-3589e^4\right)-200502d^2e^2-117284d^3e+171735d^4+104428de^3-231}{7840\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}$$

```
[Out] -((e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/((5*d^2 - 2*d*e + 3*e^2
)^3*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293
*e^2)*x)/(280*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)^2) + (171735*d^4
- 117284*d^3*e - 200502*d^2*e^2 + 104428*d*e^3 - 23189*e^4 + 5*(11015*d^4 -
 85924*d^3*e + 34698*d^2*e^2 + 10348*d*e^3 - 3589*e^4)*x)/(7840*(5*d^2 - 2*
d*e + 3*e^2)^3*(3 + 2*x + 5*x^2)) + ((211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(1568*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(40*d^5 +
83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[d + e*x])/(5*d^2
 - 2*d*e + 3*e^2)^4 - (e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*
d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rubi [A] time = 0.89268, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{5x\left(34698d^2e^2-85924d^3e+11015d^4+10348de^3-3589e^4\right)-200502d^2e^2-117284d^3e+171735d^4+104428de^3-231}{7840\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3),x]
```


```
[Out] -((e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/((5*d^2 - 2*d*e + 3*e^2
)^3*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293
*e^2)*x)/(280*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)^2) + (171735*d^4
- 117284*d^3*e - 200502*d^2*e^2 + 104428*d*e^3 - 23189*e^4 + 5*(11015*d^4 -
 85924*d^3*e + 34698*d^2*e^2 + 10348*d*e^3 - 3589*e^4)*x)/(7840*(5*d^2 - 2*
d*e + 3*e^2)^3*(3 + 2*x + 5*x^2)) + ((211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(1568*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(40*d^5 +
```

```
83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[d + e*x])/(5*d^2
 - 2*d*e + 3*e^2)^4 - (e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*
d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx = -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{2\left(3267d^4-5686d^3e+75\right.}{5\left(5d^2-\right.}}{}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}+\frac{171735d^4-117284d^3e-}{}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}+\frac{171735d^4-117284d^3e-}{}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}$$

Mathematica [A] time = 0.480602, size = 389, normalized size = 0.88

$$\frac{-\frac{392\left(5d^2-2de+3e^2\right)^2\left(d^2(423x+1367)-2de(1367x+293)+e^2(293x-703)\right)}{\left(5x^2+2x+3\right)^2}+\frac{14\left(5d^2-2de+3e^2\right)\left(6d^2e^2(28915x-33417)-4d^3e(107405x+29321)+5d^4(11015x+34\right.}{5x^2+2x+3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3)
,x]
```

```
[Out] ((-109760*e*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
2*e^4))/(d + e*x) - (392*(5*d^2 - 2*d*e + 3*e^2)^2*(e^2*(-703 + 293*x) + d^
2*(1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2)^2 + (14*(5*d^2
- 2*d*e + 3*e^2)*(5*d^4*(34347 + 11015*x) + 4*d*e^3*(26107 + 12935*x) - e^4
*(23189 + 17945*x) + 6*d^2*e^2*(-33417 + 28915*x) - 4*d^3*e*(29321 + 107405
*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]] + 109760*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 +
46*d*e^4 - 9*e^5)*Log[d + e*x] - 54880*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 -
76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(109760*(5*d^2 - 2*d*e
 + 3*e^2)^4)
```

Maple [B] time = 0.077, size = 1850, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x)
```

```
[Out] -9*e^6/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)-2*e^5/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)-
6309/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*e^6+323825/1568/(5*d^2-2*d*
e+3*e^2)^4/(5*x^2+2*x+3)^2*d^6+9/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*e^
6+116869/392/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^3*e^3-74895/1568/(5*d^
2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*e^6-91101/1568/(5*d^2-2*d*e+3*e^2)^4/(5*
x^2+2*x+3)^2*x^2*e^6-53835/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*e
^6+275375/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^6+449475/1568/(5
*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^6*x+99045/784/(5*d^2-2*d*e+3*e^2)^4/(
5*x^2+2*x+3)^2*d*e^5-530209/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^2*
e^4-161395/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^5*e-379131/1568/(5*d
^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^4*e^2+40*e/(5*d^2-2*d*e+3*e^2)^4*ln(e*x
+d)*d^5+83*e^2/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^4+12*e^3/(5*d^2-2*d*e+3*e^
2)^4*ln(e*x+d)*d^3+327265/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^
2*e^4+95555/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d*e^5-648385/784/
(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d^5*e+218053/784/(5*d^2-2*d*e+3*e^2
)^4/(5*x^2+2*x+3)^2*x^2*d*e^5+208007/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3
)^2*x*d*e^5-916595/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^5*e+6062
87/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d^4*e^2-3993/392/(5*d^2-2*d
*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d^3*e^3+380621/21952/(5*d^2-2*d*e+3*e^2)^4*14
```

```
^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2*e^4-24793/10976/(5*d^2-2*d*e+3*e^
2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^5+968825/1568/(5*d^2-2*d*e
+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^6-344285/392/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2
*x+3)^2*x^3*d^3*e^3-76*e^4/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^2+46*e^5/(5*d^
2-2*d*e+3*e^2)^4*ln(e*x+d)*d-4*e/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^4-5*e^2/(5
*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^3+211875/21952/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)
*arctan(1/28*(10*x+2)*14^(1/2))*d^6-43695/21952/(5*d^2-2*d*e+3*e^2)^4*14^(1
/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^6-6/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x
+3)*d^3*e^3+38/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^2*e^4-23/(5*d^2-2*d*
e+3*e^2)^4*ln(5*x^2+2*x+3)*d*e^5-20/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d
^5*e-83/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^4*e^2-3*e^3/(5*d^2-2*d*e+
3*e^2)^3/(e*x+d)*d^2+e^4/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d-434995/1568/(5*d^2
-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d^2*e^4-1129125/784/(5*d^2-2*d*e+3*e^2)^4
/(5*x^2+2*x+3)^2*x^3*d^5*e+504029/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^
2*x^2*d^4*e^2+1891915/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^4*e^
2+5109/392/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^3*e^3-795401/1568/(5
*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^2*e^4+1535/10976/(5*d^2-2*d*e+3*e
^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^5*e+209039/21952/(5*d^2-2*d
*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^4*e^2-230361/5488/(5*
d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3*e^3
```

Maxima [B] time = 1.7046, size = 1237, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 1/21952*sqrt(14)*(211875*d^6 + 3070*d^5*e + 209039*d^4*e^2 - 921444*d^3*e^3
 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*arctan(1/14*sqrt(14)*(5*x + 1)
)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176
*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) + (40*d^5*e + 83*d^4*e^2 + 12*
d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(e*x + d)/(625*d^8 - 1000*d^7*e
 + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6
- 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 76*d^2*e^
4 + 46*d*e^5 - 9*e^6)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6
*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
 + 81*e^8) + 1/1568*(64765*d^5 - 95100*d^4*e - 200706*d^3*e^2 + 22292*d^2*e
^3 + 12009*d*e^4 - 28224*e^5 - 5*(20345*d^4*e + 125124*d^3*e^2 - 11178*d^2*
e^3 - 18188*d*e^4 + 19269*e^5)*x^4 + (55075*d^5 - 361295*d^4*e - 272442*d^3
```

```
*e^2 - 173446*d^2*e^3 + 138539*d*e^4 - 93087*e^5)*x^3 + (193765*d^5 - 41248
5*d^4*e - 621062*d^3*e^2 - 56850*d^2*e^3 + 144973*d*e^4 - 131589*e^5)*x^2 +
 3*(29965*d^5 - 77965*d^4*e - 51590*d^3*e^2 - 21522*d^2*e^3 + 19493*d*e^4 -
 13245*e^5)*x)/(1125*d^7 - 1350*d^6*e + 2565*d^5*e^2 - 1692*d^4*e^3 + 1539*
d^3*e^4 - 486*d^2*e^5 + 243*d*e^6 + 25*(125*d^6*e - 150*d^5*e^2 + 285*d^4*e
^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^6 + 27*e^7)*x^5 + 5*(625*d^7 - 250*
d^6*e + 825*d^5*e^2 + 200*d^4*e^3 + 103*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 +
108*e^7)*x^4 + 2*(1250*d^7 + 625*d^6*e + 300*d^5*e^2 + 2965*d^4*e^3 - 1486*
d^3*e^4 + 2367*d^2*e^5 - 648*d*e^6 + 459*e^7)*x^3 + 2*(2125*d^7 - 1800*d^6*
e + 3945*d^5*e^2 - 1486*d^4*e^3 + 1779*d^3*e^4 + 108*d^2*e^5 + 135*d*e^6 +
162*e^7)*x^2 + 3*(500*d^7 - 225*d^6*e + 690*d^5*e^2 + 103*d^4*e^3 + 120*d^3
*e^4 + 297*d^2*e^5 - 54*d*e^6 + 81*e^7)*x)
```

Fricas [B] time = 3.37601, size = 4680, normalized size = 10.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/21952*(4533550*d^7 - 8470420*d^6*e - 8666490*d^5*e^2 + 3186008*d^4*e^3 -
8213198*d^3*e^4 - 1375668*d^2*e^5 + 1294650*d*e^6 - 1185408*e^7 - 70*(10172
5*d^6*e + 584930*d^5*e^2 - 245103*d^4*e^3 + 306788*d^3*e^4 + 99187*d^2*e^5
- 93102*d*e^6 + 57807*e^7)*x^4 + 14*(275375*d^7 - 1916625*d^6*e - 474395*d^
5*e^2 - 1406231*d^4*e^3 + 222261*d^3*e^4 - 1262851*d^2*e^5 + 601791*d*e^6 -
 279261*e^7)*x^3 + 14*(968825*d^7 - 2449955*d^6*e - 1699045*d^5*e^2 - 27958
1*d^4*e^3 - 1024621*d^3*e^4 - 1118441*d^2*e^5 + 698097*d*e^6 - 394767*e^7)*
x^2 + sqrt(14)*(1906875*d^7 + 27630*d^6*e + 1881351*d^5*e^2 - 8292996*d^4*e
^3 + 3425589*d^3*e^4 - 446274*d^2*e^5 - 393255*d*e^6 + 25*(211875*d^6*e + 3
070*d^5*e^2 + 209039*d^4*e^3 - 921444*d^3*e^4 + 380621*d^2*e^5 - 49586*d*e^
6 - 43695*e^7)*x^5 + 5*(1059375*d^7 + 862850*d^6*e + 1057475*d^5*e^2 - 3771
064*d^4*e^3 - 1782671*d^3*e^4 + 1274554*d^2*e^5 - 416819*d*e^6 - 174780*e^7
)*x^4 + 2*(2118750*d^7 + 3632575*d^6*e + 2142580*d^5*e^2 - 5660777*d^4*e^3
- 11858338*d^3*e^4 + 5974697*d^2*e^5 - 1279912*d*e^6 - 742815*e^7)*x^3 + 2*
(3601875*d^7 + 1323440*d^6*e + 3572083*d^5*e^2 - 14410314*d^4*e^3 + 941893*
d^3*e^4 + 1440764*d^2*e^5 - 1040331*d*e^6 - 262170*e^7)*x^2 + 3*(847500*d^7
 + 647905*d^6*e + 845366*d^5*e^2 - 3058659*d^4*e^3 - 1241848*d^3*e^4 + 9435
19*d^2*e^5 - 323538*d*e^6 - 131085*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1))
+ 42*(149825*d^7 - 449755*d^6*e - 12125*d^5*e^2 - 238325*d^4*e^3 - 14261*d^
3*e^4 - 169777*d^2*e^5 + 84969*d*e^6 - 39735*e^7)*x + 21952*(360*d^6*e + 74
```

```
7*d^5*e^2 + 108*d^4*e^3 - 684*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5
*e^2 + 83*d^4*e^3 + 12*d^3*e^4 - 76*d^2*e^5 + 46*d*e^6 - 9*e^7)*x^5 + 5*(20
0*d^6*e + 575*d^5*e^2 + 392*d^4*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6
- 36*e^7)*x^4 + 2*(400*d^6*e + 1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 -
832*d^2*e^5 + 692*d*e^6 - 153*e^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*
d^4*e^3 - 1220*d^3*e^4 + 326*d^2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6
*e + 452*d^5*e^2 + 297*d^4*e^3 - 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*
e^7)*x)*log(e*x + d) - 10976*(360*d^6*e + 747*d^5*e^2 + 108*d^4*e^3 - 684*d
^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5*e^2 + 83*d^4*e^3 + 12*d^3*e^4
- 76*d^2*e^5 + 46*d*e^6 - 9*e^7)*x^5 + 5*(200*d^6*e + 575*d^5*e^2 + 392*d^4
*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6 - 36*e^7)*x^4 + 2*(400*d^6*e +
1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 - 832*d^2*e^5 + 692*d*e^6 - 153*e
^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*d^4*e^3 - 1220*d^3*e^4 + 326*d^
2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6*e + 452*d^5*e^2 + 297*d^4*e^3
- 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*e^7)*x)*log(5*x^2 + 2*x + 3))/(
5625*d^9 - 9000*d^8*e + 18900*d^7*e^2 - 17640*d^6*e^3 + 18774*d^5*e^4 - 105
84*d^4*e^5 + 6804*d^3*e^6 - 1944*d^2*e^7 + 729*d*e^8 + 25*(625*d^8*e - 1000
*d^7*e^2 + 2100*d^6*e^3 - 1960*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*
d^2*e^7 - 216*d*e^8 + 81*e^9)*x^5 + 5*(3125*d^9 - 2500*d^8*e + 6500*d^7*e^2
 - 1400*d^6*e^3 + 2590*d^5*e^4 + 2464*d^4*e^5 - 924*d^3*e^6 + 1944*d^2*e^7
- 459*d*e^8 + 324*e^9)*x^4 + 2*(6250*d^9 + 625*d^8*e + 4000*d^7*e^2 + 16100
*d^6*e^3 - 12460*d^5*e^4 + 23702*d^4*e^5 - 12432*d^3*e^6 + 10692*d^2*e^7 -
2862*d*e^8 + 1377*e^9)*x^3 + 2*(10625*d^9 - 13250*d^8*e + 29700*d^7*e^2 - 2
0720*d^6*e^3 + 23702*d^5*e^4 - 7476*d^4*e^5 + 5796*d^3*e^6 + 864*d^2*e^7 +
81*d*e^8 + 486*e^9)*x^2 + 3*(2500*d^9 - 2125*d^8*e + 5400*d^7*e^2 - 1540*d^
6*e^3 + 2464*d^5*e^4 + 1554*d^4*e^5 - 504*d^3*e^6 + 1404*d^2*e^7 - 324*d*e^
8 + 243*e^9)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26386, size = 1029, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 1/21952*sqrt(14)*(211875*d^6*e^2 + 3070*d^5*e^3 + 209039*d^4*e^4 - 921444*d
^3*e^5 + 380621*d^2*e^6 - 49586*d*e^7 - 43695*e^8)*arctan(1/14*sqrt(14)*(5*
d - 5*d^2/(x*e + d) + 2*d*e/(x*e + d) - 3*e^2/(x*e + d) - e)*e^(-1))*e^(-2)
/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*
d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 +
12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(-10*d/(x*e + d) + 5*d^2/(x*
e + d)^2 + 2*e/(x*e + d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(625*
d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^
5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - (4*d^4*e^7/(x*e + d) + 5*d^3*e^8/(x
*e + d) + 3*d^2*e^9/(x*e + d) - d*e^10/(x*e + d) + 2*e^11/(x*e + d))/(125*d
^6*e^6 - 150*d^5*e^7 + 285*d^4*e^8 - 188*d^3*e^9 + 171*d^2*e^10 - 54*d*e^11
 + 27*e^12) + 1/1568*(275375*d^5*e - 3006775*d^4*e^2 + 1394650*d^3*e^3 + 18
35350*d^2*e^4 - 734925*d*e^5 - 5*(165225*d^6*e^2 - 1997830*d^5*e^3 + 121842
1*d^4*e^4 + 1520564*d^3*e^5 - 947049*d^2*e^6 + 93386*d*e^7 + 7963*e^8)*e^(-
1)/(x*e + d) + (826125*d^7*e^3 - 10957975*d^6*e^4 + 8449735*d^5*e^5 + 82111
75*d^4*e^6 - 7879025*d^3*e^7 + 2996315*d^2*e^8 - 443947*d*e^9 - 67267*e^10)
*e^(-2)/(x*e + d)^2 - (275375*d^8*e^4 - 3975600*d^7*e^5 + 3752280*d^6*e^6 +
 2119880*d^5*e^7 - 3655050*d^4*e^8 + 4008480*d^3*e^9 - 1453312*d^2*e^10 - 1
97784*d*e^11 + 66483*e^12)*e^(-3)/(x*e + d)^3 + 17525*e^6)/((5*d^2 - 2*d*e
+ 3*e^2)^4*(10*d/(x*e + d) - 5*d^2/(x*e + d)^2 - 2*e/(x*e + d) + 2*d*e/(x*e
 + d)^2 - 3*e^2/(x*e + d)^2 - 5)^2)
```

3.324 $\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx$

Optimal. Leaf size=143

$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4-\frac{823\left(2x^2-x+3\right)^{3/2}(2x+5)^3}{1344}+\frac{11433\left(2x^2-x+3\right)^{3/2}(2x+5)^2}{4480}-\frac{(295276x+10057\ldots}{71\ldots}$$

```
[Out] (-51435*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/32768 + (11433*(5 + 2*x)^2*(3 - x +
2*x^2)^(3/2))/4480 - (823*(5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))/1344 + (5*(5 +
 2*x)^4*(3 - x + 2*x^2)^(3/2))/112 - ((1005757 + 295276*x)*(3 - x + 2*x^2)^
(3/2))/71680 - (1183005*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

Rubi [A] time = 0.15542, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1653, 779, 612, 619, 215}

$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4-\frac{823\left(2x^2-x+3\right)^{3/2}(2x+5)^3}{1344}+\frac{11433\left(2x^2-x+3\right)^{3/2}(2x+5)^2}{4480}-\frac{(295276x+10057\ldots}{71\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```


```
[Out] (-51435*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/32768 + (11433*(5 + 2*x)^2*(3 - x +
2*x^2)^(3/2))/4480 - (823*(5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))/1344 + (5*(5 +
 2*x)^4*(3 - x + 2*x^2)^(3/2))/112 - ((1005757 + 295276*x)*(3 - x + 2*x^2)^
(3/2))/71680 - (1183005*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```


Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right) dx = \frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2} + \frac{1}{224}\int (5+2x)\sqrt{3-x+2x^2}\,(-$$

$$= -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344} + \frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2}$$

$$= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480} - \frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}$$

$$= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480} - \frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768} + \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768} + \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768} + \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}$$

Mathematica [A] time = 0.155435, size = 70, normalized size = 0.49

$$\frac{4\sqrt{2x^2-x+3}\left(4915200x^6+12984320x^5+1390592x^4+20304768x^3+11357024x^2+14742332x+6231117\right)-12421}{13762560}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(6231117 + 14742332*x + 11357024*x^2 + 20304768*x^3
+ 1390592*x^4 + 12984320*x^5 + 4915200*x^6) - 124215525*Sqrt[2]*ArcSinh[(1
- 4*x)/Sqrt[23]])/13762560
```

Maple [A] time = 0.054, size = 115, normalized size = 0.8

$$\frac{5x^4}{7}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{377x^3}{168}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{283x^2}{1120}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{5179x}{17920}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{-51435+205740x}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/7*x^4*(2*x^2-x+3)^(3/2)+377/168*x^3*(2*x^2-x+3)^(3/2)+283/1120*x^2*(2*x^2
-x+3)^(3/2)-5179/17920*x*(2*x^2-x+3)^(3/2)+51435/32768*(-1+4*x)*(2*x^2-x+3)
^(1/2)+1183005/131072*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+242329/215040*
(2*x^2-x+3)^(3/2)
```

Maxima [A] time = 1.52211, size = 170, normalized size = 1.19

$$\frac{5}{7}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{377}{168}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{283}{1120}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{5179}{17920}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x+\frac{242329}{215040}\left(2\,x^2-x+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/7*(2*x^2 - x + 3)^(3/2)*x^4 + 377/168*(2*x^2 - x + 3)^(3/2)*x^3 + 283/112
0*(2*x^2 - x + 3)^(3/2)*x^2 - 5179/17920*(2*x^2 - x + 3)^(3/2)*x + 242329/2
15040*(2*x^2 - x + 3)^(3/2) + 51435/8192*sqrt(2*x^2 - x + 3)*x + 1183005/13
1072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 51435/32768*sqrt(2*x^2 - x
+ 3)
```

Fricas [A] time = 1.30936, size = 296, normalized size = 2.07

$$\frac{1}{3440640}\left(4915200\,x^6+12984320\,x^5+1390592\,x^4+20304768\,x^3+11357024\,x^2+14742332\,x+6231117\right)\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/3440640*(4915200*x^6 + 12984320*x^5 + 1390592*x^4 + 20304768*x^3 + 113570
24*x^2 + 14742332*x + 6231117)*sqrt(2*x^2 - x + 3) + 1183005/262144*sqrt(2)
*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+5)\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((2*x + 5)*sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2), x
)
```

Giac [A] time = 1.16607, size = 105, normalized size = 0.73

$$\frac{1}{3440640}\left(4\left(8\left(4\left(16\left(20\left(120\,x+317\right)x+679\right)x+158631\right)x+354907\right)x+3685583\right)x+6231117\right)\sqrt{2\,x^2-x+3}-\frac{11830}{1310}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/3440640*(4*(8*(4*(16*(20*(120*x + 317)*x + 679)*x + 158631)*x + 354907)*x
 + 3685583)*x + 6231117)*sqrt(2*x^2 - x + 3) - 1183005/131072*sqrt(2)*log(-
2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.325 $\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

Optimal. Leaf size=124

$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3+\frac{7}{80}\left(2x^2-x+3\right)^{3/2}x^2-\frac{71\left(2x^2-x+3\right)^{3/2}x}{1280}+\frac{287\left(2x^2-x+3\right)^{3/2}}{5120}-\frac{4609(1-4x)\sqrt{2x^2-x+}}{16384}$$

```
[Out] (-4609*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 + (287*(3 - x + 2*x^2)^(3/2))/5
120 - (71*x*(3 - x + 2*x^2)^(3/2))/1280 + (7*x^2*(3 - x + 2*x^2)^(3/2))/80
+ (5*x^3*(3 - x + 2*x^2)^(3/2))/12 - (106007*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
32768*Sqrt[2])
```

Rubi [A] time = 0.0921227, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3+\frac{7}{80}\left(2x^2-x+3\right)^{3/2}x^2-\frac{71\left(2x^2-x+3\right)^{3/2}x}{1280}+\frac{287\left(2x^2-x+3\right)^{3/2}}{5120}-\frac{4609(1-4x)\sqrt{2x^2-x+}}{16384}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-4609*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 + (287*(3 - x + 2*x^2)^(3/2))/5
120 - (71*x*(3 - x + 2*x^2)^(3/2))/1280 + (7*x^2*(3 - x + 2*x^2)^(3/2))/80
+ (5*x^3*(3 - x + 2*x^2)^(3/2))/12 - (106007*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
32768*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx = \frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{12}\int \sqrt{3-x+2x^2}\left(24+12x-9x^2+\frac{21x^3}{2}\right)d$$

$$= \frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{120}\int\left(240+57x-\right.$$

$$= -\frac{71x\left(3-x+2x^2\right)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}$$

$$= \frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x+2x^2\right)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+$$

$$= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x+2x^2\right)}{1280}$$

$$= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x+2x^2\right)}{1280}$$

$$= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x+2x^2\right)}{1280}$$

Mathematica [A] time = 0.100818, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2-x+3}\left(204800x^5-59392x^4+258432x^3+105696x^2+221868x-27807\right)-1590105\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{983040}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-27807 + 221868*x + 105696*x^2 + 258432*x^3 - 59392
*x^4 + 204800*x^5) - 1590105*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/983040
```

Maple [A] time = 0.055, size = 98, normalized size = 0.8

$$\frac{5x^3}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{7x^2}{80}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{71x}{1280}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{287}{5120}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{-4609+18436x}{16384}\sqrt{2x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/12*x^3*(2*x^2-x+3)^(3/2)+7/80*x^2*(2*x^2-x+3)^(3/2)-71/1280*x*(2*x^2-x+3)
^(3/2)+287/5120*(2*x^2-x+3)^(3/2)+4609/16384*(-1+4*x)*(2*x^2-x+3)^(1/2)+106
007/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A] time = 1.57703, size = 147, normalized size = 1.19

$$\frac{5}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{7}{80}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{71}{1280}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{287}{5120}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{4609}{4096}\sqrt{2x^2-x+3}x+\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/12*(2*x^2 - x + 3)^(3/2)*x^3 + 7/80*(2*x^2 - x + 3)^(3/2)*x^2 - 71/1280*(
2*x^2 - x + 3)^(3/2)*x + 287/5120*(2*x^2 - x + 3)^(3/2) + 4609/4096*sqrt(2*
x^2 - x + 3)*x + 106007/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46
09/16384*sqrt(2*x^2 - x + 3)
```

Fricas [A] time = 1.35483, size = 258, normalized size = 2.08

$$\frac{1}{245760}\left(204800\,x^5-59392\,x^4+258432\,x^3+105696\,x^2+221868\,x-27807\right)\sqrt{2\,x^2-x+3}+\frac{106007}{131072}\,\sqrt{2}\log\left(-4\,\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/245760*(204800*x^5 - 59392*x^4 + 258432*x^3 + 105696*x^2 + 221868*x - 278
07)*sqrt(2*x^2 - x + 3) + 106007/131072*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)
```

Giac [A] time = 1.17368, size = 99, normalized size = 0.8

$$\frac{1}{245760}\left(4\left(8\left(4\left(16\left(100\,x-29\right)x+2019\right)x+3303\right)x+55467\right)x-27807\right)\sqrt{2\,x^2-x+3}-\frac{106007}{65536}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/245760*(4*(8*(4*(16*(100*x - 29)*x + 2019)*x + 3303)*x + 55467)*x - 27807
)*sqrt(2*x^2 - x + 3) - 106007/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sq
rt(2*x^2 - x + 3)) + 1)
```

3.326 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

Optimal. Leaf size=149

$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2-\frac{127}{128}\left(2x^2-x+3\right)^{3/2}(2x+5)+\frac{4535}{768}\left(2x^2-x+3\right)^{3/2}+\frac{(489587-80844x)\sqrt{2x^2-x+3}}{4096}$$

```
[Out] ((489587 - 80844*x)*Sqrt[3 - x + 2*x^2])/4096 + (4535*(3 - x + 2*x^2)^(3/2)
)/768 - (127*(5 + 2*x)*(3 - x + 2*x^2)^(3/2))/128 + ((5 + 2*x)^2*(3 - x + 2
*x^2)^(3/2))/16 + (5627989*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8192*Sqrt[2]) - (1
1001*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(16*Sqrt[2])
```

Rubi [A] time = 0.240436, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2-\frac{127}{128}\left(2x^2-x+3\right)^{3/2}(2x+5)+\frac{4535}{768}\left(2x^2-x+3\right)^{3/2}+\frac{(489587-80844x)\sqrt{2x^2-x+3}}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((489587 - 80844*x)*Sqrt[3 - x + 2*x^2])/4096 + (4535*(3 - x + 2*x^2)^(3/2)
)/768 - (127*(5 + 2*x)*(3 - x + 2*x^2)^(3/2))/128 + ((5 + 2*x)^2*(3 - x + 2
*x^2)^(3/2))/16 + (5627989*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8192*Sqrt[2]) - (1
1001*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(16*Sqrt[2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2}+\frac{1}{160}\int \frac{\sqrt{3-x+2x^2}\left(-805-6490x-930\right.}{5+2x}$$

$$= -\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2}+\frac{\int \frac{\sqrt{3-x+2}}{}}{}$$

$$= \frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}(5+2x)^2\left(3-\right.$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{127}{128}(5+2x)$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{127}{128}(5+2x)$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{127}{128}(5+2x)$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{127}{128}(5+2x)$$

Mathematica [A] time = 0.148461, size = 91, normalized size = 0.61

$$\frac{4\sqrt{2x^2-x+3}\left(6144x^4-21120x^3+79840x^2-300404x+1561161\right)-16897536\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+16883967\sqrt{2}}{49152}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(1561161 - 300404*x + 79840*x^2 - 21120*x^3 + 6144*x
^4) + 16883967*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 16897536*Sqrt[2]*ArcTa
nh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/49152
```

Maple [A] time = 0.056, size = 127, normalized size = 0.9

$$\frac{x^2}{4}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{47x}{64}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{1925}{768}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{-20211+80844x}{4096}\sqrt{2x^2-x+3}-\frac{5627989\sqrt{2}}{16384}\mathrm{Arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x)
```

```
[Out] 1/4*x^2*(2*x^2-x+3)^(3/2)-47/64*x*(2*x^2-x+3)^(3/2)+1925/768*(2*x^2-x+3)^(3
/2)-20211/4096*(-1+4*x)*(2*x^2-x+3)^(1/2)-5627989/16384*2^(1/2)*arcsinh(4/2
3*23^(1/2)*(x-1/4))+3667/32*(2*(x+5/2)^2-11*x-19/2)^(1/2)-11001/32*2^(1/2)*
arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A] time = 1.59072, size = 173, normalized size = 1.16

$$\frac{1}{4}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{47}{64}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{1925}{768}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{20211}{1024}\sqrt{2x^2-x+3}x-\frac{5627989}{16384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 1/4*(2*x^2 - x + 3)^(3/2)*x^2 - 47/64*(2*x^2 - x + 3)^(3/2)*x + 1925/768*(2
*x^2 - x + 3)^(3/2) - 20211/1024*sqrt(2*x^2 - x + 3)*x - 5627989/16384*sqrt
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 11001/32*sqrt(2)*arcsinh(22/
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 489587/4096*sqr
t(2*x^2 - x + 3)
```

Fricas [A] time = 1.38627, size = 394, normalized size = 2.64

$$\frac{1}{12288}\left(6144x^4-21120x^3+79840x^2-300404x+1561161\right)\sqrt{2x^2-x+3}+\frac{5627989}{32768}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/12288*(6144*x^4 - 21120*x^3 + 79840*x^2 - 300404*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/32768*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1
) - 32*x^2 + 16*x - 25) + 11001/64*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{2x + 5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5), x
)
```

Giac [A] time = 1.16418, size = 174, normalized size = 1.17

$$\frac{1}{12288}\left(4\left(8\left(12\left(16\,x - 55\right)x + 2495\right)x - 75101\right)x + 1561161\right)\sqrt{2\,x^2 - x + 3} + \frac{5627989}{16384}\,\sqrt{2}\log\left(-4\,\sqrt{2}x + \sqrt{2} + 4\,\sqrt{2\,x^2 -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/12288*(4*(8*(12*(16*x - 55)*x + 2495)*x - 75101)*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/16384*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2
- x + 3)) - 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*
x^2 - x + 3)))
```

3.327 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

Optimal. Leaf size=149

$$\frac{5}{64}(2x+5)\left(2x^2-x+3\right)^{3/2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}-\frac{541}{384}\left(2x^2-x+3\right)^{3/2}-\frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432}+\frac{239}{}$$

```
[Out] -((1996953 - 333380*x)*Sqrt[3 - x + 2*x^2])/18432 - (541*(3 - x + 2*x^2)^(3
/2))/384 - (3667*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 -
 x + 2*x^2)^(3/2))/64 - (2551847*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2]
) + (239201*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(384*Sqr
t[2])
```

Rubi [A] time = 0.236645, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{64}(2x+5)\left(2x^2-x+3\right)^{3/2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}-\frac{541}{384}\left(2x^2-x+3\right)^{3/2}-\frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432}+\frac{239}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

```
[Out] -((1996953 - 333380*x)*Sqrt[3 - x + 2*x^2])/18432 - (541*(3 - x + 2*x^2)^(3
/2))/384 - (3667*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 -
 x + 2*x^2)^(3/2))/64 - (2551847*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2]
) + (239201*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(384*Sqr
t[2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} - \frac{1}{72}\int \frac{\sqrt{3-x+2x^2}\left(\frac{19341}{16}-\frac{6313x}{2}+486x^2-\right.}{5+2x}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2} - \frac{\int \frac{\sqrt{3-x+2x^2}(7466}{}}{}$$

$$= -\frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)\left(3-x+2x\right.$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3\right.}{576}$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3\right.}{576}$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3\right.}{576}$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3\right.}{576}$$

Mathematica [A] time = 0.163372, size = 98, normalized size = 0.66

$$\frac{\frac{4\sqrt{2x^2-x+3}(3840x^4-17344x^3+94936x^2-728410x-3539439)}{2x+5}+7654432\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)-7655541\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{24576}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x
]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-3539439 - 728410*x + 94936*x^2 - 17344*x^3 + 3840
*x^4))/(5 + 2*x) - 7655541*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 7654432*Sq
rt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/24576
```

Maple [A] time = 0.059, size = 152, normalized size = 1.

$$\frac{5x}{32}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{391}{384}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{-6001+24004x}{2048}\sqrt{2x^2-x+3}+\frac{2551847\sqrt{2}}{8192}\mathrm{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x)
```

```
[Out] 5/32*x*(2*x^2-x+3)^(3/2)-391/384*(2*x^2-x+3)^(3/2)+6001/2048*(-1+4*x)*(2*x^
2-x+3)^(1/2)+2551847/8192*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-239201/230
4*(2*(x+5/2)^2-11*x-19/2)^(1/2)+239201/768*2^(1/2)*arctanh(1/12*(17/2-11*x)
*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-3667/1152/(x+5/2)*(2*(x+5/2)^2-11*x
-19/2)^(3/2)+3667/2304*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)
```

Maxima [A] time = 1.53635, size = 178, normalized size = 1.19

$$\frac{5}{32}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{391}{384}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{6001}{512}\sqrt{2x^2-x+3}x+\frac{2551847}{8192}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{239201}{768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/32*(2*x^2 - x + 3)^(3/2)*x - 391/384*(2*x^2 - x + 3)^(3/2) + 6001/512*sqr
t(2*x^2 - x + 3)*x + 2551847/8192*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sq
rt(23)) - 239201/768*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 182769/2048*sqrt(2*x^2 - x + 3) - 3667/32*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

Fricas [A] time = 1.41511, size = 431, normalized size = 2.89

$$\frac{7655541\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+7654432\sqrt{2}(2x+5)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22}{4}\right.}{49152\,(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="f
ricas")
```

```
[Out] 1/49152*(7655541*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 7654432*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)
) + 8*(3840*x^4 - 17344*x^3 + 94936*x^2 - 728410*x - 3539439)*sqrt(2*x^2 -
x + 3))/(2*x + 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**2,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**2
, x)
```

Giac [B] time = 1.31159, size = 717, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="g
iac")
```

```
[Out] 1/24576*sqrt(2)*(7654432*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 7655541*log(abs(sqrt(-11/(2*x + 5) +
36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 7655541*log(abs(
sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*sgn(1/(2*x + 5
)) - 1408128*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(1/(2*x + 5)) + 2*
(16367883*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^7*sgn(1/
(2*x + 5)) - 34896384*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^6*sgn(1/(2*x + 5)) - 93395*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 25574400*(sqrt(-11/(2*x + 5) + 36/(2*x +
5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 19752365*(sqrt(-11/(2*x + 5)
+ 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x + 5)) - 31921920*(sqrt(-1
1/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) - 24458
13*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)
) + 7663104*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
6/(2*x + 5))^2 - 1)^4)
```

3.328 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx$

Optimal. Leaf size=151

$$\frac{357391 (2x^2-x+3)^{3/2}}{82944(2x+5)} - \frac{3667 (2x^2-x+3)^{3/2}}{1152(2x+5)^2} + \frac{5}{48} (2x^2-x+3)^{3/2} + \frac{5(661065-110099x)\sqrt{2x^2-x+3}}{82944} - \frac{1267080}{}$$

```
[Out] (5*(661065 - 110099*x)*Sqrt[3 - x + 2*x^2])/82944 + (5*(3 - x + 2*x^2)^(3/2
))/48 - (3667*(3 - x + 2*x^2)^(3/2))/(1152*(5 + 2*x)^2) + (357391*(3 - x +
2*x^2)^(3/2))/(82944*(5 + 2*x)) + (117315*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512
*Sqrt[2]) - (12670805*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])]
)/(55296*Sqrt[2])
```

Rubi [A] time = 0.228359, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{357391 (2x^2-x+3)^{3/2}}{82944(2x+5)} - \frac{3667 (2x^2-x+3)^{3/2}}{1152(2x+5)^2} + \frac{5}{48} (2x^2-x+3)^{3/2} + \frac{5(661065-110099x)\sqrt{2x^2-x+3}}{82944} - \frac{1267080}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

```
[Out] (5*(661065 - 110099*x)*Sqrt[3 - x + 2*x^2])/82944 + (5*(3 - x + 2*x^2)^(3/2
))/48 - (3667*(3 - x + 2*x^2)^(3/2))/(1152*(5 + 2*x)^2) + (357391*(3 - x +
2*x^2)^(3/2))/(82944*(5 + 2*x)) + (117315*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512
*Sqrt[2]) - (12670805*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])]
)/(55296*Sqrt[2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{\sqrt{3-x+2x^2}\left(\frac{27681}{16}-\frac{14251x}{4}+972x^2\right.}{(5+2x)^2}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2} + \frac{357391\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{1531305}{16}-\right.}{5+2}}{103}$$

$$= \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2} + \frac{357391\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3-x\right.}{1152(5}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3-x\right.}{1152(5}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3-x\right.}{1152(5}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3-x\right.}{1152(5}$$

Mathematica [A] time = 0.155759, size = 98, normalized size = 0.65

$$\frac{\frac{24\sqrt{2x^2-x+3}\left(3840x^4-25632x^3+272520x^2+2959330x+4880551\right)}{(2x+5)^2}-12670805\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+12670020\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{110592}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(4880551 + 2959330*x + 272520*x^2 - 25632*x^3 + 38
40*x^4))/(5 + 2*x)^2 + 12670020*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 12670
805*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/110592
```

Maple [A] time = 0.063, size = 158, normalized size = 1.1

$$\frac{5}{48}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{-149+596x}{256}\sqrt{2x^2-x+3}-\frac{117315\sqrt{2}}{1024}\text{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)-\frac{3667}{4608}\left(2\left(x+5/2\right)^2-11x-\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x)
```

```
[Out] 5/48*(2*x^2-x+3)^(3/2)-149/256*(-1+4*x)*(2*x^2-x+3)^(1/2)-117315/1024*2^(1/
2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/4608/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/
2)^(3/2)+357391/165888/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+12670805/33177
6*(2*(x+5/2)^2-11*x-19/2)^(1/2)-12670805/110592*2^(1/2)*arctanh(1/12*(17/2-
11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-357391/331776*(-1+4*x)*(2*(x+5
/2)^2-11*x-19/2)^(1/2)
```

Maxima [A] time = 1.50259, size = 193, normalized size = 1.28

$$\frac{5}{48}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{149}{64}\sqrt{2x^2-x+3}x-\frac{117315}{1024}\sqrt{2}\,\text{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{12670805}{110592}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23\left|2x+5\right|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(3/2) - 149/64*sqrt(2*x^2 - x + 3)*x - 117315/1024*sqr
t(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 12670805/110592*sqrt(2)*arc
sinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 3877/14
4*sqrt(2*x^2 - x + 3) - 3667/1152*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25)
 + 357391/4608*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Fricas [A] time = 1.41095, size = 479, normalized size = 3.17

$$\frac{12670020 \sqrt{2}\left(4 x^2 + 20 x + 25\right) \log\left(4 \sqrt{2}\sqrt{2 x^2 - x + 3}(4 x - 1) - 32 x^2 + 16 x - 25\right) + 12670805 \sqrt{2}\left(4 x^2 + 20 x + 25\right)}{221184 \left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/221184*(12670020*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 12670805*sqrt(2)*(4*x^2 + 20*x + 25
)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 11
53)/(4*x^2 + 20*x + 25)) + 48*(3840*x^4 - 25632*x^3 + 272520*x^2 + 2959330*
x + 4880551)*sqrt(2*x^2 - x + 3))/(4*x^2 + 20*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{(2x + 5)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**3,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**3
, x)
```

Giac [B] time = 1.27493, size = 348, normalized size = 2.3

$$\frac{1}{768}\left(4\left(40\,x-467\right)x+19695\right)\sqrt{2\,x^{2}-x+3}+\frac{117315}{1024}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^{2}-x+3}\right)+1\right)-\frac{12670805}{110592}\,\sqrt{2}\log\left(\left|-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/768*(4*(40*x - 467)*x + 19695)*sqrt(2*x^2 - x + 3) + 117315/1024*sqrt(2)*
log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 12670805/110592*sqr
t(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 12670805/11
0592*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/9216*sqrt(2)*(10693526*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 7989
5946*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 124044603*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 80334011)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10
*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

3.329 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx$

Optimal. Leaf size=158

$$-\frac{6467659\left(2x^2-x+3\right)^{3/2}}{5971968(2x+5)}+\frac{158527\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}-\frac{(44378877-7400779x)\sqrt{2x^2-x+3}}{5971968}$$

```
[Out] -((44378877 - 7400779*x)*Sqrt[3 - x + 2*x^2])/5971968 - (3667*(3 - x + 2*x^
2)^(3/2))/(1728*(5 + 2*x)^3) + (158527*(3 - x + 2*x^2)^(3/2))/(82944*(5 + 2
*x)^2) - (6467659*(3 - x + 2*x^2)^(3/2))/(5971968*(5 + 2*x)) - (10939*ArcSi
nh[(1 - 4*x)/Sqrt[23]])/(256*Sqrt[2]) + (170114729*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3981312*Sqrt[2])
```

Rubi [A] time = 0.22581, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{6467659\left(2x^2-x+3\right)^{3/2}}{5971968(2x+5)}+\frac{158527\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}-\frac{(44378877-7400779x)\sqrt{2x^2-x+3}}{5971968}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```


```
[Out] -((44378877 - 7400779*x)*Sqrt[3 - x + 2*x^2])/5971968 - (3667*(3 - x + 2*x^
2)^(3/2))/(1728*(5 + 2*x)^3) + (158527*(3 - x + 2*x^2)^(3/2))/(82944*(5 + 2
*x)^2) - (6467659*(3 - x + 2*x^2)^(3/2))/(5971968*(5 + 2*x)) - (10939*ArcSi
nh[(1 - 4*x)/Sqrt[23]])/(256*Sqrt[2]) + (170114729*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3981312*Sqrt[2])
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\sqrt{3-x+2x^2}\left(\frac{36021}{16}-3969x+1458x^2\right.}{(5+2x)^3}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{2672127}{16}-\right.}{(5+2}}{311}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2} - \frac{6467659\left(3-x+\right.}{5971968(5+2}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{1585}{8}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{1585}{8}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{1585}{8}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{1585}{8}$$

Mathematica [A] time = 0.158555, size = 98, normalized size = 0.62

$$\frac{\frac{24\sqrt{2x^2-x+3}\left(414720x^4-5453568x^3-97682900x^2-329667508x-327735797\right)}{(2x+5)^3}+170114729\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)-170123328\sqrt{2}\sinh^{-}}{7962624}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(-327735797 - 329667508*x - 97682900*x^2 - 5453568
*x^3 + 414720*x^4))/(5 + 2*x)^3 - 170123328*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[
```

```
23]] + 170114729*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/7
962624
```

Maple [A] time = 0.064, size = 165, normalized size = 1.

$$\frac{-5+20x}{128}\sqrt{2x^2-x+3}+\frac{10939\sqrt{2}}{512}\operatorname{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)+\frac{158527}{331776}\left(2\,(x+5/2)^2-11\,x-\frac{19}{2}\right)^{\frac{3}{2}}\left(x+\frac{5}{2}\right)^{-2}-\frac{6467}{11943}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x)
```

```
[Out] 5/128*(-1+4*x)*(2*x^2-x+3)^(1/2)+10939/512*2^(1/2)*arcsinh(4/23*23^(1/2)*(x
-1/4))+158527/331776/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(3/2)-6467659/119439
36/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)-170114729/23887872*(2*(x+5/2)^2-11
*x-19/2)^(1/2)+170114729/7962624*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(
2*(x+5/2)^2-11*x-19/2)^(1/2))+6467659/23887872*(-1+4*x)*(2*(x+5/2)^2-11*x-1
9/2)^(1/2)-3667/13824/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A] time = 1.56178, size = 216, normalized size = 1.37

$$\frac{5}{32}\sqrt{2x^2-x+3}x+\frac{10939}{512}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{170114729}{7962624}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)-\frac{693}{1658}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2*x^2 - x + 3)*x + 10939/512*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 170114729/7962624*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x +
 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 693775/165888*sqrt(2*x^2 - x + 3) - 36
67/1728*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 158527/82944
*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 6467659/331776*sqrt(2*x^2 - x
+ 3)/(2*x + 5)
```

Fricas [A] time = 1.42033, size = 543, normalized size = 3.44

$$\frac{170123328\,\sqrt{2}\left(8\,x^3+60\,x^2+150\,x+125\right)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+170114729\,\sqrt{2}\left(8\,x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/15925248*(170123328*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 170114729*sqrt(2)*(8
*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17
) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(414720*x^4 - 54535
68*x^3 - 97682900*x^2 - 329667508*x - 327735797)*sqrt(2*x^2 - x + 3))/(8*x^
3 + 60*x^2 + 150*x + 125)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**4,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**4
, x)
```

Giac [B] time = 1.3983, size = 410, normalized size = 2.59

$$\frac{1}{128}\sqrt{2\,x^2-x+3}(20\,x-413)-\frac{10939}{512}\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/128*sqrt(2*x^2 - x + 3)*(20*x - 413) - 10939/512*sqrt(2)*log(-2*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 170114729/7962624*sqrt(2)*log(abs(-
2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 170114729/7962624*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/663552*sqr
t(2)*(575810908*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 9206213116*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 9688786604*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 73157325092*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 4948195
2947*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 20269228621)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^3
```

3.330 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx$

Optimal. Leaf size=165

$$-\frac{9363383\left(2x^2-x+3\right)^{3/2}}{23887872(2x+5)^2}+\frac{593771\left(2x^2-x+3\right)^{3/2}}{497664(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}+\frac{7(9616196x+52836655)\sqrt{2x^2-x+3}}{95551488(2x+5)}$$

```
[Out] (7*(52836655 + 9616196*x)*Sqrt[3 - x + 2*x^2])/(95551488*(5 + 2*x)) - (3667
*(3 - x + 2*x^2)^(3/2))/(2304*(5 + 2*x)^4) + (593771*(3 - x + 2*x^2)^(3/2))
/(497664*(5 + 2*x)^3) - (9363383*(3 - x + 2*x^2)^(3/2))/(23887872*(5 + 2*x)
^2) + (259*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) - (4640586097*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1146617856*Sqrt[2])
```

Rubi [A] time = 0.234099, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{9363383\left(2x^2-x+3\right)^{3/2}}{23887872(2x+5)^2}+\frac{593771\left(2x^2-x+3\right)^{3/2}}{497664(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}+\frac{7(9616196x+52836655)\sqrt{2x^2-x+3}}{95551488(2x+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

```
[Out] (7*(52836655 + 9616196*x)*Sqrt[3 - x + 2*x^2])/(95551488*(5 + 2*x)) - (3667
*(3 - x + 2*x^2)^(3/2))/(2304*(5 + 2*x)^4) + (593771*(3 - x + 2*x^2)^(3/2))
/(497664*(5 + 2*x)^3) - (9363383*(3 - x + 2*x^2)^(3/2))/(23887872*(5 + 2*x)
^2) + (259*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) - (4640586097*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1146617856*Sqrt[2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\sqrt{3-x+2x^2}\left(\frac{44361}{16}-\frac{17501x}{4}+1944x^2\right.}{(5+2x)^4}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{593771\left(3-x+2x^2\right)^{3/2}}{497664(5+2x)^3} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{4140069}{16}-\right.}{(5+2}}{622}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{593771\left(3-x+2x^2\right)^{3/2}}{497664(5+2x)^3} - \frac{9363383\left(3-x+\right.}{23887872(5+}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{5937}{49}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{5937}{49}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{5937}{49}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{5937}{49}$$

Mathematica [A] time = 0.178584, size = 98, normalized size = 0.59

$$\frac{\frac{24\sqrt{2x^2-x+3}\left(238878720x^4+6105343976x^3+31323229164x^2+62847867486x+44676885233\right)}{(2x+5)^4} - 4640586097\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 46402}{2293235712}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(44676885233 + 62847867486*x + 31323229164*x^2 + 6
105343976*x^3 + 238878720*x^4))/(5 + 2*x)^4 + 4640219136*Sqrt[2]*ArcSinh[(1
 - 4*x)/Sqrt[23]] - 4640586097*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x
```

```
+ 4*x^2])])/2293235712
```

Maple [A] time = 0.066, size = 167, normalized size = 1.

$$\frac{201573155}{3439853568}\left(2\,\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}\right)^{\frac{3}{2}}\left(x+\frac{5}{2}\right)^{-1}-\frac{-201573155+806292620\,x}{6879707136}\sqrt{2\,\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}}-\frac{9363383}{95551488}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x)
```

```
[Out] 201573155/3439853568/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)-201573155/687970
7136*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-9363383/95551488/(x+5/2)^2*(2*(
x+5/2)^2-11*x-19/2)^(3/2)-4640586097/2293235712*2^(1/2)*arctanh(1/12*(17/2-
11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-259/128*2^(1/2)*arcsinh(4/23*2
3^(1/2)*(x-1/4))+4640586097/6879707136*(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/3
6864/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(3/2)+593771/3981312/(x+5/2)^3*(2*(x
+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A] time = 1.59831, size = 244, normalized size = 1.48

$$-\frac{259}{128}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\,\sqrt{23}x-\frac{1}{23}\,\sqrt{23}\right)+\frac{4640586097}{2293235712}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{16828343}{47775744}\,\sqrt{2\,x^{2}-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] -259/128*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 4640586097/2293
235712*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2
*x + 5)) + 16828343/47775744*sqrt(2*x^2 - x + 3) - 3667/2304*(2*x^2 - x + 3
)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 593771/497664*(2*x^2
- x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 9363383/23887872*(2*x^2 - x
 + 3)^(3/2)/(4*x^2 + 20*x + 25) + 201573155/95551488*sqrt(2*x^2 - x + 3)/(2
*x + 5)
```

Fricas [A] time = 1.4576, size = 618, normalized size = 3.75

$$4640219136\sqrt{2}\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+464058$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="f
ricas")
```

```
[Out] 1/4586471424*(4640219136*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625
)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 46405
86097*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*
sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x +
 25)) + 48*(238878720*x^4 + 6105343976*x^3 + 31323229164*x^2 + 62847867486*
x + 44676885233)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x
+ 625)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**5,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**5
, x)
```

Giac [B] time = 1.33849, size = 441, normalized size = 2.67

$$-\frac{1}{2293235712}\sqrt{2}\left(4640586097\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+4640219136\log\left(\left|\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="g
iac")
```

```
[Out] -1/2293235712*sqrt(2)*(4640586097*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^
2 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 4640219136*log(abs(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 4640
219136*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))
*sgn(1/(2*x + 5)) + 12*(24*(144*(792072*sgn(1/(2*x + 5))/(2*x + 5) - 835793
*sgn(1/(2*x + 5)))/(2*x + 5) + 57384361*sgn(1/(2*x + 5)))/(2*x + 5) - 46456
9597*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 179159040
*(11*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x +
5)) - 12*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(
2*x + 5))^2 - 1))
```

3.331 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

Optimal. Leaf size=165

$$-\frac{38732321\left(2x^2-x+3\right)^{3/2}}{179159040(2x+5)^3}+\frac{711961\left(2x^2-x+3\right)^{3/2}}{829440(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}-\frac{(3174439702x+4583087983)\sqrt{2x^2-}}{6879707136(2x+5)^2}$$

```
[Out] -((4583087983 + 3174439702*x)*Sqrt[3 - x + 2*x^2])/(6879707136*(5 + 2*x)^2)
 - (3667*(3 - x + 2*x^2)^(3/2))/(2880*(5 + 2*x)^5) + (711961*(3 - x + 2*x^2
)^(3/2))/(829440*(5 + 2*x)^4) - (38732321*(3 - x + 2*x^2)^(3/2))/(179159040
*(5 + 2*x)^3) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2]) + (12895597463
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(82556485632*Sqrt[2
])
```

Rubi [A] time = 0.22853, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 810, 843, 619, 215, 724, 206}

$$-\frac{38732321\left(2x^2-x+3\right)^{3/2}}{179159040(2x+5)^3}+\frac{711961\left(2x^2-x+3\right)^{3/2}}{829440(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}-\frac{(3174439702x+4583087983)\sqrt{2x^2-}}{6879707136(2x+5)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] -((4583087983 + 3174439702*x)*Sqrt[3 - x + 2*x^2])/(6879707136*(5 + 2*x)^2)
 - (3667*(3 - x + 2*x^2)^(3/2))/(2880*(5 + 2*x)^5) + (711961*(3 - x + 2*x^2
)^(3/2))/(829440*(5 + 2*x)^4) - (38732321*(3 - x + 2*x^2)^(3/2))/(179159040
*(5 + 2*x)^3) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2]) + (12895597463
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(82556485632*Sqrt[2
])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
```

```
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} - \frac{1}{360}\int \frac{\sqrt{3-x+2x^2}\left(\frac{52701}{16}-\frac{9563x}{2}+2430x^2\right.}{(5+2x)^5}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{5935131}{16}-\right.}{(5+2}}{103}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} - \frac{38732321\left(3-x+\right.}{179159040(5+}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5}$$

Mathematica [A] time = 0.212094, size = 98, normalized size = 0.59

$$\frac{-\frac{24\sqrt{2x^2-x+3}\left(186470433136x^4+1285267446304x^3+3919478861832x^2+5608297138216x+3110673952831\right)}{(2x+5)^5} + 64477987315\sqrt{2}\tanh^{-1}\left(\frac{17-22}{12\sqrt{4x^2-2}}\right.}{825564856320}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x
]
```

```
[Out] ((-24*Sqrt[3 - x + 2*x^2]*(3110673952831 + 5608297138216*x + 3919478861832*
x^2 + 1285267446304*x^3 + 186470433136*x^4))/(5 + 2*x)^5 - 64497254400*Sqrt
[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 64477987315*Sqrt[2]*ArcTanh[(17 - 22*x)/(
12*Sqrt[6 - 2*x + 4*x^2])])/825564856320
```

Maple [A] time = 0.074, size = 188, normalized size = 1.1

$$-\frac{562688629}{247669456896}\left(2\,(x+5/2)^2-11\,x-\frac{19}{2}\right)^{\frac{3}{2}}\left(x+\frac{5}{2}\right)^{-1}+\frac{-562688629+2250754516\,x}{495338913792}\sqrt{2\,(x+5/2)^2-11\,x-\frac{19}{2}}+\frac{46}{687}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x)
```

```
[Out] -562688629/247669456896/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+562688629/495
338913792*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+46569601/6879707136/(x+5/2
)^2*(2*(x+5/2)^2-11*x-19/2)^(3/2)+12895597463/165112971264*2^(1/2)*arctanh(
1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+5/64*2^(1/2)*arcsin
h(4/23*23^(1/2)*(x-1/4))-12895597463/495338913792*(2*(x+5/2)^2-11*x-19/2)^(
1/2)+711961/13271040/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(3/2)-38732321/14332
72320/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(3/2)-3667/92160/(x+5/2)^5*(2*(x+5/
2)^2-11*x-19/2)^(3/2)
```

Maxima [A] time = 1.61647, size = 300, normalized size = 1.82

$$\frac{5}{64}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{12895597463}{165112971264}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{46569601}{3439853568}\sqrt{2\,x^2-x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] 5/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 12895597463/1651129
71264*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*
x + 5)) - 46569601/3439853568*sqrt(2*x^2 - x + 3) - 3667/2880*(2*x^2 - x +
3)^(3/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 711961/
```

```
829440*(2*x^2 - x + 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) -
38732321/179159040*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 4
6569601/1719926784*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 562688629/68
79707136*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Fricas [A] time = 1.40667, size = 694, normalized size = 4.21

$$\frac{64497254400\sqrt{2}\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/1651129712640*(64497254400*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^
2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
16*x - 25) + 64477987315*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(186470433136*x^4 + 1285267446304
*x^3 + 3919478861832*x^2 + 5608297138216*x + 3110673952831)*sqrt(2*x^2 - x
+ 3))/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**6,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**6
, x)
```

Giac [B] time = 1.29892, size = 522, normalized size = 3.16

$$-\frac{5}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{12895597463}{165112971264}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{12895597}{165112971}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] -5/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 12895
597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 12895597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt
(2) + 2*sqrt(2*x^2 - x + 3))) - 1/68797071360*sqrt(2)*(4368922304720*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 124570969998480*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^8 + 637804348664160*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^7 + 1828845222532320*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 37631893001870
16*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 10794416351958120*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^4 + 25049834283305880*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^3 - 34708488692384520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 +
10654664764755165*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 2507056315485
767)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^5
```

3.332 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx$

Optimal. Leaf size=169

$$\frac{87677717\left(2x^2-x+3\right)^{3/2}}{8599633920(2x+5)^3}-\frac{5703277\left(2x^2-x+3\right)^{3/2}}{39813120(2x+5)^4}+\frac{92239\left(2x^2-x+3\right)^{3/2}}{138240(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}-\frac{1172725(17}{330225}$$

```
[Out] (-1172725*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(330225942528*(5 + 2*x)^2) - (36
67*(3 - x + 2*x^2)^(3/2))/(3456*(5 + 2*x)^6) + (92239*(3 - x + 2*x^2)^(3/2)
)/(138240*(5 + 2*x)^5) - (5703277*(3 - x + 2*x^2)^(3/2))/(39813120*(5 + 2*x
)^4) + (87677717*(3 - x + 2*x^2)^(3/2))/(8599633920*(5 + 2*x)^3) - (2697267
5*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3962711310336*Sqr
t[2])
```

Rubi [A] time = 0.218244, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1650, 806, 720, 724, 206}

$$\frac{87677717\left(2x^2-x+3\right)^{3/2}}{8599633920(2x+5)^3}-\frac{5703277\left(2x^2-x+3\right)^{3/2}}{39813120(2x+5)^4}+\frac{92239\left(2x^2-x+3\right)^{3/2}}{138240(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}-\frac{1172725(17}{330225}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

```
[Out] (-1172725*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(330225942528*(5 + 2*x)^2) - (36
67*(3 - x + 2*x^2)^(3/2))/(3456*(5 + 2*x)^6) + (92239*(3 - x + 2*x^2)^(3/2)
)/(138240*(5 + 2*x)^5) - (5703277*(3 - x + 2*x^2)^(3/2))/(39813120*(5 + 2*x
)^4) + (87677717*(3 - x + 2*x^2)^(3/2))/(8599633920*(5 + 2*x)^3) - (2697267
5*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3962711310336*Sqr
t[2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
```

```
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{\sqrt{3-x+2x^2}\left(\frac{61041}{16}-\frac{20751x}{4}+2916x^2\right.}{(5+2x)^6}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{8057313}{16}-\frac{1}{}\right.}{(5+2x}}{1555}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} - \frac{5703277\left(3-x+2\right.}{39813120(5+2x}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} - \frac{5703277\left(3-x+2\right.}{39813120(5+2x}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-\right.}{138240}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-\right.}{138240}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-\right.}{138240}$$

Mathematica [A] time = 0.172825, size = 91, normalized size = 0.54

$$\frac{24\sqrt{2x^2-x+3}\left(271409942624x^5+12256250416x^4+397498825328x^3+158340720344x^2+27245373694x-21933\right.}{39627113103360(2x+5)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x
]
```

```
[Out] (24*Sqrt[3 - x + 2*x^2]*(-219337079305 + 27245373694*x + 158340720344*x^2 +
 397498825328*x^3 + 12256250416*x^4 + 271409942624*x^5) - 134863375*Sqrt[2]
*(5 + 2*x)^6*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/(396271131033
60*(5 + 2*x)^6)
```

Maple [A] time = 0.089, size = 195, normalized size = 1.2

$$-\frac{3667}{221184}\left(2\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}\right)^{\frac{3}{2}}\left(x+\frac{5}{2}\right)^{-6}-\frac{12899975}{11888133931008}\left(2\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}\right)^{\frac{3}{2}}\left(x+\frac{5}{2}\right)^{-1}+\frac{-12899975}{237762}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x)
```

```
[Out] -3667/221184/(x+5/2)^6*(2*(x+5/2)^2-11*x-19/2)^(3/2)-12899975/1188813393100
8/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+12899975/23776267862016*(-1+4*x)*(2
*(x+5/2)^2-11*x-19/2)^(1/2)-1172725/330225942528/(x+5/2)^2*(2*(x+5/2)^2-11*
x-19/2)^(3/2)-26972675/7925422620672*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/
2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+26972675/23776267862016*(2*(x+5/2)^2-11*x
-19/2)^(1/2)-5703277/637009920/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(3/2)+8767
7717/68797071360/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(3/2)+92239/4423680/(x+5
/2)^5*(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A] time = 1.60522, size = 338, normalized size = 2.

$$\frac{26972675}{7925422620672}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{1172725}{165112971264}\sqrt{2\,x^{2}-x+3}-\frac{366}{3456\left(64\,x^{6}+960\,x^{5}+6000\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 26972675/7925422620672*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/2
3*sqrt(23)/abs(2*x + 5)) + 1172725/165112971264*sqrt(2*x^2 - x + 3) - 3667/
3456*(2*x^2 - x + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500
*x^2 + 37500*x + 15625) + 92239/138240*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*
x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) - 5703277/39813120*(2*x^2 - x +
3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 87677717/8599633920*
(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 1172725/82556485632*
(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 12899975/330225942528*sqrt(2*x^
2 - x + 3)/(2*x + 5)
```

Fricas [A] time = 1.35957, size = 555, normalized size = 3.28

$$\frac{134863375\sqrt{2}\left(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-}{4x^2+20x+25}\right.}{79254226206720\left(64x^6+960\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/79254226206720*(134863375*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^
3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x
 - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(271409942624*
x^5 + 12256250416*x^4 + 397498825328*x^3 + 158340720344*x^2 + 27245373694*x
 - 219337079305)*sqrt(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*
x^3 + 37500*x^2 + 37500*x + 15625)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**7,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**7
, x)
```

Giac [B] time = 1.30763, size = 547, normalized size = 3.24

$$-\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] -26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) + 26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sq
rt(2) + 2*sqrt(2*x^2 - x + 3))) + 1/3302259425280*sqrt(2)*(16506981498400*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^11 + 389429252643040*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^10 + 2263923918689840*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3))^9 + 11663651054548560*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 90221
2326134736*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 84192729519861840*
(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 4317200555009448*sqrt(2)*(sqrt(2)*x -
 sqrt(2*x^2 - x + 3))^5 + 351543414066518760*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^4 - 376787166452923830*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 35
6306707647610982*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 82348353128195465*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 15499394004553969)/(2*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 1
1)^6
```

3.333 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8} dx$

Optimal. Leaf size=194

$$\frac{246159769\left(2x^2-x+3\right)^{3/2}}{866843099136(2x+5)^3}+\frac{19414831\left(2x^2-x+3\right)^{3/2}}{4013162496(2x+5)^4}-\frac{1464037\left(2x^2-x+3\right)^{3/2}}{13934592(2x+5)^5}+\frac{948341\left(2x^2-x+3\right)^{3/2}}{1741824(2x+5)^6}-\frac{3667}{4}$$

```
[Out] (-12568315*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(23776267862016*(5 + 2*x)^2) -
(3667*(3 - x + 2*x^2)^(3/2))/(4032*(5 + 2*x)^7) + (948341*(3 - x + 2*x^2)^(
3/2))/(1741824*(5 + 2*x)^6) - (1464037*(3 - x + 2*x^2)^(3/2))/(13934592*(5
+ 2*x)^5) + (19414831*(3 - x + 2*x^2)^(3/2))/(4013162496*(5 + 2*x)^4) + (24
6159769*(3 - x + 2*x^2)^(3/2))/(866843099136*(5 + 2*x)^3) - (289071245*ArcT
anh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(285315214344192*Sqrt[2]
)
```

Rubi [A] time = 0.267503, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{246159769\left(2x^2-x+3\right)^{3/2}}{866843099136(2x+5)^3}+\frac{19414831\left(2x^2-x+3\right)^{3/2}}{4013162496(2x+5)^4}-\frac{1464037\left(2x^2-x+3\right)^{3/2}}{13934592(2x+5)^5}+\frac{948341\left(2x^2-x+3\right)^{3/2}}{1741824(2x+5)^6}-\frac{3667}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```


```
[Out] (-12568315*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(23776267862016*(5 + 2*x)^2) -
(3667*(3 - x + 2*x^2)^(3/2))/(4032*(5 + 2*x)^7) + (948341*(3 - x + 2*x^2)^(
3/2))/(1741824*(5 + 2*x)^6) - (1464037*(3 - x + 2*x^2)^(3/2))/(13934592*(5
+ 2*x)^5) + (19414831*(3 - x + 2*x^2)^(3/2))/(4013162496*(5 + 2*x)^4) + (24
6159769*(3 - x + 2*x^2)^(3/2))/(866843099136*(5 + 2*x)^3) - (289071245*ArcT
anh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(285315214344192*Sqrt[2]
)
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
```

```
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} - \frac{1}{504}\int \frac{\sqrt{3-x+2x^2}\left(\frac{69381}{16}-5594x+3402x\right.}{(5+2x)^7}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{10506615}{16}\right.}{(5+}}{217}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} - \frac{1464037\left(3-x+\right.}{13934592(5+}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} - \frac{1464037\left(3-x+\right.}{13934592(5+}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} - \frac{1464037\left(3-x+\right.}{13934592(5+}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(\right.}{17418}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(\right.}{17418}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(\right.}{17418}$$

Mathematica [A] time = 0.206567, size = 96, normalized size = 0.49

$$\frac{24\sqrt{2x^2-x+3}\left(1574342277056x^6+27976951397184x^5+4982916071952x^4+41058010262368x^3+1471668378003\right.}{3994413000818688(2x+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x
]
```

```
[Out] (24*Sqrt[3 - x + 2*x^2]*(-20465234808721 + 590492177460*x + 14716683780036*
x^2 + 41058010262368*x^3 + 4982916071952*x^4 + 27976951397184*x^5 + 1574342
277056*x^6) - 2023498715*Sqrt[2]*(5 + 2*x)^7*ArcTanh[(17 - 22*x)/(12*Sqrt[6
 - 2*x + 4*x^2])])/(3994413000818688*(5 + 2*x)^7)
```

Maple [A] time = 0.078, size = 216, normalized size = 1.1

$$\frac{948341}{111476736}\left(2\ (x+5/2)^2-11\,x-\frac{19}{2}\right)^{\frac{3}{2}}\left(x+\frac{5}{2}\right)^{-6}-\frac{138251465}{855945643032576}\left(2\ (x+5/2)^2-11\,x-\frac{19}{2}\right)^{\frac{3}{2}}\left(x+\frac{5}{2}\right)^{-1}+\frac{-13825}{171}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x)
```

```
[Out] 948341/111476736/(x+5/2)^6*(2*(x+5/2)^2-11*x-19/2)^(3/2)-138251465/85594564
3032576/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+138251465/1711891286065152*(-
1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-12568315/23776267862016/(x+5/2)^2*(2*(
x+5/2)^2-11*x-19/2)^(3/2)-289071245/570630428688384*2^(1/2)*arctanh(1/12*(1
7/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+289071245/1711891286065152
*(2*(x+5/2)^2-11*x-19/2)^(1/2)+19414831/64210599936/(x+5/2)^4*(2*(x+5/2)^2-
11*x-19/2)^(3/2)+246159769/6934744793088/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^
(3/2)-3667/516096/(x+5/2)^7*(2*(x+5/2)^2-11*x-19/2)^(3/2)-1464037/445906944
/(x+5/2)^5*(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A] time = 1.54707, size = 406, normalized size = 2.09

$$\frac{289071245}{570630428688384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{12568315}{11888133931008}\sqrt{2\,x^2-x+3}-\frac{}{4032\left(128\,x^7+2240\,x^6+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] 289071245/570630428688384*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) + 12568315/11888133931008*sqrt(2*x^2 - x + 3) -
 3667/4032*(2*x^2 - x + 3)^(3/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^
```

```
4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125) + 948341/1741824*(2*x^2 - x
 + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x
+ 15625) - 1464037/13934592*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*x^4 + 2000*
x^3 + 5000*x^2 + 6250*x + 3125) + 19414831/4013162496*(2*x^2 - x + 3)^(3/2)
/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 246159769/866843099136*(2*x^
2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 12568315/5944066965504*(2
*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 138251465/23776267862016*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

Fricas [A] time = 1.43963, size = 649, normalized size = 3.35

$$\frac{2023498715\sqrt{2}\left(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x}}{}\right.}{79888260016}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/79888260016637376*(2023498715*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 + 70
000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-(24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)
) + 48*(1574342277056*x^6 + 27976951397184*x^5 + 4982916071952*x^4 + 410580
10262368*x^3 + 14716683780036*x^2 + 590492177460*x - 20465234808721)*sqrt(2
*x^2 - x + 3))/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 2
62500*x^2 + 218750*x + 78125)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**8,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**8
, x)
```

Giac [B] time = 1.34148, size = 616, normalized size = 3.18

$$-\frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(
2*x^2 - x + 3))) + 289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x -
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/332867750068224*sqrt(2)*(12950391
7760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 - 3320259746027840*(sqrt(
2)*x - sqrt(2*x^2 - x + 3))^12 - 23966708071916736*sqrt(2)*(sqrt(2)*x - sqr
t(2*x^2 - x + 3))^11 - 186055342532355520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^10 - 274256644494948976*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 7961
35370176031760*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 2531523139171005408*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 4610393811900786336*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^6 - 7997126854300052364*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^5 + 30842713619423538868*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4
- 21873571601855032556*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 162047
06960604668100*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 3196254593191113265*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 536799032216117911)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^7
```

3.334 $\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

Optimal. Leaf size=166

$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4-\frac{1121\left(2x^2-x+3\right)^{5/2}(2x+5)^3}{2304}+\frac{69415\left(2x^2-x+3\right)^{5/2}(2x+5)^2}{32256}-\frac{3(215900x+6613}{14}$$

```
[Out] (-6398163*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (92727*(1 - 4*x)*(3 - x
+ 2*x^2)^(3/2))/131072 + (69415*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/32256 -
(1121*(5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))/2304 + (5*(5 + 2*x)^4*(3 - x + 2*x
^2)^(5/2))/144 - (3*(661397 + 215900*x)*(3 - x + 2*x^2)^(5/2))/143360 - (14
7157749*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

Rubi [A] time = 0.194405, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1653, 779, 612, 619, 215}

$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4-\frac{1121\left(2x^2-x+3\right)^{5/2}(2x+5)^3}{2304}+\frac{69415\left(2x^2-x+3\right)^{5/2}(2x+5)^2}{32256}-\frac{3(215900x+6613}{14}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-6398163*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (92727*(1 - 4*x)*(3 - x
+ 2*x^2)^(3/2))/131072 + (69415*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/32256 -
(1121*(5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))/2304 + (5*(5 + 2*x)^4*(3 - x + 2*x
^2)^(5/2))/144 - (3*(661397 + 215900*x)*(3 - x + 2*x^2)^(5/2))/143360 - (14
7157749*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
```

```
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2}+\frac{1}{288}\int (5+2x)\left(3-x+2x^2\right.$$

$$= -\frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304}+\frac{5}{144}(5+2x)^4\left(3-x+2x^2\right.$$

$$= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2x)^3\left(3-x+2x\right.}{2304}$$

$$= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2x)^3\left(3-x+2x\right.}{2304}$$

$$= -\frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072}+\frac{69415(5+2x)^2\left(3-x+2\right.}{32256}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3-x+2x\right.}{131072}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3-x+2x\right.}{131072}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3-x+2x\right.}{131072}$$

Mathematica [A] time = 0.190871, size = 80, normalized size = 0.48

$$\frac{4\sqrt{2x^2-x+3}\left(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290112x^3\right.}{2642411520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(1592737263 + 12357760788*x + 4870637856*x^2 + 12669
290112*x^3 + 379086848*x^4 + 12117893120*x^5 + 1033175040*x^6 + 2926837760*
x^7 + 1468006400*x^8) - 46354690935*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/26
42411520
```

Maple [A] time = 0.059, size = 134, normalized size = 0.8

$$\frac{5x^4}{9}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{479x^3}{288}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{2005x^2}{8064}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{5645x}{21504}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{-6398163+25592}{2097152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

```
[Out] 5/9*x^4*(2*x^2-x+3)^(5/2)+479/288*x^3*(2*x^2-x+3)^(5/2)+2005/8064*x^2*(2*x^
2-x+3)^(5/2)+5645/21504*x*(2*x^2-x+3)^(5/2)+6398163/2097152*(-1+4*x)*(2*x^2
-x+3)^(1/2)+147157749/8388608*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+92727/
131072*(-1+4*x)*(2*x^2-x+3)^(3/2)+120809/143360*(2*x^2-x+3)^(5/2)
```

Maxima [A] time = 1.58366, size = 209, normalized size = 1.26

$$\frac{5}{9}\left(2x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{479}{288}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{2005}{8064}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{5645}{21504}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{120809}{143360}\left(2x^2-x+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 5/9*(2*x^2 - x + 3)^(5/2)*x^4 + 479/288*(2*x^2 - x + 3)^(5/2)*x^3 + 2005/80
64*(2*x^2 - x + 3)^(5/2)*x^2 + 5645/21504*(2*x^2 - x + 3)^(5/2)*x + 120809/
143360*(2*x^2 - x + 3)^(5/2) + 92727/32768*(2*x^2 - x + 3)^(3/2)*x - 92727/
131072*(2*x^2 - x + 3)^(3/2) + 6398163/524288*sqrt(2*x^2 - x + 3)*x + 14715
7749/8388608*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6398163/2097152*sqr
t(2*x^2 - x + 3)
```

Fricas [A] time = 1.32289, size = 375, normalized size = 2.26

$$\frac{1}{660602880}\left(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290112x^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 1/660602880*(1468006400*x^8 + 2926837760*x^7 + 1033175040*x^6 + 12117893120
*x^5 + 379086848*x^4 + 12669290112*x^3 + 4870637856*x^2 + 12357760788*x + 1
592737263)*sqrt(2*x^2 - x + 3) + 147157749/16777216*sqrt(2)*log(-4*sqrt(2)*
```

```
sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+5)\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

```
[Out] Integral((2*x + 5)*(2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)
, x)
```

Giac [A] time = 1.22369, size = 119, normalized size = 0.72

$$\frac{1}{660602880}\,(4\,(8\,(4\,(16\,(20\,(8\,(28\,(160\,x+319)x+3153)x+295847)x+185101)x+98978829)x+152207433)x+308$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 1/660602880*(4*(8*(4*(16*(20*(8*(28*(160*x + 319)*x + 3153)*x + 295847)*x +
 185101)*x + 98978829)*x + 152207433)*x + 3089440197)*x + 1592737263)*sqrt(
2*x^2 - x + 3) - 147157749/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3)) + 1)
```

3.335 $\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)dx$

Optimal. Leaf size=147

$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3+\frac{23}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{125\left(2x^2-x+3\right)^{5/2}x}{3584}+\frac{1167\left(2x^2-x+3\right)^{5/2}}{14336}-\frac{8597(1-4x)\left(2x^2-x\right.}{65536}$$

```
[Out] (-593193*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 - (8597*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/65536 + (1167*(3 - x + 2*x^2)^(5/2))/14336 + (125*x*(3 - x +
2*x^2)^(5/2))/3584 + (23*x^2*(3 - x + 2*x^2)^(5/2))/448 + (5*x^3*(3 - x + 2
*x^2)^(5/2))/16 - (13643439*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*Sqrt[2])
```

Rubi [A] time = 0.121456, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3+\frac{23}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{125\left(2x^2-x+3\right)^{5/2}x}{3584}+\frac{1167\left(2x^2-x+3\right)^{5/2}}{14336}-\frac{8597(1-4x)\left(2x^2-x\right.}{65536}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-593193*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 - (8597*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/65536 + (1167*(3 - x + 2*x^2)^(5/2))/14336 + (125*x*(3 - x +
2*x^2)^(5/2))/3584 + (23*x^2*(3 - x + 2*x^2)^(5/2))/448 + (5*x^3*(3 - x + 2
*x^2)^(5/2))/16 - (13643439*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)dx = \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{16}\int \left(3-x+2x^2\right)^{3/2}\left(32+16x+3x^2+\frac{23x^3}{2}\right.$$

$$= \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{224}\int \left(3-x+2x^2\right)$$

$$= \frac{125x\left(3-x+2x^2\right)^{5/2}}{3584}+\frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2}$$

$$= \frac{1167\left(3-x+2x^2\right)^{5/2}}{14336}+\frac{125x\left(3-x+2x^2\right)^{5/2}}{3584}+\frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2}$$

$$= -\frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536}+\frac{1167\left(3-x+2x^2\right)^{5/2}}{14336}+\frac{125x\left(3-x\right.}{3584}$$

$$= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536}+\frac{1167}{}$$

$$= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536}+\frac{1167}{}$$

$$= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536}+\frac{1167}{}$$

Mathematica [A] time = 0.129866, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2-x+3}\left(9175040x^7-7667712x^6+29335552x^5-7497728x^4+27023744x^3+3845856x^2+27845612x-166340\right.}{29360128}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1663407 + 27845612*x + 3845856*x^2 + 27023744*x^3
- 7497728*x^4 + 29335552*x^5 - 7667712*x^6 + 9175040*x^7) - 95504073*Sqrt[2
]*ArcSinh[(1 - 4*x)/Sqrt[23]])/29360128
```

Maple [A] time = 0.055, size = 117, normalized size = 0.8

$$\frac{5x^3}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{23x^2}{448}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{125x}{3584}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{-593193+2372772x}{1048576}\sqrt{2x^2-x+3}+\frac{13643439}{4194304}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

```
[Out] 5/16*x^3*(2*x^2-x+3)^(5/2)+23/448*x^2*(2*x^2-x+3)^(5/2)+125/3584*x*(2*x^2-x
+3)^(5/2)+593193/1048576*(-1+4*x)*(2*x^2-x+3)^(1/2)+13643439/4194304*2^(1/2
)*arcsinh(4/23*23^(1/2)*(x-1/4))+8597/65536*(-1+4*x)*(2*x^2-x+3)^(3/2)+1167
/14336*(2*x^2-x+3)^(5/2)
```

Maxima [A] time = 1.47795, size = 186, normalized size = 1.27

$$\frac{5}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{23}{448}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{125}{3584}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{1167}{14336}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{8597}{16384}\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 5/16*(2*x^2 - x + 3)^(5/2)*x^3 + 23/448*(2*x^2 - x + 3)^(5/2)*x^2 + 125/358
4*(2*x^2 - x + 3)^(5/2)*x + 1167/14336*(2*x^2 - x + 3)^(5/2) + 8597/16384*(
2*x^2 - x + 3)^(3/2)*x - 8597/65536*(2*x^2 - x + 3)^(3/2) + 593193/262144*s
qrt(2*x^2 - x + 3)*x + 13643439/4194304*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x
- 1)) - 593193/1048576*sqrt(2*x^2 - x + 3)
```

Fricas [A] time = 1.2855, size = 316, normalized size = 2.15

$$\frac{1}{7340032}\left(9175040x^7-7667712x^6+29335552x^5-7497728x^4+27023744x^3+3845856x^2+27845612x-166340\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/7340032*(9175040*x^7 - 7667712*x^6 + 29335552*x^5 - 7497728*x^4 + 2702374
4*x^3 + 3845856*x^2 + 27845612*x - 1663407)*sqrt(2*x^2 - x + 3) + 13643439/
8388608*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*
x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2x^2 - x + 3\right)^{\frac{3}{2}} \left(5x^4 - x^3 + 3x^2 + x + 2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)
```

Giac [A] time = 1.38808, size = 112, normalized size = 0.76

$$\frac{1}{7340032}(4(8(4(16(4(8(140x - 117)x + 3581)x - 3661)x + 211123)x + 120183)x + 6961403)x - 1663407)\sqrt{2x^2 - x +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/7340032*(4*(8*(4*(16*(4*(8*(140*x - 117)*x + 3581)*x - 3661)*x + 211123)*
x + 120183)*x + 6961403)*x - 1663407)*sqrt(2*x^2 - x + 3) - 13643439/419430
4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.336 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

Optimal. Leaf size=172

$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}-\frac{311}{448}(2x+5)\left(2x^2-x+3\right)^{5/2}+\frac{3505}{896}\left(2x^2-x+3\right)^{5/2}+\frac{(500141-123060x)\left(2x^2-x+\right.}{12288}$$

```
[Out] ((141051019 - 23482924*x)*Sqrt[3 - x + 2*x^2])/65536 + ((500141 - 123060*x)
*(3 - x + 2*x^2)^(3/2))/12288 + (3505*(3 - x + 2*x^2)^(5/2))/896 - (311*(5
+ 2*x)*(3 - x + 2*x^2)^(5/2))/448 + (5*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/1
12 + (1622009981*ArcSinh[(1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2]) - (99009*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(8*Sqrt[2])
```

Rubi [A] time = 0.268163, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}-\frac{311}{448}(2x+5)\left(2x^2-x+3\right)^{5/2}+\frac{3505}{896}\left(2x^2-x+3\right)^{5/2}+\frac{(500141-123060x)\left(2x^2-x+\right.}{12288}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((141051019 - 23482924*x)*Sqrt[3 - x + 2*x^2])/65536 + ((500141 - 123060*x)
*(3 - x + 2*x^2)^(3/2))/12288 + (3505*(3 - x + 2*x^2)^(5/2))/896 - (311*(5
+ 2*x)*(3 - x + 2*x^2)^(5/2))/448 + (5*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/1
12 + (1622009981*ArcSinh[(1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2]) - (99009*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(8*Sqrt[2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
```

```
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}+\frac{1}{224}\int\frac{\left(3-x+2x^2\right)^{3/2}\left(573-9926\right.}{5+2x}$$

$$= -\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2}+\frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}+\frac{\int(3}{}$$

$$= \frac{3505}{896}\left(3-x+2x^2\right)^{5/2}-\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2}+\frac{5}{112}(5+2x)^2$$

$$= \frac{(500141-123060x)\left(3-x+2x^2\right)^{3/2}}{12288}+\frac{3505}{896}\left(3-x+2x^2\right)^{5/2}-\frac{311}{448}($$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x)\left(3-x\right.}{12288}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x)\left(3-x\right.}{12288}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x)\left(3-x\right.}{12288}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x)\left(3-x\right.}{12288}$$

Mathematica [A] time = 0.187786, size = 101, normalized size = 0.59

$$\frac{4\sqrt{2x^2-x+3}\left(983040x^6-3710976x^5+14493696x^4-46476672x^3+159973408x^2-609499532x+3149403255\right)-}{5505024}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x
]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3149403255 - 609499532*x + 159973408*x^2 - 46476672
*x^3 + 14493696*x^4 - 3710976*x^5 + 983040*x^6) + 34062209601*Sqrt[2]*ArcSi
nh[(1 - 4*x)/Sqrt[23]] - 34065432576*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6
 - 2*x + 4*x^2])])/5505024
```

Maple [A] time = 0.056, size = 183, normalized size = 1.1

$$\frac{5x^2}{28}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{111x}{224}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{1395}{896}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{-10255+41020x}{4096}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{-707595+28}{65536}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

```
[Out] 5/28*x^2*(2*x^2-x+3)^(5/2)-111/224*x*(2*x^2-x+3)^(5/2)+1395/896*(2*x^2-x+3)
^(5/2)-10255/4096*(-1+4*x)*(2*x^2-x+3)^(3/2)-707595/65536*(-1+4*x)*(2*x^2-x
+3)^(1/2)-1622009981/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+3667/96*
(2*(x+5/2)^2-11*x-19/2)^(3/2)-40337/512*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1
/2)+33003/16*(2*(x+5/2)^2-11*x-19/2)^(1/2)-99009/16*2^(1/2)*arctanh(1/12*(1
7/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A] time = 1.61481, size = 212, normalized size = 1.23

$$\frac{5}{28}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2-\frac{111}{224}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{1395}{896}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{10255}{1024}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{500141}{12288}\left(2x^2-x+3\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 5/28*(2*x^2 - x + 3)^(5/2)*x^2 - 111/224*(2*x^2 - x + 3)^(5/2)*x + 1395/896
*(2*x^2 - x + 3)^(5/2) - 10255/1024*(2*x^2 - x + 3)^(3/2)*x + 500141/12288*
(2*x^2 - x + 3)^(3/2) - 5870731/16384*sqrt(2*x^2 - x + 3)*x - 1622009981/26
2144*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 99009/16*sqrt(2)*ar
csinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 141051
019/65536*sqrt(2*x^2 - x + 3)
```

Fricas [A] time = 1.44861, size = 462, normalized size = 2.69

$$\frac{1}{1376256}\left(983040\,x^6-3710976\,x^5+14493696\,x^4-46476672\,x^3+159973408\,x^2-609499532\,x+3149403255\right)\sqrt{2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/1376256*(983040*x^6 - 3710976*x^5 + 14493696*x^4 - 46476672*x^3 + 1599734
08*x^2 - 609499532*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/524288*
sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) +
 99009/32*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x
^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{2x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
, x)
```

Giac [A] time = 1.2201, size = 188, normalized size = 1.09

$$\frac{1}{1376256}\left(4\left(8\left(12\left(16\left(4\left(40\,x-151\right)x+2359\right)x-121033\right)x+4999169\right)x-152374883\right)x+3149403255\right)\sqrt{2\,x^2-x+3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/1376256*(4*(8*(12*(16*(4*(40*x - 151)*x + 2359)*x - 121033)*x + 4999169)*
x - 152374883)*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/262144*sqrt
(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 99009/16*sqrt(2)*
log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 99009/16*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

3.337 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

Optimal. Leaf size=172

$$\frac{5}{96}(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}-\frac{839}{960}\left(2x^2-x+3\right)^{5/2}-\frac{(909513-226052x)\left(2x^2-x+3\right)^{3/2}}{18432}-\frac{(8$$

```
[Out] -((85448933 - 14243732*x)*Sqrt[3 - x + 2*x^2])/32768 - ((909513 - 226052*x)
*(3 - x + 2*x^2)^(3/2))/18432 - (839*(3 - x + 2*x^2)^(5/2))/960 - (3667*(3
- x + 2*x^2)^(5/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 - x + 2*x^2)^(5/2))/9
6 - (982669459*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2]) + (959625*ArcTa
nh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(64*Sqrt[2])
```

Rubi [A] time = 0.281693, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{96}(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}-\frac{839}{960}\left(2x^2-x+3\right)^{5/2}-\frac{(909513-226052x)\left(2x^2-x+3\right)^{3/2}}{18432}-\frac{(8$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

```
[Out] -((85448933 - 14243732*x)*Sqrt[3 - x + 2*x^2])/32768 - ((909513 - 226052*x)
*(3 - x + 2*x^2)^(3/2))/18432 - (839*(3 - x + 2*x^2)^(5/2))/960 - (3667*(3
- x + 2*x^2)^(5/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 - x + 2*x^2)^(5/2))/9
6 - (982669459*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2]) + (959625*ArcTa
nh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(64*Sqrt[2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}dx = -\frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} - \frac{1}{72}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{26675}{16}-4990x+486\right.}{5+2x}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)(3-x+2x^2)^{5/2} - \frac{\int \frac{(3-x+2x^2)^{3/2}(1}{}}{}$$

$$= -\frac{839}{960}(3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)(3-x+2$$

$$= -\frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432} - \frac{839}{960}(3-x+2x^2)^{5/2} - \frac{3667(}{5}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x}{18432}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x}{18432}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x}{18432}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x}{18432}$$

Mathematica [A] time = 0.224764, size = 108, normalized size = 0.63

$$\frac{\frac{4\sqrt{2x^2-x+3}(409600x^6-1798144x^5+8283904x^4-35369408x^3+182033816x^2-1404323114x-6814208295)}{2x+5} + 14739840000\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)}{1966080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-6814208295 - 1404323114*x + 182033816*x^2 - 35369
408*x^3 + 8283904*x^4 - 1798144*x^5 + 409600*x^6))/(5 + 2*x) - 14740041885*
Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 14739840000*Sqrt[2]*ArcTanh[(17 - 22*
x)/(12*Sqrt[6 - 2*x + 4*x^2])])/1966080
```

Maple [A] time = 0.06, size = 208, normalized size = 1.2

$$\frac{5x}{48}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{589}{960}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{-9059+36236x}{6144}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{-208357+833428x}{32768}\sqrt{2x^2-x+3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

```
[Out] 5/48*x*(2*x^2-x+3)^(5/2)-589/960*(2*x^2-x+3)^(5/2)+9059/6144*(-1+4*x)*(2*x^
2-x+3)^(3/2)+208357/32768*(-1+4*x)*(2*x^2-x+3)^(1/2)+982669459/131072*2^(1/
2)*arcsinh(4/23*23^(1/2)*(x-1/4))-106625/2304*(2*(x+5/2)^2-11*x-19/2)^(3/2)
+1637/16*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-319875/128*(2*(x+5/2)^2-11*
x-19/2)^(1/2)+959625/128*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2
)^2-11*x-19/2)^(1/2))-3667/1152/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(5/2)+3667/
2304*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A] time = 1.58311, size = 217, normalized size = 1.26

$$\frac{5}{48}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{589}{960}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{9059}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{185827}{6144}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{3560933}{8192}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(5/2)*x - 589/960*(2*x^2 - x + 3)^(5/2) + 9059/1536*(2
*x^2 - x + 3)^(3/2)*x - 185827/6144*(2*x^2 - x + 3)^(3/2) + 3560933/8192*sq
rt(2*x^2 - x + 3)*x + 982669459/131072*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 959625/128*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) - 85448933/32768*sqrt(2*x^2 - x + 3) - 3667/32*
(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A] time = 1.46846, size = 504, normalized size = 2.93

$$\frac{14740041885\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+14739840000\sqrt{2}(2x+5)\log\left(\frac{24\sqrt{2}\sqrt{2}}{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3932160*(14740041885*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)
*(4*x - 1) - 32*x^2 + 16*x - 25) + 14739840000*sqrt(2)*(2*x + 5)*log((24*sq
rt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 +
20*x + 25)) + 8*(409600*x^6 - 1798144*x^5 + 8283904*x^4 - 35369408*x^3 + 18
2033816*x^2 - 1404323114*x - 6814208295)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2-x+3)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**2, x)
```

Giac [B] time = 1.36325, size = 954, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="g
iac")
```

```
[Out] 1/1966080*sqrt(2)*(14739840000*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 14740041885*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 147400
41885*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*
sgn(1/(2*x + 5)) - 2027704320*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(
1/(2*x + 5)) + 2*(45496763235*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6
/(2*x + 5))^11*sgn(1/(2*x + 5)) - 126553743360*(sqrt(-11/(2*x + 5) + 36/(2*
x + 5)^2 + 1) + 6/(2*x + 5))^10*sgn(1/(2*x + 5)) + 44062768335*(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^9*sgn(1/(2*x + 5)) + 331789824
00*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^8*sgn(1/(2*x +
5)) + 294206421582*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))
^7*sgn(1/(2*x + 5)) - 463672074240*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1
) + 6/(2*x + 5))^6*sgn(1/(2*x + 5)) + 35099942478*(sqrt(-11/(2*x + 5) + 36/
(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 171324610560*(sqrt(-11
/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 600592
81615*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x
 + 5)) - 105051009024*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^2*sgn(1/(2*x + 5)) - 5210329245*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) + 17058392064*sgn(1/(2*x + 5)))/((sqrt(-
11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^6)
```

3.338 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx$

Optimal. Leaf size=174

$$\frac{438065\left(2x^2-x+3\right)^{5/2}}{82944(2x+5)}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}+\frac{1}{16}\left(2x^2-x+3\right)^{5/2}+\frac{(2154633-534617x)\left(2x^2-x+3\right)^{3/2}}{82944}+\frac{(33741}{}$$

```
[Out] ((33741483 - 5623292*x)*Sqrt[3 - x + 2*x^2])/24576 + ((2154633 - 534617*x)*
(3 - x + 2*x^2)^(3/2))/82944 + (3 - x + 2*x^2)^(5/2)/16 - (3667*(3 - x + 2*
x^2)^(5/2))/(1152*(5 + 2*x)^2) + (438065*(3 - x + 2*x^2)^(5/2))/(82944*(5 +
 2*x)) + (129342063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2]) - (8083915
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1024*Sqrt[2])
```

Rubi [A] time = 0.272723, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{438065\left(2x^2-x+3\right)^{5/2}}{82944(2x+5)}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}+\frac{1}{16}\left(2x^2-x+3\right)^{5/2}+\frac{(2154633-534617x)\left(2x^2-x+3\right)^{3/2}}{82944}+\frac{(33741}{}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```


```
[Out] ((33741483 - 5623292*x)*Sqrt[3 - x + 2*x^2])/24576 + ((2154633 - 534617*x)*
(3 - x + 2*x^2)^(3/2))/82944 + (3 - x + 2*x^2)^(5/2)/16 - (3667*(3 - x + 2*
x^2)^(5/2))/(1152*(5 + 2*x)^2) + (438065*(3 - x + 2*x^2)^(5/2))/(82944*(5 +
 2*x)) + (129342063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2]) - (8083915
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1024*Sqrt[2])
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{35015}{16}-\frac{21585x}{4}+9\ldots\right.}{(5+2x)^2}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1152(5+2x)^2} + \frac{438065\left(3-x+2x^2\right)^{5/2}}{82944(5+2x)} + \int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{273\ldots}{}\right.}{}$$

$$= \frac{1}{16}\left(3-x+2x^2\right)^{5/2} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{1152(5+2x)^2} + \frac{438065\left(3-x+2x^2\right)^{5/\ldots}}{82944(5+2x)}$$

$$= \frac{(2154633-534617x)\left(3-x+2x^2\right)^{3/2}}{82944} + \frac{1}{16}\left(3-x+2x^2\right)^{5/2} - \frac{3667\,(\ldots}{11\ldots}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-534617x)\left(3-x\ldots\right.}{82944}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-534617x)\left(3-x\ldots\right.}{82944}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-534617x)\left(3-x\ldots\right.}{82944}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-534617x)\left(3-x\ldots\right.}{82944}$$

Mathematica [A] time = 0.212224, size = 108, normalized size = 0.62

$$\frac{\frac{4\sqrt{2x^2-x+3}\left(8192x^6-43520x^5+253312x^4-1620944x^3+16667188x^2+181223072x+298966737\right)}{(2x+5)^2} - 129342640\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 129342\ldots}{32768}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(298966737 + 181223072*x + 16667188*x^2 - 1620944*x
^3 + 253312*x^4 - 43520*x^5 + 8192*x^6))/(5 + 2*x)^2 + 129342063*Sqrt[2]*Ar
cSinh[(1 - 4*x)/Sqrt[23]] - 129342640*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[
6 - 2*x + 4*x^2])])/32768
```

Maple [A] time = 0.061, size = 214, normalized size = 1.2

$$-\frac{-343745+1374980\,x}{6144}\sqrt{2\,\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}}-\frac{3667}{4608}\left(2\,\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}\right)^{\frac{5}{2}}\left(x+\frac{5}{2}\right)^{-2}-\frac{8083915\,\sqrt{2}}{2048}\mathrm{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

```
[Out] -343745/6144*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/4608/(x+5/2)^2*(2*
(x+5/2)^2-11*x-19/2)^(5/2)-8083915/2048*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^
(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-10281/8192*(-1+4*x)*(2*x^2-x+3)^(1/2)-
129342063/32768*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-149/512*(-1+4*x)*(2*
x^2-x+3)^(3/2)+8083915/331776*(2*(x+5/2)^2-11*x-19/2)^(3/2)+8083915/6144*(2
*(x+5/2)^2-11*x-19/2)^(1/2)+1/16*(2*x^2-x+3)^(5/2)-438065/331776*(-1+4*x)*(
2*(x+5/2)^2-11*x-19/2)^(3/2)+438065/165888/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^
(5/2)
```

Maxima [A] time = 1.58913, size = 232, normalized size = 1.33

$$\frac{1}{16}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}-\frac{149}{128}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x+\frac{46691}{4608}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}}{1152\left(4\,x^{2}+20\,x+25\right)}-\frac{1405823}{6144}\sqrt{2\,x^{2}-x+3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 1/16*(2*x^2 - x + 3)^(5/2) - 149/128*(2*x^2 - x + 3)^(3/2)*x + 46691/4608*(
2*x^2 - x + 3)^(3/2) - 3667/1152*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25)
- 1405823/6144*sqrt(2*x^2 - x + 3)*x - 129342063/32768*sqrt(2)*arcsinh(4/23
*sqrt(23)*x - 1/23*sqrt(23)) + 8083915/2048*sqrt(2)*arcsinh(22/23*sqrt(23)*
x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 11247161/8192*sqrt(2*x^2 -
x + 3) + 438065/4608*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A] time = 1.39816, size = 524, normalized size = 3.01

$$\frac{129342063\sqrt{2}\left(4x^2+20x+25\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+129342640\sqrt{2}\left(4x^2+20x+25\right)\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/65536*(129342063*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 129342640*sqrt(2)*(4*x^2 + 20*x + 2
5)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1
153)/(4*x^2 + 20*x + 25)) + 8*(8192*x^6 - 43520*x^5 + 253312*x^4 - 1620944*
x^3 + 16667188*x^2 + 181223072*x + 298966737)*sqrt(2*x^2 - x + 3))/(4*x^2 +
 20*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**3, x)
```

Giac [A] time = 1.25388, size = 362, normalized size = 2.08

$$\frac{1}{8192}\left(4\left(8\left(4\left(16x-165\right)x+4879\right)x-263469\right)x+8460377\right)\sqrt{2x^2-x+3}+\frac{129342063}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/8192*(4*(8*(4*(16*x - 165)*x + 4879)*x - 263469)*x + 8460377)*sqrt(2*x^2
- x + 3) + 129342063/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3)) + 1) - 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2)
 + 2*sqrt(2*x^2 - x + 3))) + 1/512*sqrt(2)*(14243182*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^3 + 109906674*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 170
996871*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 110506087)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^2
```

3.339 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

Optimal. Leaf size=181

$$-\frac{32865365\left(2x^2-x+3\right)^{5/2}}{17915904(2x+5)}+\frac{556255\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}-\frac{(138006843-34265045x)\left(2x^2-x+\right.}{17915904}$$

```
[Out] -((135068604 - 22512089*x)*Sqrt[3 - x + 2*x^2])/331776 - ((138006843 - 3426
5045*x)*(3 - x + 2*x^2)^(3/2))/17915904 - (3667*(3 - x + 2*x^2)^(5/2))/(172
8*(5 + 2*x)^3) + (556255*(3 - x + 2*x^2)^(5/2))/(248832*(5 + 2*x)^2) - (328
65365*(3 - x + 2*x^2)^(5/2))/(17915904*(5 + 2*x)) - (19176431*ArcSinh[(1 -
4*x)/Sqrt[23]])/(8192*Sqrt[2]) + (517762327*ArcTanh[(17 - 22*x)/(12*Sqrt[2]
*Sqrt[3 - x + 2*x^2])])/(221184*Sqrt[2])
```

Rubi [A] time = 0.267145, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{32865365\left(2x^2-x+3\right)^{5/2}}{17915904(2x+5)}+\frac{556255\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}-\frac{(138006843-34265045x)\left(2x^2-x+\right.}{17915904}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```


```
[Out] -((135068604 - 22512089*x)*Sqrt[3 - x + 2*x^2])/331776 - ((138006843 - 3426
5045*x)*(3 - x + 2*x^2)^(3/2))/17915904 - (3667*(3 - x + 2*x^2)^(5/2))/(172
8*(5 + 2*x)^3) + (556255*(3 - x + 2*x^2)^(5/2))/(248832*(5 + 2*x)^2) - (328
65365*(3 - x + 2*x^2)^(5/2))/(17915904*(5 + 2*x)) - (19176431*ArcSinh[(1 -
4*x)/Sqrt[23]])/(8192*Sqrt[2]) + (517762327*ArcTanh[(17 - 22*x)/(12*Sqrt[2]
*Sqrt[3 - x + 2*x^2])])/(221184*Sqrt[2])
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
```

```
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{43355}{16}-\frac{11605x}{2}+14\cdots\right.}{(5+2x)^3}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} + \frac{\int \frac{\left(3-x+2x^2\right)^{3/2}\left(420\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} - \frac{32865365\left(3-\cdots\right.}{17915904(5\cdots}$$

$$= -\frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3}\cdots$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-34265045\cdots}{179159\cdots}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-34265045\cdots}{179159\cdots}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-34265045\cdots}{179159\cdots}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-34265045\cdots}{179159\cdots}$$

Mathematica [A] time = 0.220023, size = 108, normalized size = 0.6

$$\frac{\frac{12\sqrt{2x^2-x+3}\left(46080x^6-315648x^5+2626848x^4-33595416x^3-594798908x^2-2006873194x-1994650739\right)}{(2x+5)^3}+517762327\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)-\cdots}{442368}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-1994650739 - 2006873194*x - 594798908*x^2 - 3359
5416*x^3 + 2626848*x^4 - 315648*x^5 + 46080*x^6))/(5 + 2*x)^3 - 517763637*S
qrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 517762327*Sqrt[2]*ArcTanh[(17 - 22*x)/
(12*Sqrt[6 - 2*x + 4*x^2])])/442368
```

Maple [A] time = 0.062, size = 221, normalized size = 1.2

$$\frac{-22400309+89601236\,x}{1327104}\sqrt{2\,\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}}+\frac{556255}{995328}\left(2\,\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}\right)^{\frac{5}{2}}\left(x+\frac{5}{2}\right)^{-2}+\frac{517762327\,\sqrt{2}}{442368}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

```
[Out] 22400309/1327104*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+556255/995328/(x+5/
2)^2*(2*(x+5/2)^2-11*x-19/2)^(5/2)+517762327/442368*2^(1/2)*arctanh(1/12*(1
7/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+345/4096*(-1+4*x)*(2*x^2-x
+3)^(1/2)+19176431/16384*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+5/256*(-1+4
*x)*(2*x^2-x+3)^(3/2)-517762327/71663616*(2*(x+5/2)^2-11*x-19/2)^(3/2)-5177
62327/1327104*(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/13824/(x+5/2)^3*(2*(x+5/2)
^2-11*x-19/2)^(5/2)+32865365/71663616*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(3/2
)-32865365/35831808/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(5/2)
```

Maxima [A] time = 1.5567, size = 255, normalized size = 1.41

$$\frac{5}{64}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x-\frac{1094743}{497664}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}}{1728\left(8\,x^{3}+60\,x^{2}+150\,x+125\right)}+\frac{556255\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}}{248832\left(4\,x^{2}+20\,x+25\right)}+\frac{2251}{331}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/64*(2*x^2 - x + 3)^(3/2)*x - 1094743/497664*(2*x^2 - x + 3)^(3/2) - 3667/
1728*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 556255/248832*(
```

```
2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 22512089/331776*sqrt(2*x^2 - x +
 3)*x + 19176431/16384*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 5
17762327/442368*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) - 11255717/27648*sqrt(2*x^2 - x + 3) - 32865365/995328*(2
*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A] time = 1.48414, size = 581, normalized size = 3.21

$$517763637\sqrt{2}\left(8x^3+60x^2+150x+125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+517762327\sqrt{2}\left(8x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/884736*(517763637*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 517762327*sqrt(2)*(8*x
^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
- 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 24*(46080*x^6 - 315648*x
^5 + 2626848*x^4 - 33595416*x^3 - 594798908*x^2 - 2006873194*x - 1994650739
)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**4, x)
```

Giac [B] time = 1.24558, size = 424, normalized size = 2.34

$$\frac{1}{4096}\left(4\left(8\left(20\,x-287\right)x+23341\right)x-1004633\right)\sqrt{2\,x^2-x+3}-\frac{19176431}{16384}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/4096*(4*(8*(20*x - 287)*x + 23341)*x - 1004633)*sqrt(2*x^2 - x + 3) - 191
76431/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sq
rt(2*x^2 - x + 3))) - 1/36864*sqrt(2)*(1092794276*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 18284336132*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 2031
4214356*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 151449344092*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^2 + 102529692109*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3)) - 41882448755)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

3.340 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx$

Optimal. Leaf size=188

$$-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{23887872(2x+5)^2}+\frac{224815\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}+\frac{(67865260x+762984903)\left(2x^2-x\right.}{95551488(2x+5)}$$

```
[Out] ((2339916063 - 389975609*x)*Sqrt[3 - x + 2*x^2])/31850496 + ((762984903 + 6
7865260*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)) - (3667*(3 - x + 2*x
^2)^(5/2))/(2304*(5 + 2*x)^4) + (224815*(3 - x + 2*x^2)^(5/2))/(165888*(5 +
 2*x)^3) - (14477995*(3 - x + 2*x^2)^(5/2))/(23887872*(5 + 2*x)^2) + (43256
5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqrt[2]) - (8969688643*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(21233664*Sqrt[2])
```

Rubi [A] time = 0.265479, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{23887872(2x+5)^2}+\frac{224815\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}+\frac{(67865260x+762984903)\left(2x^2-x\right.}{95551488(2x+5)}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```


```
[Out] ((2339916063 - 389975609*x)*Sqrt[3 - x + 2*x^2])/31850496 + ((762984903 + 6
7865260*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)) - (3667*(3 - x + 2*x
^2)^(5/2))/(2304*(5 + 2*x)^4) + (224815*(3 - x + 2*x^2)^(5/2))/(165888*(5 +
 2*x)^3) - (14477995*(3 - x + 2*x^2)^(5/2))/(23887872*(5 + 2*x)^2) + (43256
5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqrt[2]) - (8969688643*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(21233664*Sqrt[2])
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
```

```
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{51695}{16}-\frac{24835x}{4}+194\right.}{(5+2x)^4}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} + \frac{224815\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^3} + \frac{\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{59950}{16}\right.}{(5}}{6}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} + \frac{224815\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^3} - \frac{14477995\left(3-x\right.}{23887872(5+}$$

$$= \frac{(762984903+67865260x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} +$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+67865260}{95551488(5}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+67865260}{95551488(5}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+67865260}{95551488(5}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+67865260}{95551488(5}$$

Mathematica [A] time = 0.239099, size = 108, normalized size = 0.57

$$\frac{\frac{24\sqrt{2x^2-x+3}\left(2949120x^6-29270016x^5+468043776x^4+11761910072x^3+60528581892x^2+121473790266x+86386856771\right)}{(2x+5)^4} - 8969688643\sqrt{2}\tanh^{-1}\left(\frac{}{12}\right.}{42467328}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5
,x]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(86386856771 + 121473790266*x + 60528581892*x^2 +
11761910072*x^3 + 468043776*x^4 - 29270016*x^5 + 2949120*x^6))/(5 + 2*x)^4
+ 8969667840*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 8969688643*Sqrt[2]*ArcTa
```

```
nh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/42467328
```

Maple [A] time = 0.062, size = 204, normalized size = 1.1

$$-\frac{-389975609+1559902436\,x}{127401984}\sqrt{2\,(x+5/2)^2-11\,x-\frac{19}{2}}-\frac{14477995}{95551488}\left(2\,(x+5/2)^2-11\,x-\frac{19}{2}\right)^{\frac{5}{2}}\left(x+\frac{5}{2}\right)^{-2}-\frac{896968}{424}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

```
[Out] -389975609/127401984*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-14477995/955514
88/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(5/2)-8969688643/42467328*2^(1/2)*arct
anh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-432565/2048*2^(
1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+8969688643/6879707136*(2*(x+5/2)^2-11*x
-19/2)^(3/2)+8969688643/127401984*(2*(x+5/2)^2-11*x-19/2)^(1/2)+224815/1327
104/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(5/2)-593321753/6879707136*(-1+4*x)*(
2*(x+5/2)^2-11*x-19/2)^(3/2)+593321753/3439853568/(x+5/2)*(2*(x+5/2)^2-11*x
-19/2)^(5/2)-3667/36864/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(5/2)
```

Maxima [A] time = 1.56479, size = 284, normalized size = 1.51

$$\frac{16966315}{47775744}\left(2\,x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{2304\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)}+\frac{224815\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{165888\left(8\,x^3+60\,x^2+150\,x+125\right)}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] 16966315/47775744*(2*x^2 - x + 3)^(3/2) - 3667/2304*(2*x^2 - x + 3)^(5/2)/(
16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 224815/165888*(2*x^2 - x + 3)^
(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 14477995/23887872*(2*x^2 - x + 3)^(5
/2)/(4*x^2 + 20*x + 25) - 389975609/31850496*sqrt(2*x^2 - x + 3)*x - 432565
/2048*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 8969688643/4246732
8*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x +
5)) + 779972021/10616832*sqrt(2*x^2 - x + 3) + 593321753/95551488*(2*x^2 -
```

```
x + 3)^(3/2)/(2*x + 5)
```

Fricas [A] time = 1.49125, size = 657, normalized size = 3.49

$$\frac{8969667840\sqrt{2}\left(16x^4+160x^3+600x^2+1000x+625\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8969688\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="f
ricas")
```

```
[Out] 1/84934656*(8969667840*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8969688
643*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*sq
rt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 2
5)) + 48*(2949120*x^6 - 29270016*x^5 + 468043776*x^4 + 11761910072*x^3 + 60
528581892*x^2 + 121473790266*x + 86386856771)*sqrt(2*x^2 - x + 3))/(16*x^4
+ 160*x^3 + 600*x^2 + 1000*x + 625)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**5, x)
```

Giac [B] time = 1.33648, size = 679, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="g
iac")
```

```
[Out] -1/42467328*sqrt(2)*(8969688643*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
+ 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 8969667840*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 896966
7840*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*s
gn(1/(2*x + 5)) + 12*(24*(1296*(29336*sgn(1/(2*x + 5))/(2*x + 5) - 42907*sg
n(1/(2*x + 5)))/(2*x + 5) + 39923563*sgn(1/(2*x + 5)))/(2*x + 5) - 54131203
9*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 13824*(80624
1*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5
)) - 1152288*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn
(1/(2*x + 5)) - 957352*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
 5))^3*sgn(1/(2*x + 5)) + 1529280*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
 + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) + 394431*(sqrt(-11/(2*x + 5) + 36/(2*x +
 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) - 620352*sgn(1/(2*x + 5)))/((sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^3)
```

3.341 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$

Optimal. Leaf size=195

$$-\frac{3730507(2x^2-x+3)^{5/2}}{11943936(2x+5)^3}+\frac{158527(2x^2-x+3)^{5/2}}{165888(2x+5)^4}-\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}+\frac{(44773976x+246012435)(2x^2-x+3}{95551488(2x+5)^2}$$

```
[Out] -((5658774871 + 1028823716*x)*Sqrt[3 - x + 2*x^2])/(127401984*(5 + 2*x)) +
((246012435 + 44773976*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)^2) - (
3667*(3 - x + 2*x^2)^(5/2))/(2880*(5 + 2*x)^5) + (158527*(3 - x + 2*x^2)^(5
/2))/(165888*(5 + 2*x)^4) - (3730507*(3 - x + 2*x^2)^(5/2))/(11943936*(5 +
2*x)^3) - (23775*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512*Sqrt[2]) + (70991525167*
ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1528823808*Sqrt[2])
```

Rubi [A] time = 0.262875, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{3730507(2x^2-x+3)^{5/2}}{11943936(2x+5)^3}+\frac{158527(2x^2-x+3)^{5/2}}{165888(2x+5)^4}-\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}+\frac{(44773976x+246012435)(2x^2-x+3}{95551488(2x+5)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```


```
[Out] -((5658774871 + 1028823716*x)*Sqrt[3 - x + 2*x^2])/(127401984*(5 + 2*x)) +
((246012435 + 44773976*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)^2) - (
3667*(3 - x + 2*x^2)^(5/2))/(2880*(5 + 2*x)^5) + (158527*(3 - x + 2*x^2)^(5
/2))/(165888*(5 + 2*x)^4) - (3730507*(3 - x + 2*x^2)^(5/2))/(11943936*(5 +
2*x)^3) - (23775*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512*Sqrt[2]) + (70991525167*
ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1528823808*Sqrt[2])
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
```

```
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} - \frac{1}{360}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{60035}{16}-6615x+243\right.}{(5+2x)^5}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} + \frac{158527\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^4} + \frac{\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{81144}{16}\right.}{(5}}{10}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} + \frac{158527\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^4} - \frac{3730507\left(3-x+\right.}{11943936(5+}$$

$$= \frac{(246012435+44773976x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} +$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+447739}{95551488}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+447739}{95551488}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+447739}{95551488}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+447739}{95551488}$$

Mathematica [A] time = 0.247036, size = 108, normalized size = 0.55

$$\frac{\frac{24\sqrt{2x^2-x+3}\left(1592524800x^6-30496849920x^5-1023534029552x^4-7117092892448x^3-21590439797064x^2-30872393829992x-17093312738327\right)}{(2x+5)^5}+3549576}{15288238080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6
```

```
,x]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(-17093312738327 - 30872393829992*x - 215904397970
64*x^2 - 7117092892448*x^3 - 1023534029552*x^4 - 30496849920*x^5 + 15925248
00*x^6))/(5 + 2*x)^5 - 354958848000*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 3
54957625835*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/152882
38080
```

Maple [A] time = 0.066, size = 225, normalized size = 1.2

$$-\frac{3667}{92160}\left(2\left(x+5/2\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}\left(x+\frac{5}{2}\right)^{-5}+\frac{-3086715581+12346862324x}{9172942848}\sqrt{2\left(x+5/2\right)^{2}-11x-\frac{19}{2}}+\frac{134077\ldots}{6879707\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

```
[Out] -3667/92160/(x+5/2)^5*(2*(x+5/2)^2-11*x-19/2)^(5/2)+3086715581/9172942848*(
-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+134077495/6879707136/(x+5/2)^2*(2*(x+
5/2)^2-11*x-19/2)^(5/2)+70991525167/3057647616*2^(1/2)*arctanh(1/12*(17/2-1
1*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+23775/1024*2^(1/2)*arcsinh(4/23
*23^(1/2)*(x-1/4))-70991525167/495338913792*(2*(x+5/2)^2-11*x-19/2)^(3/2)-7
0991525167/9172942848*(2*(x+5/2)^2-11*x-19/2)^(1/2)-3730507/95551488/(x+5/2
)^3*(2*(x+5/2)^2-11*x-19/2)^(5/2)+4698578717/495338913792*(-1+4*x)*(2*(x+5/
2)^2-11*x-19/2)^(3/2)-4698578717/247669456896/(x+5/2)*(2*(x+5/2)^2-11*x-19/
2)^(5/2)+158527/2654208/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(5/2)
```

Maxima [A] time = 1.65331, size = 339, normalized size = 1.74

$$-\frac{134077495}{3439853568}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{2880\left(32x^{5}+400x^{4}+2000x^{3}+5000x^{2}+6250x+3125\right)}+\frac{158527\left(2\ldots\right.}{165888\left(16x^{4}+160x^{3}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] -134077495/3439853568*(2*x^2 - x + 3)^(3/2) - 3667/2880*(2*x^2 - x + 3)^(5/
2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 158527/165888
*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) - 373050
7/11943936*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 134077495
/1719926784*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 3086715581/22932357
12*sqrt(2*x^2 - x + 3)*x + 23775/1024*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 70991525167/3057647616*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2
*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 6173186729/764411904*sqrt(2*x^2 -
x + 3) - 4698578717/6879707136*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A] time = 1.52147, size = 747, normalized size = 3.83

$$\frac{354958848000\sqrt{2}\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/30576476160*(354958848000*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2
 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 1
6*x - 25) + 354957625835*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(1592524800*x^6 - 30496849920*x^5
 - 1023534029552*x^4 - 7117092892448*x^3 - 21590439797064*x^2 - 30872393829
992*x - 17093312738327)*sqrt(2*x^2 - x + 3))/(32*x^5 + 400*x^4 + 2000*x^3 +
 5000*x^2 + 6250*x + 3125)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**6,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**6, x)
```

Giac [B] time = 1.30233, size = 548, normalized size = 2.81

$$\frac{1}{256}\sqrt{2x^2-x+3}(20x-633)-\frac{23775}{1024}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] 1/256*sqrt(2*x^2 - x + 3)*(20*x - 633) - 23775/1024*sqrt(2)*log(-2*sqrt(2)*
(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 70991525167/3057647616*sqrt(2)*log
(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 70991525167/3057647
616*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1
/1274019840*sqrt(2)*(8281387393360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^9 + 275661428628240*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 156038270334576
0*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 4938646760855520*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^6 - 9673562837036232*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3))^5 - 30647310393849000*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 700
60241036847960*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 97730658088823
880*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 30180638363071845*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) - 7096913381268319)/(2*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^5
```

3.342 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx$

Optimal. Leaf size=195

$$-\frac{14087245\left(2x^2-x+3\right)^{5/2}}{71663616(2x+5)^4}+\frac{182165\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}-\frac{(6793718806x+9802984711)\left(2x^2-x\right.}{13759414272(2x+5)^3}$$

```
[Out] ((151764102421 + 27596573612*x)*Sqrt[3 - x + 2*x^2])/(55037657088*(5 + 2*x)
) - ((9802984711 + 6793718806*x)*(3 - x + 2*x^2)^(3/2))/(13759414272*(5 + 2
*x)^3) - (3667*(3 - x + 2*x^2)^(5/2))/(3456*(5 + 2*x)^6) + (182165*(3 - x +
 2*x^2)^(5/2))/(248832*(5 + 2*x)^5) - (14087245*(3 - x + 2*x^2)^(5/2))/(716
63616*(5 + 2*x)^4) + (369*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2]) - (190
3976002333*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(66045188
5056*Sqrt[2])
```

Rubi [A] time = 0.267546, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1650, 810, 812, 843, 619, 215, 724, 206}

$$-\frac{14087245\left(2x^2-x+3\right)^{5/2}}{71663616(2x+5)^4}+\frac{182165\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}-\frac{(6793718806x+9802984711)\left(2x^2-x\right.}{13759414272(2x+5)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```



```
[Out] ((151764102421 + 27596573612*x)*Sqrt[3 - x + 2*x^2])/(55037657088*(5 + 2*x)
) - ((9802984711 + 6793718806*x)*(3 - x + 2*x^2)^(3/2))/(13759414272*(5 + 2
*x)^3) - (3667*(3 - x + 2*x^2)^(5/2))/(3456*(5 + 2*x)^6) + (182165*(3 - x +
 2*x^2)^(5/2))/(248832*(5 + 2*x)^5) - (14087245*(3 - x + 2*x^2)^(5/2))/(716
63616*(5 + 2*x)^4) + (369*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2]) - (190
3976002333*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(66045188
5056*Sqrt[2])
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
```

```
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
```

```
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{68375}{16}-\frac{28085x}{4}+2\right.}{(5+2x)^6}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{3456(5+2x)^6} + \frac{182165\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^5} + \int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{105}{}\right.}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{3456(5+2x)^6} + \frac{182165\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^5} - \frac{14087245\left(3-x\right.}{71663616(5}$$

$$= -\frac{(9802984711+6793718806x)\left(3-x+2x^2\right)^{3/2}}{13759414272(5+2x)^3} - \frac{3667\left(3-x+2x^2\right.}{3456(5+2x)^6}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711+679}{137594}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711+679}{137594}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711+679}{137594}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711+679}{137594}$$

Mathematica [A] time = 0.258643, size = 108, normalized size = 0.55

$$\frac{\frac{24\sqrt{2x^2-x+3}\left(275188285440x^6+11854023276320x^5+103803827945872x^4+422554114856528x^3+910256842473992x^2+1011372787716826x+45841162535458\right.}{(2x+5)^6}}{1320903770112}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7
,x]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(458411625354581 + 1011372787716826*x + 9102568424
73992*x^2 + 422554114856528*x^3 + 103803827945872*x^4 + 11854023276320*x^5
+ 275188285440*x^6))/(5 + 2*x)^6 + 1903958949888*Sqrt[2]*ArcSinh[(1 - 4*x)/
```

```
Sqrt[23]] - 1903976002333*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*
x^2])])/1320903770112
```

Maple [A] time = 0.073, size = 246, normalized size = 1.3

$$\frac{182165}{7962624}\left(2\,(x+5/2)^2-11\,x-\frac{19}{2}\right)^{\frac{5}{2}}\left(x+\frac{5}{2}\right)^{-5}-\frac{-82772668391+331090673564\,x}{3962711310336}\sqrt{2\,(x+5/2)^2-11\,x-\frac{19}{2}}-\frac{3607}{29720}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

```
[Out] 182165/7962624/(x+5/2)^5*(2*(x+5/2)^2-11*x-19/2)^(5/2)-82772668391/39627113
10336*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-3607708597/2972033482752/(x+5/
2)^2*(2*(x+5/2)^2-11*x-19/2)^(5/2)-1903976002333/1320903770112*2^(1/2)*arct
anh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-3667/221184/(x+
5/2)^6*(2*(x+5/2)^2-11*x-19/2)^(5/2)-369/256*2^(1/2)*arcsinh(4/23*23^(1/2)*
(x-1/4))+1903976002333/213986410758144*(2*(x+5/2)^2-11*x-19/2)^(3/2)+190397
6002333/3962711310336*(2*(x+5/2)^2-11*x-19/2)^(1/2)+149610673/41278242816/(
x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(5/2)-125860542215/213986410758144*(-1+4*x
)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+125860542215/106993205379072/(x+5/2)*(2*(x+
5/2)^2-11*x-19/2)^(5/2)-14087245/1146617856/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/
2)^(5/2)
```

Maxima [A] time = 1.5413, size = 401, normalized size = 2.06

$$\frac{3607708597}{1486016741376}\left(2\,x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{3456\left(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625\right)}+\frac{}{248832}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 3607708597/1486016741376*(2*x^2 - x + 3)^(3/2) - 3667/3456*(2*x^2 - x + 3)^
(5/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 1562
5) + 182165/248832*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 500
```

```
0*x^2 + 6250*x + 3125) - 14087245/71663616*(2*x^2 - x + 3)^(5/2)/(16*x^4 +
160*x^3 + 600*x^2 + 1000*x + 625) + 149610673/5159780352*(2*x^2 - x + 3)^(5
/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 3607708597/743008370688*(2*x^2 - x + 3
)^(5/2)/(4*x^2 + 20*x + 25) - 82772668391/990677827584*sqrt(2*x^2 - x + 3)*
x - 369/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 190397600233
3/1320903770112*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) + 165562389227/330225942528*sqrt(2*x^2 - x + 3) + 1258605
42215/2972033482752*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A] time = 1.44627, size = 830, normalized size = 4.26

$$\frac{1903958949888\,\sqrt{2}\left(64\,x^6 + 960\,x^5 + 6000\,x^4 + 20000\,x^3 + 37500\,x^2 + 37500\,x + 15625\right)\log\left(4\,\sqrt{2}\sqrt{2\,x^2 - x + 3}(4\,x - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/2641807540224*(1903958949888*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000
*x^3 + 37500*x^2 + 37500*x + 15625)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 1903976002333*sqrt(2)*(64*x^6 + 960*x^5 + 6000
*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2
 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48
*(275188285440*x^6 + 11854023276320*x^5 + 103803827945872*x^4 + 42255411485
6528*x^3 + 910256842473992*x^2 + 1011372787716826*x + 458411625354581)*sqrt
(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 375
00*x + 15625)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}}\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{\left(2x + 5\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**7,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**7, x)
```

Giac [B] time = 1.26596, size = 610, normalized size = 3.13

$$\frac{369}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{1903976}{1320903}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] 369/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 190
3976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*
x^2 - x + 3))) + 1903976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x -
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 5/64*sqrt(2*x^2 - x + 3) + 1/110075
314176*sqrt(2)*(159278433934432*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^1
1 + 6347903280912544*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^10 + 485445268408334
24*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 305716670132783088*(sqrt(2
)*x - sqrt(2*x^2 - x + 3))^8 + 88313821135911024*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3))^7 - 2423668581998843376*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6
 - 397211131697032056*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 1170889
7232532299576*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 12803484860728491138*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 12593033197867577234*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 - 3042533760672408875*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3)) + 589526263249780195)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2
 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^6
```

3.343 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

Optimal. Leaf size=195

$$-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{13934592(2x+5)^5}+\frac{114335\left(2x^2-x+3\right)^{5/2}}{193536(2x+5)^6}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}-\frac{(411822458x+463558457)\left(2x^2-x+\right.}{2293235712(2x+5)^4}$$

```
[Out] -((146583836191 + 101679102454*x)*Sqrt[3 - x + 2*x^2])/(440301256704*(5 + 2
*x)^2) - ((463558457 + 411822458*x)*(3 - x + 2*x^2)^(3/2))/(2293235712*(5 +
 2*x)^4) - (3667*(3 - x + 2*x^2)^(5/2))/(4032*(5 + 2*x)^7) + (114335*(3 - x
 + 2*x^2)^(5/2))/(193536*(5 + 2*x)^6) - (1930441*(3 - x + 2*x^2)^(5/2))/(13
934592*(5 + 2*x)^5) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) + (41276
0561351*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(52836150804
48*Sqrt[2])
```

Rubi [A] time = 0.261981, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 810, 843, 619, 215, 724, 206}

$$-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{13934592(2x+5)^5}+\frac{114335\left(2x^2-x+3\right)^{5/2}}{193536(2x+5)^6}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}-\frac{(411822458x+463558457)\left(2x^2-x+\right.}{2293235712(2x+5)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] -((146583836191 + 101679102454*x)*Sqrt[3 - x + 2*x^2])/(440301256704*(5 + 2
*x)^2) - ((463558457 + 411822458*x)*(3 - x + 2*x^2)^(3/2))/(2293235712*(5 +
 2*x)^4) - (3667*(3 - x + 2*x^2)^(5/2))/(4032*(5 + 2*x)^7) + (114335*(3 - x
 + 2*x^2)^(5/2))/(193536*(5 + 2*x)^6) - (1930441*(3 - x + 2*x^2)^(5/2))/(13
934592*(5 + 2*x)^5) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) + (41276
0561351*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(52836150804
48*Sqrt[2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
```

```
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7}-\frac{1}{504}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{76715}{16}-\frac{14855x}{2}+3\right.}{(5+2x)^7}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7}+\frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6}+\frac{\int \frac{\left(3-x+2x^2\right)^{3/2}\left(133\right.}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7}+\frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6}-\frac{1930441\left(3-x\right.}{13934592(5}$$

$$= -\frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4}-\frac{3667\left(3-x+2x^2\right)^{5/}}{4032(5+2x)^7}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2}-\frac{(463558457+4}{2293}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2}-\frac{(463558457+4}{2293}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2}-\frac{(463558457+4}{2293}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2}-\frac{(463558457+4}{2293}$$

Mathematica [A] time = 0.272285, size = 108, normalized size = 0.55

$$\frac{-\frac{24\sqrt{2x^2-x+3}\left(38463671680832x^6+402255822731712x^5+2069947287085104x^4+5966329646300704x^3+9976065367498188x^2+9065154700300572x+347951\right.}{(2x+5)^7}}{73970611126272}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8
,x]
```

```
[Out] ((-24*Sqrt[3 - x + 2*x^2]*(3479517268702637 + 9065154700300572*x + 99760653
67498188*x^2 + 5966329646300704*x^3 + 2069947287085104*x^4 + 40225582273171
2*x^5 + 38463671680832*x^6))/(5 + 2*x)^7 - 2889476997120*Sqrt[2]*ArcSinh[(1
 - 4*x)/Sqrt[23]] + 2889323929457*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 -
2*x + 4*x^2])])/73970611126272
```

Maple [A] time = 0.078, size = 267, normalized size = 1.4

$$-\frac{1930441}{445906944}\left(2\,(x+5/2)^2-11\,x-\frac{19}{2}\right)^{\frac{5}{2}}\left(x+\frac{5}{2}\right)^{-5}+\frac{-17957520133+71830080532\,x}{31701690482688}\sqrt{2\,(x+5/2)^2-11\,x-\frac{19}{2}}+\frac{7}{237}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

```
[Out] -1930441/445906944/(x+5/2)^5*(2*(x+5/2)^2-11*x-19/2)^(5/2)+17957520133/3170
1690482688*(-1+4*x)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+769352975/23776267862016/
(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(5/2)+412760561351/10567230160896*2^(1/2)
*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+114335/123
86304/(x+5/2)^6*(2*(x+5/2)^2-11*x-19/2)^(5/2)+5/128*2^(1/2)*arcsinh(4/23*23
^(1/2)*(x-1/4))-412760561351/1711891286065152*(2*(x+5/2)^2-11*x-19/2)^(3/2)
-412760561351/31701690482688*(2*(x+5/2)^2-11*x-19/2)^(1/2)-32967491/3302259
42528/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(5/2)-3667/516096/(x+5/2)^7*(2*(x+5
/2)^2-11*x-19/2)^(5/2)+27452157541/1711891286065152*(-1+4*x)*(2*(x+5/2)^2-1
1*x-19/2)^(3/2)-27452157541/855945643032576/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)
^(5/2)+7861079/9172942848/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(5/2)
```

Maxima [B] time = 1.61073, size = 470, normalized size = 2.41

$$-\frac{769352975}{11888133931008}\left(2\,x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{4032\left(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] -769352975/11888133931008*(2*x^2 - x + 3)^(3/2) - 3667/4032*(2*x^2 - x + 3)
^(5/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^
2 + 218750*x + 78125) + 114335/193536*(2*x^2 - x + 3)^(5/2)/(64*x^6 + 960*x
^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625) - 1930441/13934592
*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3
125) + 7861079/573308928*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2
+ 1000*x + 625) - 32967491/41278242816*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^
2 + 150*x + 125) + 769352975/5944066965504*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 2
0*x + 25) + 17957520133/7925422620672*sqrt(2*x^2 - x + 3)*x + 5/128*sqrt(2)
*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 412760561351/10567230160896*sqr
t(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) -
 35893173457/2641807540224*sqrt(2*x^2 - x + 3) - 27452157541/23776267862016
*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A] time = 1.53481, size = 915, normalized size = 4.69

$$\frac{2889476997120\sqrt{2}\left(128x^7 + 2240x^6 + 16800x^5 + 70000x^4 + 175000x^3 + 262500x^2 + 218750x + 78125\right)\log\left(-4\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/147941222252544*(2889476997120*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 +
70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-4*sqrt(2)*sqrt
(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 2889323929457*sqrt(2)*(12
8*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750
*x + 78125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 10
36*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(38463671680832*x^6 + 40225582273171
2*x^5 + 2069947287085104*x^4 + 5966329646300704*x^3 + 9976065367498188*x^2
+ 9065154700300572*x + 3479517268702637)*sqrt(2*x^2 - x + 3))/(128*x^7 + 22
40*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**8,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**8, x)
```

Giac [B] time = 1.26089, size = 660, normalized size = 3.38

$$-\frac{5}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{412760561351}{10567230160896}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{4127}{10567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -5/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 4127
60561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) - 412760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x -
11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/6164217593856*sqrt(2)*(11218973984
12224*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 + 48260296303776704*(sqr
t(2)*x - sqrt(2*x^2 - x + 3))^12 + 444673458321712704*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^11 + 3996455936659982656*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^10 + 6725227967167489360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 -
 1713266102848394808O*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 - 637130120947372
46112*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 106515880136064432096*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 + 226947197958946260516*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^5 - 856601202771483308188*(sqrt(2)*x - sqrt(2*x^2
 - x + 3))^4 + 617998258357377713732*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^3 - 467121785339763351756*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 9229208
0735560562227*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 15161716093827501
349)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^7
```

3.344 $\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4-\frac{105}{128}\sqrt{2x^2-x+3}(2x+5)^3+\frac{761}{256}\sqrt{2x^2-x+3}(2x+5)^2-\frac{(4676x+19227)\sqrt{2x^2-x+3}}{2048}-$$

```
[Out] (761*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/256 - (105*(5 + 2*x)^3*Sqrt[3 - x + 2
*x^2])/128 + ((5 + 2*x)^4*Sqrt[3 - x + 2*x^2])/16 - ((19227 + 4676*x)*Sqrt[
3 - x + 2*x^2])/2048 - (85429*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2])
```

Rubi [A] time = 0.135734, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1653, 779, 619, 215}

$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4-\frac{105}{128}\sqrt{2x^2-x+3}(2x+5)^3+\frac{761}{256}\sqrt{2x^2-x+3}(2x+5)^2-\frac{(4676x+19227)\sqrt{2x^2-x+3}}{2048}-$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (761*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/256 - (105*(5 + 2*x)^3*Sqrt[3 - x + 2
*x^2])/128 + ((5 + 2*x)^4*Sqrt[3 - x + 2*x^2])/16 - ((19227 + 4676*x)*Sqrt[
3 - x + 2*x^2])/2048 - (85429*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2}+\frac{1}{160}\int \frac{(5+2x)\left(-5055-4390x-5580x^2-4200\right.}{\sqrt{3-x+2x^2}}$$

$$= -\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2}+\frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2}+\frac{\int \frac{(5+2x)\left(327480+10544\right.}{\sqrt{3-x+2}}}{10240}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2}-\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2}+\frac{1}{16}(5+2x)^4\sqrt{3-x}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2}-\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2}+\frac{1}{16}(5+2x)^4\sqrt{3-x}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2}-\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2}+\frac{1}{16}(5+2x)^4\sqrt{3-x}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2}-\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2}+\frac{1}{16}(5+2x)^4\sqrt{3-x}$$

Mathematica [A] time = 0.115793, size = 60, normalized size = 0.5

$$\frac{4\sqrt{2x^2-x+3}\left(2048x^4+7040x^3+352x^2-6916x+2973\right)-85429\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(2973 - 6916*x + 352*x^2 + 7040*x^3 + 2048*x^4) - 85
429*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/8192
```

Maple [A] time = 0.05, size = 95, normalized size = 0.8

$$x^4\sqrt{2x^2-x+3}+\frac{55x^3}{16}\sqrt{2x^2-x+3}+\frac{11x^2}{64}\sqrt{2x^2-x+3}-\frac{1729x}{512}\sqrt{2x^2-x+3}+\frac{2973}{2048}\sqrt{2x^2-x+3}+\frac{85429\sqrt{2}}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] x^4*(2*x^2-x+3)^(1/2)+55/16*x^3*(2*x^2-x+3)^(1/2)+11/64*x^2*(2*x^2-x+3)^(1/
2)-1729/512*x*(2*x^2-x+3)^(1/2)+2973/2048*(2*x^2-x+3)^(1/2)+85429/8192*2^(1
/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A] time = 1.47478, size = 130, normalized size = 1.08

$$\sqrt{2x^2-x+3}x^4+\frac{55}{16}\sqrt{2x^2-x+3}x^3+\frac{11}{64}\sqrt{2x^2-x+3}x^2-\frac{1729}{512}\sqrt{2x^2-x+3}x+\frac{85429}{8192}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] sqrt(2*x^2 - x + 3)*x^4 + 55/16*sqrt(2*x^2 - x + 3)*x^3 + 11/64*sqrt(2*x^2
- x + 3)*x^2 - 1729/512*sqrt(2*x^2 - x + 3)*x + 85429/8192*sqrt(2)*arcsinh(
1/23*sqrt(23)*(4*x - 1)) + 2973/2048*sqrt(2*x^2 - x + 3)
```

Fricas [A] time = 1.29906, size = 223, normalized size = 1.86

$$\frac{1}{2048}\left(2048\,x^4+7040\,x^3+352\,x^2-6916\,x+2973\right)\sqrt{2\,x^2-x+3}+\frac{85429}{16384}\,\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2048*(2048*x^4 + 7040*x^3 + 352*x^2 - 6916*x + 2973)*sqrt(2*x^2 - x + 3)
+ 85429/16384*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
 + 16*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/sqrt(2*x**2 - x + 3), x
)
```

Giac [A] time = 1.20803, size = 92, normalized size = 0.77

$$\frac{1}{2048}\left(4\left(8\left(4\left(16\,x+55\right)x+11\right)x-1729\right)x+2973\right)\sqrt{2\,x^2-x+3}-\frac{85429}{8192}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/2048*(4*(8*(4*(16*x + 55)*x + 11)*x - 1729)*x + 2973)*sqrt(2*x^2 - x + 3)
 - 85429/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.345 $\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=101

$$\frac{5}{8}\sqrt{2x^2-x+3}x^3+\frac{19}{96}\sqrt{2x^2-x+3}x^2-\frac{409}{768}\sqrt{2x^2-x+3}x-\frac{505\sqrt{2x^2-x+3}}{1024}-\frac{6863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

```
[Out] (-505*Sqrt[3 - x + 2*x^2])/1024 - (409*x*Sqrt[3 - x + 2*x^2])/768 + (19*x^2
*Sqrt[3 - x + 2*x^2])/96 + (5*x^3*Sqrt[3 - x + 2*x^2])/8 - (6863*ArcSinh[(1
 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rubi [A] time = 0.0801967, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1661, 640, 619, 215}

$$\frac{5}{8}\sqrt{2x^2-x+3}x^3+\frac{19}{96}\sqrt{2x^2-x+3}x^2-\frac{409}{768}\sqrt{2x^2-x+3}x-\frac{505\sqrt{2x^2-x+3}}{1024}-\frac{6863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-505*Sqrt[3 - x + 2*x^2])/1024 - (409*x*Sqrt[3 - x + 2*x^2])/768 + (19*x^2
*Sqrt[3 - x + 2*x^2])/96 + (5*x^3*Sqrt[3 - x + 2*x^2])/8 - (6863*ArcSinh[(1
 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{8}\int \frac{16+8x-21x^2+\frac{19x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{48}\int \frac{96-9x-\frac{409x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{192}\int \frac{\frac{2763}{4}-\frac{1515x}{8}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{686\cdots}{\cdots}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{686\cdots}{\cdots}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}-\frac{686\cdots}{\cdots}$$

Mathematica [A] time = 0.0750094, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2-x+3}\left(1920x^3+608x^2-1636x-1515\right)-20589\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1515 - 1636*x + 608*x^2 + 1920*x^3) - 20589*Sqrt[2
]*ArcSinh[(1 - 4*x)/Sqrt[23]])/12288
```

Maple [A] time = 0.053, size = 79, normalized size = 0.8

$$\frac{5\,x^3}{8}\sqrt{2\,x^2-x+3}+\frac{19\,x^2}{96}\sqrt{2\,x^2-x+3}-\frac{409\,x}{768}\sqrt{2\,x^2-x+3}-\frac{505}{1024}\sqrt{2\,x^2-x+3}+\frac{6863\,\sqrt{2}}{4096}\mathrm{Arcsinh}\left(\frac{4\,\sqrt{23}}{23}\left(x-\frac{1}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/8*x^3*(2*x^2-x+3)^(1/2)+19/96*x^2*(2*x^2-x+3)^(1/2)-409/768*x*(2*x^2-x+3)
^(1/2)-505/1024*(2*x^2-x+3)^(1/2)+6863/4096*2^(1/2)*arcsinh(4/23*23^(1/2)*(
x-1/4))
```

Maxima [A] time = 1.47592, size = 108, normalized size = 1.07

$$\frac{5}{8}\sqrt{2\,x^2-x+3}x^3+\frac{19}{96}\sqrt{2\,x^2-x+3}x^2-\frac{409}{768}\sqrt{2\,x^2-x+3}x+\frac{6863}{4096}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{505}{1024}\sqrt{2\,x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/8*sqrt(2*x^2 - x + 3)*x^3 + 19/96*sqrt(2*x^2 - x + 3)*x^2 - 409/768*sqrt(
2*x^2 - x + 3)*x + 6863/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 505
/1024*sqrt(2*x^2 - x + 3)
```

Fricas [A] time = 1.30573, size = 205, normalized size = 2.03

$$\frac{1}{3072}\left(1920\,x^3+608\,x^2-1636\,x-1515\right)\sqrt{2\,x^2-x+3}+\frac{6863}{8192}\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(1920*x^3 + 608*x^2 - 1636*x - 1515)*sqrt(2*x^2 - x + 3) + 6863/8192
*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{\sqrt{2x^2 - x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/sqrt(2*x**2 - x + 3), x)
```

Giac [A] time = 1.15413, size = 85, normalized size = 0.84

$$\frac{1}{3072}\left(4\left(8\left(60\,x + 19\right)x - 409\right)x - 1515\right)\sqrt{2\,x^2 - x + 3} - \frac{6863}{4096}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x - \sqrt{2\,x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3072*(4*(8*(60*x + 19)*x - 409)*x - 1515)*sqrt(2*x^2 - x + 3) - 6863/4096
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.346 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=126

$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2-\frac{337}{192}\sqrt{2x^2-x+3}(2x+5)+\frac{1669}{128}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}}+\frac{9657\,\mathrm{sinh}}{256}$$

```
[Out] (1669*Sqrt[3 - x + 2*x^2])/128 - (337*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/192 +
(5*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/48 + (9657*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])
])/(96*Sqrt[2])
```

Rubi [A] time = 0.210483, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1653, 843, 619, 215, 724, 206}

$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2-\frac{337}{192}\sqrt{2x^2-x+3}(2x+5)+\frac{1669}{128}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}}+\frac{9657\,\mathrm{sinh}}{256}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (1669*Sqrt[3 - x + 2*x^2])/128 - (337*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/192 +
(5*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/48 + (9657*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])
])/(96*Sqrt[2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{1}{96}\int \frac{-2183-3054x-4092x^2-2696x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{\int \frac{24504+128736x+160224x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{3072}$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{\int \frac{997152-}{(5+2x)\sqrt{}}}{24}$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}-\frac{9657}{256}\int$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}-\frac{3667}{8}\text{Sul}$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{9657\sinh}{25}$$

Mathematica [A] time = 0.103197, size = 81, normalized size = 0.64

$$\frac{4\sqrt{2x^2-x+3}\left(160x^2-548x+2637\right)-29336\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+28971\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1536}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(2637 - 548*x + 160*x^2) + 28971*Sqrt[2]*ArcSinh[(1
- 4*x)/Sqrt[23]] - 29336*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x
^2])])/1536
```

Maple [A] time = 0.056, size = 92, normalized size = 0.7

$$\frac{5\,x^2}{12}\sqrt{2\,x^2-x+3}-\frac{137\,x}{96}\sqrt{2\,x^2-x+3}+\frac{879}{128}\sqrt{2\,x^2-x+3}-\frac{9657\,\sqrt{2}}{512}\text{Arcsinh}\left(\frac{4\,\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)-\frac{3667\,\sqrt{2}}{192}\text{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/12*x^2*(2*x^2-x+3)^(1/2)-137/96*x*(2*x^2-x+3)^(1/2)+879/128*(2*x^2-x+3)^(
1/2)-9657/512*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/192*2^(1/2)*arcta
nh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A] time = 1.52173, size = 134, normalized size = 1.06

$$\frac{5}{12}\sqrt{2x^2-x+3}x^2-\frac{137}{96}\sqrt{2x^2-x+3}x-\frac{9657}{512}\sqrt{2}\,\text{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{3667}{192}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{1}{23|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/12*sqrt(2*x^2 - x + 3)*x^2 - 137/96*sqrt(2*x^2 - x + 3)*x - 9657/512*sqrt
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 3667/192*sqrt(2)*arcsinh(22/
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 879/128*sqrt(2*
x^2 - x + 3)
```

Fricas [A] time = 1.4016, size = 344, normalized size = 2.73

$$\frac{1}{384}\left(160x^2-548x+2637\right)\sqrt{2x^2-x+3}+\frac{9657}{1024}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+\frac{3667}{384}\sqrt{2}\,\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/384*(160*x^2 - 548*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/1024*sqrt(2)*log(
4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 3667/384*sq
rt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x
+ 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)\sqrt{2x^2 - x + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*sqrt(2*x**2 - x + 3)),
 x)
```

Giac [A] time = 1.16402, size = 161, normalized size = 1.28

$$\frac{1}{384}\left(4\left(40\,x - 137\right)x + 2637\right)\sqrt{2\,x^2 - x + 3} + \frac{9657}{512}\,\sqrt{2}\log\left(-4\,\sqrt{2}x + \sqrt{2} + 4\,\sqrt{2\,x^2 - x + 3}\right) - \frac{3667}{192}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/384*(4*(40*x - 137)*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/512*sqrt(2)*log(
-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/192*sqrt(2)*log(abs(
-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/192*sqrt(2)*log(abs
(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

3.347 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=126

$$\frac{5}{32}\sqrt{2x^2-x+3}(2x+5)-\frac{243}{64}\sqrt{2x^2-x+3}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}+\frac{158527\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6912\sqrt{2}}-\frac{2943\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

```
[Out] (-243*Sqrt[3 - x + 2*x^2])/64 - (3667*Sqrt[3 - x + 2*x^2])/(576*(5 + 2*x))
+ (5*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/32 - (2943*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(128*Sqrt[2]) + (158527*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2
])])/(6912*Sqrt[2])
```

Rubi [A] time = 0.202046, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 1653, 843, 619, 215, 724, 206}

$$\frac{5}{32}\sqrt{2x^2-x+3}(2x+5)-\frac{243}{64}\sqrt{2x^2-x+3}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}+\frac{158527\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6912\sqrt{2}}-\frac{2943\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-243*Sqrt[3 - x + 2*x^2])/64 - (3667*Sqrt[3 - x + 2*x^2])/(576*(5 + 2*x))
+ (5*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/32 - (2943*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(128*Sqrt[2]) + (158527*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2
])])/(6912*Sqrt[2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} - \frac{1}{72}\int \frac{\frac{12007}{16}-1323x+486x^2-180x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{30314-27216x+34992x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{2304}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{417472-847584x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{18432}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} + \frac{2943}{128}\int \frac{1}{\sqrt{3-x+\ldots}}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} + \frac{158527}{576}\,\mathrm{Subst}\left(\int \ldots\right.$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{2943\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right.}{128\sqrt{2}}$$

Mathematica [A] time = 0.114362, size = 88, normalized size = 0.7

$$\frac{\frac{48\sqrt{2x^2-x+3}\left(180x^2-1287x-6176\right)}{2x+5}+158527\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)-158922\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{13824}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((48*Sqrt[3 - x + 2*x^2]*(-6176 - 1287*x + 180*x^2))/(5 + 2*x) - 158922*Sqr
t[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 158527*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*S
qrt[6 - 2*x + 4*x^2])])/13824
```

Maple [A] time = 0.056, size = 96, normalized size = 0.8

$$\frac{5x}{16}\sqrt{2x^2-x+3} - \frac{193}{64}\sqrt{2x^2-x+3} + \frac{2943\sqrt{2}}{256}\mathrm{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right) + \frac{158527\sqrt{2}}{13824}\mathrm{Artanh}\left(\frac{\sqrt{2}}{12}\left(\frac{17}{2}-11x\right)\frac{1}{\sqrt{2(\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/16*x*(2*x^2-x+3)^(1/2)-193/64*(2*x^2-x+3)^(1/2)+2943/256*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))+158527/13824*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2
)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-3667/1152/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^
(1/2)
```

Maxima [A] time = 1.52076, size = 139, normalized size = 1.1

$$\frac{5}{16}\sqrt{2x^2-x+3}x+\frac{2943}{256}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{158527}{13824}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)-\frac{193}{64}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2*x^2 - x + 3)*x + 2943/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 158527/13824*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) -
17/23*sqrt(23)/abs(2*x + 5)) - 193/64*sqrt(2*x^2 - x + 3) - 3667/576*sqrt(2
*x^2 - x + 3)/(2*x + 5)
```

Fricas [A] time = 1.44426, size = 389, normalized size = 3.09

$$\frac{158922\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+158527\sqrt{2}(2x+5)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-}{4x^2+}\right.}{27648\,(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/27648*(158922*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x -
 1) - 32*x^2 + 16*x - 25) + 158527*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt(2
*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25))
```

```
+ 96*(180*x^2 - 1287*x - 6176)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^2 \sqrt{2x^2 - x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*sqrt(2*x**2 - x + 3
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^4 - x^3 + 3\,x^2 + x + 2}{\sqrt{2\,x^2 - x + 3}(2\,x + 5)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((5*x^4 - x^3 + 3*x^2 + x + 2)/(sqrt(2*x^2 - x + 3)*(2*x + 5)^2),
x)
```

3.348 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=128

$$\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}+\frac{5}{16}\sqrt{2x^2-x+3}-\frac{1546507\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}}+\frac{149\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] (5*Sqrt[3 - x + 2*x^2])/16 - (3667*Sqrt[3 - x + 2*x^2])/(1152*(5 + 2*x)^2)
+ (92239*Sqrt[3 - x + 2*x^2])/(27648*(5 + 2*x)) + (149*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(32*Sqrt[2]) - (1546507*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x
 + 2*x^2])])/(331776*Sqrt[2])
```

Rubi [A] time = 0.207631, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 1653, 843, 619, 215, 724, 206}

$$\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}+\frac{5}{16}\sqrt{2x^2-x+3}-\frac{1546507\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}}+\frac{149\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x]
```


```
[Out] (5*Sqrt[3 - x + 2*x^2])/16 - (3667*Sqrt[3 - x + 2*x^2])/(1152*(5 + 2*x)^2)
+ (92239*Sqrt[3 - x + 2*x^2])/(27648*(5 + 2*x)) + (149*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(32*Sqrt[2]) - (1546507*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x
 + 2*x^2])])/(331776*Sqrt[2])
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{\frac{20347}{16}-\frac{6917x}{4}+972x^2-360x^3}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int \frac{\frac{647841}{16}-67392x+12960x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{10368}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int \frac{\frac{777441}{2}-772416x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{82944}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{149}{32}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{1546507\,\text{Subst}\left(\int \frac{1}{288-x}\right.}{27648}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{149\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}} - \frac{154}{}$$

Mathematica [A] time = 0.129727, size = 88, normalized size = 0.69

$$\frac{\frac{24\sqrt{2x^2-x+3}(34560x^2+357278x+589187)}{(2x+5)^2} - 1546507\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 1544832\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{663552}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(589187 + 357278*x + 34560*x^2))/(5 + 2*x)^2 + 154
4832*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 1546507*Sqrt[2]*ArcTanh[(17 - 22
*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/663552
```

Maple [A] time = 0.065, size = 102, normalized size = 0.8

$$\frac{5}{16}\sqrt{2x^2-x+3}-\frac{149\sqrt{2}}{64}\mathrm{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)-\frac{3667}{4608}\sqrt{2\,(x+5/2)^2-11\,x-\frac{19}{2}}\left(x+\frac{5}{2}\right)^{-2}+\frac{92239}{55296}\sqrt{2\,(x+5/2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/16*(2*x^2-x+3)^(1/2)-149/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/4
608/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(1/2)+92239/55296/(x+5/2)*(2*(x+5/2)^
2-11*x-19/2)^(1/2)-1546507/663552*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/
(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A] time = 1.50934, size = 154, normalized size = 1.2

$$-\frac{149}{64}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{1546507}{663552}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{5}{16}\sqrt{2\,x^2-x+3}-\frac{3667\,\sqrt{}}{1152\,(4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -149/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 1546507/663552*s
qrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5))
 + 5/16*sqrt(2*x^2 - x + 3) - 3667/1152*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x +
 25) + 92239/27648*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Fricas [A] time = 1.42837, size = 443, normalized size = 3.46

$$\frac{1544832\sqrt{2}\left(4\,x^2+20\,x+25\right)\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+1546507\sqrt{2}\left(4\,x^2+20\,x+25\right)\log\left(\right.}{1327104\left(4\,x^2+20\,x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/1327104*(1544832*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 1546507*sqrt(2)*(4*x^2 + 20*x + 25)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) + 48*(34560*x^2 + 357278*x + 589187)*sqrt(2*x^2 - x
 + 3))/(4*x^2 + 20*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^3 \sqrt{2x^2 - x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*sqrt(2*x**2 - x + 3
)), x)
```

Giac [B] time = 1.20277, size = 335, normalized size = 2.62

$$\frac{149}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) - \frac{1546507}{663552}\sqrt{2}\log\left(\left|-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) + \frac{1546507}{663552}\sqrt{2}\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 149/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1546
507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
 + 1546507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 5/16*sqrt(2*x^2 - x + 3) + 1/55296*sqrt(2)*(2381290*sqrt(2)*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 16628406*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^2 - 25697445*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 16720645)/(2*(
```

```
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3)) - 11)^2
```

3.349 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=135

$$-\frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)}+\frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}+\frac{22389491\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}}-\frac{5\sinh^{-1}\left(1\cdots\right.}{16\sqrt{2}}$$

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(1728*(5 + 2*x)^3) + (394907*Sqrt[3 - x + 2*x^2
])/(248832*(5 + 2*x)^2) - (3163415*Sqrt[3 - x + 2*x^2])/(5971968*(5 + 2*x))
 - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) + (22389491*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(71663616*Sqrt[2])
```

Rubi [A] time = 0.204716, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1650, 843, 619, 215, 724, 206}

$$-\frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)}+\frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}+\frac{22389491\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}}-\frac{5\sinh^{-1}\left(1\cdots\right.}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(1728*(5 + 2*x)^3) + (394907*Sqrt[3 - x + 2*x^2
])/(248832*(5 + 2*x)^2) - (3163415*Sqrt[3 - x + 2*x^2])/(5971968*(5 + 2*x))
 - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) + (22389491*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(71663616*Sqrt[2])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\frac{28687}{16}-\frac{4271x}{2}+1458x^2-540x^3}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} + \frac{\int \frac{\frac{1464275}{16}-\frac{413797x}{4}+38880x^2}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{31104}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} - \frac{\int \frac{\frac{11181273}{16}-139968}{(5+2x)\sqrt{3-x+2}}}{2239488}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} + \frac{5}{16}\int \frac{1}{\sqrt{3-x+}}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} + \frac{22389491\,\mathrm{Sub}}{}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} - \frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

Mathematica [A] time = 0.151497, size = 88, normalized size = 0.65

$$\frac{-\frac{24\sqrt{2x^2-x+3}(12653660x^2+44312764x+44369687)}{(2x+5)^3}+22389491\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)-22394880\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{143327232}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((-24*Sqrt[3 - x + 2*x^2]*(44369687 + 44312764*x + 12653660*x^2))/(5 + 2*x)
^3 - 22394880*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 22389491*Sqrt[2]*ArcTan
h[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/143327232
```

Maple [A] time = 0.059, size = 109, normalized size = 0.8

$$\frac{5\sqrt{2}}{32}\mathrm{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)+\frac{394907}{995328}\sqrt{2\,(x+5/2)^2-11\,x-\frac{19}{2}}\left(x+\frac{5}{2}\right)^{-2}-\frac{3163415}{11943936}\sqrt{2\,(x+5/2)^2-11\,x-\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/32*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+394907/995328/(x+5/2)^2*(2*(x+5
/2)^2-11*x-19/2)^(1/2)-3163415/11943936/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(1/
2)+22389491/143327232*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2
-11*x-19/2)^(1/2))-3667/13824/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(1/2)
```

Maxima [A] time = 1.49446, size = 177, normalized size = 1.31

$$\frac{5}{32}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{22389491}{143327232}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{3667\sqrt{2\,x^2-x+3}}{1728\left(8\,x^3+60\,x^2+150\,x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 22389491/143327232*
sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)
) - 3667/1728*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2 + 150*x + 125) + 394907/2
48832*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25) - 3163415/5971968*sqrt(2*x^2
- x + 3)/(2*x + 5)
```

Fricas [A] time = 1.34501, size = 502, normalized size = 3.72

$$\frac{22394880\sqrt{2}\left(8\,x^3+60\,x^2+150\,x+125\right)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+22389491\sqrt{2}\left(8\,x^3+6\right.}{286654464\left(8\,x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/286654464*(22394880*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 22389491*sqrt(2)*(8*
```

```
x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
 - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(12653660*x^2 + 4431
2764*x + 44369687)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^4\sqrt{2x^2 - x + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*sqrt(2*x**2 - x + 3
)), x)
```

Giac [B] time = 1.20272, size = 385, normalized size = 2.85

$$-\frac{5}{32}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{22389491}{143327232}\sqrt{2}\log\left(\left|-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) - \frac{22389491}{143327232}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] -5/32*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 22389
491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3
))) - 22389491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt
(2*x^2 - x + 3))) - 1/11943936*sqrt(2)*(215012404*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 3010410772*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 27408
02468*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 21459328844*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 14434519361*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3)) - 5957650879)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

3.350 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=139

$$\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)}-\frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2}+\frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}+\frac{2053207\tanh^{-1}\left(\frac{1}{12\sqrt{2}}\right.}{20639121408\sqrt{}}$$

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(2304*(5 + 2*x)^4) + (513097*Sqrt[3 - x + 2*x^2
])/(497664*(5 + 2*x)^3) - (16295969*Sqrt[3 - x + 2*x^2])/(71663616*(5 + 2*x
)^2) + (26800085*Sqrt[3 - x + 2*x^2])/(1719926784*(5 + 2*x)) + (2053207*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(20639121408*Sqrt[2])
```

Rubi [A] time = 0.191478, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1650, 806, 724, 206}

$$\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)}-\frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2}+\frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}+\frac{2053207\tanh^{-1}\left(\frac{1}{12\sqrt{2}}\right.}{20639121408\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x]
```


```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(2304*(5 + 2*x)^4) + (513097*Sqrt[3 - x + 2*x^2
])/(497664*(5 + 2*x)^3) - (16295969*Sqrt[3 - x + 2*x^2])/(71663616*(5 + 2*x
)^2) + (26800085*Sqrt[3 - x + 2*x^2])/(1719926784*(5 + 2*x)) + (2053207*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(20639121408*Sqrt[2])
```


Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}-\frac{1}{288}\int \frac{\frac{37027}{16}-\frac{10167x}{4}+1944x^2-720x^3}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3}+\frac{\int \frac{\frac{2607829}{16}-\frac{295607x}{2}+77760x^2}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx}{62208} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3}-\frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}-\frac{\int \frac{\frac{19411145}{16}-\frac{60989\cdots}{4}}{(5+2x)^2\sqrt{3-x+\cdots}}}{8957952\cdots} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3}-\frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}+\frac{26800085\sqrt{3\cdots}}{1719926784\cdots} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3}-\frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}+\frac{26800085\sqrt{3\cdots}}{1719926784\cdots} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3}-\frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}+\frac{26800085\sqrt{3\cdots}}{1719926784\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.136726, size = 81, normalized size = 0.58

$$\frac{24\sqrt{2x^2-x+3}\left(214400680x^3+43592076x^2-255525906x-298655447\right)+2053207\sqrt{2}(2x+5)^4\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)}{41278242816(2x+5)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] (24*Sqrt[3 - x + 2*x^2]*(-298655447 - 255525906*x + 43592076*x^2 + 21440068
0*x^3) + 2053207*Sqrt[2]*(5 + 2*x)^4*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x +
 4*x^2])])/(41278242816*(5 + 2*x)^4)
```

Maple [A] time = 0.07, size = 116, normalized size = 0.8

$$-\frac{16295969}{286654464}\sqrt{2\left(x+5/2\right)^2-11x-\frac{19}{2}}\left(x+\frac{5}{2}\right)^{-2}+\frac{26800085}{3439853568}\sqrt{2\left(x+5/2\right)^2-11x-\frac{19}{2}}\left(x+\frac{5}{2}\right)^{-1}+\frac{2053207\sqrt{2}}{41278242816}A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x)
```

```
[Out] -16295969/286654464/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(1/2)+26800085/343985
3568/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+2053207/41278242816*2^(1/2)*arct
anh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+513097/3981312/
(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/36864/(x+5/2)^4*(2*(x+5/2)^2-1
1*x-19/2)^(1/2)
```

Maxima [A] time = 1.56057, size = 201, normalized size = 1.45

$$-\frac{2053207}{41278242816}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)-\frac{3667\sqrt{2x^2-x+3}}{2304\left(16x^4+160x^3+600x^2+1000x+625\right)}+\frac{51309}{497664\left(8x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2053207/41278242816*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 3667/2304*sqrt(2*x^2 - x + 3)/(16*x^4 + 160*x^3 +
600*x^2 + 1000*x + 625) + 513097/497664*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2
 + 150*x + 125) - 16295969/71663616*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25)
 + 26800085/1719926784*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Fricas [A] time = 1.33975, size = 405, normalized size = 2.91

$$\frac{2053207\,\sqrt{2}\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)\log\left(\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)-1060\,x^2+1036\,x-1153}{4\,x^2+20\,x+25}\right)+48\left(214400680\,x^3\right.}{82556485632\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/82556485632*(2053207*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)
/(4*x^2 + 20*x + 25)) + 48*(214400680*x^3 + 43592076*x^2 - 255525906*x - 29
8655447)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^5\,\sqrt{2x^2 - x + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**5*sqrt(2*x**2 - x + 3
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^4 - x^3 + 3\,x^2 + x + 2}{\sqrt{2\,x^2 - x + 3}(2\,x+5)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((5*x^4 - x^3 + 3*x^2 + x + 2)/(sqrt(2*x^2 - x + 3)*(2*x + 5)^5),
x)
```

3.351 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{5}{4}\sqrt{2x^2-x+3}x^3+\frac{153}{16}\sqrt{2x^2-x+3}x^2+\frac{2645}{128}\sqrt{2x^2-x+3}x-\frac{13153}{512}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{144217\sinh}{1024}$$

```
[Out] (-4*(346 - 533*x))/(23*Sqrt[3 - x + 2*x^2]) - (13153*Sqrt[3 - x + 2*x^2])/5
12 + (2645*x*Sqrt[3 - x + 2*x^2])/128 + (153*x^2*Sqrt[3 - x + 2*x^2])/16 +
(5*x^3*Sqrt[3 - x + 2*x^2])/4 + (144217*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*
Sqrt[2])
```

Rubi [A] time = 0.152252, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{4}\sqrt{2x^2-x+3}x^3+\frac{153}{16}\sqrt{2x^2-x+3}x^2+\frac{2645}{128}\sqrt{2x^2-x+3}x-\frac{13153}{512}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{144217\sinh}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```


```
[Out] (-4*(346 - 533*x))/(23*Sqrt[3 - x + 2*x^2]) - (13153*Sqrt[3 - x + 2*x^2])/5
12 + (2645*x*Sqrt[3 - x + 2*x^2])/128 + (153*x^2*Sqrt[3 - x + 2*x^2])/16 +
(5*x^3*Sqrt[3 - x + 2*x^2])/4 + (144217*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*
Sqrt[2])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(5+2x)^2 \left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-759-\frac{575x}{2}+805x^2+\frac{1219x^3}{2}+115x^4}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{5}{4}x^3\sqrt{3-x+2x^2} + \frac{1}{92}\int \frac{-6072-2300x+5405x^2+}{\sqrt{3-x+2x^2}}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{153}{16}x^2\sqrt{3-x+2x^2} + \frac{5}{4}x^3\sqrt{3-x+2x^2} + \frac{1}{552}\int \frac{-364}{}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{153}{16}x^2\sqrt{3-x+2x^2} + \frac{5}{4}x^3\sqrt{3-}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{153}{16}x^2\sqrt{3}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{153}{16}x^2\sqrt{3}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{153}{16}x^2\sqrt{3}$$

Mathematica [A] time = 0.491766, size = 74, normalized size = 0.6

$$\frac{4\left(29440x^5+210496x^4+418232x^3-510554x^2+2124123x-1616165\right)+3316991\sqrt{4x^2-2x+6}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{47104\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2)
,x]
```

```
[Out] (4*(-1616165 + 2124123*x - 510554*x^2 + 418232*x^3 + 210496*x^4 + 29440*x^5
) + 3316991*Sqrt[6 - 2*x + 4*x^2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/(47104*Sqrt[
3 - x + 2*x^2])
```

Maple [A] time = 0.056, size = 132, normalized size = 1.1

$$\frac{5x^5}{2}\frac{1}{\sqrt{2x^2-x+3}}+\frac{143x^4}{8}\frac{1}{\sqrt{2x^2-x+3}}-\frac{144217\sqrt{2}}{2048}\mathrm{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)+\frac{-931255+3725020x}{94208}\frac{1}{\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/2*x^5/(2*x^2-x+3)^(1/2)+143/8*x^4/(2*x^2-x+3)^(1/2)-144217/2048*2^(1/2)*a
rcsinh(4/23*23^(1/2)*(x-1/4))+931255/94208*(-1+4*x)/(2*x^2-x+3)^(1/2)+14421
7/1024*x/(2*x^2-x+3)^(1/2)+2273/64*x^3/(2*x^2-x+3)^(1/2)-11099/256*x^2/(2*x
^2-x+3)^(1/2)-521655/4096/(2*x^2-x+3)^(1/2)
```

Maxima [A] time = 1.64788, size = 154, normalized size = 1.24

$$\frac{5x^5}{2\sqrt{2x^2-x+3}}+\frac{143x^4}{8\sqrt{2x^2-x+3}}+\frac{2273x^3}{64\sqrt{2x^2-x+3}}-\frac{11099x^2}{256\sqrt{2x^2-x+3}}-\frac{144217}{2048}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/2*x^5/sqrt(2*x^2 - x + 3) + 143/8*x^4/sqrt(2*x^2 - x + 3) + 2273/64*x^3/s
qrt(2*x^2 - x + 3) - 11099/256*x^2/sqrt(2*x^2 - x + 3) - 144217/2048*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2124123/11776*x/sqrt(2*x^2 - x + 3) -
1616165/11776/sqrt(2*x^2 - x + 3)
```

Fricas [A] time = 1.36305, size = 300, normalized size = 2.42

$$\frac{3316991\sqrt{2}\left(2x^2-x+3\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(29440x^5+210496x^4+418232x^3-\right.}{94208\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/94208*(3316991*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*
(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(29440*x^5 + 210496*x^4 + 418232*x^3 -
510554*x^2 + 2124123*x - 1616165)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+5)^2 \left(5x^4 - x^3 + 3x^2 + x + 2\right)}{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3
/2), x)
```

Giac [A] time = 1.15011, size = 97, normalized size = 0.78

$$\frac{144217}{2048}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(46(4(8(20x + 143)x + 2273)x - 11099)x + 2124123)x - 161616}{11776\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] 144217/2048*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 1/11776*((46*(4*(8*(20*x + 143)*x + 2273)*x - 11099)*x + 2124123)*x - 1616
165)/sqrt(2*x^2 - x + 3)
```

3.352 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{5}{6}\sqrt{2x^2-x+3}x^2+\frac{193}{48}\sqrt{2x^2-x+3}x+\frac{33}{64}\sqrt{2x^2-x+3}-\frac{53-373x}{23\sqrt{2x^2-x+3}}+\frac{3111\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

```
[Out] -(53 - 373*x)/(23*Sqrt[3 - x + 2*x^2]) + (33*Sqrt[3 - x + 2*x^2])/64 + (193
*x*Sqrt[3 - x + 2*x^2])/48 + (5*x^2*Sqrt[3 - x + 2*x^2])/6 + (3111*ArcSinh[
(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rubi [A] time = 0.101666, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{6}\sqrt{2x^2-x+3}x^2+\frac{193}{48}\sqrt{2x^2-x+3}x+\frac{33}{64}\sqrt{2x^2-x+3}-\frac{53-373x}{23\sqrt{2x^2-x+3}}+\frac{3111\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] -(53 - 373*x)/(23*Sqrt[3 - x + 2*x^2]) + (33*Sqrt[3 - x + 2*x^2])/64 + (193
*x*Sqrt[3 - x + 2*x^2])/48 + (5*x^2*Sqrt[3 - x + 2*x^2])/6 + (3111*ArcSinh[
(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{575}{4}+161x^2+\frac{115x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{5}{6}x^2\sqrt{3-x+2x^2} + \frac{1}{69}\int \frac{-\frac{1725}{2}-345x+\frac{4439x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3-x+2x^2} + \frac{1}{276}\int \frac{-\frac{27117}{4}+}{\sqrt{3-x}}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3-x+2x^2}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3-x+2x^2}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3-x+2x^2}$$

Mathematica [A] time = 0.18094, size = 60, normalized size = 0.58

$$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}} - \frac{3111\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x
]
```

```
[Out] (-3345 + 122607*x - 2162*x^2 + 31832*x^3 + 7360*x^4)/(4416*Sqrt[3 - x + 2*x
^2]) - (3111*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Maple [A] time = 0.064, size = 115, normalized size = 1.1

$$\frac{5\,x^4}{3}\frac{1}{\sqrt{2\,x^2-x+3}} - \frac{3111\,\sqrt{2}}{256}\mathrm{Arcsinh}\left(\frac{4\,\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right) + \frac{-10185+40740\,x}{11776}\frac{1}{\sqrt{2\,x^2-x+3}} + \frac{3111\,x}{128}\frac{1}{\sqrt{2\,x^2-x+3}} + \frac{17}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/3*x^4/(2*x^2-x+3)^(1/2)-3111/256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+1
0185/11776*(-1+4*x)/(2*x^2-x+3)^(1/2)+3111/128*x/(2*x^2-x+3)^(1/2)+173/24*x
^3/(2*x^2-x+3)^(1/2)-47/96*x^2/(2*x^2-x+3)^(1/2)+55/512/(2*x^2-x+3)^(1/2)
```

Maxima [A] time = 1.4559, size = 131, normalized size = 1.27

$$\frac{5\,x^4}{3\,\sqrt{2\,x^2-x+3}}+\frac{173\,x^3}{24\,\sqrt{2\,x^2-x+3}}-\frac{47\,x^2}{96\,\sqrt{2\,x^2-x+3}}-\frac{3111}{256}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{40869\,x}{1472\,\sqrt{2\,x^2-x+3}}-\frac{1}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")
```

```
[Out] 5/3*x^4/sqrt(2*x^2 - x + 3) + 173/24*x^3/sqrt(2*x^2 - x + 3) - 47/96*x^2/sq
rt(2*x^2 - x + 3) - 3111/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 408
69/1472*x/sqrt(2*x^2 - x + 3) - 1115/1472/sqrt(2*x^2 - x + 3)
```

Fricas [A] time = 1.32192, size = 270, normalized size = 2.62

$$\frac{214659\,\sqrt{2}\left(2\,x^2-x+3\right)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+8\left(7360\,x^4+31832\,x^3-2162\,x^2+1226\right.}{35328\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/35328*(214659*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 8*(7360*x^4 + 31832*x^3 - 2162*x^2 + 12260
7*x - 3345)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2)
, x)
```

Giac [A] time = 1.14613, size = 90, normalized size = 0.87

$$\frac{3111}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(4(40x+173)x-47)x+122607)x-3345}{4416\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")
```

```
[Out] 3111/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
4416*((46*(4*(40*x + 173)*x - 47)*x + 122607)*x - 3345)/sqrt(2*x^2 - x + 3)
```

3.353 $\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{5}{8}\sqrt{2x^2-x+3}x+\frac{27}{32}\sqrt{2x^2-x+3}+\frac{219x+89}{92\sqrt{2x^2-x+3}}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

```
[Out] (89 + 219*x)/(92*Sqrt[3 - x + 2*x^2]) + (27*Sqrt[3 - x + 2*x^2])/32 + (5*x*
Sqrt[3 - x + 2*x^2])/8 + (213*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

Rubi [A] time = 0.0551878, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{8}\sqrt{2x^2-x+3}x+\frac{27}{32}\sqrt{2x^2-x+3}+\frac{219x+89}{92\sqrt{2x^2-x+3}}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (89 + 219*x)/(92*Sqrt[3 - x + 2*x^2]) + (27*Sqrt[3 - x + 2*x^2])/32 + (5*x*
Sqrt[3 - x + 2*x^2])/8 + (213*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx &= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{345}{16}+\frac{69x}{8}+\frac{115x^2}{4}}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{5}{8}x\sqrt{3-x+2x^2}+\frac{1}{46}\int \frac{-\frac{345}{2}+\frac{621x}{8}}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}-\frac{213}{64}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}-\frac{213\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+\right.}{64\sqrt{46}}\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.128901, size = 55, normalized size = 0.67

$$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}}-\frac{213\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (2575 + 2511*x + 782*x^2 + 920*x^3)/(736*Sqrt[3 - x + 2*x^2]) - (213*ArcSin
h[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

Maple [A] time = 0.051, size = 98, normalized size = 1.2

$$\frac{5\,x^3}{4}\frac{1}{\sqrt{2\,x^2-x+3}}+\frac{17\,x^2}{16}\frac{1}{\sqrt{2\,x^2-x+3}}+\frac{213\,x}{64}\frac{1}{\sqrt{2\,x^2-x+3}}+\frac{901}{256}\frac{1}{\sqrt{2\,x^2-x+3}}+\frac{-123+492\,x}{5888}\frac{1}{\sqrt{2\,x^2-x+3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/4*x^3/(2*x^2-x+3)^(1/2)+17/16*x^2/(2*x^2-x+3)^(1/2)+213/64*x/(2*x^2-x+3)^
(1/2)+901/256/(2*x^2-x+3)^(1/2)+123/5888*(-1+4*x)/(2*x^2-x+3)^(1/2)-213/128
*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A] time = 1.54162, size = 108, normalized size = 1.32

$$\frac{5\,x^3}{4\,\sqrt{2\,x^2-x+3}}+\frac{17\,x^2}{16\,\sqrt{2\,x^2-x+3}}-\frac{213}{128}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)+\frac{2511\,x}{736\,\sqrt{2\,x^2-x+3}}+\frac{2575}{736\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/4*x^3/sqrt(2*x^2 - x + 3) + 17/16*x^2/sqrt(2*x^2 - x + 3) - 213/128*sqrt(
2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2511/736*x/sqrt(2*x^2 - x + 3) + 2575
/736/sqrt(2*x^2 - x + 3)
```

Fricas [A] time = 1.35129, size = 244, normalized size = 2.98

$$\frac{4899 \sqrt{2}\left(2 x^2-x+3\right) \log \left(4 \sqrt{2} \sqrt{2 x^2-x+3}(4 x-1)-32 x^2+16 x-25\right)+8\left(920 x^3+782 x^2+2511 x+2575\right) \sqrt{2 x^2 \cdots}}{5888\left(2 x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5888*(4899*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25) + 8*(920*x^3 + 782*x^2 + 2511*x + 2575)*sqrt(2*
x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

Giac [A] time = 1.15944, size = 84, normalized size = 1.02

$$\frac{213}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(46(20x + 17)x + 2511)x + 2575}{736\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 213/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/7
36*((46*(20*x + 17)*x + 2511)*x + 2575)/sqrt(2*x^2 - x + 3)
```

3.354 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{917x+1191}{3312\sqrt{2x^2-x+3}}+\frac{5}{8}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

```
[Out] (1191 + 917*x)/(3312*Sqrt[3 - x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/8 + (39
*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1728*Sqrt[2])
```

Rubi [A] time = 0.151246, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1646, 1653, 843, 619, 215, 724, 206}

$$\frac{917x+1191}{3312\sqrt{2x^2-x+3}}+\frac{5}{8}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (1191 + 917*x)/(3312*Sqrt[3 - x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/8 + (39
*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1728*Sqrt[2])
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{6739}{576}+\frac{69x}{8}+\frac{115x^2}{4}}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$=\frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}+\frac{1}{92}\int \frac{\frac{3611}{72}-\frac{897x}{2}}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$=\frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}-\frac{39}{16}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx+\frac{3667}{288}\int \frac{1}{(5+2x)\sqrt{3}}$$

$$=\frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}-\frac{3667}{144}\operatorname{Subst}\left(\int \frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)$$

$$=\frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1728\sqrt{2}}$$

Mathematica [A] time = 0.349433, size = 86, normalized size = 0.85

$$\frac{\frac{12\left(4140x^2-1153x+7401\right)}{23\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}}-3667\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)+3667\log(2x+5)-4212\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{1728\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x
]
```

```
[Out] ((12*(7401 - 1153*x + 4140*x^2))/(23*Sqrt[3/2 - x/2 + x^2]) - 4212*ArcSinh[
(-1 + 4*x)/Sqrt[23]] + 3667*Log[5 + 2*x] - 3667*Log[17 - 22*x + 12*Sqrt[6 -
 2*x + 4*x^2]])/(1728*Sqrt[2])
```

Maple [A] time = 0.061, size = 148, normalized size = 1.5

$$\frac{5x^2}{4}\frac{1}{\sqrt{2x^2-x+3}}+\frac{39x}{16}\frac{1}{\sqrt{2x^2-x+3}}-\frac{309}{64}\frac{1}{\sqrt{2x^2-x+3}}-\frac{-5507+22028x}{1472}\frac{1}{\sqrt{2x^2-x+3}}-\frac{39\sqrt{2}}{32}\operatorname{Arcsinh}\left(\frac{4\sqrt{}}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/4*x^2/(2*x^2-x+3)^(1/2)+39/16*x/(2*x^2-x+3)^(1/2)-309/64/(2*x^2-x+3)^(1/2
)-5507/1472*(-1+4*x)/(2*x^2-x+3)^(1/2)-39/32*2^(1/2)*arcsinh(4/23*23^(1/2)*
(x-1/4))+3667/576/(2*(x+5/2)^2-11*x-19/2)^(1/2)+40337/13248*(-1+4*x)/(2*(x+
5/2)^2-11*x-19/2)^(1/2)-3667/3456*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/
(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A] time = 1.52708, size = 134, normalized size = 1.33

$$\frac{5x^2}{4\sqrt{2x^2-x+3}} - \frac{39}{32}\sqrt{2}\,\text{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{3667}{3456}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{1153x}{3312\sqrt{2x^2-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")
```

```
[Out] 5/4*x^2/sqrt(2*x^2 - x + 3) - 39/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*
sqrt(23)) + 3667/3456*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23
*sqrt(23)/abs(2*x + 5)) - 1153/3312*x/sqrt(2*x^2 - x + 3) + 2467/1104/sqrt(
2*x^2 - x + 3)
```

Fricas [A] time = 1.38289, size = 413, normalized size = 4.09

$$\frac{96876\sqrt{2}\left(2x^2-x+3\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+84341\sqrt{2}\left(2x^2-x+3\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x}}{}\right.}{158976\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/158976*(96876*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 84341*sqrt(2)*(2*x^2 - x + 3)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
```

```
0*x + 25)) + 48*(4140*x^2 - 1153*x + 7401)*sqrt(2*x^2 - x + 3))/(2*x^2 - x
+ 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)\left(2x^2 - x + 3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(3/2
)), x)
```

Giac [A] time = 1.16943, size = 159, normalized size = 1.57

$$\frac{39}{32}\sqrt{2}\log\left(-4\sqrt{2}x + \sqrt{2} + 4\sqrt{2x^2 - x + 3}\right) - \frac{3667}{3456}\sqrt{2}\log\left(\left|-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) + \frac{3667}{3456}\sqrt{2}\log\left(\left|-2\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")
```

```
[Out] 39/32*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/34
56*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/
3456*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/3312*((4140*x - 1153)*x + 7401)/sqrt(2*x^2 - x + 3)
```

3.355 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{10368(2x+5)}+\frac{25951\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

```
[Out] (9897 + 2203*x)/(119232*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(
10368*(5 + 2*x)) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8*Sqrt[2]) + (25951*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(41472*Sqrt[2])
```

Rubi [A] time = 0.152743, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1646, 1650, 843, 619, 215, 724, 206}

$$\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{10368(2x+5)}+\frac{25951\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)),x]
```


```
[Out] (9897 + 2203*x)/(119232*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(
10368*(5 + 2*x)) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8*Sqrt[2]) + (25951*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(41472*Sqrt[2])
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{33649}{20736}+\frac{131215x}{10368}+\frac{115x^2}{4}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} - \frac{1}{828}\int \frac{\frac{100073}{192}-1035x}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} + \frac{5}{8}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx - \frac{25951\int \frac{1}{(5+2x)\sqrt{3}}}{6912}$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} + \frac{25951\,\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{3456} +$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} - \frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}} + \frac{25951\tanh^{-1}\left(\frac{17-}{12\sqrt{2}\sqrt{3}}\right.}{41472\sqrt{2}}$$

Mathematica [A] time = 0.317964, size = 104, normalized size = 0.96

$$\frac{\frac{8(2203x+9897)}{23\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}} - \frac{14668\sqrt{4x^2-2x+6}}{2x+5} + 25951\log\left(12\sqrt{4x^2-2x+6}-22x+17\right) - 25951\log(2x+5) + 25920\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{41472\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((8*(9897 + 2203*x))/(23*Sqrt[3/2 - x/2 + x^2]) - (14668*Sqrt[6 - 2*x + 4*x
^2])/(5 + 2*x) + 25920*ArcSinh[(-1 + 4*x)/Sqrt[23]] - 25951*Log[5 + 2*x] +
25951*Log[17 - 22*x + 12*Sqrt[6 - 2*x + 4*x^2]])/(41472*Sqrt[2])
```

Maple [A] time = 0.057, size = 152, normalized size = 1.4

$$-\frac{5x}{8}\frac{1}{\sqrt{2x^2-x+3}} + \frac{99}{32}\frac{1}{\sqrt{2x^2-x+3}} + \frac{-1529+6116x}{736}\frac{1}{\sqrt{2x^2-x+3}} + \frac{5\sqrt{2}}{16}\text{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right) - \frac{25951}{13824}\frac{}{\sqrt{2\,(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/8*x/(2*x^2-x+3)^(1/2)+99/32/(2*x^2-x+3)^(1/2)+1529/736*(-1+4*x)/(2*x^2-x
+3)^(1/2)+5/16*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-25951/13824/(2*(x+5/2
)^2-11*x-19/2)^(1/2)-637493/317952*(-1+4*x)/(2*(x+5/2)^2-11*x-19/2)^(1/2)+2
5951/82944*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)
^(1/2))-3667/1152/(x+5/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2)
```

Maxima [A] time = 1.52622, size = 157, normalized size = 1.45

$$\frac{5}{16}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{25951}{82944}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{26645\,x}{79488\sqrt{2\,x^2-x+3}}+\frac{3}{26496\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 25951/82944*sqrt(2)
*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 266
45/79488*x/sqrt(2*x^2 - x + 3) + 30313/26496/sqrt(2*x^2 - x + 3) - 3667/576
/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt(2*x^2 - x + 3))
```

Fricas [A] time = 1.37733, size = 458, normalized size = 4.24

$$\frac{596160\sqrt{2}\left(4\,x^3+8\,x^2+x+15\right)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+596873\sqrt{2}\left(4\,x^3+8\,x^2+x+\right.}{3815424\left(4\,x^3+8\,x^2+x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/3815424*(596160*sqrt(2)*(4*x^3 + 8*x^2 + x + 15)*log(-4*sqrt(2)*sqrt(2*x^
2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 596873*sqrt(2)*(4*x^3 + 8*x^2
+ x + 15)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036
*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(53290*x^2 - 48653*x + 51351)*sqrt(2*x
```

```
^2 - x + 3))/(4*x^3 + 8*x^2 + x + 15)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^2 (2x^2 - x + 3)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^4 - x^3 + 3\,x^2 + x + 2}{(2\,x^2 - x + 3)^{\frac{3}{2}} (2\,x+5)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((5*x^4 - x^3 + 3*x^2 + x + 2)/((2*x^2 - x + 3)^(3/2)*(2*x + 5)^2)
, x)
```

3.356 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}+\frac{115369\sqrt{2x^2-x+3}}{1492992(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{20736(2x+5)^2}-\frac{52631\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}$$

```
[Out] (65991 - 8779*x)/(4292352*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])
/(20736*(5 + 2*x)^2) + (115369*Sqrt[3 - x + 2*x^2])/(1492992*(5 + 2*x)) - (
52631*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(5971968*Sqrt[
2])
```

Rubi [A] time = 0.145885, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}+\frac{115369\sqrt{2x^2-x+3}}{1492992(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{20736(2x+5)^2}-\frac{52631\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2)),x]
```


```
[Out] (65991 - 8779*x)/(4292352*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])
/(20736*(5 + 2*x)^2) + (115369*Sqrt[3 - x + 2*x^2])/(1492992*(5 + 2*x)) - (
52631*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(5971968*Sqrt[
2])
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
```

```
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{\frac{5168261}{746496}+\frac{3637795x}{186624}+\frac{5620625x^2}{186624}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}-\frac{\int \frac{\frac{842237}{1296}-\frac{4102487x}{2592}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{1656}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}+\frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)}+\frac{52631\int \frac{}{(5+2x)}}{9953}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}+\frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)}-\frac{52631\,\text{Subst}\,(}{}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}+\frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)}-\frac{52631\tanh^{-1}}{5971}$$

Mathematica [A] time = 0.296961, size = 84, normalized size = 0.75

$$\frac{\frac{12\left(3444340x^3+3263288x^2+5842933x+11594283\right)}{23(2x+5)^2\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}}-52631\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)+52631\log(2x+5)}{5971968\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((12*(11594283 + 5842933*x + 3263288*x^2 + 3444340*x^3))/(23*(5 + 2*x)^2*Sq
rt[3/2 - x/2 + x^2]) + 52631*Log[5 + 2*x] - 52631*Log[17 - 22*x + 12*Sqrt[6
 - 2*x + 4*x^2]])/(5971968*Sqrt[2])
```

Maple [A] time = 0.058, size = 144, normalized size = 1.3

$$-\frac{5}{16}\frac{1}{\sqrt{2x^2-x+3}}-\frac{-149+596x}{368}\frac{1}{\sqrt{2x^2-x+3}}-\frac{3667}{4608}\left(x+\frac{5}{2}\right)^{-2}\frac{1}{\sqrt{2\left(x+5/2\right)^2-11x-\frac{19}{2}}}+\frac{196043}{165888}\left(x+\frac{5}{2}\right)^{-1}\frac{}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/16/(2*x^2-x+3)^(1/2)-149/368*(-1+4*x)/(2*x^2-x+3)^(1/2)-3667/4608/(x+5/2
)^2/(2*(x+5/2)^2-11*x-19/2)^(1/2)+196043/165888/(x+5/2)/(2*(x+5/2)^2-11*x-1
9/2)^(1/2)+52631/1990656/(2*(x+5/2)^2-11*x-19/2)^(1/2)+19399069/45785088*(-
1+4*x)/(2*(x+5/2)^2-11*x-19/2)^(1/2)-52631/11943936*2^(1/2)*arctanh(1/12*(1
7/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A] time = 1.51339, size = 201, normalized size = 1.79

$$\frac{52631}{11943936}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{861085\,x}{11446272\sqrt{2\,x^2-x+3}}-\frac{1163201}{3815424\sqrt{2\,x^2-x+3}}-\frac{}{1152\left(4\sqrt{2\,x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 52631/11943936*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(2
3)/abs(2*x + 5)) + 861085/11446272*x/sqrt(2*x^2 - x + 3) - 1163201/3815424/
sqrt(2*x^2 - x + 3) - 3667/1152/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2
- x + 3)*x + 25*sqrt(2*x^2 - x + 3)) + 196043/82944/(2*sqrt(2*x^2 - x + 3)*
x + 5*sqrt(2*x^2 - x + 3))
```

Fricas [A] time = 1.3407, size = 379, normalized size = 3.38

$$\frac{1210513\sqrt{2}\left(8\,x^4+36\,x^3+42\,x^2+35\,x+75\right)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)+48\left(3444340\,x^3+3263288\right.}{549421056\left(8\,x^4+36\,x^3+42\,x^2+35\,x+75\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/549421056*(1210513*sqrt(2)*(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)*log(-(24
*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2
 + 20*x + 25)) + 48*(3444340*x^3 + 3263288*x^2 + 5842933*x + 11594283)*sqrt
```

```
(2*x^2 - x + 3))/(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^3 \left(2x^2 - x + 3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
3/2)), x)
```

Giac [B] time = 1.15733, size = 297, normalized size = 2.65

$$-\frac{52631}{11943936}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x + \sqrt{2} + 2\,\sqrt{2\,x^2 - x + 3}\right|\right) + \frac{52631}{11943936}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x - 11\,\sqrt{2} + 2\,\sqrt{2\,x^2 - x + 3}\right|\right) - \frac{}{429}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x +
 3))) + 52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2
*x^2 - x + 3))) - 1/4292352*(8779*x - 65991)/sqrt(2*x^2 - x + 3) + 1/298598
4*sqrt(2)*(3594214*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 19874490*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 30140067*sqrt(2)*(sqrt(2)*x - sqrt(2*x
^2 - x + 3)) + 19989859)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

3.357 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}+\frac{430799\sqrt{2x^2-x+3}}{107495424(2x+5)}+\frac{152885\sqrt{2x^2-x+3}}{4478976(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{31104(2x+5)^3}-\frac{3505819\tanh^{-1}\left(\frac{17-}{12\sqrt{2}\sqrt{}}\right.}{1289945088\sqrt{2}}$$

```
[Out] (369609 - 175877*x)/(154524672*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*
x^2])/(31104*(5 + 2*x)^3) + (152885*Sqrt[3 - x + 2*x^2])/(4478976*(5 + 2*x)
^2) + (430799*Sqrt[3 - x + 2*x^2])/(107495424*(5 + 2*x)) - (3505819*ArcTanh
[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1289945088*Sqrt[2])
```

Rubi [A] time = 0.203512, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}+\frac{430799\sqrt{2x^2-x+3}}{107495424(2x+5)}+\frac{152885\sqrt{2x^2-x+3}}{4478976(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{31104(2x+5)^3}-\frac{3505819\tanh^{-1}\left(\frac{17-}{12\sqrt{2}\sqrt{}}\right.}{1289945088\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2)),x]
```


```
[Out] (369609 - 175877*x)/(154524672*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*
x^2])/(31104*(5 + 2*x)^3) + (152885*Sqrt[3 - x + 2*x^2])/(4478976*(5 + 2*x)
^2) + (430799*Sqrt[3 - x + 2*x^2])/(107495424*(5 + 2*x)) - (3505819*ArcTanh
[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1289945088*Sqrt[2])
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
```

```
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{\frac{348877271}{26873856}+\frac{119871055x}{4478976}+\frac{73960295x^2}{2239488}+\frac{1302559x^3}{3359232}}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}-\frac{\int \frac{\frac{79609325}{124416}-\frac{71248733x}{31104}-\frac{1302559x^2}{31104}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx}{2484}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{\int \frac{\frac{29340847}{1728}+\frac{148}{}}{(5+2x)^2\sqrt{3-x}}}{35769}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430799\sqrt{3-}}{107495424(}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430799\sqrt{3-}}{107495424(}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430799\sqrt{3-}}{107495424(}$$

Mathematica [A] time = 0.164399, size = 95, normalized size = 0.69

$$\frac{24\left(56754760x^4+572739684x^3+441046842x^2+1257975811x+1873786587\right)-80633837(2x+5)^3\sqrt{4x^2-2x+6}\tan}{59337474048(2x+5)^3\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] (24*(1873786587 + 1257975811*x + 441046842*x^2 + 572739684*x^3 + 56754760*x
^4) - 80633837*(5 + 2*x)^3*Sqrt[6 - 2*x + 4*x^2]*ArcTanh[(17 - 22*x)/(12*Sq
rt[6 - 2*x + 4*x^2])])/(59337474048*(5 + 2*x)^3*Sqrt[3 - x + 2*x^2])
```

Maple [A] time = 0.063, size = 151, normalized size = 1.1

$$\frac{-5+20\,x}{184}\frac{1}{\sqrt{2\,x^2-x+3}}+\frac{314233}{995328}\left(x+\frac{5}{2}\right)^{-2}\frac{1}{\sqrt{2\,(x+5/2)^2-11\,x-\frac{19}{2}}}-\frac{3127169}{35831808}\left(x+\frac{5}{2}\right)^{-1}\frac{1}{\sqrt{2\,(x+5/2)^2-11\,x-\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/184*(-1+4*x)/(2*x^2-x+3)^(1/2)+314233/995328/(x+5/2)^2/(2*(x+5/2)^2-11*x-
19/2)^(1/2)-3127169/35831808/(x+5/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2)+3505819/
429981696/(2*(x+5/2)^2-11*x-19/2)^(1/2)-261644215/9889579008*(-1+4*x)/(2*(x
+5/2)^2-11*x-19/2)^(1/2)-3505819/2579890176*2^(1/2)*arctanh(1/12*(17/2-11*x
)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-3667/13824/(x+5/2)^3/(2*(x+5/2)^2-
11*x-19/2)^(1/2)
```

Maxima [A] time = 1.60093, size = 293, normalized size = 2.14

$$\frac{3505819}{2579890176}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{7094345\,x}{2472394752\sqrt{2\,x^2-x+3}}+\frac{6128291}{824131584\sqrt{2\,x^2-x+3}}-\frac{}{1728\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 3505819/2579890176*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sq
rt(23)/abs(2*x + 5)) + 7094345/2472394752*x/sqrt(2*x^2 - x + 3) + 6128291/8
24131584/sqrt(2*x^2 - x + 3) - 3667/1728/(8*sqrt(2*x^2 - x + 3)*x^3 + 60*sq
rt(2*x^2 - x + 3)*x^2 + 150*sqrt(2*x^2 - x + 3)*x + 125*sqrt(2*x^2 - x + 3)
) + 314233/248832/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2 - x + 3)*x + 2
5*sqrt(2*x^2 - x + 3)) - 3127169/17915904/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt
(2*x^2 - x + 3))
```

Fricas [A] time = 1.33425, size = 458, normalized size = 3.34

$$\frac{80633837\sqrt{2}\left(16\,x^5+112\,x^4+264\,x^3+280\,x^2+325\,x+375\right)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)+48\,(567}{118674948096\,(16\,x^5+112\,x^4+264\,x^3+280\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/118674948096*(80633837*sqrt(2)*(16*x^5 + 112*x^4 + 264*x^3 + 280*x^2 + 32
5*x + 375)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 10
36*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(56754760*x^4 + 572739684*x^3 + 4410
46842*x^2 + 1257975811*x + 1873786587)*sqrt(2*x^2 - x + 3))/(16*x^5 + 112*x
^4 + 264*x^3 + 280*x^2 + 325*x + 375)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^4 \left(2x^2 - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(
3/2)), x)
```

Giac [B] time = 1.18457, size = 366, normalized size = 2.67

$$-\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) + \frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) + 3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
2*sqrt(2*x^2 - x + 3))) - 1/154524672*(175877*x - 369609)/sqrt(2*x^2 - x +
3) - 1/214990848*sqrt(2)*(10398764*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^5 - 303070900*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 529738052*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3))^3 + 3644644652*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^2 - 2612608649*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1052284471)
/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3)) - 11)^3
```

3.358 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$\frac{4(18982-20383x)}{1587\sqrt{2x^2-x+3}}+\frac{5}{4}x\sqrt{2x^2-x+3}+\frac{247}{16}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] (-4*(346 - 533*x))/(69*(3 - x + 2*x^2)^(3/2)) + (4*(18982 - 20383*x))/(1587
*Sqrt[3 - x + 2*x^2]) + (247*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*
x^2])/4 - (1471*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rubi [A] time = 0.130773, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{4(18982-20383x)}{1587\sqrt{2x^2-x+3}}+\frac{5}{4}x\sqrt{2x^2-x+3}+\frac{247}{16}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-4*(346 - 533*x))/(69*(3 - x + 2*x^2)^(3/2)) + (4*(18982 - 20383*x))/(1587
*Sqrt[3 - x + 2*x^2]) + (247*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*
x^2])/4 - (1471*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{-145-\frac{1725x}{2}+2415x^2+\frac{3657x^3}{2}+345x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{4\int \frac{\frac{33327}{2}+\frac{46023x}{4}+\frac{7935x^2}{4}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{\int \frac{\frac{242811}{4}+\frac{39}{}}{\sqrt{3-x+2}}}{158}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{247}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{247}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{247}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x}$$

Mathematica [A] time = 0.711421, size = 65, normalized size = 0.62

$$\frac{126960x^5+1440996x^4-3764360x^3+8639625x^2-6410082x+6663133}{25392\left(2x^2-x+3\right)^{3/2}} - \frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2)
,x]
```

```
[Out] (6663133 - 6410082*x + 8639625*x^2 - 3764360*x^3 + 1440996*x^4 + 126960*x^5
)/(25392*(3 - x + 2*x^2)^(3/2)) - (1471*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sq
rt[2])
```

Maple [B] time = 0.059, size = 180, normalized size = 1.7

$$5\,\frac{x^{5}}{\left(2\,x^{2}-x+3\right)^{3/2}}-\frac{1471\,x^{3}}{48}\left(2\,x^{2}-x+3\right)^{-\frac{3}{2}}+\frac{19073\,x^{2}}{64}\left(2\,x^{2}-x+3\right)^{-\frac{3}{2}}+\frac{1471\,\sqrt{2}}{64}\mathrm{Arcsinh}\left(\frac{4\,\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)-\frac{-1629}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 5*x^5/(2*x^2-x+3)^(3/2)-1471/48*x^3/(2*x^2-x+3)^(3/2)+19073/64*x^2/(2*x^2-x
+3)^(3/2)+1471/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-162931/50784*(-1+4
*x)/(2*x^2-x+3)^(1/2)-753223/141312*(-1+4*x)/(2*x^2-x+3)^(3/2)-32257/512*x/
(2*x^2-x+3)^(3/2)+227/4*x^4/(2*x^2-x+3)^(3/2)-1471/32*x/(2*x^2-x+3)^(1/2)-1
471/128/(2*x^2-x+3)^(1/2)+577397/2048/(2*x^2-x+3)^(3/2)
```

Maxima [B] time = 1.55226, size = 296, normalized size = 2.82

$$\frac{5\,x^{5}}{\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}}+\frac{227\,x^{4}}{4\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}}+\frac{1471}{50784}\,x\left(\frac{284\,x}{\sqrt{2\,x^{2}-x+3}}-\frac{3174\,x^{2}}{\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^{2}-x+3}}+\frac{805\,x}{\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{}{(2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 5*x^5/(2*x^2 - x + 3)^(3/2) + 227/4*x^4/(2*x^2 - x + 3)^(3/2) + 1471/50784*
x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x
^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 1
471/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 104441/25392*sqrt(2*x^2 -
 x + 3) - 383581/12696*x/sqrt(2*x^2 - x + 3) + 321*x^2/(2*x^2 - x + 3)^(3/2
) - 15965/4232/sqrt(2*x^2 - x + 3) - 4147/46*x/(2*x^2 - x + 3)^(3/2) + 4288
3/138/(2*x^2 - x + 3)^(3/2)
```

Fricas [A] time = 1.33287, size = 359, normalized size = 3.42

$$\frac{2334477\,\sqrt{2}\left(4\,x^{4}-4\,x^{3}+13\,x^{2}-6\,x+9\right)\log\left(-4\,\sqrt{2}\sqrt{2\,x^{2}-x+3}(4\,x-1)-32\,x^{2}+16\,x-25\right)+8\left(126960\,x^{5}+14409\right.}{203136\left(4\,x^{4}-4\,x^{3}+13\,x^{2}-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/203136*(2334477*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(126960*x^5 + 1440
996*x^4 - 3764360*x^3 + 8639625*x^2 - 6410082*x + 6663133)*sqrt(2*x^2 - x +
 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+5)^2\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5
/2), x)
```

Giac [A] time = 1.16211, size = 96, normalized size = 0.91

$$-\frac{1471}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(1587(20x+227)x-941090)x+8639625)x-6410082)x+66}{25392\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] -1471/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
25392*(((4*(1587*(20*x + 227)*x - 941090)*x + 8639625)*x - 6410082)*x + 666
3133)/(2*x^2 - x + 3)^(3/2)
```

3.359 $\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{6055-28981x}{3174\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}-\frac{71\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

```
[Out] -(53 - 373*x)/(69*(3 - x + 2*x^2)^(3/2)) + (6055 - 28981*x)/(3174*Sqrt[3 - x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/4 - (71*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8*Sqrt[2])
```

Rubi [A] time = 0.0822592, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1660, 640, 619, 215}

$$\frac{6055-28981x}{3174\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}-\frac{71\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] -(53 - 373*x)/(69*(3 - x + 2*x^2)^(3/2)) + (6055 - 28981*x)/(3174*Sqrt[3 - x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/4 - (71*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{233}{4}+483x^2+\frac{345x^3}{2}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{52371}{16}+\frac{7935x}{8}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}+\frac{71}{8}\int \frac{1}{\sqrt{3-x}\ldots}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}+\frac{71\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}-\frac{71\sinh^{-1}\left(\frac{1-\ldots}{\sqrt{2\ldots}}\right.}{8\sqrt{2}}$$

Mathematica [A] time = 0.259808, size = 60, normalized size = 0.7

$$\frac{31740x^4-147664x^3+185337x^2-199290x+102869}{6348\left(2x^2-x+3\right)^{3/2}}+\frac{71\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x
]
```

```
[Out] (102869 - 199290*x + 185337*x^2 - 147664*x^3 + 31740*x^4)/(6348*(3 - x + 2*
x^2)^(3/2)) + (71*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8*Sqrt[2])
```

Maple [B] time = 0.055, size = 163, normalized size = 1.9

$$-\frac{71\,x^3}{12}\left(2\,x^2-x+3\right)^{-\frac{3}{2}}+\frac{401\,x^2}{16}\left(2\,x^2-x+3\right)^{-\frac{3}{2}}+\frac{71\,\sqrt{2}}{16}\mathrm{Arcsinh}\left(\frac{4\,\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)+\frac{-643+2572\,x}{12696}\frac{1}{\sqrt{2\,x^2-x+3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -71/12*x^3/(2*x^2-x+3)^(3/2)+401/16*x^2/(2*x^2-x+3)^(3/2)+71/16*2^(1/2)*arc
sinh(4/23*23^(1/2)*(x-1/4))+643/12696*(-1+4*x)/(2*x^2-x+3)^(1/2)-2327/35328
*(-1+4*x)/(2*x^2-x+3)^(3/2)-945/128*x/(2*x^2-x+3)^(3/2)+5*x^4/(2*x^2-x+3)^(
3/2)-71/8*x/(2*x^2-x+3)^(1/2)-71/32/(2*x^2-x+3)^(1/2)+11749/512/(2*x^2-x+3)
^(3/2)
```

Maxima [B] time = 1.50182, size = 273, normalized size = 3.17

$$\frac{5\,x^4}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{71}{12696}x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\right)+\frac{71}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 5*x^4/(2*x^2 - x + 3)^(3/2) + 71/12696*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*
x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^
(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 71/16*sqrt(2)*arcsinh(1/23*sqrt(23)*(
4*x - 1)) - 5041/6348*sqrt(2*x^2 - x + 3) - 10007/3174*x/sqrt(2*x^2 - x + 3
```

```
) + 59/2*x^2/(2*x^2 - x + 3)^(3/2) - 2959/2116/sqrt(2*x^2 - x + 3) - 807/92
*x/(2*x^2 - x + 3)^(3/2) + 7603/276/(2*x^2 - x + 3)^(3/2)
```

Fricas [A] time = 1.34327, size = 331, normalized size = 3.85

$$\frac{112677\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(31740x^4-147664\right.}{50784\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/50784*(112677*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(31740*x^4 - 147664*
x^3 + 185337*x^2 - 199290*x + 102869)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 +
 13*x^2 - 6*x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2)
, x)
```

Giac [A] time = 1.17698, size = 89, normalized size = 1.03

$$-\frac{71}{16}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(7935x-36916)x+185337)x-199290)x+102869}{6348\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
```

```
[Out] -71/16*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/63
48*(((4*(7935*x - 36916)*x + 185337)*x - 199290)*x + 102869)/(2*x^2 - x + 3
)^(3/2)
```

3.360 $\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{219x+89}{276\left(2x^2-x+3\right)^{3/2}}-\frac{2604x+1465}{2116\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

```
[Out] (89 + 219*x)/(276*(3 - x + 2*x^2)^(3/2)) - (1465 + 2604*x)/(2116*Sqrt[3 - x
 + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rubi [A] time = 0.0518717, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1660, 12, 619, 215}

$$\frac{219x+89}{276\left(2x^2-x+3\right)^{3/2}}-\frac{2604x+1465}{2116\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89 + 219*x)/(276*(3 - x + 2*x^2)^(3/2)) - (1465 + 2604*x)/(2116*Sqrt[3 - x
 + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx &= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{159}{16}+\frac{207x}{8}+\frac{345x^2}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{4\int \frac{7935}{16\sqrt{3-x+2x^2}}\,dx}{1587}\\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5}{4}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{4\sqrt{46}}\\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.226112, size = 55, normalized size = 0.81

$$\frac{5\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{7812x^3+489x^2+7002x+5569}{3174\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] -(5569 + 7002*x + 489*x^2 + 7812*x^3)/(3174*(3 - x + 2*x^2)^(3/2)) + (5*Arc
Sinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Maple [B] time = 0.053, size = 146, normalized size = 2.2

$$-\frac{5x^3}{6}\left(2x^2-x+3\right)^{-\frac{3}{2}}-\frac{x^2}{8}\left(2x^2-x+3\right)^{-\frac{3}{2}}-\frac{47x}{64}\left(2x^2-x+3\right)^{-\frac{3}{2}}-\frac{271}{768}\left(2x^2-x+3\right)^{-\frac{3}{2}}+\frac{-2423+9692x}{17664}\left(2x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/6*x^3/(2*x^2-x+3)^(3/2)-1/8*x^2/(2*x^2-x+3)^(3/2)-47/64*x/(2*x^2-x+3)^(3
/2)-271/768/(2*x^2-x+3)^(3/2)+2423/17664*(-1+4*x)/(2*x^2-x+3)^(3/2)+173/158
7*(-1+4*x)/(2*x^2-x+3)^(1/2)-5/4*x/(2*x^2-x+3)^(1/2)-5/16/(2*x^2-x+3)^(1/2)
+5/8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [B] time = 1.46495, size = 250, normalized size = 3.68

$$\frac{5}{6348}x\left(\frac{284x}{\sqrt{2x^2-x+3}}-\frac{3174x^2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2x^2-x+3}}+\frac{805x}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\right)+\frac{5}{8}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/s
qrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/
2)) + 5/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 355/3174*sqrt(2*x^2 -
x + 3) - 58/1587*x/sqrt(2*x^2 - x + 3) + 1/2*x^2/(2*x^2 - x + 3)^(3/2) - 18
97/6348/sqrt(2*x^2 - x + 3) - 95/276*x/(2*x^2 - x + 3)^(3/2) + 41/276/(2*x^
2 - x + 3)^(3/2)
```

Fricas [B] time = 1.34762, size = 300, normalized size = 4.41

$$\frac{7935\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)-8\left(7812x^3+489x^2+700\right.}{25392\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25392*(7935*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sqr
t(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 8*(7812*x^3 + 489*x^2 +
7002*x + 5569)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{5x^4-x^3+3x^2+x+2}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

Giac [A] time = 1.14788, size = 84, normalized size = 1.24

$$-\frac{5}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{3\left(\left(2604x+163\right)x+2334\right)x+5569}{3174\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1/3174
*(3*((2604*x + 163)*x + 2334)*x + 5569)/(2*x^2 - x + 3)^(3/2)
```

3.361 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{146729x+335337}{1371168\sqrt{2x^2-x+3}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}$$

```
[Out] (1191 + 917*x)/(9936*(3 - x + 2*x^2)^(3/2)) - (335337 + 146729*x)/(1371168*
Sqrt[3 - x + 2*x^2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2
*x^2])])/(31104*Sqrt[2])
```

Rubi [A] time = 0.128261, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1646, 12, 724, 206}

$$\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{146729x+335337}{1371168\sqrt{2x^2-x+3}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x]
```


```
[Out] (1191 + 917*x)/(9936*(3 - x + 2*x^2)^(3/2)) - (335337 + 146729*x)/(1371168*
Sqrt[3 - x + 2*x^2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2
*x^2])])/(31104*Sqrt[2])
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
```

```
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx &= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{1877}{576}+\frac{695x}{18}+\frac{345x^2}{4}}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{4\int \frac{1939843}{6912(5+2x)\sqrt{3-x+2x^2}}\,dx}{1587}\\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{3667\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{5184}\\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\,\mathrm{Subst}\left(\int \frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{2592}\\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{31104\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.466597, size = 80, normalized size = 0.94

$$\frac{-\frac{12\sqrt{2}(293458x^3+523945x^2-21696x+841653)}{529(2x^2-x+3)^{3/2}}-3667\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)+3667\log(2x+5)}{31104\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x
]
```

```
[Out] ((-12*Sqrt[2]*(841653 - 21696*x + 523945*x^2 + 293458*x^3))/(529*(3 - x + 2
*x^2)^(3/2)) + 3667*Log[5 + 2*x] - 3667*Log[17 - 22*x + 12*Sqrt[6 - 2*x + 4
*x^2]])/(31104*Sqrt[2])
```

Maple [B] time = 0.056, size = 190, normalized size = 2.2

$$-\frac{5x^2}{4}\left(2x^2-x+3\right)^{-\frac{3}{2}}+\frac{59x}{32}\left(2x^2-x+3\right)^{-\frac{3}{2}}-\frac{1597}{384}\left(2x^2-x+3\right)^{-\frac{3}{2}}-\frac{-3817+15268x}{2944}\left(2x^2-x+3\right)^{-\frac{3}{2}}-\frac{-3817+}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/4*x^2/(2*x^2-x+3)^(3/2)+59/32*x/(2*x^2-x+3)^(3/2)-1597/384/(2*x^2-x+3)^(
3/2)-3817/2944*(-1+4*x)/(2*x^2-x+3)^(3/2)-3817/4232*(-1+4*x)/(2*x^2-x+3)^(1
/2)+3667/1728/(2*(x+5/2)^2-11*x-19/2)^(3/2)+40337/39744*(-1+4*x)/(2*(x+5/2)
^2-11*x-19/2)^(3/2)+4800103/5484672*(-1+4*x)/(2*(x+5/2)^2-11*x-19/2)^(1/2)+
3667/10368/(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/62208*2^(1/2)*arctanh(1/12*(1
7/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A] time = 1.65677, size = 149, normalized size = 1.75

$$\frac{3667}{62208}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)-\frac{146729\,x}{1371168\sqrt{2x^2-x+3}}-\frac{5x^2}{4\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{173881}{457056\sqrt{2x^2-x+3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 3667/62208*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/a
bs(2*x + 5)) - 146729/1371168*x/sqrt(2*x^2 - x + 3) - 5/4*x^2/(2*x^2 - x +
3)^(3/2) + 173881/457056/sqrt(2*x^2 - x + 3) + 7127/9936*x/(2*x^2 - x + 3)^
(3/2) - 5813/3312/(2*x^2 - x + 3)^(3/2)
```

Fricas [A] time = 1.31877, size = 362, normalized size = 4.26

$$\frac{1939843\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)-48\left(293458x^3+523945x^2-\right.}{65816064\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/65816064*(1939843*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
0*x + 25)) - 48*(293458*x^3 + 523945*x^2 - 21696*x + 841653)*sqrt(2*x^2 - x
 + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(5/2
)), x)
```

Giac [A] time = 1.17316, size = 124, normalized size = 1.46

$$-\frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{((293458x}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
```

```
[Out] -3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))
) + 3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 1/1371168*(((293458*x + 523945)*x - 21696)*x + 841653)/(2*x^2 - x
 + 3)^(3/2)
```

3.362 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{1255878-62021x}{24681024\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{186624(2x+5)}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}-\frac{2821\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}$$

```
[Out] (9897 + 2203*x)/(357696*(3 - x + 2*x^2)^(3/2)) - (1255878 - 62021*x)/(24681
024*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(186624*(5 + 2*x)) -
(2821*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(2239488*Sqrt[
2])
```

Rubi [A] time = 0.152912, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1646, 806, 724, 206}

$$-\frac{1255878-62021x}{24681024\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{186624(2x+5)}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}-\frac{2821\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (9897 + 2203*x)/(357696*(3 - x + 2*x^2)^(3/2)) - (1255878 - 62021*x)/(24681
024*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(186624*(5 + 2*x)) -
(2821*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(2239488*Sqrt[
2])
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
```

```
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{119353}{20736}+\frac{481765x}{10368}+\frac{113983x^2}{1296}}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}} - \frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}} + \frac{4\int \frac{\frac{10109719}{124416}-\frac{4961491x}{62208}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}} - \frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)} + \frac{2821\int \frac{}{(5+2}}{373}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}} - \frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)} - \frac{2821\,\mathrm{Subst}}{}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}} - \frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)} - \frac{2821\tanh^{}}{223}$$

Mathematica [A] time = 0.406562, size = 92, normalized size = 0.84

$$\frac{-\frac{12\sqrt{2}\left(6767036x^4+10350004x^3+63941915x^2-18840090x+79153407\right)}{529(2x+5)\left(2x^2-x+3\right)^{3/2}} - 2821\log\left(12\sqrt{4x^2-2x+6}-22x+17\right) + 2821\log(2x+5)}{2239488\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((-12*Sqrt[2]*(79153407 - 18840090*x + 63941915*x^2 + 10350004*x^3 + 676703
6*x^4))/(529*(5 + 2*x)*(3 - x + 2*x^2)^(3/2)) + 2821*Log[5 + 2*x] - 2821*Lo
g[17 - 22*x + 12*Sqrt[6 - 2*x + 4*x^2]])/(2239488*Sqrt[2])
```

Maple [B] time = 0.058, size = 194, normalized size = 1.8

$$-\frac{5\,x}{16}\left(2\,x^2-x+3\right)^{-\frac{3}{2}} + \frac{203}{192}\left(2\,x^2-x+3\right)^{-\frac{3}{2}} + \frac{-3173+12692\,x}{4416}\left(2\,x^2-x+3\right)^{-\frac{3}{2}} + \frac{-3173+12692\,x}{6348}\frac{1}{\sqrt{2\,x^2-x+3}} + \frac{}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/16*x/(2*x^2-x+3)^(3/2)+203/192/(2*x^2-x+3)^(3/2)+3173/4416*(-1+4*x)/(2*x
^2-x+3)^(3/2)+3173/6348*(-1+4*x)/(2*x^2-x+3)^(1/2)+2821/124416/(2*(x+5/2)^2
-11*x-19/2)^(3/2)-2081161/2861568*(-1+4*x)/(2*(x+5/2)^2-11*x-19/2)^(3/2)-19
9077743/394896384*(-1+4*x)/(2*(x+5/2)^2-11*x-19/2)^(1/2)+2821/746496/(2*(x+
5/2)^2-11*x-19/2)^(1/2)-2821/4478976*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/
2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-3667/1152/(x+5/2)/(2*(x+5/2)^2-11*x-19/2)
^(3/2)
```

Maxima [A] time = 1.55799, size = 171, normalized size = 1.55

$$\frac{2821}{4478976}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{1691759\,x}{98724096\sqrt{2\,x^2-x+3}}+\frac{265339}{32908032\sqrt{2\,x^2-x+3}}-\frac{24861\cdots}{715392\,(2\,x^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 2821/4478976*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)
/abs(2*x + 5)) - 1691759/98724096*x/sqrt(2*x^2 - x + 3) + 265339/32908032/s
qrt(2*x^2 - x + 3) - 248617/715392*x/(2*x^2 - x + 3)^(3/2) - 3667/576/(2*(2
*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x + 3)^(3/2)) + 259621/238464/(2*x^2 - x
 + 3)^(3/2)
```

Fricas [A] time = 1.31711, size = 425, normalized size = 3.86

$$\frac{1492309\sqrt{2}\left(8\,x^5+12\,x^4+6\,x^3+53\,x^2-12\,x+45\right)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)-48\left(6767036\,x^4+\cdots\right.}{4738756608\left(8\,x^5+12\,x^4+6\,x^3+53\,x^2-12\,x+4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/4738756608*(1492309*sqrt(2)*(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2 - 12*x + 45)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) - 48*(6767036*x^4 + 10350004*x^3 + 63941915*x^2 - 1
8840090*x + 79153407)*sqrt(2*x^2 - x + 3))/(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2
 - 12*x + 45)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^2 \left(2x^2 - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^4 - x^3 + 3\,x^2 + x + 2}{\left(2\,x^2 - x + 3\right)^{\frac{5}{2}} (2\,x + 5)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((5*x^4 - x^3 + 3*x^2 + x + 2)/((2*x^2 - x + 3)^(5/2)*(2*x + 5)^2)
, x)
```

3.363 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{4679797-2148263x}{592344576\sqrt{2x^2-x+3}}-\frac{45979\sqrt{2x^2-x+3}}{26873856(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{373248(2x+5)^2}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}+\frac{774079\tanh^{-1}\left(\right.}{3224862}$$

```
[Out] (65991 - 8779*x)/(12877056*(3 - x + 2*x^2)^(3/2)) - (4679797 - 2148263*x)/(
592344576*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(373248*(5 + 2*
x)^2) - (45979*Sqrt[3 - x + 2*x^2])/(26873856*(5 + 2*x)) + (774079*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(322486272*Sqrt[2])
```

Rubi [A] time = 0.220896, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$-\frac{4679797-2148263x}{592344576\sqrt{2x^2-x+3}}-\frac{45979\sqrt{2x^2-x+3}}{26873856(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{373248(2x+5)^2}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}+\frac{774079\tanh^{-1}\left(\right.}{3224862}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2)),x]
```



```
[Out] (65991 - 8779*x)/(12877056*(3 - x + 2*x^2)^(3/2)) - (4679797 - 2148263*x)/(
592344576*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(373248*(5 + 2*
x)^2) - (45979*Sqrt[3 - x + 2*x^2])/(26873856*(5 + 2*x)) + (774079*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(322486272*Sqrt[2])
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
```

```
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{11115283}{746496}+\frac{3198845x}{62208}+\frac{605005x^2}{6912}-\frac{8779x^3}{23328}}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} + \frac{4\int \frac{-\frac{171639869}{2985984}-\frac{142392517x}{746496}-\frac{165709\cdots}{7464\cdots}\cdots}{(5+2x)^3\sqrt{3-x+2x^2}}}{1587}\cdots$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} - \frac{\int \frac{\frac{34040\cdots}{103\cdots}\cdots}{(5+\cdots}}{\cdots}\cdots$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} - \frac{45979\cdots}{26873\cdots}\cdots$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} - \frac{45979\cdots}{26873\cdots}\cdots$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} - \frac{45979\cdots}{26873\cdots}\cdots$$

<u>Mathematica</u> [A] time = 0.306919, size = 97, normalized size = 0.72

$$\frac{\frac{12\sqrt{2}\left(217883368x^5+107028732x^4-1503926130x^3-5919924791x^2+2280511668x-8953831359\right)}{529(2x+5)^2\left(2x^2-x+3\right)^{3/2}} + 774079\log\left(12\sqrt{4x^2-2x+6}-22x+17\right) - \cdots}{322486272\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((12*Sqrt[2]*(-8953831359 + 2280511668*x - 5919924791*x^2 - 1503926130*x^3
+ 107028732*x^4 + 217883368*x^5))/(529*(5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)) -
 774079*Log[5 + 2*x] + 774079*Log[17 - 22*x + 12*Sqrt[6 - 2*x + 4*x^2]])/(3
22486272*Sqrt[2])
```

Maple [A] time = 0.062, size = 200, normalized size = 1.5

$$-\frac{5}{48}\left(2x^2-x+3\right)^{-\frac{3}{2}}-\frac{-149+596x}{1104}\left(2x^2-x+3\right)^{-\frac{3}{2}}-\frac{-149+596x}{1587}\frac{1}{\sqrt{2x^2-x+3}}-\frac{3667}{4608}\left(x+\frac{5}{2}\right)^{-2}\left(2\left(x+5/2\right)^2-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/48/(2*x^2-x+3)^(3/2)-149/1104*(-1+4*x)/(2*x^2-x+3)^(3/2)-149/1587*(-1+4*
x)/(2*x^2-x+3)^(1/2)-3667/4608/(x+5/2)^2/(2*(x+5/2)^2-11*x-19/2)^(3/2)+1153
69/165888/(x+5/2)/(2*(x+5/2)^2-11*x-19/2)^(3/2)-774079/17915904/(2*(x+5/2)^
2-11*x-19/2)^(3/2)+57937675/412065792*(-1+4*x)/(2*(x+5/2)^2-11*x-19/2)^(3/2
)+5366174813/56865079296*(-1+4*x)/(2*(x+5/2)^2-11*x-19/2)^(1/2)-774079/1074
95424/(2*(x+5/2)^2-11*x-19/2)^(1/2)+774079/644972544*2^(1/2)*arctanh(1/12*(
17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A] time = 1.61319, size = 240, normalized size = 1.78

$$-\frac{774079}{644972544}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\left|2x+5\right|}-\frac{17\sqrt{23}}{23\left|2x+5\right|}\right)+\frac{27235421x}{14216269824\sqrt{2x^2-x+3}}-\frac{36393601}{4738756608\sqrt{2x^2-x+3}}+\frac{}{1030}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -774079/644972544*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqr
t(23)/abs(2*x + 5)) + 27235421/14216269824*x/sqrt(2*x^2 - x + 3) - 36393601
/4738756608/sqrt(2*x^2 - x + 3) + 2323723/103016448*x/(2*x^2 - x + 3)^(3/2)
 - 3667/1152/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25
*(2*x^2 - x + 3)^(3/2)) + 115369/82944/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^
2 - x + 3)^(3/2)) - 5254255/34338816/(2*x^2 - x + 3)^(3/2)
```

Fricas [A] time = 1.40361, size = 502, normalized size = 3.72

$$\frac{409487791\sqrt{2}\left(16x^6+64x^5+72x^4+136x^3+241x^2+30x+225\right)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right)+48\,(}{682380951552\left(16x^6+64x^5+72x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/682380951552*(409487791*sqrt(2)*(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241
*x^2 + 30*x + 225)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x
^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(217883368*x^5 + 107028732*x^
4 - 1503926130*x^3 - 5919924791*x^2 + 2280511668*x - 8953831359)*sqrt(2*x^2
 - x + 3))/(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241*x^2 + 30*x + 225)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^3 \left(2x^2 - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
5/2)), x)
```

Giac [B] time = 1.21714, size = 308, normalized size = 2.28

$$\frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) - \frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) + -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
+ 3))) - 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 1/53747712*sqrt(2)*(44558*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 10136238*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 16812201*s
```

```
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 10182217)/(2*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2 + 1
/592344576*(((4296526*x - 11507857)*x + 10720752)*x - 11003805)/(2*x^2 - x
+ 3)^(3/2)
```

3.364 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{27754539-31190998x}{31986607104\sqrt{2x^2-x+3}}+\frac{475357\sqrt{2x^2-x+3}}{1934917632(2x+5)}-\frac{89137\sqrt{2x^2-x+3}}{80621568(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{559872(2x+5)^3}+\frac{369609-1758}{463574016\left(2x^2-x\right.}$$

```
[Out] (369609 - 175877*x)/(463574016*(3 - x + 2*x^2)^(3/2)) - (27754539 - 3119099
8*x)/(31986607104*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(559872
*(5 + 2*x)^3) - (89137*Sqrt[3 - x + 2*x^2])/(80621568*(5 + 2*x)^2) + (47535
7*Sqrt[3 - x + 2*x^2])/(1934917632*(5 + 2*x)) + (4778789*ArcTanh[(17 - 22*x
)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(7739670528*Sqrt[2])
```

Rubi [A] time = 0.282999, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$-\frac{27754539-31190998x}{31986607104\sqrt{2x^2-x+3}}+\frac{475357\sqrt{2x^2-x+3}}{1934917632(2x+5)}-\frac{89137\sqrt{2x^2-x+3}}{80621568(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{559872(2x+5)^3}+\frac{369609-1758}{463574016\left(2x^2-x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2)),x]
```


```
[Out] (369609 - 175877*x)/(463574016*(3 - x + 2*x^2)^(3/2)) - (27754539 - 3119099
8*x)/(31986607104*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(559872
*(5 + 2*x)^3) - (89137*Sqrt[3 - x + 2*x^2])/(80621568*(5 + 2*x)^2) + (47535
7*Sqrt[3 - x + 2*x^2])/(1934917632*(5 + 2*x)) + (4778789*ArcTanh[(17 - 22*x
)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(7739670528*Sqrt[2])
```


Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{606939313}{26873856}+\frac{727085495x}{13436928}+\frac{186705485x^2}{2239488}-\frac{10162483x^3}{3359232}}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} + \frac{4\int \frac{-\frac{4811736919}{40310784}-\frac{3560904781x}{13436928}}{(5+2x)^4\sqrt{3}}}{158}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} - \int$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} - \frac{89}{80}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} - \frac{89}{80}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} - \frac{89}{80}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} - \frac{89}{80}$$

Mathematica [A] time = 0.232494, size = 89, normalized size = 0.56

$$\frac{\frac{24\left(6664404208x^6+34872810880x^5+46210466520x^4+27484986184x^3-6702882569x^2+73621973154x-95241881529\right)}{(2x+5)^3\left(2x^2-x+3\right)^{3/2}}+2527979381\sqrt{2}\tanh^{-1}\left(\frac{}{12}\right.}{8188571418624}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((24*(-95241881529 + 73621973154*x - 6702882569*x^2 + 27484986184*x^3 + 462
10466520*x^4 + 34872810880*x^5 + 6664404208*x^6))/((5 + 2*x)^3*(3 - x + 2*x
^2)^(3/2)) + 2527979381*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^
2])])/8188571418624
```

Maple [A] time = 0.062, size = 207, normalized size = 1.3

$$-\frac{-72646615+290586460\,x}{9889579008}\left(2\,\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}\right)^{-\frac{3}{2}}+\frac{25951}{110592}\left(x+\frac{5}{2}\right)^{-2}\left(2\,\left(x+5/2\right)^{2}-11\,x-\frac{19}{2}\right)^{-\frac{3}{2}}+\frac{4778789}{15479341}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -72646615/9889579008*(-1+4*x)/(2*(x+5/2)^2-11*x-19/2)^(3/2)+25951/110592/(x
+5/2)^2/(2*(x+5/2)^2-11*x-19/2)^(3/2)+4778789/15479341056*2^(1/2)*arctanh(1
/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-3667/13824/(x+5/2)^3
/(2*(x+5/2)^2-11*x-19/2)^(3/2)-8183108657/1364761903104*(-1+4*x)/(2*(x+5/2)
^2-11*x-19/2)^(1/2)-4778789/429981696/(2*(x+5/2)^2-11*x-19/2)^(3/2)+10/1587
*(-1+4*x)/(2*x^2-x+3)^(1/2)+5/552*(-1+4*x)/(2*x^2-x+3)^(3/2)-4778789/257989
0176/(2*(x+5/2)^2-11*x-19/2)^(1/2)-34861/3981312/(x+5/2)/(2*(x+5/2)^2-11*x-
19/2)^(3/2)
```

Maxima [A] time = 1.59192, size = 332, normalized size = 2.08

$$-\frac{4778789}{15479341056}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{416525263\,x}{341190475776\,\sqrt{2\,x^2-x+3}}-\frac{245375387}{113730158592\,\sqrt{2\,x^2-x+3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -4778789/15479341056*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) + 416525263/341190475776*x/sqrt(2*x^2 - x + 3) - 245
375387/113730158592/sqrt(2*x^2 - x + 3) + 16932905/2472394752*x/(2*x^2 - x
+ 3)^(3/2) - 3667/1728/(8*(2*x^2 - x + 3)^(3/2)*x^3 + 60*(2*x^2 - x + 3)^(3
/2)*x^2 + 150*(2*x^2 - x + 3)^(3/2)*x + 125*(2*x^2 - x + 3)^(3/2)) + 25951/
27648/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25*(2*x^2
 - x + 3)^(3/2)) - 34861/1990656/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x
+ 3)^(3/2)) - 10570421/824131584/(2*x^2 - x + 3)^(3/2)
```

Fricas [A] time = 1.39946, size = 582, normalized size = 3.64

$$\frac{2527979381\sqrt{2}\left(32x^7+208x^6+464x^5+632x^4+1162x^3+1265x^2+600x+1125\right)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2}{4x^2+20x+25}\right.}{16377142837248\left(32x^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/16377142837248*(2527979381*sqrt(2)*(32*x^7 + 208*x^6 + 464*x^5 + 632*x^4
+ 1162*x^3 + 1265*x^2 + 600*x + 1125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(
22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(666440420
8*x^6 + 34872810880*x^5 + 46210466520*x^4 + 27484986184*x^3 - 6702882569*x^
2 + 73621973154*x - 95241881529)*sqrt(2*x^2 - x + 3))/(32*x^7 + 208*x^6 + 4
64*x^5 + 632*x^4 + 1162*x^3 + 1265*x^2 + 600*x + 1125)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24861, size = 377, normalized size = 2.36

$$\frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
 2*sqrt(2*x^2 - x + 3))) + 1/7996651776*(((15595499*x - 21675019)*x + 27298
005)*x - 14440149)/(2*x^2 - x + 3)^(3/2) + 1/3869835264*sqrt(2)*(38030012*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 734231900*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^4 + 122834956*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 2
154595396*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 1659431083*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) - 760577429)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

3.365 $\int \frac{f+gx+hx^2+ix^3+jx^4}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=354

$$-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f\right)-4bc^2\left(8a^2j+ach+2c^2f\right)+24a^2c\right.}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g - a*i) - a*b^3*j - b*c*(c^2*f + a*c*h - 3*a^2*
j) - (2*c^4*f - c^3*(b*g + 2*a*h) + b^4*j - b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g - 3*a*i) - b^5*j - b^3*c*(c*
h - 10*a*j) - 4*b*c^2*(2*c^2*f + a*c*h + 8*a^2*j) - c*(16*c^4*f - c^3*(8*b*
g - 8*a*h) - 4*b^4*j + b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a
^2*j))*x))/(3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (j*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

Rubi [A] time = 0.377244, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1660, 12, 621, 206}

$$-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f\right)-4bc^2\left(8a^2j+ach+2c^2f\right)+24a^2c\right.}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g - a*i) - a*b^3*j - b*c*(c^2*f + a*c*h - 3*a^2*
j) - (2*c^4*f - c^3*(b*g + 2*a*h) + b^4*j - b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g - 3*a*i) - b^5*j - b^3*c*(c*
h - 10*a*j) - 4*b*c^2*(2*c^2*f + a*c*h + 8*a^2*j) - c*(16*c^4*f - c^3*(8*b*
g - 8*a*h) - 4*b^4*j + b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a
^2*j))*x))/(3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (j*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{f+gx+hx^2+365x^3+jx^4}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-cg)-b^4j\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-cg)-b^4j\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-cg)-b^4j\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-cg)-b^4j\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-cg)-b^4j\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Mathematica [A] time = 1.23512, size = 316, normalized size = 0.89

$$\frac{-\frac{2\left(bc\left(-3a^2j+ac(h+3ix)+c^2(f-gx)\right)+2c^2\left(a^2(i+jx)-ac(g+hx)+c^2fx\right)+b^2c(chx-a(i+4jx))+b^3(aj-cix)+b^4jx\right)}{\left(b^2-4ac\right)(a+x(b+cx))^{3/2}}+\frac{2\left(4bc^2\left(8a^2j+ac(h-3ix)+2c^2(f-gx)\right)+8c^3\left(a^2(-(3i\right.\right.}{}}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] ((-2*(b^4*j*x + b^3*(a*j - c*i*x) + b*c*(-3*a^2*j + c^2*(f - g*x) + a*c*(h
+ 3*i*x)) + 2*c^2*(c^2*f*x - a*c*(g + h*x) + a^2*(i + j*x)) + b^2*c*(c*h*x
- a*(i + 4*j*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))^(3/2)) + (2*(b^5*j - b^
4*c*(i + 4*j*x) + 2*b^2*c^2*(-2*c*g + 3*a*i + c*h*x + 14*a*j*x) + 4*b*c^2*(
8*a^2*j + 2*c^2*(f - g*x) + a*c*(h - 3*i*x)) + b^3*c*(-10*a*j + c*(h + i*x)
) + 8*c^3*(2*c^2*f*x + a*c*h*x - a^2*(3*i + 4*j*x))))/((b^2 - 4*a*c)^2*Sqrt
[a + x*(b + c*x)]) + 3*Sqrt[c]*j*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b +
c*x)]])/(3*c^3)
```

Maple [B] time = 0.056, size = 1406, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] j/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+4/3*f/(4*a*c-b^2)/(c*
x^2+b*x+a)^(3/2)*x*c+2/3*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b-1/2*h*x/c/(c*x
^2+b*x+a)^(3/2)+1/12*h*b/c^2/(c*x^2+b*x+a)^(3/2)-8/3*g*b^2/(4*a*c-b^2)^2/(c
*x^2+b*x+a)^(1/2)-1/3*j*x^3/c/(c*x^2+b*x+a)^(3/2)-1/48*j*b^3/c^4/(c*x^2+b*x
+a)^(3/2)-j/c^2*x/(c*x^2+b*x+a)^(1/2)+1/2*j/c^3*b/(c*x^2+b*x+a)^(1/2)-i*x^2
/c/(c*x^2+b*x+a)^(3/2)-i*b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/2*j*b^2/
c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+4*j*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*
x+a)^(1/2)*x-1/3*g/c/(c*x^2+b*x+a)^(3/2)+32/3*f*c^2/(4*a*c-b^2)^2/(c*x^2+b*
x+a)^(1/2)*x+1/24*i*b^2/c^3/(c*x^2+b*x+a)^(3/2)-2/3*i*a/c^2/(c*x^2+b*x+a)^(
3/2)+1/24*i*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+8/3*h*a/(4*a*c-b^2)^2/(
c*x^2+b*x+a)^(1/2)*b+2/3*h*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/3*i*b^4/c^
2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/2*j*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)+1/8
*j*b^2/c^3*x/(c*x^2+b*x+a)^(3/2)-1/48*j*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(
3/2)+2/3*h*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-1/4*i*b/c^2*x/(c*x^2+b*x
+a)^(3/2)+1/2*j/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/3*g*b^2/c/(4*a*c-
b^2)/(c*x^2+b*x+a)^(3/2)-1/6*j*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/
3*j*b/c^3*a/(c*x^2+b*x+a)^(3/2)+16/3*f*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*
b+4/3*h*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-2/3*g*b/(4*a*c-b^2)/(c*x^2+
b*x+a)^(3/2)*x+1/12*h*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/4*j*b^3/c^3
*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2*j*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a
)^(1/2)+j/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/12*i*b^3/c^2/(4*a*c-b
^2)/(c*x^2+b*x+a)^(3/2)*x+2/3*i*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-1
/2*i*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-8*i*b*a/(4*a*c-b^2)^2/(c*x^2
+b*x+a)^(1/2)*x-4*i*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-1/24*j*b^4/c^
3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-1/3*j*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+
a)^(1/2)*x+1/6*h*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/3*h*a/c/(4*a*c-b
^2)/(c*x^2+b*x+a)^(3/2)*b+16/3*h*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-16
/3*g*b*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16615, size = 628, normalized size = 1.77

$$\frac{2\left(\left(\left(\frac{\left(16\,c^5f-8\,bc^4g+2\,b^2c^3h+8\,ac^4h+b^3c^2i-12\,abc^3i-4\,b^4cj+28\,ab^2c^2j-32\,a^2c^3j\right)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{3\left(8\,bc^4f-4\,b^2c^3g+b^3c^2h+4\,abc^3h-2\,ab^2c^2i-8\,a^2c^3i-b^5j+6\,ab^3cj\right)}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x+\right.\right.\right.}{3\left(cx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac"
)
```

```
[Out] 2/3*((((16*c^5*f - 8*b*c^4*g + 2*b^2*c^3*h + 8*a*c^4*h + b^3*c^2*i - 12*a*b
*c^3*i - 4*b^4*c*j + 28*a*b^2*c^2*j - 32*a^2*c^3*j)*x/(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4) + 3*(8*b*c^4*f - 4*b^2*c^3*g + b^3*c^2*h + 4*a*b*c^3*h - 2*
a*b^2*c^2*i - 8*a^2*c^3*i - b^5*j + 6*a*b^3*c*j)/(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4))*x + 3*(2*b^2*c^3*f + 8*a*c^4*f - b^3*c^2*g - 4*a*b*c^3*g + 4*a*
b^2*c^2*h - 8*a^2*b*c^2*i - 2*a*b^4*j + 14*a^2*b^2*c*j - 8*a^3*c^2*j)/(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*c^2*f - 12*a*b*c^3*f + 2*a*b^2*c^
2*g + 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i + 3*a^2*b^3*j - 20*a^3*b*c
*j)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2) - j*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

3.366 $\int \frac{f+gx+hx^2+ix^3+jx^4}{\left(a+bx-cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=353

$$-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+2c^2f\right)+24a^2c^3\right.}{3c^3\left(4ac+b^2\right)^2\sqrt{a+bx-cx^2}}$$

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g + a*i) + a*b^3*j - b*c*(c^2*f - a*c*h - 3*a^2*
j) + (2*c^4*f + c^3*(b*g + 2*a*h) + b^4*j + b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 + 4*a*c)*(a + b*x - c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g + 3*a*i) + b^5*j + b^3*c*(c*
h + 10*a*j) + 4*b*c^2*(2*c^2*f - a*c*h + 8*a^2*j) - c*(16*c^4*f + 8*c^3*(b*
g - a*h) - 4*b^4*j - b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a^2
*j))*x))/(3*c^3*(b^2 + 4*a*c)^2*Sqrt[a + b*x - c*x^2]) - (j*ArcTan[(b - 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x - c*x^2])])/c^(5/2)
```

Rubi [A] time = 0.3857, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1660, 12, 621, 204}

$$-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+2c^2f\right)+24a^2c^3\right.}{3c^3\left(4ac+b^2\right)^2\sqrt{a+bx-cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g + a*i) + a*b^3*j - b*c*(c^2*f - a*c*h - 3*a^2*
j) + (2*c^4*f + c^3*(b*g + 2*a*h) + b^4*j + b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 + 4*a*c)*(a + b*x - c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g + 3*a*i) + b^5*j + b^3*c*(c*
h + 10*a*j) + 4*b*c^2*(2*c^2*f - a*c*h + 8*a^2*j) - c*(16*c^4*f + 8*c^3*(b*
g - a*h) - 4*b^4*j - b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a^2
*j))*x))/(3*c^3*(b^2 + 4*a*c)^2*Sqrt[a + b*x - c*x^2]) - (j*ArcTan[(b - 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x - c*x^2])])/c^(5/2)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{f+gx+hx^2+366x^3+jx^4}{\left(a+bx-cx^2\right)^{5/2}}\,dx = -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+cg)+b^4j\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+cg)+b^4j\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+cg)+b^4j\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+cg)+b^4j\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+cg)+b^4j\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

Mathematica [C] time = 1.17812, size = 319, normalized size = 0.9

$$-\frac{2\left(b^3\left(3a^2j+18acjx^2+c^2\left(f+3gx+x^2(-(3h+ix))\right)\right)+2b^2c\left(21a^2jx+ac\left(g+x\left(-6h+3ix-14jx^2\right)\right)+c^2x(3f+x(h\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (-2*(3*b^5*j*x^2 + b^4*(6*a*j*x - 4*c*j*x^3) + b^3*(3*a^2*j + 18*a*c*j*x^2
+ c^2*(f + 3*g*x - x^2*(3*h + i*x))) + 8*c^2*(2*c^3*f*x^3 + a^3*(2*i + 3*j*
x) - a*c^2*x*(3*f + h*x^2) - a^2*c*(g + x^2*(3*i + 4*j*x))) + 4*b*c*(5*a^3*
j + 2*c^3*x^2*(-3*f + g*x) - 2*a^2*c*(h - 3*i*x) + 3*a*c^2*(f - x*(g - h*x
+ i*x^2))) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f + x*(-6*g + h*x)) + a*c*(g +
x*(-6*h + 3*i*x - 14*j*x^2)))))/(3*c^2*(b^2 + 4*a*c)^2*(a + x*(b - c*x))^(3
/2)) + (I*j*Log[(I*(b - 2*c*x))/Sqrt[c] + 2*Sqrt[a + x*(b - c*x)]])/c^(5/2)
```

Maple [B] time = 0.059, size = 1453, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -1/24*i*b^4/c^3/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+1/3*i*b^4/c^2/(-4*a*c-b^2
)^2/(-c*x^2+b*x+a)^(1/2)+i*b/c*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-4*j*b^
2/c*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x+1/6*j*b^5/c^3/(-4*a*c-b^2)^2/(-
c*x^2+b*x+a)^(1/2)-1/3*j*b/c^3*a/(-c*x^2+b*x+a)^(3/2)-1/2*j/c^3*b^3/(-4*a*c
-b^2)/(-c*x^2+b*x+a)^(1/2)-2/3*g*b/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x+1/3*
g*b^2/c/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+2/3*h*a/(-4*a*c-b^2)/(-c*x^2+b*x+
a)^(3/2)*x+8/3*h*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*b-4/3*f/(-4*a*c-b^2)
/(-c*x^2+b*x+a)^(3/2)*x*c+32/3*f*c^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-
1/4*i*b/c^2*x/(-c*x^2+b*x+a)^(3/2)+1/2*j*b/c^2*x^2/(-c*x^2+b*x+a)^(3/2)-1/8
*j*b^2/c^3*x/(-c*x^2+b*x+a)^(3/2)-1/48*j*b^5/c^4/(-4*a*c-b^2)/(-c*x^2+b*x+a
)^(3/2)+1/2*j*b^2/c^2*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x+1/2*h*x/c/(-c*x
^2+b*x+a)^(3/2)+1/12*h*b/c^2/(-c*x^2+b*x+a)^(3/2)+1/3*j*x^3/c/(-c*x^2+b*x+a
)^(3/2)-1/48*j*b^3/c^4/(-c*x^2+b*x+a)^(3/2)-2/3*h*b^3/c/(-4*a*c-b^2)^2/(-c*
x^2+b*x+a)^(1/2)+1/12*h*b^3/c^2/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-j/c^2*x/(
-c*x^2+b*x+a)^(1/2)-1/2*j/c^3*b/(-c*x^2+b*x+a)^(1/2)-2/3*i*a/c^2/(-c*x^2+b*
x+a)^(3/2)+i*x^2/c/(-c*x^2+b*x+a)^(3/2)-1/24*i*b^2/c^3/(-c*x^2+b*x+a)^(3/2)
+2/3*f/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*b-8/3*g*b^2/(-4*a*c-b^2)^2/(-c*x^2
+b*x+a)^(1/2)+4/3*h*b^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x+j/c^(5/2)*arc
tan(c^(1/2)*(x-1/2*b/c)/(-c*x^2+b*x+a)^(1/2))+1/3*g/c/(-c*x^2+b*x+a)^(3/2)-
16/3*f*c/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*b+1/24*j*b^4/c^3/(-4*a*c-b^2)/
(-c*x^2+b*x+a)^(3/2)*x-1/3*j*b^4/c^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-
1/4*j*b^3/c^3*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+2*j*b^3/c^2*a/(-4*a*c-b^2
)^2/(-c*x^2+b*x+a)^(1/2)+j/c^2*b^2/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(1/2)*x+1/12
*i*b^3/c^2/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-2/3*i*b^3/c/(-4*a*c-b^2)^2/(
-c*x^2+b*x+a)^(1/2)*x-1/2*i*b^2/c^2*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-8*i
*b*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x+4*i*b^2/c*a/(-4*a*c-b^2)^2/(-c*x
^2+b*x+a)^(1/2)-1/6*h*b^2/c/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-1/3*h*a/c/(
-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*b-16/3*h*a*c/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)
^(1/2)*x+16/3*g*b*c/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(-c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1967, size = 659, normalized size = 1.87

$$-\frac{2\sqrt{-cx^2+bx+a}\left(\left(\left(\frac{\left(16c^5f+8bc^4g+2b^2c^3h-8ac^4h-b^3c^2i-12abc^3i-4b^4cj-28ab^2c^2j-32a^2c^3j\right)x}{b^4c^2+8ab^2c^3+16a^2c^4}-\frac{3\left(8bc^4f+4b^2c^3g+b^3c^2h-4abc^3h-2ab^2c^2i+8a^2c\right.}{b^4c^2+8ab^2c^3+16a^2c^4}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="giac
")
```

```
[Out] -2/3*sqrt(-c*x^2 + b*x + a)*((((16*c^5*f + 8*b*c^4*g + 2*b^2*c^3*h - 8*a*c^
4*h - b^3*c^2*i - 12*a*b*c^3*i - 4*b^4*c*j - 28*a*b^2*c^2*j - 32*a^2*c^3*j)
*x/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4) - 3*(8*b*c^4*f + 4*b^2*c^3*g + b^3*
c^2*h - 4*a*b*c^3*h - 2*a*b^2*c^2*i + 8*a^2*c^3*i - b^5*j - 6*a*b^3*c*j)/(b
^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c^3*f - 8*a*c^4*f + b^3*c^
2*g - 4*a*b*c^3*g - 4*a*b^2*c^2*h + 8*a^2*b*c^2*i + 2*a*b^4*j + 14*a^2*b^2*
c*j + 8*a^3*c^2*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + (b^3*c^2*f + 1
2*a*b*c^3*f + 2*a*b^2*c^2*g - 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i +
3*a^2*b^3*j + 20*a^3*b*c*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 -
b*x - a)^2 - j*log(abs(2*(sqrt(-c)*x - sqrt(-c*x^2 + b*x + a))*sqrt(-c) + b
))/(sqrt(-c)*c^2)
```

3.367 $\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=588

$$\frac{\left(5d^2-2de+3e^2\right)^3\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^{11}(m+1)}-\frac{\left(5d^2-2de+3e^2\right)^2\left(108d^3e^2-20d^2e^3+169d^4e+20\right.}{e^{11}(m+2)}$$

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^11*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169
*d^4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x)^(2 + m))/(
e^11*(2 + m)) + (3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*
e^2 + 58*d^3*e^3 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^(3 + m))/(e^
11*(3 + m)) - (2*(30000*d^7 + 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9
990*d^3*e^4 - 2550*d^2*e^5 + 2218*d*e^6 - 287*e^7)*(d + e*x)^(4 + m))/(e^11
*(4 + m)) + ((105000*d^6 + 3150*d^5*e + 53550*d^4*e^2 + 3430*d^3*e^3 + 1498
5*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^(5 + m))/(e^11*(5 + m)) - (6*(
21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*e^4 - 85*e^5)*(d
 + e*x)^(6 + m))/(e^11*(6 + m)) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*e^2
 + 686*d*e^3 + 999*e^4)*(d + e*x)^(7 + m))/(e^11*(7 + m)) - (2*(30000*d^3 +
 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^(8 + m))/(e^11*(8 + m)) + (45*(
500*d^2 + 5*d*e + 17*e^2)*(d + e*x)^(9 + m))/(e^11*(9 + m)) - (25*(200*d +
e)*(d + e*x)^(10 + m))/(e^11*(10 + m)) + (500*(d + e*x)^(11 + m))/(e^11*(11
 + m))
```

Rubi [A] time = 0.364184, antiderivative size = 588, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{\left(5d^2-2de+3e^2\right)^3\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^{11}(m+1)}-\frac{\left(5d^2-2de+3e^2\right)^2\left(108d^3e^2-20d^2e^3+169d^4e+20\right.}{e^{11}(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```



```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^11*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169
*d^4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x)^(2 + m))/(
e^11*(2 + m)) + (3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*
e^2 + 58*d^3*e^3 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^(3 + m))/(e^
11*(3 + m)) - (2*(30000*d^7 + 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9
```

```
990*d^3*e^4 - 2550*d^2*e^5 + 2218*d*e^6 - 287*e^7)*(d + e*x)^(4 + m))/(e^11
*(4 + m)) + ((105000*d^6 + 3150*d^5*e + 53550*d^4*e^2 + 3430*d^3*e^3 + 1498
5*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^(5 + m))/(e^11*(5 + m)) - (6*(
21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*e^4 - 85*e^5)*(d
 + e*x)^(6 + m))/(e^11*(6 + m)) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*e^2
 + 686*d*e^3 + 999*e^4)*(d + e*x)^(7 + m))/(e^11*(7 + m)) - (2*(30000*d^3 +
 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^(8 + m))/(e^11*(8 + m)) + (45*(
500*d^2 + 5*d*e + 17*e^2)*(d + e*x)^(9 + m))/(e^11*(9 + m)) - (25*(200*d +
e)*(d + e*x)^(10 + m))/(e^11*(10 + m)) + (500*(d + e*x)^(11 + m))/(e^11*(11
 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+}{e^{10}}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)}{e^{11}(1+m)}$$

Mathematica [A] time = 0.410217, size = 537, normalized size = 0.91

$$\frac{(d+ex)^{m+1}\left(\frac{45\left(500d^2+5de+17e^2\right)(d+ex)^8}{m+9}-\frac{2\left(450d^2e+30000d^3+3060de^2+49e^3\right)(d+ex)^7}{m+8}+\frac{\left(21420d^2e^2+2100d^3e+105000d^4+686de^3+999e^4\right)(d+ex)^6}{m+7}-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2
 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169*d^4*
e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x))/(2 + m) + (3*(
5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*e^2 + 58*d^3*e^3 + 5
47*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^2)/(3 + m) - (2*(30000*d^7 + 105
```

```
0*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9990*d^3*e^4 - 2550*d^2*e^5 + 2218
*d*e^6 - 287*e^7)*(d + e*x)^3)/(4 + m) + ((105000*d^6 + 3150*d^5*e + 53550*
d^4*e^2 + 3430*d^3*e^3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^4
)/(5 + m) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*
e^4 - 85*e^5)*(d + e*x)^5)/(6 + m) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*
e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^6)/(7 + m) - (2*(30000*d^3 + 450*d^2*e
 + 3060*d*e^2 + 49*e^3)*(d + e*x)^7)/(8 + m) + (45*(500*d^2 + 5*d*e + 17*e^
2)*(d + e*x)^8)/(9 + m) - (25*(200*d + e)*(d + e*x)^9)/(10 + m) + (500*(d +
 e*x)^10)/(11 + m)))/e^11
```

Maple [B] time = 0.082, size = 5924, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84756, size = 14657, normalized size = 24.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (54*d*e^10*m^10 + 500*(e^11*m^10 + 55*e^11*m^9 + 1320*e^11*m^8 + 18150*e^11
*m^7 + 157773*e^11*m^6 + 902055*e^11*m^5 + 3416930*e^11*m^4 + 8409500*e^11*
m^3 + 12753576*e^11*m^2 + 10628640*e^11*m + 3628800*e^11)*x^11 + 1814400000
*d^11 + 99792000*d^10*e + 3392928000*d^9*e^2 + 488980800*d^8*e^3 + 56966976
00*d^7*e^4 - 3392928000*d^6*e^5 + 8853546240*d^5*e^6 - 5728060800*d^4*e^7 +
 6346771200*d^3*e^8 - 2694384000*d^2*e^9 + 2155507200*d*e^10 - 25*(3991680*
e^11 - (20*d*e^10 - e^11)*m^10 - 4*(225*d*e^10 - 14*e^11)*m^9 - 15*(1160*d*
e^10 - 91*e^11)*m^8 - 60*(3150*d*e^10 - 317*e^11)*m^7 - 21*(60260*d*e^10 -
7963*e^11)*m^6 - 84*(64125*d*e^10 - 11492*e^11)*m^5 - 5*(2894720*d*e^10 - 7
37251*e^11)*m^4 - 20*(1172700*d*e^10 - 456659*e^11)*m^3 - 36*(570320*d*e^10
 - 386841*e^11)*m^2 - 144*(50400*d*e^10 - 80939*e^11)*m)*x^10 - 135*(d^2*e^
9 - 26*d*e^10)*m^9 + 5*(678585600*e^11 - (5*d*e^10 - 153*e^11)*m^10 - (1000
*d^2*e^9 + 235*d*e^10 - 8721*e^11)*m^9 - 6*(6000*d^2*e^9 + 785*d*e^10 - 360
06*e^11)*m^8 - 6*(91000*d^2*e^9 + 8785*d*e^10 - 509031*e^11)*m^7 - 105*(432
00*d^2*e^9 + 3445*d*e^10 - 259029*e^11)*m^6 - 21*(1069000*d^2*e^9 + 74815*d
*e^10 - 7560189*e^11)*m^5 - 2*(33642000*d^2*e^9 + 2145620*d*e^10 - 30603656
7*e^11)*m^4 - 4*(29531000*d^2*e^9 + 1761185*d*e^10 - 382172121*e^11)*m^3 -
72*(1522000*d^2*e^9 + 86510*d*e^10 - 32587351*e^11)*m^2 - 1440*(28000*d^2*e
^9 + 1540*d*e^10 - 1370727*e^11)*m)*x^9 + 9*(106*d^3*e^8 - 945*d^2*e^9 + 11
160*d*e^10)*m^8 - (488980800*e^11 - (765*d*e^10 - 98*e^11)*m^10 - (225*d^2*
e^9 + 37485*d*e^10 - 5684*e^11)*m^9 - 3*(15000*d^3*e^8 + 2925*d^2*e^9 + 260
100*d*e^10 - 47726*e^11)*m^8 - 42*(30000*d^3*e^8 + 3375*d^2*e^9 + 214965*d*
e^10 - 48958*e^11)*m^7 - 63*(230000*d^3*e^8 + 19650*d^2*e^9 + 1012095*d*e^1
0 - 294882*e^11)*m^6 - 63*(1400000*d^3*e^8 + 101175*d^2*e^9 + 4503555*d*e^1
0 - 1743812*e^11)*m^5 - (304605000*d^3*e^8 + 19707975*d^2*e^9 + 790573950*d
*e^10 - 428393182*e^11)*m^4 - 4*(147735000*d^3*e^8 + 8860500*d^2*e^9 + 3297
12705*d*e^10 - 270109021*e^11)*m^3 - 36*(16335000*d^3*e^8 + 929925*d^2*e^9
+ 32795550*d*e^10 - 46438966*e^11)*m^2 - 5040*(45000*d^3*e^8 + 2475*d^2*e^9
 + 84150*d*e^10 - 280861*e^11)*m)*x^8 - 6*(574*d^4*e^7 - 9540*d^3*e^8 + 390
15*d^2*e^9 - 277290*d*e^10)*m^7 + (5696697600*e^11 - (98*d*e^10 - 999*e^11)
*m^10 - 3*(2040*d^2*e^9 + 1666*d*e^10 - 19647*e^11)*m^9 - 24*(75*d^3*e^8 +
10710*d^2*e^9 + 4508*d*e^10 - 62937*e^11)*m^8 - 6*(60000*d^4*e^7 + 9600*d^3
*e^8 + 740520*d^2*e^9 + 216482*d*e^10 - 3677319*e^11)*m^7 - 3*(2520000*d^4*
e^7 + 243600*d^3*e^8 + 13708800*d^2*e^9 + 3161774*d*e^10 - 67539393*e^11)*m
^6 - 21*(3000000*d^4*e^7 + 228000*d^3*e^8 + 10581480*d^2*e^9 + 2069662*d*e^
10 - 57933009*e^11)*m^5 - 2*(132300000*d^4*e^7 + 8738100*d^3*e^8 + 35715708
0*d^2*e^9 + 62076434*d*e^10 - 2405021571*e^11)*m^4 - 36*(16240000*d^4*e^7 +
 981400*d^3*e^8 + 36788680*d^2*e^9 + 5871278*d*e^10 - 341095341*e^11)*m^3 -
 72*(8820000*d^4*e^7 + 503100*d^3*e^8 + 17778600*d^2*e^9 + 2670010*d*e^10 -
 266622111*e^11)*m^2 - 12960*(20000*d^4*e^7 + 1100*d^3*e^8 + 37400*d^2*e^9
+ 5390*d*e^10 - 1264623*e^11)*m)*x^7 + 6*(4436*d^5*e^6 - 32144*d^4*e^7 + 24
```

7086*d^3*e^8 - 615195*d^2*e^9 + 2939517*d*e^10)*m^6 + (3392928000*e^11 + 3*
(333*d*e^10 + 170*e^11)*m^10 + (686*d^2*e^9 + 52947*d*e^10 + 30600*e^11)*m^
9 + 6*(7140*d^3*e^8 + 5145*d^2*e^9 + 198801*d*e^10 + 133025*e^11)*m^8 + 6*(
2100*d^4*e^7 + 257040*d^3*e^8 + 95354*d^2*e^9 + 2484513*d*e^10 + 1978800*e^
11)*m^7 + 3*(840000*d^5*e^6 + 109200*d^4*e^7 + 7282800*d^3*e^8 + 1886500*d^
2*e^9 + 37725237*d*e^10 + 37016310*e^11)*m^6 + 3*(12600000*d^5*e^6 + 105000
0*d^4*e^7 + 52264800*d^3*e^8 + 10813418*d^2*e^9 + 179179641*d*e^10 + 226287
000*e^11)*m^5 + 42*(5100000*d^5*e^6 + 348000*d^4*e^7 + 14635980*d^3*e^8 + 2
609495*d^2*e^9 + 37733562*d*e^10 + 64999925*e^11)*m^4 + 4*(141750000*d^5*e^
6 + 8659350*d^4*e^7 + 327983040*d^3*e^8 + 52869334*d^2*e^9 + 692643663*d*e^
10 + 1769460300*e^11)*m^3 + 120*(5754000*d^5*e^6 + 329070*d^4*e^7 + 1165962
0*d^3*e^8 + 1755817*d^2*e^9 + 21444534*d*e^10 + 93454763*e^11)*m^2 + 7200*(
42000*d^5*e^6 + 2310*d^4*e^7 + 78540*d^3*e^8 + 11319*d^2*e^9 + 131868*d*e^1
0 + 1344547*e^11)*m)*x^6 - 3*(20400*d^6*e^5 - 452472*d^5*e^6 + 1526840*d^4*
e^7 - 7212240*d^3*e^8 + 12236805*d^2*e^9 - 41597010*d*e^10)*m^5 + (88535462
40*e^11 + (510*d*e^10 + 1109*e^11)*m^10 - (5994*d^2*e^9 - 28050*d*e^10 - 67
649*e^11)*m^9 - 12*(343*d^3*e^8 + 23976*d^2*e^9 - 54825*d*e^10 - 149715*e^1
1)*m^8 - 6*(42840*d^4*e^7 + 27440*d^3*e^8 + 953046*d^2*e^9 - 1430550*d*e^10
 - 4541355*e^11)*m^7 - 3*(25200*d^5*e^6 + 2656080*d^4*e^7 + 869848*d^3*e^8
 + 20283696*d^2*e^9 - 22710810*d*e^10 - 86713819*e^11)*m^6 - 3*(5040000*d^6*
e^5 + 529200*d^5*e^6 + 30416400*d^4*e^7 + 6969760*d^3*e^8 + 124932942*d^2*e
^9 - 112732950*d*e^10 - 541448179*e^11)*m^5 - 2*(75600000*d^6*e^5 + 5481000
*d^5*e^6 + 242260200*d^4*e^7 + 45047562*d^3*e^8 + 675619704*d^2*e^9 - 51950
1300*d*e^10 - 3335910815*e^11)*m^4 - 4*(132300000*d^6*e^5 + 8221500*d^5*e^6
 + 316416240*d^4*e^7 + 51779280*d^3*e^8 + 688165146*d^2*e^9 - 470707050*d*e
^10 - 4412539105*e^11)*m^3 - 72*(10500000*d^6*e^5 + 602700*d^5*e^6 + 214342
80*d^4*e^7 + 3239978*d^3*e^8 + 39724236*d^2*e^9 - 25005980*d*e^10 - 3955614
47*e^11)*m^2 - 288*(1260000*d^6*e^5 + 69300*d^5*e^6 + 2356200*d^4*e^7 + 339
570*d^3*e^8 + 3956040*d^2*e^9 - 2356200*d*e^10 - 86687203*e^11)*m)*x^5 + 3*
(239760*d^7*e^4 - 918000*d^6*e^5 + 9537400*d^5*e^6 - 19929280*d^4*e^7 + 648
36702*d^3*e^8 - 79518915*d^2*e^9 + 198514620*d*e^10)*m^4 + (5728060800*e^11
 + (1109*d*e^10 + 574*e^11)*m^10 - (2550*d^2*e^9 - 63213*d*e^10 - 35588*e^1
1)*m^9 + 6*(4995*d^3*e^8 - 21675*d^2*e^9 + 257288*d*e^10 + 160433*e^11)*m^8
 + 6*(3430*d^4*e^7 + 219780*d^3*e^8 - 461550*d^2*e^9 + 3512203*d*e^10 + 248
3698*e^11)*m^7 + 15*(85680*d^5*e^6 + 49392*d^4*e^7 + 1554444*d^3*e^8 - 2122
620*d^2*e^9 + 11723239*d*e^10 + 9703470*e^11)*m^6 + 3*(126000*d^6*e^5 + 115
66800*d^5*e^6 + 3361400*d^4*e^7 + 70329600*d^3*e^8 - 71101650*d^2*e^9 + 306
983399*d*e^10 + 310583364*e^11)*m^5 + 2*(37800000*d^7*e^4 + 3213000*d^6*e^5
 + 158722200*d^5*e^6 + 32104800*d^4*e^7 + 515019465*d^3*e^8 - 418887225*d^2
*e^9 + 1494010421*d*e^10 + 1964946361*e^11)*m^4 + 4*(113400000*d^7*e^4 + 72
76500*d^6*e^5 + 288206100*d^5*e^6 + 48409305*d^4*e^7 + 659010330*d^3*e^8 -
460978800*d^2*e^9 + 1424518263*d*e^10 + 2670494533*e^11)*m^3 + 72*(11550000
*d^7*e^4 + 666750*d^6*e^5 + 23847600*d^5*e^6 + 3625510*d^4*e^7 + 44710245*d
^3*e^8 - 28312225*d^2*e^9 + 79001833*d*e^10 + 245697543*e^11)*m^2 + 720*(63
0000*d^7*e^4 + 34650*d^6*e^5 + 1178100*d^5*e^6 + 169785*d^4*e^7 + 1978020*d

^3*e^8 - 1178100*d^2*e^9 + 3074148*d*e^10 + 22036147*e^11)*m)*x^4 + 12*(411
60*d^8*e^3 + 2277720*d^7*e^4 - 4105500*d^6*e^5 + 26582730*d^5*e^6 - 3858686
3*d^4*e^7 + 91855890*d^3*e^8 - 84312180*d^2*e^9 + 157352130*d*e^10)*m^3 + (
6346771200*e^11 + (574*d*e^10 + 477*e^11)*m^10 - (4436*d^2*e^9 - 33866*d*e^
10 - 30051*e^11)*m^9 + 24*(425*d^3*e^8 - 9981*d^2*e^9 + 35875*d*e^10 + 3450
3*e^11)*m^8 - 6*(19980*d^4*e^7 - 81600*d^3*e^8 + 909380*d^2*e^9 - 2053198*d
*e^10 - 2183229*e^11)*m^7 - 3*(27440*d^5*e^6 + 1638360*d^4*e^7 - 3202800*d^
3*e^8 + 22641344*d^2*e^9 - 36198162*d*e^10 - 43730883*e^11)*m^6 - 3*(171360
0*d^6*e^5 + 905520*d^5*e^6 + 26173800*d^4*e^7 - 32844000*d^3*e^8 + 16654074
8*d^2*e^9 - 201988878*d*e^10 - 288179073*e^11)*m^5 - 2*(756000*d^7*e^4 + 61
689600*d^6*e^5 + 16093560*d^5*e^6 + 304195500*d^4*e^7 - 278811900*d^3*e^8 +
 1092467028*d^2*e^9 - 1055996410*d*e^10 - 1884673269*e^11)*m^4 - 4*(7560000
0*d^8*e^3 + 5292000*d^7*e^4 + 224910000*d^6*e^5 + 40069260*d^5*e^6 + 573745
680*d^4*e^7 - 419556600*d^3*e^8 + 1349320300*d^2*e^9 - 1086499918*d*e^10 -
2657980899*e^11)*m^3 - 24*(37800000*d^8*e^3 + 2205000*d^7*e^4 + 79682400*d^
6*e^5 + 12238240*d^5*e^6 + 152467380*d^4*e^7 - 97540900*d^3*e^8 + 275018692
*d^2*e^9 - 193842670*d*e^10 - 763013811*e^11)*m^2 - 4320*(140000*d^8*e^3 +
7700*d^7*e^4 + 261800*d^6*e^5 + 37730*d^5*e^6 + 439560*d^4*e^7 - 261800*d^3
*e^8 + 683144*d^2*e^9 - 441980*d*e^10 - 3946963*e^11)*m)*x^3 + 12*(2570400*
d^9*e^2 + 1234800*d^8*e^3 + 32307660*d^7*e^4 - 36490500*d^6*e^5 + 165294232
*d^5*e^6 - 177258088*d^4*e^7 + 320238402*d^3*e^8 - 224755965*d^2*e^9 + 3163
09212*d*e^10)*m^2 + 3*(898128000*e^11 + 3*(53*d*e^10 + 15*e^11)*m^10 - (574
*d^2*e^9 - 9699*d*e^10 - 2880*e^11)*m^9 + (4436*d^3*e^8 - 32718*d^2*e^9 + 2
56626*d*e^10 + 80865*e^11)*m^8 - 2*(5100*d^4*e^7 - 115336*d^3*e^8 + 397782*
d^2*e^9 - 1926603*d*e^10 - 654210*e^11)*m^7 + (119880*d^5*e^6 - 469200*d^4*
e^7 + 4994936*d^3*e^8 - 10728060*d^2*e^9 + 36024471*d*e^10 + 13467195*e^11)
*m^6 + (82320*d^6*e^5 + 4675320*d^5*e^6 - 8670000*d^4*e^7 + 57934160*d^3*e^
8 - 87138366*d^2*e^9 + 216130131*d*e^10 + 91755720*e^11)*m^5 + (5140800*d^7
*e^4 + 2551920*d^6*e^5 + 69170760*d^5*e^6 - 81192000*d^4*e^7 + 383753924*d^
3*e^8 - 431689902*d^2*e^9 + 824188584*d*e^10 + 416767635*e^11)*m^4 + 4*(378
000*d^8*e^3 + 28274400*d^7*e^4 + 6770820*d^6*e^5 + 117512370*d^5*e^6 - 9880
9950*d^4*e^7 + 354356552*d^3*e^8 - 312153254*d^2*e^9 + 473899341*d*e^10 + 3
09068145*e^11)*m^3 + 12*(25200000*d^9*e^2 + 1512000*d^8*e^3 + 56120400*d^7*
e^4 + 8842540*d^6*e^5 + 112906980*d^5*e^6 - 73978900*d^4*e^7 + 213535732*d^
3*e^8 - 154064470*d^2*e^9 + 192742980*d*e^10 + 188672355*e^11)*m^2 + 2160*(
140000*d^9*e^2 + 7700*d^8*e^3 + 261800*d^7*e^4 + 37730*d^6*e^5 + 439560*d^5
*e^6 - 261800*d^4*e^7 + 683144*d^3*e^8 - 441980*d^2*e^9 + 489720*d*e^10 + 1
047765*e^11)*m)*x^2 + 144*(63000*d^10*e + 4498200*d^9*e^2 + 1025570*d^8*e^3
 + 16893090*d^7*e^4 - 13427450*d^6*e^5 + 45284906*d^5*e^6 - 37254035*d^4*e^
7 + 52296690*d^3*e^8 - 28438425*d^2*e^9 + 30235140*d*e^10)*m + 3*(718502400
e^11 + 9(5*d*e^10 + 2*e^11)*m^10 - 3*(106*d^2*e^9 - 945*d*e^10 - 390*e^11
)*m^9 + 2*(574*d^3*e^8 - 9540*d^2*e^9 + 39015*d*e^10 + 16740*e^11)*m^8 - 2*
(4436*d^4*e^7 - 32144*d^3*e^8 + 247086*d^2*e^9 - 615195*d*e^10 - 277290*e^1
1)*m^7 + (20400*d^5*e^6 - 452472*d^4*e^7 + 1526840*d^3*e^8 - 7212240*d^2*e^
9 + 12236805*d*e^10 + 5879034*e^11)*m^6 - (239760*d^6*e^5 - 918000*d^5*e^6

```
+ 9537400*d^4*e^7 - 19929280*d^3*e^8 + 64836702*d^2*e^9 - 79518915*d*e^10 -
 41597010*e^11)*m^5 - 4*(41160*d^7*e^4 + 2277720*d^6*e^5 - 4105500*d^5*e^6
+ 26582730*d^4*e^7 - 38586863*d^3*e^8 + 91855890*d^2*e^9 - 84312180*d*e^10
- 49628655*e^11)*m^4 - 4*(2570400*d^8*e^3 + 1234800*d^7*e^4 + 32307660*d^6*
e^5 - 36490500*d^5*e^6 + 165294232*d^4*e^7 - 177258088*d^3*e^8 + 320238402*
d^2*e^9 - 224755965*d*e^10 - 157352130*e^11)*m^3 - 48*(63000*d^9*e^2 + 4498
200*d^8*e^3 + 1025570*d^7*e^4 + 16893090*d^6*e^5 - 13427450*d^5*e^6 + 45284
906*d^4*e^7 - 37254035*d^3*e^8 + 52296690*d^2*e^9 - 28438425*d*e^10 - 26359
101*e^11)*m^2 - 8640*(70000*d^10*e + 3850*d^9*e^2 + 130900*d^8*e^3 + 18865*
d^7*e^4 + 219780*d^6*e^5 - 130900*d^5*e^6 + 341572*d^4*e^7 - 220990*d^3*e^8
 + 244860*d^2*e^9 - 103950*d*e^10 - 167973*e^11)*m)*x)*(e*x + d)^m/(e^11*m^
11 + 66*e^11*m^10 + 1925*e^11*m^9 + 32670*e^11*m^8 + 357423*e^11*m^7 + 2637
558*e^11*m^6 + 13339535*e^11*m^5 + 45995730*e^11*m^4 + 105258076*e^11*m^3 +
 150917976*e^11*m^2 + 120543840*e^11*m + 39916800*e^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**3*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40895, size = 14796, normalized size = 25.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (500*(x*e + d)^m*m^10*x^11*e^11 + 500*(x*e + d)^m*d*m^10*x^10*e^10 - 25*(x*
e + d)^m*m^10*x^10*e^11 + 27500*(x*e + d)^m*m^9*x^11*e^11 - 25*(x*e + d)^m*
d*m^10*x^9*e^10 + 22500*(x*e + d)^m*d*m^9*x^10*e^10 - 5000*(x*e + d)^m*d^2*
m^9*x^9*e^9 + 765*(x*e + d)^m*m^10*x^9*e^11 - 1400*(x*e + d)^m*m^9*x^10*e^1
1 + 660000*(x*e + d)^m*m^8*x^11*e^11 + 765*(x*e + d)^m*d*m^10*x^8*e^10 - 11
```

75*(x*e + d)^m*d*m^9*x^9*e^10 + 435000*(x*e + d)^m*d*m^8*x^10*e^10 + 225*(x
*e + d)^m*d^2*m^9*x^8*e^9 - 180000*(x*e + d)^m*d^2*m^8*x^9*e^9 + 45000*(x*e
 + d)^m*d^3*m^8*x^8*e^8 - 98*(x*e + d)^m*m^10*x^8*e^11 + 43605*(x*e + d)^m*
m^9*x^9*e^11 - 34125*(x*e + d)^m*m^8*x^10*e^11 + 9075000*(x*e + d)^m*m^7*x^
11*e^11 - 98*(x*e + d)^m*d*m^10*x^7*e^10 + 37485*(x*e + d)^m*d*m^9*x^8*e^10
 - 23550*(x*e + d)^m*d*m^8*x^9*e^10 + 4725000*(x*e + d)^m*d*m^7*x^10*e^10 -
 6120*(x*e + d)^m*d^2*m^9*x^7*e^9 + 8775*(x*e + d)^m*d^2*m^8*x^8*e^9 - 2730
000*(x*e + d)^m*d^2*m^7*x^9*e^9 - 1800*(x*e + d)^m*d^3*m^8*x^7*e^8 + 126000
0*(x*e + d)^m*d^3*m^7*x^8*e^8 - 360000*(x*e + d)^m*d^4*m^7*x^7*e^7 + 999*(x
*e + d)^m*m^10*x^7*e^11 - 5684*(x*e + d)^m*m^9*x^8*e^11 + 1080180*(x*e + d)
^m*m^8*x^9*e^11 - 475500*(x*e + d)^m*m^7*x^10*e^11 + 78886500*(x*e + d)^m*m
^6*x^11*e^11 + 999*(x*e + d)^m*d*m^10*x^6*e^10 - 4998*(x*e + d)^m*d*m^9*x^7
e^10 + 780300(x*e + d)^m*d*m^8*x^8*e^10 - 263550*(x*e + d)^m*d*m^7*x^9*e^
10 + 31636500*(x*e + d)^m*d*m^6*x^10*e^10 + 686*(x*e + d)^m*d^2*m^9*x^6*e^9
 - 257040*(x*e + d)^m*d^2*m^8*x^7*e^9 + 141750*(x*e + d)^m*d^2*m^7*x^8*e^9
- 22680000*(x*e + d)^m*d^2*m^6*x^9*e^9 + 42840*(x*e + d)^m*d^3*m^8*x^6*e^8
- 57600*(x*e + d)^m*d^3*m^7*x^7*e^8 + 14490000*(x*e + d)^m*d^3*m^6*x^8*e^8
+ 12600*(x*e + d)^m*d^4*m^7*x^6*e^7 - 7560000*(x*e + d)^m*d^4*m^6*x^7*e^7 +
 2520000*(x*e + d)^m*d^5*m^6*x^6*e^6 + 510*(x*e + d)^m*m^10*x^6*e^11 + 5894
1*(x*e + d)^m*m^9*x^7*e^11 - 143178*(x*e + d)^m*m^8*x^8*e^11 + 15270930*(x*
e + d)^m*m^7*x^9*e^11 - 4180575*(x*e + d)^m*m^6*x^10*e^11 + 451027500*(x*e
+ d)^m*m^5*x^11*e^11 + 510*(x*e + d)^m*d*m^10*x^5*e^10 + 52947*(x*e + d)^m*
d*m^9*x^6*e^10 - 108192*(x*e + d)^m*d*m^8*x^7*e^10 + 9028530*(x*e + d)^m*d*
m^7*x^8*e^10 - 1808625*(x*e + d)^m*d*m^6*x^9*e^10 + 134662500*(x*e + d)^m*d
*m^5*x^10*e^10 - 5994*(x*e + d)^m*d^2*m^9*x^5*e^9 + 30870*(x*e + d)^m*d^2*m
^8*x^6*e^9 - 4443120*(x*e + d)^m*d^2*m^7*x^7*e^9 + 1237950*(x*e + d)^m*d^2*
m^6*x^8*e^9 - 112245000*(x*e + d)^m*d^2*m^5*x^9*e^9 - 4116*(x*e + d)^m*d^3*
m^8*x^5*e^8 + 1542240*(x*e + d)^m*d^3*m^7*x^6*e^8 - 730800*(x*e + d)^m*d^3*
m^6*x^7*e^8 + 88200000*(x*e + d)^m*d^3*m^5*x^8*e^8 - 257040*(x*e + d)^m*d^4
*m^7*x^5*e^7 + 327600*(x*e + d)^m*d^4*m^6*x^6*e^7 - 63000000*(x*e + d)^m*d^
4*m^5*x^7*e^7 - 75600*(x*e + d)^m*d^5*m^6*x^5*e^6 + 37800000*(x*e + d)^m*d^
5*m^5*x^6*e^6 - 15120000*(x*e + d)^m*d^6*m^5*x^5*e^5 + 1109*(x*e + d)^m*m^1
0*x^5*e^11 + 30600*(x*e + d)^m*m^9*x^6*e^11 + 1510488*(x*e + d)^m*m^8*x^7*e
^11 - 2056236*(x*e + d)^m*m^7*x^8*e^11 + 135990225*(x*e + d)^m*m^6*x^9*e^11
 - 24133200*(x*e + d)^m*m^5*x^10*e^11 + 1708465000*(x*e + d)^m*m^4*x^11*e^1
1 + 1109*(x*e + d)^m*d*m^10*x^4*e^10 + 28050*(x*e + d)^m*d*m^9*x^5*e^10 + 1
192806*(x*e + d)^m*d*m^8*x^6*e^10 - 1298892*(x*e + d)^m*d*m^7*x^7*e^10 + 63
761985*(x*e + d)^m*d*m^6*x^8*e^10 - 7855575*(x*e + d)^m*d*m^5*x^9*e^10 + 36
1840000*(x*e + d)^m*d*m^4*x^10*e^10 - 2550*(x*e + d)^m*d^2*m^9*x^4*e^9 - 28
7712*(x*e + d)^m*d^2*m^8*x^5*e^9 + 572124*(x*e + d)^m*d^2*m^7*x^6*e^9 - 411
26400*(x*e + d)^m*d^2*m^6*x^7*e^9 + 6374025*(x*e + d)^m*d^2*m^5*x^8*e^9 - 3
36420000*(x*e + d)^m*d^2*m^4*x^9*e^9 + 29970*(x*e + d)^m*d^3*m^8*x^4*e^8 -
164640*(x*e + d)^m*d^3*m^7*x^5*e^8 + 21848400*(x*e + d)^m*d^3*m^6*x^6*e^8 -
 4788000*(x*e + d)^m*d^3*m^5*x^7*e^8 + 304605000*(x*e + d)^m*d^3*m^4*x^8*e^
8 + 20580*(x*e + d)^m*d^4*m^7*x^4*e^7 - 7968240*(x*e + d)^m*d^4*m^6*x^5*e^7

+ 3150000*(x*e + d)^m*d^4*m^5*x^6*e^7 - 264600000*(x*e + d)^m*d^4*m^4*x^7*
e^7 + 1285200*(x*e + d)^m*d^5*m^6*x^4*e^6 - 1587600*(x*e + d)^m*d^5*m^5*x^5
e^6 + 214200000(x*e + d)^m*d^5*m^4*x^6*e^6 + 378000*(x*e + d)^m*d^6*m^5*x
^4*e^5 - 151200000*(x*e + d)^m*d^6*m^4*x^5*e^5 + 75600000*(x*e + d)^m*d^7*m
^4*x^4*e^4 + 574*(x*e + d)^m*m^10*x^4*e^11 + 67649*(x*e + d)^m*m^9*x^5*e^11
+ 798150*(x*e + d)^m*m^8*x^6*e^11 + 22063914*(x*e + d)^m*m^7*x^7*e^11 - 18
577566*(x*e + d)^m*m^6*x^8*e^11 + 793819845*(x*e + d)^m*m^5*x^9*e^11 - 9215
6375*(x*e + d)^m*m^4*x^10*e^11 + 4204750000*(x*e + d)^m*m^3*x^11*e^11 + 574
*(x*e + d)^m*d*m^10*x^3*e^10 + 63213*(x*e + d)^m*d*m^9*x^4*e^10 + 657900*(x
*e + d)^m*d*m^8*x^5*e^10 + 14907078*(x*e + d)^m*d*m^7*x^6*e^10 - 9485322*(x
*e + d)^m*d*m^6*x^7*e^10 + 283723965*(x*e + d)^m*d*m^5*x^8*e^10 - 21456200*
(x*e + d)^m*d*m^4*x^9*e^10 + 586350000*(x*e + d)^m*d*m^3*x^10*e^10 - 4436*(
x*e + d)^m*d^2*m^9*x^3*e^9 - 130050*(x*e + d)^m*d^2*m^8*x^4*e^9 - 5718276*(
x*e + d)^m*d^2*m^7*x^5*e^9 + 5659500*(x*e + d)^m*d^2*m^6*x^6*e^9 - 22221108
0*(x*e + d)^m*d^2*m^5*x^7*e^9 + 19707975*(x*e + d)^m*d^2*m^4*x^8*e^9 - 5906
20000*(x*e + d)^m*d^2*m^3*x^9*e^9 + 10200*(x*e + d)^m*d^3*m^8*x^3*e^8 + 131
8680*(x*e + d)^m*d^3*m^7*x^4*e^8 - 2609544*(x*e + d)^m*d^3*m^6*x^5*e^8 + 15
6794400*(x*e + d)^m*d^3*m^5*x^6*e^8 - 17476200*(x*e + d)^m*d^3*m^4*x^7*e^8
+ 590940000*(x*e + d)^m*d^3*m^3*x^8*e^8 - 119880*(x*e + d)^m*d^4*m^7*x^3*e^
7 + 740880*(x*e + d)^m*d^4*m^6*x^4*e^7 - 91249200*(x*e + d)^m*d^4*m^5*x^5*e
^7 + 14616000*(x*e + d)^m*d^4*m^4*x^6*e^7 - 584640000*(x*e + d)^m*d^4*m^3*x
^7*e^7 - 82320*(x*e + d)^m*d^5*m^6*x^3*e^6 + 34700400*(x*e + d)^m*d^5*m^5*x
^4*e^6 - 10962000*(x*e + d)^m*d^5*m^4*x^5*e^6 + 567000000*(x*e + d)^m*d^5*m
^3*x^6*e^6 - 5140800*(x*e + d)^m*d^6*m^5*x^3*e^5 + 6426000*(x*e + d)^m*d^6*
m^4*x^4*e^5 - 529200000*(x*e + d)^m*d^6*m^3*x^5*e^5 - 1512000*(x*e + d)^m*d
^7*m^4*x^3*e^4 + 453600000*(x*e + d)^m*d^7*m^3*x^4*e^4 - 302400000*(x*e + d
)^m*d^8*m^3*x^3*e^3 + 477*(x*e + d)^m*m^10*x^3*e^11 + 35588*(x*e + d)^m*m^9
*x^4*e^11 + 1796580*(x*e + d)^m*m^8*x^5*e^11 + 11872800*(x*e + d)^m*m^7*x^6
e^11 + 202618179(x*e + d)^m*m^6*x^7*e^11 - 109860156*(x*e + d)^m*m^5*x^8*
e^11 + 3060365670*(x*e + d)^m*m^4*x^9*e^11 - 228329500*(x*e + d)^m*m^3*x^10
e^11 + 6376788000(x*e + d)^m*m^2*x^11*e^11 + 477*(x*e + d)^m*d*m^10*x^2*e
^10 + 33866*(x*e + d)^m*d*m^9*x^3*e^10 + 1543728*(x*e + d)^m*d*m^8*x^4*e^10
+ 8583300*(x*e + d)^m*d*m^7*x^5*e^10 + 113175711*(x*e + d)^m*d*m^6*x^6*e^1
0 - 43462902*(x*e + d)^m*d*m^5*x^7*e^10 + 790573950*(x*e + d)^m*d*m^4*x^8*e
^10 - 35223700*(x*e + d)^m*d*m^3*x^9*e^10 + 513288000*(x*e + d)^m*d*m^2*x^1
0*e^10 - 1722*(x*e + d)^m*d^2*m^9*x^2*e^9 - 239544*(x*e + d)^m*d^2*m^8*x^3*
e^9 - 2769300*(x*e + d)^m*d^2*m^7*x^4*e^9 - 60851088*(x*e + d)^m*d^2*m^6*x^
5*e^9 + 32440254*(x*e + d)^m*d^2*m^5*x^6*e^9 - 714314160*(x*e + d)^m*d^2*m^
4*x^7*e^9 + 35442000*(x*e + d)^m*d^2*m^3*x^8*e^9 - 547920000*(x*e + d)^m*d^
2*m^2*x^9*e^9 + 13308*(x*e + d)^m*d^3*m^8*x^2*e^8 + 489600*(x*e + d)^m*d^3*
m^7*x^3*e^8 + 23316660*(x*e + d)^m*d^3*m^6*x^4*e^8 - 20909280*(x*e + d)^m*d
^3*m^5*x^5*e^8 + 614711160*(x*e + d)^m*d^3*m^4*x^6*e^8 - 35330400*(x*e + d)
^m*d^3*m^3*x^7*e^8 + 588060000*(x*e + d)^m*d^3*m^2*x^8*e^8 - 30600*(x*e + d
)^m*d^4*m^7*x^2*e^7 - 4915080*(x*e + d)^m*d^4*m^6*x^3*e^7 + 10084200*(x*e +
d)^m*d^4*m^5*x^4*e^7 - 484520400*(x*e + d)^m*d^4*m^4*x^5*e^7 + 34637400*(x

*e + d)^m*d^4*m^3*x^6*e^7 - 635040000*(x*e + d)^m*d^4*m^2*x^7*e^7 + 359640*
(x*e + d)^m*d^5*m^6*x^2*e^6 - 2716560*(x*e + d)^m*d^5*m^5*x^3*e^6 + 3174444
00*(x*e + d)^m*d^5*m^4*x^4*e^6 - 32886000*(x*e + d)^m*d^5*m^3*x^5*e^6 + 690
480000*(x*e + d)^m*d^5*m^2*x^6*e^6 + 246960*(x*e + d)^m*d^6*m^5*x^2*e^5 - 1
23379200*(x*e + d)^m*d^6*m^4*x^3*e^5 + 29106000*(x*e + d)^m*d^6*m^3*x^4*e^5
 - 756000000*(x*e + d)^m*d^6*m^2*x^5*e^5 + 15422400*(x*e + d)^m*d^7*m^4*x^2
e^4 - 21168000(x*e + d)^m*d^7*m^3*x^3*e^4 + 831600000*(x*e + d)^m*d^7*m^2
*x^4*e^4 + 4536000*(x*e + d)^m*d^8*m^3*x^2*e^3 - 907200000*(x*e + d)^m*d^8*
m^2*x^3*e^3 + 907200000*(x*e + d)^m*d^9*m^2*x^2*e^2 + 135*(x*e + d)^m*m^10*
x^2*e^11 + 30051*(x*e + d)^m*m^9*x^3*e^11 + 962598*(x*e + d)^m*m^8*x^4*e^11
 + 27248130*(x*e + d)^m*m^7*x^5*e^11 + 111048930*(x*e + d)^m*m^6*x^6*e^11 +
 1216593189*(x*e + d)^m*m^5*x^7*e^11 - 428393182*(x*e + d)^m*m^4*x^8*e^11 +
 7643442420*(x*e + d)^m*m^3*x^9*e^11 - 348156900*(x*e + d)^m*m^2*x^10*e^11
+ 5314320000*(x*e + d)^m*m*x^11*e^11 + 135*(x*e + d)^m*d*m^10*x*e^10 + 2909
7*(x*e + d)^m*d*m^9*x^2*e^10 + 861000*(x*e + d)^m*d*m^8*x^3*e^10 + 21073218
*(x*e + d)^m*d*m^7*x^4*e^10 + 68132430*(x*e + d)^m*d*m^6*x^5*e^10 + 5375389
23*(x*e + d)^m*d*m^5*x^6*e^10 - 124152868*(x*e + d)^m*d*m^4*x^7*e^10 + 1318
850820*(x*e + d)^m*d*m^3*x^8*e^10 - 31143600*(x*e + d)^m*d*m^2*x^9*e^10 + 1
81440000*(x*e + d)^m*d*m*x^10*e^10 - 954*(x*e + d)^m*d^2*m^9*x*e^9 - 98154*
(x*e + d)^m*d^2*m^8*x^2*e^9 - 5456280*(x*e + d)^m*d^2*m^7*x^3*e^9 - 3183930
0*(x*e + d)^m*d^2*m^6*x^4*e^9 - 374798826*(x*e + d)^m*d^2*m^5*x^5*e^9 + 109
598790*(x*e + d)^m*d^2*m^4*x^6*e^9 - 1324392480*(x*e + d)^m*d^2*m^3*x^7*e^9
 + 33477300*(x*e + d)^m*d^2*m^2*x^8*e^9 - 201600000*(x*e + d)^m*d^2*m*x^9*e
^9 + 3444*(x*e + d)^m*d^3*m^8*x*e^8 + 692016*(x*e + d)^m*d^3*m^7*x^2*e^8 +
9608400*(x*e + d)^m*d^3*m^6*x^3*e^8 + 210988800*(x*e + d)^m*d^3*m^5*x^4*e^8
 - 90095124*(x*e + d)^m*d^3*m^4*x^5*e^8 + 1311932160*(x*e + d)^m*d^3*m^3*x^
6*e^8 - 36223200*(x*e + d)^m*d^3*m^2*x^7*e^8 + 226800000*(x*e + d)^m*d^3*m*
x^8*e^8 - 26616*(x*e + d)^m*d^4*m^7*x*e^7 - 1407600*(x*e + d)^m*d^4*m^6*x^2
e^7 - 78521400(x*e + d)^m*d^4*m^5*x^3*e^7 + 64209600*(x*e + d)^m*d^4*m^4*
x^4*e^7 - 1265664960*(x*e + d)^m*d^4*m^3*x^5*e^7 + 39488400*(x*e + d)^m*d^4
*m^2*x^6*e^7 - 259200000*(x*e + d)^m*d^4*m*x^7*e^7 + 61200*(x*e + d)^m*d^5*
m^6*x*e^6 + 14025960*(x*e + d)^m*d^5*m^5*x^2*e^6 - 32187120*(x*e + d)^m*d^5
*m^4*x^3*e^6 + 1152824400*(x*e + d)^m*d^5*m^3*x^4*e^6 - 43394400*(x*e + d)^
m*d^5*m^2*x^5*e^6 + 302400000*(x*e + d)^m*d^5*m*x^6*e^6 - 719280*(x*e + d)^
m*d^6*m^5*x*e^5 + 7655760*(x*e + d)^m*d^6*m^4*x^2*e^5 - 899640000*(x*e + d)
^m*d^6*m^3*x^3*e^5 + 48006000*(x*e + d)^m*d^6*m^2*x^4*e^5 - 362880000*(x*e
+ d)^m*d^6*m*x^5*e^5 - 493920*(x*e + d)^m*d^7*m^4*x*e^4 + 339292800*(x*e +
d)^m*d^7*m^3*x^2*e^4 - 52920000*(x*e + d)^m*d^7*m^2*x^3*e^4 + 453600000*(x*
e + d)^m*d^7*m*x^4*e^4 - 30844800*(x*e + d)^m*d^8*m^3*x*e^3 + 54432000*(x*e
 + d)^m*d^8*m^2*x^2*e^3 - 604800000*(x*e + d)^m*d^8*m*x^3*e^3 - 9072000*(x*
e + d)^m*d^9*m^2*x*e^2 + 907200000*(x*e + d)^m*d^9*m*x^2*e^2 - 1814400000*(
x*e + d)^m*d^10*m*x*e + 54*(x*e + d)^m*m^10*x*e^11 + 8640*(x*e + d)^m*m^9*x
^2*e^11 + 828072*(x*e + d)^m*m^8*x^3*e^11 + 14902188*(x*e + d)^m*m^7*x^4*e^
11 + 260141457*(x*e + d)^m*m^6*x^5*e^11 + 678861000*(x*e + d)^m*m^5*x^6*e^1
1 + 4810043142*(x*e + d)^m*m^4*x^7*e^11 - 1080436084*(x*e + d)^m*m^3*x^8*e^

11 + 11731446360*(x*e + d)^m*m^2*x^9*e^11 - 291380400*(x*e + d)^m*m*x^10*e^
11 + 1814400000*(x*e + d)^m*x^11*e^11 + 54*(x*e + d)^m*d*m^10*e^10 + 8505*(
x*e + d)^m*d*m^9*x*e^10 + 769878*(x*e + d)^m*d*m^8*x^2*e^10 + 12319188*(x*e
 + d)^m*d*m^7*x^3*e^10 + 175848585*(x*e + d)^m*d*m^6*x^4*e^10 + 338198850*(
x*e + d)^m*d*m^5*x^5*e^10 + 1584809604*(x*e + d)^m*d*m^4*x^6*e^10 - 2113660
08*(x*e + d)^m*d*m^3*x^7*e^10 + 1180639800*(x*e + d)^m*d*m^2*x^8*e^10 - 110
88000*(x*e + d)^m*d*m*x^9*e^10 - 135*(x*e + d)^m*d^2*m^9*e^9 - 57240*(x*e +
 d)^m*d^2*m^8*x*e^9 - 2386692*(x*e + d)^m*d^2*m^7*x^2*e^9 - 67924032*(x*e +
 d)^m*d^2*m^6*x^3*e^9 - 213304950*(x*e + d)^m*d^2*m^5*x^4*e^9 - 1351239408*
(x*e + d)^m*d^2*m^4*x^5*e^9 + 211477336*(x*e + d)^m*d^2*m^3*x^6*e^9 - 12800
59200*(x*e + d)^m*d^2*m^2*x^7*e^9 + 12474000*(x*e + d)^m*d^2*m*x^8*e^9 + 95
4*(x*e + d)^m*d^3*m^8*e^8 + 192864*(x*e + d)^m*d^3*m^7*x*e^8 + 14984808*(x*
e + d)^m*d^3*m^6*x^2*e^8 + 98532000*(x*e + d)^m*d^3*m^5*x^3*e^8 + 103003893
0*(x*e + d)^m*d^3*m^4*x^4*e^8 - 207117120*(x*e + d)^m*d^3*m^3*x^5*e^8 + 139
9154400*(x*e + d)^m*d^3*m^2*x^6*e^8 - 14256000*(x*e + d)^m*d^3*m*x^7*e^8 -
3444*(x*e + d)^m*d^4*m^7*e^7 - 1357416*(x*e + d)^m*d^4*m^6*x*e^7 - 26010000
*(x*e + d)^m*d^4*m^5*x^2*e^7 - 608391000*(x*e + d)^m*d^4*m^4*x^3*e^7 + 1936
37220*(x*e + d)^m*d^4*m^3*x^4*e^7 - 1543268160*(x*e + d)^m*d^4*m^2*x^5*e^7
+ 16632000*(x*e + d)^m*d^4*m*x^6*e^7 + 26616*(x*e + d)^m*d^5*m^6*e^6 + 2754
000*(x*e + d)^m*d^5*m^5*x*e^6 + 207512280*(x*e + d)^m*d^5*m^4*x^2*e^6 - 160
277040*(x*e + d)^m*d^5*m^3*x^3*e^6 + 1717027200*(x*e + d)^m*d^5*m^2*x^4*e^6
 - 19958400*(x*e + d)^m*d^5*m*x^5*e^6 - 61200*(x*e + d)^m*d^6*m^5*e^5 - 273
32640*(x*e + d)^m*d^6*m^4*x*e^5 + 81249840*(x*e + d)^m*d^6*m^3*x^2*e^5 - 19
12377600*(x*e + d)^m*d^6*m^2*x^3*e^5 + 24948000*(x*e + d)^m*d^6*m*x^4*e^5 +
 719280*(x*e + d)^m*d^7*m^4*e^4 - 14817600*(x*e + d)^m*d^7*m^3*x*e^4 + 2020
334400*(x*e + d)^m*d^7*m^2*x^2*e^4 - 33264000*(x*e + d)^m*d^7*m*x^3*e^4 + 4
93920*(x*e + d)^m*d^8*m^3*e^3 - 647740800*(x*e + d)^m*d^8*m^2*x*e^3 + 49896
000*(x*e + d)^m*d^8*m*x^2*e^3 + 30844800*(x*e + d)^m*d^9*m^2*e^2 - 99792000
*(x*e + d)^m*d^9*m*x*e^2 + 9072000*(x*e + d)^m*d^10*m*e + 1814400000*(x*e +
 d)^m*d^11 + 3510*(x*e + d)^m*m^9*x*e^11 + 242595*(x*e + d)^m*m^8*x^2*e^11
+ 13099374*(x*e + d)^m*m^7*x^3*e^11 + 145552050*(x*e + d)^m*m^6*x^4*e^11 +
1624344537*(x*e + d)^m*m^5*x^5*e^11 + 2729996850*(x*e + d)^m*m^4*x^6*e^11 +
 12279432276*(x*e + d)^m*m^3*x^7*e^11 - 1671802776*(x*e + d)^m*m^2*x^8*e^11
 + 9869234400*(x*e + d)^m*m*x^9*e^11 - 99792000*(x*e + d)^m*x^10*e^11 + 351
0*(x*e + d)^m*d*m^9*e^10 + 234090*(x*e + d)^m*d*m^8*x*e^10 + 11559618*(x*e
+ d)^m*d*m^7*x^2*e^10 + 108594486*(x*e + d)^m*d*m^6*x^3*e^10 + 920950197*(x
*e + d)^m*d*m^5*x^4*e^10 + 1039002600*(x*e + d)^m*d*m^4*x^5*e^10 + 27705746
52*(x*e + d)^m*d*m^3*x^6*e^10 - 192240720*(x*e + d)^m*d*m^2*x^7*e^10 + 4241
16000*(x*e + d)^m*d*m*x^8*e^10 - 8505*(x*e + d)^m*d^2*m^8*e^9 - 1482516*(x*
e + d)^m*d^2*m^7*x*e^9 - 32184180*(x*e + d)^m*d^2*m^6*x^2*e^9 - 499622244*(
x*e + d)^m*d^2*m^5*x^3*e^9 - 837774450*(x*e + d)^m*d^2*m^4*x^4*e^9 - 275266
0584*(x*e + d)^m*d^2*m^3*x^5*e^9 + 210698040*(x*e + d)^m*d^2*m^2*x^6*e^9 -
484704000*(x*e + d)^m*d^2*m*x^7*e^9 + 57240*(x*e + d)^m*d^3*m^7*e^8 + 45805
20*(x*e + d)^m*d^3*m^6*x*e^8 + 173802480*(x*e + d)^m*d^3*m^5*x^2*e^8 + 5576
23800*(x*e + d)^m*d^3*m^4*x^3*e^8 + 2636041320*(x*e + d)^m*d^3*m^3*x^4*e^8

- 233278416*(x*e + d)^m*d^3*m^2*x^5*e^8 + 565488000*(x*e + d)^m*d^3*m*x^6*e
^8 - 192864*(x*e + d)^m*d^4*m^6*e^7 - 28612200*(x*e + d)^m*d^4*m^5*x*e^7 -
243576000*(x*e + d)^m*d^4*m^4*x^2*e^7 - 2294982720*(x*e + d)^m*d^4*m^3*x^3*
e^7 + 261036720*(x*e + d)^m*d^4*m^2*x^4*e^7 - 678585600*(x*e + d)^m*d^4*m*x
^5*e^7 + 1357416*(x*e + d)^m*d^5*m^5*e^6 + 49266000*(x*e + d)^m*d^5*m^4*x*e
^6 + 1410148440*(x*e + d)^m*d^5*m^3*x^2*e^6 - 293717760*(x*e + d)^m*d^5*m^2
*x^3*e^6 + 848232000*(x*e + d)^m*d^5*m*x^4*e^6 - 2754000*(x*e + d)^m*d^6*m^
4*e^5 - 387691920*(x*e + d)^m*d^6*m^3*x*e^5 + 318331440*(x*e + d)^m*d^6*m^2
*x^2*e^5 - 1130976000*(x*e + d)^m*d^6*m*x^3*e^5 + 27332640*(x*e + d)^m*d^7*
m^3*e^4 - 147682080*(x*e + d)^m*d^7*m^2*x*e^4 + 1696464000*(x*e + d)^m*d^7*
m*x^2*e^4 + 14817600*(x*e + d)^m*d^8*m^2*e^3 - 3392928000*(x*e + d)^m*d^8*m
*x*e^3 + 647740800*(x*e + d)^m*d^9*m*e^2 + 99792000*(x*e + d)^m*d^10*e + 10
0440*(x*e + d)^m*m^8*x*e^11 + 3925260*(x*e + d)^m*m^7*x^2*e^11 + 131192649*
(x*e + d)^m*m^6*x^3*e^11 + 931750092*(x*e + d)^m*m^5*x^4*e^11 + 6671821630*
(x*e + d)^m*m^4*x^5*e^11 + 7077841200*(x*e + d)^m*m^3*x^6*e^11 + 1919679199
2*(x*e + d)^m*m^2*x^7*e^11 - 1415539440*(x*e + d)^m*m*x^8*e^11 + 3392928000
*(x*e + d)^m*x^9*e^11 + 100440*(x*e + d)^m*d*m^8*e^10 + 3691170*(x*e + d)^m
*d*m^7*x*e^10 + 108073413*(x*e + d)^m*d*m^6*x^2*e^10 + 605966634*(x*e + d)^
m*d*m^5*x^3*e^10 + 2988020842*(x*e + d)^m*d*m^4*x^4*e^10 + 1882828200*(x*e
+ d)^m*d*m^3*x^5*e^10 + 2573344080*(x*e + d)^m*d*m^2*x^6*e^10 - 69854400*(x
*e + d)^m*d*m*x^7*e^10 - 234090*(x*e + d)^m*d^2*m^7*e^9 - 21636720*(x*e + d
)^m*d^2*m^6*x*e^9 - 261415098*(x*e + d)^m*d^2*m^5*x^2*e^9 - 2184934056*(x*e
 + d)^m*d^2*m^4*x^3*e^9 - 1843915200*(x*e + d)^m*d^2*m^3*x^4*e^9 - 28601449
92*(x*e + d)^m*d^2*m^2*x^5*e^9 + 81496800*(x*e + d)^m*d^2*m*x^6*e^9 + 14825
16*(x*e + d)^m*d^3*m^6*e^8 + 59787840*(x*e + d)^m*d^3*m^5*x*e^8 + 115126177
2*(x*e + d)^m*d^3*m^4*x^2*e^8 + 1678226400*(x*e + d)^m*d^3*m^3*x^3*e^8 + 32
19137640*(x*e + d)^m*d^3*m^2*x^4*e^8 - 97796160*(x*e + d)^m*d^3*m*x^5*e^8 -
 4580520*(x*e + d)^m*d^4*m^5*e^7 - 318992760*(x*e + d)^m*d^4*m^4*x*e^7 - 11
85719400*(x*e + d)^m*d^4*m^3*x^2*e^7 - 3659217120*(x*e + d)^m*d^4*m^2*x^3*e
^7 + 122245200*(x*e + d)^m*d^4*m*x^4*e^7 + 28612200*(x*e + d)^m*d^5*m^4*e^6
 + 437886000*(x*e + d)^m*d^5*m^3*x*e^6 + 4064651280*(x*e + d)^m*d^5*m^2*x^2
e^6 - 162993600(x*e + d)^m*d^5*m*x^3*e^6 - 49266000*(x*e + d)^m*d^6*m^3*e
^5 - 2432604960*(x*e + d)^m*d^6*m^2*x*e^5 + 244490400*(x*e + d)^m*d^6*m*x^2
e^5 + 387691920(x*e + d)^m*d^7*m^2*e^4 - 488980800*(x*e + d)^m*d^7*m*x*e^
4 + 147682080*(x*e + d)^m*d^8*m*e^3 + 3392928000*(x*e + d)^m*d^9*e^2 + 1663
740*(x*e + d)^m*m^7*x*e^11 + 40401585*(x*e + d)^m*m^6*x^2*e^11 + 864537219*
(x*e + d)^m*m^5*x^3*e^11 + 3929892722*(x*e + d)^m*m^4*x^4*e^11 + 1765015642
0*(x*e + d)^m*m^3*x^5*e^11 + 11214571560*(x*e + d)^m*m^2*x^6*e^11 + 1638951
4080*(x*e + d)^m*m*x^7*e^11 - 488980800*(x*e + d)^m*x^8*e^11 + 1663740*(x*e
 + d)^m*d*m^7*e^10 + 36710415*(x*e + d)^m*d*m^6*x*e^10 + 648390393*(x*e + d
)^m*d*m^5*x^2*e^10 + 2111992820*(x*e + d)^m*d*m^4*x^3*e^10 + 5698073052*(x*
e + d)^m*d*m^3*x^4*e^10 + 1800430560*(x*e + d)^m*d*m^2*x^5*e^10 + 949449600
*(x*e + d)^m*d*m*x^6*e^10 - 3691170*(x*e + d)^m*d^2*m^6*e^9 - 194510106*(x*
e + d)^m*d^2*m^5*x*e^9 - 1295069706*(x*e + d)^m*d^2*m^4*x^2*e^9 - 539728120
0*(x*e + d)^m*d^2*m^3*x^3*e^9 - 2038480200*(x*e + d)^m*d^2*m^2*x^4*e^9 - 11

39339520*(x*e + d)^m*d^2*m*x^5*e^9 + 21636720*(x*e + d)^m*d^3*m^5*e^8 + 463042356*(x*e + d)^m*d^3*m^4*x*e^8 + 4252278624*(x*e + d)^m*d^3*m^3*x^2*e^8 + 2340981600*(x*e + d)^m*d^3*m^2*x^3*e^8 + 1424174400*(x*e + d)^m*d^3*m*x^4*e^8 - 59787840*(x*e + d)^m*d^4*m^4*e^7 - 1983530784*(x*e + d)^m*d^4*m^3*x*e^7 - 2663240400*(x*e + d)^m*d^4*m^2*x^2*e^7 - 1898899200*(x*e + d)^m*d^4*m*x^3*e^7 + 318992760*(x*e + d)^m*d^5*m^3*e^6 + 1933552800*(x*e + d)^m*d^5*m^2*x*e^6 + 2848348800*(x*e + d)^m*d^5*m*x^2*e^6 - 437886000*(x*e + d)^m*d^6*m^2*e^5 - 5696697600*(x*e + d)^m*d^6*m*x*e^5 + 2432604960*(x*e + d)^m*d^7*m*e^4 + 488980800*(x*e + d)^m*d^8*e^3 + 17637102*(x*e + d)^m*m^6*x*e^11 + 275267160*(x*e + d)^m*m^5*x^2*e^11 + 3769346538*(x*e + d)^m*m^4*x^3*e^11 + 10681978132*(x*e + d)^m*m^3*x^4*e^11 + 28480424184*(x*e + d)^m*m^2*x^5*e^11 + 9680738400*(x*e + d)^m*m*x^6*e^11 + 5696697600*(x*e + d)^m*x^7*e^11 + 17637102*(x*e + d)^m*d*m^6*e^10 + 238556745*(x*e + d)^m*d*m^5*x*e^10 + 2472565752*(x*e + d)^m*d*m^4*x^2*e^10 + 4345999672*(x*e + d)^m*d*m^3*x^3*e^10 + 5688131976*(x*e + d)^m*d*m^2*x^4*e^10 + 678585600*(x*e + d)^m*d*m*x^5*e^10 - 36710415*(x*e + d)^m*d^2*m^5*e^9 - 1102270680*(x*e + d)^m*d^2*m^4*x*e^9 - 3745839048*(x*e + d)^m*d^2*m^3*x^2*e^9 - 6600448608*(x*e + d)^m*d^2*m^2*x^3*e^9 - 848232000*(x*e + d)^m*d^2*m*x^4*e^9 + 194510106*(x*e + d)^m*d^3*m^4*e^8 + 2127097056*(x*e + d)^m*d^3*m^3*x*e^8 + 7687286352*(x*e + d)^m*d^3*m^2*x^2*e^8 + 1130976000*(x*e + d)^m*d^3*m*x^3*e^8 - 463042356*(x*e + d)^m*d^4*m^3*e^7 - 6521026464*(x*e + d)^m*d^4*m^2*x*e^7 - 1696464000*(x*e + d)^m*d^4*m*x^2*e^7 + 1983530784*(x*e + d)^m*d^5*m^2*e^6 + 3392928000*(x*e + d)^m*d^5*m*x*e^6 - 1933552800*(x*e + d)^m*d^6*m*e^5 + 5696697600*(x*e + d)^m*d^7*e^4 + 124791030*(x*e + d)^m*m^5*x*e^11 + 1250302905*(x*e + d)^m*m^4*x^2*e^11 + 10631923596*(x*e + d)^m*m^3*x^3*e^11 + 17690223096*(x*e + d)^m*m^2*x^4*e^11 + 24965914464*(x*e + d)^m*m*x^5*e^11 + 3392928000*(x*e + d)^m*x^6*e^11 + 124791030*(x*e + d)^m*d*m^5*e^10 + 1011746160*(x*e + d)^m*d*m^4*x*e^10 + 5686792092*(x*e + d)^m*d*m^3*x^2*e^10 + 4652224080*(x*e + d)^m*d*m^2*x^3*e^10 + 2213386560*(x*e + d)^m*d*m*x^4*e^10 - 238556745*(x*e + d)^m*d^2*m^4*e^9 - 3842860824*(x*e + d)^m*d^2*m^3*x*e^9 - 5546320920*(x*e + d)^m*d^2*m^2*x^2*e^9 - 2951182080*(x*e + d)^m*d^2*m*x^3*e^9 + 1102270680*(x*e + d)^m*d^3*m^3*e^8 + 5364581040*(x*e + d)^m*d^3*m^2*x*e^8 + 4426773120*(x*e + d)^m*d^3*m*x^2*e^8 - 2127097056*(x*e + d)^m*d^4*m^2*e^7 - 8853546240*(x*e + d)^m*d^4*m*x*e^7 + 6521026464*(x*e + d)^m*d^5*m*e^6 - 3392928000*(x*e + d)^m*d^6*e^5 + 595543860*(x*e + d)^m*m^4*x*e^11 + 3708817740*(x*e + d)^m*m^3*x^2*e^11 + 18312331464*(x*e + d)^m*m^2*x^3*e^11 + 15866025840*(x*e + d)^m*m*x^4*e^11 + 8853546240*(x*e + d)^m*x^5*e^11 + 595543860*(x*e + d)^m*d*m^4*e^10 + 2697071580*(x*e + d)^m*d*m^3*x*e^10 + 6938747280*(x*e + d)^m*d*m^2*x^2*e^10 + 1909353600*(x*e + d)^m*d*m*x^3*e^10 - 1011746160*(x*e + d)^m*d^2*m^3*e^9 - 7530723360*(x*e + d)^m*d^2*m^2*x*e^9 - 2864030400*(x*e + d)^m*d^2*m*x^2*e^9 + 3842860824*(x*e + d)^m*d^3*m^2*e^8 + 5728060800*(x*e + d)^m*d^3*m*x*e^8 - 5364581040*(x*e + d)^m*d^4*m*e^7 + 8853546240*(x*e + d)^m*d^5*e^6 + 1888225560*(x*e + d)^m*m^3*x*e^11 + 6792204780*(x*e + d)^m*m^2*x^2*e^11 + 17050880160*(x*e + d)^m*m*x^3*e^11 + 5728060800*(x*e + d)^m*x^4*e^11 + 1888225560*(x*e + d)^m*d*m^3*e^10 + 4095133200*(x*e + d)^m*d*m^2*x*e^10 + 3173385600*

```
(x*e + d)^m*d*m*x^2*e^10 - 2697071580*(x*e + d)^m*d^2*m^2*e^9 - 6346771200*
(x*e + d)^m*d^2*m*x*e^9 + 7530723360*(x*e + d)^m*d^3*m*e^8 - 5728060800*(x*
e + d)^m*d^4*e^7 + 3795710544*(x*e + d)^m*m^2*x*e^11 + 6789517200*(x*e + d)
^m*m*x^2*e^11 + 6346771200*(x*e + d)^m*x^3*e^11 + 3795710544*(x*e + d)^m*d*
m^2*e^10 + 2694384000*(x*e + d)^m*d*m*x*e^10 - 4095133200*(x*e + d)^m*d^2*m
*e^9 + 6346771200*(x*e + d)^m*d^3*e^8 + 4353860160*(x*e + d)^m*m*x*e^11 + 2
694384000*(x*e + d)^m*x^2*e^11 + 4353860160*(x*e + d)^m*d*m*e^10 - 26943840
00*(x*e + d)^m*d^2*e^9 + 2155507200*(x*e + d)^m*x*e^11 + 2155507200*(x*e +
d)^m*d*e^10)/(m^11*e^11 + 66*m^10*e^11 + 1925*m^9*e^11 + 32670*m^8*e^11 + 3
57423*m^7*e^11 + 2637558*m^6*e^11 + 13339535*m^5*e^11 + 45995730*m^4*e^11 +
 105258076*m^3*e^11 + 150917976*m^2*e^11 + 120543840*m*e^11 + 39916800*e^11
)
```

3.368 $\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=432

$$\frac{\left(5d^2-2de+3e^2\right)^2 \left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^9(m+1)}-\frac{\left(5d^2-2de+3e^2\right)\left(88d^3e^2-4d^2e^3+127d^4e+160d^5\right.}{e^9(m+2)}$$

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^9*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^
4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^(2 + m))/(e^9*(
2 + m)) + ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4
 - 195*d*e^5 + 107*e^6)*(d + e*x)^(3 + m))/(e^9*(3 + m)) - ((5600*d^5 + 157
5*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^(4 + m
))/(e^9*(4 + m)) + ((7000*d^4 + 1575*d^3*e + 1665*d^2*e^2 + 185*d*e^3 + 148
*e^4)*(d + e*x)^(5 + m))/(e^9*(5 + m)) - ((5600*d^3 + 945*d^2*e + 666*d*e^2
 + 37*e^3)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + ((2800*d^2 + 315*d*e + 111*e^
2)*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (5*(160*d + 9*e)*(d + e*x)^(8 + m))/(
e^9*(8 + m)) + (100*(d + e*x)^(9 + m))/(e^9*(9 + m))
```

Rubi [A] time = 0.242795, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{\left(5d^2-2de+3e^2\right)^2 \left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^9(m+1)}-\frac{\left(5d^2-2de+3e^2\right)\left(88d^3e^2-4d^2e^3+127d^4e+160d^5\right.}{e^9(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^9*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^
4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^(2 + m))/(e^9*(
2 + m)) + ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4
 - 195*d*e^5 + 107*e^6)*(d + e*x)^(3 + m))/(e^9*(3 + m)) - ((5600*d^5 + 157
5*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^(4 + m
))/(e^9*(4 + m)) + ((7000*d^4 + 1575*d^3*e + 1665*d^2*e^2 + 185*d*e^3 + 148
*e^4)*(d + e*x)^(5 + m))/(e^9*(5 + m)) - ((5600*d^3 + 945*d^2*e + 666*d*e^2
 + 37*e^3)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + ((2800*d^2 + 315*d*e + 111*e^
2)*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (5*(160*d + 9*e)*(d + e*x)^(8 + m))/(
```

```
e^9*(8 + m)) + (100*(d + e*x)^(9 + m))/(e^9*(9 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+}{e^8}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)}{e^9(1+m)}$$

Mathematica [A] time = 0.255382, size = 391, normalized size = 0.91

$$\frac{(d+ex)^{m+1}\left(\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^6}{m+7}-\frac{\left(945d^2e+5600d^3+666de^2+37e^3\right)(d+ex)^5}{m+6}+\frac{\left(1665d^2e^2+1575d^3e+7000d^4+185de^3+148e^4\right)(d+ex)^4}{m+5}-\frac{(2220}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2
 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e
+ 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x))/(2 + m) + ((2800*d
^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107
*e^6)*(d + e*x)^2)/(3 + m) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d
^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^3)/(4 + m) + ((7000*d^4 + 1575*d^3*e
 + 1665*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^4)/(5 + m) - ((5600*d^3 +
945*d^2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^5)/(6 + m) + ((2800*d^2 + 315*d*e
 + 111*e^2)*(d + e*x)^6)/(7 + m) - (5*(160*d + 9*e)*(d + e*x)^7)/(8 + m) +
(100*(d + e*x)^8)/(9 + m)))/e^9
```

Maple [B] time = 0.062, size = 3222, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] (e*x+d)^(1+m)*(100*e^8*m^8*x^8-45*e^8*m^8*x^7+3600*e^8*m^7*x^8-800*d*e^7*m^
7*x^7+111*e^8*m^8*x^6-1665*e^8*m^7*x^7+54600*e^8*m^6*x^8+315*d*e^7*m^7*x^6-
22400*d*e^7*m^6*x^7-37*e^8*m^8*x^5+4218*e^8*m^7*x^6-25830*e^8*m^6*x^7+45360
0*e^8*m^5*x^8+5600*d^2*e^6*m^6*x^6-666*d*e^7*m^7*x^5+9450*d*e^7*m^6*x^6-257
600*d*e^7*m^5*x^7+148*e^8*m^8*x^4-1443*e^8*m^7*x^5+67044*e^8*m^6*x^6-218610
*e^8*m^5*x^7+2244900*e^8*m^4*x^8-1890*d^2*e^6*m^6*x^5+117600*d^2*e^6*m^5*x^
6+185*d*e^7*m^7*x^4-21312*d*e^7*m^6*x^5+114660*d*e^7*m^5*x^6-1568000*d*e^7*
m^4*x^7+65*e^8*m^8*x^3+5920*e^8*m^7*x^4-23532*e^8*m^6*x^5+579642*e^8*m^5*x^
6-1098405*e^8*m^4*x^7+6728400*e^8*m^3*x^8-33600*d^3*e^5*m^5*x^5+3330*d^2*e^
6*m^6*x^4-45360*d^2*e^6*m^5*x^5+980000*d^2*e^6*m^4*x^6-592*d*e^7*m^7*x^3+62
90*d*e^7*m^6*x^4-274392*d*e^7*m^5*x^5+727650*d*e^7*m^4*x^6-5415200*d*e^7*m^
3*x^7+107*e^8*m^8*x^2+2665*e^8*m^7*x^3+99160*e^8*m^6*x^4-208458*e^8*m^5*x^5
+2965809*e^8*m^4*x^6-3332385*e^8*m^3*x^7+11812400*e^8*m^2*x^8+9450*d^3*e^5*
m^5*x^4-504000*d^3*e^5*m^4*x^5-740*d^2*e^6*m^6*x^3+89910*d^2*e^6*m^5*x^4-41
5800*d^2*e^6*m^4*x^5+4116000*d^2*e^6*m^3*x^6-195*d*e^7*m^7*x^2-21312*d*e^7*
m^6*x^3+86210*d*e^7*m^5*x^4-1831500*d*e^7*m^4*x^5+2595285*d*e^7*m^3*x^6-105
05600*d*e^7*m^2*x^7+33*e^8*m^8*x+4494*e^8*m^7*x^2+45890*e^8*m^6*x^3+902800*
e^8*m^5*x^4-1090353*e^8*m^4*x^5+9134412*e^8*m^3*x^6-5906520*e^8*m^2*x^7+109
58400*e^8*m*x^8+168000*d^4*e^4*m^4*x^4-13320*d^3*e^5*m^5*x^3+179550*d^3*e^5
*m^4*x^4-2856000*d^3*e^5*m^3*x^5+1776*d^2*e^6*m^6*x^2-22200*d^2*e^6*m^5*x^3
+922410*d^2*e^6*m^4*x^4-1871100*d^2*e^6*m^3*x^5+9094400*d^2*e^6*m^2*x^6-214
*d*e^7*m^7*x-7410*d*e^7*m^6*x^2-311392*d*e^7*m^5*x^3+611240*d*e^7*m^4*x^4-6
805854*d*e^7*m^3*x^5+5159700*d*e^7*m^2*x^6-10454400*d*e^7*m*x^7+18*e^8*m^8+
1419*e^8*m^7*x+79608*e^8*m^6*x^2+430690*e^8*m^5*x^3+4850404*e^8*m^4*x^4-342
2907*e^8*m^3*x^5+16387596*e^8*m^2*x^6-5519340*e^8*m*x^7+4032000*e^8*x^8-378
00*d^4*e^4*m^4*x^3+1680000*d^4*e^4*m^3*x^4+2220*d^3*e^5*m^5*x^2-306360*d^3*
e^5*m^4*x^3+1181250*d^3*e^5*m^3*x^4-7560000*d^3*e^5*m^2*x^5+390*d^2*e^6*m^6
*x+58608*d^2*e^6*m^5*x^2-256040*d^2*e^6*m^4*x^3+4545450*d^2*e^6*m^3*x^4-434
5110*d^2*e^6*m^2*x^5+9878400*d^2*e^6*m*x^6-33*d*e^7*m^7-8560*d*e^7*m^6*x-11
5440*d*e^7*m^5*x^2-2365632*d*e^7*m^4*x^3+2395565*d*e^7*m^3*x^4-13971348*d*e
^7*m^2*x^5+5227740*d*e^7*m*x^6-4032000*d*e^7*x^7+792*e^8*m^7+25872*e^8*m^6*
x+772326*e^8*m^5*x^2+2389985*e^8*m^4*x^3+15608080*e^8*m^3*x^4-6238718*e^8*m
^2*x^5+15456528*e^8*m*x^6-2041200*e^8*x^7-672000*d^5*e^3*m^3*x^3+39960*d^4*
e^4*m^4*x^2-567000*d^4*e^4*m^3*x^3+5880000*d^4*e^4*m^2*x^4-3552*d^3*e^5*m^5
*x+59940*d^3*e^5*m^4*x^2-2464200*d^3*e^5*m^3*x^3+3449250*d^3*e^5*m^2*x^4-92
06400*d^3*e^5*m*x^5+214*d^2*e^6*m^6+14040*d^2*e^6*m^5*x+758352*d^2*e^6*m^4*
```

x^2-1420800*d^2*e^6*m^3*x^3+11302020*d^2*e^6*m^2*x^4-4887540*d^2*e^6*m*x^5+
4032000*d^2*e^6*x^6-1386*d*e^7*m^6-142096*d*e^7*m^5*x-945750*d*e^7*m^4*x^2-
9939088*d*e^7*m^3*x^3+5136710*d*e^7*m^2*x^4-14497488*d*e^7*m*x^5+2041200*d*
e^7*x^6+14868*e^8*m^6+260106*e^8*m^5*x+4453233*e^8*m^4*x^2+7946185*e^8*m^3*
x^3+29064240*e^8*m^2*x^4-5957592*e^8*m*x^5+5754240*e^8*x^6+113400*d^5*e^3*m
^3*x^2-4032000*d^5*e^3*m^2*x^3-4440*d^4*e^4*m^4*x+799200*d^4*e^4*m^3*x^2-24
57000*d^4*e^4*m^2*x^3+8400000*d^4*e^4*m*x^4-390*d^3*e^5*m^5-110112*d^3*e^5*
m^4*x+588300*d^3*e^5*m^3*x^2-8325000*d^3*e^5*m^2*x^3+4479300*d^3*e^5*m*x^4-
4032000*d^3*e^5*x^5+8346*d^2*e^6*m^5+202800*d^2*e^6*m^4*x+4821840*d^2*e^6*m
^3*x^2-3899060*d^2*e^6*m^2*x^3+13346640*d^2*e^6*m*x^4-2041200*d^2*e^6*x^5-2
4486*d*e^7*m^5-1260460*d*e^7*m^4*x-4332705*d*e^7*m^3*x^2-22675968*d*e^7*m^2
*x^3+5510040*d*e^7*m*x^4-5754240*d*e^7*x^5+155232*e^8*m^5+1567797*e^8*m^4*x
+15458076*e^8*m^3*x^2+15254460*e^8*m^2*x^3+28238400*e^8*m*x^4-2237760*e^8*x
^5+2016000*d^6*e^2*m^2*x^2-79920*d^5*e^3*m^3*x+1360800*d^5*e^3*m^2*x^2-7392
000*d^5*e^3*m*x^3+3552*d^4*e^4*m^4-111000*d^4*e^4*m^3*x+4995000*d^4*e^4*m^2
*x^2-3969000*d^4*e^4*m*x^3+4032000*d^4*e^4*x^4-13650*d^3*e^5*m^4-1296480*d^
3*e^5*m^3*x+2497500*d^3*e^5*m^2*x^2-11908080*d^3*e^5*m*x^3+2041200*d^3*e^5*
x^4+133750*d^2*e^6*m^4+1485900*d^2*e^6*m^3*x+15351744*d^2*e^6*m^2*x^2-49506
00*d^2*e^6*m*x^3+5754240*d^2*e^6*x^4-235620*d*e^7*m^4-6385546*d*e^7*m^3*x-1
0840440*d*e^7*m^2*x^2-25553088*d*e^7*m*x^3+2237760*d*e^7*x^4+983682*e^8*m^4
+5752131*e^8*m^3*x+31059532*e^8*m^2*x^2+15207660*e^8*m*x^3+10741248*e^8*x^4
-226800*d^6*e^2*m^2*x+6048000*d^6*e^2*m*x^2+4440*d^5*e^3*m^3-1438560*d^5*e^
3*m^2*x+3288600*d^5*e^3*m*x^2-4032000*d^5*e^3*x^3+106560*d^4*e^4*m^3-954600
*d^4*e^4*m^2*x+9990000*d^4*e^4*m*x^2-2041200*d^4*e^4*x^3-189150*d^3*e^5*m^3
-7050720*d^3*e^5*m^2*x+4204680*d^3*e^5*m*x^2-5754240*d^3*e^5*x^3+1126710*d^
2*e^6*m^3+5693610*d^2*e^6*m^2*x+21972672*d^2*e^6*m*x^2-2237760*d^2*e^6*x^3-
1332177*d*e^7*m^3-18145060*d*e^7*m^2*x-13242060*d*e^7*m*x^2-10741248*d*e^7*
x^3+3864168*e^8*m^3+12377178*e^8*m^2*x+32300304*e^8*m*x^2+5896800*e^8*x^3-4
032000*d^7*e*m*x+79920*d^6*e^2*m^2-2268000*d^6*e^2*m*x+4032000*d^6*e^2*x^2+
106560*d^5*e^3*m^2-7112880*d^5*e^3*m*x+2041200*d^5*e^3*x^2+1189920*d^4*e^4*
m^2-3085800*d^4*e^4*m*x+5754240*d^4*e^4*x^2-1296750*d^3*e^5*m^2-16602048*d^
3*e^5*m*x+2237760*d^3*e^5*x^2+5258836*d^2*e^6*m^2+10293660*d^2*e^6*m*x+1074
1248*d^2*e^6*x^2-4419954*d*e^7*m^2-25828944*d*e^7*m*x-5896800*d*e^7*x^2+916
2072*e^8*m^2+13944744*e^8*m*x+12942720*e^8*x^2+226800*d^7*e*m-4032000*d^7*e
*x+1358640*d^6*e^2*m-2041200*d^6*e^2*x+848040*d^5*e^3*m-5754240*d^5*e^3*x+5
860800*d^4*e^4*m-2237760*d^4*e^4*x-4396860*d^3*e^5*m-10741248*d^3*e^5*x+128
86224*d^2*e^6*m+5896800*d^2*e^6*x-7957224*d*e^7*m-12942720*d*e^7*x+11946528
*e^8*m+5987520*e^8*x+4032000*d^8+2041200*d^7*e+5754240*d^6*e^2+2237760*d^5*
e^3+10741248*d^4*e^4-5896800*d^3*e^5+12942720*d^2*e^6-5987520*d*e^7+6531840
*e^8)/e^9/(m^9+45*m^8+870*m^7+9450*m^6+63273*m^5+269325*m^4+723680*m^3+1172
700*m^2+1026576*m+362880)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.58728, size = 7602, normalized size = 17.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (18*d*e^8*m^8 + 100*(e^9*m^8 + 36*e^9*m^7 + 546*e^9*m^6 + 4536*e^9*m^5 + 22
449*e^9*m^4 + 67284*e^9*m^3 + 118124*e^9*m^2 + 109584*e^9*m + 40320*e^9)*x^
9 + 4032000*d^9 + 2041200*d^8*e + 5754240*d^7*e^2 + 2237760*d^6*e^3 + 10741
248*d^5*e^4 - 5896800*d^4*e^5 + 12942720*d^3*e^6 - 5987520*d^2*e^7 + 653184
0*d*e^8 - 5*(408240*e^9 - (20*d*e^8 - 9*e^9)*m^8 - (560*d*e^8 - 333*e^9)*m^
7 - 14*(460*d*e^8 - 369*e^9)*m^6 - 14*(2800*d*e^8 - 3123*e^9)*m^5 - 7*(1934
0*d*e^8 - 31383*e^9)*m^4 - 7*(37520*d*e^8 - 95211*e^9)*m^3 - 216*(1210*d*e^
8 - 5469*e^9)*m^2 - 36*(2800*d*e^8 - 30663*e^9)*m)*x^8 - 33*(d^2*e^7 - 24*d
*e^8)*m^7 + (5754240*e^9 - 3*(15*d*e^8 - 37*e^9)*m^8 - 2*(400*d^2*e^7 + 675
*d*e^8 - 2109*e^9)*m^7 - 12*(1400*d^2*e^7 + 1365*d*e^8 - 5587*e^9)*m^6 - 14
*(10000*d^2*e^7 + 7425*d*e^8 - 41403*e^9)*m^5 - 21*(28000*d^2*e^7 + 17655*d
*e^8 - 141229*e^9)*m^4 - 28*(46400*d^2*e^7 + 26325*d*e^8 - 326229*e^9)*m^3
- 36*(39200*d^2*e^7 + 20745*d*e^8 - 455211*e^9)*m^2 - 144*(4000*d^2*e^7 + 2
025*d*e^8 - 107337*e^9)*m)*x^7 + 2*(107*d^3*e^6 - 693*d^2*e^7 + 7434*d*e^8)
*m^6 - (2237760*e^9 - 37*(3*d*e^8 - e^9)*m^8 - 3*(105*d^2*e^7 + 1184*d*e^8
- 481*e^9)*m^7 - 4*(1400*d^3*e^6 + 1890*d^2*e^7 + 11433*d*e^8 - 5883*e^9)*m
^6 - 6*(14000*d^3*e^6 + 11550*d^2*e^7 + 50875*d*e^8 - 34743*e^9)*m^5 - (476
000*d^3*e^6 + 311850*d^2*e^7 + 1134309*d*e^8 - 1090353*e^9)*m^4 - 3*(420000
*d^3*e^6 + 241395*d^2*e^7 + 776186*d*e^8 - 1140969*e^9)*m^3 - 2*(767200*d^3
*e^6 + 407295*d^2*e^7 + 1208124*d*e^8 - 3119359*e^9)*m^2 - 24*(28000*d^3*e^
6 + 14175*d^2*e^7 + 39960*d*e^8 - 248233*e^9)*m)*x^6 - 6*(65*d^4*e^5 - 1391
*d^3*e^6 + 4081*d^2*e^7 - 25872*d*e^8)*m^5 + (10741248*e^9 - 37*(d*e^8 - 4*
```

e^9)*m^8 - 74*(9*d^2*e^7 + 17*d*e^8 - 80*e^9)*m^7 - 2*(945*d^3*e^6 + 8991*d^2*e^7 + 8621*d*e^8 - 49580*e^9)*m^6 - 2*(16800*d^4*e^5 + 17955*d^3*e^6 + 92241*d^2*e^7 + 61124*d*e^8 - 451400*e^9)*m^5 - (336000*d^4*e^5 + 236250*d^3*e^6 + 909090*d^2*e^7 + 479113*d*e^8 - 4850404*e^9)*m^4 - 2*(588000*d^4*e^5 + 344925*d^3*e^6 + 1130202*d^2*e^7 + 513671*d*e^8 - 7804040*e^9)*m^3 - 12*(140000*d^4*e^5 + 74655*d^3*e^6 + 222444*d^2*e^7 + 91834*d*e^8 - 2422020*e^9)*m^2 - 144*(5600*d^4*e^5 + 2835*d^3*e^6 + 7992*d^2*e^7 + 3108*d*e^8 - 196100*e^9)*m)*x^5 + 2*(1776*d^5*e^4 - 6825*d^4*e^5 + 66875*d^3*e^6 - 117810*d^2*e^7 + 491841*d*e^8)*m^4 + (5896800*e^9 + (148*d*e^8 + 65*e^9)*m^8 + (185*d^2*e^7 + 5328*d*e^8 + 2665*e^9)*m^7 + 2*(1665*d^3*e^6 + 2775*d^2*e^7 + 38924*d*e^8 + 22945*e^9)*m^6 + 2*(4725*d^4*e^5 + 38295*d^3*e^6 + 32005*d^2*e^7 + 295704*d*e^8 + 215345*e^9)*m^5 + (168000*d^5*e^4 + 141750*d^4*e^5 + 616050*d^3*e^6 + 355200*d^2*e^7 + 2484772*d*e^8 + 2389985*e^9)*m^4 + (1008000*d^5*e^4 + 614250*d^4*e^5 + 2081250*d^3*e^6 + 974765*d^2*e^7 + 5668992*d*e^8 + 7946185*e^9)*m^3 + 6*(308000*d^5*e^4 + 165375*d^4*e^5 + 496170*d^3*e^6 + 206275*d^2*e^7 + 1064712*d*e^8 + 2542410*e^9)*m^2 + 36*(28000*d^5*e^4 + 14175*d^4*e^5 + 39960*d^3*e^6 + 15540*d^2*e^7 + 74592*d*e^8 + 422435*e^9)*m)*x^4 + 3*(1480*d^6*e^3 + 35520*d^5*e^4 - 63050*d^4*e^5 + 375570*d^3*e^6 - 444059*d^2*e^7 + 1288056*d*e^8)*m^3 + (12942720*e^9 + (65*d*e^8 + 107*e^9)*m^8 - 2*(296*d^2*e^7 - 1235*d*e^8 - 2247*e^9)*m^7 - 4*(185*d^3*e^6 + 4884*d^2*e^7 - 9620*d*e^8 - 19902*e^9)*m^6 - 2*(6660*d^4*e^5 + 9990*d^3*e^6 + 126392*d^2*e^7 - 157625*d*e^8 - 386163*e^9)*m^5 - (37800*d^5*e^4 + 266400*d^4*e^5 + 196100*d^3*e^6 + 1607280*d^2*e^7 - 1444235*d*e^8 - 4453233*e^9)*m^4 - 4*(168000*d^6*e^3 + 113400*d^5*e^4 + 416250*d^4*e^5 + 208125*d^3*e^6 + 1279312*d^2*e^7 - 903370*d*e^8 - 3864519*e^9)*m^3 - 4*(504000*d^6*e^3 + 274050*d^5*e^4 + 832500*d^4*e^5 + 350390*d^3*e^6 + 1831056*d^2*e^7 - 1103505*d*e^8 - 7764883*e^9)*m^2 - 48*(28000*d^6*e^3 + 14175*d^5*e^4 + 39960*d^4*e^5 + 15540*d^3*e^6 + 74592*d^2*e^7 - 40950*d*e^8 - 672923*e^9)*m)*x^3 + 2*(39960*d^7*e^2 + 53280*d^6*e^3 + 594960*d^5*e^4 - 648375*d^4*e^5 + 2629418*d^3*e^6 - 2209977*d^2*e^7 + 4581036*d*e^8)*m^2 + (5987520*e^9 + (107*d*e^8 + 33*e^9)*m^8 - (195*d^2*e^7 - 4280*d*e^8 - 1419*e^9)*m^7 + 4*(444*d^3*e^6 - 1755*d^2*e^7 + 17762*d*e^8 + 6468*e^9)*m^6 + 2*(1110*d^4*e^5 + 27528*d^3*e^6 - 50700*d^2*e^7 + 315115*d*e^8 + 130053*e^9)*m^5 + (39960*d^5*e^4 + 55500*d^4*e^5 + 648240*d^3*e^6 - 742950*d^2*e^7 + 3192773*d*e^8 + 1567797*e^9)*m^4 + (113400*d^6*e^3 + 719280*d^5*e^4 + 477300*d^4*e^5 + 3525360*d^3*e^6 - 2846805*d^2*e^7 + 9072530*d*e^8 + 5752131*e^9)*m^3 + 6*(336000*d^7*e^2 + 189000*d^6*e^3 + 592740*d^5*e^4 + 257150*d^4*e^5 + 1383504*d^3*e^6 - 857805*d^2*e^7 + 2152412*d*e^8 + 2062863*e^9)*m^2 + 72*(28000*d^7*e^2 + 14175*d^6*e^3 + 39960*d^5*e^4 + 15540*d^4*e^5 + 74592*d^3*e^6 - 40950*d^2*e^7 + 89880*d*e^8 + 193677*e^9)*m)*x^2 + 12*(18900*d^8*e + 113220*d^7*e^2 + 70670*d^6*e^3 + 488400*d^5*e^4 - 366405*d^4*e^5 + 1073852*d^3*e^6 - 663102*d^2*e^7 + 995544*d*e^8)*m + (6531840*e^9 + 3*(11*d*e^8 + 6*e^9)*m^8 - 2*(107*d^2*e^7 - 693*d*e^8 - 396*e^9)*m^7 + 6*(65*d^3*e^6 - 1391*d^2*e^7 + 4081*d*e^8 + 2478*e^9)*m^6 - 2*(1776*d^4*e^5 - 6825*d^3*e^6 + 66875*d^2*e^7 - 117810*d*e^8 - 77616*e^9)*m^5 - 3*(1480*d^5*e^4 + 35520*d^4*e^5 - 63050*d^3*e^6 + 375570*d^2*e

```
^7 - 444059*d*e^8 - 327894*e^9)*m^4 - 2*(39960*d^6*e^3 + 53280*d^5*e^4 + 59
4960*d^4*e^5 - 648375*d^3*e^6 + 2629418*d^2*e^7 - 2209977*d*e^8 - 1932084*e
^9)*m^3 - 12*(18900*d^7*e^2 + 113220*d^6*e^3 + 70670*d^5*e^4 + 488400*d^4*e
^5 - 366405*d^3*e^6 + 1073852*d^2*e^7 - 663102*d*e^8 - 763506*e^9)*m^2 - 14
4*(28000*d^8*e + 14175*d^7*e^2 + 39960*d^6*e^3 + 15540*d^5*e^4 + 74592*d^4*
e^5 - 40950*d^3*e^6 + 89880*d^2*e^7 - 41580*d*e^8 - 82962*e^9)*m)*x)*(e*x +
 d)^m/(e^9*m^9 + 45*e^9*m^8 + 870*e^9*m^7 + 9450*e^9*m^6 + 63273*e^9*m^5 +
269325*e^9*m^4 + 723680*e^9*m^3 + 1172700*e^9*m^2 + 1026576*e^9*m + 362880*
e^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31855, size = 8401, normalized size = 19.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (100*(x*e + d)^m*m^8*x^9*e^9 + 100*(x*e + d)^m*d*m^8*x^8*e^8 - 45*(x*e + d)
^m*m^8*x^8*e^9 + 3600*(x*e + d)^m*m^7*x^9*e^9 - 45*(x*e + d)^m*d*m^8*x^7*e^
8 + 2800*(x*e + d)^m*d*m^7*x^8*e^8 - 800*(x*e + d)^m*d^2*m^7*x^7*e^7 + 111*
(x*e + d)^m*m^8*x^7*e^9 - 1665*(x*e + d)^m*m^7*x^8*e^9 + 54600*(x*e + d)^m*
m^6*x^9*e^9 + 111*(x*e + d)^m*d*m^8*x^6*e^8 - 1350*(x*e + d)^m*d*m^7*x^7*e^
8 + 32200*(x*e + d)^m*d*m^6*x^8*e^8 + 315*(x*e + d)^m*d^2*m^7*x^6*e^7 - 168
00*(x*e + d)^m*d^2*m^6*x^7*e^7 + 5600*(x*e + d)^m*d^3*m^6*x^6*e^6 - 37*(x*e
 + d)^m*m^8*x^6*e^9 + 4218*(x*e + d)^m*m^7*x^7*e^9 - 25830*(x*e + d)^m*m^6*
x^8*e^9 + 453600*(x*e + d)^m*m^5*x^9*e^9 - 37*(x*e + d)^m*d*m^8*x^5*e^8 + 3
552*(x*e + d)^m*d*m^7*x^6*e^8 - 16380*(x*e + d)^m*d*m^6*x^7*e^8 + 196000*(x
```

*e + d)^m*d*m^5*x^8*e^8 - 666*(x*e + d)^m*d^2*m^7*x^5*e^7 + 7560*(x*e + d)^
m*d^2*m^6*x^6*e^7 - 140000*(x*e + d)^m*d^2*m^5*x^7*e^7 - 1890*(x*e + d)^m*d
^3*m^6*x^5*e^6 + 84000*(x*e + d)^m*d^3*m^5*x^6*e^6 - 33600*(x*e + d)^m*d^4*
m^5*x^5*e^5 + 148*(x*e + d)^m*m^8*x^5*e^9 - 1443*(x*e + d)^m*m^7*x^6*e^9 +
67044*(x*e + d)^m*m^6*x^7*e^9 - 218610*(x*e + d)^m*m^5*x^8*e^9 + 2244900*(x
*e + d)^m*m^4*x^9*e^9 + 148*(x*e + d)^m*d*m^8*x^4*e^8 - 1258*(x*e + d)^m*d*
m^7*x^5*e^8 + 45732*(x*e + d)^m*d*m^6*x^6*e^8 - 103950*(x*e + d)^m*d*m^5*x^
7*e^8 + 676900*(x*e + d)^m*d*m^4*x^8*e^8 + 185*(x*e + d)^m*d^2*m^7*x^4*e^7
- 17982*(x*e + d)^m*d^2*m^6*x^5*e^7 + 69300*(x*e + d)^m*d^2*m^5*x^6*e^7 - 5
88000*(x*e + d)^m*d^2*m^4*x^7*e^7 + 3330*(x*e + d)^m*d^3*m^6*x^4*e^6 - 3591
0*(x*e + d)^m*d^3*m^5*x^5*e^6 + 476000*(x*e + d)^m*d^3*m^4*x^6*e^6 + 9450*(
x*e + d)^m*d^4*m^5*x^4*e^5 - 336000*(x*e + d)^m*d^4*m^4*x^5*e^5 + 168000*(x
*e + d)^m*d^5*m^4*x^4*e^4 + 65*(x*e + d)^m*m^8*x^4*e^9 + 5920*(x*e + d)^m*m
^7*x^5*e^9 - 23532*(x*e + d)^m*m^6*x^6*e^9 + 579642*(x*e + d)^m*m^5*x^7*e^9
 - 1098405*(x*e + d)^m*m^4*x^8*e^9 + 6728400*(x*e + d)^m*m^3*x^9*e^9 + 65*(
x*e + d)^m*d*m^8*x^3*e^8 + 5328*(x*e + d)^m*d*m^7*x^4*e^8 - 17242*(x*e + d)
^m*d*m^6*x^5*e^8 + 305250*(x*e + d)^m*d*m^5*x^6*e^8 - 370755*(x*e + d)^m*d*
m^4*x^7*e^8 + 1313200*(x*e + d)^m*d*m^3*x^8*e^8 - 592*(x*e + d)^m*d^2*m^7*x
^3*e^7 + 5550*(x*e + d)^m*d^2*m^6*x^4*e^7 - 184482*(x*e + d)^m*d^2*m^5*x^5*
e^7 + 311850*(x*e + d)^m*d^2*m^4*x^6*e^7 - 1299200*(x*e + d)^m*d^2*m^3*x^7*
e^7 - 740*(x*e + d)^m*d^3*m^6*x^3*e^6 + 76590*(x*e + d)^m*d^3*m^5*x^4*e^6 -
 236250*(x*e + d)^m*d^3*m^4*x^5*e^6 + 1260000*(x*e + d)^m*d^3*m^3*x^6*e^6 -
 13320*(x*e + d)^m*d^4*m^5*x^3*e^5 + 141750*(x*e + d)^m*d^4*m^4*x^4*e^5 - 1
176000*(x*e + d)^m*d^4*m^3*x^5*e^5 - 37800*(x*e + d)^m*d^5*m^4*x^3*e^4 + 10
08000*(x*e + d)^m*d^5*m^3*x^4*e^4 - 672000*(x*e + d)^m*d^6*m^3*x^3*e^3 + 10
7*(x*e + d)^m*m^8*x^3*e^9 + 2665*(x*e + d)^m*m^7*x^4*e^9 + 99160*(x*e + d)^
m*m^6*x^5*e^9 - 208458*(x*e + d)^m*m^5*x^6*e^9 + 2965809*(x*e + d)^m*m^4*x^
7*e^9 - 3332385*(x*e + d)^m*m^3*x^8*e^9 + 11812400*(x*e + d)^m*m^2*x^9*e^9
+ 107*(x*e + d)^m*d*m^8*x^2*e^8 + 2470*(x*e + d)^m*d*m^7*x^3*e^8 + 77848*(x
*e + d)^m*d*m^6*x^4*e^8 - 122248*(x*e + d)^m*d*m^5*x^5*e^8 + 1134309*(x*e +
 d)^m*d*m^4*x^6*e^8 - 737100*(x*e + d)^m*d*m^3*x^7*e^8 + 1306800*(x*e + d)^
m*d*m^2*x^8*e^8 - 195*(x*e + d)^m*d^2*m^7*x^2*e^7 - 19536*(x*e + d)^m*d^2*m
^6*x^3*e^7 + 64010*(x*e + d)^m*d^2*m^5*x^4*e^7 - 909090*(x*e + d)^m*d^2*m^4
*x^5*e^7 + 724185*(x*e + d)^m*d^2*m^3*x^6*e^7 - 1411200*(x*e + d)^m*d^2*m^2
*x^7*e^7 + 1776*(x*e + d)^m*d^3*m^6*x^2*e^6 - 19980*(x*e + d)^m*d^3*m^5*x^3
e^6 + 616050(x*e + d)^m*d^3*m^4*x^4*e^6 - 689850*(x*e + d)^m*d^3*m^3*x^5*
e^6 + 1534400*(x*e + d)^m*d^3*m^2*x^6*e^6 + 2220*(x*e + d)^m*d^4*m^5*x^2*e^
5 - 266400*(x*e + d)^m*d^4*m^4*x^3*e^5 + 614250*(x*e + d)^m*d^4*m^3*x^4*e^5
 - 1680000*(x*e + d)^m*d^4*m^2*x^5*e^5 + 39960*(x*e + d)^m*d^5*m^4*x^2*e^4
- 453600*(x*e + d)^m*d^5*m^3*x^3*e^4 + 1848000*(x*e + d)^m*d^5*m^2*x^4*e^4
+ 113400*(x*e + d)^m*d^6*m^3*x^2*e^3 - 2016000*(x*e + d)^m*d^6*m^2*x^3*e^3
+ 2016000*(x*e + d)^m*d^7*m^2*x^2*e^2 + 33*(x*e + d)^m*m^8*x^2*e^9 + 4494*(
x*e + d)^m*m^7*x^3*e^9 + 45890*(x*e + d)^m*m^6*x^4*e^9 + 902800*(x*e + d)^m
*m^5*x^5*e^9 - 1090353*(x*e + d)^m*m^4*x^6*e^9 + 9134412*(x*e + d)^m*m^3*x^
7*e^9 - 5906520*(x*e + d)^m*m^2*x^8*e^9 + 10958400*(x*e + d)^m*m*x^9*e^9 +

33*(x*e + d)^m*d*m^8*x*e^8 + 4280*(x*e + d)^m*d*m^7*x^2*e^8 + 38480*(x*e + d)^m*d*m^6*x^3*e^8 + 591408*(x*e + d)^m*d*m^5*x^4*e^8 - 479113*(x*e + d)^m*d*m^4*x^5*e^8 + 2328558*(x*e + d)^m*d*m^3*x^6*e^8 - 746820*(x*e + d)^m*d*m^2*x^7*e^8 + 504000*(x*e + d)^m*d*m*x^8*e^8 - 214*(x*e + d)^m*d^2*m^7*x*e^7 - 7020*(x*e + d)^m*d^2*m^6*x^2*e^7 - 252784*(x*e + d)^m*d^2*m^5*x^3*e^7 + 355200*(x*e + d)^m*d^2*m^4*x^4*e^7 - 2260404*(x*e + d)^m*d^2*m^3*x^5*e^7 + 814590*(x*e + d)^m*d^2*m^2*x^6*e^7 - 576000*(x*e + d)^m*d^2*m*x^7*e^7 + 390*(x*e + d)^m*d^3*m^6*x*e^6 + 55056*(x*e + d)^m*d^3*m^5*x^2*e^6 - 196100*(x*e + d)^m*d^3*m^4*x^3*e^6 + 2081250*(x*e + d)^m*d^3*m^3*x^4*e^6 - 895860*(x*e + d)^m*d^3*m^2*x^5*e^6 + 672000*(x*e + d)^m*d^3*m*x^6*e^6 - 3552*(x*e + d)^m*d^4*m^5*x*e^5 + 55500*(x*e + d)^m*d^4*m^4*x^2*e^5 - 1665000*(x*e + d)^m*d^4*m^3*x^3*e^5 + 992250*(x*e + d)^m*d^4*m^2*x^4*e^5 - 806400*(x*e + d)^m*d^4*m*x^5*e^5 - 4440*(x*e + d)^m*d^5*m^4*x*e^4 + 719280*(x*e + d)^m*d^5*m^3*x^2*e^4 - 1096200*(x*e + d)^m*d^5*m^2*x^3*e^4 + 1008000*(x*e + d)^m*d^5*m*x^4*e^4 - 79920*(x*e + d)^m*d^6*m^3*x*e^3 + 1134000*(x*e + d)^m*d^6*m^2*x^2*e^3 - 1344000*(x*e + d)^m*d^6*m*x^3*e^3 - 226800*(x*e + d)^m*d^7*m^2*x*e^2 + 2016000*(x*e + d)^m*d^7*m*x^2*e^2 - 4032000*(x*e + d)^m*d^8*m*x*e + 18*(x*e + d)^m*m^8*x*e^9 + 1419*(x*e + d)^m*m^7*x^2*e^9 + 79608*(x*e + d)^m*m^6*x^3*e^9 + 430690*(x*e + d)^m*m^5*x^4*e^9 + 4850404*(x*e + d)^m*m^4*x^5*e^9 - 3422907*(x*e + d)^m*m^3*x^6*e^9 + 16387596*(x*e + d)^m*m^2*x^7*e^9 - 5519340*(x*e + d)^m*m*x^8*e^9 + 4032000*(x*e + d)^m*x^9*e^9 + 18*(x*e + d)^m*d*m^8*e^8 + 1386*(x*e + d)^m*d*m^7*x*e^8 + 71048*(x*e + d)^m*d*m^6*x^2*e^8 + 315250*(x*e + d)^m*d*m^5*x^3*e^8 + 2484772*(x*e + d)^m*d*m^4*x^4*e^8 - 1027342*(x*e + d)^m*d*m^3*x^5*e^8 + 2416248*(x*e + d)^m*d*m^2*x^6*e^8 - 291600*(x*e + d)^m*d*m*x^7*e^8 - 33*(x*e + d)^m*d^2*m^7*e^7 - 8346*(x*e + d)^m*d^2*m^6*x*e^7 - 101400*(x*e + d)^m*d^2*m^5*x^2*e^7 - 1607280*(x*e + d)^m*d^2*m^4*x^3*e^7 + 974765*(x*e + d)^m*d^2*m^3*x^4*e^7 - 2669328*(x*e + d)^m*d^2*m^2*x^5*e^7 + 340200*(x*e + d)^m*d^2*m*x^6*e^7 + 214*(x*e + d)^m*d^3*m^6*e^6 + 13650*(x*e + d)^m*d^3*m^5*x*e^6 + 648240*(x*e + d)^m*d^3*m^4*x^2*e^6 - 832500*(x*e + d)^m*d^3*m^3*x^3*e^6 + 2977020*(x*e + d)^m*d^3*m^2*x^4*e^6 - 408240*(x*e + d)^m*d^3*m*x^5*e^6 - 390*(x*e + d)^m*d^4*m^5*e^5 - 106560*(x*e + d)^m*d^4*m^4*x*e^5 + 477300*(x*e + d)^m*d^4*m^3*x^2*e^5 - 3330000*(x*e + d)^m*d^4*m^2*x^3*e^5 + 510300*(x*e + d)^m*d^4*m*x^4*e^5 + 3552*(x*e + d)^m*d^5*m^4*e^4 - 106560*(x*e + d)^m*d^5*m^3*x*e^4 + 3556440*(x*e + d)^m*d^5*m^2*x^2*e^4 - 680400*(x*e + d)^m*d^5*m*x^3*e^4 + 4440*(x*e + d)^m*d^6*m^3*e^3 - 1358640*(x*e + d)^m*d^6*m^2*x*e^3 + 1020600*(x*e + d)^m*d^6*m*x^2*e^3 + 79920*(x*e + d)^m*d^7*m^2*e^2 - 2041200*(x*e + d)^m*d^7*m*x*e^2 + 226800*(x*e + d)^m*d^8*m*e + 4032000*(x*e + d)^m*d^9 + 792*(x*e + d)^m*m^7*x*e^9 + 25872*(x*e + d)^m*m^6*x^2*e^9 + 772326*(x*e + d)^m*m^5*x^3*e^9 + 2389985*(x*e + d)^m*m^4*x^4*e^9 + 15608080*(x*e + d)^m*m^3*x^5*e^9 - 6238718*(x*e + d)^m*m^2*x^6*e^9 + 15456528*(x*e + d)^m*m*x^7*e^9 - 2041200*(x*e + d)^m*x^8*e^9 + 792*(x*e + d)^m*d*m^7*e^8 + 24486*(x*e + d)^m*d*m^6*x*e^8 + 630230*(x*e + d)^m*d*m^5*x^2*e^8 + 1444235*(x*e + d)^m*d*m^4*x^3*e^8 + 5668992*(x*e + d)^m*d*m^3*x^4*e^8 - 1102008*(x*e + d)^m*d*m^2*x^5*e^8 + 959040*(x*e + d)^m*d*m*x^6*e^8 - 1386*(x*e + d)^m*d^2*m^6*e^7 - 133750*(x*e + d)^m*d^2*m^5* x*e^7 - 742950*(x*e + d)^m*d^2*m^4*x^2*e^7 - 5117248*(x*e + d)^m*d^2*m^3*x^3*e^7 + 1237650*(x*e + d)^m*d^2*m^2*x^4*e^7 - 1150848*(x*e + d)^m*d^2*m*x^5*e^7 + 8346*(x*e + d)^m*d^3*m^5*e^6 + 189150*(x*e + d)^m*d^3*m^4*x*e^6 + 3525360*(x*e + d)^m*d^3*m^3*x^2*e^6 - 1401560*(x*e + d)^m*d^3*m^2*x^3*e^6 + 1438560*(x*e + d)^m*d^3*m*x^4*e^6 - 13650*(x*e + d)^m*d^4*m^4*e^5 - 1189920*(x*e + d)^m*d^4*m^3*x*e^5 + 1542900*(x*e + d)^m*d^4*m^2*x^2*e^5 - 1918080*(x*e + d)^m*d^4*m*x^3*e^5 + 106560*(x*e + d)^m*d^5*m^3*e^4 - 848040*(x*e + d)^m*d^5*m^2*x*e^4 + 2877120*(x*e + d)^m*d^5*m*x^2*e^4 + 106560*(x*e + d)^m*d^6*m^2*e^3 - 5754240*(x*e + d)^m*d^6*m*x*e^3 + 1358640*(x*e + d)^m*d^7*m*e^2 + 2041200*(x*e + d)^m*d^8*e + 14868*(x*e + d)^m*m^6*x*e^9 + 260106*(x*e + d)^m*m^5*x^2*e^9 + 4453233*(x*e + d)^m*m^4*x^3*e^9 + 7946185*(x*e + d)^m*m^3*x^4*e^9 + 29064240*(x*e + d)^m*m^2*x^5*e^9 - 5957592*(x*e + d)^m*m*x^6*e^9 + 5754240*(x*e + d)^m*x^7*e^9 + 14868*(x*e + d)^m*d*m^6*e^8 + 235620*(x*e + d)^m*d*m^5*x*e^8 + 3192773*(x*e + d)^m*d*m^4*x^2*e^8 + 3613480*(x*e + d)^m*d*m^3*x^3*e^8 + 6388272*(x*e + d)^m*d*m^2*x^4*e^8 - 447552*(x*e + d)^m*d*m*x^5*e^8 - 24486*(x*e + d)^m*d^2*m^5*e^7 - 1126710*(x*e + d)^m*d^2*m^4*x*e^7 - 2846805*(x*e + d)^m*d^2*m^3*x^2*e^7 - 7324224*(x*e + d)^m*d^2*m^2*x^3*e^7 + 559440*(x*e + d)^m*d^2*m*x^4*e^7 + 133750*(x*e + d)^m*d^3*m^4*e^6 + 1296750*(x*e + d)^m*d^3*m^3*x*e^6 + 8301024*(x*e + d)^m*d^3*m^2*x^2*e^6 - 745920*(x*e + d)^m*d^3*m*x^3*e^6 - 189150*(x*e + d)^m*d^4*m^3*e^5 - 5860800*(x*e + d)^m*d^4*m^2*x*e^5 + 1118880*(x*e + d)^m*d^4*m*x^2*e^5 + 1189920*(x*e + d)^m*d^5*m^2*e^4 - 2237760*(x*e + d)^m*d^5*m*x*e^4 + 848040*(x*e + d)^m*d^6*m*e^3 + 5754240*(x*e + d)^m*d^7*e^2 + 155232*(x*e + d)^m*m^5*x*e^9 + 1567797*(x*e + d)^m*m^4*x^2*e^9 + 15458076*(x*e + d)^m*m^3*x^3*e^9 + 15254460*(x*e + d)^m*m^2*x^4*e^9 + 28238400*(x*e + d)^m*m*x^5*e^9 - 2237760*(x*e + d)^m*x^6*e^9 + 155232*(x*e + d)^m*d*m^5*e^8 + 1332177*(x*e + d)^m*d*m^4*x*e^8 + 9072530*(x*e + d)^m*d*m^3*x^2*e^8 + 4414020*(x*e + d)^m*d*m^2*x^3*e^8 + 2685312*(x*e + d)^m*d*m*x^4*e^8 - 235620*(x*e + d)^m*d^2*m^4*e^7 - 5258836*(x*e + d)^m*d^2*m^3*x*e^7 - 5146830*(x*e + d)^m*d^2*m^2*x^2*e^7 - 3580416*(x*e + d)^m*d^2*m*x^3*e^7 + 1126710*(x*e + d)^m*d^3*m^3*e^6 + 4396860*(x*e + d)^m*d^3*m^2*x*e^6 + 5370624*(x*e + d)^m*d^3*m*x^2*e^6 - 1296750*(x*e + d)^m*d^4*m^2*e^5 - 10741248*(x*e + d)^m*d^4*m*x*e^5 + 5860800*(x*e + d)^m*d^5*m*e^4 + 2237760*(x*e + d)^m*d^6*e^3 + 983682*(x*e + d)^m*m^4*x*e^9 + 5752131*(x*e + d)^m*m^3*x^2*e^9 + 31059532*(x*e + d)^m*m^2*x^3*e^9 + 15207660*(x*e + d)^m*m*x^4*e^9 + 10741248*(x*e + d)^m*x^5*e^9 + 983682*(x*e + d)^m*d*m^4*e^8 + 4419954*(x*e + d)^m*d*m^3*x*e^8 + 12914472*(x*e + d)^m*d*m^2*x^2*e^8 + 1965600*(x*e + d)^m*d*m*x^3*e^8 - 1332177*(x*e + d)^m*d^2*m^3*e^7 - 12886224*(x*e + d)^m*d^2*m^2*x*e^7 - 2948400*(x*e + d)^m*d^2*m*x^2*e^7 + 5258836*(x*e + d)^m*d^3*m^2*e^6 + 5896800*(x*e + d)^m*d^3*m*x*e^6 - 4396860*(x*e + d)^m*d^4*m*e^5 + 10741248*(x*e + d)^m*d^5*e^4 + 3864168*(x*e + d)^m*m^3*x*e^9 + 12377178*(x*e + d)^m*m^2*x^2*e^9 + 32300304*(x*e + d)^m*m*x^3*e^9 + 5896800*(x*e + d)^m*x^4*e^9 + 3864168*(x*e + d)^m*d*m^3*e^8 + 7957224*(x*e + d)^m*d*m^2*x*e^8 + 6471360*(x*e + d)^m*d*m*x^2*e^8 - 4419954*(x*e + d)^m*d^2*m^2*e^7 - 12942720*(x*e + d)^m*d^2*m*x*e^7 + 12886224*(x*e + d)^m*d^3*m*e^6 - 5896800*(x*e + d)^m*d^4*e^5 + 9162072*(x*e + d)^m*m^2*x*e^9 +

13944744*(x*e + d)^m*m*x^2*e^9 + 12942720*(x*e + d)^m*x^3*e^9 + 9162072*(x*e + d)^m*d*m^2*e^8 + 5987520*(x*e + d)^m*d*m*x*e^8 - 7957224*(x*e + d)^m*d^2*m*e^7 + 12942720*(x*e + d)^m*d^3*e^6 + 11946528*(x*e + d)^m*m*x*e^9 + 5987520*(x*e + d)^m*x^2*e^9 + 11946528*(x*e + d)^m*d*m*e^8 - 5987520*(x*e + d)^m*d^2*e^7 + 6531840*(x*e + d)^m*x*e^9 + 6531840*(x*e + d)^m*d*e^8)/(m^9*e^9 + 45*m^8*e^9 + 870*m^7*e^9 + 9450*m^6*e^9 + 63273*m^5*e^9 + 269325*m^4*e^9 + 723680*m^3*e^9 + 1172700*m^2*e^9 + 1026576*m*e^9 + 362880*e^9)

3.369 $\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=292

$$\frac{\left(5d^2-2de+3e^2\right)\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^7(m+1)}-\frac{\left(68d^3e^2+12d^2e^3+85d^4e+120d^5+42de^4-7e^5\right)(d+$$

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^(1 + m))/(e^7*(1 + m)) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e
^3 + 42*d*e^4 - 7*e^5)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((300*d^4 + 170*d
^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^(3 + m))/(e^7*(3 + m)) -
(2*(200*d^3 + 85*d^2*e + 34*d*e^2 + 2*e^3)*(d + e*x)^(4 + m))/(e^7*(4 + m))
 + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - ((120*d
+ 17*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (20*(d + e*x)^(7 + m))/(e^7*(7 +
 m))
```

Rubi [A] time = 0.189003, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{\left(5d^2-2de+3e^2\right)\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^7(m+1)}-\frac{\left(68d^3e^2+12d^2e^3+85d^4e+120d^5+42de^4-7e^5\right)(d+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```



```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^(1 + m))/(e^7*(1 + m)) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e
^3 + 42*d*e^4 - 7*e^5)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((300*d^4 + 170*d
^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^(3 + m))/(e^7*(3 + m)) -
(2*(200*d^3 + 85*d^2*e + 34*d*e^2 + 2*e^3)*(d + e*x)^(4 + m))/(e^7*(4 + m))
 + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - ((120*d
+ 17*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (20*(d + e*x)^(7 + m))/(e^7*(7 +
 m))
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(\frac{\left(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6\right)}{e^6}\right.$$
$$= \frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)}{e^7(1+m)}$$

Mathematica [A] time = 0.181669, size = 261, normalized size = 0.89

$$\frac{(d+ex)^{m+1}\left(\frac{\left(300d^2+85de+17e^2\right)(d+ex)^4}{m+5}-\frac{2\left(85d^2e+200d^3+34de^2+2e^3\right)(d+ex)^3}{m+4}+\frac{\left(102d^2e^2+170d^3e+300d^4+12de^3+21e^4\right)(d+ex)^2}{m+3}-\frac{\left(68d^3e^2+12d^2e^3\right.}{}\right.}{e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 -
 d*e^3 + 2*e^4))/(1 + m) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 +
 42*d*e^4 - 7*e^5)*(d + e*x))/(2 + m) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2
 + 12*d*e^3 + 21*e^4)*(d + e*x)^2)/(3 + m) - (2*(200*d^3 + 85*d^2*e + 34*d*
e^2 + 2*e^3)*(d + e*x)^3)/(4 + m) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^
4)/(5 + m) - ((120*d + 17*e)*(d + e*x)^5)/(6 + m) + (20*(d + e*x)^6)/(7 + m
)))/e^7
```

Maple [B] time = 0.052, size = 1504, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] (e*x+d)^(1+m)*(20*e^6*m^6*x^6-17*e^6*m^6*x^5+420*e^6*m^5*x^6-120*d*e^5*m^5*
x^5+17*e^6*m^6*x^4-374*e^6*m^5*x^5+3500*e^6*m^4*x^6+85*d*e^5*m^5*x^4-1800*d
*e^5*m^4*x^5-4*e^6*m^6*x^3+391*e^6*m^5*x^4-3230*e^6*m^4*x^5+14700*e^6*m^3*x
^6+600*d^2*e^4*m^4*x^4-68*d*e^5*m^5*x^3+1445*d*e^5*m^4*x^4-10200*d*e^5*m^3*
x^5+21*e^6*m^6*x^2-96*e^6*m^5*x^3+3519*e^6*m^4*x^4-13940*e^6*m^3*x^5+32480*
```

```
e^6*m^2*x^6-340*d^2*e^4*m^4*x^3+6000*d^2*e^4*m^3*x^4+12*d*e^5*m^5*x^2-1292*
d*e^5*m^4*x^3+8925*d*e^5*m^3*x^4-27000*d*e^5*m^2*x^5+7*e^6*m^6*x+525*e^6*m^
5*x^2-904*e^6*m^4*x^3+15725*e^6*m^3*x^4-31433*e^6*m^2*x^5+35280*e^6*m*x^6-2
400*d^3*e^3*m^3*x^3+204*d^2*e^4*m^4*x^2-4420*d^2*e^4*m^3*x^3+21000*d^2*e^4*
m^2*x^4-42*d*e^5*m^5*x+252*d*e^5*m^4*x^2-8908*d*e^5*m^3*x^3+25075*d*e^5*m^2
*x^4-32880*d*e^5*m*x^5+6*e^6*m^6+182*e^6*m^5*x+5187*e^6*m^4*x^2-4224*e^6*m^
3*x^3+36448*e^6*m^2*x^4-34646*e^6*m*x^5+14400*e^6*x^6+1020*d^3*e^3*m^3*x^2-
14400*d^3*e^3*m^2*x^3-24*d^2*e^4*m^4*x+3264*d^2*e^4*m^3*x^2-18020*d^2*e^4*m
^2*x^3+30000*d^2*e^4*m*x^4-7*d*e^5*m^5-966*d*e^5*m^4*x+1956*d*e^5*m^3*x^2-2
7268*d*e^5*m^2*x^3+31790*d*e^5*m*x^4-14400*d*e^5*x^5+162*e^6*m^5+1890*e^6*m
^4*x+25599*e^6*m^3*x^2-10180*e^6*m^2*x^3+41004*e^6*m*x^4-14280*e^6*x^5+7200
*d^4*e^2*m^2*x^2-408*d^3*e^3*m^3*x+10200*d^3*e^3*m^2*x^2-26400*d^3*e^3*m*x^
3+42*d^2*e^4*m^4-456*d^2*e^4*m^3*x+16932*d^2*e^4*m^2*x^2-28220*d^2*e^4*m*x^
3+14400*d^2*e^4*x^4-175*d*e^5*m^4-8442*d*e^5*m^3*x+6804*d*e^5*m^2*x^2-36720
*d*e^5*m*x^3+14280*d*e^5*x^4+1770*e^6*m^4+9940*e^6*m^3*x+65352*e^6*m^2*x^2-
11808*e^6*m*x^3+17136*e^6*x^4-2040*d^4*e^2*m^2*x+21600*d^4*e^2*m*x^2+24*d^3
*e^3*m^3-5712*d^3*e^3*m^2*x+23460*d^3*e^3*m*x^2-14400*d^3*e^3*x^3+924*d^2*e
^4*m^3-3000*d^2*e^4*m^2*x+31008*d^2*e^4*m*x^2-14280*d^2*e^4*x^3-1715*d*e^5*
m^3-34314*d*e^5*m^2*x+10128*d*e^5*m*x^2-17136*d*e^5*x^3+9990*e^6*m^3+27503*
e^6*m^2*x+79716*e^6*m*x^2-5040*e^6*x^3-14400*d^5*e*m*x+408*d^4*e^2*m^2-1632
0*d^4*e^2*m*x+14400*d^4*e^2*x^2+432*d^3*e^3*m^2-22440*d^3*e^3*m*x+14280*d^3
*e^3*x^2+7518*d^2*e^4*m^2-7608*d^2*e^4*m*x+17136*d^2*e^4*x^2-8225*d*e^5*m^2
-62076*d*e^5*m*x+5040*d*e^5*x^2+30624*e^6*m^2+36918*e^6*m*x+35280*e^6*x^2+2
040*d^5*e*m-14400*d^5*e*x+5304*d^4*e^2*m-14280*d^4*e^2*x+2568*d^3*e^3*m-171
36*d^3*e^3*x+26796*d^2*e^4*m-5040*d^2*e^4*x-19278*d*e^5*m-35280*d*e^5*x+481
68*e^6*m+17640*e^6*x+14400*d^6+14280*d^5*e+17136*d^4*e^2+5040*d^3*e^3+35280
*d^2*e^4-17640*d*e^5+30240*e^6)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+1
3132*m^2+13068*m+5040)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.44759, size = 3494, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] (6*d*e^6*m^6 + 20*(e^7*m^6 + 21*e^7*m^5 + 175*e^7*m^4 + 735*e^7*m^3 + 1624*
e^7*m^2 + 1764*e^7*m + 720*e^7)*x^7 + 14400*d^7 + 14280*d^6*e + 17136*d^5*e
^2 + 5040*d^4*e^3 + 35280*d^3*e^4 - 17640*d^2*e^5 + 30240*d*e^6 - (14280*e^
7 - (20*d*e^6 - 17*e^7)*m^6 - 2*(150*d*e^6 - 187*e^7)*m^5 - 170*(10*d*e^6 -
 19*e^7)*m^4 - 20*(225*d*e^6 - 697*e^7)*m^3 - (5480*d*e^6 - 31433*e^7)*m^2
- 2*(1200*d*e^6 - 17323*e^7)*m)*x^6 - (7*d^2*e^5 - 162*d*e^6)*m^5 + (17136*
e^7 - 17*(d*e^6 - e^7)*m^6 - (120*d^2*e^5 + 289*d*e^6 - 391*e^7)*m^5 - 3*(4
00*d^2*e^5 + 595*d*e^6 - 1173*e^7)*m^4 - 5*(840*d^2*e^5 + 1003*d*e^6 - 3145
*e^7)*m^3 - 2*(3000*d^2*e^5 + 3179*d*e^6 - 18224*e^7)*m^2 - 12*(240*d^2*e^5
 + 238*d*e^6 - 3417*e^7)*m)*x^5 + (42*d^3*e^4 - 175*d^2*e^5 + 1770*d*e^6)*m
^4 - (5040*e^7 - (17*d*e^6 - 4*e^7)*m^6 - (85*d^2*e^5 + 323*d*e^6 - 96*e^7)
*m^5 - (600*d^3*e^4 + 1105*d^2*e^5 + 2227*d*e^6 - 904*e^7)*m^4 - (3600*d^3*
e^4 + 4505*d^2*e^5 + 6817*d*e^6 - 4224*e^7)*m^3 - 5*(1320*d^3*e^4 + 1411*d^
2*e^5 + 1836*d*e^6 - 2036*e^7)*m^2 - 6*(600*d^3*e^4 + 595*d^2*e^5 + 714*d*e
^6 - 1968*e^7)*m)*x^4 + (24*d^4*e^3 + 924*d^3*e^4 - 1715*d^2*e^5 + 9990*d*e
^6)*m^3 + (35280*e^7 - (4*d*e^6 - 21*e^7)*m^6 - (68*d^2*e^5 + 84*d*e^6 - 52
5*e^7)*m^5 - (340*d^3*e^4 + 1088*d^2*e^5 + 652*d*e^6 - 5187*e^7)*m^4 - (240
0*d^4*e^3 + 3400*d^3*e^4 + 5644*d^2*e^5 + 2268*d*e^6 - 25599*e^7)*m^3 - 4*(
1800*d^4*e^3 + 1955*d^3*e^4 + 2584*d^2*e^5 + 844*d*e^6 - 16338*e^7)*m^2 - 4
*(1200*d^4*e^3 + 1190*d^3*e^4 + 1428*d^2*e^5 + 420*d*e^6 - 19929*e^7)*m)*x^
3 + (408*d^5*e^2 + 432*d^4*e^3 + 7518*d^3*e^4 - 8225*d^2*e^5 + 30624*d*e^6)
*m^2 + (17640*e^7 + 7*(3*d*e^6 + e^7)*m^6 + (12*d^2*e^5 + 483*d*e^6 + 182*e
^7)*m^5 + 3*(68*d^3*e^4 + 76*d^2*e^5 + 1407*d*e^6 + 630*e^7)*m^4 + (1020*d^
4*e^3 + 2856*d^3*e^4 + 1500*d^2*e^5 + 17157*d*e^6 + 9940*e^7)*m^3 + (7200*d
^5*e^2 + 8160*d^4*e^3 + 11220*d^3*e^4 + 3804*d^2*e^5 + 31038*d*e^6 + 27503*
e^7)*m^2 + 6*(1200*d^5*e^2 + 1190*d^4*e^3 + 1428*d^3*e^4 + 420*d^2*e^5 + 29
40*d*e^6 + 6153*e^7)*m)*x^2 + 6*(340*d^6*e + 884*d^5*e^2 + 428*d^4*e^3 + 44
66*d^3*e^4 - 3213*d^2*e^5 + 8028*d*e^6)*m + (30240*e^7 + (7*d*e^6 + 6*e^7)*
m^6 - (42*d^2*e^5 - 175*d*e^6 - 162*e^7)*m^5 - (24*d^3*e^4 + 924*d^2*e^5 -
1715*d*e^6 - 1770*e^7)*m^4 - (408*d^4*e^3 + 432*d^3*e^4 + 7518*d^2*e^5 - 82
25*d*e^6 - 9990*e^7)*m^3 - 6*(340*d^5*e^2 + 884*d^4*e^3 + 428*d^3*e^4 + 446
6*d^2*e^5 - 3213*d*e^6 - 5104*e^7)*m^2 - 24*(600*d^6*e + 595*d^5*e^2 + 714*
d^4*e^3 + 210*d^3*e^4 + 1470*d^2*e^5 - 735*d*e^6 - 2007*e^7)*m)*x)*(e*x + d
)^m/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 131
32*e^7*m^2 + 13068*e^7*m + 5040*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21134, size = 4182, normalized size = 14.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] (20*(x*e + d)^m*m^6*x^7*e^7 + 20*(x*e + d)^m*d*m^6*x^6*e^6 - 17*(x*e + d)^m
*m^6*x^6*e^7 + 420*(x*e + d)^m*m^5*x^7*e^7 - 17*(x*e + d)^m*d*m^6*x^5*e^6 +
 300*(x*e + d)^m*d*m^5*x^6*e^6 - 120*(x*e + d)^m*d^2*m^5*x^5*e^5 + 17*(x*e
+ d)^m*m^6*x^5*e^7 - 374*(x*e + d)^m*m^5*x^6*e^7 + 3500*(x*e + d)^m*m^4*x^7
*e^7 + 17*(x*e + d)^m*d*m^6*x^4*e^6 - 289*(x*e + d)^m*d*m^5*x^5*e^6 + 1700*
(x*e + d)^m*d*m^4*x^6*e^6 + 85*(x*e + d)^m*d^2*m^5*x^4*e^5 - 1200*(x*e + d)
^m*d^2*m^4*x^5*e^5 + 600*(x*e + d)^m*d^3*m^4*x^4*e^4 - 4*(x*e + d)^m*m^6*x^
4*e^7 + 391*(x*e + d)^m*m^5*x^5*e^7 - 3230*(x*e + d)^m*m^4*x^6*e^7 + 14700*
(x*e + d)^m*m^3*x^7*e^7 - 4*(x*e + d)^m*d*m^6*x^3*e^6 + 323*(x*e + d)^m*d*m
^5*x^4*e^6 - 1785*(x*e + d)^m*d*m^4*x^5*e^6 + 4500*(x*e + d)^m*d*m^3*x^6*e^
6 - 68*(x*e + d)^m*d^2*m^5*x^3*e^5 + 1105*(x*e + d)^m*d^2*m^4*x^4*e^5 - 420
0*(x*e + d)^m*d^2*m^3*x^5*e^5 - 340*(x*e + d)^m*d^3*m^4*x^3*e^4 + 3600*(x*e
 + d)^m*d^3*m^3*x^4*e^4 - 2400*(x*e + d)^m*d^4*m^3*x^3*e^3 + 21*(x*e + d)^m
*m^6*x^3*e^7 - 96*(x*e + d)^m*m^5*x^4*e^7 + 3519*(x*e + d)^m*m^4*x^5*e^7 -
13940*(x*e + d)^m*m^3*x^6*e^7 + 32480*(x*e + d)^m*m^2*x^7*e^7 + 21*(x*e + d
)^m*d*m^6*x^2*e^6 - 84*(x*e + d)^m*d*m^5*x^3*e^6 + 2227*(x*e + d)^m*d*m^4*x
^4*e^6 - 5015*(x*e + d)^m*d*m^3*x^5*e^6 + 5480*(x*e + d)^m*d*m^2*x^6*e^6 +
12*(x*e + d)^m*d^2*m^5*x^2*e^5 - 1088*(x*e + d)^m*d^2*m^4*x^3*e^5 + 4505*(x
*e + d)^m*d^2*m^3*x^4*e^5 - 6000*(x*e + d)^m*d^2*m^2*x^5*e^5 + 204*(x*e + d
)^m*d^3*m^4*x^2*e^4 - 3400*(x*e + d)^m*d^3*m^3*x^3*e^4 + 6600*(x*e + d)^m*d
^3*m^2*x^4*e^4 + 1020*(x*e + d)^m*d^4*m^3*x^2*e^3 - 7200*(x*e + d)^m*d^4*m^
```

2*x^3*e^3 + 7200*(x*e + d)^m*d^5*m^2*x^2*e^2 + 7*(x*e + d)^m*m^6*x^2*e^7 +
525*(x*e + d)^m*m^5*x^3*e^7 - 904*(x*e + d)^m*m^4*x^4*e^7 + 15725*(x*e + d)
^m*m^3*x^5*e^7 - 31433*(x*e + d)^m*m^2*x^6*e^7 + 35280*(x*e + d)^m*m*x^7*e^
7 + 7*(x*e + d)^m*d*m^6*x*e^6 + 483*(x*e + d)^m*d*m^5*x^2*e^6 - 652*(x*e +
d)^m*d*m^4*x^3*e^6 + 6817*(x*e + d)^m*d*m^3*x^4*e^6 - 6358*(x*e + d)^m*d*m^
2*x^5*e^6 + 2400*(x*e + d)^m*d*m*x^6*e^6 - 42*(x*e + d)^m*d^2*m^5*x*e^5 + 2
28*(x*e + d)^m*d^2*m^4*x^2*e^5 - 5644*(x*e + d)^m*d^2*m^3*x^3*e^5 + 7055*(x
*e + d)^m*d^2*m^2*x^4*e^5 - 2880*(x*e + d)^m*d^2*m*x^5*e^5 - 24*(x*e + d)^m
*d^3*m^4*x*e^4 + 2856*(x*e + d)^m*d^3*m^3*x^2*e^4 - 7820*(x*e + d)^m*d^3*m^
2*x^3*e^4 + 3600*(x*e + d)^m*d^3*m*x^4*e^4 - 408*(x*e + d)^m*d^4*m^3*x*e^3
+ 8160*(x*e + d)^m*d^4*m^2*x^2*e^3 - 4800*(x*e + d)^m*d^4*m*x^3*e^3 - 2040*
(x*e + d)^m*d^5*m^2*x*e^2 + 7200*(x*e + d)^m*d^5*m*x^2*e^2 - 14400*(x*e + d
)^m*d^6*m*x*e + 6*(x*e + d)^m*m^6*x*e^7 + 182*(x*e + d)^m*m^5*x^2*e^7 + 518
7*(x*e + d)^m*m^4*x^3*e^7 - 4224*(x*e + d)^m*m^3*x^4*e^7 + 36448*(x*e + d)^
m*m^2*x^5*e^7 - 34646*(x*e + d)^m*m*x^6*e^7 + 14400*(x*e + d)^m*x^7*e^7 + 6
*(x*e + d)^m*d*m^6*e^6 + 175*(x*e + d)^m*d*m^5*x*e^6 + 4221*(x*e + d)^m*d*m
^4*x^2*e^6 - 2268*(x*e + d)^m*d*m^3*x^3*e^6 + 9180*(x*e + d)^m*d*m^2*x^4*e^
6 - 2856*(x*e + d)^m*d*m*x^5*e^6 - 7*(x*e + d)^m*d^2*m^5*e^5 - 924*(x*e + d
)^m*d^2*m^4*x*e^5 + 1500*(x*e + d)^m*d^2*m^3*x^2*e^5 - 10336*(x*e + d)^m*d^
2*m^2*x^3*e^5 + 3570*(x*e + d)^m*d^2*m*x^4*e^5 + 42*(x*e + d)^m*d^3*m^4*e^4
 - 432*(x*e + d)^m*d^3*m^3*x*e^4 + 11220*(x*e + d)^m*d^3*m^2*x^2*e^4 - 4760
*(x*e + d)^m*d^3*m*x^3*e^4 + 24*(x*e + d)^m*d^4*m^3*e^3 - 5304*(x*e + d)^m*
d^4*m^2*x*e^3 + 7140*(x*e + d)^m*d^4*m*x^2*e^3 + 408*(x*e + d)^m*d^5*m^2*e^
2 - 14280*(x*e + d)^m*d^5*m*x*e^2 + 2040*(x*e + d)^m*d^6*m*e + 14400*(x*e +
 d)^m*d^7 + 162*(x*e + d)^m*m^5*x*e^7 + 1890*(x*e + d)^m*m^4*x^2*e^7 + 2559
9*(x*e + d)^m*m^3*x^3*e^7 - 10180*(x*e + d)^m*m^2*x^4*e^7 + 41004*(x*e + d)
^m*m*x^5*e^7 - 14280*(x*e + d)^m*x^6*e^7 + 162*(x*e + d)^m*d*m^5*e^6 + 1715
*(x*e + d)^m*d*m^4*x*e^6 + 17157*(x*e + d)^m*d*m^3*x^2*e^6 - 3376*(x*e + d)
^m*d*m^2*x^3*e^6 + 4284*(x*e + d)^m*d*m*x^4*e^6 - 175*(x*e + d)^m*d^2*m^4*e
^5 - 7518*(x*e + d)^m*d^2*m^3*x*e^5 + 3804*(x*e + d)^m*d^2*m^2*x^2*e^5 - 57
12*(x*e + d)^m*d^2*m*x^3*e^5 + 924*(x*e + d)^m*d^3*m^3*e^4 - 2568*(x*e + d)
^m*d^3*m^2*x*e^4 + 8568*(x*e + d)^m*d^3*m*x^2*e^4 + 432*(x*e + d)^m*d^4*m^2
e^3 - 17136(x*e + d)^m*d^4*m*x*e^3 + 5304*(x*e + d)^m*d^5*m*e^2 + 14280*(
x*e + d)^m*d^6*e + 1770*(x*e + d)^m*m^4*x*e^7 + 9940*(x*e + d)^m*m^3*x^2*e^
7 + 65352*(x*e + d)^m*m^2*x^3*e^7 - 11808*(x*e + d)^m*m*x^4*e^7 + 17136*(x*
e + d)^m*x^5*e^7 + 1770*(x*e + d)^m*d*m^4*e^6 + 8225*(x*e + d)^m*d*m^3*x*e^
6 + 31038*(x*e + d)^m*d*m^2*x^2*e^6 - 1680*(x*e + d)^m*d*m*x^3*e^6 - 1715*(
x*e + d)^m*d^2*m^3*e^5 - 26796*(x*e + d)^m*d^2*m^2*x*e^5 + 2520*(x*e + d)^m
*d^2*m*x^2*e^5 + 7518*(x*e + d)^m*d^3*m^2*e^4 - 5040*(x*e + d)^m*d^3*m*x*e^
4 + 2568*(x*e + d)^m*d^4*m*e^3 + 17136*(x*e + d)^m*d^5*e^2 + 9990*(x*e + d)
^m*m^3*x*e^7 + 27503*(x*e + d)^m*m^2*x^2*e^7 + 79716*(x*e + d)^m*m*x^3*e^7
- 5040*(x*e + d)^m*x^4*e^7 + 9990*(x*e + d)^m*d*m^3*e^6 + 19278*(x*e + d)^m
*d*m^2*x*e^6 + 17640*(x*e + d)^m*d*m*x^2*e^6 - 8225*(x*e + d)^m*d^2*m^2*e^5
 - 35280*(x*e + d)^m*d^2*m*x*e^5 + 26796*(x*e + d)^m*d^3*m*e^4 + 5040*(x*e
+ d)^m*d^4*e^3 + 30624*(x*e + d)^m*m^2*x*e^7 + 36918*(x*e + d)^m*m*x^2*e^7

```
+ 35280*(x*e + d)^m*x^3*e^7 + 30624*(x*e + d)^m*d*m^2*e^6 + 17640*(x*e + d)
^m*d*m*x*e^6 - 19278*(x*e + d)^m*d^2*m*e^5 + 35280*(x*e + d)^m*d^3*e^4 + 48
168*(x*e + d)^m*m*x*e^7 + 17640*(x*e + d)^m*x^2*e^7 + 48168*(x*e + d)^m*d*m
*e^6 - 17640*(x*e + d)^m*d^2*e^5 + 30240*(x*e + d)^m*x*e^7 + 30240*(x*e + d
)^m*d*e^6)/(m^7*e^7 + 28*m^6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^
7 + 13132*m^2*e^7 + 13068*m*e^7 + 5040*e^7)
```

3.370 $\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$

Optimal. Leaf size=255

$$\frac{(100d^2+165de+81e^2)(d+ex)^{m+1}}{125e^3(m+1)}-\frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)}+\frac{4(d+ex)^{m+3}}{5e^3(m+3)}-\frac{(-423\sqrt{14}+6412i)(d+ex)^{m+1}\,{}_2F_1(1}{3500(m+1)(5id-(\sqrt{}}$$

```
[Out] ((100*d^2 + 165*d*e + 81*e^2)*(d + e*x)^(1 + m))/(125*e^3*(1 + m)) - ((40*d
 + 33*e)*(d + e*x)^(2 + m))/(25*e^3*(2 + m)) + (4*(d + e*x)^(3 + m))/(5*e^3
*(3 + m)) - ((6412*I - 423*Sqrt[14])*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(3500*((5*I)*d - (I
 + Sqrt[14])*e)*(1 + m)) - ((6412*I + 423*Sqrt[14])*(d + e*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(3
500*((5*I)*d - (I - Sqrt[14])*e)*(1 + m))
```

Rubi [A] time = 0.480741, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1628, 68}

$$\frac{(100d^2+165de+81e^2)(d+ex)^{m+1}}{125e^3(m+1)}-\frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)}+\frac{4(d+ex)^{m+3}}{5e^3(m+3)}-\frac{(-423\sqrt{14}+6412i)(d+ex)^{m+1}\,{}_2F_1(1}{3500(m+1)(5id-(\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```



```
[Out] ((100*d^2 + 165*d*e + 81*e^2)*(d + e*x)^(1 + m))/(125*e^3*(1 + m)) - ((40*d
 + 33*e)*(d + e*x)^(2 + m))/(25*e^3*(2 + m)) + (4*(d + e*x)^(3 + m))/(5*e^3
*(3 + m)) - ((6412*I - 423*Sqrt[14])*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(3500*((5*I)*d - (I
 + Sqrt[14])*e)*(1 + m)) - ((6412*I + 423*Sqrt[14])*(d + e*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(3
500*((5*I)*d - (I - Sqrt[14])*e)*(1 + m))
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int\left(\frac{\left(100d^2+165de+81e^2\right)(d+ex)^m}{125e^2}+\frac{\left(\frac{458}{125}+\frac{423i}{125\sqrt{14}}\right)(d+ex)^m}{2-2i\sqrt{14}+10x}+\frac{\left(\frac{458}{125}-\right.}{2+}\right.$$

$$=\frac{\left(100d^2+165de+81e^2\right)(d+ex)^{1+m}}{125e^3(1+m)}-\frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)}+\frac{4(d+ex)^{3}}{5e^3(3+m}$$

$$=\frac{\left(100d^2+165de+81e^2\right)(d+ex)^{1+m}}{125e^3(1+m)}-\frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)}+\frac{4(d+ex)^{3}}{5e^3(3+m}$$

Mathematica [A] time = 0.672754, size = 221, normalized size = 0.87

$$\frac{(d+ex)^{m+1}\left(\frac{28\left(100d^2+165de+81e^2\right)}{e^3(m+1)}+\frac{2800(d+ex)^2}{e^3(m+3)}-\frac{140(40d+33e)(d+ex)}{e^3(m+2)}-\frac{\left(423\sqrt{14}+6412i\right){}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{(m+1)\left(5id+\left(\sqrt{14}-i\right)e\right)}-\frac{\left(423\sqrt{14}-6412i\right){}_2F}{(m+1)(}\right.}{3500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] ((d + e*x)^(1 + m)*((28*(100*d^2 + 165*d*e + 81*e^2))/(e^3*(1 + m)) - (140*
(40*d + 33*e)*(d + e*x))/(e^3*(2 + m)) + (2800*(d + e*x)^2)/(e^3*(3 + m)) -
 ((6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/
(5*d + (-1 - I*Sqrt[14])*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) - ((-
6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*
d + I*(I + Sqrt[14])*e)])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m))))/3500
```

Maple [F] time = 3.62, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m \left(4x^4-5x^3+3x^2+x+2\right)}{5x^2+2x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{5x^2+2x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{5x^2+2x+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x^4 - 5x^3 + 3x^2 + x + 2)(ex + d)^m}{5x^2 + 2x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

3.371 $\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=377

$$\frac{\left(i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)+80360d^2-5922dem-32144de+19138e^2m+48216e^2\right)(d+}{19600(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)\left(5d^2-2de+3e^2\right)}$$

```
[Out] (4*(d + e*x)^(1 + m))/(25*e*(1 + m)) - ((1367*d - 293*e + (423*d - 1367*e)*
x)*(d + e*x)^(1 + m))/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + ((8
0360*d^2 - 32144*d*e + 48216*e^2 + I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 320
6*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(1
9600*(5*d + I*(I + Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m)) + ((80360*
d^2 - 32144*d*e + 48216*e^2 - I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 3206*m)
+ e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hypergeo
metric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(1960
0*(5*d - (1 + I*Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m))
```

Rubi [A] time = 0.899862, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {1648, 1628, 68}

$$\frac{\left(i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)+80360d^2-5922dem-32144de+19138e^2m+48216e^2\right)(d+}{19600(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)\left(5d^2-2de+3e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```



```
[Out] (4*(d + e*x)^(1 + m))/(25*e*(1 + m)) - ((1367*d - 293*e + (423*d - 1367*e)*
x)*(d + e*x)^(1 + m))/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + ((8
0360*d^2 - 32144*d*e + 48216*e^2 + I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 320
6*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(1
9600*(5*d + I*(I + Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m)) + ((80360*
d^2 - 32144*d*e + 48216*e^2 - I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 3206*m)
+ e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hypergeo
```

```
metric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(1960
0*(5*d - (1 + I*Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m))
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int \frac{(d+ex)^m\left(\frac{2}{25}\left(1845d^2-de(738\right.\right.}{}}{}$$

$$= -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int \left(\frac{224}{25}\left(5d^2-2de+3e^2\right.\right.}{}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} - \frac{\left(80360d^2\right.}{}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\left(80360d^2\right.}{}$$

Mathematica [A] time = 1.75703, size = 441, normalized size = 1.17

$$(d+ex)^{m+1}\left(-\frac{\sqrt{14}\left(\frac{\left(2115d^2+de\left(-846+\left(-6412+423i\sqrt{14}\right)m\right)+e^2\left(1269+\left(98-1367i\sqrt{14}\right)m\right)\right){}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{5id+\left(\sqrt{14}-i\right)e} - \frac{\left(2115d^2-de\left(846+\left(6412+423i\sqrt{14}\right)m\right)+e^2\left(1269+\left(9\right.\right.\right.}{5id-\left(\sqrt{\right.}}\right.}{(m+1)\left(5d^2-2de+3e^2\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] ((d + e*x)^(1 + m)*(3136/(e + e*m) - (28*(d*(1367 + 423*x) - e*(293 + 1367*
x)))/((5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + (56*(287*I + 31*Sqrt[14]
)*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[14])
*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) + (56*(-287*I + 31*Sqrt[14])*
Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)
])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m)) - (Sqrt[14]*(((2115*d^2 + d*e*(-
846 + (-6412 + (423*I)*Sqrt[14])*m) + e^2*(1269 + (98 - (1367*I)*Sqrt[14])*
m))*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[14
])*e)])/((5*I)*d + (-I + Sqrt[14])*e) - ((2115*d^2 - d*e*(846 + (6412 + (42
3*I)*Sqrt[14])*m) + e^2*(1269 + (98 + (1367*I)*Sqrt[14])*m))*Hypergeometric
```

```
2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)])/((5*I)*d -
(I + Sqrt[14])*e)))/((5*d^2 - 2*d*e + 3*e^2)*(1 + m))))/19600
```

Maple [F] time = 2.375, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m \left(4x^4-5x^3+3x^2+x+2\right)}{\left(5x^2+2x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{\left(5x^2+2x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{25x^4+20x^3+34x^2+12x+9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4\,x^4 - 5\,x^3 + 3\,x^2 + x + 2)(ex + d)^m}{(5\,x^2 + 2\,x + 3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

3.372 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3} \, dx$

Optimal. Leaf size=528

$$\frac{2cx\left(c^3\left(-10a^2h-3abg+b^2f\right)-b^3c(15ai+bh)-c^4(3be-2af)+abc^2(25ai+8bh)+2b^5i+6c^5d\right)-b^2c^2\left(39a^2i-5acg+\right.}{2c^4\left(b^2-4ac\right)^2\left(a+bx\right.}$$

```
[Out] -(a*b^3*c*h + b*c^2*(c^2*d + a*c*f - 3*a^2*h) - a*b^4*i - a*b^2*c*(c*g - 4*
a*i) - 2*a*c^2*(c^2*e - a*c*g + a^2*i) + (2*c^5*d - c^4*(b*e + 2*a*f) + c^3
*(b^2*f + 3*a*b*g + 2*a^2*h) - b^5*i + b^3*c*(b*h + 5*a*i) - b*c^2*(b^2*g +
 4*a*b*h + 5*a^2*i))*x)/(2*c^4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (b^5*c*
h + b^3*c^2*(c*f - 8*a*h) + 2*b*c^3*(3*c^2*d + a*c*f + 11*a^2*h) - b^6*i -
b^4*c*(c*g - 11*a*i) - 16*a^2*c^3*(c*g - 2*a*i) - b^2*c^2*(3*c^2*e - 5*a*c*
g + 39*a^2*i) + 2*c*(6*c^5*d - c^4*(3*b*e - 2*a*f) + c^3*(b^2*f - 3*a*b*g -
 10*a^2*h) + 2*b^5*i - b^3*c*(b*h + 15*a*i) + a*b*c^2*(8*b*h + 25*a*i))*x)/
(2*c^4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - ((12*c^5*d - c^4*(6*b*e - 4*a*f
) + 2*c^3*(b^2*f - 3*a*b*g + 6*a^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2
*i)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + (i*
Log[a + b*x + c*x^2])/(2*c^3)
```

Rubi [A] time = 1.31158, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1660, 634, 618, 206, 628}

$$\frac{2cx\left(c^3\left(-10a^2h-3abg+b^2f\right)-b^3c(15ai+bh)-c^4(3be-2af)+abc^2(25ai+8bh)+2b^5i+6c^5d\right)-b^2c^2\left(39a^2i-5acg+\right.}{2c^4\left(b^2-4ac\right)^2\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(a*b^3*c*h + b*c^2*(c^2*d + a*c*f - 3*a^2*h) - a*b^4*i - a*b^2*c*(c*g - 4*
a*i) - 2*a*c^2*(c^2*e - a*c*g + a^2*i) + (2*c^5*d - c^4*(b*e + 2*a*f) + c^3
*(b^2*f + 3*a*b*g + 2*a^2*h) - b^5*i + b^3*c*(b*h + 5*a*i) - b*c^2*(b^2*g +
 4*a*b*h + 5*a^2*i))*x)/(2*c^4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (b^5*c*
h + b^3*c^2*(c*f - 8*a*h) + 2*b*c^3*(3*c^2*d + a*c*f + 11*a^2*h) - b^6*i -
b^4*c*(c*g - 11*a*i) - 16*a^2*c^3*(c*g - 2*a*i) - b^2*c^2*(3*c^2*e - 5*a*c*
g + 39*a^2*i) + 2*c*(6*c^5*d - c^4*(3*b*e - 2*a*f) + c^3*(b^2*f - 3*a*b*g -
```

```
 10*a^2*h) + 2*b^5*i - b^3*c*(b*h + 15*a*i) + a*b*c^2*(8*b*h + 25*a*i))*x)/
(2*c^4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - ((12*c^5*d - c^4*(6*b*e - 4*a*f
) + 2*c^3*(b^2*f - 3*a*b*g + 6*a^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2
*i)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + (i*
Log[a + b*x + c*x^2])/(2*c^3)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+372x^5}{\left(a+bx+cx^2\right)^3}\,dx = \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3f+b^2c^2\right.}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3f+b^2c^2\right.}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3f+b^2c^2\right.}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3f+b^2c^2\right.}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3f+b^2c^2\right.}{}$$

Mathematica [A] time = 1.31935, size = 488, normalized size = 0.92

$$\frac{\frac{b^2c\left(-4a^2i+ac(g+4hx)-c^2fx\right)+bc^2\left(a^2(3h+5ix)-ac(f+3gx)+c^2(ex-d)\right)+2c^2\left(-a^2c(g+hx)+a^3i+ac^2(e+fx)-c^3dx\right)+b^3c(cgx-a(h+5ix))+b^4(ai-chx)+b^5ix}{\left(b^2-4ac\right)(a+x(b+cx))^2}+\frac{b^2c^2\left(-39\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((b^5*i*x + b^4*(a*i - c*h*x) + 2*c^2*(a^3*i - c^3*d*x + a*c^2*(e + f*x) -
a^2*c*(g + h*x)) + b^2*c*(-4*a^2*i - c^2*f*x + a*c*(g + 4*h*x)) + b^3*c*(c*
g*x - a*(h + 5*i*x)) + b*c^2*(c^2*(-d + e*x) - a*c*(f + 3*g*x) + a^2*(3*h +
 5*i*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (-(b^6*i) + b^5*c*(h + 4*i*
x) + b^3*c^2*(c*f - 8*a*h - 30*a*i*x) - b^4*c*(-11*a*i + c*(g + 2*h*x)) + 4
*c^3*(8*a^3*i + 3*c^3*d*x + a*c^2*f*x - a^2*c*(4*g + 5*h*x)) + b^2*c^2*(-39
*a^2*i + c^2*(-3*e + 2*f*x) + a*c*(5*g + 16*h*x)) + 2*b*c^3*(3*c^2*(d - e*x
) + a*c*(f - 3*g*x) + a^2*(11*h + 25*i*x)))/((b^2 - 4*a*c)^2*(a + x*(b + c*
x))) + (2*c*(12*c^5*d + c^4*(-6*b*e + 4*a*f) + 2*c^3*(b^2*f - 3*a*b*g + 6*a
^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2*i)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*i*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [B] time = 0.191, size = 1244, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x)
```

```
[Out] ((25*a^2*b*c^2*i-10*a^2*c^3*h-15*a*b^3*c*i+8*a*b^2*c^2*h-3*a*b*c^3*g+2*a*c^
4*f+2*b^5*i-b^4*c*h+b^2*c^3*f-3*b*c^4*e+6*c^5*d)/c^2/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^3+1/2*(32*a^3*c^3*i+11*a^2*b^2*c^2*i+2*a^2*b*c^3*h-16*a^2*c^4*g-19*a
*b^4*c*i+8*a*b^3*c^2*h-a*b^2*c^3*g+6*a*b*c^4*f+3*b^6*i-b^5*c*h-b^4*c^2*g+3*
b^3*c^3*f-9*b^2*c^4*e+18*b*c^5*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^2+(31*a^
3*b*c^2*i-6*a^3*c^3*h-22*a^2*b^3*c*i+10*a^2*b^2*c^2*h-5*a^2*b*c^3*g-2*a^2*c
^4*f+3*a*b^5*i-a*b^4*c*h-a*b^3*c^2*g+5*a*b^2*c^3*f-5*a*b*c^4*e+10*a*c^5*d-b
^3*c^3*e+2*b^2*c^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+1/2/c^3*(24*a^4*c^2*
i-21*a^3*b^2*c*i+10*a^3*b*c^2*h-8*a^3*c^3*g+3*a^2*b^4*i-a^2*b^3*c*h-a^2*b^2
*c^2*g+6*a^2*b*c^3*f-8*a^2*c^4*e-a*b^2*c^3*e+10*a*b*c^4*d-b^3*c^3*d)/(16*a^
2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+8/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x
^2+b*x+a)*a^2*i-4/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a*b^2*i+1/
2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^4*i-30/c/(16*a^2*c^2-8*a
*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*i+1
2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*a^2*h+10/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*i-6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*g+4*c/(16*a^2*c^2-8*a*b^2*c+
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*f+2/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*
f-6*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b*e+12*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*d-1/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*i
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84246, size = 7274, normalized size = 13.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="fri
cas")
```

```
[Out] [1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^4*c^4 - 2
*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c^2 - 12*a
*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^2 + 85*a^
2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d - 9*(b^4*c^
4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (b^6*c^2 -
 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*c^2 + 30*
a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^
3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - (12*a^2*c^5*d - 6*a^2*b*c^4*e - 6*a^3*b*c
^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a^2*c^5*h +
2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*i)*x^4 +
2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2*(b^3*c^4 +
 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3 + (12*(b^2*c
^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a*b^2*c^4 + 4*
a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3 + 2*a^3*c^4)*h
 - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2 + 2*(a^2*b^2*c^
3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*i + 2*(12*a*b*c^
5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2*(a*b^3*c^3 + 2*a
^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*x)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/
(c*x^2 + b*x + a)) - (b^5*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d - (a*b^4*c^3
 + 4*a^2*b^2*c^4 - 32*a^3*c^5)*e + 6*(a^2*b^3*c^3 - 4*a^3*b*c^4)*f - (a^2*b
^4*c^2 + 4*a^3*b^2*c^3 - 32*a^4*c^4)*g - (a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a
^4*b*c^3)*h + 3*(a^2*b^6 - 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*i +
2*(2*(b^4*c^4 + a*b^2*c^5 - 20*a^2*c^6)*d - (b^5*c^3 + a*b^3*c^4 - 20*a^2*b
*c^5)*e + (5*a*b^4*c^3 - 22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b
^3*c^3 - 20*a^3*b*c^4)*g - (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*
a^4*c^4)*h + (3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)*
x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2
*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*
```

a^4*b^2*c^2 - 64*a^5*c^3)*i)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x), 1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c^2 - 12*a*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d - 9*(b^4*c^4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (b^6*c^2 - 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*c^2 + 30*a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - 2*(12*a^2*c^5*d - 6*a^2*b*c^4*e - 6*a^3*b*c^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a^2*c^5*h + 2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*i)*x^4 + 2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2*(b^3*c^4 + 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3 + (12*(b^2*c^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a*b^2*c^4 + 4*a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3 + 2*a^3*c^4)*h - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2 + 2*(a^2*b^2*c^3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*i + 2*(12*a*b*c^5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2*(a*b^3*c^3 + 2*a^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d - (a*b^4*c^3 + 4*a^2*b^2*c^4 - 32*a^3*c^5)*e + 6*(a^2*b^3*c^3 - 4*a^3*b*c^4)*f - (a^2*b^4*c^2 + 4*a^3*b^2*c^3 - 32*a^4*c^4)*g - (a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a^4*b*c^3)*h + 3*(a^2*b^6 - 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*i + 2*(2*(b^4*c^4 + a*b^2*c^5 - 20*a^2*c^6)*d - (b^5*c^3 + a*b^3*c^4 - 20*a^2*b*c^5)*e + (5*a*b^4*c^3 - 22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b^3*c^3 - 20*a^3*b*c^4)*g - (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*a^4*c^4)*h + (3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*i)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15391, size = 887, normalized size = 1.68

$$\frac{\left(12\,c^5di + 2\,b^2c^3fi + 4\,ac^4fi - 6\,abc^3gi + 12\,a^2c^3hi - 6\,bc^4ie + b^5 - 10\,ab^3c + 30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4c^3i - 8\,ab^2c^4i + 16\,a^2c^5i\right)\sqrt{-b^2+4\,ac}} + \frac{i\log\left(cx^2 + \right.}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="gia
c")
```

```
[Out] (12*c^5*d*i + 2*b^2*c^3*f*i + 4*a*c^4*f*i - 6*a*b*c^3*g*i + 12*a^2*c^3*h*i
- 6*b*c^4*i*e + b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b
^2 + 4*a*c))/((b^4*c^3*i - 8*a*b^2*c^4*i + 16*a^2*c^5*i)*sqrt(-b^2 + 4*a*c)
) + 1/2*i*log(c*x^2 + b*x + a)/c^3 - 1/2*(b^3*c^3*d - 10*a*b*c^4*d - 6*a^2*
b*c^3*f + a^2*b^2*c^2*g + 8*a^3*c^3*g + a^2*b^3*c*h - 10*a^3*b*c^2*h - 3*a^
2*b^4*i + 21*a^3*b^2*c*i - 24*a^4*c^2*i + a*b^2*c^3*e + 8*a^2*c^4*e - 2*(6*
c^6*d + b^2*c^4*f + 2*a*c^5*f - 3*a*b*c^4*g - b^4*c^2*h + 8*a*b^2*c^3*h - 1
0*a^2*c^4*h + 2*b^5*c*i - 15*a*b^3*c^2*i + 25*a^2*b*c^3*i - 3*b*c^5*e)*x^3
- (18*b*c^5*d + 3*b^3*c^3*f + 6*a*b*c^4*f - b^4*c^2*g - a*b^2*c^3*g - 16*a^
2*c^4*g - b^5*c*h + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h + 3*b^6*i - 19*a*b^4*c*i
+ 11*a^2*b^2*c^2*i + 32*a^3*c^3*i - 9*b^2*c^4*e)*x^2 - 2*(2*b^2*c^4*d + 10*
a*c^5*d + 5*a*b^2*c^3*f - 2*a^2*c^4*f - a*b^3*c^2*g - 5*a^2*b*c^3*g - a*b^4
*c*h + 10*a^2*b^2*c^2*h - 6*a^3*c^3*h + 3*a*b^5*i - 22*a^2*b^3*c*i + 31*a^3
*b*c^2*i - b^3*c^3*e - 5*a*b*c^4*e)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2
*c^3)
```

3.373 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$

Optimal. Leaf size=765

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^6\left(2a^2h+3abg+b^2f\right)-c^5\left(5a^2bj+2a^3k+4ab^2h+b^3g\right)+c^4\left(9a^2b^2k+7a^3bl+2a^4m+5ab^3j+b\right.\right.}{c^8\sqrt{b^2}}$$

```
[Out] ((c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b^4*c*(
b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a*b^2*k
 + 3*a^2*b*l + a^3*m))*x)/c^7 + ((c^5*g - c^4*(b*h + a*j) + c^3*(b^2*j + 2*
a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b*l + 3*a
^2*m))*x^2)/(2*c^6) + ((c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l + 3*a*
m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3)/(3*c^5) + ((c^3*j - c^2*(b*k + a*l
) - b^3*m + b*c*(b*l + 2*a*m))*x^4)/(4*c^4) + ((c^2*k + b^2*m - c*(b*l + a*
m))*x^5)/(5*c^3) + ((c*l - b*m)*x^6)/(6*c^2) + (m*x^7)/(7*c) - ((2*c^8*d -
c^7*(b*e + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*
h + 5*a^2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7
*a*b*l + 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) +
c^4*(b^4*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(c^8*Sqrt[b^2 - 4*a*c]) + ((c^7*e - c^6*(b*f + a*
g) + c^5*(b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a
^3*l) - b^7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m)
+ b*c^3*(b^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + b*x + c*x^2])/(2
*c^8)
```

Rubi [A] time = 5.82519, antiderivative size = 765, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^6\left(2a^2h+3abg+b^2f\right)-c^5\left(5a^2bj+2a^3k+4ab^2h+b^3g\right)+c^4\left(9a^2b^2k+7a^3bl+2a^4m+5ab^3j+b\right.\right.}{c^8\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x + c*x^2),x]
```


```
[Out] ((c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b^4*c*(
b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a*b^2*k
 + 3*a^2*b*l + a^3*m))*x)/c^7 + ((c^5*g - c^4*(b*h + a*j) + c^3*(b^2*j + 2*
```

```
a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b*l + 3*a
^2*m))*x^2)/(2*c^6) + ((c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l + 3*a*
m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3)/(3*c^5) + ((c^3*j - c^2*(b*k + a*l
) - b^3*m + b*c*(b*l + 2*a*m))*x^4)/(4*c^4) + ((c^2*k + b^2*m - c*(b*l + a*
m))*x^5)/(5*c^3) + ((c*l - b*m)*x^6)/(6*c^2) + (m*x^7)/(7*c) - ((2*c^8*d -
c^7*(b*e + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*
h + 5*a^2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7
*a*b*l + 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) +
c^4*(b^4*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(c^8*Sqrt[b^2 - 4*a*c]) + ((c^7*e - c^6*(b*f + a*
g) + c^5*(b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a
^3*l) - b^7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m)
+ b*c^3*(b^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + b*x + c*x^2])/(2
*c^8)
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{a + bx + cx^2}\,dx = \int \Bigg(\frac{c^6 f - c^5(bg + ah) + c^4\left(b^2h + 2abj + a^2k\right) + b^6m - b^4c(b}{}$$

$$= \frac{\left(c^6 f - c^5(bg + ah) + c^4\left(b^2h + 2abj + a^2k\right) + b^6m - b^4c(bl +\right.}{}$$

$$= \frac{\left(c^6 f - c^5(bg + ah) + c^4\left(b^2h + 2abj + a^2k\right) + b^6m - b^4c(bl +\right.}{}$$

$$= \frac{\left(c^6 f - c^5(bg + ah) + c^4\left(b^2h + 2abj + a^2k\right) + b^6m - b^4c(bl +\right.}{}$$

$$= \frac{\left(c^6 f - c^5(bg + ah) + c^4\left(b^2h + 2abj + a^2k\right) + b^6m - b^4c(bl +\right.}{}$$

Mathematica [A] time = 0.778456, size = 754, normalized size = 0.99

$$\frac{\frac{420\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c^6\left(2a^2h+3abg+b^2f\right)-c^5\left(5a^2bj+2a^3k+4ab^2h+b^3g\right)+c^4\left(9a^2b^2k+7a^3bl+2a^4m+5ab^3j+b^4h\right)-b^2c^3\left(14a^2bl+16a^3m+6ab^2k+b^3j\right)+b^4c^2\left(20a^2m+\right.\right.}{\sqrt{4ac-b^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x + c*x^2),x]
```

```
[Out] (420*c*(c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b
^4*c*(b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a
*b^2*k + 3*a^2*b*l + a^3*m))*x + 210*c^2*(c^5*g - c^4*(b*h + a*j) + c^3*(b^
2*j + 2*a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b
*l + 3*a^2*m))*x^2 + 140*c^3*(c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l
+ 3*a*m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3 + 105*c^4*(c^3*j - c^2*(b*k +
 a*l) - b^3*m + b*c*(b*l + 2*a*m))*x^4 + 84*c^5*(c^2*k + b^2*m - c*(b*l + a
*m))*x^5 + 70*c^6*(c*l - b*m)*x^6 + 60*c^7*m*x^7 + (420*(2*c^8*d - c^7*(b*e
 + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*h + 5*a^
2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7*a*b*l +
 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) + c^4*(b^4
*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTan[(b + 2*c*x)/Sqr
```

```
t[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 210*(c^7*e - c^6*(b*f + a*g) + c^5*(
b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a^3*l) - b^
7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m) + b*c^3*(b
^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + x*(b + c*x)])/(420*c^8)
```

Maple [B] time = 0.192, size = 1960, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] -1/2/c^2*x^2*a*j+1/2/c^3*x^2*a^2*l+1/2/c^5*ln(c*x^2+b*x+a)*b^4*j-1/2/c^4*ln
(c*x^2+b*x+a)*b^3*h-1/2/c^8*ln(c*x^2+b*x+a)*b^7*m+1/2/c^7*ln(c*x^2+b*x+a)*b
^6*l-1/2/c^4*ln(c*x^2+b*x+a)*a^3*l+1/2/c^3*ln(c*x^2+b*x+a)*b^2*g-1/2/c^2*ln
(c*x^2+b*x+a)*b*f+1/2/c^3*ln(c*x^2+b*x+a)*a^2*j-1/2/c^2*ln(c*x^2+b*x+a)*a*g
-1/2/c^6*x^2*b^5*m+1/c^5*b^4*k*x-1/3/c^2*x^3*a*k-1/5/c^2*x^5*b*l+1/3/c^3*x^
3*a^2*m-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*g+1
/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*f-2/c^3/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*k-1/c^5/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*j+1/c^4/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*h-1/c^4*a^3*m*x+1/c^3*a^2*k*x-1/c^7/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^7*l-1/c/(4*a*c-b^2)^(1
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e+1/c^6/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6*k+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*a^2*h-3/2/c^4*ln(c*x^2+b*x+a)*a*b^2*j-5/2/c^6*ln(c*x^2+
b*x+a)*a*b^4*l+2/c^5*ln(c*x^2+b*x+a)*a*b^3*k-2/c/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*a*f-1/c^2*b*g*x-1/c^2*a*h*x-1/c^6*b^5*l*x+3/c^5
*ln(c*x^2+b*x+a)*a^2*b^2*l+2/c^5*ln(c*x^2+b*x+a)*a^3*b*m-1/4/c^2*x^4*b*k-1/
c^4*b^3*j*x+1/c^3*b^2*h*x-1/4/c^2*x^4*a*l-1/4/c^4*x^4*b^3*m+1/4/c^3*x^4*b^2
*l-1/2/c^6*ln(c*x^2+b*x+a)*b^5*k+2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))*a^4*m+1/c^8/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2))*b^8*m-3/2/c^4*ln(c*x^2+b*x+a)*a^2*b*k-5/c^6*ln(c*x^2+b*x+a)*a^2*b^
3*m+1/c^3*x^2*a*b*k+6/c^5*a^2*b^2*m*x+1/c^3*ln(c*x^2+b*x+a)*a*b*h-3/2/c^4*x
^2*a*b^2*l-3/2/c^4*x^2*a^2*b*m+1/2/c*x^2*g+1/3/c*x^3*h+1/5/c*x^5*k+1/4/c*x^
4*j+1/6/c*x^6*l+2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d+1
/2/c*ln(c*x^2+b*x+a)*e+1/c*f*x-1/3/c^2*x^3*b*j-1/6/c^2*x^6*b*m-1/5/c^2*x^5*
a*m-1/3/c^4*x^3*b^3*l+1/3/c^3*x^3*b^2*k+1/5/c^3*x^5*b^2*m+1/3/c^5*x^3*b^4*m
+1/c^7*b^6*m*x+1/2/c^5*x^2*b^4*l-1/2/c^4*x^2*b^3*k+1/2/c^3*x^2*b^2*j-1/2/c^
2*x^2*b*h-3/c^4*a^2*b*l*x-5/c^6*a*b^4*m*x+2/c^5*x^2*a*b^3*m-1/c^4*x^3*a*b^2
*m+2/3/c^3*x^3*a*b*l+1/2/c^3*x^4*a*b*m+4/c^5*a*b^3*l*x-3/c^4*a*b^2*k*x+2/c^
```

```
3*a*b*j*x+3/c^7*ln(c*x^2+b*x+a)*a*b^5*m+1/7*m*x^7/c+7/c^6/(4*a*c-b^2)^(1/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^5*l-6/c^5/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4*k-14/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*a^2*b^3*l-8/c^7/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*a*b^6*m+5/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*a*b^3*j-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*a*b^2*h+9/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^
2*b^2*k-5/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*j
+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*g-16/c^5/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*b^2*m+7/c^4/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*b*l+20/c^6/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^4*m
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.61538, size = 5392, normalized size = 7.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="fricas")
```

```
[Out] [1/420*(60*(b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6
 - 4*a*b*c^7)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l
+ (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j -
 (b^3*c^6 - 4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4
 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^
6 - 4*a*b*c^7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b
^3*c^5 + 8*a^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c
```

```
^6)*m)*x^3 + 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^
5 - 5*a*b^2*c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k +
(b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5
*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 + 210*(2*c^8*d - b*c^7*e + (b^
2*c^6 - 2*a*c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a
^2*c^6)*h - (b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^
3 + 9*a^2*b^2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 -
7*a^3*b*c^4)*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4
*c^4)*m)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 420*((b^2*c^7 - 4*a*c^8)*f - (
b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*h - (b^5*c^4 -
 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4
*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*l + (
b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*m)*x + 2
10*((b^2*c^7 - 4*a*c^8)*e - (b^3*c^6 - 4*a*b*c^7)*f + (b^4*c^5 - 5*a*b^2*c^
6 + 4*a^2*c^7)*g - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*h + (b^6*c^3 - 7*a
*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*
b^3*c^4 - 12*a^3*b*c^5)*k + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*
b^2*c^4 + 4*a^4*c^5)*l - (b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 44*a^3*b^3*c^
3 + 16*a^4*b*c^4)*m)*log(c*x^2 + b*x + a))/(b^2*c^8 - 4*a*c^9), 1/420*(60*(
b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6 - 4*a*b*c^7
)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l + (b^4*c^5 -
 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j - (b^3*c^6 -
4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4 - 6*a*b^3*c
^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^6 - 4*a*b*c^
7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b^3*c^5 + 8*a
^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*m)*x^3 +
 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^5 - 5*a*b^2*
c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k + (b^6*c^3 - 7
*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^
2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 - 420*(2*c^8*d - b*c^7*e + (b^2*c^6 - 2*a*
c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*h -
(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^3 + 9*a^2*b^
2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)
*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*m)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 420
*((b^2*c^7 - 4*a*c^8)*f - (b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6
+ 4*a^2*c^7)*h - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b
^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^
3*c^4 - 12*a^3*b*c^5)*l + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^
2*c^4 + 4*a^4*c^5)*m)*x + 210*((b^2*c^7 - 4*a*c^8)*e - (b^3*c^6 - 4*a*b*c^7
)*f + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*g - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^
2*b*c^6)*h + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j - (b^7*
c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*k + (b^8*c - 9*a*b^6*c^2
 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*l - (b^9 - 10*a*b^7*c + 34*
```

```
a^2*b^5*c^2 - 44*a^3*b^3*c^3 + 16*a^4*b*c^4)*m)*log(c*x^2 + b*x + a))/(b^2*
c^8 - 4*a*c^9)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+
b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29237, size = 1326, normalized size = 1.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="giac")
```

```
[Out] 1/420*(60*c^6*m*x^7 + 70*c^6*l*x^6 - 70*b*c^5*m*x^6 + 84*c^6*k*x^5 - 84*b*c
^5*l*x^5 + 84*b^2*c^4*m*x^5 - 84*a*c^5*m*x^5 + 105*c^6*j*x^4 - 105*b*c^5*k*
x^4 + 105*b^2*c^4*l*x^4 - 105*a*c^5*l*x^4 - 105*b^3*c^3*m*x^4 + 210*a*b*c^4
*m*x^4 + 140*c^6*h*x^3 - 140*b*c^5*j*x^3 + 140*b^2*c^4*k*x^3 - 140*a*c^5*k*
x^3 - 140*b^3*c^3*l*x^3 + 280*a*b*c^4*l*x^3 + 140*b^4*c^2*m*x^3 - 420*a*b^2
*c^3*m*x^3 + 140*a^2*c^4*m*x^3 + 210*c^6*g*x^2 - 210*b*c^5*h*x^2 + 210*b^2*
c^4*j*x^2 - 210*a*c^5*j*x^2 - 210*b^3*c^3*k*x^2 + 420*a*b*c^4*k*x^2 + 210*b
^4*c^2*l*x^2 - 630*a*b^2*c^3*l*x^2 + 210*a^2*c^4*l*x^2 - 210*b^5*c*m*x^2 +
840*a*b^3*c^2*m*x^2 - 630*a^2*b*c^3*m*x^2 + 420*c^6*f*x - 420*b*c^5*g*x + 4
20*b^2*c^4*h*x - 420*a*c^5*h*x - 420*b^3*c^3*j*x + 840*a*b*c^4*j*x + 420*b^
4*c^2*k*x - 1260*a*b^2*c^3*k*x + 420*a^2*c^4*k*x - 420*b^5*c*l*x + 1680*a*b
^3*c^2*l*x - 1260*a^2*b*c^3*l*x + 420*b^6*m*x - 2100*a*b^4*c*m*x + 2520*a^2
*b^2*c^2*m*x - 420*a^3*c^3*m*x)/c^7 - 1/2*(b*c^6*f - b^2*c^5*g + a*c^6*g +
b^3*c^4*h - 2*a*b*c^5*h - b^4*c^3*j + 3*a*b^2*c^4*j - a^2*c^5*j + b^5*c^2*k
 - 4*a*b^3*c^3*k + 3*a^2*b*c^4*k - b^6*c*l + 5*a*b^4*c^2*l - 6*a^2*b^2*c^3*
l + a^3*c^4*l + b^7*m - 6*a*b^5*c*m + 10*a^2*b^3*c^2*m - 4*a^3*b*c^3*m - c^
```

```
7*e)*log(c*x^2 + b*x + a)/c^8 + (2*c^8*d + b^2*c^6*f - 2*a*c^7*f - b^3*c^5*
g + 3*a*b*c^6*g + b^4*c^4*h - 4*a*b^2*c^5*h + 2*a^2*c^6*h - b^5*c^3*j + 5*a
*b^3*c^4*j - 5*a^2*b*c^5*j + b^6*c^2*k - 6*a*b^4*c^3*k + 9*a^2*b^2*c^4*k -
2*a^3*c^5*k - b^7*c*l + 7*a*b^5*c^2*l - 14*a^2*b^3*c^3*l + 7*a^3*b*c^4*l +
b^8*m - 8*a*b^6*c*m + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 2*a^4*c^4*m - b
*c^7*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^8)
```

3.374 $\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx$

Optimal. Leaf size=208

$$-\frac{343}{50}\left(5x^2+2x+3\right)^{3/2}x^7-\frac{50519\left(5x^2+2x+3\right)^{3/2}x^6}{2250}+\frac{190939\left(5x^2+2x+3\right)^{3/2}x^5}{3000}-\frac{888751\left(5x^2+2x+3\right)^{3/2}x^4}{105000}-$$

```
[Out] (-77159983*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/31250000 - (1968340667*(3 + 2*x
 + 5*x^2)^(3/2))/131250000 + (1045360143*x*(3 + 2*x + 5*x^2)^(3/2))/4375000
0 + (98060877*x^2*(3 + 2*x + 5*x^2)^(3/2))/4375000 - (90960857*x^3*(3 + 2*x
 + 5*x^2)^(3/2))/1575000 - (888751*x^4*(3 + 2*x + 5*x^2)^(3/2))/105000 + (1
90939*x^5*(3 + 2*x + 5*x^2)^(3/2))/3000 - (50519*x^6*(3 + 2*x + 5*x^2)^(3/2
))/2250 - (343*x^7*(3 + 2*x + 5*x^2)^(3/2))/50 - (540119881*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(15625000*Sqrt[5])
```

Rubi [A] time = 0.352348, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{343}{50}\left(5x^2+2x+3\right)^{3/2}x^7-\frac{50519\left(5x^2+2x+3\right)^{3/2}x^6}{2250}+\frac{190939\left(5x^2+2x+3\right)^{3/2}x^5}{3000}-\frac{888751\left(5x^2+2x+3\right)^{3/2}x^4}{105000}-$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```


```
[Out] (-77159983*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/31250000 - (1968340667*(3 + 2*x
 + 5*x^2)^(3/2))/131250000 + (1045360143*x*(3 + 2*x + 5*x^2)^(3/2))/4375000
0 + (98060877*x^2*(3 + 2*x + 5*x^2)^(3/2))/4375000 - (90960857*x^3*(3 + 2*x
 + 5*x^2)^(3/2))/1575000 - (888751*x^4*(3 + 2*x + 5*x^2)^(3/2))/105000 + (1
90939*x^5*(3 + 2*x + 5*x^2)^(3/2))/3000 - (50519*x^6*(3 + 2*x + 5*x^2)^(3/2
))/2250 - (343*x^7*(3 + 2*x + 5*x^2)^(3/2))/50 - (540119881*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(15625000*Sqrt[5])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
```

```
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx = -\frac{343}{50} x^7 \left(3+2x+5x^2\right)^{3/2} + \frac{1}{50} \int \sqrt{3+2x+5x^2} \left(100+1450x+\right.$$

$$= -\frac{50519x^6 \left(3+2x+5x^2\right)^{3/2}}{2250} - \frac{343}{50} x^7 \left(3+2x+5x^2\right)^{3/2} + \frac{\int \sqrt{3+}}{}$$

$$= \frac{190939x^5 \left(3+2x+5x^2\right)^{3/2}}{3000} - \frac{50519x^6 \left(3+2x+5x^2\right)^{3/2}}{2250} - \frac{343}{50} x^7$$

$$= -\frac{888751x^4 \left(3+2x+5x^2\right)^{3/2}}{105000} + \frac{190939x^5 \left(3+2x+5x^2\right)^{3/2}}{3000} - \frac{505}{}$$

$$= -\frac{90960857x^3 \left(3+2x+5x^2\right)^{3/2}}{1575000} - \frac{888751x^4 \left(3+2x+5x^2\right)^{3/2}}{105000} + $$

$$= \frac{98060877x^2 \left(3+2x+5x^2\right)^{3/2}}{4375000} - \frac{90960857x^3 \left(3+2x+5x^2\right)^{3/2}}{1575000} - $$

$$= \frac{1045360143x \left(3+2x+5x^2\right)^{3/2}}{43750000} + \frac{98060877x^2 \left(3+2x+5x^2\right)^{3/2}}{4375000}$$

$$= -\frac{1968340667 \left(3+2x+5x^2\right)^{3/2}}{131250000} + \frac{1045360143x \left(3+2x+5x^2\right)^{3/2}}{43750000}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667 \left(3+2x+5x^2\right)^{3}}{131250000}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667 \left(3+2x+5x^2\right)^{3}}{131250000}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667 \left(3+2x+5x^2\right)^{3}}{131250000}$$

Mathematica [A] time = 0.315134, size = 85, normalized size = 0.41

$$\frac{-5\sqrt{5x^2+2x+3}\left(67528125000x^9 + 248031875000x^8 - 497593468750x^7 - 34674656250x^6 + 225922362500x^5 + 567\right.}{984375}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(93436408944 - 57768004650*x - 78839046795*x^2 +
17642392275*x^3 + 56757413000*x^4 + 225922362500*x^5 - 34674656250*x^6 - 49
```

```
7593468750*x^7 + 248031875000*x^8 + 67528125000*x^9) - 68055105006*Sqrt[5]*
ArcSinh[(1 + 5*x)/Sqrt[14]])/9843750000
```

Maple [A] time = 0.075, size = 166, normalized size = 0.8

$$\frac{190939\,x^5}{3000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}-\frac{888751\,x^4}{105000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}-\frac{90960857\,x^3}{1575000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}+\frac{98060877\,x^2}{4375000}\left(5\,x^2+2\,x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] 190939/3000*x^5*(5*x^2+2*x+3)^(3/2)-888751/105000*x^4*(5*x^2+2*x+3)^(3/2)-9
0960857/1575000*x^3*(5*x^2+2*x+3)^(3/2)+98060877/4375000*x^2*(5*x^2+2*x+3)^
(3/2)+1045360143/43750000*x*(5*x^2+2*x+3)^(3/2)-77159983/62500000*(10*x+2)*
(5*x^2+2*x+3)^(1/2)-540119881/78125000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5
))-1968340667/131250000*(5*x^2+2*x+3)^(3/2)-343/50*x^7*(5*x^2+2*x+3)^(3/2)-
50519/2250*x^6*(5*x^2+2*x+3)^(3/2)
```

Maxima [A] time = 1.49426, size = 239, normalized size = 1.15

$$-\frac{343}{50}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^7-\frac{50519}{2250}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^6+\frac{190939}{3000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^5-\frac{888751}{105000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^4-\frac{909}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/50*(5*x^2 + 2*x + 3)^(3/2)*x^7 - 50519/2250*(5*x^2 + 2*x + 3)^(3/2)*x^
6 + 190939/3000*(5*x^2 + 2*x + 3)^(3/2)*x^5 - 888751/105000*(5*x^2 + 2*x +
3)^(3/2)*x^4 - 90960857/1575000*(5*x^2 + 2*x + 3)^(3/2)*x^3 + 98060877/4375
000*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 1045360143/43750000*(5*x^2 + 2*x + 3)^(3/
2)*x - 1968340667/131250000*(5*x^2 + 2*x + 3)^(3/2) - 77159983/6250000*sqrt
(5*x^2 + 2*x + 3)*x - 540119881/78125000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x
 + 1)) - 77159983/31250000*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.13303, size = 417, normalized size = 2.

$$-\frac{1}{1968750000}\left(67528125000\,x^9 + 248031875000\,x^8 - 497593468750\,x^7 - 34674656250\,x^6 + 225922362500\,x^5 + 5675\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/1968750000*(67528125000*x^9 + 248031875000*x^8 - 497593468750*x^7 - 3467
4656250*x^6 + 225922362500*x^5 + 56757413000*x^4 + 17642392275*x^3 - 788390
46795*x^2 - 57768004650*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/
156250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -29x\sqrt{5x^2+2x+3}\,dx - \int -115x^2\sqrt{5x^2+2x+3}\,dx - \int 61x^3\sqrt{5x^2+2x+3}\,dx - \int 871x^4\sqrt{5x^2+2x+3}\,dx - \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-29*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-115*x**2*sqrt(5*x**2
 + 2*x + 3), x) - Integral(61*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(87
1*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(-127*x**5*sqrt(5*x**2 + 2*x +
3), x) - Integral(-2065*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(1127*x**
7*sqrt(5*x**2 + 2*x + 3), x) - Integral(343*x**8*sqrt(5*x**2 + 2*x + 3), x)
 - Integral(-2*sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A] time = 1.28566, size = 124, normalized size = 0.6

$$-\frac{1}{1968750000}\,(5\,((5\,(10\,(25\,(5\,(49\,(140\,(315\,x+1157)x-324959)x-1109589)x+36147578)x+227029652)x+70569$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/1968750000*(5*((5*(10*(25*(5*(49*(140*(315*x + 1157)*x - 324959)*x - 110
9589)*x + 36147578)*x + 227029652)*x + 705695691)*x - 15767809359)*x - 1155
3600930)*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/78125000*sqrt(5
)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.375 $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx$

Optimal. Leaf size=166

$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5+\frac{989}{200}\left(5x^2+2x+3\right)^{3/2}x^4-\frac{25277\left(5x^2+2x+3\right)^{3/2}x^3}{3000}-\frac{77509\left(5x^2+2x+3\right)^{3/2}x^2}{25000}+\frac{17816}{}$$

```
[Out] (-2521723*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/1250000 + (198439*(3 + 2*x + 5*x
^2)^(3/2))/750000 + (1781669*x*(3 + 2*x + 5*x^2)^(3/2))/250000 - (77509*x^2
*(3 + 2*x + 5*x^2)^(3/2))/25000 - (25277*x^3*(3 + 2*x + 5*x^2)^(3/2))/3000
+ (989*x^4*(3 + 2*x + 5*x^2)^(3/2))/200 + (49*x^5*(3 + 2*x + 5*x^2)^(3/2))/
40 - (17652061*ArcSinh[(1 + 5*x)/Sqrt[14]])/(625000*Sqrt[5])
```

Rubi [A] time = 0.203479, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5+\frac{989}{200}\left(5x^2+2x+3\right)^{3/2}x^4-\frac{25277\left(5x^2+2x+3\right)^{3/2}x^3}{3000}-\frac{77509\left(5x^2+2x+3\right)^{3/2}x^2}{25000}+\frac{17816}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```


```
[Out] (-2521723*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/1250000 + (198439*(3 + 2*x + 5*x
^2)^(3/2))/750000 + (1781669*x*(3 + 2*x + 5*x^2)^(3/2))/250000 - (77509*x^2
*(3 + 2*x + 5*x^2)^(3/2))/25000 - (25277*x^3*(3 + 2*x + 5*x^2)^(3/2))/3000
+ (989*x^4*(3 + 2*x + 5*x^2)^(3/2))/200 + (49*x^5*(3 + 2*x + 5*x^2)^(3/2))/
40 - (17652061*ArcSinh[(1 + 5*x)/Sqrt[14]])/(625000*Sqrt[5])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx = \frac{49}{40}x^5 \left(3+2x+5x^2\right)^{3/2} + \frac{1}{40}\int \sqrt{3+2x+5x^2}\left(80+840x+1800\right.$$

$$= \frac{989}{200}x^4 \left(3+2x+5x^2\right)^{3/2} + \frac{49}{40}x^5 \left(3+2x+5x^2\right)^{3/2} + \frac{\int \sqrt{3+2x+}}{}$$

$$= -\frac{25277x^3 \left(3+2x+5x^2\right)^{3/2}}{3000} + \frac{989}{200}x^4 \left(3+2x+5x^2\right)^{3/2} + \frac{49}{40}x^5 \left(3\right.$$

$$= -\frac{77509x^2 \left(3+2x+5x^2\right)^{3/2}}{25000} - \frac{25277x^3 \left(3+2x+5x^2\right)^{3/2}}{3000} + \frac{989}{200}x$$

$$= \frac{1781669x \left(3+2x+5x^2\right)^{3/2}}{250000} - \frac{77509x^2 \left(3+2x+5x^2\right)^{3/2}}{25000} - \frac{25277}{}$$

$$= \frac{198439 \left(3+2x+5x^2\right)^{3/2}}{750000} + \frac{1781669x \left(3+2x+5x^2\right)^{3/2}}{250000} - \frac{77509x}{}$$

$$= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439 \left(3+2x+5x^2\right)^{3/2}}{750000} + \frac{1}{}$$

$$= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439 \left(3+2x+5x^2\right)^{3/2}}{750000} + \frac{1}{}$$

$$= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439 \left(3+2x+5x^2\right)^{3/2}}{750000} + \frac{1}{}$$

Mathematica [A] time = 0.194699, size = 75, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3}\left(22968750x^7+101906250x^6-107112500x^5-65693000x^4+15583725x^3+23531995x^2+44333650x\right.}{18750000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(-4588584 + 44333650*x + 23531995*x^2 + 15583725*x
^3 - 65693000*x^4 - 107112500*x^5 + 101906250*x^6 + 22968750*x^7) - 1059123
66*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/18750000
```

Maple [A] time = 0.06, size = 132, normalized size = 0.8

$$\frac{49 x^5}{40}\left(5 x^2+2 x+3\right)^{\frac{3}{2}}+\frac{989 x^4}{200}\left(5 x^2+2 x+3\right)^{\frac{3}{2}}-\frac{25277 x^3}{3000}\left(5 x^2+2 x+3\right)^{\frac{3}{2}}-\frac{77509 x^2}{25000}\left(5 x^2+2 x+3\right)^{\frac{3}{2}}+\frac{1781669}{250000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] 49/40*x^5*(5*x^2+2*x+3)^(3/2)+989/200*x^4*(5*x^2+2*x+3)^(3/2)-25277/3000*x^
3*(5*x^2+2*x+3)^(3/2)-77509/25000*x^2*(5*x^2+2*x+3)^(3/2)+1781669/250000*x*
(5*x^2+2*x+3)^(3/2)-2521723/2500000*(10*x+2)*(5*x^2+2*x+3)^(1/2)-17652061/3
125000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+198439/750000*(5*x^2+2*x+3)^(
3/2)
```

Maxima [A] time = 1.49802, size = 193, normalized size = 1.16

$$\frac{49}{40}\left(5 x^2+2 x+3\right)^{\frac{3}{2}} x^5+\frac{989}{200}\left(5 x^2+2 x+3\right)^{\frac{3}{2}} x^4-\frac{25277}{3000}\left(5 x^2+2 x+3\right)^{\frac{3}{2}} x^3-\frac{77509}{25000}\left(5 x^2+2 x+3\right)^{\frac{3}{2}} x^2+\frac{1781669}{250000}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/40*(5*x^2 + 2*x + 3)^(3/2)*x^5 + 989/200*(5*x^2 + 2*x + 3)^(3/2)*x^4 - 2
5277/3000*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 77509/25000*(5*x^2 + 2*x + 3)^(3/2)
*x^2 + 1781669/250000*(5*x^2 + 2*x + 3)^(3/2)*x + 198439/750000*(5*x^2 + 2*
x + 3)^(3/2) - 2521723/250000*sqrt(5*x^2 + 2*x + 3)*x - 17652061/3125000*sq
rt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2521723/1250000*sqrt(5*x^2 + 2*x +
 3)
```

Fricas [A] time = 1.10473, size = 324, normalized size = 1.95

$$\frac{1}{3750000}\left(22968750 x^7+101906250 x^6-107112500 x^5-65693000 x^4+15583725 x^3+23531995 x^2+44333650 x-45\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/3750000*(22968750*x^7 + 101906250*x^6 - 107112500*x^5 - 65693000*x^4 + 15
583725*x^3 + 23531995*x^2 + 44333650*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 1
7652061/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^
2 - 10*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+5x+2\right)\sqrt{5x^2+2x+3}\left(7x^2-4x-1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2, x)
```

Giac [A] time = 1.17767, size = 111, normalized size = 0.67

$$\frac{1}{3750000}\left(5\left(\left(5\left(10\left(25\left(15\left(245x+1087\right)x-17138\right)x-262772\right)x+623349\right)x+4706399\right)x+8866730\right)x-4588584\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/3750000*(5*((5*(10*(25*(15*(245*x + 1087)*x - 17138)*x - 262772)*x + 6233
49)*x + 4706399)*x + 8866730)*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 17652061
/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.376 $\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{7}{30}\left(5x^2+2x+3\right)^{3/2}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{3/2}x^2+\frac{2149\left(5x^2+2x+3\right)^{3/2}x}{2500}+\frac{7819\left(5x^2+2x+3\right)^{3/2}}{7500}-\frac{4633(5x+1}{12}$$

```
[Out] (-4633*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/12500 + (7819*(3 + 2*x + 5*x^2)^(3/
2))/7500 + (2149*x*(3 + 2*x + 5*x^2)^(3/2))/2500 - (289*x^2*(3 + 2*x + 5*x^
2)^(3/2))/250 - (7*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 - (32431*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(6250*Sqrt[5])
```

Rubi [A] time = 0.115772, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{7}{30}\left(5x^2+2x+3\right)^{3/2}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{3/2}x^2+\frac{2149\left(5x^2+2x+3\right)^{3/2}x}{2500}+\frac{7819\left(5x^2+2x+3\right)^{3/2}}{7500}-\frac{4633(5x+1}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-4633*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/12500 + (7819*(3 + 2*x + 5*x^2)^(3/
2))/7500 + (2149*x*(3 + 2*x + 5*x^2)^(3/2))/2500 - (289*x^2*(3 + 2*x + 5*x^
2)^(3/2))/250 - (7*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 - (32431*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(6250*Sqrt[5])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}+\frac{1}{30}\int\sqrt{3+2x+5x^2}\left(60+390x+273x\right.$$

$$= -\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}-\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}+\frac{1}{750}\int\sqrt{3+\right.$$

$$= \frac{2149x\left(3+2x+5x^2\right)^{3/2}}{2500}-\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}-\frac{7}{30}x^3\left(3+2x\right.$$

$$= \frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}+\frac{2149x\left(3+2x+5x^2\right)^{3/2}}{2500}-\frac{289}{250}x^2\left(3+2x\right.$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}+\frac{2149x\left(3\right.}{}$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}+\frac{2149x\left(3\right.}{}$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}+\frac{2149x\left(3\right.}{}$$

Mathematica [A] time = 0.107163, size = 65, normalized size = 0.52

$$\frac{5\sqrt{5x^2+2x+3}\left(-43750x^5-234250x^4+48225x^3+129895x^2+105400x+103386\right)-194586\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{187500}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(103386 + 105400*x + 129895*x^2 + 48225*x^3 - 2342
50*x^4 - 43750*x^5) - 194586*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/187500
```

Maple [A] time = 0.055, size = 98, normalized size = 0.8

$$-\frac{7x^3}{30}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{289x^2}{250}\left(5x^2+2x+3\right)^{\frac{3}{2}}+\frac{2149x}{2500}\left(5x^2+2x+3\right)^{\frac{3}{2}}+\frac{7819}{7500}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{46330x+926}{25000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -7/30*x^3*(5*x^2+2*x+3)^(3/2)-289/250*x^2*(5*x^2+2*x+3)^(3/2)+2149/2500*x*(
5*x^2+2*x+3)^(3/2)+7819/7500*(5*x^2+2*x+3)^(3/2)-4633/25000*(10*x+2)*(5*x^2
+2*x+3)^(1/2)-32431/31250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Maxima [A] time = 1.59115, size = 147, normalized size = 1.19

$$-\frac{7}{30}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{2149}{2500}\left(5x^2+2x+3\right)^{\frac{3}{2}}x+\frac{7819}{7500}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{4633}{2500}\sqrt{5x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/30*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 289/250*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 2
149/2500*(5*x^2 + 2*x + 3)^(3/2)*x + 7819/7500*(5*x^2 + 2*x + 3)^(3/2) - 46
33/2500*sqrt(5*x^2 + 2*x + 3)*x - 32431/31250*sqrt(5)*arcsinh(1/14*sqrt(14)
```

```
*(5*x + 1)) - 4633/12500*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.03031, size = 255, normalized size = 2.06

$$-\frac{1}{37500}\left(43750\,x^5 + 234250\,x^4 - 48225\,x^3 - 129895\,x^2 - 105400\,x - 103386\right)\sqrt{5\,x^2 + 2\,x + 3} + \frac{32431}{62500}\,\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/37500*(43750*x^5 + 234250*x^4 - 48225*x^3 - 129895*x^2 - 105400*x - 1033
86)*sqrt(5*x^2 + 2*x + 3) + 32431/62500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*
x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -13x\sqrt{5x^2 + 2x + 3}\,dx - \int -7x^2\sqrt{5x^2 + 2x + 3}\,dx - \int 31x^3\sqrt{5x^2 + 2x + 3}\,dx - \int 7x^4\sqrt{5x^2 + 2x + 3}\,dx - \int -2\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-13*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-7*x**2*sqrt(5*x**2 +
 2*x + 3), x) - Integral(31*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(7*x*
*4*sqrt(5*x**2 + 2*x + 3), x) - Integral(-2*sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A] time = 1.24684, size = 97, normalized size = 0.78

$$-\frac{1}{37500}\left(5\left(\left(5\left(10\left(175\,x + 937\right)x - 1929\right)x - 25979\right)x - 21080\right)x - 103386\right)\sqrt{5\,x^2 + 2\,x + 3} + \frac{32431}{31250}\,\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/37500*(5*((5*(10*(175*x + 937)*x - 1929)*x - 25979)*x - 21080)*x - 10338
6)*sqrt(5*x^2 + 2*x + 3) + 32431/31250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sq
rt(5*x^2 + 2*x + 3)) - 1)
```

3.377 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$

Optimal. Leaf size=187

$$-\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397)-\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{343}$$

```
[Out] -((397 + 35*x)*Sqrt[3 + 2*x + 5*x^2])/490 - (8233*ArcSinh[(1 + 5*x)/Sqrt[14
]])/(1715*Sqrt[5]) - (3*Sqrt[(497041 - 146555*Sqrt[11])/11]*ArcTanh[(23 - S
qrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])])/343 + (3*Sqrt[(497041 + 146555*Sqrt[11])/11]*ArcTanh[(23 + Sqrt[1
1] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2
])])/343
```

Rubi [A] time = 0.35472, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1066, 1076, 619, 215, 1032, 724, 206}

$$-\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397)-\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{343}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```


```
[Out] -((397 + 35*x)*Sqrt[3 + 2*x + 5*x^2])/490 - (8233*ArcSinh[(1 + 5*x)/Sqrt[14
]])/(1715*Sqrt[5]) - (3*Sqrt[(497041 - 146555*Sqrt[11])/11]*ArcTanh[(23 - S
qrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])])/343 + (3*Sqrt[(497041 + 146555*Sqrt[11])/11]*ArcTanh[(23 + Sqrt[1
1] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2
])])/343
```


Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
```

```
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{1}{490}\int \frac{-3442-13408x-16466x^2}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} + \frac{\int \frac{40560+159720x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{3430} - \frac{8233\int \frac{1}{\sqrt{3+2x+5x^2}}\,d}{1715}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{3430\sqrt{70}} + \frac{\left(12\left(1\right.\right.}{}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{\left(24\left(14641-5028\sqrt{11}\right)\right.}{}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{3\sqrt{5467451-1612105}}{}$$

Mathematica [A] time = 0.91179, size = 189, normalized size = 1.01

$$\frac{-385\sqrt{5x^2+2x+3}(35x+397) - 75\sqrt{250-34\sqrt{11}}\left(61\sqrt{11}-143\right)\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right) + 75\sqrt{250+34\sqrt{11}}\,(}{188650}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-385*(397 + 35*x)*Sqrt[3 + 2*x + 5*x^2] - 181126*Sqrt[5]*ArcSinh[(1 + 5*x)
/Sqrt[14]] - 75*Sqrt[250 - 34*Sqrt[11]]*(-143 + 61*Sqrt[11])*ArcTanh[(23 -
```

```
Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x
^2])] + 75*Sqrt[250 + 34*Sqrt[11]]*(143 + 61*Sqrt[11])*ArcTanh[(23 + Sqrt[1
1] + (17 + 5*Sqrt[11])*x)/(Sqrt[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])])
/188650
```

Maple [B] time = 0.142, size = 403, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x)
```

```
[Out] -1/140*(10*x+2)*(5*x^2+2*x+3)^(1/2)-1/25*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1
/5))-3/154*(61+13*11^(1/2))*11^(1/2)*(1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/70*(34/7+
10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10
/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-3/154*(-61+13*11^(1/
2))*11^(1/2)*(1/49*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/70*(34/7-10/7*11^(1/2))*5^(1/2)*a
rcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x
+1/5))-(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49
-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2)
)^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2))+250-34*11^(1/2))^(1/2)))
```

Maxima [B] time = 1.77033, size = 675, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/188650*sqrt(11)*(975*sqrt(11)*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7
*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(
```

```
2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2
*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 1225*sq
rt(11)*sqrt(5*x^2 + 2*x + 3)*x - 16466*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7
)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 6825*sqrt(11)*sqrt(-34/49*sqrt(11) +
250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) -
 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(
11) - 4)) + 4575*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7*sqrt(11)*sqrt(
7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x -
 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)
+ 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 32025*sqrt(-34/49*sqrt
(11) + 250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4)) - 13895*sqrt(11)*sqrt(5*x^2 + 2*x + 3))
```

Fricas [B] time = 1.23336, size = 1148, normalized size = 6.14

$$\frac{3}{7546}\sqrt{11}\sqrt{146555\sqrt{11}+497041}\log\left(\frac{6\left(\sqrt{5x^2+2x+3}\sqrt{146555\sqrt{11}+497041}\left(87\sqrt{11}-265\right)+6517\sqrt{11}(x+3)+\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log(6*(sqrt(5*x^2 + 2*x + 3)
*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 265) + 6517*sqrt(11)*(x + 3)
 + 19551*x - 32585)/x) - 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log
(-6*(sqrt(5*x^2 + 2*x + 3)*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 26
5) - 6517*sqrt(11)*(x + 3) - 19551*x + 32585)/x) - 1/15092*sqrt(11)*sqrt(-5
275980*sqrt(11) + 17893476)*log(-(sqrt(5*x^2 + 2*x + 3)*(87*sqrt(11) + 265)
*sqrt(-5275980*sqrt(11) + 17893476) + 39102*sqrt(11)*(x + 3) - 117306*x + 1
95510)/x) + 1/15092*sqrt(11)*sqrt(-5275980*sqrt(11) + 17893476)*log((sqrt(5
*x^2 + 2*x + 3)*(87*sqrt(11) + 265)*sqrt(-5275980*sqrt(11) + 17893476) - 39
102*sqrt(11)*(x + 3) + 117306*x - 195510)/x) - 1/490*sqrt(5*x^2 + 2*x + 3)*
(35*x + 397) + 8233/17150*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x +
1) - 25*x^2 - 10*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{5x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1),x)
```

```
[Out] -Integral(2*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(5*x*sq
rt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(x**2*sqrt(5*x**2 + 2
*x + 3)/(7*x**2 - 4*x - 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.378 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

Optimal. Leaf size=199

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)}-\frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2156}+\frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\tanh^{-1}}{2156}$$

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(154*(1 + 4*x - 7*x^2)) + (Sqrt[5]*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/49 - (Sqrt[(325022311 + 39132731*Sqrt[11])/1397]*
ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/2156 + (Sqrt[(325022311 - 39132731*Sqrt[11])/1397]
*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]
*Sqrt[3 + 2*x + 5*x^2])])/2156
```

Rubi [A] time = 0.25007, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1054, 1076, 619, 215, 1032, 724, 206}

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)}-\frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2156}+\frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\tanh^{-1}}{2156}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(154*(1 + 4*x - 7*x^2)) + (Sqrt[5]*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/49 - (Sqrt[(325022311 + 39132731*Sqrt[11])/1397]*
ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/2156 + (Sqrt[(325022311 - 39132731*Sqrt[11])/1397]
*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]
*Sqrt[3 + 2*x + 5*x^2])])/2156
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
```

```
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}}{\left(1+4x-7x^2\right)^2}\,dx = \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} - \frac{1}{308}\int \frac{-948-188x+220x^2}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{\int \frac{6416+436x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{2156} + \frac{5}{49}\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{1}{98}\sqrt{\frac{5}{14}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right) + \frac{(1199}{}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{1}{49}\sqrt{5}\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{\left(2\left(1199-11446\sqrt{11}\right)\right)\text{Sul}}{}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{1}{49}\sqrt{5}\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\tan}{2}$$

Mathematica [A] time = 1.29952, size = 354, normalized size = 1.78

$$\frac{\frac{56364\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1} + \frac{2772\sqrt{5x^2+2x+3}}{-7x^2+4x+1} + 22892\sqrt{\frac{22}{125+17\sqrt{11}}}\log\left(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3} + \left(55+17\sqrt{11}\right)x + 23\sqrt{11} +\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((2772*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (56364*x*Sqrt[3 + 2*x + 5
*x^2])/(1 + 4*x - 7*x^2) + 968*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]] + 2*Sqrt
[2/(125 - 17*Sqrt[11])]*(-1199 + 11446*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqr
t[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] - 17*x + 5*Sqrt[11]*x)] - 239
8*Sqrt[2/(125 + 17*Sqrt[11])]*Log[2 + Sqrt[11] - 7*x] - 22892*Sqrt[22/(125
```

```
+ 17*Sqrt[11])]*Log[2 + Sqrt[11] - 7*x] + 2398*Sqrt[2/(125 + 17*Sqrt[11])]*
Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqr
t[3 + 2*x + 5*x^2]] + 22892*Sqrt[22/(125 + 17*Sqrt[11])]*Log[11 + 23*Sqrt[1
1] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]
])/47432
```

Maple [B] time = 0.124, size = 1084, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x)
```

```
[Out] -161/484*11^(1/2)*(1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))
*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/70*(34/7+10/7*11^(1/2))*5^(1
/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/
2)*(x+1/5))-(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(5
00/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^
(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7
*11^(1/2))+250+34*11^(1/2))^(1/2)))+(183/44+39/44*11^(1/2))*(-1/49/(250/49+
34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/98*(34/7+10/7*
11^(1/2))/(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7
+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7
*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*1
1^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2
)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1
/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^
(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+10/49/(250/49+34/49*11
^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/
7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)
-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/2
0*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))))+161/484*11^(1/2)*(1/49*(245*(x-2
/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(
1/2))^(1/2)+1/70*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49
*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49-34/49*11^(1/2
))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(
1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)
))+(183/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/
2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250
```

/49-34/49*11^(1/2))^(3/2)+1/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+10/49/(250/49-34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5x^2+2x+3}\left(x^2+5x+2\right)}{\left(7x^2-4x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

Fricas [B] time = 1.17428, size = 1499, normalized size = 7.53

$$-\frac{\sqrt{1397}\left(7x^2-4x-1\right)\sqrt{39132731\sqrt{11}+325022311}\log\left(-\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{39132731\sqrt{11}+325022311}\left(16943\sqrt{11}+235367\right)+26119}{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] -1/6023864*(sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) + 325022311
)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 325022311
```

```
)*(16943*sqrt(11) + 235367) + 26119953475*sqrt(11)*(x + 3) - 78359860425*x
+ 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) +
325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 3
25022311)*(16943*sqrt(11) + 235367) - 26119953475*sqrt(11)*(x + 3) + 783598
60425*x - 130599767375)/x) + sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-39132731*sq
rt(11) + 325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943*sqrt(11) -
 235367)*sqrt(-39132731*sqrt(11) + 325022311) + 26119953475*sqrt(11)*(x + 3
) + 78359860425*x - 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-3
9132731*sqrt(11) + 325022311)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943
*sqrt(11) - 235367)*sqrt(-39132731*sqrt(11) + 325022311) - 26119953475*sqrt
(11)*(x + 3) - 78359860425*x + 130599767375)/x) - 61468*sqrt(5)*(7*x^2 - 4*
x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) +
117348*sqrt(5*x^2 + 2*x + 3)*(61*x + 3))/(7*x^2 - 4*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(7x^2-4x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**2,x)
```

```
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.379 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$

Optimal. Leaf size=213

$$-\frac{\sqrt{5x^2+2x+3}(272941-813113x)}{1721104\left(-7x^2+4x+1\right)}+\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308\left(-7x^2+4x+1\right)^2}-\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\cdots}{\sqrt{2(125-17\sqrt{11})}\cdots}\right.}{491744}$$

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(308*(1 + 4*x - 7*x^2)^2) - ((272941 -
 813113*x)*Sqrt[3 + 2*x + 5*x^2])/(1721104*(1 + 4*x - 7*x^2)) - (Sqrt[(6492
253020949 - 11879169071*Sqrt[11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sq
rt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/491744 + (
Sqrt[(6492253020949 + 11879169071*Sqrt[11])/1397]*ArcTanh[(23 + Sqrt[11] +
(17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/
491744
```

Rubi [A] time = 0.236551, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1054, 1060, 1032, 724, 206}

$$-\frac{\sqrt{5x^2+2x+3}(272941-813113x)}{1721104\left(-7x^2+4x+1\right)}+\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308\left(-7x^2+4x+1\right)^2}-\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\cdots}{\sqrt{2(125-17\sqrt{11})}\cdots}\right.}{491744}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(308*(1 + 4*x - 7*x^2)^2) - ((272941 -
 813113*x)*Sqrt[3 + 2*x + 5*x^2])/(1721104*(1 + 4*x - 7*x^2)) - (Sqrt[(6492
253020949 - 11879169071*Sqrt[11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sq
rt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/491744 + (
Sqrt[(6492253020949 + 11879169071*Sqrt[11])/1397]*ArcTanh[(23 + Sqrt[11] +
(17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/
491744
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx = \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{1}{616}\int \frac{-3012-1564x-3220x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} + \frac{\int \frac{47581408+283\ldots}{(1+4x-7x^2)\sqrt{3+\ldots}}}{275376\ldots}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} + \frac{(1391962-17\ldots}{\ldots}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} + \frac{(-1391962+1\ldots}{\ldots}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} - \frac{\sqrt{\frac{6492253020949\ldots}{1\ldots}}}{\ldots}$$

Mathematica [A] time = 1.49292, size = 334, normalized size = 1.57

$$\frac{-\frac{44\sqrt{5x^2+2x+3}(813113x^3-737577x^2-106279x+31807)}{(-7x^2+4x+1)^2} - \sqrt{\frac{22}{125-17\sqrt{11}}}\left(126542\sqrt{11}-1740003\right)\log\left(49x^2+14\left(\sqrt{11}-2\right)x-4\sqrt{11}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] ((-44*Sqrt[3 + 2*x + 5*x^2]*(31807 - 106279*x - 737577*x^2 + 813113*x^3))/(
1 + 4*x - 7*x^2)^2 - 2*Sqrt[22/(125 - 17*Sqrt[11])]*(-1740003 + 126542*Sqrt
[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[1
1] + (-17 + 5*Sqrt[11])*x)] - 2*Sqrt[22/(125 + 17*Sqrt[11])]*(1740003 + 126
542*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + Sqrt[22/(125 - 17*Sqrt[11])]*(-1740
003 + 126542*Sqrt[11])*Log[(-2 + Sqrt[11] + 7*x)^2] - Sqrt[22/(125 - 17*Sqr
t[11])]*(-1740003 + 126542*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2 + Sqrt[11
])*x + 49*x^2] + 2*Sqrt[22/(125 + 17*Sqrt[11])]*(1740003 + 126542*Sqrt[11])
*Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sq
rt[3 + 2*x + 5*x^2]])/10818368
```

Maple [B] time = 0.127, size = 2342, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x)
```

```
[Out] -(-3535/1936-273/1936*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*1
1^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)
)+250/49+34/49*11^(1/2))^(3/2)+1/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(
1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*1
1^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5
^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7
*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49
*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2
)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+
250+34*11^(1/2))^(1/2)))+10/49/(250/49+34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x
-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49
*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(
1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1
/2)*(x+1/5))))-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(
1/2))/(x-2/7+1/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))
*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)-1/1372*(34/7-10/7*11^(1/
2))/(250/49-34/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2)
)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/4
9-34/49*11^(1/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1
/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))
+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(
```

250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49
-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2
)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(
x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*1
1^(1/2))^(1/2)))+10/(250/49-34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11
^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^
(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsin
h(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)
)))+5/686/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7
-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7
*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*1
1^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2
)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1
/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-3535/21296*11^(1/2)*(
1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2
))+250+34*11^(1/2))^(1/2)+1/70*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)
/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49
+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2
)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(
x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*1
1^(1/2))^(1/2)))+3535/21296*11^(1/2)*(1/49*(245*(x-2/7+1/7*11^(1/2))^2+49*(
34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/70*(34/7-
10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10
/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^
(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-21/968*(61+13*11^(1/
2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))^2*(5*(x-2
/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*1
1^(1/2))^(3/2)-1/1372*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(-1/(250
/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10
/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(3
4/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+1
0/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/
7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(
1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11
^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*
11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+10/(250/49+34/49*11
^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/
7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)
-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/2
0*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))))+5/686/(250/49+34/49*11^(1/2))*(1
/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))

```
+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(
250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49
+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2
)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(
x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*1
1^(1/2))^(1/2))))-(-3535/1936+273/1936*11^(1/2))*(-1/49/(250/49-34/49*11^(1
/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x
-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/98*(34/7-10/7*11^(1/2))/(
250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1
/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*
5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)
^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(4
9/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-
34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+10/49/(250/49-34/49*11^(1/2))*(1/
20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1
/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7
*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/
7*11^(1/2))^2)^(1/2)*(x+1/5))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{5x^2+2x+3}\left(x^2+5x+2\right)}{\left(7x^2-4x-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

Fricas [B] time = 1.25648, size = 1700, normalized size = 7.98

$$-\frac{\sqrt{1397}\left(49x^4-56x^3+2x^2+8x+1\right)\sqrt{11879169071\sqrt{11}+6492253020949}\log\left(\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{11879169071\sqrt{11}+6492253\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1373932736*(sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169
071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11
879169071*sqrt(11) + 6492253020949)*(4822219*sqrt(11) - 37335441) + 5690716
98870455*sqrt(11)*(x + 3) + 1707215096611365*x - 2845358494352275)/x) - sqr
t(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169071*sqrt(11) + 649
2253020949)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11879169071*sqrt(11
) + 6492253020949)*(4822219*sqrt(11) - 37335441) - 569071698870455*sqrt(11)
*(x + 3) - 1707215096611365*x + 2845358494352275)/x) + sqrt(1397)*(49*x^4 -
 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log(
-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-1187
9169071*sqrt(11) + 6492253020949) + 569071698870455*sqrt(11)*(x + 3) - 1707
215096611365*x + 2845358494352275)/x) - sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2
 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqr
t(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-11879169071*sqrt(11)
 + 6492253020949) - 569071698870455*sqrt(11)*(x + 3) + 1707215096611365*x -
 2845358494352275)/x) + 5588*(813113*x^3 - 737577*x^2 - 106279*x + 31807)*s
qrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.380 $\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx$

Optimal. Leaf size=231

$$-\frac{343}{60}\left(5x^2+2x+3\right)^{5/2}x^7-\frac{61103\left(5x^2+2x+3\right)^{5/2}x^6}{3300}+\frac{1031177\left(5x^2+2x+3\right)^{5/2}x^5}{20625}-\frac{796559\left(5x^2+2x+3\right)^{5/2}x^4}{123750}$$

```
[Out] (-479652579*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/312500000 - (22840599*(1 + 5*x
)*(3 + 2*x + 5*x^2)^(3/2))/62500000 - (6133820867*(3 + 2*x + 5*x^2)^(5/2))/
1203125000 + (837379699*x*(3 + 2*x + 5*x^2)^(5/2))/72187500 + (2173004363*x
^2*(3 + 2*x + 5*x^2)^(5/2))/173250000 - (190236913*x^3*(3 + 2*x + 5*x^2)^(5
/2))/4950000 - (796559*x^4*(3 + 2*x + 5*x^2)^(5/2))/123750 + (1031177*x^5*(
3 + 2*x + 5*x^2)^(5/2))/20625 - (61103*x^6*(3 + 2*x + 5*x^2)^(5/2))/3300 -
(343*x^7*(3 + 2*x + 5*x^2)^(5/2))/60 - (3357568053*ArcSinh[(1 + 5*x)/Sqrt[1
4]])/(156250000*Sqrt[5])
```

Rubi [A] time = 0.363843, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{343}{60}\left(5x^2+2x+3\right)^{5/2}x^7-\frac{61103\left(5x^2+2x+3\right)^{5/2}x^6}{3300}+\frac{1031177\left(5x^2+2x+3\right)^{5/2}x^5}{20625}-\frac{796559\left(5x^2+2x+3\right)^{5/2}x^4}{123750}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```


```
[Out] (-479652579*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/312500000 - (22840599*(1 + 5*x
)*(3 + 2*x + 5*x^2)^(3/2))/62500000 - (6133820867*(3 + 2*x + 5*x^2)^(5/2))/
1203125000 + (837379699*x*(3 + 2*x + 5*x^2)^(5/2))/72187500 + (2173004363*x
^2*(3 + 2*x + 5*x^2)^(5/2))/173250000 - (190236913*x^3*(3 + 2*x + 5*x^2)^(5
/2))/4950000 - (796559*x^4*(3 + 2*x + 5*x^2)^(5/2))/123750 + (1031177*x^5*(
3 + 2*x + 5*x^2)^(5/2))/20625 - (61103*x^6*(3 + 2*x + 5*x^2)^(5/2))/3300 -
(343*x^7*(3 + 2*x + 5*x^2)^(5/2))/60 - (3357568053*ArcSinh[(1 + 5*x)/Sqrt[1
4]])/(156250000*Sqrt[5])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} dx = -\frac{343}{60}x^7 \left(3+2x+5x^2\right)^{5/2} + \frac{1}{60}\int \left(3+2x+5x^2\right)^{3/2} \left(120+174\right.$$

$$= -\frac{61103x^6 \left(3+2x+5x^2\right)^{5/2}}{3300} - \frac{343}{60}x^7 \left(3+2x+5x^2\right)^{5/2} + \frac{\int \left(3\right.}{}$$

$$= \frac{1031177x^5 \left(3+2x+5x^2\right)^{5/2}}{20625} - \frac{61103x^6 \left(3+2x+5x^2\right)^{5/2}}{3300} - \frac{34}{60}$$

$$= -\frac{796559x^4 \left(3+2x+5x^2\right)^{5/2}}{123750} + \frac{1031177x^5 \left(3+2x+5x^2\right)^{5/2}}{20625} -$$

$$= -\frac{190236913x^3 \left(3+2x+5x^2\right)^{5/2}}{4950000} - \frac{796559x^4 \left(3+2x+5x^2\right)^{5/2}}{123750}$$

$$= \frac{2173004363x^2 \left(3+2x+5x^2\right)^{5/2}}{173250000} - \frac{190236913x^3 \left(3+2x+5x^2\right.}{4950000}$$

$$= \frac{837379699x \left(3+2x+5x^2\right)^{5/2}}{72187500} + \frac{2173004363x^2 \left(3+2x+5x^2\right.}{173250000}$$

$$= -\frac{6133820867 \left(3+2x+5x^2\right)^{5/2}}{1203125000} + \frac{837379699x \left(3+2x+5x^2\right)}{72187500}$$

$$= -\frac{22840599(1+5x) \left(3+2x+5x^2\right)^{3/2}}{62500000} - \frac{6133820867 \left(3+2x+\right.}{1203125000}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x) \left(3+2\right.}{62500000}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x) \left(3+2\right.}{62500000}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x) \left(3+2\right.}{62500000}$$

Mathematica [A] time = 0.398603, size = 95, normalized size = 0.41

$$-5\sqrt{5x^2+2x+3} \left(30950390625000x^{11} + 125007421875000x^{10} - 148393743750000x^9 - 30505457500000x^8 - 729182\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(10506617068392 - 6352777129950*x - 1586584440868
5*x^2 - 19041688239675*x^3 - 2573089891000*x^4 + 85130334087500*x^5 + 52106
830406250*x^6 - 72918247281250*x^7 - 30505457500000*x^8 - 148393743750000*x
^9 + 125007421875000*x^10 + 30950390625000*x^11) - 4653589321458*Sqrt[5]*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/1082812500000
```

Maple [A] time = 0.074, size = 185, normalized size = 0.8

$$\frac{1031177\,x^5}{20625}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}-\frac{796559\,x^4}{123750}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}-\frac{190236913\,x^3}{4950000}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}+\frac{2173004363\,x^2}{173250000}\left(5\,x^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 1031177/20625*x^5*(5*x^2+2*x+3)^(5/2)-796559/123750*x^4*(5*x^2+2*x+3)^(5/2)
-190236913/4950000*x^3*(5*x^2+2*x+3)^(5/2)+2173004363/173250000*x^2*(5*x^2+
2*x+3)^(5/2)-479652579/625000000*(10*x+2)*(5*x^2+2*x+3)^(1/2)-3357568053/78
1250000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+837379699/72187500*x*(5*x^2+
2*x+3)^(5/2)-22840599/125000000*(10*x+2)*(5*x^2+2*x+3)^(3/2)-6133820867/120
3125000*(5*x^2+2*x+3)^(5/2)-61103/3300*x^6*(5*x^2+2*x+3)^(5/2)-343/60*x^7*(
5*x^2+2*x+3)^(5/2)
```

Maxima [A] time = 1.53308, size = 278, normalized size = 1.2

$$-\frac{343}{60}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^7-\frac{61103}{3300}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^6+\frac{1031177}{20625}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^5-\frac{796559}{123750}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^4-\frac{19}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/60*(5*x^2 + 2*x + 3)^(5/2)*x^7 - 61103/3300*(5*x^2 + 2*x + 3)^(5/2)*x^
6 + 1031177/20625*(5*x^2 + 2*x + 3)^(5/2)*x^5 - 796559/123750*(5*x^2 + 2*x
+ 3)^(5/2)*x^4 - 190236913/4950000*(5*x^2 + 2*x + 3)^(5/2)*x^3 + 2173004363
/173250000*(5*x^2 + 2*x + 3)^(5/2)*x^2 + 837379699/72187500*(5*x^2 + 2*x +
3)^(5/2)*x - 6133820867/1203125000*(5*x^2 + 2*x + 3)^(5/2) - 22840599/12500
000*(5*x^2 + 2*x + 3)^(3/2)*x - 22840599/62500000*(5*x^2 + 2*x + 3)^(3/2) -
```

```
479652579/62500000*sqrt(5*x^2 + 2*x + 3)*x - 3357568053/781250000*sqrt(5)*
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 479652579/312500000*sqrt(5*x^2 + 2*x + 3
)
```

Fricas [A] time = 1.48695, size = 518, normalized size = 2.24

$$-\frac{1}{216562500000}\left(30950390625000\,x^{11}+125007421875000\,x^{10}-148393743750000\,x^{9}-30505457500000\,x^{8}-72918\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/216562500000*(30950390625000*x^11 + 125007421875000*x^10 - 1483937437500
00*x^9 - 30505457500000*x^8 - 72918247281250*x^7 + 52106830406250*x^6 + 851
30334087500*x^5 - 2573089891000*x^4 - 19041688239675*x^3 - 15865844408685*x
^2 - 6352777129950*x + 10506617068392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/1
562500000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -91x\sqrt{5x^2+2x+3}\,dx-\int -413x^2\sqrt{5x^2+2x+3}\,dx-\int -192x^3\sqrt{5x^2+2x+3}\,dx-\int 2160x^4\sqrt{5x^2+2x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-91*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-413*x**2*sqrt(5*x**2
 + 2*x + 3), x) - Integral(-192*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(
2160*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(1666*x**5*sqrt(5*x**2 + 2*x
 + 3), x) - Integral(-2094*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-1384
*x**7*sqrt(5*x**2 + 2*x + 3), x) - Integral(-7042*x**8*sqrt(5*x**2 + 2*x +
3), x) - Integral(6321*x**9*sqrt(5*x**2 + 2*x + 3), x) - Integral(1715*x**1
0*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A] time = 1.2009, size = 138, normalized size = 0.6

$$-\frac{1}{216562500000}\left(5\left(\left(5\left(10\left(25\left(5\left(7\left(20\left(105\left(875\left(77x+311\right)x-323034\right)x-6972676\right)x-333340559\right)x+1667418573\right)x\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/216562500000*(5*((5*(10*(25*(5*(7*(20*(105*(875*(77*x + 311)*x - 323034)
*x - 6972676)*x - 333340559)*x + 1667418573)*x + 13620853454)*x - 102923595
64)*x - 761667529587)*x - 3173168881737)*x - 1270555425990)*x + 10506617068
392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/781250000*sqrt(5)*log(-sqrt(5)*(sqr
t(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.381 $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx$

Optimal. Leaf size=189

$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5+\frac{581}{150}\left(5x^2+2x+3\right)^{5/2}x^4-\frac{18379\left(5x^2+2x+3\right)^{5/2}x^3}{3000}-\frac{219271\left(5x^2+2x+3\right)^{5/2}x^2}{105000}+\frac{86721}{}$$

```
[Out] (-14501781*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/6250000 - (690561*(1 + 5*x)*(3
+ 2*x + 5*x^2)^(3/2))/1250000 + (505667*(3 + 2*x + 5*x^2)^(5/2))/2187500 +
(86721*x*(3 + 2*x + 5*x^2)^(5/2))/21875 - (219271*x^2*(3 + 2*x + 5*x^2)^(5/
2))/105000 - (18379*x^3*(3 + 2*x + 5*x^2)^(5/2))/3000 + (581*x^4*(3 + 2*x +
 5*x^2)^(5/2))/150 + (49*x^5*(3 + 2*x + 5*x^2)^(5/2))/50 - (101512467*ArcSi
nh[(1 + 5*x)/Sqrt[14]])/(3125000*Sqrt[5])
```

Rubi [A] time = 0.226532, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5+\frac{581}{150}\left(5x^2+2x+3\right)^{5/2}x^4-\frac{18379\left(5x^2+2x+3\right)^{5/2}x^3}{3000}-\frac{219271\left(5x^2+2x+3\right)^{5/2}x^2}{105000}+\frac{86721}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```



```
[Out] (-14501781*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/6250000 - (690561*(1 + 5*x)*(3
+ 2*x + 5*x^2)^(3/2))/1250000 + (505667*(3 + 2*x + 5*x^2)^(5/2))/2187500 +
(86721*x*(3 + 2*x + 5*x^2)^(5/2))/21875 - (219271*x^2*(3 + 2*x + 5*x^2)^(5/
2))/105000 - (18379*x^3*(3 + 2*x + 5*x^2)^(5/2))/3000 + (581*x^4*(3 + 2*x +
 5*x^2)^(5/2))/150 + (49*x^5*(3 + 2*x + 5*x^2)^(5/2))/50 - (101512467*ArcSi
nh[(1 + 5*x)/Sqrt[14]])/(3125000*Sqrt[5])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
```

```
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} dx = \frac{49}{50} x^5 \left(3+2x+5x^2\right)^{5/2} + \frac{1}{50} \int \left(3+2x+5x^2\right)^{3/2} (100+1050x$$

$$= \frac{581}{150} x^4 \left(3+2x+5x^2\right)^{5/2} + \frac{49}{50} x^5 \left(3+2x+5x^2\right)^{5/2} + \frac{\int (3+2x}{}$$

$$= -\frac{18379x^3 \left(3+2x+5x^2\right)^{5/2}}{3000} + \frac{581}{150} x^4 \left(3+2x+5x^2\right)^{5/2} + \frac{49}{50} x^5$$

$$= -\frac{219271x^2 \left(3+2x+5x^2\right)^{5/2}}{105000} - \frac{18379x^3 \left(3+2x+5x^2\right)^{5/2}}{3000} + \frac{58}{15}$$

$$= \frac{86721x \left(3+2x+5x^2\right)^{5/2}}{21875} - \frac{219271x^2 \left(3+2x+5x^2\right)^{5/2}}{105000} - \frac{1837}{}$$

$$= \frac{505667 \left(3+2x+5x^2\right)^{5/2}}{2187500} + \frac{86721x \left(3+2x+5x^2\right)^{5/2}}{21875} - \frac{21927}{}$$

$$= -\frac{690561(1+5x) \left(3+2x+5x^2\right)^{3/2}}{1250000} + \frac{505667 \left(3+2x+5x^2\right)^{5/2}}{2187500}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x) \left(3+2x+\right.}{1250000}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x) \left(3+2x+\right.}{1250000}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x) \left(3+2x+\right.}{1250000}$$

Mathematica [A] time = 0.25241, size = 85, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3} \left(3215625000x^9 + 15281875000x^8 - 5561281250x^7 - 4105593750x^6 - 12554262500x^5 - 3227597000x\right.}{656250000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(-249003936 + 2291675850*x + 3721040355*x^2 + 5959
365525*x^3 - 3227597000*x^4 - 12554262500*x^5 - 4105593750*x^6 - 5561281250
*x^7 + 15281875000*x^8 + 3215625000*x^9) - 4263523614*Sqrt[5]*ArcSinh[(1 +
5*x)/Sqrt[14]])/656250000
```

Maple [A] time = 0.061, size = 151, normalized size = 0.8

$$\frac{49x^5}{50}\left(5x^2+2x+3\right)^{\frac{5}{2}}+\frac{581x^4}{150}\left(5x^2+2x+3\right)^{\frac{5}{2}}-\frac{18379x^3}{3000}\left(5x^2+2x+3\right)^{\frac{5}{2}}-\frac{219271x^2}{105000}\left(5x^2+2x+3\right)^{\frac{5}{2}}-\frac{1450178}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 49/50*x^5*(5*x^2+2*x+3)^(5/2)+581/150*x^4*(5*x^2+2*x+3)^(5/2)-18379/3000*x^
3*(5*x^2+2*x+3)^(5/2)-219271/105000*x^2*(5*x^2+2*x+3)^(5/2)-14501781/125000
00*(10*x+2)*(5*x^2+2*x+3)^(1/2)-101512467/15625000*5^(1/2)*arcsinh(5/14*14^
(1/2)*(x+1/5))+86721/21875*x*(5*x^2+2*x+3)^(5/2)-690561/2500000*(10*x+2)*(5
*x^2+2*x+3)^(3/2)+505667/2187500*(5*x^2+2*x+3)^(5/2)
```

Maxima [A] time = 1.58564, size = 232, normalized size = 1.23

$$\frac{49}{50}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^5+\frac{581}{150}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^4-\frac{18379}{3000}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^3-\frac{219271}{105000}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^2+\frac{86721}{21875}\left(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/50*(5*x^2 + 2*x + 3)^(5/2)*x^5 + 581/150*(5*x^2 + 2*x + 3)^(5/2)*x^4 - 1
8379/3000*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 219271/105000*(5*x^2 + 2*x + 3)^(5/
2)*x^2 + 86721/21875*(5*x^2 + 2*x + 3)^(5/2)*x + 505667/2187500*(5*x^2 + 2*
x + 3)^(5/2) - 690561/250000*(5*x^2 + 2*x + 3)^(3/2)*x - 690561/1250000*(5*
x^2 + 2*x + 3)^(3/2) - 14501781/1250000*sqrt(5*x^2 + 2*x + 3)*x - 101512467
/15625000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 14501781/6250000*sqrt(
5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.28599, size = 397, normalized size = 2.1

$$\frac{1}{131250000}\left(3215625000x^9+15281875000x^8-5561281250x^7-4105593750x^6-12554262500x^5-3227597000x^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/131250000*(3215625000*x^9 + 15281875000*x^8 - 5561281250*x^7 - 4105593750
*x^6 - 12554262500*x^5 - 3227597000*x^4 + 5959365525*x^3 + 3721040355*x^2 +
 2291675850*x - 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/31250000*sqrt(
5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+5x+2\right)\left(5x^2+2x+3\right)^{\frac{3}{2}}\left(7x^2-4x-1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2,
x)
```

Giac [A] time = 1.21031, size = 124, normalized size = 0.66

$$\frac{1}{131250000}\,(5\,((5\,(10\,(25\,(5\,(7\,(140\,(105\,x+499)x-25423)x-131379)x-2008682)x-12910388)x+238374621)x+7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/131250000*(5*((5*(10*(25*(5*(7*(140*(105*x + 499)*x - 25423)*x - 131379)*
x - 2008682)*x - 12910388)*x + 238374621)*x + 744208071)*x + 458335170)*x -
 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/15625000*sqrt(5)*log(-sqrt(5)
*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.382 $\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx$

Optimal. Leaf size=147

$$-\frac{7}{40}\left(5x^2+2x+3\right)^{5/2}x^3-\frac{1163\left(5x^2+2x+3\right)^{5/2}x^2}{1400}+\frac{2809\left(5x^2+2x+3\right)^{5/2}x}{5250}+\frac{149509\left(5x^2+2x+3\right)^{5/2}}{262500}-\frac{18397(5x$$

```
[Out] (-128779*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/250000 - (18397*(1 + 5*x)*(3 + 2*
x + 5*x^2)^(3/2))/150000 + (149509*(3 + 2*x + 5*x^2)^(5/2))/262500 + (2809*
x*(3 + 2*x + 5*x^2)^(5/2))/5250 - (1163*x^2*(3 + 2*x + 5*x^2)^(5/2))/1400 -
 (7*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 - (901453*ArcSinh[(1 + 5*x)/Sqrt[14]])/
(125000*Sqrt[5])
```

Rubi [A] time = 0.129781, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{7}{40}\left(5x^2+2x+3\right)^{5/2}x^3-\frac{1163\left(5x^2+2x+3\right)^{5/2}x^2}{1400}+\frac{2809\left(5x^2+2x+3\right)^{5/2}x}{5250}+\frac{149509\left(5x^2+2x+3\right)^{5/2}}{262500}-\frac{18397(5x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-128779*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/250000 - (18397*(1 + 5*x)*(3 + 2*
x + 5*x^2)^(3/2))/150000 + (149509*(3 + 2*x + 5*x^2)^(5/2))/262500 + (2809*
x*(3 + 2*x + 5*x^2)^(5/2))/5250 - (1163*x^2*(3 + 2*x + 5*x^2)^(5/2))/1400 -
 (7*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 - (901453*ArcSinh[(1 + 5*x)/Sqrt[14]])/
(125000*Sqrt[5])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = -\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+\frac{1}{40}\int \left(3+2x+5x^2\right)^{3/2}\left(80+520x+\right.$$

$$= -\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+\frac{\int \left(3+2x\right.}{}$$

$$= \frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}-\frac{7}{40}x^3\left(3+\right.$$

$$= \frac{149509\left(3+2x+5x^2\right)^{5/2}}{262500}+\frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-\frac{1163x^2\left(3\right.}{}$$

$$= -\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/2}}{150000}+\frac{149509\left(3+2x+5x^2\right)^{5/2}}{262500}+$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/}}{150000}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/}}{150000}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/}}{150000}$$

Mathematica [A] time = 0.148884, size = 75, normalized size = 0.51

$$\frac{-5\sqrt{5x^2+2x+3}\left(22968750x^7+127406250x^6+48237500x^5+28373000x^4-78608475x^3-86464445x^2-36695150x\right.}{26250000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(-22275576 - 36695150*x - 86464445*x^2 - 78608475
*x^3 + 28373000*x^4 + 48237500*x^5 + 127406250*x^6 + 22968750*x^7) - 378610
26*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/26250000
```

Maple [A] time = 0.056, size = 117, normalized size = 0.8

$$-\frac{7x^3}{40}\left(5x^2+2x+3\right)^{\frac{5}{2}}-\frac{1163x^2}{1400}\left(5x^2+2x+3\right)^{\frac{5}{2}}-\frac{1287790x+257558}{500000}\sqrt{5x^2+2x+3}-\frac{901453\sqrt{5}}{625000}\mathrm{Arcsinh}\left(\frac{5\sqrt{14}}{14}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -7/40*x^3*(5*x^2+2*x+3)^(5/2)-1163/1400*x^2*(5*x^2+2*x+3)^(5/2)-128779/5000
00*(10*x+2)*(5*x^2+2*x+3)^(1/2)-901453/625000*5^(1/2)*arcsinh(5/14*14^(1/2)
*(x+1/5))+2809/5250*x*(5*x^2+2*x+3)^(5/2)-18397/300000*(10*x+2)*(5*x^2+2*x+
3)^(3/2)+149509/262500*(5*x^2+2*x+3)^(5/2)
```

Maxima [A] time = 1.47904, size = 186, normalized size = 1.27

$$-\frac{7}{40}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^3-\frac{1163}{1400}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^2+\frac{2809}{5250}\left(5x^2+2x+3\right)^{\frac{5}{2}}x+\frac{149509}{262500}\left(5x^2+2x+3\right)^{\frac{5}{2}}-\frac{18397}{30000}\left(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/40*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 1163/1400*(5*x^2 + 2*x + 3)^(5/2)*x^2 +
 2809/5250*(5*x^2 + 2*x + 3)^(5/2)*x + 149509/262500*(5*x^2 + 2*x + 3)^(5/2
) - 18397/30000*(5*x^2 + 2*x + 3)^(3/2)*x - 18397/150000*(5*x^2 + 2*x + 3)^
(3/2) - 128779/50000*sqrt(5*x^2 + 2*x + 3)*x - 901453/625000*sqrt(5)*arcsin
h(1/14*sqrt(14)*(5*x + 1)) - 128779/250000*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.26446, size = 323, normalized size = 2.2

$$-\frac{1}{5250000}\left(22968750x^7+127406250x^6+48237500x^5+28373000x^4-78608475x^3-86464445x^2-36695150x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/5250000*(22968750*x^7 + 127406250*x^6 + 48237500*x^5 + 28373000*x^4 - 78
608475*x^3 - 86464445*x^2 - 36695150*x - 22275576)*sqrt(5*x^2 + 2*x + 3) +
901453/1250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
 - 10*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -43x\sqrt{5x^2+2x+3}\,dx - \int -57x^2\sqrt{5x^2+2x+3}\,dx - \int 14x^3\sqrt{5x^2+2x+3}\,dx - \int 48x^4\sqrt{5x^2+2x+3}\,dx - \int 16$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-43*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-57*x**2*sqrt(5*x**2
+ 2*x + 3), x) - Integral(14*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(48*
x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(169*x**5*sqrt(5*x**2 + 2*x + 3),
 x) - Integral(35*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2
 + 2*x + 3), x)
```

Giac [A] time = 1.30397, size = 111, normalized size = 0.76

$$-\frac{1}{5250000}\left(5\left(\left(5\left(10\left(25\left(15\left(245x+1359\right)x+7718\right)x+113492\right)x-3144339\right)x-17292889\right)x-7339030\right)x-22275576\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -1/5250000*(5*((5*(10*(25*(15*(245*x + 1359)*x + 7718)*x + 113492)*x - 3144
339)*x - 17292889)*x - 7339030)*x - 22275576)*sqrt(5*x^2 + 2*x + 3) + 90145
3/625000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.383 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx$

Optimal. Leaf size=210

$$-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}-\frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100}-\frac{6\sqrt{\frac{2}{11}}\left(8098902607-2434122235\sqrt{11}\right)\text{ta}}{16807}$$

```
[Out] (-3*(571621 + 196105*x)*Sqrt[3 + 2*x + 5*x^2])/240100 - ((267 + 35*x)*(3 +
2*x + 5*x^2)^(3/2))/980 - (34425687*ArcSinh[(1 + 5*x)/Sqrt[14]])/(840350*Sq
rt[5]) - (6*Sqrt[(2*(8098902607 - 2434122235*Sqrt[11]))/11]*ArcTanh[(23 - S
qrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])]])/16807 + (6*Sqrt[(2*(8098902607 + 2434122235*Sqrt[11]))/11]*ArcTan
h[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3
 + 2*x + 5*x^2])]])/16807
```

Rubi [A] time = 0.304076, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1066, 1076, 619, 215, 1032, 724, 206}

$$-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}-\frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100}-\frac{6\sqrt{\frac{2}{11}}\left(8098902607-2434122235\sqrt{11}\right)\text{ta}}{16807}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-3*(571621 + 196105*x)*Sqrt[3 + 2*x + 5*x^2])/240100 - ((267 + 35*x)*(3 +
2*x + 5*x^2)^(3/2))/980 - (34425687*ArcSinh[(1 + 5*x)/Sqrt[14]])/(840350*Sq
rt[5]) - (6*Sqrt[(2*(8098902607 - 2434122235*Sqrt[11]))/11]*ArcTanh[(23 - S
qrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])]])/16807 + (6*Sqrt[(2*(8098902607 + 2434122235*Sqrt[11]))/11]*ArcTan
h[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3
 + 2*x + 5*x^2])]])/16807
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
```

```
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}}{1+4x-7x^2}\,dx = -\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - \frac{\int \frac{\left(-20358-79272x-100854x^2\right)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx}{2940}$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} + -$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - -$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - -$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - -$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - -$$

Mathematica [A] time = 0.948347, size = 202, normalized size = 0.96

$$-5\left(77\sqrt{5x^2+2x+3}\left(42875x^3+344225x^2+744870x+1911108\right)+600\sqrt{1572625-425459\sqrt{11}}\left(61\sqrt{11}-143\right)\tanh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-757365114*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]] - 5*(77*Sqrt[3 + 2*x + 5*x^
2]*(1911108 + 744870*x + 344225*x^2 + 42875*x^3) + 600*Sqrt[1572625 - 42545
9*Sqrt[11]]*(-143 + 61*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + 17*x - 5*Sqrt[11]
*x)/(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])] - 600*(143 + 61*Sqrt[1
1])*Sqrt[1572625 + 425459*Sqrt[11]]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[1
1])*x)/(Sqrt[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])]))/92438500
```

Maple [B] time = 0.116, size = 730, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x)
```

```
[Out] -1/280*(10*x+2)*(5*x^2+2*x+3)^(3/2)-3/200*(10*x+2)*(5*x^2+2*x+3)^(1/2)-21/2
50*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-3/154*(61+13*11^(1/2))*11^(1/2)*(
1/21*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0/49+34/49*11^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2
/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*1
1^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/
2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2
)*(x+1/5)))+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49
*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/
7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+
10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2)
)^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7
*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))-3/154*(-61+13*1
1^(1/2))*11^(1/2)*(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(1/
20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1
/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7
*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/
7*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7
+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/
2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*1
1^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2
))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(
```

```
1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)
)))
```

Maxima [B] time = 1.87847, size = 722, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/92438500*sqrt(11)*(19500*sqrt(11)*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 3
00125*sqrt(11)*(5*x^2 + 2*x + 3)^(3/2)*x - 3344250*sqrt(11)*(-34/49*sqrt(11
) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(
11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x
 + 2*sqrt(11) - 4)) + 91500*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsinh(5/7*s
qrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)
*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*s
qrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 15692250*
(-34/49*sqrt(11) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs
(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4)
 + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4)) - 2289525*sqrt(11)*(5*x^2 + 2*x + 3)^(3/
2) - 20591025*sqrt(11)*sqrt(5*x^2 + 2*x + 3)*x - 68851374*sqrt(11)*sqrt(5)*
arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 60020205*sqrt(11)*
sqrt(5*x^2 + 2*x + 3))
```

Fricas [B] time = 1.49094, size = 1442, normalized size = 6.87

$$\frac{3}{184877}\sqrt{11}\sqrt{2}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(\frac{12\left(\sqrt{2}\sqrt{5x^2+2x+3}\sqrt{2434122235\sqrt{11}+8098902607}\left(7690\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(12*(sq
rt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*sq
rt(11) - 24697) + 40555291*sqrt(11)*(x + 3) + 121665873*x - 202776455)/x) -
 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(-12*(
sqrt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*
sqrt(11) - 24697) - 40555291*sqrt(11)*(x + 3) - 121665873*x + 202776455)/x)
 - 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log(-(sqr
t(5*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23
32483950816) + 486663492*sqrt(11)*(x + 3) - 1459990476*x + 2433317460)/x) +
 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log((sqrt(5
*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23324
83950816) - 486663492*sqrt(11)*(x + 3) + 1459990476*x - 2433317460)/x) - 1/
240100*(42875*x^3 + 344225*x^2 + 744870*x + 1911108)*sqrt(5*x^2 + 2*x + 3)
+ 34425687/8403500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25
*x^2 - 10*x - 8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.384 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx$

Optimal. Leaf size=222

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}+\frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773}-\frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\tanh^{-1}\left(\frac{(1}{\sqrt{2(1}}\right.}{26411}$$

```
[Out] ((5826 + 3395*x)*Sqrt[3 + 2*x + 5*x^2])/3773 + (3*(3 + 61*x)*(3 + 2*x + 5*x
^2)^(3/2))/(154*(1 + 4*x - 7*x^2)) + (16691*ArcSinh[(1 + 5*x)/Sqrt[14]])/(2
401*Sqrt[5]) - (Sqrt[(52175400311 - 13155376531*Sqrt[11])/22]*ArcTanh[(23 -
 Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x
+ 5*x^2])])/26411 - (Sqrt[(52175400311 + 13155376531*Sqrt[11])/22]*ArcTanh[
(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 +
 2*x + 5*x^2])])/26411
```

Rubi [A] time = 0.3216, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1054, 1066, 1076, 619, 215, 1032, 724, 206}

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}+\frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773}-\frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\tanh^{-1}\left(\frac{(1}{\sqrt{2(1}}\right.}{26411}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((5826 + 3395*x)*Sqrt[3 + 2*x + 5*x^2])/3773 + (3*(3 + 61*x)*(3 + 2*x + 5*x
^2)^(3/2))/(154*(1 + 4*x - 7*x^2)) + (16691*ArcSinh[(1 + 5*x)/Sqrt[14]])/(2
401*Sqrt[5]) - (Sqrt[(52175400311 - 13155376531*Sqrt[11])/22]*ArcTanh[(23 -
 Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x
+ 5*x^2])])/26411 - (Sqrt[(52175400311 + 13155376531*Sqrt[11])/22]*ArcTanh[
(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 +
 2*x + 5*x^2])])/26411
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} - \frac{1}{308}\int \frac{\sqrt{3+2x+5x^2}\,(-912+724x+3880x^2)}{1+4x-7x^2}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{\int \frac{700200-4304880}{(1+4x-7x^2)\sqrt{}}}{1509}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} - \frac{\int \frac{2442640+59510}{(1+4x-7x^2)\sqrt{3+}}}{105644}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691\,\mathrm{Subst}\Big(}{4}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691\sinh^{-1}\Big(}{2401\sqrt{5}}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691\sinh^{-1}\Big(}{2401\sqrt{5}}$$

Mathematica [A] time = 1.8726, size = 354, normalized size = 1.59

$$\frac{\frac{770\sqrt{5x^2+2x+3}(2695x^3+34265x^2-81181x-12975)}{7x^2-4x-1} + 5\sqrt{\frac{22}{125-17\sqrt{11}}}\left(743879\sqrt{11}-1701489\right)\log\left(49x^2+14\left(\sqrt{11}-2\right)x-4\sqrt{11}+15\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((770*Sqrt[3 + 2*x + 5*x^2]*(-12975 - 81181*x + 34265*x^2 + 2695*x^3))/(-1
- 4*x + 7*x^2) + 8078444*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]] + 10*Sqrt[22/(
125 - 17*Sqrt[11])]*(-1701489 + 743879*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqr
t[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] + 10
*Sqrt[22/(125 + 17*Sqrt[11])]*(1701489 + 743879*Sqrt[11])*Log[2 + Sqrt[11]
- 7*x] - 5*Sqrt[22/(125 - 17*Sqrt[11])]*(-1701489 + 743879*Sqrt[11])*Log[(-
2 + Sqrt[11] + 7*x)^2] + 5*Sqrt[22/(125 - 17*Sqrt[11])]*(-1701489 + 743879*
Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2 + Sqrt[11])*x + 49*x^2] - 10*Sqrt[22
```

```
/(125 + 17*Sqrt[11])]*(1701489 + 743879*Sqrt[11])*Log[11 + 23*Sqrt[11] + (5
5 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]])/5810
420
```

Maple [B] time = 0.118, size = 1828, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x)
```

```
[Out] -161/484*11^(1/2)*(1/21*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/
20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1
/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7
*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/
7*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7
-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/
2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*1
1^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2
))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(
1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)
)))+(183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1
/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0/49+34/49*11^(1/2))^(5/2)+3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2)
)*(1/3*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+
250/49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-
2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*
11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1
/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/
2)*(x+1/5)))+(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(3
4/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+1
0/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/
7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(
1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11
^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*
11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))+20/49/(250/49+34/4
9*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(
x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+3/80*(5000/49+680/49*11^(1
/2)-(34/7+10/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+
```

```
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(500
0/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/4
9+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))+161/484*11^
(1/2)*(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1
/2))+250/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*
(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-
34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2
)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^
2)^(1/2)*(x+1/5)))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2
))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/
10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20
*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*1
1^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-
2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(
34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))+(183/44-
39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2
/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*1
1^(1/2))^(5/2)+3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/3*(5*(x
-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49
*11^(1/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(
1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1
/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(
5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))
+(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2)
)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^
2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh
(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(25
0-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x
-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))+20/49/(250/49-34/49*11^(1/2))*
(1/40*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2))+250/49-34/49*11^(1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/2)-(34/7-10
/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2
))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*
11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(
1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}\left(x^2+5\,x+2\right)}{\left(7\,x^2-4\,x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
```

```
[Out] integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

Fricas [B] time = 1.54977, size = 1515, normalized size = 6.82

$$\frac{5\sqrt{11}\left(7x^2-4x-1\right)\sqrt{26310753062\sqrt{11}+104350800622}\log\left(\frac{\sqrt{5x^2+2x+3}\sqrt{26310753062\sqrt{11}+104350800622}\left(16206\sqrt{11}-68441\right)+17}{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] 1/5810420*(5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 1043508
00622)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 104350800622)
*(16206*sqrt(11) - 68441) + 1795191685*sqrt(11)*(x + 3) + 5385575055*x - 89
75958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 104
350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 10435080
0622)*(16206*sqrt(11) - 68441) - 1795191685*sqrt(11)*(x + 3) - 5385575055*x
 + 8975958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*sqrt(11)
 + 104350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)*sqrt(
-26310753062*sqrt(11) + 104350800622) + 1795191685*sqrt(11)*(x + 3) - 53855
75055*x + 8975958425)/x) + 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*s
qrt(11) + 104350800622)*log((sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)
*sqrt(-26310753062*sqrt(11) + 104350800622) - 1795191685*sqrt(11)*(x + 3) +
 5385575055*x - 8975958425)/x) + 4039222*sqrt(5)*(7*x^2 - 4*x - 1)*log(-sqr
t(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 770*(2695*x^3 +
 34265*x^2 - 81181*x - 12975)*sqrt(5*x^2 + 2*x + 3))/(7*x^2 - 4*x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.385 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx$

Optimal. Leaf size=234

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}-\frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}}\right.}{332024}$$

```
[Out] -((9495 - 37088*x)*Sqrt[3 + 2*x + 5*x^2])/(23716*(1 + 4*x - 7*x^2)) + (3*(3
 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(308*(1 + 4*x - 7*x^2)^2) - (5*Sqrt[5]*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/343 - (Sqrt[(62294197250171 - 2085440742055*Sqrt
[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17
*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024 + (Sqrt[(62294197250171 + 20854
40742055*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqr
t[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024
```

Rubi [A] time = 0.318617, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1054, 1076, 619, 215, 1032, 724, 206}

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}-\frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}}\right.}{332024}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] -((9495 - 37088*x)*Sqrt[3 + 2*x + 5*x^2])/(23716*(1 + 4*x - 7*x^2)) + (3*(3
 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(308*(1 + 4*x - 7*x^2)^2) - (5*Sqrt[5]*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/343 - (Sqrt[(62294197250171 - 2085440742055*Sqrt
[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17
*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024 + (Sqrt[(62294197250171 + 20854
40742055*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqr
t[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{1}{616}\int \frac{\sqrt{3+2x+5x^2}\,(-2976-652x+440x^2) \cdots}{(1+4x-7x^2)^2}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} + \frac{\int \frac{1024152+7\cdots}{(1+4x-7\cdots}}{1\cdots}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{\int \frac{-7265864\cdots}{(1+4x-7x^2\cdots}}{132\cdots}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{1}{686}\left(5\sqrt{\frac{5\cdots}{14\cdots}}\cdots\right.$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{5}{343}\sqrt{5}\sin\cdots$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{5}{343}\sqrt{5}\sin\cdots$$

Mathematica [A] time = 2.34292, size = 376, normalized size = 1.61

$$\frac{\frac{88\sqrt{5x^2+2x+3}(138372-189161x)}{7x^2-4x-1} + \frac{11616(5028x+655)\sqrt{5x^2+2x+3}}{(-7x^2+4x+1)^2} - \sqrt{\frac{22}{125-17\sqrt{11}}}\left(674221\sqrt{11}-7706073\right)\log\left(49x^2+14\left(\sqrt{11}-2\right)x\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] ((11616*(655 + 5028*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (88*(13
8372 - 189161*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 212960*Sqrt[5]
*ArcSinh[(1 + 5*x)/Sqrt[14]] - 2*Sqrt[22/(125 - 17*Sqrt[11])]*(-7706073 + 6
74221*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-2
3 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 2*Sqrt[22/(125 + 17*Sqrt[11])]*(770
6073 + 674221*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + Sqrt[22/(125 - 17*Sqrt[11
])]*(-7706073 + 674221*Sqrt[11])*Log[(-2 + Sqrt[11] + 7*x)^2] - Sqrt[22/(12
5 - 17*Sqrt[11])]*(-7706073 + 674221*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2
 + Sqrt[11])*x + 49*x^2] + 2*Sqrt[22/(125 + 17*Sqrt[11])]*(7706073 + 674221
*Sqrt[11])*Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sq
rt[11]]*Sqrt[3 + 2*x + 5*x^2]])/14609056
```

Maple [B] time = 0.125, size = 3828, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x)
```

```
[Out] -21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/
7*11^(1/2))^2*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2))+250/49+34/49*11^(1/2))^(5/2)+1/1372*(34/7+10/7*11^(1/2))/(250/49+34/
49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(5/2)+3/2*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7-1/
7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/
2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2
+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/2
00*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)
/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/4
9+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*
(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/
2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2
)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(
500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11
^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/
7*11^(1/2))+250+34*11^(1/2))^(1/2))))+20/(250/49+34/49*11^(1/2))*(1/40*(10*
x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0/49+34/49*11^(1/2))^(3/2)+3/80*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2
```

```
))^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-
1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(
34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*
(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))+15/686/(250/49+34/49*11^(1/2))*(1
/3*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/
49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-
1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(
1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*
arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(
x+1/5)))+(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*
11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11
^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)
*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))))-3535/21296*11^(1/2)*(
1/21*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0/49+34/49*11^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2
/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*1
1^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/
2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2
)*(x+1/5)))+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49
*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/
7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+
10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2)
)^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7
*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))-21/968*(-61+13*
11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))^2*(
5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-3
4/49*11^(1/2))^(5/2)+1/1372*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(-
1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(3
4/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(5/2)+3/2*(3
4/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/7*11^(1/2))^2+(
34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/2*(
34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680
/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*
11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49-34/49*11^(1/2
))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1
/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(2
50/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11
^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(
245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250
```

-34*11^(1/2))^(1/2))))+20/(250/49-34/49*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7+
1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(
1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*(1/20*(10
x+2)(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1
/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(
1/2))^2)^(1/2)*(x+1/5)))))+15/686/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/
7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/
2))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2
+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/2
00*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)
/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/4
9-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*
(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/
2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2
)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(
500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11
^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/
7*11^(1/2))+250-34*11^(1/2))^(1/2)))))+3535/21296*11^(1/2)*(1/21*(5*(x-2/7+
1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(
1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2)
)^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+
1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1
/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7
*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2)
)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^
2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh
(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(25
0-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x
-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-(-3535/1936+273/1936*11^(1/2))
*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2)
)^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(5/2)+
3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/7*11^(1/
2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2
)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000
/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49
-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49-34/49*
11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1
/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsi
nh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5
))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-6
8/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^
(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/

```
2))+250-34*11^(1/2))^(1/2))))+20/49/(250/49-34/49*11^(1/2))*(1/40*(10*x+2)*
(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-
34/49*11^(1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)
*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*1
1^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-
10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7
-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))-(-3535/1936-273/1936*11^(1/2))*(-1/49/
(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/
7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(5/2)+3/98*(34
/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7-1/7*11^(1/2))^2+(3
4/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/2*(3
4/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/
49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*1
1^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49+34/49*11^(1/2)
)*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1
/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/
2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(25
0/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^
(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(2
45*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+
34*11^(1/2))^(1/2))))+20/49/(250/49+34/49*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/
7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11
^(1/2))^(3/2)+3/80*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*(1/20*(
10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^
(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11
^(1/2))^2)^(1/2)*(x+1/5)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(5 x^2+2 x+3\right)^{\frac{3}{2}}\left(x^2+5 x+2\right)}{\left(7 x^2-4 x-1\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

Fricas [B] time = 1.72209, size = 1914, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1855350112*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(20854407
42055*sqrt(11) + 62294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt
(2085440742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) +
5426671202560069*sqrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345
)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(2085440742055*sq
rt(11) + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(208544
0742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) - 5426671
202560069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) +
sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11)
 + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11
) + 83479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) + 542667120256
0069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) - sqrt(
2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11) + 62
294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11) + 83
479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) - 5426671202560069*s
qrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345)/x) - 13522960*sq
rt(5)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)
*(5*x + 1) - 25*x^2 - 10*x - 8) + 78232*(189161*x^3 - 246464*x^2 - 42767*x
+ 7416)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.386 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=185

$$-\frac{343}{40}\sqrt{5x^2+2x+3}x^7-\frac{1141}{40}\sqrt{5x^2+2x+3}x^6+\frac{26159}{300}\sqrt{5x^2+2x+3}x^5-\frac{47807\sqrt{5x^2+2x+3}x^4}{3750}-\frac{5160533\sqrt{5x^2+2}}{50000}$$

```
[Out] (-16515809*Sqrt[3 + 2*x + 5*x^2])/156250 + (5793077*x*Sqrt[3 + 2*x + 5*x^2]
)/75000 + (40722851*x^2*Sqrt[3 + 2*x + 5*x^2])/750000 - (5160533*x^3*Sqrt[3
 + 2*x + 5*x^2])/50000 - (47807*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 + (26159*x^
5*Sqrt[3 + 2*x + 5*x^2])/300 - (1141*x^6*Sqrt[3 + 2*x + 5*x^2])/40 - (343*x
^7*Sqrt[3 + 2*x + 5*x^2])/40 - (77513689*ArcSinh[(1 + 5*x)/Sqrt[14]])/(6250
00*Sqrt[5])
```

Rubi [A] time = 0.312332, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1661, 640, 619, 215}

$$-\frac{343}{40}\sqrt{5x^2+2x+3}x^7-\frac{1141}{40}\sqrt{5x^2+2x+3}x^6+\frac{26159}{300}\sqrt{5x^2+2x+3}x^5-\frac{47807\sqrt{5x^2+2x+3}x^4}{3750}-\frac{5160533\sqrt{5x^2+2}}{50000}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```



```
[Out] (-16515809*Sqrt[3 + 2*x + 5*x^2])/156250 + (5793077*x*Sqrt[3 + 2*x + 5*x^2]
)/75000 + (40722851*x^2*Sqrt[3 + 2*x + 5*x^2])/750000 - (5160533*x^3*Sqrt[3
 + 2*x + 5*x^2])/50000 - (47807*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 + (26159*x^
5*Sqrt[3 + 2*x + 5*x^2])/300 - (1141*x^6*Sqrt[3 + 2*x + 5*x^2])/40 - (343*x
^7*Sqrt[3 + 2*x + 5*x^2])/40 - (77513689*ArcSinh[(1 + 5*x)/Sqrt[14]])/(6250
00*Sqrt[5])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
```

```
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{343}{40}x^7\sqrt{3+2x+5x^2}+\frac{1}{40}\int \frac{80+1160x+4600x^2-2440x^3-34840x^4+5080}{\sqrt{3+2x+5x^2}}$$

$$= -\frac{1141}{40}x^6\sqrt{3+2x+5x^2}-\frac{343}{40}x^7\sqrt{3+2x+5x^2}+\frac{\int \frac{2800+40600x+161000x^2-85400x^3-12}{\sqrt{3+2x+5}}}{1400}$$

$$= \frac{26159}{300}x^5\sqrt{3+2x+5x^2}-\frac{1141}{40}x^6\sqrt{3+2x+5x^2}-\frac{343}{40}x^7\sqrt{3+2x+5x^2}+\frac{\int \frac{84000}{}}{}$$

$$= -\frac{47807x^4\sqrt{3+2x+5x^2}}{3750}+\frac{26159}{300}x^5\sqrt{3+2x+5x^2}-\frac{1141}{40}x^6\sqrt{3+2x+5x^2}-\frac{343}{40}$$

$$= -\frac{5160533x^3\sqrt{3+2x+5x^2}}{50000}-\frac{47807x^4\sqrt{3+2x+5x^2}}{3750}+\frac{26159}{300}x^5\sqrt{3+2x+5x^2}-$$

$$= \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000}-\frac{5160533x^3\sqrt{3+2x+5x^2}}{50000}-\frac{47807x^4\sqrt{3+2x+5x^2}}{3750}$$

$$= \frac{5793077x\sqrt{3+2x+5x^2}}{75000}+\frac{40722851x^2\sqrt{3+2x+5x^2}}{750000}-\frac{5160533x^3\sqrt{3+2x+5x}}{50000}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250}+\frac{5793077x\sqrt{3+2x+5x^2}}{75000}+\frac{40722851x^2\sqrt{3+2x+5}}{750000}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250}+\frac{5793077x\sqrt{3+2x+5x^2}}{75000}+\frac{40722851x^2\sqrt{3+2x+5}}{750000}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250}+\frac{5793077x\sqrt{3+2x+5x^2}}{75000}+\frac{40722851x^2\sqrt{3+2x+5}}{750000}$$

Mathematica [A] time = 0.261152, size = 75, normalized size = 0.41

$$\frac{-5\sqrt{5x^2+2x+3}\left(32156250x^7+106968750x^6-326987500x^5+47807000x^4+387039975x^3-203614255x^2-289653\right.}{18750000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(396379416 - 289653850*x - 203614255*x^2 + 387039
975*x^3 + 47807000*x^4 - 326987500*x^5 + 106968750*x^6 + 32156250*x^7) - 46
```

```
5082134*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/18750000
```

Maple [A] time = 0.067, size = 147, normalized size = 0.8

$$-\frac{343 x^7}{40}\sqrt{5 x^2+2 x+3}-\frac{1141 x^6}{40}\sqrt{5 x^2+2 x+3}+\frac{26159 x^5}{300}\sqrt{5 x^2+2 x+3}-\frac{47807 x^4}{3750}\sqrt{5 x^2+2 x+3}-\frac{77513689}{3125000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -343/40*x^7*(5*x^2+2*x+3)^(1/2)-1141/40*x^6*(5*x^2+2*x+3)^(1/2)+26159/300*x
^5*(5*x^2+2*x+3)^(1/2)-47807/3750*x^4*(5*x^2+2*x+3)^(1/2)-77513689/3125000*
5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-5160533/50000*x^3*(5*x^2+2*x+3)^(1/2
)+5793077/75000*x*(5*x^2+2*x+3)^(1/2)+40722851/750000*x^2*(5*x^2+2*x+3)^(1/
2)-16515809/156250*(5*x^2+2*x+3)^(1/2)
```

Maxima [A] time = 1.53147, size = 200, normalized size = 1.08

$$-\frac{343}{40}\sqrt{5 x^2+2 x+3}x^7-\frac{1141}{40}\sqrt{5 x^2+2 x+3}x^6+\frac{26159}{300}\sqrt{5 x^2+2 x+3}x^5-\frac{47807}{3750}\sqrt{5 x^2+2 x+3}x^4-\frac{5160533}{50000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/40*sqrt(5*x^2 + 2*x + 3)*x^7 - 1141/40*sqrt(5*x^2 + 2*x + 3)*x^6 + 261
59/300*sqrt(5*x^2 + 2*x + 3)*x^5 - 47807/3750*sqrt(5*x^2 + 2*x + 3)*x^4 - 5
160533/50000*sqrt(5*x^2 + 2*x + 3)*x^3 + 40722851/750000*sqrt(5*x^2 + 2*x +
 3)*x^2 + 5793077/75000*sqrt(5*x^2 + 2*x + 3)*x - 77513689/3125000*sqrt(5)*
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 16515809/156250*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.40984, size = 332, normalized size = 1.79

$$-\frac{1}{3750000}\left(32156250 x^7+106968750 x^6-326987500 x^5+47807000 x^4+387039975 x^3-203614255 x^2-28965385\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/3750000*(32156250*x^7 + 106968750*x^6 - 326987500*x^5 + 47807000*x^4 + 3
87039975*x^3 - 203614255*x^2 - 289653850*x + 396379416)*sqrt(5*x^2 + 2*x +
3) + 77513689/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) -
 25*x^2 - 10*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{29x}{\sqrt{5x^2+2x+3}}\,dx - \int -\frac{115x^2}{\sqrt{5x^2+2x+3}}\,dx - \int \frac{61x^3}{\sqrt{5x^2+2x+3}}\,dx - \int \frac{871x^4}{\sqrt{5x^2+2x+3}}\,dx - \int -\frac{127x^5}{\sqrt{5x^2+2x+3}}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-29*x/sqrt(5*x**2 + 2*x + 3), x) - Integral(-115*x**2/sqrt(5*x**2
 + 2*x + 3), x) - Integral(61*x**3/sqrt(5*x**2 + 2*x + 3), x) - Integral(87
1*x**4/sqrt(5*x**2 + 2*x + 3), x) - Integral(-127*x**5/sqrt(5*x**2 + 2*x +
3), x) - Integral(-2065*x**6/sqrt(5*x**2 + 2*x + 3), x) - Integral(1127*x**
7/sqrt(5*x**2 + 2*x + 3), x) - Integral(343*x**8/sqrt(5*x**2 + 2*x + 3), x)
 - Integral(-2/sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A] time = 1.16367, size = 111, normalized size = 0.6

$$-\frac{1}{3750000}\,(5\,((5\,(10\,(175\,(15\,(49\,x+163)x-7474)x+191228)x+15481599)x-40722851)x-57930770)x+39637941$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/3750000*(5*((5*(10*(175*(15*(49*x + 163)*x - 7474)*x + 191228)*x + 15481
599)*x - 40722851)*x - 57930770)*x + 396379416)*sqrt(5*x^2 + 2*x + 3) + 775
13689/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.387 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=143

$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5+\frac{5131}{750}\sqrt{5x^2+2x+3}x^4-\frac{33259\sqrt{5x^2+2x+3}x^3}{2500}-\frac{207427\sqrt{5x^2+2x+3}x^2}{37500}+\frac{36073\sqrt{5x^2+2x}}{1875}$$

```
[Out] (-22053*Sqrt[3 + 2*x + 5*x^2])/31250 + (36073*x*Sqrt[3 + 2*x + 5*x^2])/1875
 - (207427*x^2*Sqrt[3 + 2*x + 5*x^2])/37500 - (33259*x^3*Sqrt[3 + 2*x + 5*x
^2])/2500 + (5131*x^4*Sqrt[3 + 2*x + 5*x^2])/750 + (49*x^5*Sqrt[3 + 2*x + 5
*x^2])/30 - (1719097*ArcSinh[(1 + 5*x)/Sqrt[14]])/(31250*Sqrt[5])
```

Rubi [A] time = 0.202813, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1661, 640, 619, 215}

$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5+\frac{5131}{750}\sqrt{5x^2+2x+3}x^4-\frac{33259\sqrt{5x^2+2x+3}x^3}{2500}-\frac{207427\sqrt{5x^2+2x+3}x^2}{37500}+\frac{36073\sqrt{5x^2+2x}}{1875}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```


```
[Out] (-22053*Sqrt[3 + 2*x + 5*x^2])/31250 + (36073*x*Sqrt[3 + 2*x + 5*x^2])/1875
 - (207427*x^2*Sqrt[3 + 2*x + 5*x^2])/37500 - (33259*x^3*Sqrt[3 + 2*x + 5*x
^2])/2500 + (5131*x^4*Sqrt[3 + 2*x + 5*x^2])/750 + (49*x^5*Sqrt[3 + 2*x + 5
*x^2])/30 - (1719097*ArcSinh[(1 + 5*x)/Sqrt[14]])/(31250*Sqrt[5])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)}{\sqrt{3+2x+5x^2}}\,dx = \frac{49}{30}x^5\sqrt{3+2x+5x^2}+\frac{1}{30}\int \frac{60+630x+1350x^2-2820x^3-6135x^4+5131x^5}{\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{5131}{750}x^4\sqrt{3+2x+5x^2}+\frac{49}{30}x^5\sqrt{3+2x+5x^2}+\frac{1}{750}\int \frac{1500+15750x+33750x^2}{\sqrt{3+2x}}$$

$$= -\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5131}{750}x^4\sqrt{3+2x+5x^2}+\frac{49}{30}x^5\sqrt{3+2x+5x^2}+\frac{\int \frac{3000}{}}{}$$

$$= -\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}-\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5131}{750}x^4\sqrt{3+2x+5x^2}+\frac{4}{3}$$

$$= \frac{36073x\sqrt{3+2x+5x^2}}{1875}-\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}-\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{51}{75}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875}-\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}-\frac{332}{}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875}-\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}-\frac{332}{}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875}-\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}-\frac{332}{}$$

Mathematica [A] time = 0.153379, size = 65, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3}\left(306250x^5+1282750x^4-2494425x^3-1037135x^2+3607300x-132318\right)-10314582\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{937500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(-132318 + 3607300*x - 1037135*x^2 - 2494425*x^3 +
 1282750*x^4 + 306250*x^5) - 10314582*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/
937500
```

Maple [A] time = 0.058, size = 113, normalized size = 0.8

$$\frac{49\,x^5}{30}\sqrt{5\,x^2+2\,x+3}+\frac{5131\,x^4}{750}\sqrt{5\,x^2+2\,x+3}-\frac{1719097\,\sqrt{5}}{156250}\mathrm{Arcsinh}\left(\frac{5\,\sqrt{14}}{14}\left(x+\frac{1}{5}\right)\right)-\frac{33259\,x^3}{2500}\sqrt{5\,x^2+2\,x+3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] 49/30*x^5*(5*x^2+2*x+3)^(1/2)+5131/750*x^4*(5*x^2+2*x+3)^(1/2)-1719097/1562
50*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-33259/2500*x^3*(5*x^2+2*x+3)^(1/2
)+36073/1875*x*(5*x^2+2*x+3)^(1/2)-207427/37500*x^2*(5*x^2+2*x+3)^(1/2)-220
53/31250*(5*x^2+2*x+3)^(1/2)
```

Maxima [A] time = 1.45245, size = 154, normalized size = 1.08

$$\frac{49}{30}\sqrt{5\,x^2+2\,x+3}x^5+\frac{5131}{750}\sqrt{5\,x^2+2\,x+3}x^4-\frac{33259}{2500}\sqrt{5\,x^2+2\,x+3}x^3-\frac{207427}{37500}\sqrt{5\,x^2+2\,x+3}x^2+\frac{36073}{1875}\sqrt{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/30*sqrt(5*x^2 + 2*x + 3)*x^5 + 5131/750*sqrt(5*x^2 + 2*x + 3)*x^4 - 3325
9/2500*sqrt(5*x^2 + 2*x + 3)*x^3 - 207427/37500*sqrt(5*x^2 + 2*x + 3)*x^2 +
```

```
36073/1875*sqrt(5*x^2 + 2*x + 3)*x - 1719097/156250*sqrt(5)*arcsinh(1/14*s
qrt(14)*(5*x + 1)) - 22053/31250*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.36288, size = 267, normalized size = 1.87

$$\frac{1}{187500}\left(306250\,x^5+1282750\,x^4-2494425\,x^3-1037135\,x^2+3607300\,x-132318\right)\sqrt{5\,x^2+2\,x+3}+\frac{1719097}{312500}\,\sqrt{5}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/187500*(306250*x^5 + 1282750*x^4 - 2494425*x^3 - 1037135*x^2 + 3607300*x
- 132318)*sqrt(5*x^2 + 2*x + 3) + 1719097/312500*sqrt(5)*log(sqrt(5)*sqrt(5
*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+5x+2\right)\left(7x^2-4x-1\right)^2}{\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A] time = 1.19185, size = 97, normalized size = 0.68

$$\frac{1}{187500}\left(5\left(\left(5\left(70\left(175\,x+733\right)x-99777\right)x-207427\right)x+721460\right)x-132318\right)\sqrt{5\,x^2+2\,x+3}+\frac{1719097}{156250}\,\sqrt{5}\log\left(-\sqrt{5}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/187500*(5*((5*(70*(175*x + 733)*x - 99777)*x - 207427)*x + 721460)*x - 13
2318)*sqrt(5*x^2 + 2*x + 3) + 1719097/156250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.388 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{7}{20}\sqrt{5x^2+2x+3}\,x^3-\frac{571}{300}\sqrt{5x^2+2x+3}\,x^2+\frac{59}{30}\sqrt{5x^2+2x+3}\,x+\frac{463}{125}\sqrt{5x^2+2x+3}-\frac{1901\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}$$

```
[Out] (463*Sqrt[3 + 2*x + 5*x^2])/125 + (59*x*Sqrt[3 + 2*x + 5*x^2])/30 - (571*x^
2*Sqrt[3 + 2*x + 5*x^2])/300 - (7*x^3*Sqrt[3 + 2*x + 5*x^2])/20 - (1901*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/(250*Sqrt[5])
```

Rubi [A] time = 0.145073, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1661, 640, 619, 215}

$$-\frac{7}{20}\sqrt{5x^2+2x+3}\,x^3-\frac{571}{300}\sqrt{5x^2+2x+3}\,x^2+\frac{59}{30}\sqrt{5x^2+2x+3}\,x+\frac{463}{125}\sqrt{5x^2+2x+3}-\frac{1901\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (463*Sqrt[3 + 2*x + 5*x^2])/125 + (59*x*Sqrt[3 + 2*x + 5*x^2])/30 - (571*x^
2*Sqrt[3 + 2*x + 5*x^2])/300 - (7*x^3*Sqrt[3 + 2*x + 5*x^2])/20 - (1901*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/(250*Sqrt[5])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx &= -\frac{7}{20}x^3\sqrt{3+2x+5x^2}+\frac{1}{20}\int \frac{40+260x+203x^2-571x^3}{\sqrt{3+2x+5x^2}}\,dx\\ &= -\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x+5x^2}+\frac{1}{300}\int \frac{600+7326x+5900x^2}{\sqrt{3+2x+5x^2}}\,dx\\ &= \frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x+5x^2}+\frac{\int \frac{-11700+5556}{\sqrt{3+2x+5x^2}}}{3000}\\ &= \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x}\\ &= \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x}\\ &= \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x}\end{aligned}$$

Mathematica [A] time = 0.0860674, size = 55, normalized size = 0.54

$$\frac{-5\sqrt{5x^2+2x+3}\left(525x^3+2855x^2-2950x-5556\right)-11406\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{7500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(-5556 - 2950*x + 2855*x^2 + 525*x^3) - 11406*Sqr
t[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/7500
```

Maple [A] time = 0.052, size = 79, normalized size = 0.8

$$-\frac{7x^3}{20}\sqrt{5x^2+2x+3}-\frac{571x^2}{300}\sqrt{5x^2+2x+3}+\frac{59x}{30}\sqrt{5x^2+2x+3}+\frac{463}{125}\sqrt{5x^2+2x+3}-\frac{1901\sqrt{5}}{1250}\mathrm{Arcsinh}\left(\frac{5\sqrt{14}}{14}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -7/20*x^3*(5*x^2+2*x+3)^(1/2)-571/300*x^2*(5*x^2+2*x+3)^(1/2)+59/30*x*(5*x^
2+2*x+3)^(1/2)+463/125*(5*x^2+2*x+3)^(1/2)-1901/1250*5^(1/2)*arcsinh(5/14*1
4^(1/2)*(x+1/5))
```

Maxima [A] time = 1.53065, size = 108, normalized size = 1.07

$$-\frac{7}{20}\sqrt{5x^2+2x+3}x^3-\frac{571}{300}\sqrt{5x^2+2x+3}x^2+\frac{59}{30}\sqrt{5x^2+2x+3}x-\frac{1901}{1250}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{463}{125}\sqrt{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/20*sqrt(5*x^2 + 2*x + 3)*x^3 - 571/300*sqrt(5*x^2 + 2*x + 3)*x^2 + 59/30
*sqrt(5*x^2 + 2*x + 3)*x - 1901/1250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1
)) + 463/125*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.24932, size = 207, normalized size = 2.05

$$-\frac{1}{1500}\left(525x^3+2855x^2-2950x-5556\right)\sqrt{5x^2+2x+3}+\frac{1901}{2500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/1500*(525*x^3 + 2855*x^2 - 2950*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/2
500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{13x}{\sqrt{5x^2+2x+3}}\,dx-\int -\frac{7x^2}{\sqrt{5x^2+2x+3}}\,dx-\int \frac{31x^3}{\sqrt{5x^2+2x+3}}\,dx-\int \frac{7x^4}{\sqrt{5x^2+2x+3}}\,dx-\int -\frac{2}{\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-13*x/sqrt(5*x**2 + 2*x + 3), x) - Integral(-7*x**2/sqrt(5*x**2 +
 2*x + 3), x) - Integral(31*x**3/sqrt(5*x**2 + 2*x + 3), x) - Integral(7*x*
*4/sqrt(5*x**2 + 2*x + 3), x) - Integral(-2/sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A] time = 1.19552, size = 84, normalized size = 0.83

$$-\frac{1}{1500}\left(5\left(\left(105\,x+571\right)x-590\right)x-5556\right)\sqrt{5\,x^2+2\,x+3}+\frac{1901}{1250}\,\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/1500*(5*((105*x + 571)*x - 590)*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/1
250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.389 $\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=164

$$-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}}\right.$$

```
[Out] -ArcSinh[(1 + 5*x)/Sqrt[14]]/(7*Sqrt[5]) - (3*Sqrt[(4091 - 1055*Sqrt[11])/2
794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[1
1])]*Sqrt[3 + 2*x + 5*x^2])])/14 + (3*Sqrt[(4091 + 1055*Sqrt[11])/2794]*Arc
Tanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqr
t[3 + 2*x + 5*x^2])])/14
```

Rubi [A] time = 0.230132, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1076, 619, 215, 1032, 724, 206}

$$-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+}{\sqrt{2\left(125+17\sqrt{11}\right)}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] -ArcSinh[(1 + 5*x)/Sqrt[14]]/(7*Sqrt[5]) - (3*Sqrt[(4091 - 1055*Sqrt[11])/2
794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[1
1])]*Sqrt[3 + 2*x + 5*x^2])])/14 + (3*Sqrt[(4091 + 1055*Sqrt[11])/2794]*Arc
Tanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqr
t[3 + 2*x + 5*x^2])])/14
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = -\left(\frac{1}{7}\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx\right) - \frac{1}{7}\int \frac{-15-39x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{14\sqrt{70}} + \frac{1}{77}\left(3\left(143-61\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{\vphantom{x}}}$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} - \frac{1}{77}\left(6\left(143-61\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(\right.}\right.$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} - \frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x}}\right.$$

Mathematica [A] time = 0.484516, size = 157, normalized size = 0.96

$$-\frac{3\left(\sqrt{4091-1055\sqrt{11}}\tanh^{-1}\left(\frac{-5\sqrt{11}x+17x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right)-\sqrt{4091+1055\sqrt{11}}\tanh^{-1}\left(\frac{5\sqrt{11}x+17x+\sqrt{11}+23}{\sqrt{250+34\sqrt{11}}\sqrt{5x^2+2x+3}}\right)\right)}{14\sqrt{2794}} - \frac{\sinh^{-1}\left(\frac{5x+}{\sqrt{1}}\right.}{7\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] -ArcSinh[(1 + 5*x)/Sqrt[14]]/(7*Sqrt[5]) - (3*(Sqrt[4091 - 1055*Sqrt[11]]*A
rcTanh[(23 - Sqrt[11] + 17*x - 5*Sqrt[11]*x)/(Sqrt[250 - 34*Sqrt[11]]*Sqrt[
3 + 2*x + 5*x^2])] - Sqrt[4091 + 1055*Sqrt[11]]*ArcTanh[(23 + Sqrt[11] + 17
*x + 5*Sqrt[11]*x)/(Sqrt[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])]))/(14*S
qrt[2794])
```

Maple [A] time = 0.106, size = 204, normalized size = 1.2

$$-\frac{\sqrt{5}}{35}\text{Arcsinh}\left(\frac{5\sqrt{14}}{14}\left(x+\frac{1}{5}\right)\right) + \frac{\left(183+39\sqrt{11}\right)\sqrt{11}}{154\sqrt{250+34\sqrt{11}}}\text{Artanh}\left(\frac{49}{2\sqrt{250+34\sqrt{11}}}\left(\frac{500}{49}+\frac{68\sqrt{11}}{49}+\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -1/35*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+3/154*(61+13*11^(1/2))*11^(1/2
)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11
^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1
/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
+3/154*(-61+13*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500
/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1
/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*1
1^(1/2))+250-34*11^(1/2))^(1/2))
```

Maxima [B] time = 1.65272, size = 628, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/10780*sqrt(11)*(28*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/1
4*sqrt(7)*sqrt(2)) - 1365*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqr
t(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqr
t(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7
*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11) + 125) + 390*
sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4)
- 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*
sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt
(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49) - 6405*sqrt(2)*arcsinh(5/7*sqrt(1
1)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/ab
s(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(1
1) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11)
 + 125) - 1830*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49))
```

Fricas [B] time = 1.54786, size = 1126, normalized size = 6.87

$$-\frac{3}{78232}\sqrt{2794}\sqrt{1055\sqrt{11}+4091}\log\left(\frac{3\left(\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{1055\sqrt{11}+4091}\left(172\sqrt{11}-715\right)+185801\sqrt{11}(x+3\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -3/78232*sqrt(2794)*sqrt(1055*sqrt(11) + 4091)*log(3*(sqrt(2794)*sqrt(5*x^2
 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)*(172*sqrt(11) - 715) + 185801*sqrt(1
1)*(x + 3) + 557403*x - 929005)/x) + 3/78232*sqrt(2794)*sqrt(1055*sqrt(11)
+ 4091)*log(-3*(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)
*(172*sqrt(11) - 715) - 185801*sqrt(11)*(x + 3) - 557403*x + 929005)/x) - 1
/78232*sqrt(2794)*sqrt(-9495*sqrt(11) + 36819)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(172*sqrt(11) + 715)*sqrt(-9495*sqrt(11) + 36819) + 557403*sqrt(
11)*(x + 3) - 1672209*x + 2787015)/x) + 1/78232*sqrt(2794)*sqrt(-9495*sqrt(
11) + 36819)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(172*sqrt(11) + 715)*sqr
t(-9495*sqrt(11) + 36819) - 557403*sqrt(11)*(x + 3) + 1672209*x - 2787015)/
x) + 1/70*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{5x}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx-\int\frac{x^2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(5*x/(7*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) -
 sqrt(5*x**2 + 2*x + 3)), x) - Integral(x**2/(7*x**2*sqrt(5*x**2 + 2*x + 3)
 - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(7
*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2
*x + 3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.390 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=178

$$-\frac{3\sqrt{5x^2+2x+3}(40-371x)}{5588\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{11176}+\frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}}\tanh\ldots}{111\ldots}$$

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) - (Sqrt[(3
027900955 + 14035681*Sqrt[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[
11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176 + (Sqrt
[(3027900955 - 14035681*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sq
rt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176
```

Rubi [A] time = 0.195938, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{3\sqrt{5x^2+2x+3}(40-371x)}{5588\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{11176}+\frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}}\tanh\ldots}{111\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```


```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) - (Sqrt[(3
027900955 + 14035681*Sqrt[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[
11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176 + (Sqrt
[(3027900955 - 14035681*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sq
rt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176
```


Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
```

```
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\int \frac{-52136-29544x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{44704}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\left(-40623+53005\sqrt{11}\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,d}{61468}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\left(40623-53005\sqrt{11}\right)\operatorname{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+}\right.}{30}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\right.}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3}}\right.}{11176}$$

Mathematica [A] time = 1.02412, size = 313, normalized size = 1.76

$$\frac{\frac{48972\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1} + \frac{5280\sqrt{5x^2+2x+3}}{7x^2-4x-1} + 53005\sqrt{\frac{22}{125+17\sqrt{11}}}\log\left(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3} + \left(55+17\sqrt{11}\right)x + 23\sqrt{11} + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] ((48972*x*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (5280*Sqrt[3 + 2*x + 5
*x^2])/(-1 - 4*x + 7*x^2) + Sqrt[2/(125 - 17*Sqrt[11])]*(-40623 + 53005*Sqr
t[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[
11] + (-17 + 5*Sqrt[11])*x)] - Sqrt[2/(125 + 17*Sqrt[11])]*(40623 + 53005*S
qrt[11])*Log[2 + Sqrt[11] - 7*x] + 40623*Sqrt[2/(125 + 17*Sqrt[11])]*Log[11
 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 +
2*x + 5*x^2]] + 53005*Sqrt[22/(125 + 17*Sqrt[11])]*Log[11 + 23*Sqrt[11] + (
55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]])/245
872
```

Maple [B] time = 0.117, size = 510, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] 161/484*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2
)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(
x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*1
1^(1/2))^(1/2))+(183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2
/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*
11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/14*(34/7+10/7*11^(1/2))/(250/49+34
/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2
/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(
1/2))^(1/2)))-161/484*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49
-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2)
)^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2))+250-34*11^(1/2))^(1/2))+(183/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*
11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2
))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/14*(34/7-10/7*11^(1/
2))/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68
/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(
1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2
))+250-34*11^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+5x+2}{\left(7x^2-4x-1\right)^2\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*sqrt(5*x^2 + 2*x + 3)), x)
```

Fricas [B] time = 1.36634, size = 1393, normalized size = 7.83

$$-\frac{\sqrt{2794}\left(7\,x^2-4\,x-1\right)\sqrt{14035681\,\sqrt{11}+3027900955}\log\left(-\frac{\sqrt{2794}\sqrt{5\,x^2+2\,x+3}\sqrt{14035681\,\sqrt{11}+3027900955}\left(71796\,\sqrt{11}+567523\right)+2653}{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/62451488*(sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(11) + 30279009
55)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(11) + 3027900
955)*(71796*sqrt(11) + 567523) + 265381033753*sqrt(11)*(x + 3) - 7961431012
59*x + 1326905168765)/x) - sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(
11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(
11) + 3027900955)*(71796*sqrt(11) + 567523) - 265381033753*sqrt(11)*(x + 3)
 + 796143101259*x - 1326905168765)/x) + sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(-
14035681*sqrt(11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(7179
6*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955) + 265381033753*s
qrt(11)*(x + 3) + 796143101259*x - 1326905168765)/x) - sqrt(2794)*(7*x^2 -
4*x - 1)*sqrt(-14035681*sqrt(11) + 3027900955)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955)
- 265381033753*sqrt(11)*(x + 3) - 796143101259*x + 1326905168765)/x) + 3352
8*sqrt(5*x^2 + 2*x + 3)*(371*x - 40))/(7*x^2 - 4*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+5x+2}{\sqrt{5x^2+2x+3}\left(7x^2-4x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)/(sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2), x
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.391 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=227

$$-\frac{7\sqrt{5x^2+2x+3}(409769-1189370x)}{62451488\left(-7x^2+4x+1\right)}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}-\frac{7\left(39370231-2538725\sqrt{11}\right)\tanh^{-1}\left(\frac{(17-5\sqrt{1}}{\sqrt{2(125-17}}\right.}{124902976\sqrt{22\left(125-17\sqrt{11}\right)}}$$

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(11176*(1 + 4*x - 7*x^2)^2) - (7*(4
09769 - 1189370*x)*Sqrt[3 + 2*x + 5*x^2])/(62451488*(1 + 4*x - 7*x^2)) - (7
*(39370231 - 2538725*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x
)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(124902976*Sqrt[22*
(125 - 17*Sqrt[11])]) + (7*(39370231 + 2538725*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(124902976*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rubi [A] time = 0.271138, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{7\sqrt{5x^2+2x+3}(409769-1189370x)}{62451488\left(-7x^2+4x+1\right)}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}-\frac{7\left(39370231-2538725\sqrt{11}\right)\tanh^{-1}\left(\frac{(17-5\sqrt{1}}{\sqrt{2(125-17}}\right.}{124902976\sqrt{22\left(125-17\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```



```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(11176*(1 + 4*x - 7*x^2)^2) - (7*(4
09769 - 1189370*x)*Sqrt[3 + 2*x + 5*x^2])/(62451488*(1 + 4*x - 7*x^2)) - (7
*(39370231 - 2538725*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x
)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(124902976*Sqrt[22*
(125 - 17*Sqrt[11])]) + (7*(39370231 + 2538725*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(124902976*Sqrt[22*(125 + 17*Sqrt[11])])
```


Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\sqrt{3+2x+5x^2}}\,dx = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{\int \frac{-130024-81000x-89040x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx}{89408}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} + \frac{\int \frac{219473}{(1+4x}}{3}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} + \frac{(7(279}{}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} - \frac{(7(2792}{}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} - \frac{7(3937}{}$$

Mathematica [A] time = 1.28187, size = 371, normalized size = 1.63

$$\frac{\frac{732651920\sqrt{5x^2+2x+3}x}{-7x^2+4x+1} + \frac{547311072\sqrt{5x^2+2x+3}x}{\left(-7x^2+4x+1\right)^2} - \frac{59009280\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2} + \frac{252417704\sqrt{5x^2+2x+3}}{7x^2-4x-1} + 551183234\sqrt{\frac{22}{125+17\sqrt{11}}}\log\left(\sqrt{2750+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] ((-59009280*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (547311072*x*Sqrt[
3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (732651920*x*Sqrt[3 + 2*x + 5*x^2])
/(1 + 4*x - 7*x^2) + (252417704*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) +
 14*Sqrt[2/(125 - 17*Sqrt[11])]*(-27925975 + 39370231*Sqrt[11])*ArcTanh[(Sq
rt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] + (-17 + 5*Sqr
t[11])*x)] - 14*Sqrt[2/(125 + 17*Sqrt[11])]*(27925975 + 39370231*Sqrt[11])*
Log[2 + Sqrt[11] - 7*x] + 390963650*Sqrt[2/(125 + 17*Sqrt[11])]*Log[11 + 23
*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x +
 5*x^2]] + 551183234*Sqrt[22/(125 + 17*Sqrt[11])]*Log[11 + 23*Sqrt[11] + (5
5 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]])/5495
```

730944

Maple [B] time = 0.131, size = 1194, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] 3535/21296*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(
1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(24
5*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+3
4*11^(1/2))^(1/2))-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*
11^(1/2))/(x-2/7+1/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1
/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-3/1372*(34/7-10/7*11
^(1/2))/(250/49-34/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(
1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50/49-34/49*11^(1/2))^(1/2)+7/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2)
)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1
/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
)+5/98/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49
-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2)
)^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2))+250-34*11^(1/2))^(1/2)))-21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(25
0/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))^2*(5*(x-2/7-1/7*11^(1/2))^2+(34/7
+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-3/1372*(3
4/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2
/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*
11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+7/2*(34/7+10/7*11^(1/2))/(250/49+34/
49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(3
4/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/
7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1
/2))^(1/2)))+5/98/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(4
9/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+
34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-3535/21296*11^(1/2)/(250-34*11^(1
/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+
1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7
-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))-(-3535/1936-27
3/1936*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-
```

```
2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*
11^(1/2))^(1/2)+1/14*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))/(250+34*1
1^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-
2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-(-3535/19
36+273/1936*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(
5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-3
4/49*11^(1/2))^(1/2)+1/14*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))/(250
-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2)
)*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2
+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2+5x+2}{\left(7x^2-4x-1\right)^3\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*sqrt(5*x^2 + 2*x + 3)), x)
```

Fricas [B] time = 1.56052, size = 1897, normalized size = 8.36

$$-\frac{\sqrt{2794}\left(49x^4-56x^3+2x^2+8x+1\right)\sqrt{1283973697005131\sqrt{11}+82616280769148425}\log\left(-\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{128397369}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/697957829888*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(128397
3697005131*sqrt(11) + 82616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x
+ 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(358684877*sqrt(11
) + 2940638404) + 7232150972206110797*sqrt(11)*(x + 3) - 216964529166183323
```

```
91*x + 36160754861030553985)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x
 + 1)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*log((sqrt(2794)*s
qrt(5*x^2 + 2*x + 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(3
58684877*sqrt(11) + 2940638404) - 7232150972206110797*sqrt(11)*(x + 3) + 21
696452916618332391*x - 36160754861030553985)/x) + sqrt(2794)*(49*x^4 - 56*x
^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425)*
log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11) - 2940638404)*sqr
t(-1283973697005131*sqrt(11) + 82616280769148425) + 7232150972206110797*sqr
t(11)*(x + 3) + 21696452916618332391*x - 36160754861030553985)/x) - sqrt(27
94)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 8
2616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11
) - 2940638404)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425) - 7232
150972206110797*sqrt(11)*(x + 3) - 21696452916618332391*x + 361607548610305
53985)/x) + 11176*(58279130*x^3 - 53381041*x^2 - 3071502*x + 3538943)*sqrt(
5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.392 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{343}{150}\sqrt{5x^2+2x+3}x^5-\frac{25921\sqrt{5x^2+2x+3}x^4}{3750}+\frac{393659\sqrt{5x^2+2x+3}x^3}{12500}-\frac{2583293\sqrt{5x^2+2x+3}x^2}{187500}-\frac{3192602\sqrt{5x}}{468}$$

```
[Out] (16*(6122807 - 5338217*x))/(546875*Sqrt[3 + 2*x + 5*x^2]) + (15715799*Sqrt[
3 + 2*x + 5*x^2])/156250 - (3192602*x*Sqrt[3 + 2*x + 5*x^2])/46875 - (25832
93*x^2*Sqrt[3 + 2*x + 5*x^2])/187500 + (393659*x^3*Sqrt[3 + 2*x + 5*x^2])/1
2500 - (25921*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 - (343*x^5*Sqrt[3 + 2*x + 5*x
^2])/150 + (50047657*ArcSinh[(1 + 5*x)/Sqrt[14]])/(156250*Sqrt[5])
```

Rubi [A] time = 0.241002, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1660, 1661, 640, 619, 215}

$$-\frac{343}{150}\sqrt{5x^2+2x+3}x^5-\frac{25921\sqrt{5x^2+2x+3}x^4}{3750}+\frac{393659\sqrt{5x^2+2x+3}x^3}{12500}-\frac{2583293\sqrt{5x^2+2x+3}x^2}{187500}-\frac{3192602\sqrt{5x}}{468}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```



```
[Out] (16*(6122807 - 5338217*x))/(546875*Sqrt[3 + 2*x + 5*x^2]) + (15715799*Sqrt[
3 + 2*x + 5*x^2])/156250 - (3192602*x*Sqrt[3 + 2*x + 5*x^2])/46875 - (25832
93*x^2*Sqrt[3 + 2*x + 5*x^2])/187500 + (393659*x^3*Sqrt[3 + 2*x + 5*x^2])/1
2500 - (25921*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 - (343*x^5*Sqrt[3 + 2*x + 5*x
^2])/150 + (50047657*ArcSinh[(1 + 5*x)/Sqrt[14]])/(156250*Sqrt[5])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
```

```
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)}{\left(3+2x+5x^2\right)^{3/2}}\,dx = \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{1}{28}\int \frac{\frac{473724104}{78125}+\frac{94462228x}{15625}-\frac{40822404x^2}{3125}-\frac{1210328x^3}{625}+}{\sqrt{3+2x+5x^2}}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{1}{840}\int \frac{\frac{2842344624}{15625}+\frac{566773368x}{3125}}{}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \int$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} -$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875}$$

Mathematica [A] time = 0.460733, size = 75, normalized size = 0.45

$$\frac{2102001594\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)-\frac{5\left(75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-315576961\right.}{\sqrt{5x^2+2x+3}}}{32812500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] ((-5*(-3155769618 + 1045703388*x - 2135143465*x^2 + 1795638985*x^3 + 174819
575*x^4 - 897612625*x^5 + 256821250*x^6 + 75031250*x^7))/Sqrt[3 + 2*x + 5*x
```

```
^2] + 2102001594*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/32812500
```

Maple [A] time = 0.069, size = 166, normalized size = 1.

$$\frac{175268451}{390625}\frac{1}{\sqrt{5x^2+2x+3}}+\frac{1025843x^5}{7500}\frac{1}{\sqrt{5x^2+2x+3}}-\frac{998969x^4}{37500}\frac{1}{\sqrt{5x^2+2x+3}}+\frac{61004099x^2}{187500}\frac{1}{\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 175268451/390625/(5*x^2+2*x+3)^(1/2)+1025843/7500*x^5/(5*x^2+2*x+3)^(1/2)-9
98969/37500*x^4/(5*x^2+2*x+3)^(1/2)+61004099/187500*x^2/(5*x^2+2*x+3)^(1/2)
+50047657/781250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+176049701/10937500*
(10*x+2)/(5*x^2+2*x+3)^(1/2)-50047657/156250*x/(5*x^2+2*x+3)^(1/2)-51303971
/187500*x^3/(5*x^2+2*x+3)^(1/2)-343/30*x^7/(5*x^2+2*x+3)^(1/2)-29351/750*x^
6/(5*x^2+2*x+3)^(1/2)
```

Maxima [A] time = 1.48461, size = 200, normalized size = 1.2

$$-\frac{343x^7}{30\sqrt{5x^2+2x+3}}-\frac{29351x^6}{750\sqrt{5x^2+2x+3}}+\frac{1025843x^5}{7500\sqrt{5x^2+2x+3}}-\frac{998969x^4}{37500\sqrt{5x^2+2x+3}}-\frac{51303971x^3}{187500\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/30*x^7/sqrt(5*x^2 + 2*x + 3) - 29351/750*x^6/sqrt(5*x^2 + 2*x + 3) + 1
025843/7500*x^5/sqrt(5*x^2 + 2*x + 3) - 998969/37500*x^4/sqrt(5*x^2 + 2*x +
 3) - 51303971/187500*x^3/sqrt(5*x^2 + 2*x + 3) + 61004099/187500*x^2/sqrt(
5*x^2 + 2*x + 3) + 50047657/781250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1))
 - 87141949/546875*x/sqrt(5*x^2 + 2*x + 3) + 525961603/1093750/sqrt(5*x^2 +
 2*x + 3)
```

Fricas [A] time = 1.44196, size = 386, normalized size = 2.33

$$\frac{1051000797\sqrt{5}\left(5x^2+2x+3\right)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)-5\left(75031250x^7+256821250x^6\right.}{32812500\left(5x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/32812500*(1051000797*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 +
2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(75031250*x^7 + 256821250*x^6 -
 897612625*x^5 + 174819575*x^4 + 1795638985*x^3 - 2135143465*x^2 + 10457033
88*x - 3155769618)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{29x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx-\int -\frac{115x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-29*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-115*x**2/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(61*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(871*x**4/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(-127*x**5/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-2065*x**6/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(1127*x**7/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(343*x**8/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*s
qrt(5*x**2 + 2*x + 3)), x)
```

Giac [A] time = 1.18346, size = 109, normalized size = 0.66

$$-\frac{50047657}{781250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)-\frac{(35((5(35(70(175x+599)x-146549)x+998969)x+513039}{6562500\sqrt{5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -50047657/781250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) -
 1) - 1/6562500*((35*((5*(35*(70*(175*x + 599)*x - 146549)*x + 998969)*x +
51303971)*x - 61004099)*x + 1045703388)*x - 3155769618)/sqrt(5*x^2 + 2*x +
3)
```

3.393 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{49}{100}\sqrt{5x^2+2x+3}x^3+\frac{203}{100}\sqrt{5x^2+2x+3}x^2-\frac{8749\sqrt{5x^2+2x+3}x}{1250}-\frac{5086\sqrt{5x^2+2x+3}}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}+\frac{89}{}$$

```
[Out] (-8*(12983 + 136602*x))/(21875*Sqrt[3 + 2*x + 5*x^2]) - (5086*Sqrt[3 + 2*x
+ 5*x^2])/3125 - (8749*x*Sqrt[3 + 2*x + 5*x^2])/1250 + (203*x^2*Sqrt[3 + 2*
x + 5*x^2])/100 + (49*x^3*Sqrt[3 + 2*x + 5*x^2])/100 + (89583*ArcSinh[(1 +
5*x)/Sqrt[14]])/(1250*Sqrt[5])
```

Rubi [A] time = 0.160835, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{49}{100}\sqrt{5x^2+2x+3}x^3+\frac{203}{100}\sqrt{5x^2+2x+3}x^2-\frac{8749\sqrt{5x^2+2x+3}x}{1250}-\frac{5086\sqrt{5x^2+2x+3}}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}+\frac{89}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```


```
[Out] (-8*(12983 + 136602*x))/(21875*Sqrt[3 + 2*x + 5*x^2]) - (5086*Sqrt[3 + 2*x
+ 5*x^2])/3125 - (8749*x*Sqrt[3 + 2*x + 5*x^2])/1250 + (203*x^2*Sqrt[3 + 2*
x + 5*x^2])/100 + (49*x^3*Sqrt[3 + 2*x + 5*x^2])/100 + (89583*ArcSinh[(1 +
5*x)/Sqrt[14]])/(1250*Sqrt[5])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{1}{28}\int \frac{\frac{4291112}{3125} - \frac{296716x}{625} - \frac{194012x^2}{125} + \frac{23716x^3}{25} + \frac{1372x^4}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{49}{100}x^3\sqrt{3+2x+5x^2} + \frac{1}{560}\int \frac{\frac{17164448}{625} - \frac{1186864x}{125} - \frac{8377}{2}}{\sqrt{3+2x+5}}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{203}{100}x^2\sqrt{3+2x+5x^2} + \frac{49}{100}x^3\sqrt{3+2x+5x^2} + \frac{\int \frac{5149334}{125}}{}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} + \frac{203}{100}x^2\sqrt{3+2x+5x^2} + \frac{49}{100}x^3\sqrt{}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} + \frac{203}{100}x^2\sqrt{3}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} + \frac{203}{100}x^2\sqrt{3}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} + \frac{203}{100}x^2\sqrt{3}$$

Mathematica [A] time = 0.274322, size = 65, normalized size = 0.52

$$\frac{\frac{5(42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536)}{\sqrt{5x^2+2x+3}} + 1254162\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{87500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] ((5*(-168536 - 1298674*x - 280805*x^2 - 515655*x^3 + 194775*x^4 + 42875*x^5
))/Sqrt[3 + 2*x + 5*x^2] + 1254162*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/875
00
```

Maple [A] time = 0.054, size = 132, normalized size = 1.1

$$-\frac{28506}{3125}\frac{1}{\sqrt{5x^2+2x+3}}+\frac{49x^5}{20}\frac{1}{\sqrt{5x^2+2x+3}}+\frac{1113x^4}{100}\frac{1}{\sqrt{5x^2+2x+3}}-\frac{8023x^2}{500}\frac{1}{\sqrt{5x^2+2x+3}}+\frac{89583\sqrt{5}}{6250}\mathrm{Arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -28506/3125/(5*x^2+2*x+3)^(1/2)+49/20*x^5/(5*x^2+2*x+3)^(1/2)+1113/100*x^4/
(5*x^2+2*x+3)^(1/2)-8023/500*x^2/(5*x^2+2*x+3)^(1/2)+89583/6250*5^(1/2)*arc
sinh(5/14*14^(1/2)*(x+1/5))-5564/21875*(10*x+2)/(5*x^2+2*x+3)^(1/2)-89583/1
250*x/(5*x^2+2*x+3)^(1/2)-14733/500*x^3/(5*x^2+2*x+3)^(1/2)
```

Maxima [A] time = 1.57881, size = 154, normalized size = 1.24

$$\frac{49x^5}{20\sqrt{5x^2+2x+3}}+\frac{1113x^4}{100\sqrt{5x^2+2x+3}}-\frac{14733x^3}{500\sqrt{5x^2+2x+3}}-\frac{8023x^2}{500\sqrt{5x^2+2x+3}}+\frac{89583}{6250}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/20*x^5/sqrt(5*x^2 + 2*x + 3) + 1113/100*x^4/sqrt(5*x^2 + 2*x + 3) - 1473
3/500*x^3/sqrt(5*x^2 + 2*x + 3) - 8023/500*x^2/sqrt(5*x^2 + 2*x + 3) + 8958
3/6250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 649337/8750*x/sqrt(5*x^2
+ 2*x + 3) - 42134/4375/sqrt(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.30554, size = 305, normalized size = 2.46

$$\frac{627081\sqrt{5}\left(5x^2+2x+3\right)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)+5\left(42875x^5+194775x^4-515655x^3\right.}{87500\left(5x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/87500*(627081*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3
)*(5*x + 1) - 25*x^2 - 10*x - 8) + 5*(42875*x^5 + 194775*x^4 - 515655*x^3 -
 280805*x^2 - 1298674*x - 168536)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+5x+2\right)\left(7x^2-4x-1\right)^2}{\left(5x^2+2x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/(5*x**2 + 2*x + 3)**(3/2),
x)
```

Giac [A] time = 1.16135, size = 96, normalized size = 0.77

$$-\frac{89583}{6250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)+\frac{(35\,((35\,(35\,(35\,x+159)x-14733)x-8023)x-1298674)x-168536}{17500\,\sqrt{5\,x^2+2\,x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -89583/6250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) +
 1/17500*((35*((35*(35*x + 159)*x - 14733)*x - 8023)*x - 1298674)*x - 16853
6)/sqrt(5*x^2 + 2*x + 3)
```

3.394 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{7}{50}\sqrt{5x^2+2x+3}x-\frac{261}{250}\sqrt{5x^2+2x+3}-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}+\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}$$

```
[Out] (-2*(2321 + 2449*x))/(875*Sqrt[3 + 2*x + 5*x^2]) - (261*Sqrt[3 + 2*x + 5*x^
2])/250 - (7*x*Sqrt[3 + 2*x + 5*x^2])/50 + (149*ArcSinh[(1 + 5*x)/Sqrt[14]]
)/(25*Sqrt[5])
```

Rubi [A] time = 0.0865919, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1660, 1661, 640, 619, 215}

$$-\frac{7}{50}\sqrt{5x^2+2x+3}x-\frac{261}{250}\sqrt{5x^2+2x+3}-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}+\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```


```
[Out] (-2*(2321 + 2449*x))/(875*Sqrt[3 + 2*x + 5*x^2]) - (261*Sqrt[3 + 2*x + 5*x^
2])/250 - (7*x*Sqrt[3 + 2*x + 5*x^2])/50 + (149*ArcSinh[(1 + 5*x)/Sqrt[14]]
)/(25*Sqrt[5])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)\left(2+5x+x^2\right)}{\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}+\frac{1}{28}\int \frac{\frac{15736}{125}-\frac{3948x}{25}-\frac{196x^2}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{1}{280}\int \frac{\frac{34412}{25}-\frac{7308x}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{261}{250}\sqrt{3+2x+5x^2}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{149}{25}\int \frac{1}{\sqrt{3+2x}}$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{261}{250}\sqrt{3+2x+5x^2}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{149\,\mathrm{Subst}\left(\int -\right.}{50}$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{261}{250}\sqrt{3+2x+5x^2}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{149\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right.}{25\sqrt{5}}$$

Mathematica [A] time = 0.155307, size = 55, normalized size = 0.67

$$\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] -(2953 + 2837*x + 1925*x^2 + 245*x^3)/(350*Sqrt[3 + 2*x + 5*x^2]) + (149*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/(25*Sqrt[5])
```

Maple [A] time = 0.05, size = 98, normalized size = 1.2

$$-\frac{7x^3}{10}\frac{1}{\sqrt{5x^2+2x+3}}-\frac{11x^2}{2}\frac{1}{\sqrt{5x^2+2x+3}}-\frac{149x}{25}\frac{1}{\sqrt{5x^2+2x+3}}-\frac{1001}{125}\frac{1}{\sqrt{5x^2+2x+3}}-\frac{7510x+1502}{3500}\frac{1}{\sqrt{5x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -7/10*x^3/(5*x^2+2*x+3)^(1/2)-11/2*x^2/(5*x^2+2*x+3)^(1/2)-149/25*x/(5*x^2+
2*x+3)^(1/2)-1001/125/(5*x^2+2*x+3)^(1/2)-751/3500*(10*x+2)/(5*x^2+2*x+3)^(
1/2)+149/125*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Maxima [A] time = 1.5036, size = 108, normalized size = 1.32

$$-\frac{7x^3}{10\sqrt{5x^2+2x+3}}-\frac{11x^2}{2\sqrt{5x^2+2x+3}}+\frac{149}{125}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{2837x}{350\sqrt{5x^2+2x+3}}-\frac{2953}{350\sqrt{5x^2+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/10*x^3/sqrt(5*x^2 + 2*x + 3) - 11/2*x^2/sqrt(5*x^2 + 2*x + 3) + 149/125*
sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2837/350*x/sqrt(5*x^2 + 2*x + 3)
```

```
- 2953/350/sqrt(5*x^2 + 2*x + 3)
```

Fricas [A] time = 1.38676, size = 254, normalized size = 3.1

$$\frac{1043\sqrt{5}\left(5x^2+2x+3\right)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)-5\left(245x^3+1925x^2+2837x+2953\right)\sqrt{5}}{1750\left(5x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/1750*(1043*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(
5*x + 1) - 25*x^2 - 10*x - 8) - 5*(245*x^3 + 1925*x^2 + 2837*x + 2953)*sqrt
(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{13x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx-\int -\frac{7x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-13*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-7*x**2/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(31*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) +
3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(7*x**4/(5*x**2*sqrt(5*x**2 + 2*x +
 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral
(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*
x**2 + 2*x + 3)), x)
```

Giac [A] time = 1.18432, size = 84, normalized size = 1.02

$$-\frac{149}{125}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)-\frac{(35(7x+55)x+2837)x+2953}{350\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -149/125*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) - 1/
350*((35*(7*x + 55)*x + 2837)*x + 2953)/sqrt(5*x^2 + 2*x + 3)
```

3.395 $\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}-\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016}+\frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\dots}{\sqrt{2(125+17\sqrt{11})}\dots}\right)}{1016}$$

```
[Out] -(131 - 605*x)/(3556*Sqrt[3 + 2*x + 5*x^2]) - (3*Sqrt[(281693 - 25015*Sqrt[
11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*
Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016 + (3*Sqrt[(281693 + 25015*Sqrt[11]
)/1397]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016
```

Rubi [A] time = 0.215755, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}-\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016}+\frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\dots}{\sqrt{2(125+17\sqrt{11})}\dots}\right)}{1016}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] -(131 - 605*x)/(3556*Sqrt[3 + 2*x + 5*x^2]) - (3*Sqrt[(281693 - 25015*Sqrt[
11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*
Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016 + (3*Sqrt[(281693 + 25015*Sqrt[11]
)/1397]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
```

```
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} + \frac{\int \frac{13776+14112x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{28448}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} + \frac{\left(21\left(66-53\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{2794} + \frac{\left(21\left(6\right.\right.}{}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{\left(21\left(66-53\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)}\right.}{1397}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016} + $$

Mathematica [A] time = 1.17675, size = 174, normalized size = 1.05

$$\frac{\frac{2794(605x-131)}{\sqrt{5x^2+2x+3}} - 21\sqrt{127\left(125+17\sqrt{11}\right)}\left(53\sqrt{11}-66\right)\tanh^{-1}\left(\frac{-5\sqrt{11}x+17x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right) + 21\sqrt{127\left(125-17\sqrt{11}\right)}\left(66+53\sqrt{\right.}}{9935464}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] ((2794*(-131 + 605*x))/Sqrt[3 + 2*x + 5*x^2] - 21*Sqrt[127*(125 + 17*Sqrt[1
1])]*(-66 + 53*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + 17*x - 5*Sqrt[11]*x)/(Sqr
t[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])] + 21*Sqrt[127*(125 - 17*Sqrt[1
1])]*(66 + 53*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt
[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])])/9935464
```

Maple [B] time = 0.104, size = 489, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -1/196*(10*x+2)/(5*x^2+2*x+3)^(1/2)-3/154*(61+13*11^(1/2))*11^(1/2)*(1/7/(2
50/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7
-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/7*(34/7+10/7*11^(1/2))/(250/4
9+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)
/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49
+34/49*11^(1/2))^(1/2)-1/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*ar
ctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
)/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-3/154*(-61+13*11^(1/2))*11
^(1/2)*(1/7/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/7*(34/7-10/7*11
^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/
7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(1/2))/(250-34*11^(
1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7
+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/
7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))
```

Maxima [B] time = 1.80126, size = 1049, normalized size = 6.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -1/4312*sqrt(11)*(20*sqrt(11)*x/sqrt(5*x^2 + 2*x + 3) - 7890*sqrt(11)*x/(17
*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 7890*sqrt(11
)*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 13377
*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(
11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x
 - 2*sqrt(11) - 4))/(17*sqrt(11) + 125)^(3/2) + 4*sqrt(11)/sqrt(5*x^2 + 2*x
 + 3) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x +
 3)) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x +
3)) + 156*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt
(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11
)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*
x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) - 62769*sqrt(2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
```

```
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/(17
*sqrt(11) + 125)^(3/2) + 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) +
 125*sqrt(5*x^2 + 2*x + 3)) - 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x +
 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 732*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)
*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11
) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqr
t(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) +
 12678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 12
678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)))
```

Fricas [B] time = 1.40603, size = 1269, normalized size = 7.64

$$-\frac{21\sqrt{1397}\left(5x^2+2x+3\right)\sqrt{25015\sqrt{11}+281693}\log\left(\frac{3\left(\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{25015\sqrt{11}+281693}\left(1335\sqrt{11}-8173\right)+23596727\sqrt{11}(x+3)+70\right.}{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/19870928*(21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*
log(3*(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335
*sqrt(11) - 8173) + 23596727*sqrt(11)*(x + 3) + 70790181*x - 117983635)/x)
- 21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*log(-3*(sqr
t(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335*sqrt(11)
- 8173) - 23596727*sqrt(11)*(x + 3) - 70790181*x + 117983635)/x) + 7*sqrt(1
397)*(5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log(-(sqrt(1397)*sq
rt(5*x^2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237)
 + 70790181*sqrt(11)*(x + 3) - 212370543*x + 353950905)/x) - 7*sqrt(1397)*(
5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log((sqrt(1397)*sqrt(5*x^
2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237) - 7079
0181*sqrt(11)*(x + 3) + 212370543*x - 353950905)/x) - 5588*sqrt(5*x^2 + 2*x
 + 3)*(605*x - 131))/(5*x^2 + 2*x + 3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.396 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{22755x+76567}{19870928\sqrt{5x^2+2x+3}}-\frac{7\left(541543-5144\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x}}\right.}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}}$$

```
[Out] -(76567 + 22755*x)/(19870928*Sqrt[3 + 2*x + 5*x^2]) - (3*(40 - 371*x))/(558
8*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(541543 - 5144*Sqrt[11])*Ar
cTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sq
rt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 - 17*Sqrt[11])]) + (7*(541543
+ 5144*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 + 17*Sqrt[1
1])])
```

Rubi [A] time = 0.316306, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{22755x+76567}{19870928\sqrt{5x^2+2x+3}}-\frac{7\left(541543-5144\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x}}\right.}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```


```
[Out] -(76567 + 22755*x)/(19870928*Sqrt[3 + 2*x + 5*x^2]) - (3*(40 - 371*x))/(558
8*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(541543 - 5144*Sqrt[11])*Ar
cTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sq
rt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 - 17*Sqrt[11])]) + (7*(541543
+ 5144*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 + 17*Sqrt[1
1])])
```


Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

<u>Rule 1032</u>

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

<u>Rule 724</u>

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

<u>Rule 206</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{\int \frac{-50216-37752x-89040x^2}{\left(1+4x-7x^2\right)\left(3+2x+5x^2\right)^{3/2}}\,dx}{44704}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{\int \frac{-47600448}{\left(1+4x-7x^2\right.}}{1271}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} + \frac{\left(7\left(56584\right.\right.}{}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{\left(7\left(56584\right.\right.}{}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{7\left(541543\right.}{}$$

Mathematica [A] time = 1.19835, size = 351, normalized size = 1.63

$$\frac{\frac{5084772\sqrt{5x^2+2x+3}x}{-7x^2+4x+1} + \frac{24422640x}{7\sqrt{5x^2+2x+3}} + \frac{12968296}{7\sqrt{5x^2+2x+3}} + \frac{1672044\sqrt{5x^2+2x+3}}{7x^2-4x-1} + 7581602\sqrt{\frac{22}{125+17\sqrt{11}}}\log\left(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (12968296/(7*Sqrt[3 + 2*x + 5*x^2]) + (24422640*x)/(7*Sqrt[3 + 2*x + 5*x^2]
) + (5084772*x*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (1672044*Sqrt[3 +
 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) + 14*Sqrt[2/(125 - 17*Sqrt[11])]*(-56584
+ 541543*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/
(-23 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 14*Sqrt[2/(125 + 17*Sqrt[11])]*(
56584 + 541543*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + 792176*Sqrt[2/(125 + 17*
Sqrt[11])]*Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sq
rt[11]]*Sqrt[3 + 2*x + 5*x^2]] + 7581602*Sqrt[22/(125 + 17*Sqrt[11])]*Log[1
1 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 +
 2*x + 5*x^2]])/124902976
```

Maple [B] time = 0.111, size = 1214, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -161/484*11^(1/2)*(1/7/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(3
4/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/7*(3
4/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2
)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x
-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/(250/49+34/49*11^(1/2))/(
250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1
/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2)
)^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+(
183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))/
(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+
34/49*11^(1/2))^(1/2)-3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/
(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/7*11^(1/2))/(250/49+
34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(
5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+3
4/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arct
anh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/
(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))
*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/49/(250/49+34/49*11^(1/2)
)*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*1
1^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))
^(1/2))+161/484*11^(1/2)*(1/7/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2
))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)
-1/7*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*
11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1
/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(
1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/
7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*1
1^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1
/2)))+(183/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^
(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250/49-34/49*11^(1/2))^(1/2)-3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/
2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2
))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(
250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2
```

```
))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/
2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-20/49/(250/49-34/49*1
1^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/
7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11
^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+5x+2}{\left(7x^2-4x-1\right)^2\left(5x^2+2x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*(5*x^2 + 2*x + 3)^(3/2)), x)
```

Fricas [B] time = 1.48231, size = 1759, normalized size = 8.18

$$-\frac{7\sqrt{1397}\left(35x^4-6x^3+8x^2-14x-3\right)\sqrt{4294093814065\sqrt{11}+35653135368317}\log\left(-\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{4294093814065\sqrt{\cdots}}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/111038745664*(7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(4294
093814065*sqrt(11) + 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)
*sqrt(4294093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905
) + 2865029444171587*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857
935)/x) - 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(42940938140
65*sqrt(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(42
```

```
94093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905) - 2865
029444171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x)
+ 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt
(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(
11) - 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) + 2865029444
171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x) - 7*sq
rt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt(11) +
 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(11) -
 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) - 286502944417158
7*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857935)/x) + 5588*(159
285*x^3 + 444949*x^2 + 3628805*x - 503287)*sqrt(5*x^2 + 2*x + 3))/(35*x^4 -
 6*x^3 + 8*x^2 - 14*x - 3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(3/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")

[Out] Exception raised: TypeError
```

3.397 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\left(3+2x+5x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{2701733-9148874x}{62451488\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{5(1118731375x+461370781)}{222077491328\sqrt{5x^2+2x+3}}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}-$$

```
[Out] (-5*(461370781 + 1118731375*x))/(222077491328*Sqrt[3 + 2*x + 5*x^2]) - (3*(
40 - 371*x))/(11176*(1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]) - (2701733 -
 9148874*x)/(62451488*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(279286
0024 - 84865895*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(31725355904*Sqrt[22*(12
5 - 17*Sqrt[11])]) + (7*(2792860024 + 84865895*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(31725355904*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rubi [A] time = 0.323972, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{2701733-9148874x}{62451488\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{5(1118731375x+461370781)}{222077491328\sqrt{5x^2+2x+3}}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}-$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```


```
[Out] (-5*(461370781 + 1118731375*x))/(222077491328*Sqrt[3 + 2*x + 5*x^2]) - (3*(
40 - 371*x))/(11176*(1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]) - (2701733 -
 9148874*x)/(62451488*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(279286
0024 - 84865895*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(31725355904*Sqrt[22*(12
5 - 17*Sqrt[11])]) + (7*(2792860024 + 84865895*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(31725355904*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{\int \frac{-128104-89208x-178080x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx}{89408}$$

$$= -\frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{2701733-9148874x}{62451488\left(1+4x-7x^2\right)\sqrt{3+2x+5}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{}{624}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{}{624}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{}{624}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{}{624}$$

Mathematica [A] time = 1.57375, size = 381, normalized size = 1.52

$$\frac{\frac{44\sqrt{5x^2+2x+3}(507770113-1167248019x)}{7x^2-4x-1} + \frac{737616(38521x-12667)\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2} + \frac{21296(501205x+1702037)}{7\sqrt{5x^2+2x+3}} - 7\sqrt{\frac{22}{125-17\sqrt{11}}}\left(84865895\sqrt{11}-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] ((21296*(1702037 + 501205*x))/(7*Sqrt[3 + 2*x + 5*x^2]) + (737616*(-12667 +
 38521*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (44*(507770113 - 116
7248019*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 14*Sqrt[22/(125 - 17
*Sqrt[11])]*(-2792860024 + 84865895*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[1
```

```
1]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 14*Sq
rt[22/(125 + 17*Sqrt[11])]*(2792860024 + 84865895*Sqrt[11])*Log[2 + Sqrt[11
] - 7*x] + 7*Sqrt[22/(125 - 17*Sqrt[11])]*(-2792860024 + 84865895*Sqrt[11])
*Log[(-2 + Sqrt[11] + 7*x)^2] - 7*Sqrt[22/(125 - 17*Sqrt[11])]*(-2792860024
 + 84865895*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2 + Sqrt[11])*x + 49*x^2]
+ 14*Sqrt[22/(125 + 17*Sqrt[11])]*(2792860024 + 84865895*Sqrt[11])*Log[11 +
 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*
x + 5*x^2]])/1395915659776
```

Maple [B] time = 0.12, size = 2600, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/
7*11^(1/2))^2/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2))+250/49+34/49*11^(1/2))^(1/2)-5/1372*(34/7+10/7*11^(1/2))/(250/49+34/
49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))/(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(1/2)-3/2*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/(250/49+34/49*
11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2
))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2)
)*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*1
1^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))
^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/
49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/
2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11
^(1/2))+250+34*11^(1/2))^(1/2)))-20/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/
49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2))-15/686/(2
50/49+34/49*11^(1/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+
(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/
7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-
(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(25
0+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^
2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-35
35/21296*11^(1/2)*(1/7/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(3
```

4/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/7*(3
4/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2
)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x
-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/(250/49+34/49*11^(1/2))/(
250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1
/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2)
)^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-2
1/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+1/7
11^(1/2))^2/(5(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2))+250/49-34/49*11^(1/2))^(1/2)-5/1372*(34/7-10/7*11^(1/2))/(250/49-34/4
9*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))/(5*(x-2/7+1/7*
11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2)
)^(1/2)-3/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/(250/49-34/49*1
1^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)
)+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))
*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11
^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^
(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/4
9-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2
))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^
(1/2))+250-34*11^(1/2))^(1/2)))-20/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/4
9-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2))-15/686/(25
0/49-34/49*11^(1/2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(
34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7
-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(
34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250
-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2)
)*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2
+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))+353
5/21296*11^(1/2)*(1/7/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34
/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/7*(34
/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)
-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-
2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(1/2))/(2
50-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))
^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-(-
3535/1936-273/1936*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(
1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+2
50/49+34/49*11^(1/2))^(1/2)-3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2
))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2)
)*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/7*11^(1/2))/(2
50/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2)

```
)^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2
)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1
/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^
(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/49/(250/49+34/49*11
^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7
-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^
(1/2))^(1/2))-(-3535/1936+273/1936*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))
/(x-2/7+1/7*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7
+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-3/98*(34/7-10/7*11^(1/2))/(250/
49-34/49*11^(1/2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34
/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-1
0/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34
/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+
1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-3
4*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*
(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+4
9*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-20/49/
(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/
2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250/49-34/49*11^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2+5x+2}{\left(7x^2-4x-1\right)^3\left(5x^2+2x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*(5*x^2 + 2*x + 3)^(3/2)), x
)
```

Fricas [B] time = 2.54705, size = 2311, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/1240969021540864*(7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 2
7*x^2 + 26*x + 3)*sqrt(74693314710639641467*sqrt(11) + 896266498377233657855
5)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*sqrt(11
) + 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) + 75502120
686844055144479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 37751060343
4220275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27
*x^2 + 26*x + 3)*sqrt(74693314710639641467*sqrt(11) + 896266498377233657855
)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*sqrt(11)
+ 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) - 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) + 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855)
*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 407780707037
)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855) + 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855)
*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 40778070703
7)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855) - 755021206
86844055144479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 377510603434
220275722395)/x) + 5588*(274089186875*x^5 - 200208943655*x^4 + 109737266678
*x^3 - 148022158802*x^2 + 7828199499*x + 14298727813)*sqrt(5*x^2 + 2*x + 3)
)/(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x^2 + 26*x + 3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.398 $\int (a+cx^2)^p (A+Cx^2) (d+fx^2)^q \, dx$

Optimal. Leaf size=166

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right.$$

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)
```

Rubi [A] time = 0.14659, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {531, 430, 429, 511, 510}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx + C\int x^2 \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx$$
$$= \left(A\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \left(1+\frac{cx^2}{a}\right)^p \left(d+fx^2\right)^q \, dx + \left(C\left(a+cx^2\right)^p \left(1+\right.\right.$$
$$= \left(A\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q}\right)\int \left(1+\frac{cx^2}{a}\right)^p \left(1+\frac{fx^2}{d}\right)^q$$
$$= Ax\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{f}{a}\right.$$

Mathematica [A] time = 0.42378, size = 242, normalized size = 1.46

$$\frac{1}{3}x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \left(\frac{9aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)+3adF_1\left(\frac{1}{2};-p,\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((9*a*A*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^
2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*
x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2, -((c*x^2)/a), -((f*x^
2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2)/a), -((f*x^2)/d)]))
+ (C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/((1 + (c*x
^2)/a)^p*(1 + (f*x^2)/d)^q)))/3
```

Maple [F] time = 0.681, size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p \left(Cx^2+A\right)\left(fx^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
```

```
[Out] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cx^2+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cx^2+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p*(C*x**2+A)*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (Cx^2 + A)(cx^2 + a)^p (fx^2 + d)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.399 $\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx$

Optimal. Leaf size=167

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{p+1}\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}}{2c(p+}$$

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p
)*(d + f*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d
- a*f))])/(2*c*(1 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rubi [A] time = 0.152508, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1010, 430, 429, 444, 70, 69}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{p+1}\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}}{2c(p+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```


```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p
)*(d + f*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d
- a*f))])/(2*c*(1 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```


Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
```

```
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx = A\int \left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx + B\int x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx$$

$$= \frac{1}{2}B\,\mathrm{Subst}\left(\int (a+cx)^p(d+fx)^q\,dx, x, x^2\right) + \left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \left(1+\frac{c}{}\right.$$

$$= \frac{1}{2}\left(B\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right)\mathrm{Subst}\left(\int (a+cx)^p\left(\frac{cd}{cd-af}+\frac{cfx}{cd-af}\right)^q\,dx, x,\right.$$

$$= Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

Mathematica [A] time = 0.330761, size = 236, normalized size = 1.41

$$\frac{1}{2}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\left(\frac{6aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)+3adF_1\left(\frac{1}{2};-p,-\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (6*a*A*d*AppellF1[1/2
, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/
2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2,
 -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2
)/a), -((f*x^2)/d)]))))/2
```

Maple [F] time = 0.631, size = 0, normalized size = 0.

$$\int (Bx+A)\left(cx^2+a\right)^p\left(fx^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

```
[Out] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx + A)\left(cx^2 + a\right)^p \left(fx^2 + d\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((Bx + A)\left(cx^2 + a\right)^p \left(fx^2 + d\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+a)**p*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx + A)\left(cx^2 + a\right)^p \left(fx^2 + d\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.400 $\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \,dx$

Optimal. Leaf size=252

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+f\right.$$

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p)*(d + f*x^2)^q*H
ypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d - a*f))])/(2*c*(1
 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rubi [A] time = 0.480678, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6742, 430, 429, 444, 70, 69, 511, 510}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+f\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p)*(d + f*x^2)^q*H
ypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d - a*f))])/(2*c*(1
 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right)\left(d+fx^2\right)^q dx = \int \left(A\left(a+cx^2\right)^p\left(d+fx^2\right)^q + Bx\left(a+cx^2\right)^p\left(d+fx^2\right)^q + Cx^2\left(a+cx^2\right)^p\right.$$

$$= A\int \left(a+cx^2\right)^p\left(d+fx^2\right)^q dx + B\int x\left(a+cx^2\right)^p\left(d+fx^2\right)^q dx + C\int x$$

$$= \frac{1}{2}B\,\text{Subst}\left(\int (a+cx)^p(d+fx)^q\,dx, x, x^2\right) + \left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)$$

$$= \frac{1}{2}\left(B\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right)\text{Subst}\left(\int (a+cx)^p\left(\frac{cd}{cd-af}+\frac{cfx}{cd-af}\right.\right.$$

$$= Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a}\right.$$

Mathematica [A] time = 0.530799, size = 302, normalized size = 1.2

$$\frac{1}{6}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\left(\frac{18aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)+3adF_1\left(\frac{1}{2};-p,\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((3*B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a),
 -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (18*a*A*d*AppellF1[
1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q,
 3/2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5
/2, -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*
x^2)/a), -((f*x^2)/d)])) + (2*C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a)
, -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)))/6
```

Maple [F] time = 0.618, size = 0, normalized size = 0.

$$\int \left(c x^2+a\right)^p \left(C x^2+B x+A\right) \left(f x^2+d\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

```
[Out] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C x^2+B x+A\right) \left(c x^2+a\right)^p \left(f x^2+d\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C x^2+B x+A\right) \left(c x^2+a\right)^p \left(f x^2+d\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p*(C*x**2+B*x+A)*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cx^2 + Bx + A\right)\left(cx^2 + a\right)^p \left(fx^2 + d\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                 #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```python


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```